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THE AUSTRIAN THEORY OF THE MARGINAL USE AND OF ORDINAL MARGINAL UTILITY

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ABSTRACT

The Austrian Theory of the Marginal Use
and of Ordinal Marginal Utility

J. Huston McCulloch

The Austrian theory of the "marginal use" is restated and extended. It is found that the Austrian concept of marginal utility (as derived from the marginal use) is not dependent on cardinal utility, and indeed is consistent with "intrinsically ordinal" utility. In this system, diminishing (ordinal) marginal utility is an implication of rational choice, rather than an assumption. Examples of the rank-ordering on commodity space, derived from the underlying rank ordering on want-set space in conjunction with the technological relationship between goods and wants, are given in the cases of independent, rival, and complementary goods. In each case the derived commodity preferences are quasi-concave, which suggests that the Hicksian assumption of quasi-concavity is superfluous. In each case, the Auspitz and Lieben-Edgeworth-Pareto criterion for net complementarity or rivalness emerges. It is shown that while a negative cross substitution elasticity is not a necessary condition for net complementarity, it _is_ a sufficient condition under not very restrictive conditions.

INTRODUCTION

The Austrian theory of the marginal use and of ordinal marginal utility has not stood still since its original development in the hands of Menger, Wieser, and Böhm-Bawerk. Over the past hundred years, it has moved far beyond their statement of it, even though this movement sometimes proceeded at a rather leisurely pace.

This paper brings the old theory up to date. We do this by restating it, insisting on a new English translation for one of its most important technical terms. We call attention to two crucial assumptions which were implicit in the old theory, but which were never stated explicitly until the past decade. We point out that a recent mathematical finding implies that the Austrian marginal utility concept is not just ordinal, but in a sense is "intrinsically ordinal."

The restated theory has many important implications for the structure of preferences over commodities, implications which do not follow from the Jevonian, Walrasian, or Hicksian traditions. The theory indicates that preferences over commodities are indeed quasi-concave (as Hicks and Allen merely assume), that marginal utility does diminish, even in an ordinalist framework, and that rival and complementary interactions between goods do lead to the Auspitz and Lieben-Edgeworth-Pareto criterion. From this criterion we are then able to deduce that a negative cross substitution elasticity, while not a necessary condition for net complementarity, is a sufficient condition when there are three non-inferior goods, provided the third good is on net independent of the two under consideration.

WANTS AND UTILITY

What distinguishes the Austrian approach from that of Jevons or Walras is that the Austrians did not accept the utility or subjective value of commodities as a given, but rather derived it from the importance of the wants (sometimes translated as "needs") that the goods can be used to satisfy. Menger described this relation as follows:

> Value is therefore nothing inherent in goods, no property of them, but merely the importance that we attribute to the satisfaction of our needs, that is, to our lives and well-being, and in consequence carry over to economic goods as the exclusive causes of the satisfaction of our needs. (1950, 116)

The starting point for inferences about the subjective importance of goods is a subjective rank-ordering of the set of all wants which arranges them in the order of their importance to the individual. According to Böhm-Bawerk,

> It is a matter of common knowledge that our wants vary widely in importance. ....[I]t is possible to construct a progression or graduated scale of wants in point of importance. That scale will of course vary from person to person because their varying physical and intellectual propensities, amount of education, and the like will result in widely varying wants. Even the same individual will vary widely in his wants at different times. And yet every practical economizing person, if he is to make a wise choice in the application of his limited means will have his scale of wants more or less clearly in mind. (1959 II, 137)

Given this scale, if we can determine which want is dependent upon the possession of a certain good, we may ascribe the importance of that want to the good. Thus, the utility of a good will essentially be a position on this scale of wants.[1]

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[1] Böhm-Bawerk pointed out that "The expression ['the ranking of wants'] may mean the rank and order of categories of wants [Bedürfnisgattungen], or may mean concrete wants [konkreten Budurfnisse], that is to say, the individual feelings of want." (1959 II, p. 137, and 1909 A, p. 237) He went on to make it explicit that he had in mind a ranking on concrete wants. Thus, we are to enter nothing so general as "the want for food" in the scale of wants, but are to break wants down into specific uses for each portion and type of food.

THE DEPENDENT WANT AND THE IMPLICIT ASSUMPTIONS.

The Austrian determination of the dependent want that determines value, as given in the classic expositions of Menger's farmer and Böhm-Bawerk's hunter, runs essentially as follows: Suppose there are three wants, a, b, and c, any one of which can be satisfied by a unit of a certain good, and that the indivual prefers a to b and b to c. We represent these preferences by $a \succ b \succ c$, using the symbol $\succ$ rather than $>$, in order to emphasize that this is a preference ordering rather than a numerical inequality. Obviously if the individual has only one unit of the good he will use it to satisfy want a, if he has two units, he will satisfy wants a and b, and if he has three units he will satisfy all three wants. Therefore the value of the first unit is the importance of want a, the value of the second unit is the importance of want b, the value of the third unit is the importance of want c, and any additional units are worthless unless the individual can come up with more wants the good can be used to satisfy.

