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OPTIMAL INFLATION POLICY

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ABSTRACT

This paper considers the problem of optimal long run monetary policy. It shows that optimal inflation policy involves trading off two quite different considerations. First, increases in the rate of inflation tax the holding of many balances, leading to a deadweight loss as excessive resources are devoted to economizing on cash balances. Second, increases in the rate of inflation raise capital intensity. As long as the economy has a capital stock short of the golden rule level, increases in capital intensity raise the level of consumption. Ignoring the second consideration leads to the common recommendation that the money growth rate be set so that the nominal interest rate is zero. Taking it into account can lead to significant modifications in the "full liquidity rule." Interactions of inflation policy with financial intermediation and taxation are also considered. The results taken together suggest that inflation can have important welfare effects, and that optimal inflation policy is an empirical question, which depends on the structure of the economy.

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The widely accepted "Natural Rate" hypothesis implies that monetary policy can have no long run effect on the level of employment. The unemployment rate deviates from the natural rate only when economic actors are "fooled" by the rate of inflation. In the long run, it is not possible to cause rational agents to systematically under(over)estimate the rate of inflation, so the level of employment must return to its natural level. Some have interpreted these results as implying that monetary policy has no long-run real effects. For example, Sargent and Wallace (1975) write, "In the rational expectations version of the model, one deterministic money supply rule is as good as any other, insofar as concerns the probability distribution of real output." However, as Fischer (1978) has demonstrated, monetary policy does have real effects when inflation's long run impact on capital intensity is considered A faster rate of monetary growth raises the rate of inflation, leading individuals to economize on cash balances, and raising aggregate capital intensity through the Tobin effect. Since monetary policy has real effects, acceptance of the rational expectations hypothesis alters but does not eliminate the problem of optimal monetary policy.

The long run monetary policy problem involves trading off two quite different considerations. First, increases in the rate of inflation tax the holding of money balances, leading to a deadweight loss as excessive resources are devoted to economizing on cash balances. Second, increases in the rate of inflation raise capital intensity affecting steady state consumption. As long as the economy has a capital stock short of the golden rule level, increases in capital intensity raise the level of consumption. Ignoring the second of these factors leads to Friedman's (1969) recommendation that the rate of money growth be set so that the nominal rate of interest is zero. Feldstein (1977), using the same basic framework, has argued that the cost of excessive inflation can be very large. Neither of these analyses considers the impact of inflation on capital intensity. Studies of optimal capital accumulation typically neglect the optimization of money balances. This paper attempts an analysis of optimal monetary policy in which capital intensity is explicitly considered within the context of a simple monetary growth model. The analysis is then extended to examine how optimal inflation policy is affected by the presence of inside money, taxation, and a government budget constraint.

Section I outlines the basic model, a variant of Tobin's (1965) formulation of the monetary growth problem, and uses it to derive an explicit solution for the optimal rate of inflation. The model differs from Tobin's in that money holding is treated as a consumption good and its demand is derived from utility maximization. In Section II, the role of inside money and the implications of paying interest on money balances are examined. The impact of capital income taxes on the optimal rate of inflation, as well as the problem of indexing taxes, is considered in Section III. Section IV discusses the implications of the government budget constraint, and the revenue gains from inflationary finance. The fifth and final section summarizes the results of the paper and discusses their implications.

I. The Model

In this section a variant of the Tobin (1965) monetary growth model suitable for examining optimum monetary policy is outlined. Since only the problem of finding the optimal steady state path is considered, the dynamics of expectation formation and convergence to equilibrium are ignored.

As is standard in the literature, we assume that population growth is given by

$$N = N_o e^{nt} \qquad (1)$$

In numerical examples it will be convenient to treat n as the growth rate of labor measured in efficiency units, that is, the sum of the rates of population growth and labor augmenting technical change. The fraction of the population in the labor force is assumed to be constant.

Production is described by an aggregate production function with constant returns to scale. The relationship between per capita output and capital stock is given by

$$y = f(k) \qquad (2)$$

with $f'>0$ and $f''<0$. Depreciation is ignored to highlight the essential features, though none of the qualitative results would be affected by its inclusion.

Along a steady state growth path, k is constant. All output not invested is consumed. It follows immediately that per capita consumption is:

$$c = f(k) - nk \qquad (3)$$

We will make extensive use of the golden rule relationship apparent from (3). Increases in capital intensity raise per capita steady state consumption so long as $f' - n > 0$. A maximum level of consumption is achieved where $f' = n$ along the golden rule path. When k is further increased, $f' < n$, and per capita consumption is reduced. It will be assumed throughout the paper that $f' > n$, so consumption is below its maximum sustainable level. It should be emphasized that when the utility of money balances is considered, achievement of maximum consumption may be neither necessary nor sufficient for welfare maximization.

