

NBER WORKING PAPER SERIES

FORECASTING WITH THE INDEX OF LEADING INDICATORS

Beatrice N. Vaccara
U.S. Department of the Treasury

and

Victor Zarnowitz
University of Chicago and NBER

Working Paper No. 244

NATIONAL BUREAU OF ECONOMIC RESEARCH, INC.
1050 Massachusetts Avenue
Cambridge, Massachusetts 02138

May 1978

This paper is a revised version of "How Good Are the Leading Indicators?" presented at the annual meeting of the American Statistical Association in Chicago on August 15, 1977. Our thanks go to Gregg A. Jarrell and Thomas T. Kutzen for skillful research assistance, to Feliks Tamm for valuable aid with the data, and to Geoffrey H. Moore and William E. Wecker for helpful comments.

Forecasting with the Index of Leading Indicators

ABSTRACT

The composite index of leading indicators is found to be a valuable tool for predicting not only the direction but also the size of near-term changes in aggregate economic activity. This conclusion is based on assessments of the leading index as a predictor of (1) business cycle turning points as dated by the National Bureau of Economic Research and (2) quantitative changes in real GNP and the composite index of coincident indicators. Specific smoothing rules are identified which reduce the frequency of false signals but still provide adequate early warning of cyclical turning points. Simple regression models based on first differences in the logarithms produce a comparatively good record of forecasts one and two quarters ahead. The best results are obtained by using predictive chains whereby, e.g., quarterly changes in the lagging index (inverted) for Q_t are used to forecast changes in the leading index in quarter Q_{t+1} which in turn are used to forecast changes in real GNP (or the coincident index) in Q_{t+2} .

Beatrice N. Vaccara
Deputy Assistant Secretary for
Economic Policy
U.S. Department of the Treasury
Main Treasury Building, Rm. 3445
Washington, D.C. 20220
(202) 566-2768

Victor Zarnowitz
Professor of Economics and
Finance
Graduate School of Business
University of Chicago
5836 Greenwood Avenue
Chicago, Illinois 60637
(312) 753-3615

1. INTRODUCTION

Experienced analysts and forecasters consult a variety of leading and other cyclical indicators when they seek to judge the future course of economic activity. Moreover, they rely on a variety of techniques, not just the cyclical indicator approach. All of this is wise, for no single forecasting method can be considered foolproof and indeed in this uncertain world we need all the help we can get.¹ In this paper, however, practical considerations compel us to restrict our subject drastically to manageable proportions. Thus, we shall deal here only with the cyclical approach and indeed only with the predictive properties of the composite index of leading indicators, not with those of its individual components or other time series characterized by early cyclical timing.

Because the Bureau of Economic Analysis of the Department of Commerce has recently completed a comprehensive revision of the cyclical indicators, changing both the content and method of construction of the composite indexes,² this seemed like an appropriate time to seek to answer the question, "How good is the leading composite index as a predictor of change in aggregate economic activity?" This formulation raises yet another question concerning the definition of aggregate economic activity. Some people have regarded this as synonymous with real GNP. However, in the classical writings of Burns and Mitchell a broader concept is expounded; aggregate economic activity is defined to encompass measures of aggregate input as well as of output.³ Accordingly, we consider the coincident index, which contains a measure of labor input as well as measures of output and income, an

alternative and possibly superior measure of aggregate economic activity. Thus, in order to evaluate the leading composite index, we will relate changes in that index to changes in both the coincident composite index and gross national product in 1972 dollars.

The first part of this report attempts to evaluate the performance of the composite index of leading indicators as a predictor of cyclical turning points, that is, of sustained changes of direction in aggregate economic activity. Here we concentrate on the direction and duration, and abstract from the size, of general economic change. The second part is an assessment of the leading index as a predictor of short-term changes in real GNP and in the coincident index. Here the degree of movement, as well as its direction and duration, are taken into account.

2. PREDICTING CYCLICAL TURNING POINTS

The leading composite index was designed primarily as a tool for providing early warning of cyclical turning points. Before one can evaluate the performance of the index in this regard, however, a set of ground rules must be established. Because it is recognized that each monthly wiggle in the leading index may not be significant, and indeed excessive concern with a single month's movement is to be severely discouraged, we have defined a signal of change in direction as having occurred when the leading composite index shows a change in direction of movement for at least three consecutive months. As

shown in Table 1, under this set of ground rules, and using a time series beginning in January 1948 and ending March 1977, we find that the leading index provided warning of 11 reference cycle turns. No turns were missed (see Table 1, fn. a). The mean lead was 4.9 months; 2.8 months at troughs and 7.4 months at peaks (Table 3). However, because our rule requires change in direction of at least three consecutive months before a turning point is identified, a lead of at least four months is required before one can consider the leading index as providing an early warning of even a single month. Only two of the six troughs were identified with leads of at least four months, but leads of at least this long were characteristic of all five reference peaks. Thus, the leading index identified all reference turns, provided early warning at all peaks, but failed to provide advance indication for two-thirds of the reference troughs.

Although the leading index did indeed identify all reference cycle turns, it also gave indications of five extra cycles--that is, it forewarned of turning points which were not matched by turning points in either the coincident index⁴ or the reference cycle chronology. However, two of the "extra" cycles involved phases of only three months' duration. Thus, if the ground rules were extended to require at least four months of change in direction, there would be only three extra cycles. Moreover, two of these extra cycles, the one in 1950-1951 and the one in 1966 were associated with "growth cycles"--periods of marked retardation in the rate of growth in aggregate economic activity.⁵ On the other hand, if the ground rules were relaxed so that changes in

direction of movement of two months or more were considered as indications of turning points, the leading composite index would have signaled 14 extra cycles.

Because the leading index is highly sensitive, and frequently reacts to random changes in economic events, such as strikes, abnormal weather conditions, etc., if one considers changes in direction of short duration (two months or less) as indications of cyclical turning points, many more false signals than correct ones are identified. For this reason, various techniques have been adopted which, in effect, smooth the monthly movements in the index. However, smoothing devices (such as requiring three or more consecutive months of change in direction of movement or measuring changes over longer spans than a one-month interval) generally involve trade-offs. They result in the identification of fewer "false" or "extra" turns, but cut down the effective lead time at the "true" turns.

One of the most fruitful approaches was to measure changes in the leading index over a six-, rather than a one-month span,⁶ but to treat even a single month's change in direction as providing a signal of a turning point, and thus to minimize the required lead time. The results of the application of such a procedure are shown in Table 2. As is expected, measuring the changes over a six-month span (which is equivalent to a trailing six-month moving average of changes over a one-month span) considerably smooths the index and eliminates many erratic movements. Even with rules as stringent as considering changes of a single month's duration as indicating a turning point, there are only five extra cycles (or 10 extra turns) identified. (If

such a rule were applied to changes measured over a one-month span, there would have been 70 false turns identified, if applied to changes measured over a three-month span, 34 false turns would have been identified.) Of the five extra cycles identified, three (12/50-11/51; 4/62-10/62; and 6/66-2/67) were associated with periods of growth retardation in aggregate economic activity.

The mean lead at the 11 matched reference turns was 3.1 months; 5.6 months at peaks, and 1.0 months at troughs. All five reference peaks were identified with leads of at least two months (the requisite early warning under a one-month directional change rule) but only three of the six reference troughs were identified with leads of at least two months.

If one associates turns in the leading index with turns in the coincident index, rather than with reference turns, a somewhat different and more favorable picture emerges. As can be seen from column 5 of Table 2, during the period January 1948 - March 1977, the coincident index indicated 13 (rather than 11) cyclical turning points⁷ and thus only four rather than five extra cycles were identified. All of the turning points in the coincident index were identified by the leading index with a mean lead of 3.7 months at troughs as well as at peaks. Moreover, at only two turns was the lead less than the requisite two months.⁸ Thus, measuring changes over a six-month span (rather than over a one- or three-month span) is an effective smoothing device. It cuts down considerably the degree of random month-to-month movements and drastically reduces the number of false signals.

Another way of evaluating the turning point performance of the leading composite index is to count the number of months during a cyclical phase which evidenced directional movements counter to that of the appropriate reference phase--allowing, of course, for the leading tendency of the index. Thus, for example, any downward movement which occurred earlier than eight months before a reference peak (or any upward movement earlier than three months before a reference trough) would be considered a counter movement. Under this method of evaluation, we again see progressive improvement in the performance of the indicators as we extend the length of the smoothing period. With changes measured over a one-month span, there were 61 out of 346 monthly movements which were counter to the appropriate reference cycle phase. With changes measured over a three-month span, there were 49 counter movements; this was reduced to 42 when changes were measured over a six-month span. As might be expected, most of those months of false signals occurred during reference cycle expansion phases, which had an average duration in the postwar period of 61 months as compared to 11 months for the contraction phases.

3. QUANTITATIVE FORECASTS WITH LEADING INDICATORS

Correlations between the levels of the composite indexes of leading indicators (I) and of coincident indicators (C) are very high, particularly for short leads of I and simultaneous timing, and comparably close associations exist also between the quarterly data for real GNP (Y) and the composite indexes. But regressions

of C on I and of Y on I produce highly autocorrelated residuals and hence generally unreliable estimates. Because of pervasiveness of major business fluctuations and trends, many economic series, and particularly comprehensive aggregates and sensitive cyclical indicators, show strong dependencies over time within themselves and with each other.

Taking first differences of the indexes and of real GNP essentially eliminates the trends and converts the data into stationary series. Studying the relationships between the series in this form is much more informative, particularly in the context of short-term movements (the method brings out such movements, including noise, at the expense of the longer trends). Figure A shows the crosscorrelograms for the monthly and quarterly changes in I and C, and for the quarterly changes in I and Y. Absolute first differences are denoted by D, first differences in logs by DL. The correlations $R(i)$ are shown for spans i varying from -12 to +12, where "-" refers to leads and "+" to lags of DI relative to DC or DY (or of DLI relative to DLC or DLY), in months.