This conclusion may be obvious, but it is not really warranted, given only the traditional rank-ordering on wants. It is true that if the individual has only one unit he will, by assumption, use it to satisfy want a. However, if he has two units, he may satisfy <u>any two</u> wants, that is, he may choose from a and b, and a and c, and b and c. In fact, if he feels like it, he may satisfy only one want, a, b, c, or for that matter, he may satisfy no wants at all if he is so inclined. If W is the set of all wants, in this case $W = \{a, b, c\}$, then with two units he may choose from any subset of W with two or fewer elements. In order to infer which subset he will choose, we must be given a preference ordering not just on W, but on W*, the set of all subsets of W: $W^* = \{P | P \subset W\}$. In our example, $W^* = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$, where "$\emptyset$" is the empty set, the subset of W which corresponds to the satisfaction of no wants at all. To eliminate unnecessary clutter, we will omit the braces and commas from the designation of elements of W*, so that $W^* = \{\emptyset, a, b, c, ab, ac, bc, abc\}$. If W is finite and has n elements, then W* has 2^n elements, in this case $2^3 = 8$. The first implicit assumption that the Austrians made is therefore that the individual's preferences define a linear ordering on W* , such that an individual with a quantity of some good or goods will use these goods to satisfy the highest rated subset which is feasible, given the supply. This implicit assumption was noted by Georgescu-Roegen (1968, 251), and was also independently discovered by Young (1969) and the present author at about the same time.

And the traditional Austrian formulation makes a second implicit assumption, which somehow implies that if $a \succ b \succ c$ and all are "desirable" so that $a \succ b \succ c \succ \emptyset$, then it is ab that will be the highest rated subject with two or fewer elements. An article by the later Austrian-school economist Bilimovič (1934, esp. p. 183) provides a clue to what they had in mind. He argues in effect that $b \succ c$ would imply $ab \succ ac$, that $a \succ b$ would imply $ac \succ bc$, and that $c \succ \emptyset$ would imply $ac \succ a$. These inferences, together with the transitivity of the ordering on W*, imply $ab \succ ac \succ bc$ and $ab \succ ac \succ a \succ b \succ c \succ \emptyset$, so that ab is indeed the highest feasible subset when two units are available. Bilimovič argues as if these inferences were valid deductions from a rank-ordering on W, but that is not the case unless we assume that if an additional want or set or set of wants is "added" to both sides of a relationship, the elements of the additional set not being contained in either of the sets involved in the original relationship, then the relationship remains undisturbed. We will call this property "additivity," following the terminology of Kraft et al. (1959, 408). Formally defined, a set of subsets W* is additively ordered if for any two subsets P and Q of W, we have $P \succ Q$ if and only if $P - Q \succ Q - P$. In economics, this assumption was first made explicit by Young (1969) and by the present author, working independently at about the same time.

Additivity is illustrated in the Venn diagram of Figure 1. The two circles circles P and Q represent subsets of the universal set W. The set difference P-Q is the set of all wants in P but not in Q, and Q-P is the set of all wants in Q but not in P. The intersection $P \cap Q$ is the set of all wants in both P and Q. The additivity assumption states that $P \cap Q$, the wants P and Q have in common, are irrelevant to the relative ordering of P and Q. All that matters is the relative importance of P-Q and Q-P, the wants P and Q do not have in common.

THE LAW OF THE MARGINAL USE

Given that W* is additively ordered and that $a \succ b \succ c \succ \emptyset$, it follows that an individual in possession of one unit of our good will use it to satisfy want a, that the use of two units would be the satisfaction of wants a and b, and that the use of three or more units will be the satisfaction of all three wants. Hence, if the individual has only one unit, the use which depends on possession of the last unit will be a, and therefore the value or utility of one loaf will be that of a, that is, the place of the set

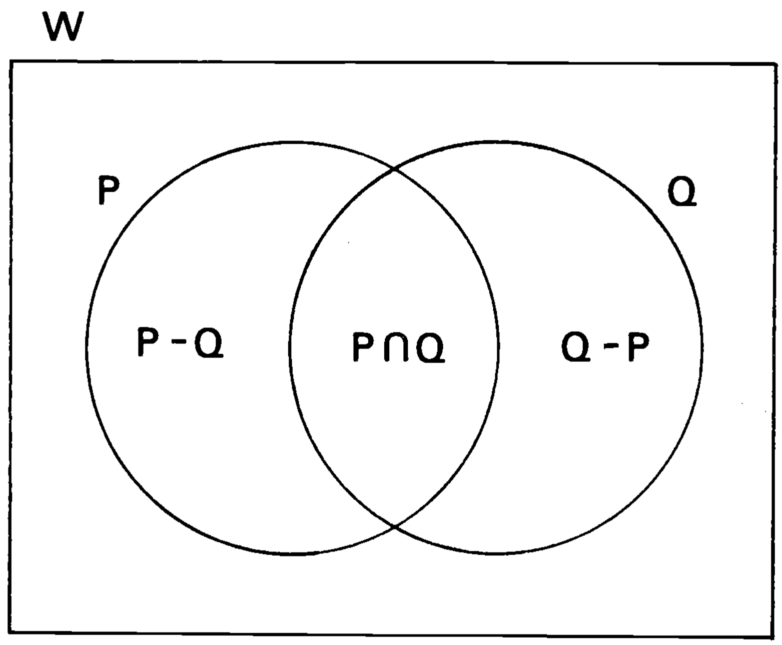

Figure 1.

Additivity means that P is preferred to Q if and only if P - Q is preferred to Q - P.

containing only a in the rank-ordering of W*. If he has two units, the use dependent on either of these units will be the satisfaction of b, the less important of the two uses covered by two units, and hence the utility of one unit will be the position of b on the scale. And if he has three units, the dependent use is the satisfaction of c, the least important of the three uses covered by three units, and hence the utility of the third unit will be the position of c on the scale.