Following Tobin, we assume that savings are a fixed proportion of disposable income.[1] None of the results in the first two sections would be altered by allowing the savings rate to depend on the real rate of interest. Savings are represented by

$$S = \sigma Y^D \qquad (4)$$

Disposable income is equal to national income, less the fall in real money balances due to inflation, $\Pi M/P$, (where Π is the rate of inflation), plus the value of government transfers of newly printed money (DM/P). Disposable income is therefore

$$Y^D = Y + (\alpha - \Pi)M/P \qquad (5)$$

where α is the rate of money growth, DM/M.

Along a balanced growth path, monetary velocity is constant. This insures that the rate of inflation equals the excess of the rate of money growth over output growth or

$$\Pi = \alpha - n \qquad (6)$$

Using (6), (5) can be rewritten in per-capita terms as

$$y^d = y + nm \qquad (7)$$

where m represents per-capita holdings of real money balances.

In order to close the model, the demand for real money balances must be specified. The conventional approach is to let the money-capital ratio depend on the nominal rate of interest. Since inflation's role as a tax on money balances is of central concern here, this approach is rejected in favor of one based on utility maximization.

In making steady state welfare comparisons, it is postulated that money holdings are chosen so that at each instant a concave utility function U(c,m) is maximized. The utility function has the property that: U_c, $U_m > 0$ and that: $\lim_{i \to 0} U_i = \infty$ for $i = c,m$.[2] It is also assumed that real balance satiation occurs, that is $U_2(c,m) = 0$ for m sufficiently large.[3] These conditions insure that both consumption and money holding occur in all steady states. By holding an extra dollar, individuals forego consumption equal to the nominal interest rate i, which they could have earned by holding their wealth in the form of capital. It follows that money holdings will be chosen to satisfy:[4]

$$U_2(c,m) = iU_1(c,m) \qquad (8)$$

It is assumed that U is homothetic,[5] and that all individuals have identical utility functions. These conditions imply:

$$m = L(i)c \qquad \lim_{i \to 0} L = \bar{L} \qquad (9)$$

That is, the ratio of individual money holdings to consumption varies with the nominal interest rate.

Competition insures that factors are paid their marginal products. The interest rate, which equals the nominal return to capital is,

$$i = r + \Pi = f'(k) + \Pi \qquad (10)$$

All savings must be absorbed in money or capital holdings. Hence the requirement for equilibrium growth is

$$s = nk + nm \qquad (11)$$

where s represents per-capita savings. Using (4), (7), (9), (10), and (11), the steady state condition for the model may be derived as

$$\sigma f(k) - nk - (1 - \sigma)nL[f'(k) + \Pi][f(k) - nk] = 0 \qquad (12)$$

By differentiating (12) we can find the impact of inflation on steady state capital and money holdings. The effect of a change in the rate of inflation on capital intensity is given by

$$\frac{dk}{d\Pi} = \frac{(1 - \sigma)nL'(f - nk)}{\sigma f' - n - (1 - \sigma)nL(f' - n) - (1 - \sigma)nL'f''(f - nk)} \tag{13}$$

The denominator of (13) is unambiguously negative so long as $\sigma f' < n < f'$. This assumption, holding that capital intensity is below the golden rule level but above the point where $f' = \frac{n}{\sigma}$ is maintained throughout the paper.[6] The numerator is unambiguously negative. Hence the model exhibits the Tobin effect, an increase in the rate of inflation raises steady state capital intensity.[7]

Equations (4), (7) and (11) imply that changes in capital and money holdings are related by the identity

$$\frac{dm}{d\Pi} = \frac{\sigma f' - n}{(1 - \sigma)n} \frac{dk}{d\Pi} \tag{14}$$

Hence, under the condition noted above, that $\sigma f' - n < 0$, money and capital holdings move in opposite directions. Since they are alternative portfolio assets, this is to be expected. An increase in the rate of inflation thus reduces per-capita real money holdings.

At this point we are ready to consider the problem of optimal monetary policy in this simple model. In subsequent sections more realism is achieved as inside money and taxation are introduced into the model. Equation (6) demonstrates that the problem of choosing an optimal growth rate of money is equivalent to the choice of an optimal rate of inflation. We will be concerned here only with the comparison of steady states. Problems of transition are ignored. However, Feldstein (1977) has argued that it is always desirable to drive the inflation rate to its optimal level regardless of the initial conditions and transition costs. The maximand is steady state utility which can be represented by

$$U(c,m) = U(f - nk, L(f' + \Pi)(f - nk)) \qquad (15)$$

It remains only to specify the policy instruments at the government's disposal.

We first consider the case in which the government can control both the rate of inflation Π, and the level of the steady state capital stock k, by controlling capital formation. This second instrument might be taken to represent the capabilities of open market operations or public investment. In this case, the first order conditions for maximizing (15) are

$$U_2 L'(\cdot)[(f - nk)] = 0 \qquad (16)$$

$$U_1(f' - n) + U_2((f' - n)L + L'f''(f - nk)) = 0 \qquad (17)$$

Condition (16) implies that $U_2 = 0$, which along with (8) indicates that $\Pi = -r$, so the nominal rate of interest is zero. This is the same result found in Friedman (1969); however, his paper does not make clear the dependence of this monetary rule on the simultaneous optimization of the capital stock.