The quarterly correlograms linking the changes in the leading to those in the coincident index reach peaks of over 0.7 at one-quarter (-3) leads of DLI relative to DLC and of DI relative to DC. The $R(i)$ coefficients are small or near zero for $i = -12$, significantly positive for shorter leads and $i = 0$, and zero or small negative for lags, i.e., for $i > 0$. The monthly correlograms run lower for the leads but show the same general contrast between $R > 0$ for leads and

$R \leq 0$ for lags of DI (DLI) vis-à-vis DC (DLC). These asymmetries confirm the "leading" characteristic of the I index.

The quarterly correlograms linking the changes in the leading index to those in real GNP are similar but somewhat less asymmetrical. Both for absolute changes and changes in logs, $R(0)$ is slightly larger than $R(-3)$ and $R(+3)$ is positive.

We shall next present an analysis of regressions and forecasts based on the data used in Figure A. Only the results for the relationships between the changes in logs will be shown. Somewhat better sample-period fits are obtained for the regressions of DY_t on DI_{t-i} than for the corresponding regressions of DLY_t on DLI_{t-i} , but the latter produce slope coefficients of the constant-elasticity type which are easier to interpret. On the whole, the differences between the two sets of estimates are small.

3.1 Estimation and Prediction of Real GNP

Regressions on Lagged Changes in the Leading Index. As shown in Table 4, lines 1-3, about 4/10 of the variance of the quarterly change in the log of real GNP (DLY_t) can be statistically "explained" by the concurrent change in the log of the index of leading indicators (DLI_t). The addition of the lagged value of the dependent variable (DLY_{t-1}) raises the corrected coefficient of determination \bar{R}^2 from .40 to .48. (DLY_{t-1} alone accounts for nearly .2 of the variance of DLY_t .)

When DLI_t (which, of course, is not available for one-step-ahead forecasts of DLY_t) is replaced by DLI_{t-1} , a slightly lower R^2 of .39 is obtained but the Durbin-Watson (DW) statistic increases from 1.6

to 2 (cf. lines 4 and 1) suggesting that the residuals in the equation are free of any first-order autocorrelation. Further, DLY_{t-1} makes no contribution at all to the regression when included along with DLI_{t-1} (lines 5 and 4).

Relating DLY_t to both DLI_t and DLI_{t-1} increases \bar{R}^2 to .51 and reduces appreciably the standard error of regression SEE (line 6), but adding further prior values of DLI_{t-i} ($i = 2$ and 3 quarters) results only in marginal gains (lines 7 and 8). This is so especially when DLI_t is not included (cf. lines 4 and 9). In the predictive equations, then, longer leads in DLI_{t-i} , i.e., terms with $i > 1$, appear to be of little use.

Sample Estimates and Predictions. Following a widely used procedure, we next test the equations of the type just described by fitting them to some of the data and predicting the rest. Here good results are obtained for one-quarter-ahead predictions from the simple relation between the first differences in the logarithms of real GNP and of the leading index (Table 4, line 4). We regress DLY_t on DLI_{t-1} using quarterly data for 1948-69 (85 observations) and use the resulting sample-period estimates of the intercept and slope parameters to compute forecasts \widehat{DLY}_t for 1970-76 (28 quarters). Then we reestimate the regression for 1948-70 and predict with it \widehat{DLY}_t in 1971-76, and so on, extending the sample period by one year in each step and shortening the forecast period correspondingly. The full results are presented in Table 5 (part A), where each column lists first the sample-period estimates (lines 1-10) and then the corresponding forecast-period estimates (lines 11-19).

This exceedingly simple forecasting model can be seen to have performed well and consistently so. The goodness-of-fit statistics are reasonably high for variables cast in the volatile form of rates of change. The squared correlations between the actual and predicted changes in the forecast periods fall in the narrow range of .46 to .55 (line 11); perhaps surprisingly, they exceed the R^2 coefficients for the sample periods (line 5). The average error statistics are also satisfactory and quite stable over time: the root mean square errors are only slightly higher than the standard errors of regression and the mean errors are near zero (cf. lines 12-14 and line 9). Theil's inequality coefficient U equals zero for a perfect forecast and unity for a forecast that performs no better than a naive model extrapolating the last recorded change.^{9/} The U values of .6-.7 (line 15) are higher (less favorable) than those achieved by the best known econometric model and other forecasts of real GNP in the quarter ahead; however, the differences here are not large and their significance is not clear.^{10/} The proportions of the mean square error due to bias and inefficiency are very small indeed; between 91 and 98 percent of MSE is accounted for by residual variation rather than errors of systematic nature (lines 16-18).

No significant improvements in the one-step-ahead DLY forecasts result from adding longer leads (DLI_{t-2} , DLI_{t-3}) to the predictive equation. As would be expected from the fact that our distributed-lag regressions have very few effective terms (only the shortest leads "work"), simple indicator forecasts of changes in real GNP two and three quarters ahead are much worse than for one quarter ahead. Thus, the equations for GNP changes in the second quarter ahead have $\bar{R}^2 \approx .15$ and $DW \approx 1.5$, and there is further, though much slower, deterioration in the estimates for GNP changes in the more distant future quarters. However, substantially more

accurate forecasts of real GNP changes over longer spans can be made with the leading index data in other ways.

The simplest procedure is to lengthen the unit period, e.g., forecast DLY_t from DLI_{t-1} , where t and $t-1$ refer to successive six-month intervals. This works better (but is also less informative) than forecasting DLY for each of the next two quarters from DLI data available in the current quarter. Part B of Table 5, which has exactly the same format as part A but covers semiannual (instead of quarterly) regressions and predictions, shows results that are still reasonably satisfactory. The sample period correlations are slightly higher for the semiannual than for the quarterly data, but the forecast period R^2 coefficients are significantly lower (averaging .33 for the semiannual, .50 for the quarterly predictions. The absolute errors are nearly twice as high (compare lines 31-33 and 12-14), but the relative errors are not much higher (the average U coefficients are .76 and .66 in lines 34 and 15, respectively). There is again very little evidence of any bias or inefficiency (lines 35-37). Finally, here too, the regressions and predictions show a great deal of stability between the different periods covered.

Errors in Forecasts of Changes and Levels. Taking antilogs of the estimated values of \widehat{DLY}_t results in ratios \hat{Y}_{t+1}/\hat{Y}_t for $t = 1, 2, \dots, n$ time units contained in any of our "forecast periods." Starting with the actual value of Y for the last quarter of a sample period (call it Y_0), we use successive multiplication to cumulate such ratios into a series of the implicit forecasts of real GNP levels; that is, we compute the series to be labeled \hat{Y}_a as follows:

$$Y_0(\hat{Y}_1/Y_0) = \hat{Y}_1, \quad \hat{Y}_1(\hat{Y}_2/\hat{Y}_1) = \hat{Y}_2, \quad \dots, \quad \hat{Y}_{n-1}(\hat{Y}_n/\hat{Y}_{n-1}) = \hat{Y}_n.$$

The left-hand panel in Figure B illustrates the results for 1972-76 as derived from equation (2) in Table 5, part A (column 3). In assessing the relationship between the estimated (\hat{Y}_a) and the actual (Y) levels of GNP in 1972 dollars, it is important to remember that \hat{Y}_a represents a 20-quarter simulation based on data for 1948-71 as summarized in the statistics from a simple regression of $\Delta \ln Y_t$ on $\Delta \ln I_{t-1}$. The intercept \hat{a} and slope coefficient \hat{b} are called upon to perform a heavy duty, since they are not reestimated but kept constant throughout. Furthermore, one would expect that cumulation of the predicted changes over a stretch of five turbulent years will result in a substantial cumulation of the errors as well, causing deviations from the initial (1971:4) level. Hence, it is not to be presumed that \hat{Y}_a and Y will be closely associated, and they are not. The squared correlation between the two series is .722 and the U coefficient is very small (.021), but the bias and inefficiency proportions are high (.100 and .303, respectively) and so are the average errors in billions of 1972 dollars (RMSE = 25.2, MAE = 22.8, and ME = 8.0). \hat{Y}_a underestimated Y in 1972 and 1973; it overestimated Y since mid-1974, particularly late in 1974 and early in 1975.

The major part of these errors represents simply the effect of cumulation of the predicted changes over the 20-quarter period. This can be demonstrated as follows. The actual values of Y_{t-1} are known at time t ; add $\ln Y_{t-1}$ to predicted DLY_t for $t = 1, 2, \dots, n$ to get $\ln Y_1 + (\ln \hat{Y}_2 - \ln \hat{Y}_1)$, $\ln Y_2 + (\ln \hat{Y}_3 - \ln \hat{Y}_2)$, \dots , $\ln Y_{n-1} + (\ln \hat{Y}_n - \ln \hat{Y}_{n-1})$.

Note that in this form our change forecasts $(\ln \hat{Y}_t - \ln \hat{Y}_{t-1})$ are attached directly to the previous actual values $\ln Y_{t-1}$; this should reduce the errors due to the cumulation of the $\Delta \ln Y$ terms. Next antilogs are taken to obtain the following series which will be called \hat{Y}_b :

$$Y_1(\hat{Y}_2/\hat{Y}_1), \quad Y_2(\hat{Y}_3/\hat{Y}_2), \quad \dots, \quad Y_{n-1}(\hat{Y}_n/\hat{Y}_{n-1}).$$

Each of the above expressions can also be written as $\hat{Y}_t(Y_{t-1}/\hat{Y}_{t-1})$, so that this procedure may as well be viewed as adjusting each of our forecasts for the error in the previous forecast.