Menger had no name for this use which determines utility, but Weiser proposed one which was subsequently adopted by Böhm-Bawerk:

> I will henceforth refer to that use of a good which is decisive for the value of a single unit of that good as the economically marginal use, or simply as the marginal use, since it stands at the margin of the economically permissible employments.... It will be shown that in every instance in which we are concerned with the value of a single unit which is part of a supply of a good, the marginal use determines the magnitude of the value. Economic value is marginal value.[1]

If P_n is the set of wants that will be satisfied by n units and P_{n-1} the set

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[1] "Ich werde im Folgenden den für den Werth der Gütereinheit entscheidenden Güternutzen, weil er an den Grenze der wirthschaftlich zugelassenen Verwendungen steht, den wirthschaftlichen Grenznutzen oder auch kurzweg den Grenznutzen nennen (vergl. die Ausdrücke "final degree of utility" und "terminal utility" bei Jevons). Es wird sich zeigen, dass in allen Verhältnissen, in denen es sich um den Werth der einzelnen, einen Vorrath bildenden Guter handelt, der Grenznutzen den Ausschlag für die Größe des Werthes giebt. Der wirtschaftliche Werth ist Grenzwerth." (Wieser 1884, 128) Although Wieser invites his reader to compare his Grenznutzen to Jevon's Final Utility, "Nutzen" and "Verwendung" can mean "utility" only in the common English sense of the word, "usefulness," and cannot refer to the economist's "utility," which, as Jevons (1931,xxxiv) and Wieser himself five years later (1889, 26n.) confirmed, corresponds to the Austrians' Wert. Wieser's term "Grenzwerth" does, however, correspond roughly to "marginal utility." Our choice of "marginal use" for "Grenznutzen" is discussed below.

that will be satisfied by n-1 units, then the set difference $P_n - P_{n-1}$ will be the dependent set of wants or the marginal use of the nth unit. If the individual has n units of the good, "the marginal use of one unit" is somewhat ambiguous, since it can refer either to the marginal use of the last (nth) unit or the marginal use of one additional (the n+1st) unit. When necessary, the former may be referred to as the inner marginal use and the latter as the outer marginal use. The Austrians' Law of the Marginal Use, then, which does not appear in the English literature, is that the value or utility of a goods-increment is determined by the position of its marginal use on the scale of sets of wants. A generalized proof of this theorem is given by McCulloch and Smith (1975).

Weiser's Grenzwerth or marginal value is the closest term the Austrians had to "marginal utility."[1] It corresponds exactly to Bernardelli's "conditional utility" (1938). Thus, it makes sense in their framework to speak of the (marginal) value of two units of a good, which is determined by the marginal use of two units, in turn the satisfaction of the two least important wants covered by the total supply. Because the Austrians thought in terms of realistic discretely divisible goods instead of hypothetical continuously divisible goods, their "value" corresponds to a non-infinitessimal increment. When only a single unit is at stake, their "value" can be thought of as "marginal utility," provided a distinction is kept in mind between outer marginal utility, corresponding to the outer marginal use, and inner marginal utility, corresponding to the inner marginal use.

Wieser turns the law of the marginal use about to get what might be called

[1] Alt's concept of Grenzwert (1936, 163) has no relation to Wieser's.

a "law of marginal utility," which determines which uses are permissible and which are not:

> Each desire whose importance lies above or is equal to the [marginal] value will be permitted, each whose importance lies below it will be rejected. All economically permissible employments will be included by the marginal value [<u>Grenzwerth</u>], and all impermissible ones excluded. (1884, 136, my trans.)

THE LAW OF DIMINISHING MARGINAL UTILITY

An immediate consequence of the Law of the Marginal Use is the fact that if we have a greater quantity of a good, the dependent want will have a lower rank on the scale, and therefore the (marginal) utility of one unit will be lower. As Menger put it,

> If a quantity of goods stands opposite needs of varying importance to men, they will first satisfy, or provide for, those needs whose satisfaction has the greatest importance to them. If there are any goods remaining, they will direct them to the satisfaction of needs that are next in degree of importance to those already satisfied. Any further remainder will be applied to the satisfaction of needs that come next in degree of importance. (1950, 151)

When Menger here says that the most important wants will be satisfied first, he of course means first logically (<u>i.e.</u> economically), and not necessarily chronologically. Thus, although Böhm-Bawerk's famous hunter will always feed himself (satisfy want a) "first," if he has two loaves of bread available he may in fact give one to his dog (satisfying want b) before he actually takes a bite of his own loaf.

If the marginal uses decline in importance as the available quantity increases, and it is the importance of the marginal use which determines marginal utility, then marginal utility must decrease as the available quantity increases. To illustrate this Law of Diminishing Marginal Utility, let us suppose that there is one good, "X," and that there are four wants, a, b, c and d that can be satisfied by a unit of X. (An imaginative individual might be able to think up an infinite number of wants he would like to satisfy with a certain commodity. For the sake of brevity, however, we will restrict our examples to finite cases.) An additive ranking of W* which could describe an individual's subjective preferences is given in Table 1. The sixteen positions on this scale have been numbered from "0th" to "15th," starting with the lowest position and proceeding up to the highest. These numbers comprise an ordinal utility index, where each number designates a certain utility level. These utility indices are <u>not</u> meant to mean that the twelfth utility level is in any sense "twice" as high as the sixth level, or that the utility of the tenth level equals that of the third level "plus" that of the seventh level. The indices simply give us a convenient method of referring to higher or lower positions on the scale. It seems appropriate to give the empty set ∅ (the set with no elements) the zeroeth position, though it could just as logically be assigned the ninety-seventh, or any other position.