Equation (17) implies that $f' - n = 0$ since $U_2 = 0$. This is the famous golden rule. When both capital intensity and liquidity are subject to control, optimality requires full liquidity and golden rule capital intensity. It is clear then that $\Pi = -n$ and so (6) implies that $\alpha = 0$ is required in order to maximize steady state utility. That is, the money supply should be fixed, allowing prices to fall at a rate equal to the economy's growth rate. This result closely parallels Samueleson's (1958) finding that a constant money stock is optimal in a consumption loan economy. It is also in accordance with Friedman's (1969) actual policy prescription.

The analysis so far has assumed that the monetary authority can simultaneously optimize the rate of inflation and the capital stock. A more realistic assumption is that the monetary authority takes σ, the private savings rate, as given, and can control only the rate of inflation.[8] In this case, the full liquidity golden rule optimum described above will not, in general, be feasible. Optimal monetary policy becomes a second best problem.

The full impact of a change in inflation on steady state welfare may be found using (8):

$$\frac{dU}{d\Pi} = U_1 \left(\frac{dc}{d\Pi} + i\frac{dm}{d\Pi}\right) \qquad (18)$$

Substituting from equations (3) and (14) into (18) yields:

$$\frac{dU}{d\Pi} = U_1\frac{dk}{d\Pi} \left((f' - n) + \frac{(f' + \Pi)(\sigma f' - n)}{n(1 - \sigma)}\right) = 0 \qquad (19)$$

Given the assumption made above, that $f' > n$, we see that optimality requires a capital stock below the golden rule level. The second term in (19) reflects the "liquidity cost" of pushing the capital stock towards the golden rule level. The welfare gain from raising capital intensity to its first best level is insufficient to offset the loss from reduced liquidity.

Solving (19), we can find the optimum rate of inflation:

$$\Pi^* = \frac{(f' - n)n(1 - \sigma)}{n - \sigma f'} - f' \qquad (20)$$

This equation makes clear a strong virtue of the approach adopted here. The optimal rate of inflation can be expressed in terms of observables and does not depend on the form of the utility function. In particular, the elasticity of the money demand function does not enter the expression for the optimal rate of inflation.

The first term of equation (20) reflects the increased capital intensity brought about by inflation. When capital is optimized, at the golden rule level, it equals zero, and so (19) implies the familiar zero nominal interest rate rule. In the plausible case where $f' > n$, and capital intensity is too low, a higher rate of inflation is implied. The larger the divergence from the golden rule, the greater is the optimal rate of inflation.

Feldstein and Summers (1977) present evidence that the U.S. capital stock is well below the golden rule level. The net marginal product of capital f' is estimated to be about .10,[9] in contrast to an output growth rate of about .03.[10] Assuming $\sigma = .1$, which is consistent with the observed capital output ratio of 3, (20) implies that the optimal rate of inflation is -.005. This is consistent with a nominal return to capital of about .105, significantly above the zero nominal interest rate rule. These conclusions are quite sensitive to the parameter values chosen. Table 1 presents the optimal rate of inflation for various values of f', σ, and n.

Table 1

Optimal Rates of Inflation

	n		
	.02	.03	.04
$\sigma = .10$, $f' = .1$	.044	-.005	-.028
$\sigma = .15$, $f' = .1$	.172	.019	-.018
$\sigma = .10$, $f' = .05$	-.014	-.028	-.049
$\sigma = .15$, $f' = .05$	-.009	-.027	-.023

While the exact rate of inflation which should be set as a target varies with the parameter values, it is almost always well above the full liquidity level. Indeed with $\sigma = .15$, and $n = .02$, the optimal inflation rate is very large, 17.2%. The existence of the Tobin effect seems to be an important factor in determining the optimal rate of inflation. Surprisingly however, its magnitude does not affect optimal monetary policy.[11] These calculations indicate that consideration of capital intensity effects can lead to significant modifications in the full liquidity rule. Below we explore the implications of taxation and the existence of outside money for the optimal monetary policy problem.

II. Inside Money

The standard monetary growth model treats all money as an asset to the private sector, and liability of the public sector. Inside money, provided by financial intermediation, is ignored. Inside money holdings are taxed, and hence reduced by inflation. Since inside money is a liability of the banking system, the revenues accrue to banks rather than to the government. However, the Tobin effect involves only the substitution of capital for outside money. It would therefore seem likely that the extent of intermediation would affect optimal monetary policy.