The right-hand panel in Figure B shows the \hat{Y}_b series corresponding to the \hat{Y}_a series on the left. It is evident from a comparison of the two graphs that the \hat{Y}_b predictions of real GNP are vastly superior to the \hat{Y}_a predictions. In contrast to the large underestimation and overestimation errors of \hat{Y}_a in 1972-73 and 1974-76, \hat{Y}_b underpredicts Y in 1972 and overpredicts Y in 1974 by relatively small amounts. The performance of \hat{Y}_b at the 1973 peak, while not very satisfactory, is much better than the much more sluggish behavior of \hat{Y}_a (at the 1975 trough both \hat{Y}_a and \hat{Y}_b lag Y by one quarter). Statistics confirm the visual impressions: R^2 for the relation between \hat{Y}_b and Y is .924; U is .009 and U^B , U^I , and U^V are .02, .19, and .79, respectively; RMSE = 11.4, MAE = 9.7, and ME = 1.6 billion 1972 dollars.

In the last test of the same simple model, we compute 20 one-quarter-ahead forecasts of real GNP from an equal number of regressions of DLY_t on DLI_{t-1} . That is, \hat{Y}_1 for 1972:1 is based on the regres-

sion for 1948:1-1971:4, \hat{Y}_2 for 1972:2 on that for 1948:1-1972:1, and so on, ending with \hat{Y}_{20} which refers to 1976:4 and is derived from the regression covering the period 1948:1-1976:3. The coefficients in the equation $DLY_t = a + b DLI_{t-1} + u_t$ are thus reestimated for each of the twenty predictions.

The estimates \hat{a} vary between .006 and .007; \hat{b} between .27 and .31; R^2 between .33 and .43; and DW between 1.95 and 2.02; they are all significant and stable, showing no systematic variation over time. When the level predictions (let us call the set \hat{Y}_c) are calculated from the DLY figures and related to the corresponding actual values Y , the comparison yields the following measures of absolute and relative accuracy:

$$\begin{array}{llll} R^2 = .924 & RMSE = 11.6 & MAE = 9.6 & ME = 1.3 \\ U = .010 & U^B = .01 & U^I = .21 & U^V = .77 \end{array}$$

These statistics are very close to those reported above for the corrected forecasts from the single regression for 1948-71 (Y_b). The graph comparing Y_c and Y looks almost like a replica of the right-hand panel in Figure B and is not reproduced here. We conclude that reestimation produces little, if any, gain in this case.

To sum up the story up to this point, the simple bivariate model linking real GNP to prior values of the leading index suggests a remarkably stable association, and a close one as economic relationships go. For predictive purposes, it is at least a good benchmark to be used for assessments of the results that reputable forecasters get by employing a variety of complex and comprehensive approaches. Of course, including other relevant variables and judgment based on experience

and current observation would improve forecasts with the leading index; most genuine macropredictions combine many determinants of general economic activity and are aided by elements of judgment.

On the negative side, the lags at turning points in 1973 and 1975 stand out. However, it is an implication of first-difference equations that the predicted levels depend on the lagged levels of the dependent variable, and this tends to produce lags of this sort. Regressions of LY (log of real GNP) on the lagged values of LI (log of the leading index), having highly autocorrelated residuals, produce of course much poorer parameter estimates and predictions than do our regressions of DLY on lagged DLI; but the forecasts from the latter, unlike those from the former, do not lag behind the actual values. Further work with monthly data may give better results here, conforming more to the leads that the index did actually have relative to real GNP.¹¹

Improvements and Extensions--Predictive Chains. If the leading index itself can be reasonably well predicted (say, over the next quarter), then these predictions, along with the past observations on the index, can be used to develop forecasts of real GNP for more than one period ahead. This is a simple application of the device that has been called the "chain principle of forecasting."¹² Moreover, it will be shown that the same approach yields improved one-step-ahead forecasts as well.

Several autoregressive, moving average, and mixed autoregressive-moving average models were fitted to the leading index, with moderately

good results.¹³ Further work with such time-series models, possibly in combination with regression analysis, is warranted. However, the best results so far were obtained with models including other variables in addition to past values of the leading index itself. The composite of lagging indicators (G) includes several series reflecting largely the costs of doing business (interest rates, unit labor costs, inventories, business loans). The inverse of that index, $1/G$, tends to lead other important indicators, including the leaders.¹⁴ We therefore regress DLI on prior values of DLV , i.e., of the quarter-to-quarter change in $\ln(1/G)$, as well as on its own past values (Table 6). Good results are obtained simply by relating DLI_t to DLV_{t-1} and DLI_{t-1} (line 4); the longer lags contribute little or nothing to the quality of the \widehat{DLI} estimates (lines 1-3).¹⁵

Lines 1 and 2 of Table 7 present regressions of DLY_t on DLI_t and DLI_{t-1} and on DLI_{t-1} alone for 1948-71. (These parallel the results for 1948-76 reported in Table 5, lines 15 and 13.) Also shown for the 1948-71 period is the regression of DLI_t on DLI_{t-1} and DLV_{t-1} (line 3; cf. line 4 in Table 6). This last equation serves as the basis for the predictions \widehat{DLI}_t , 1972-76 (line 4). Next, the predicted \widehat{DLI}_t is used along with the last known value of the same variable, DLI_{t-1} , to compute the one-step-ahead forecasts \widehat{DLY}_t in line 5; here the estimates of the constant term and the slope coefficients are taken from the equation in line 1 above, and the equation can be written out as

$$(5) \quad \widehat{DLY}_t = .006 + .134 (.010 + .421 DLI_{t-1} + .544 DLV_{t-1}) + .188 DLI_{t-1}.$$

Finally, two-steps-ahead forecasts are obtained in line 6 by linking \widehat{DLY}_t to \widehat{DLI}_{t-1} which is estimated from line 4. That is, we have

$$(6) \quad \widehat{DLY}_t = .006 + .267 (.010 + .421 DLI_{t-2} + .544 DLV_{t-2}).$$

The forecasts of real GNP change in the next quarter are improved by including the \widehat{DLI}_t term in addition to DLI_{t-1} (compare the R^2 and error measures in Table 7, line 5, with those in Table 5, column 3, lines 11-19). The forecasts for the second quarter ahead have, of course, larger errors but are still reasonable in relative terms. Thus they are far better than the predictions that could be obtained by relating DLY_t directly to DLI_{t-2} . Also, a combination of the forecasts for quarters $t + 1$ and $t + 2$ (from Table 7, lines 5 and 6) not only conveys more information but produces smaller average errors than do the one-step-ahead semiannual forecasts (Table 5, column 3, lines 30-38).

The implicit forecasts of real GNP levels are calculated from the above forecasts of change by the procedure described in the preceding section, again in two versions, one unadjusted and the other adjusted for the effects of cumulation of the predicted changes. The results for 1972-76 as derived from equation (5) are as follows (\hat{Y}_a , denotes the unadjusted, \hat{Y}_b , the adjusted forecasts: they are comparable to the \hat{Y}_a and \hat{Y}_b series shown in Figure B).

| | | | | |
|-------------------|--------------|--------------|--------------|--------------|
| For \hat{Y}_a : | $R^2 = .734$ | RMSE = 25.7 | MAE = 23.0 | ME = 6.2 |
| | $U = .021$ | $U^B = -.59$ | $U^I = .395$ | $U^V = .545$ |
| For \hat{Y}_b : | $R^2 = .928$ | RMSE = 10.8 | MAE = 9.1 | ME = 2.0 |
| | $U = .009$ | $U^B = .034$ | $U^I = .122$ | $U^V = .837$ |

These measures indicate that the predictive performance of \hat{Y}_a , and \hat{Y}_b , is on the whole better than that of \hat{Y}_a and \hat{Y}_b , respectively, but by small margins. The largest errors still occur in the same periods, namely in 1974 and in 1975, near the critical turning points.

Reestimation quarter by quarter yields a series of 20 forecasts \hat{Y}_c , comparable to \hat{Y}_c discussed in the previous section; it again fails to yield any significant gain.

3.2 Estimation and Prediction of the Coincident Index

Only a few first steps were taken so far to exploit for quantitative forecasting the monthly data on the composite indexes. These series can be used to update monthly the forecasts of quarterly real GNP or the forecasts of the quarterly index of coincident indicators; it is the latter predictions that will be discussed here.¹⁶

Our estimates are again based on first differences in logarithmic form. Table 8 shows the regressions of ΔLC on three lagged ΔLI terms. The second of these is not significant (column 3) but its retention is harmless; multicollinearity is clearly present but it does not affect the predictions. The \bar{R}^2 coefficients are substantially higher here (column 5) than for the relationships with real GNP changes (see, e.g., Table 5, line 5). The DW statistics, however, are low, particularly for the shortest forecasts, $i = 1$ (column 6).

The assessment of the ΔLC predictions in lines 4-6 is definitely favorable. The average error measures and U statistics are rela-

tively low, the R^2 and U^V statistics relatively high. The indications are strong that the leading index is more closely associated with the coincident index than with real GNP, and that it predicts the former variable better than it does the latter. This is consistent with the cross-correlograms in Figure A, which peak at one-quarter lead of DLI vs. DLC but at a point of coincident timing for the relation between DLI and DLY. It also helps that the regressions for DLC include two effective DLI_{t-i} terms (those for DLY included only one, DLI_{t-1}).

The updating, which shortens the measured lead i from 3 to 1 months, reduces the average forecast errors and increases the correlation between the actual values DLC and the predicted values \widehat{DLC} (lines 4-6). The improvement in the forecast so achieved is not large, however, considering the costs in terms of the rather drastic reduction in the effective span of the predictions. The quarter-ahead ($i = 3$) forecasts are already quite good and virtually free of any systematic errors (line 4).