If an individual has n units of X, additivity implies his use of them will be to satisfy the n most important wants, as indicated in Table 2. The utility level of this total use naturally increases with X, as long as we have additional "desirable" wants (that is, ones that are preferred to the empty set).

The marginal use of one unit of X for different quantities is shown in Table 3, along with the utility of this use, which in turn is the marginal utility of a unit of the good. The marginal utility of one additional unit is found to decline from fifth to third to second to first to zeroeth as X increases from 0 to 4.

ONE GOOD

Hypothetical Preference Ordering of W*, with Assigned Ordinal Utility Levels

Set of Wants	Ordinal Utility
abcd	15th
abc	14th
abd	13th
acd	12th
ab	11th
bcd	10th
ac	9th
ad	8th
bc	7th
bd	6th
a	5th
cd	4th
b	3rd
c	2nd
d	1st
Ø	0th

Table 1

Total Use and Total Utility of Various Quantities of X

Units of X	Use	Ordinal Utility
0	Ø	0th
1	a	5th
2	ab	11th
3	abc	14th
4	abcd	15th
5	abcd	15th

Table 2

Marginal Use and Marginal Utility of one Unit of X

Unit of X	Marginal Use of 1X	Ordinal Marginal Utility of 1X
1st	a	5th
2nd	b	3rd
3rd	c	2nd
4th	d	1st
5th	Ø	0th

Table 3

Notice that the marginal utility is not the arithmetic difference in the utility level. Rather, the Austrian concept of marginal utility is the utility level of the set difference of the respective uses. When von Mises insists, "There are in the sphere of values and valuations no arithmetical operations; there is no such thing as a calculation of values," (1963, 122) he has therefore only gotten at half the truth, for there are, we argue, set operations implicit in the Austrian utility analysis. Since the algebra of set manipulation is only a formalization of elementary categories of logic and since it has only recently come into fashion to use set notation, even in mathematics, it is understandable that the Austrians did not make these operations explicit, and in fact, were probably not even consciously aware that they were using them.

The Austrian principle of diminishing ordinal marginal utility points up the substantial difference between the Austrian theory of utility and the theory of utility as it has grown up in the English language literature. Hicks tells us that if we reject cardinal utility and purge our analysis of

> all concepts which are tainted by quantitative utility, ...the first victim must be marginal utility itself. If total utility is arbitrary so is marginal utility.... The second victim (a more serious one this time) must be the principle of Diminishing Marginal Utility. If marginal utility has no exact sense, diminishing marginal utility can have no exact sense either. (1946, 19-20)

Yet the Austrians had an ordinal concept of utility in which marginal utility does have a meaning, and furthermore, their marginal utility does diminish. For example, in Table 1 we could square each of the ordinal utility index values so that from the top down they read 15^2 = 225th, 14^2 = 196th, etc. The marginal utilities in Table 3 would still decline, from 25th to 9th to 4th to 1st to 0th. The Austrian law of diminishing marginal utility is thus invulnerable to monotonic transformations of the utility index.

THE UTILITY OF TWO INDEPENDENT GOODS

Let us suppose that there are two kinds of goods, X and Y, and that one unit of X will satisfy want a, c or e, and that one unit of Y will satisfy b or d. We then have W = {a, b, c, d, e}.

A conceivable additive preference ordering of W* is shown in Table 4, along with an ordinal utility index identifying the positions on the scale from zeroth to thirty-first. If an individual with the preferences of Table 4 has m units of X and n units of Y, additivity obviously implies that he will use them to satisfy the m most important elements of the set {a, c, e}, and the n most important elements of the set {b, d}. Table 5 shows the optimal use which would be made of various combinations of X and Y and the respective utility levels. These utility levels imply a derived preference ordering on the commodity bundles. In Table 6 the commodity bundles are arranged in decreasing order of utility. Table 7 lays out the total use and total utility of these bundles in two-dimensional tabular form. In Figure 2 the horizontal axis represents units of X and the vertical axis units of Y. Lines have been drawn on this graph corresponding to different utility levels. These lines have the property that any point below and to the left of the line has a utility level lower than that corresponding to that of the line, while all points on the line have exactly this utility, and all points above and to the right have at least this utility. Our lines roughly correspond to the "indifference curves" of conventional utility theory. The only difference

INDEPENDENT GOODS

Hypothetical Preference
Ordering of W*

Set of Wants	Ordinal Utility
abcde	31st
abcd	30th
abce	29th
abc	28th
abde	27th
acde	26th
abd	25th
abe	24th
acd	23rd
bcde	22nd
ace	21st
ab	20th
bcd	19th
ac	18th
ade	17th
ad	16th
bce	15th
bc	14th
bde	13th
ae	12th
cde	11th
bd	10th
a	9th
be	8th
cd	7th
ce	6th
b	5th
c	4th
de	3rd
d	2nd
e	1st
∅	0th

Table 4

Derived Utility of Combinations
of X and Y

Units of		Use	Ordinal Utility
X	Y		
0	0	∅	0th
1	0	a	9th
2	0	ac	18th
3	0	ace	21st
0	1	b	5th
1	1	ab	20th
2	1	abc	28th
3	1	abce	29th
0	2	bd	10th
1	2	abd	25th
2	2	abcd	30th
3	2	abcde	31st