In order to consider inside money, we modify the basic model of Section I by allowing for financial intermediation. In particular, we distinguish inside money, m_i, the liabilities of banks, and outside money, m_o, the government's liabilities. It is assume that individuals are indifferent between holding money in its inside or outside form. Banks accept individual demand deposits and then use the funds to purchase capital. We can represent the balance sheet of the private sector in per-capita terms as follows:

<u>Households</u>

<u>Assets</u>	<u>Liabilities</u>
$m_i + m_o^h$	
k_h	

<u>Banks</u>

$k_b + m_o^b$	m_i
$m_i + m_o + k$	m_i

Total private net worth is unaffected by the presence of inside money since it is an asset to the household sector, and a liability to the banking sector.[12] Initially we assume that households hold a fixed proportion d, of their money in the outside form (currency) and that banks hold a fraction r of demand deposits in the form of cash reserves. It follow that outside money represents a fixed fraction $v = \frac{r + d}{1 + r + d}$ of total money holding.

The steady state condition (11) must be modified to take account of inside money. It becomes

$$s = nk + vnm \qquad (21)$$

This reflects the fact that only outside money is a source of net worth. Using (21) the balanced growth condition (12) may be altered to take account of inside money as follows:

$$f - nk - (1 - \sigma)nvL(f' + \Pi)(f - nk) = 0 \qquad (22)$$

where it is recognized that only the increase in outside money is treated by households as disposable income. This condition is the same as (12) except that it takes account of the fact that financial intermediation makes it possible for savings to yield money services, and simultaneously be channelled into real capital formation.

Maximizing steady state utility as in the preceding section yields a condition for the optimum rate of inflation:

$$\Pi^* = \frac{(f' - n)n(1 - \sigma)v}{n - \sigma f'} - f' \qquad (23)$$

Comparing (23) and (20) we see that the "capital creation" term is smaller by a factor v, the proportion of outside money. This is because an inflation induced reduction in inside money holdings does not increase capital intensity. It is also clear that the presence of inside money reduces the optimal inflation rate.

In the U.S., the monetary base, or outside money stock, represents about 1/3 of M_1, and about 10% of M_3, the money stock including savings accounts, at commercial banks and savings and loans. Even using the narrower money stock defintion, consideration of inside money significantly reduces the optimal rate of inflation. For the parameter values assumed earlier, and a value of v = 1/3, the optimal rate of inflation is -.068 rather than -.005, as calculated above ignoring inside money. Table 2 presents a calculation of optimal inflation for various parameter values. Almost all plausible assumptions imply the optimality of a negative rate of inflation, though not necessarily a negative rate of money growth. The Tobin effect cannot be used to justify rates of inflation which approach current levels once inside money is considered.

Table 2

Optimum Rates of Inflation with Inside Money

	n		
	.02	.03	.04
σ = .10, f' = .1	-.052	-.068	-.076
σ = .15, f' = .1	-.009	-.060	-.072
σ = .10, f' = .05	-.038	-.042	-.049
σ = .15, f' = .05	-.036	-.042	-.041

It is frequently proposed that interest be paid on money holdings. This suggestion is justified as eliminating the distortion due to the taxation of money balances. With interest at rate μ paid on all money balances, steady state condition (22) becomes:

$$f(k) - nk - (1 - \sigma)nvL(f'(k) + \mu - \Pi)(f(k) - nk) = 0 \qquad (24)$$

after modifying (9) to let money holding depend on the differences between the return on money and capital. It is clear from (26) that capital intensity depends only on the differences between μ and Π. Hence in this model, inflation is completely neutral and has no welfare effects if all money balances are fully indexed. However, payment of interest on currency is not feasible, or at least not normally proposed, so it is useful to consider the case where interest is paid only on inside money holdings.

In order to study the effect of allowing interest to be paid only on inside money, it will first be useful to study the consequences of changes in v. Changes in v can result either from changes in the household propensity to hold currency or banks' reserve holdings. Differentiating (22) with respect to v yields:

$$\frac{dk}{dv} = \frac{L(f - nk)\ (1 - \sigma)n}{f' - n - (1 - \quad)nv[L(f' - n) + L'f''(f-nk)]} \qquad (25)$$

This expression is unambiguously negative, under the maintained assumptions. Since inside money does not represent wealth to the private sector, it does not displace capital. Hence the substitution of inside for outside money raises capital intensity.

Using equation (21), it can be shown that

$$\frac{dm}{dv} = \frac{-dk}{dv}\frac{(\sigma f' - n)}{(1 - \sigma)nv} \qquad (26)$$

Hence a reduction in the proportion of outside money raises real money holdings as well as capital intensity. The welfare effect is therefore unambiguously positive. Using (18) along with (26), we find

$$\frac{dU}{dv} = U_1\frac{dk}{dv}(f' - n + \frac{(f' + \pi)(\sigma f' - n)}{n(1 - \sigma)v}) \qquad (27)$$

which is unambiguously negative. It follows immediately that there are gains from increasing financial intermediation. If we assume that L' = 0, an approximation to their magnitude can be found. Using the parameter values assumed above, and taking π as .06, $dU/dv = .029cU_1$. This figure implies that a reduction in the proportion of outside money from 1/3 to 1/6, corresponding to a doubling of financial intermediation, would raise welfare by about $5 billion based on current U.S. values of L = .3 and c = $1 trillion. This calculation does of course ignore the direct costs of intermediation.[13]