The predicted changes in the log of the coincident index, \widehat{DLC} , are converted into level forecasts, \hat{C} , by procedures that were described before in the section on the estimates for real GNP. Figure C shows the results for the one-quarter-ahead predictions derived from the equation in Table 8, line 1. It parallels Figure B: again, the panel on the left represents unadjusted forecasts, that on the right represents forecasts adjusted to reduce the cumulation errors (\hat{C}_a and \hat{C}_b , respectively). The measures of predictive performance are

for \hat{C}_a : $R^2 = .910$ $RMSE = 2.6$ $MAE = 2.2$ $ME = -2.0$
 $U = .021$ $U^B = .628$ $U^I = .035$ $U^V = .338$

for \hat{C}_b : $R^2 = .939$ $RMSE = 1.5$ $MAE = 1.1$ $ME = -.06$
 $U = .012$ $U^B = .001$ $U^I = .284$ $U^V = .713$

It is rather remarkable that \hat{C}_a agrees perfectly with C in direction of each quarterly movement; furthermore, the two series have about equal amplitudes of cyclical rises (in 1972-73 and 1975-76) and declines (in 1975-76). However, it is clear that \hat{C}_a strongly underpredicts the levels of C . This bias is completely eliminated in \hat{C}_b . On the other hand, regression of C on \hat{C}_a yields a slope coefficient of .91 and regression of C on \hat{C}_b one of .86; the fraction of error due to the divergence of these coefficients from unity (U^I) is considerably larger for \hat{C}_b than for \hat{C}_a . There are some directional errors in \hat{C}_b near the 1973 peak, but the quantitative errors involved are relatively small. These are the costs of the correction which, let us recall, makes use of the lagged values of the predicted variable (here C_{t-1}) but they are small compared to the benefits. Certainly there are good reasons to conclude that the overall accuracy of \hat{C}_b is quite satisfactory.

4. SUMMARY

An objective and rather elaborate evaluation leads to the conclusion that the leading composite index is a valuable, but not fool-proof, tool for predicting both the degree and direction of changes in

aggregate economic activity. There was not a single episode in the postwar period when the index failed to provide early warning of major downturns in economic activity. Although the index also indicated the advent of all major upturns in economic activity, in two-thirds of these upturns it failed to provide early warning.

One disadvantage of the leading composite index, however, is its extreme sensitivity to random or short-lived changes in economic movements. As a consequence, the composite index evidenced several "extra" cycles, that is, it gave indications of changes in direction in aggregate economic activity which were not followed by cyclical episodes in GNP or the coincident index. In some cases, however, these extra cycles corresponded to periods of growth retardation in aggregate activity.

Devices for smoothing the monthly movements in the leading index, such as measuring changes over a six- rather than a one-month span, while cutting down somewhat on the length of the "early warning" of impending turning points, provide effective mechanisms for minimizing the number of false signals. There was not a single episode during the entire postwar period when the leading index failed to provide at least two months early warning of cyclical downturns and the mean warning was almost six months. Moreover, when smoothed in this manner there were only two episodes in the entire postwar period, and none in the last ten years, when the index signaled a downturn of even a single month which was not followed either by sustained periods of actual decline in economic activity or by episodes of marked retardation in its rate of growth.

The evidence examined in this paper indicates that the leading composite index has a good performance record not only in predicting major turning points, but also in forecasting all quarterly changes in aggregate economic activity. Its record as a predictor of quarterly changes in the coincident index and real GNP is remarkably stable and favorable in comparison with the corresponding goodness-of-fit statistics. The predictive performance of the leading index is particularly good when the coincident index is used as a measure of aggregate economic activity.

The leading index model for predicting changes in the coincident index and real GNP one quarter ahead is considerably superior to autoregressive schemes which relate change of the dependent variable in the current quarter to change in this variable one quarter earlier. In the case of GNP, for example, such a scheme explains only about 18 percent of the variance in the quarterly changes during the postwar period 1948-1976 as compared to a 39 percent when the previous quarter's movement in the leading index is used as the single explanatory variable.

The introduction of additional changes in the leading index (those of two and three quarters earlier) adds only marginally to the ability of the regression equation to explain (or forecast) changes in quarterly GNP but considerably improves the ability of the regression equation to predict changes in the coincident index.

If attempts are made to lengthen the forecast period, that is to explain changes in aggregate economic activity two-, three-, and four-quarters ahead, the predictive performance of the leading composite

index deteriorates considerably. A more effective method for extending the forecast period is by employing the chain principle. Under such a procedure, quarterly changes in the lagging index (inverted) for Q_t are used to forecast changes in the leading index in quarter Q_{t+1} which in turn are used to forecast changes in real GNP (or the coincident index) in Q_{t+2} . Use of the chain principle not only improves the predictive ability of the leading index in two-quarter-ahead forecasts, but also substantially improves its ability to forecast one quarter ahead. Forecast models which employ actual changes in the leading index for the prior quarter along with predicted changes in this index for the given quarter considerably increase the predictive power of the equations.

Despite the relatively favorable findings resulting from this study, many questions remain to be answered by further research efforts. These call for attempts to devise more effective methods of using indicator data in longer-span and multiple-period forecasting; for examination of how changes in real GNP are related to changes in the coincident index; and for a study of ways to combine the information from the indicator system with other tools so as to produce more accurate forecasts. On the strength of the results reported in this paper, such future studies appear to be well justified.

Table 1. Cycle Turning Points Based on Criterion
of Change in Direction of 3 Consecutive Months or More
(measured over 1 month span)

| P(Peak) T(Trough) | | | | Timing of Leading Index in Relation to | |
|----------------------|--------------------|---------------------|--------------------|---|----------|
| Leading Index | Reference Turns | Coincident Index | Reference Turns | Turns in Coincident Index | |
| P | a/ | 11/48 | 10/48 | | |
| T | 6/49 | 10/49 | 10/49 | -4 | -4 |
| P | 8/50 | | | { extra | { extra |
| T | 11/51 | | | | |
| P | 3/53 | 7/53 | 5/53 | -4 | -2 |
| T | 11/53 | 5/54 | 8/54 | -6 | -9 |
| P | 9/55 | | | { extra | { extra |
| T | 6/56 | | | | |
| P | 11/56 | 8/57 | 2/57 | -9 | -3 |
| T | 1/58 | 4/58 | 4/58 | -3 | -3 |
| P | 5/59 | 4/60 | 6/59 | -11 | -1 |
| T | 6/60 | | 10/59 | { extra* | { extra* |
| P | 9/60 | | 1/60 | | |
| T | 12/60 | 2/61 | 2/61 | -2 | -2 |
| P | 3/62 | | | { extra* | { extra* |
| T | 6/62 | | | | |
| P | 3/66 | | | { extra | { extra |
| T | 12/66 | | | | |
| P | 4/69 | 12/69 | 10/69 | -8 | -6 |
| T | 10/70 | 11/70 | 2/71 | -1 | -4 |
| P | 6/73 | 11/73 | 11/73 | -5 | -5 |
| T | 2/75 | 3/75 | 3/75 | -1 | -1 |

*Involves phases of only 3 months duration.

^aThe leading index declined almost continuously in 1948 and showed no episodes of "reversal of direction" lasting at least three months; it was therefore not possible to identify a cyclical turning point under our "3 month rule."

Table 2. Cycle Turning Points Based on Criterion
of Change in Direction of a Single Month
(measured over 6 month span)

| P(Peak) T(Trough) | Leading Index | Reference Turns | Coincident ^a Index | Timing of Leading Index in Relation to | |
|----------------------|------------------|--------------------|----------------------------------|---|------------------------------|
| | | | | Reference Turns | Turns in Coincident Index |
| P | b/ | 11/48 | 11/48 | | |
| T | 7/49 | 10/49 | 12/49 | -3 | -5 |
| P | 12/50 | | 6/51 | { extra | -6 |
| T | 11/51 | | 12/51 | | -1 |
| P | 5/53 | 7/53 | 7/53 | -2 | -2 |
| T | 3/54 | 5/54 | 9/54 | -2 | -6 |
| P | 12/55 | | | { extra | { extra |
| T | 10/56 | | | | |
| P | 1/57 | 8/57 | 3/57 | -7 | -2 |
| T | 4/58 | 4/58 | 7/58 | 0 | -3 |
| P | 8/59 | 4/60 | 8/59 | -8 | 0 |
| T | 8/60 | | 11/59 | { extra* | { extra* |
| P | 10/60 | | 5/60 | | |
| T | 2/61 | 2/61 | 4/61 | 0 | -2 |
| P | 4/62 | | | { extra | { extra |
| T | 10/62 | | | | |
| P | 6/66 | | | { extra | { extra |
| T | 2/67 | | | | |
| P | 5/69 | 12/69 | 12/69 | -7 | -7 |
| T | 8/70 | 11/70 | 3/71 | -3 | -7 |
| P | 7/73 | 11/73 | 12/73 | -4 | -5 |
| T | 5/75 | 3/75 | 7/75 | +2 | -2 |

*Involves phase of only two months duration.

^aThe coincident index shows three episodes of change in direction of a single month's duration in addition to the cycles shown.

^bNo cyclical peak in the leading index could be identified because data on changes measured over a six-month span were not available prior to July 1948. (The July 1948 and subsequent changes during the year were all negative.)

Note: The cyclical turning points are determined from changes measured over a six-month span and thus are not necessarily the same as those measured over a one-month span.

Table 3. Mean Timing of Matched Turns in Leading Index

in Relation to

| | Reference Cycle Turns | | | Turns in Coincident Index | | |
|------------------------------------|-----------------------|---------------|---------------|---------------------------|---------------|---------------|
| | Peaks | Troughs | All Turns | Peaks | Frougths | All Turns |
| Changes measured over 1 month span | -7.4 (2.6) | -2.8 (1.8) | -4.9 (3.2) | -3.4 (1.9) | -3.8 (2.6) | -3.6 (2.3) |
| Changes measured over 3 month span | -6.4 (2.9) | -2.3 (1.5) | -4.2 (3.0) | -3.2 (1.3) | -3.6 (2.2) | -3.4 (1.8) |
| Changes measured over 6 month span | -5.6 (2.3) | -1.0 (1.8) | -3.1 (3.1) | -3.7 (2.5) | -3.7 (2.1) | -3.7 (2.3) |

Note: Figures in parentheses indicate standard deviation.