Table 5

Derived Preference Ordering
of Combinations of X and Y

Units of		Use	Ordinal Utility
X	Y		
3	2	abcde	31st
2	2	abcd	30th
3	1	abce	29th
2	1	abc	28th
1	2	abd	25th
3	0	ace	21st
1	1	ab	20th
2	0	ac	18th
0	2	bd	10th
1	0	a	9th
0	1	b	5th
0	0	∅	0th

Table 6

INDEPENDENT GOODS

Total Use and Total Utility
of Combinations of X and Y

Units of Y					
3	bd 10th	abd 25th	abcd 30th	abcde 31st	abcde 31st
2	bd 10th	abd 25th	abcd 30th	abcde 31st	abcde 31st
1	b 5th	ab 20th	abc 28th	abce 29th	abce 29th
0	∅ 0th	a 9th	ac 18th	ace 21st	ace 21st
Units of X	0	1	2	3	4

Table 7

"Indifference Curves" Separating
More Preferred Combinations from
Less Preferred Combinations

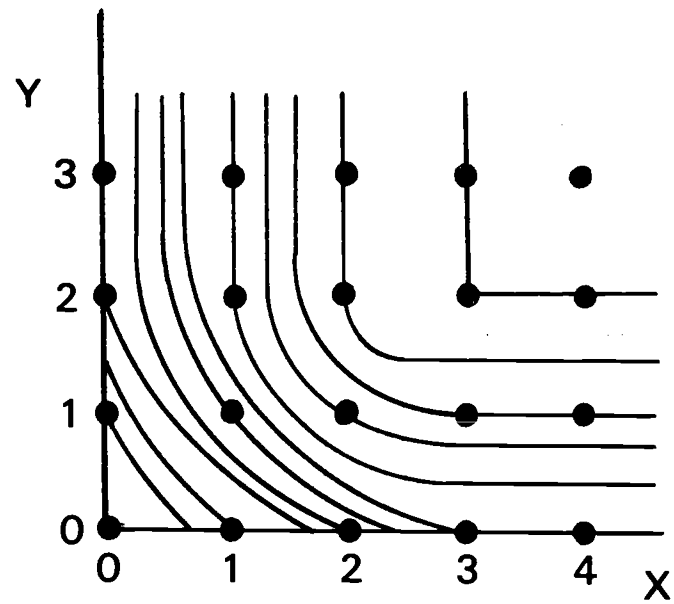

Figure 2

Marginal Use and Marginal
Utility of 1 unit of X

Units of Y				
3	a 9th	c 4th	e 1st	∅ 0th
2	a 9th	c 4th	e 1st	∅ 0th
1	a 9th	c 4th	e 1st	∅ 0th
0	a 9th	c 4th	e 1st	∅ 0th
Unit of X	1st	2nd	3rd	4th

Table 8

Marginal Use and Marginal
Utility of 1 unit of Y

Unit of Y					
3rd	∅ 0th	∅ 0th	∅ 0th	∅ 0th	∅ 0th
2nd	d 2nd	d 2nd	d 2nd	d 2nd	d 2nd
1st	b 5th	b 5th	b 5th	b 5th	b 5th
Units of X	0	1	2	3	4

Table 9

is that our commodities are "lumpy," rather than infinitely divisible, and therefore the lines usually go through only one point. The reader may, if he likes, think of these lines as "preference curves."

Table 8 shows the marginal use and marginal utility of one unit of X as the total quantities of X and Y vary. Table 9 shows the marginal use and marginal utility of one unit of Y. The marginal utility of X is found to diminish from 9th to 4th to 1st to 0th as X increases from 1 to 4, regardless of the quantity of Y available. Similarly, the marginal utility of Y diminishes from 5th to 2nd to 0th, regardless of the quantity of X available. It could not be otherwise in this case, for the quantity of one good has no bearing on the use that will be made of the other, and therefore no effect on the marginal use. We may therefore state as a general rule that when X and Y are independent in consumption, i.e., when W may be partitioned into two categories of wants such that a unit of X and only a unit of X will satisfy the wants in one category, and a unit of Y and only a unit of Y will satisfy the wants in the other category, the marginal utility of one good will be independent of the quantity available of the other.[1]

It should be noted that when there is only one good, the concept of marginal utility has no operational significance. So what if a unit of a good has a certain desirability, if there is nothing to compare it to? But when there is more than one good, we have the seemingly trivial but actually important rule, that if an individual is offered a choice between a unit of

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[1]Strotz's concept of a "utility tree" [1957] is undoubtedly related to independence of the goods in question. The exact connection deserves to be explained in greater detail.

one good or a unit of another, he will always choose the one with the higher marginal utility, as determined by the marginal use.