If interest is paid on inside money but not on outside money, inflation will change the proportion v of money held in the outside form. In this case, inflation will have real effects even with indexation. The change in steady state welfare resulting from a change in the rate of inflation may be written as:

$$\frac{dU}{d\Pi} = \frac{\partial U}{\partial \pi} + \frac{\partial U}{\partial v}\frac{\partial v}{\partial \pi} \qquad (28)$$

The first term of (28) is zero, as long as $\frac{d\mu}{d\pi} = 1$. The second term is positive, since an increase in the rate of inflation will promote intermediation, which as (27) shows, raises welfare. Fully indexed inflation's only real effect is to raise the proportion of inside money, and so it is desirable. Under these conditions there is no optimal rate of inflation. Inflation should be raised until outside money is wrung entirely out of the economy.

It has thus been shown that the welfare effects of steady state inflation in an economy with inside money depend critically on the nature of indexation. If no interest can be paid on inside money, a negative rate of inflation is optimum. If however, inside money yields are indexed, optimal policy may call for very high rates of inflation.

III. Inflation and Taxation

This section begins consideration of the influence of taxation on optimal inflation policy. Four separate interactions of inflation and taxation may usefully be distinguished. First, inflationary finance is a source of government revenue and so, ceteris paribus, makes possible a reduction in other distortionary taxes. Second, inflation, by altering the size and composition of output will change the size of the real tax base. Third, as emphasized by Green and Sheshinski, inflation affects tax collections by altering the effective tax rate on capital income in a non-indexed tax system. Fourth, as stressed by Feldstein (1976), inflation may substantially reduce after tax yields, leading to large effects on savings and portfolio decisions. While the interaction of taxes and inflation may reduce incentives for capital accumulation, taxation of interest income does tend to offset the inflation tax on money balances. In this section, the last of these interactions is analyzed. The first three are taken up in the next section.

The appropriate way to model the taxation of capital income is not at all clear. Feldstein (1976) and Green and Sheshinski assume that all capital is financed by corporate debt. As Feldstein and Summers (1978) demonstrate, the assumption of all debt corporate finance may be very misleading. Observed interest rates are more consistent with the view that at the margin, debt accounts for only about one third of corporate capital. Moreover almost half the capital stock is non-corporate, consisting largely of residential housing. Its tax treatment is radically different than that of corporate capital.

A full analysis of the impact of capital income taxation on the size and composition of the capital stock is far beyond the scope of this paper. We assume that the net impact of the tax system is to tax the real return on capital, $f'(k)$, at rate t_1 and the return due to inflation at rate t_2. The after tax return to capital as a porfolio asset is then

$$i_n = (1 - t_1)f'(k) + (1 - t_2)\Pi \qquad (29)$$

It is important to understand that the values of t_1 and t_2 are to be interpreted as averages of the very different effective tax rates on the real and inflationary parts of various forms of capital income. The non-indexed character of our tax system does not imply that $t_1 = t_2$. In order to illustrate this point, and to shed light on realistic interpretations of the model, we derive below effective tax rates, t_1 and t_2, under various assumptions about the nature of capital.

Consider first the model of Feldstein and Green and Sheshinski in which firms finance all capital investment through the sale of bonds to individuals. Here firms equate the nominal return from capital to its nominal cost

$$(1 - \tau)f'(k) + \Pi = (1 - \tau)i \qquad (30)$$

where τ is the corporate tax rate. Individuals pay tax at a rate Θ on nominal interest income. Substituting from (30) the after tax return to debt holders is found to be:

$$i_n = (1 - \Theta)f'(k) + (1 - \frac{\Theta - \tau}{1 - \tau})\Pi \qquad (31)$$

Hence in this case $t_1 = \Theta$, the individual income tax rate. The tax on inflationary income, t_2, in the all debt world is at rate $\frac{\Theta - \tau}{1 - \tau}$ which is negative as long as the corporate tax rate exceeds the individual rate. The tax treatment of owner occupied housing is quite similar to debt financed corporate capital. The difference is that homeowners deduct interest payments at the individual rate while creditors pay taxes at the corporate rate. Capital gains on homes are taxes at the individual level.

The results in an all equity world are radically different. We assume that a fixed proportion γ of operating profits $f'(k)$ are retained by the corporation, and taken by individuals in the form of capital gains, taxable at rate Θ_g. The remainder are paid out in dividends taxable at rate Θ. Individuals are taxed at the capital gains rate on the appreciation of their equity, due to inflation. In this case the nominal after tax return to individuals is

$$i_n = 1 - (1 - (1 - \tau)(1 - \gamma\Theta - (1 - \gamma)\Theta_g))f' + (1 - \Theta_g)\Pi \qquad (32)$$

Equation (32) implies that while both inflationary and real gains are taxed, real gains are taxed at much heavier rates. The effective rate of tax on real gains t_1 is $1 - (1 - \tau)(1 - \gamma\Theta - (1 - \gamma)\Theta_g)$, while inflationary gains are taxed at a rate of only Θ_g. In an economy with mixed debt and equity finance, the effective tax rates on real and inflationary capital income are weighted averages of the expressions derived above, with weights being the shares of capital financed by debt and equity.