TABLE 4
CHANGES IN REAL GNP RELATED TO CORRESPONDING CHANGES
IN THE COMPOSITE INDEX OF LEADING INDICATORS,
LOGARITHMIC FORM, QUARTERLY, 1948-76

| Line | Constant | Regression Coefficients of | | | | | R^2 | DW | SEE |
|------|----------------|----------------------------|----------------|----------------|----------------|----------------|-----------------|------|-------|
| | Term | DLI_t | DLI_{t-1} | DLI_{t-2} | DLI_{t-3} | DLY_{t-1} | (\bar{R}^2) | | |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| 1 | .006 (7.20) | .286 (8.59) | | | | | .397 | 1.57 | .0090 |
| 2 | .005 (3.95) | | | | | .425 (4.99) | .182 | 2.02 | .0105 |
| 3 | .004 (4.04) | .256 (8.06) | | | | .300 (4.30) | .484 (.479) | 2.39 | .0084 |
| 4 | .006 (7.23) | | .283 (8.45) | | | | .389 | 2.02 | .0090 |
| 5 | .006 (5.58) | | .267 (6.18) | | | .058 (.61) | .391 (.386) | 2.11 | .0091 |
| 6 | .006 (6.75) | .192 (4.94) | .173 (4.51) | | | | .506 (.502) | 1.86 | .0082 |
| 7 | .005 (6.27) | .234 (5.90) | .074 (1.51) | .123 (3.12) | | | .546 (.538) | 1.97 | .0079 |
| 8 | .005 (5.80) | .239 (6.15) | .103 (2.09) | .047 (.96) | .095 (2.46) | | .571 (.559) | 2.00 | .0078 |
| 9 | .006 (6.37) | | .289 (6.47) | -.024 (.44) | .082 (1.85) | | .419 (.408) | 2.03 | .0090 |

NOTE: Figures in parentheses underneath the entries in columns 1-6 are t-statistics (with signs omitted) for the corresponding least-squares estimates of constant terms and regression coefficients. R^2 is the squared correlation coefficient; \bar{R}^2 (in parentheses, given only where $k > 1$ so $\bar{R}^2 \neq R^2$) is the R^2 corrected for the number of degrees of freedom; i.e., $\bar{R}^2 = 1 - (1 - R^2) \frac{n-1}{n-k}$, where n is the number of observations and k is the number of independent variables. DW is the Durbin-Watson statistic. SEE is the standard error of estimate. $n = 114$ for lines 1-6, 10-15; $n = 112$ for each of the remaining equations.

DLY = quarter-to-quarter change in the natural logarithm of GNP in 1972 dollars.
DLI = quarter-to-quarter change in the natural logarithm of the composite index of leading indicators.

TABLE 5

RATE OF CHANGE IN REAL GNP (DLY) RELATED TO THE PRIOR RATE OF CHANGE
IN THE COMPOSITE INDEX OF LEADING INDICATORS (DLI)
Regression Estimates and Forecasts for Different Periods

A. Quarterly, 1948-1976

| Line | Statistic ^a | Estimates for Sample Periods | | | | | |
|------|------------------------|---|---------|---------|---------|---------|---------|
| | | 1948-69 | 1948-70 | 1948-71 | 1948-72 | 1948-73 | 1948-74 |
| | | (1) | (2) | (3) | (4) | (5) | (6) |
| | | (1) $DLY_t = a + bDLI_{t-1} + u_t$ | | | | | |
| 1 | \hat{a} | .007 | .007 | .007 | .007 | .007 | .006 |
| 2 | t_a | 6.75 | 6.65 | 6.77 | 7.10 | 7.22 | 6.77 |
| 3 | \hat{b} | .271 | .276 | .268 | .272 | .274 | .296 |
| 4 | t_b | 6.66 | 6.88 | 6.77 | 7.02 | 7.19 | 8.03 |
| 5 | R^2 | .348 | .352 | .335 | .342 | .343 | .385 |
| 6 | $F[1/(n-2)]$ | 44.4 | 47.3 | 45.8 | 49.3 | 51.7 | 64.5 |
| 7 | DW | 1.94 | 1.91 | 2.01 | 2.00 | 2.01 | 1.97 |
| 8 | $\sum \hat{u}_t^2$ | .007 | .007 | .007 | .007 | .007 | .008 |
| 9 | SEE | .009 | .009 | .009 | .009 | .009 | .009 |
| 10 | n | 85 | 89 | 93 | 97 | 101 | 105 |
| | | Estimates for Forecast Periods | | | | | |
| | Statistic ^b | 1970-76 | 1971-76 | 1972-76 | 1973-76 | 1974-76 | 1975-76 |
| | | (2) $\widehat{DLY}_t = \hat{a} + \hat{b} DLI_{t-1}$ | | | | | |
| 11 | R^2 | .482 | .494 | .551 | .522 | .519 | .456 |
| 12 | RMSE | .009 | .009 | .010 | .010 | .011 | .011 |
| 13 | MAE | .008 | .008 | .008 | .009 | .010 | .009 |
| 14 | ME | .002 | .001 | .001 | .003 | .003 | -.001 |
| 15 | U | .668 | .625 | .619 | .697 | .718 | .653 |
| 16 | U^B | .051 | .016 | .016 | .077 | .087 | .010 |
| 17 | U^I | .010 | .006 | .030 | .008 | .003 | .054 |
| 18 | U^V | .939 | .978 | .953 | .915 | .910 | .936 |
| 19 | n | 28 | 24 | 20 | 16 | 12 | 8 |

TABLE 5
(continued)B. Semiannual, 1948-1976

| Line | Statistic ^a | Estimates for Sample Periods | | | | | |
|------|------------------------|--|---------|---------|---------|---------|---------|
| | | 1948-69 | 1948-70 | 1948-71 | 1948-72 | 1948-73 | 1948-74 |
| | | (1) | (2) | (3) | (4) | (5) | (6) |
| | | (1) $DLY_t = a + bDLI_{t-1} + u_t$ | | | | | |
| 20 | \hat{a} | .014 | .014 | .014 | .014 | .014 | .012 |
| 21 | t_a | 5.99 | 6.06 | 6.18 | 6.46 | 6.46 | 5.81 |
| 22 | \hat{b} | .270 | .281 | .273 | .277 | .277 | .301 |
| 23 | t_b | 4.92 | 5.32 | 5.28 | 5.47 | 5.54 | 5.92 |
| 24 | R^2 | .383 | .408 | .394 | .399 | .395 | .417 |
| 25 | $F[1/(n-2)]$ | 24.2 | 28.3 | 27.9 | 29.9 | 30.7 | 35.0 |
| 26 | DW | 1.92 | 1.90 | 1.88 | 1.94 | 1.92 | 1.77 |
| 27 | $\sum \hat{u}_t^2$ | .007 | .008 | .008 | .008 | .008 | .009 |
| 28 | SEE | .014 | .014 | .013 | .013 | .013 | .014 |
| 29 | n | 41 | 43 | 45 | 47 | 49 | 51 |
| | | Estimates for Forecast Periods | | | | | |
| | Statistic ^b | 1970-76 | 1971-76 | 1972-76 | 1973-76 | 1974-76 | 1975-76 |
| | | (2) $\hat{DLY}_t = \hat{a} + \hat{b}DLI_{t-1}$ | | | | | |
| 30 | R^2 | .375 | .345 | .366 | .318 | .260 | .309 |
| 31 | RMSE | .017 | .018 | .019 | .021 | .023 | .023 |
| 32 | MAE | .014 | .014 | .015 | .017 | .019 | .017 |
| 33 | ME | .004 | .003 | .003 | .006 | .005 | -.005 |
| 34 | U | .709 | .688 | .697 | .810 | .880 | .780 |
| 35 | U^B | .064 | .029 | .018 | .071 | .057 | .049 |
| 36 | U^I | .0002 | .006 | .0002 | .010 | .021 | .099 |
| 37 | U^V | .936 | .965 | .982 | .919 | .922 | .852 |
| 38 | n | 14 | 12 | 10 | 8 | 6 | 4 |

Notes to Table 5

\hat{a} and \hat{b} are least squares estimates of the parameters a and b in equation (1) $DLY_t = a + b DLI_{t-1} + u_t$, where $DLY = \Delta \log RGNP$ and $DLI = \Delta \log CIL$ ($RGNP = GNP$ in 1972 dollars, $CIL =$ composite index of leading indicators, 1967 = 100); t_a and t_b are t -statistics for \hat{a} and \hat{b} , respectively; R^2 is the squared coefficient of correlation between DLY_t and DLI_{t-1} ; F is the ratio of the "explained" variation in DLY (regression sum of squares) to the "unexplained" variance, with 1 and $n-2$ degrees of freedom in the numerator and denominator, respectively;

$DW = \frac{\sum_{t=2}^T (\hat{u}_t - u_{t-1})^2}{\sum_{t=1}^T \hat{u}_t^2}$ is the Durbin-Watson statistic;

$\sum \hat{u}_t^2 = \sum (DLY_t - \hat{a} - \hat{b} DLI_{t-1})^2$ is the sum of squared residuals (estimated error terms); $SEE = \hat{\sigma}^2 = \sum \hat{u}_t^2 / (n-2)$ is the standard error of estimate (s.e. of the regression); and n is the number of observations (quarterly in line 10, semiannual in line 29).

b/R^2 is the squared correlation coefficient between the predicted values $P_t = \widehat{DLY}_t$ and the actual values $A_t = DLY_t$. Forecast \widehat{DLY}_t is based on equation (2) in which \hat{a} and \hat{b} are taken at values estimated from the corresponding equation (1) and shown in the same column (lines 1 and 3 for the quarterly, lines 20 and 22 for the semiannual estimates).