THE AUSTRIAN RESOLUTION OF THE PARADOX OF VALUE

Before the Austrians came on the scene, economists were troubled by the so-called paradox of value. As Adam Smith expressed it,

> The things which have the greatest value in use have frequently little or no value in exchange; and on the contrary, those which have the greatest value in exchange have frequently little or no value in use. Nothing is more useful than water: but it will purchase scarce any thing; scarce any thing can be had in exchange for it. A diamond, on the contrary, has scarce any value in use; but a very great quantity of other goods may frequently be had for it. (1776/1937, 28)

The Austrians made this paradox a central part of their theory. According to Böhm-Bawerk,

> This unquestionably astonishing phenomenon became a troublesome bone of contention for the theory of value. Supreme utility [Nutzen: utility in the sense of "usefulness"] and minimal value -- what a strange paradox! ....It is therefore not difficult to understand that from the days of Adam Smith down to our own numberless theorists have despaired completely of finding the essence and the measure of value in a relation to human welfare.[1]

The alternate theories that did spring up, theories based on labor or on costs, had, according to the Austrians, grossly erroneous implications for the interpretation of economic phenomena. They argued that, if rightly qualified, the value of a good is in fact determined by the importance of its usefulness. In Weiser's words, or rather in our translation of Wieser's words,

> For most goods a distinction must be made between the magnitude of their value [ihres Werthes] and the magnitude of their use [ihres Nutzens]. Only for those goods that are actually employed to bring about the marginal use-performance will the good's own use be the source of its

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[1]Böhm-Bawerk (1959 I, 135-6 and 1909 A, 234). Note however that Lindgren (1976) puts a completely different interpretation on Smith's meaning.

> value and will there be agreement between the two judgments. For any other good a different use, which must nevertheless be a use characteristic of that sort of good, will be the basis for the estimate of its value, which accordingly will differ from the estimate of the use-effect it actually brings about; for such a good, the actual use is higher than the dependent use and therefore higher than its value.[1]

To illustrate the paradox and its resolution, let us look again at the individual of Tables 4 through 9. Suppose he has 3X and 2Y. The total use of X (ace, twenty-first position) is more important that the total use of Y (bd, tenth position). Furthermore, the highest use of X (a, ninth position) is more important than the highest use of Y (b, fifth position). Yet the subjective value, the utility, of a unit of X, even of the very unit that will satisfy want a, is lower (first position) than the utility of a unit of Y (second position). The Austrians' answer to this paradox is that the value of a unit of a good is determined, not by the total use of goods of that sort, and not necessarily even by its own use, but rather by its marginal use. Goods do not obtain value from the labor they "contain." Rather, labor derives its value from use-value of the goods it is used to produce.

RIVAL GOODS

In the example given above, it was assumed that the two goods were used

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[1] "Es muss bei den meisten Gütern ein Underschied zwischen der Größe ihres Werthes und der Größe ihres Nutzens entstehen. Nur bei denjenigen Gütern, welche gerade zur Herbeiführungen der Grenznutzleistungen verwendet werden, wird der eigene Nutzen die Grundlage des Werthes und besteht Uebereinstimmung beider Beurtheilungen. Bei allen übrigen wird ein fremder Nutzen, der aber allerdings der Nutzsphäre derselben Gütergattung angehören muss, zur Grundlage des Werthurtheiles genommen, welches daher von dem Urtheile über den herbeigeführten Nutzeffect abweichen wird; bei allen diesen ist der herbeigeführten Nutzen größer, als der abhängig gefülte, and mithin größer, als der Werth." (Wieser 1884, 128)

independently of one another. However, the law of the marginal use is also applicable if the goods must be used together to satisfy some wants or if they can be utilized in place of one another.

Suppose there are two goods, X and Y, and that a unit of either may be used to satisfy some want or wants, say c. Let there be other wants, a and e, which a unit of X can satisfy, and still others, b and d, which may be satisfied by a unit of Y. X and Y are then rivals, at least with respect to want c.

Let an individual preference-rank the subsets of W = {a, b, c, d, e} as shown in Table 10. For various combinations of X and Y, Table 11 shows the wants X will be used to satisfy, the wants Y will be used to satisfy, the collective use of X and Y, and the corresponding total utility. Want c is sometimes satisfied by X and sometimes by Y. Table 12 shows the implied preference ordering on the commodity space. (Note that this ordering is now only semi-linear; it sometimes happens that two different commodity bundles have the same utility.) Tables 13-15 and Figure 3 are constructed in the same manner as Tables 7-9 and Figure 2.

As in the case of independent goods, the marginal utility of each good decreases with its own quantity. However, three phenomena in Tables 14 and 15 are different from the case of independent goods and are worthy of note. First, the marginal utility of one good is not independent of the quantity of the other good available. For example, the marginal utility of the second unit of X falls from the third to the second to the first position as Y increases from 1 to 3. Similarly, the marginal utility of the third unit of Y falls from second to first to zeroeth as X increases from 1 to 3. Thus,

RIVAL GOODS

Hypothetical
Preference Ordering

Set of Wants	Ordinal Utility
abcde	31st
abcd	30th
abce	29th
abde	28th
abc	27th
abd	26th
acde	25th
bcde	24th
acd	23rd
abe	22nd
bcd	21st
ab	20th
ace	19th
ade	18th
bce	17th
bde	16th
ac	15th
ad	14th
bc	13th
bd	12th
cde	11th
ae	10th
cd	9th
be	8th
a	7th
b	6th
ce	5th
de	4th
c	3rd
d	2nd
e	1st
∅	0th

Table 10

Use of X, Use of Y, Total Use and Total Utility

Units of		Use of		Total Use	Total Utility
X	Y	X	Y		
0	0	∅	∅	∅	0th
1	0	a	∅	a	7th
2	0	ac	∅	ac	15th
3	0	ace	∅	ace	19th
0	1	∅	b	b	6th
1	1	a	b	ab	20th
2	1	ac	b	abc	27th
3	1	ace	b	abce	29th
0	2	∅	bc	bc	13th
1	2	a	bc	abc	27th
2	2	ac	bd	abcd	30th
3	2	ace	bd	abcde	31st
0	3	∅	bcd	bcd	21st
1	3	a	bcd	abcd	30th
2	3	ae	bcd	abcde	31th