The real and inflationary capital income from different sources are taxed at very different rates. Effective tax rates are presumably some average of rates on the different forms of capital income. It is difficult to estimate such effective rates. In particular, the sign of t_2 is unclear. Inflation reduces tax liabilities on debt financed corporate capital, and on housing, but increases liabilities on equity. The absence of replacement cost depreciation also causes inflation to raise effective tax rates.

In the presence of taxation, steady state condition (22) becomes:

$$f - nk - (1 - \sigma)nvL(i_n)(f - nk) = 0 \qquad (33)$$

where it is assumed that all revenues are returned to households in a lump sum manner. It is immediately apparent that any combination of changes in Π, t_1 and t_2 which leaves i_n unchanged, also does not affect steady state capital intensity or money holding. The optimum value of i_n is therefore independent of the tax system. Increases in i_n will increase capital intensity, while decreases will reduce capital accumulation. Thus if taxes may be adjusted, optimality can be achieved at any rate of inflation. Likewise, any tax effects may be undone by changing the rate of inflation.

Since the optimal value of i_n does not depend on the tax system, it is apparent that

$$i_n = f'(k) + \Pi^* = (1 - t_1)f'(k) + (1 - t_2)\Pi \qquad (34)$$

or

$$\Pi = \frac{\Pi^* + t_1 f'(k)}{1 - t_2} \qquad (35)$$

where Π^* represents the optimal rate of inflation with no taxes. Equation (37) implies that the optimal rate of inflation rises with t_1.[14] A greater tax on real capital income tends to encourage money holding at the expense of capital. Increased inflation offsets this distortion. The effect of an increase in t_2 on the optimal rate of inflation is ambiguous. If $\Pi + t_1 f'(k) > 0$, increases in t_2 raise the optimum rate of inflation, else they decrease it. On balance, it appears likely that the conclusions of the previous section must be modified upwards to take account of taxes, though lacking estimates of t_1 and t_2, it is impossible to say by how much.

Optimal rates of inflation for various combinations of tax parameters are presented in Table 3, for the standard case considered in previous sections. For all reasonable combinations of tax parameters, the optimal rate of inflation is negative. The presence of capital taxes cannot justify positive inflation. It is important to note, however, that the optimal rates found here are far above those implied by the zero nominal interest rate rule.

Table 3

Optimal Inflation with Taxation

	$t_2 =$ 0	-.2	.2
$t_1 =$ 0	-.068	-.056	-.085
.2	-.048	-.040	-.060
.4	-.028	-.023	-.035

Equation (36) may also be used to evaluate proposals for indexing the tax system. It is often suggested that neutrality and optimality require that $t_2 = 0$, so real tax liabilities are independent of the rate of inflation. This is not optimal in our model. If t_1 is fixed, t_2 should vary with the rate of inflation so that $i_n = i_n{}^*$. The greater the rate of inflation, the larger is the optimal value of t_2. Not only should capital income due to inflation be taxed, the tax rate should rise with the rate of inflation. This result follows from the need to offset the inflation tax on money balances.

These results reflect the fact that in this model capital intensity depends basically on portfolio allocation. The assumption of a constant savings rate eliminates any tax effects on the rate of accumulation. While relaxation of this assumption would be valuable, little could be said without more definitive empirical evidence than is now available. The direction of the real interest rate effect on savings is ambiguous on theoretical grounds, since substitution and income effects oppose each other. Moreover, the effect of inflation on the real after tax return to savers is unclear. In the debt finance and owner occupied housing cases considered above, increases in inflation actually raise the real after tax return. The opposite result occurs with equity financed corporate capital.

So far it has been assumed that tax revenues are distributed in a lump sum fashion. Government revenue needs have been ignored as have the distortionary effects of alternative sources of revenue. In the next section we explicitly introduce a government budget constraint.

IV. Inflationary Finance as a Revenue Source

This section first considers the revenue yield from the inflation tax and then studies how the optimal rate of inflation is affected by the existence of a government budget constraint. The revenue yield from increases in inflation has received much attention in the literature. Cagan (1956) argued that to maximize its revenue, the government should raise the rate of inflation to the point where money had unitary demand elasticity. Friedman (1972) showed how this condition must be modified to take account of economic growth. Both these contributions ignore inflation's effect on revenue from pre-existing taxes. While Feldstein (1976) and Green and Sheshinski (1977) have considered these revenues, their analyses neglect the effect of inflation on the size of the monetary base stressed by Friedman and Cagan. The model developed in previous sections permits simultaneous consideration of all these issues.