$RMSE = \sqrt{\frac{1}{n} \sum e_t^2}$, where $e_t = P_t - A_t$ is the root mean square error of the forecast; $MAE = \frac{1}{n} \sum |e_t|$ is the mean absolute error; $ME = \frac{1}{n} \sum e_t$ is the mean error. U is Theil's inequality coefficient defined as

$\sqrt{\sum e^2 / \sum (\Delta A)^2}$, where the summations are over the n quarterly periods covered (see line 19) and $\Delta A = A_t - A_{t-1}$. Decomposition of the mean square

error yields $MSE = \frac{1}{n} \sum e^2 = (\bar{P} - \bar{A})^2 + (S_P - S_A)^2 + 2(1-r)S_P S_A = B + I + V$, where \bar{P} , \bar{A} , S_P , and S_A are the means and standard deviations of the time series P (here \widehat{DLY}_t) and A (here DLY_t), respectively, and r is their correlation coefficient. $U^B = B/MSE$, $U^I = I/MSE$, and $U^V = V/MSE$ are the fractions of MSE due to bias, inefficiency, and residual variation, respectively.

TABLE 6

REGRESSIONS OF CHANGES IN THE COMPOSITE INDEX OF LEADING
INDICATORS ON LAGGED CHANGES IN THE SAME INDEX AND
IN THE INVERTED INDEX OF LAGGING INDICATORS,
LOGARITHMIC FORM, QUARTERLY, 1948-76

| Line | Constant Term (1) | Regression Coefficients of | | | | | R^2 (\bar{R}^2) (7) | DW (8) | SEE (9) |
|------|-------------------------|----------------------------|--------------------|--------------------|--------------------|--------------------|---------------------------------|-----------|------------|
| | | DLI_{t-1} (2) | DLI_{t-2} (3) | DLI_{t-3} (4) | DLV_{t-1} (5) | DLV_{t-2} (6) | | | |
| 1 | .009 (4.32) | .517 (5.22) | -.195 (1.53) | .126 (1.35) | .445 (3.35) | .069 (.50) | .557 (.540) | 1.97 | .0172 |
| 2 | .008 (4.51) | .524 (5.35) | -.162 (1.50) | .115 (1.27) | .493 (5.36) | | .556 (.543) | 2.00 | .0171 |
| 3 | .009 (4.55) | .509 (5.22) | -.091 (.98) | | .452 (5.22) | | .549 (.541) | 1.98 | .0172 |
| 4 | .009 (4.68) | .441 (6.49) | | | .496 (6.67) | | .545 (.541) | 1.88 | .0172 |

NOTE: DLI = quarter-to-quarter change in the natural logarithm of the composite index of leading indicators. DLI_t is the dependent variable.

DLV = quarter-to-quarter change in the natural logarithm of $1/G$, the inverse of the composite index of lagging indicators. (G, like I, is an index, 1967 = 100, seasonally adjusted, monthly rate.)

The number of observations (n) is 112 for each equation.

For other explanations, see notes to Table 4.

TABLE 7

SAMPLE-PERIOD REGRESSIONS AND FORECAST-PERIOD
ESTIMATES FOR DLY AND DLI, QUARTERLY,
1948-71 AND 1972-76

A. Regressions, 1948-71^a

$$(1) \quad DLY_t = .006 + .134 DLI_t + .188 DLI_{t-1} + u_{1t} \quad n = 94$$

(6.56) (2.84) (3.98)

$$\bar{R}^2 = .382 \quad DW = 1.86 \quad SEE = .0086$$

$$(2) \quad DLY_t = .006 + .267 DLI_{t-1} + u_{2t} \quad n = 94$$

(6.89) (6.79)

$$\bar{R}^2 = .334 \quad DW = 2.01 \quad SEE = .0089$$

$$(3) \quad DLI_t = .010 + .421 DLI_{t-1} + .544 DLY_{t-1} + u_{3t} \quad n = 94$$

(4.64) (5.38) (5.75)

$$\bar{R}^2 = .520 \quad DW = 1.93 \quad SEE = .0164$$

B. Predictions, 1972-76^b

$$(4) \quad \widehat{DLI}_t = .010 + .421 DLI_{t-1} + .544 DLY_{t-1} \quad n = 20$$

$$R^2 = .619 \quad RMSE = .020 \quad MAE = .016 \quad ME = .004$$

$$U = .626 \quad U^B = .040 \quad U^I = .004 \quad U^V = .955$$

$$(5) \quad \widehat{DLY}_t = .006 + .134 \widehat{DLI}_t + .188 DLI_{t-1} \quad n = 20$$

$$R^2 = .616 \quad RMSE = .009 \quad MAE = .007 \quad ME = .002$$

$$U = .582 \quad U^B = .031 \quad U^I = .046 \quad U^V = .923$$

$$(6) \quad \widehat{DLY}_t = .006 + .267 \widehat{DLI}_{t-1} \quad n = 20$$

$$R^2 = .347 \quad RMSE = .011 \quad MAE = .010 \quad ME = .002$$

$$U = .746 \quad U^B = .039 \quad U^I = .008 \quad U^V = .953$$

^aFigures in parentheses are t-statistics. See also notes to Table 4.

^bSee text and Table 5, note b, for explanations.

TABLE 8

CHANGES IN THE COMPOSITE INDEX OF COINCIDENT INDICATORS (DLC)
RELATED TO PRIOR CHANGES IN THE COMPOSITE INDEX OF LEADING INDICATORS (DLI),
SAMPLE-PERIOD REGRESSIONS AND FORECAST-PERIOD ESTIMATES, 1948-71 AND 1972-76

| Line | Lag of DLC Relative to First DLI Term i^a | Estimates for Sample Period 1948-71 ^b | | | | | | | |
|--|---|--|-------------------------|----------------|----------------|---------------------------------|----------------|----------------|----------------|
| | | Constant Term a (1) | Regression Coefficients | | | R^2 (\bar{R}^2) (5) | DW (6) | SEE (7) | (8) |
| | | | b_1 (2) | b_2 (3) | b_3 (4) | | | | |
| 1 | 3 months | .0013 (.75) | .556 (6.52) | .002 (.01) | .292 (3.42) | .546 (.536) | 1.84 | .0146 | |
| 2 | 2 months | .0004 (.27) | .624 (7.88) | -.079 (.80) | .394 (5.00) | .637 (.629) | 1.62 | .0131 | |
| 3 | 1 month | .0004 (.23) | .514 (6.79) | .053 (.58) | .366 (4.89) | .627 (.619) | 1.48 | .0133 | |
| Estimates for Forecast Period 1972-76 ^c | | | | | | | | | |
| | | R^2 | RMSE | MAE | ME | U | U ^B | U ^I | U ^V |
| 4 | 3 months | .717 | .013 | .009 | -.0007 | .274 .520 | .003 | .009 | .988 |
| 5 | 2 months | .797 | .011 | .009 | -.001 | .228 .447 | .014 | .026 | .960 |
| 6 | 1 month | .814 | .010 | .008 | -.001 | .219 .427 | .019 | .021 | .960 |

^a Measured between the midpoints of the quarterly terms DLC_t and DLI_{t-i} .

^b Based on regression equation

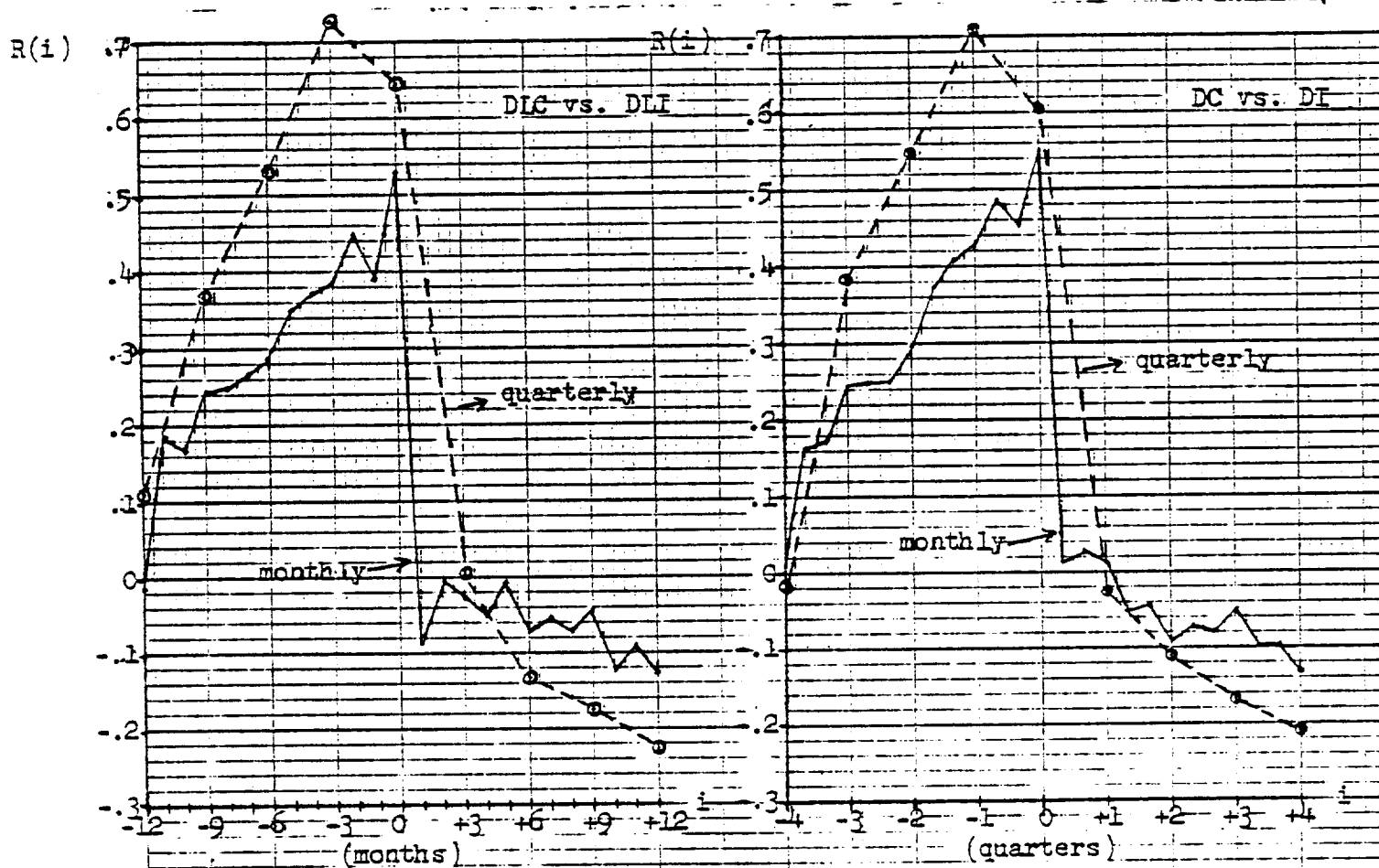
$$DLC_t = a + b_1 DLI_{t-1} + b_2 DLI_{t-3} + b_3 DLI_{t-6} + u_t$$

Figures in parentheses are t-statistics for the corresponding estimates of constant term and regression coefficients. The number of observations $n = 92$. For other explanation see notes to Table 4.