Table 11

Implied Preference Ordering on
Combinations of X and Y

X	Y	Utility
2	3	31st
3	2	31st
1	3	30th
2	2	30th
3	1	29th
1	2	27th
2	1	27th
0	3	21st
1	1	20th
3	0	19th
2	0	15th
0	2	13th
1	0	7th
0	1	6th
0	0	0th

Table 12

RIVAL GOODS

Total Use and Total Utility of Combinations of X and Y

Units of Y \ Units of X	0	1	2	3	4
4	bcd 21st	abcd 30th	abcde 31st	abcde 31st	abcde 31st
3	bcd 21st	abcd 30th	abcde 31st	abcde 31st	abcde 31st
2	bc 13th	abc 27th	abcd 30th	abcde 31st	abcde 31st
1	b 6th	ab 20th	abc 27th	abce 29th	abce 29th
0	Ø 0th	a 7th	ac 15th	ace 19th	ace 19th

Table 13

Indifference Curves

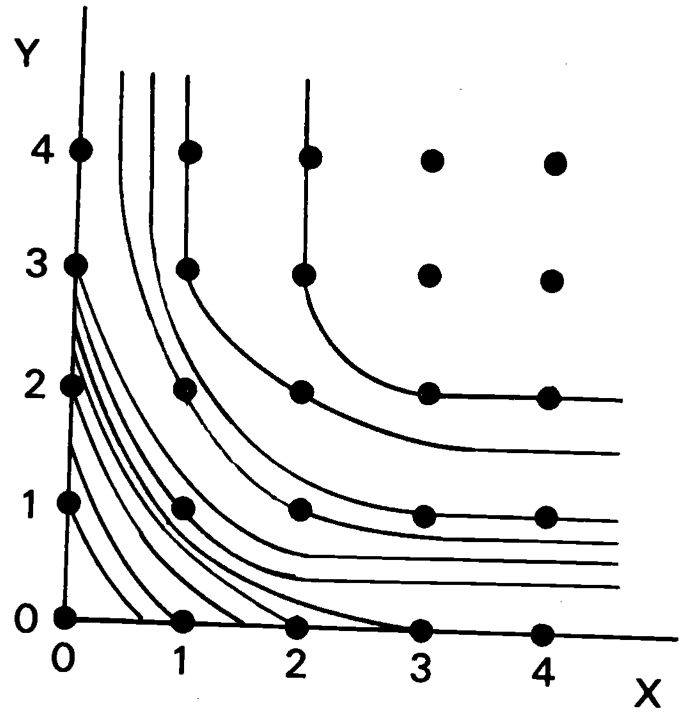

Figure 3

Marginal Use and Marginal Utility of X

Units of Y \ Unit of X	1st	2nd	3rd	4th
4	a 7th	e 1st	Ø 0th	Ø 0th
3	a 7th	e 1st	Ø 0th	Ø 0th
2	a 7th	d 2nd	e 1st	Ø 0th
1	a 7th	c 3rd	e 1st	Ø 0th
0	a 7th	c 3rd	e 1st	Ø 0th

Table 14

Marginal Use and Marginal Utility of Y

Unit of Y \ Units of X	0	1	2	3	4
4th	Ø 0th	Ø 0th	Ø 0th	Ø 0th	Ø 0th
3rd	d 2nd	d 2nd	e 1st	Ø 0th	Ø 0th
2nd	c 3rd	c 3rd	d 2nd	d 2nd	d 2nd
1st	b 6th	b 6th	b 6th	b 6th	b 6th

Table 15

when goods are rivals in consumption, the marginal utility of one good tends to fall off as the quantity of the other increases.

Second, it sometimes happens that the marginal use of one good is the satisfaction of a want which that good cannot itself satisfy. For instance, when the individual has (1X, 2Y), the marginal use of one additional X is d, a want that can only be satisfied by Y. When he has (2X, 2Y), the marginal use of one more Y is e, a want that can only be satisfied by X. Thus Wieser's assertion above (p. 18), that the marginal use must be a use characteristic of the good in question, is not always true.

And third, when goods are rivals, it often happens that their marginal uses coincide. Thus, when (1X, 1Y) is available, the outer marginal use of both X and Y is the satisfaction of want e. When (2X, 2Y) is available, the outer marginal use of both X and Y is the satisfaction of want e.

COMPLEMENTARY GOODS

Böhm-Bawerk attributes the origin of the expression "complementary goods" to Menger:

> It frequently occurs that the cooperation of several goods is required for the achievement of an economic use, in such a manner that if one of the goods was lacking, the use could be achieved not at all or only incompletely. We designate goods whose useful services thus supplement one another, "complementary goods" [*komplementäre Güter*], following the precedent of Menger. As examples of complementary goods we have paper, pen and ink, needle and thread, horse and wagon, bow and arrow, the two shoes or gloves that make up a pair, and the like. (1909 A, 276, my trans.)

Although the Austrians frequently gave examples of complementary production goods, which combine to produce a material product which in turn satisfies wants, it is not hard to think of complementary goods which satisfy wants directly, such as an hour or two of one's leisure time and a ticket to a play. We will therefore illustrate with an example of complementarity in consumption.