In this section we assume that all real income is taxed at an effective rate t_1,[15] and that the inflationary part of capital income is taxed at rate t_2. Total tax revenue T is

$$T = (\Pi + n)vm + t_1 f + t_2 k\Pi \tag{36}$$

where the first term represents the revenue yield from printing money. By differentiating (36), the revenue impact of changes in the rate of inflation may be calculated. Using the steady state condition (22) and letting $\beta = \frac{(1 - \sigma)nv}{\sigma f' - n}$ we find that

$$\frac{dT}{d\Pi} = vm + t_2 k + [(\Pi + n)v + \beta(t_2 + t_1 f')]\frac{dm}{d\Pi} \tag{37}$$

The sign of this expression is indeterminate even with specific assumptions about the values of t_1 and t_2. It is however clear from (22) that inflation's effect on revenue from other taxes dwarfs any revenue derived directly from inflation. Consider only the second term of (37). In the U.S., k /vm ≃ 40. Hence if $|t_2| > .025$,[16] this term will be more important than the first term which reflects the direct revenue gain from inflation.

Even in a fully indexed tax system, $t_2 = 0$, inflation has an impact on income tax revenues represented by the term $\beta t_1 f' \frac{dm}{d\Pi}$ in (37). Using the parameter values assumed above $\beta = -.45$. The interest elasticity of demand for money in the U.S. is estimated to be about .3 (by, e.g., Goldfeld, 1974), implying that $\frac{dm}{d\Pi} = -1800$ billion. An income tax rate of 40% implies that an extra point of inflation raises revenue by about \$325 million. While this is not a large effect, it is comparable to the direct revenue from the inflation tax. Starting with an inflation rate of 6%, the values assumed above imply that an extra point of inflation raises revenues from the inflation tax by about \$460 million. Again, it is important to realize that both these effects are very small relative to the revenue effects on the non-indexed parts of the tax system.

Phelps (1973(has argued that a high rate of inflation may be optimal because raising revenue through the inflation tax makes possible reductions in other distortionary taxes. As the above discussion makes clear, this argument is dubious since if $t_2 < 0$, inflation may reduce revenues requiring increases in other distortionary taxes.[17] In our model it is possible to calculate the ratio of the welfare loss to the revenue yield from increases in the rate of inflation. The calculation here improves on past calculations of this sort in that the welfare consequences of inflation-induced changes in capital intensity and the full revenue implications of changes in inflation are incorporated. In finding the optimal rate of inflation, the excess burden from inflationary finance can be compared to that from alternative revenue sources.

The revenue raised from increases in the rate is given by equation (37). The welfare cost of changes in the rate of inflation is:

$$\frac{dU}{d\Pi} = U_1\frac{dk}{d\Pi}(f' - n + [(1 - t_1)f' + (1 - t_2)\Pi][\frac{(f' - n)}{(1 - \sigma)nv} \tag{38}$$

To measure welfare loss in consumption units, U_1 is set equal to 1. Table 4 records the ratio of the welfare loss to the revenue gain from raising the rate of inflation for various values of Π and t_2. All other parameters take on their previously assumed values. (σ = .1, f' = .1, n = .03, v = 1/3, t_1 = .4, β = .45, $\frac{dm}{d\Pi}$ = \$1800).

Table 4

Welfare Loss per $ Revenue Raised

t_2 =	-.2	0	.2
Π = -.02	-.01	.12	.03
0	-.06	.44	.06
.04	-.19	1.36	.13
.08	-.325	2.70	.21

The first column illustrates what is apparent from the previous discussion. When $t_2 < 0$, increases in inflation reduce revenues, as well as welfare. In this case, consideration of the revenue constraint will reduce the optimal rate of inflation. In the case $t_2 = 0$, where the tax system is indexed, inflation does not appear to be a viable revenue source. Even at an inflation rate of zero, the welfare loss from increases in inflation approaches half the revenue gain. At current rates of inflation, the welfare loss is about twice the revenue gain. These figures must be compared with loss from increases in other tax instruments. While such estimates are difficult to make, it seems clear that the loss from increases in income tax rates is much smaller than the losses implied by moderate rates of inflation. The analysis here thus confirms Phelps' conclusion that consideration of inflation policy in the context of the government's public finance problem can substantially alter the optimum rate. However, the results here emphasize the importance of taking account of inflation's effect on revenues from pre-existing taxes.

In the case where $t_2 = .2$, inflation is a viable revenue source. Even at 8% inflation, the welfare loss-revenue ratio is a very moderate .21. This reflects the fact that when the tax system is not indexed, inflation will have very large revenue effects. With a non-indexed tax system, the optimal rate of inflation may be very high though the probable responsiveness of savings to very low real yields limits the potential of inflationary finance.

V. Conclusions and Implications

The results presented here show that at least in certain circumstances the full liquidity rule may be a very poor guide for inflation policy. The effect of monetary policy on capital intensity must also be considered. This can lead to substantial increases in the optimal rate of inflation. The results imply that the presence of taxes may further increase the optimal rate of inflation. It is also shown that dynamic considerations change entirely the nature of the optimal public finance problem in which the government chooses among inflation and other revenue sources.