^c Refer to the predictions \hat{DLC}_t derived from the equation given in note b above. See Table 5, note b, for explanation of the symbols.

FIGURE A

CROSS CORRELOGRAMS FOR THE RELATIONS BETWEEN THE LEADING INDEX
AND THE COINCIDENT INDEX AND REAL GROSS NATIONAL PRODUCT



Notes to Figure A

DI -- change in the leading index
DLI -- change in the logarithm of the leading index
DC -- change in the coincident index
DCI -- change in the logarithm of the coincident index
DY -- change in real GNP
DLY -- change in the logarithm of real GNP

- refers to leads, + to lags of DI (DLI) relative to DC (DLC) or
relative to DY (DLY).

FIGURE B

REAL GNP PREDICTIONS BASED ON THE COMPOSITE INDEX
OF LEADING INDICATORS, QUARTERLY, 1972-76

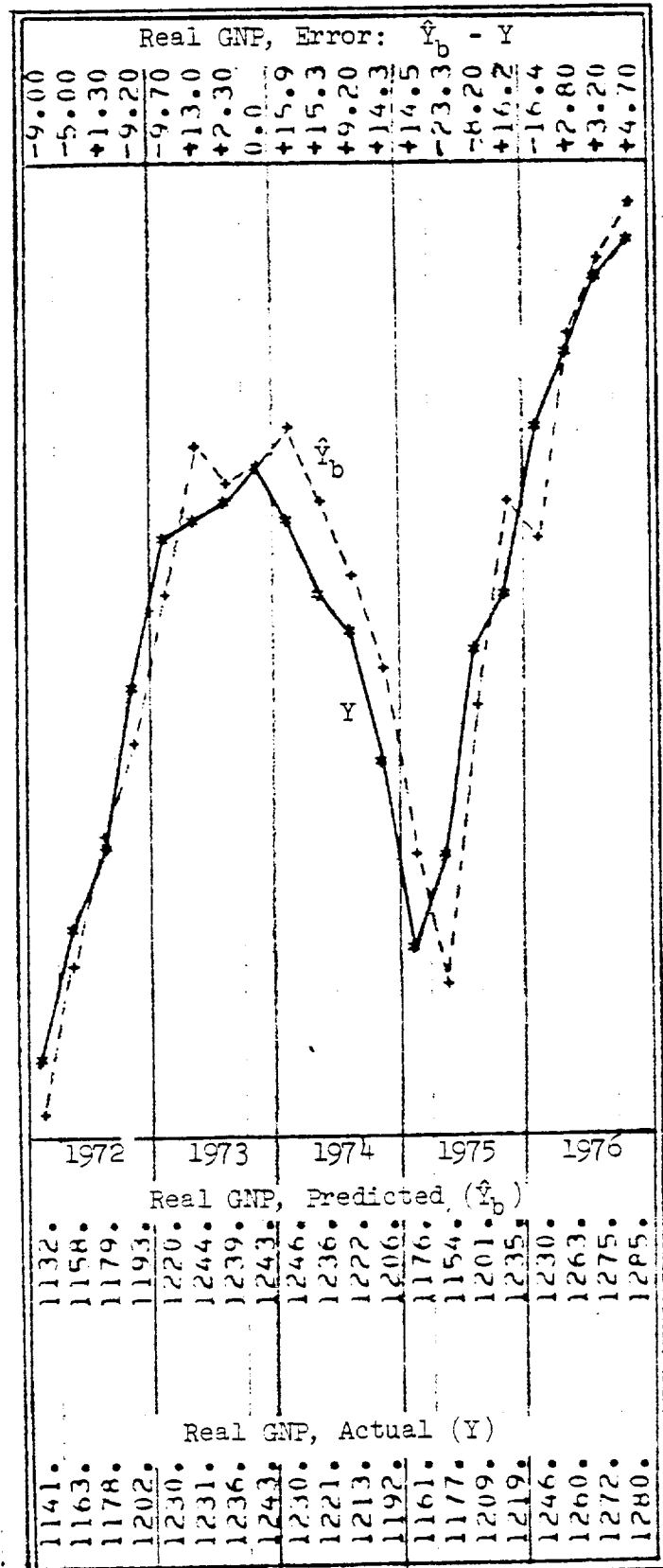
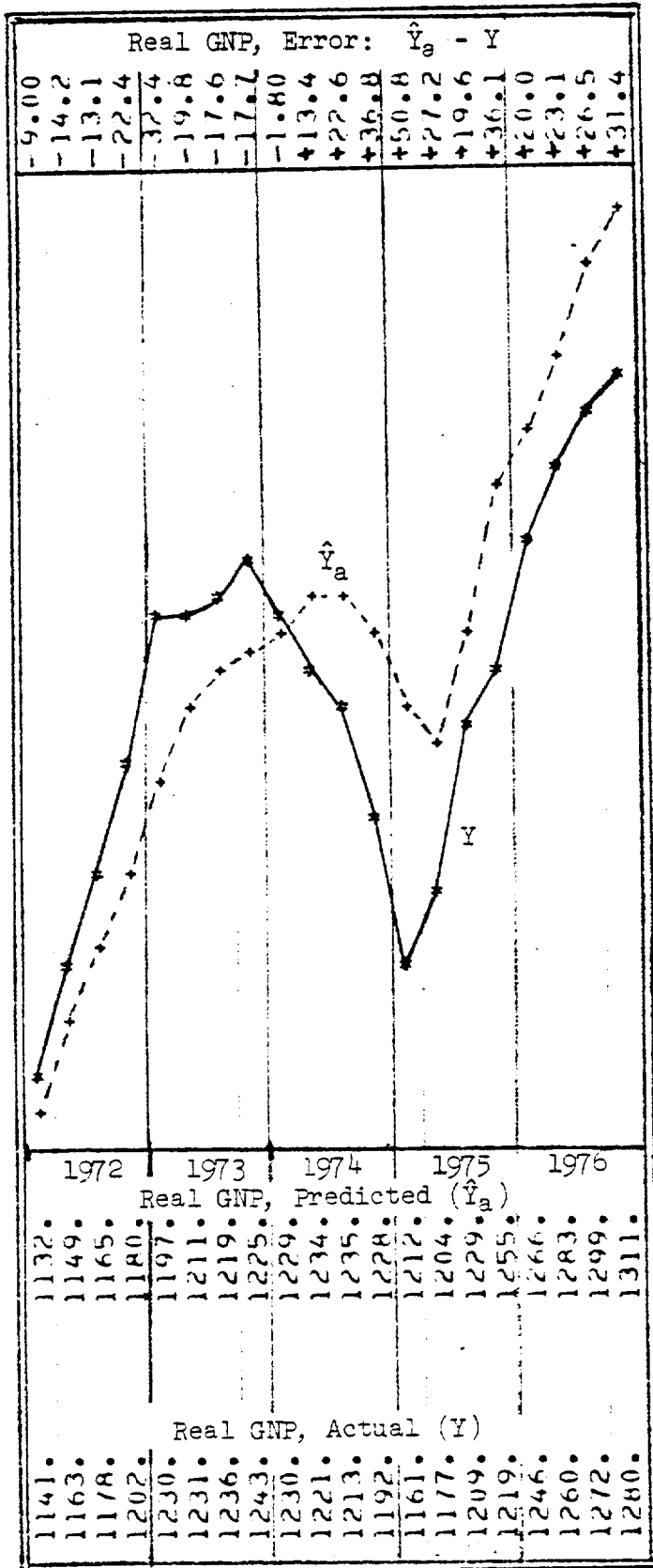
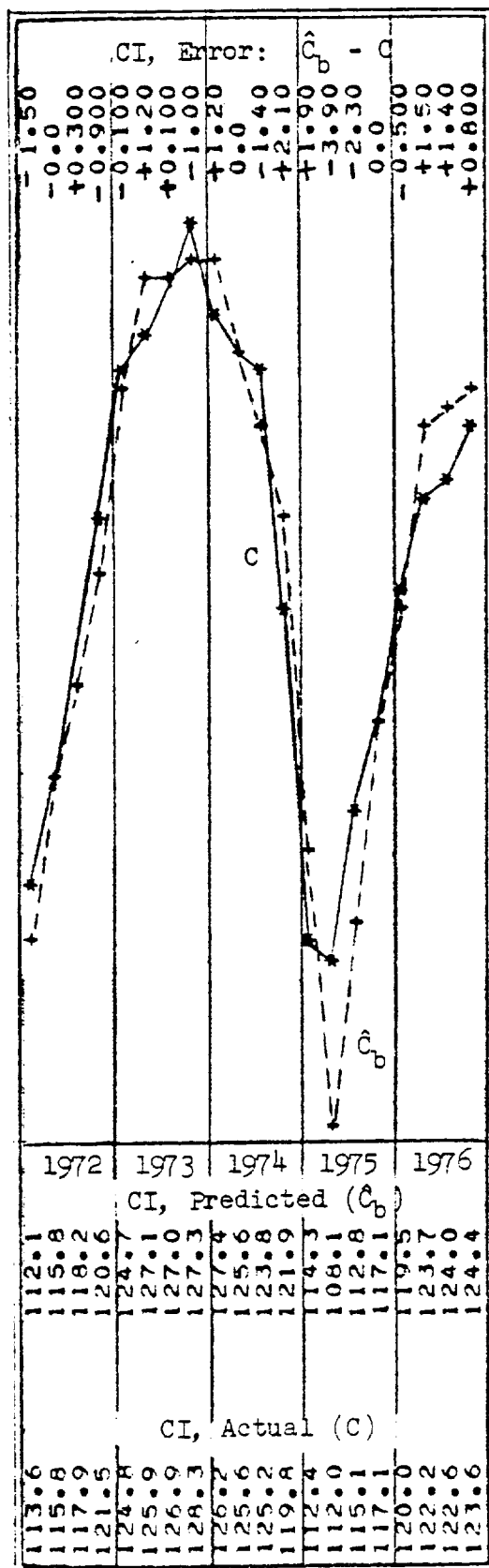
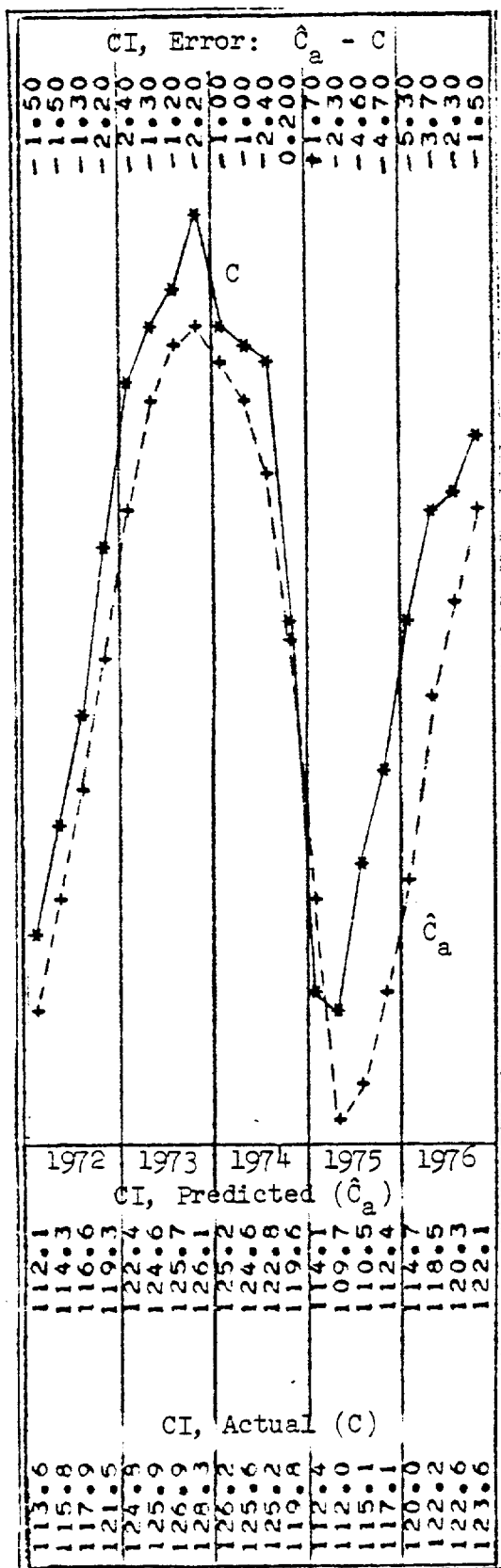