This analysis of optimal long run inflation policy could usefully be extended in several directions. Most importantly, the analysis here has relied entirely on comparison of steady states. This procedure ignores issues surrounding the transition to an optimal path. The steady state character of the analysis also precludes any consideration of unemployment. It would be valuable to consider the monetary policy problem in an explicit optimal control framework. The model employed here has relied on ad-hoc formulations of savings behavior. Altering the assumptions, or using a savings function derived from utility maximization, might alter the conclusions. Finally, it would be useful to explore in more detail the effect of inflation on the composition of the capital stock. Section III implies that inflation may significantly exacerbate distortions already present due to differential taxation of corporate and non-corporate capital.

It is not likely, however, that theoretical analysis can shed much light on the optimal rate of inflation, until more empirical evidence is available. Reliable estimates of the impact of inflation on capital formation do not yet exist. Likewise, little is known about the effect of inflation on financial intermediation or on effective tax rates. Until these gaps in our empirical knowledge are filled, calculations regarding optimal inflation will remain highly speculative exercises.

Footnotes

1. A natural alternative assumption would be to allow the savings rate to vary as individuals maximize an intertemporal utility function. If the horizon is infinite, Sidrauski (1967) showed that the rate of money growth has no effect on steady state capital intensity. Barro (1974) has argued that inter-generational bequest motives make the infinite horizon assumption tenable. Drazen (1977) notes several considerations including corner solutions, liquidity constraints, and the non-fungibility of human capital, which suggest that a finite saving horizon is a more reasonable assumption. In this case, as Diamond (1965) showed, there is no reason to assume that the capital stock will be driven to the optimal level. Our assumption of a fixed, not necessarily optimal savings rate, σ, captures this phenomenon. It also comports with observed constancy of the American savings rate (David and Scadding, 1974).

   A constant savings rate out of disposable income can result from utility maximization in a two-period overlapping generations framework under quite restrictive assumptions. If the elasticity of substitution between present and future consumption is one, a constant share of first period income will be saved. Typically, the share of total income received by the young generation will not be constant. If, however, the production function is Cobb-Douglas as well, labor and hence the young, will receive a fixed share of real output so savings can be expressed approximately as a constant fraction of income. An approximation is still involved since the savings rate will depend on which generation receives the transfers of newly created money.

2. Specification of a utility function containing real money balances is a simple way of explaining why individuals hold money. An analysis of conditions under which this procedure is legitimate may be found in Fischer (1974). The existence of such a utility function is implicit in past analyses of inflation's welfare cost, which employ the money demand function to measure the value of money services.

3. It is plausible that individuals derive utility from the holding of capital apart from any return which it renders. This possibility is excluded from consideration here.

4. Samuelson (1947), p. 117-122, derives an equivalent first order condition for real money balances in a somewhat more general context. The equation in the text follows directly from the observation that the "price" of holding money in terms of foregone consumption is the interest rate.

5. This condition is sufficient for the existence of a steady state path.

6. This condition is sufficient but not necessary for the negativity of the denominator. It is satisfied by most plausible combinations of parameter values. The same condition arises in Sidrauski's (1967) consideration of inflation's effect on capital intensity.

7. This result is a property of many but not all monetary growth models. Levhari and Patinkin (1968) show that inflation may reduce capital intensity, when savings out of the "imputed real income" from money balances and money as a factor of production are allowed for. A general discussion of the conditions under which inflation raises capital intensity may be found in Dornbusch and Frenkel (1973).

8. This conclusion is motivated by the separation of fiscal and monetary authorities, the multiplicity of targets at which policy instruments must be aimed, and the observed constancy of the savings rate over the past 70 years.

9. Actual real interest rates are far less than this for a host of reasons, the most important of which are taxation and the existence of equity finance. These issues are discussed in detail in Feldstein and Summers (1978).

10. This figure represents the growth rate of output, which is partially due to technical change. The maximum steady state welfare criteria used here does lose some of its appeal when technological change is introduced.

11. This is essentially because both the "capital intensity" and "money balance" effect are proportional to the reduction in real money balances brought about by inflation.

12. Note that the value of private ownership claims on financial intermediaries will in a steady state, equal the value of the capital owned by intermediaries.

13. The analysis obviously has implications for various forms of banking regulation, i.e., reserve requirements, but this topic is not pursued here.

14. The results in this and the next paragraph follow immediately from total differentiation of (36).

15. Note that the notation has changed from that of the preceding section.

16. Recall that the sign of t_2 depends on assumptions about how capital is financed.

17. The argument is not clear-cut even within Phelps' static framework. Money holding is likely to be a substitute for leisure. Since leisure cannot be taxed, this creates a presumption that it should be subsidized, as long as there exist other taxable goods more complementary with leisure and revenue needs are not too great.

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