FIGURE C

COINCIDENT INDEX PREDICTIONS BASED ON THE COMPOSITE INDEX
OF LEADING INDICATORS, QUARTERLY, 1972-76



CI = Coincident Index

Footnotes

¹ Indicators are likely to prove most useful when combined with other informational tools. Indeed, the evidence of forecasters' behavior and performance supports that view not only for the cyclical indicators but for other economic forecasting techniques as well. See, e.g., V. Zarnowitz, "How Accurate Have the Forecasts Been?" and other essays in W. F. Butler, R. A. Kavesh, and R. B. Platt, Methods and Techniques of Business Forecasting, Prentice-Hall, Englewood Cliffs, N.J., 1974.

² For the current concepts, data, and techniques used in the construction of the indexes, see V. Zarnowitz and C. Boschan, "Cyclical Indicators: An Evaluation and New Leading Indexes," Business Conditions Digest, May 1975, and "New Composite Indexes of Coincident and Lagging Indexes," Business Conditions Digest, November 1975; also, the revised edition issue of Business Conditions Digest, November 1976, and U.S. Department of Commerce, Bureau of Economic Analysis, Handbook of Cyclical Indicators: A Supplement to the Business Conditions Digest, May 1977.

³ Arthur F. Burns and Wesley C. Mitchell, Measuring Business Cycles, National Bureau of Economic Research, Inc., New York, 1947.

⁴ Although there was one extra cycle in the coincident index under the three consecutive month rule (October 1959-January 1960), it reflected a long steel strike and the extra cycle in the leading index did not match it.

⁵ According to a reference chronology for "growth cycles" established by the National Bureau of Economic Research (see V. Zarnowitz and G. H. Moore, "The Recession and Recovery of 1973-1976," Explorations in Economic Research, vol. 4., no. 4, Fall 1977), there were eight complete growth cycles in the 1948-1973 period; five of these cycles correspond to the business cycle, although they tend to show earlier timing at peaks.

⁶ Under such a procedure, the change for, say, June 1960 represents a comparison of the June 1960 index level with that of December 1959.

⁷ These turns were identified by applying a rule requiring change of direction of at least two months with the changes measured over a six-month span. If the rule were relaxed to include changes of only one-month duration, 16 turning points would have been identified.

⁸ The primary reason for this improved performance is the fact that turning points in the coincident index selected on the basis of changes measured over a six-month span are consistently later than those selected on the basis of changes measured over a one-month span. This fact is readily observable from a comparison of the dates listed in Tables 1 and 2.

⁹ See Henri Theil, Applied Economic Forecasting, Rand McNally, Chicago, 1966, p. 28. The formula is $U = \sqrt{\Sigma e^2} / \sqrt{\Sigma \Delta A^2}$. Previously, in his Economic Forecasts and Policy, North-Holland, Amsterdam, 1958, p. 32, Theil had a different definition of the inequality coefficient

(call it U'), namely $\sqrt{E^2}/(\sqrt{\Sigma \Delta A^2} + \sqrt{\Sigma \Delta P^2})$. The formula for U' was incorporated in the computer program which we used in "How Good Are the Leading Indicators," 1977 Proceedings of Business and Economic Statistics Section, American Statistical Association, Washington, D.C. Regrettably, there the results were mistakenly interpreted as representing U .

¹⁰ For example, the U coefficients for the "early quarter" forecasts--from the ASA-NBER surveys and the Wharton, DRI and Chase models--are all close to .5 (they vary from .47 to .51) for the 1970:3 to 1975:4 period; see Stephen K. McNees, "An Evaluation of Economic Forecasts: Extension and Update," New England Economic Review, September/October 1976, p. 32. The margins of advantage relative to the measures in Table 5, line 15, thus appear to be small, particularly when one considers that the indicator predictions are much simpler and less costly to make than the others. Of course, the differences between the forecasts are such that comparisons of this kind are inevitably very crude. A more interesting question, to be left for future research, concerns the possible gains from combining the indicator forecasts with the others: to what extent are the methods complementary so that the combination would result in more accurate composite predictions?

¹¹ The index reached a peak in June 1973 (or 1973:2 on quarterly basis), that is, two quarters ahead of the peak in real GNP. The subsequent decline, while initially moderate, was continuous and so would have provided an advance warning to a current observer willing

to rely on a sequence of several signals from the monthly index data. In 1975, the leading index had a trough in February, real GNP in the first quarter, a more nearly coincident timing.

¹² See Herman Wold, "Forecasting by the Chain Principle" in H. Wold, ed., Econometric Model Building: Essays on the Causal Chain Approach, Amsterdam, North-Holland Publishing Company, 1964, pp. 5-36.

¹³ For example, a second-order autoregressive model (2, 1, 0) applied to the first differences of monthly, seasonally adjusted data for 1948-75, works fairly well, as suggested by the following results:

$$DI_t = .524 DI_{t-1} + .168 DI_{t-2} + .058 + \epsilon_t \quad n = 335$$

(9.72) (3.11) (1.47)

$\bar{R}^2 = .408$; $Q = n \sum_{j=1}^K \hat{r}_j^2 = 18.5, 33.3, 48.7$ for $K = 12, 24, 36$ lags, respectively ($\hat{r}_1, \dots, \hat{r}_K$ denote the first K residual autocorrelations); the corresponding P values (significance levels of Q according to χ^2 distribution) are .047, .058, and .049. On the techniques of analyzing time series as discrete linear stochastic processes, see G. E. P. Box and G. M. Jenkins, Time Series Analysis, San Francisco, Holden-Day, Inc., 1970.

¹⁴ See V. Zarnowitz and C. Boschan, "New Composite Indexes of Coincident and Lagging Indicators," BCD, November 1976, pp. XV-XVIII.

¹⁵ An alternative to $1/G$ is the ratio of the coincident to the lagging index C/G (also discussed in the BCD article cited in note 13), but we find the former preferable to the latter series in the present context.

¹⁶ Prediction of monthly values of the coincident index is, of course, also possible, but it appears to us both less promising and less interesting.