Let there be two goods, X and Y, and suppose that one unit of each is required to satisfy some want, b. Let both goods have alternative uses in which they are not complements: a and d for X, and c and e for Y. Table 16 shows an additive ranking of W* which might reflect an individual's preferences. For various combinations of X and Y, Table 17 shows the best use that can be made of the combination if the satisfaction of b is excluded, the best use if b is included, and the utilities or subjective values of both these uses. The best overall use is the better of these two and is shown with its utility, the derived utility of the combination, in the last two columns.

When we try to derive the marginal uses of X and Y from Table 18, we encounter a new difficulty. For instance when our individual has (2X, 1Y) we find that there is not a simple want dependent on the possession of another unit of X. Rather, an additional unit of X enables him to replace want c with the higher rated want b. We represent this sort of marginal use by the ordered pair (b, c), where the first entry (b) represents the additional want satisfied and the second entry (c) represents the want (if any) whose satisfaction is omitted. Clearly in this case, the unit of X will have higher utility to the individual, the more important b is, and the less important c is. Menger carelessly describes such a utility as the difference between the utility of b and the utility of c, without telling us what we are to make of this concept:

> "...the value of a concrete quantity of a good of higher order [or of a good which is complementary in consumption in our example] is equal to the difference in importance between the satisfactions that can be attained when we have command of the given quantity of

COMPLEMENTARY GOODS

Hypothetical
·eference Ordering

Set of Wants	Ordinal Utility
abcde	31th
abcd	30th
abce	29th
abde	28th
abc	27th
abd	26th
acde	25th
acd	24th
abe	23rd
bcde	22nd
ab	21st
ace	20th
ade	19th
bcd	18th
ac	17th
bce	16th
ad	15th
bde	14th
ae	13th
bc	12th
bd	11th
cde	10th
a	9th
cd	8th
be	7th
b	6th
ce	5th
de	4th
c	3rd
d	2nd
e	1st
∅	0th

Table 16

Best Use of X and Y with and without Want b,
and Best Overall Use

Units of		Best Use if b--				Best Use	Utility of Best Use
		Excluded		Included			
X	Y	Use	Utility				
0	0	∅	0th	---	---	∅	0th
1	0	a	9th	---	---	a	9th
2	0	ad	15th	---	---	ad	15th
3	0	ad	15th	---	---	ad	15th
0	1	c	3rd	---	---	c	3rd
1	1	ac	17th	b	6th	ac	17th
2	1	acd	24th	ab	21st	acd	24th
3	1	acd	24th	abd	26th	abd	26th
0	2	ce	5th	---	---	ce	5th
1	2	ace	20th	bc	12th	ace	20th
2	2	acde	25th	abc	27th	abc	27th
3	2	acde	25th	abcd	30th	abcd	30th
0	3	ce	5th	---	---	ce	5th
1	3	ace	20th	bce	16th	ace	20th
2	3	acde	25th	abce	29th	abce	29th
3	3	acde	25th	abcde	31st	abcde	31st

Table 17

COMPLEMENTARY GOODS

Total Use and Total Utility of Combinations of X and Y

Units of Y					
4	ce 5th	ace 20th	abce 29th	abcde 31st	abcde 31st
3	ce 5th	ace 20th	abce 29th	abcde 31st	abcde 31st
2	ce 5th	ace 20th	abc 27th	abcd 30th	abcd 30th
1	c 3rd	ac 17th	acd 24th	abd 26th	abd 26th
0	∅ 0th	a 9th	ad 15th	ad 15th	ad 15th
Units of X	0	1	2	3	4

Table 18

Indifference Curves

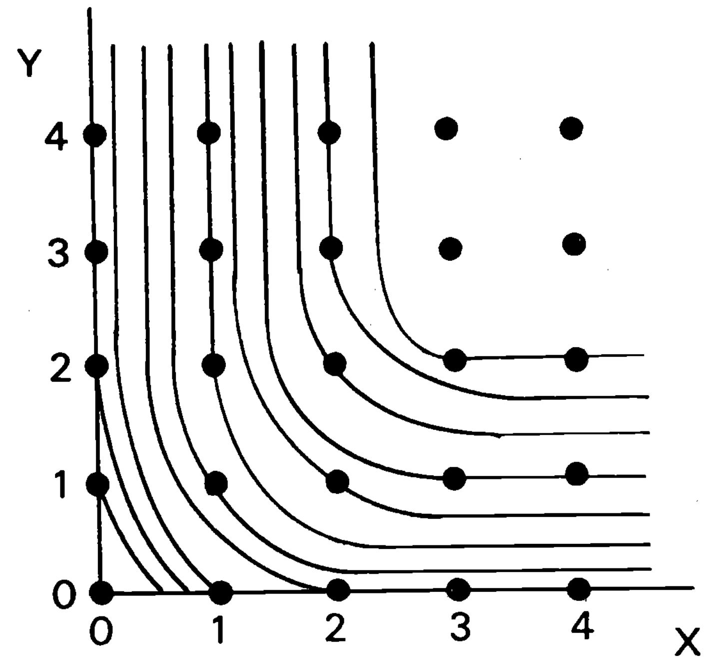

Figure 4

Marginal Use and Marginal Utility of 1 Unit of X

Units of Y				
4	a 9th	b 6th	d 2nd	∅ 0th
3	a 9th	b 6th	d 2nd	∅ 0th
2	a 9th	(b,e) 3.5th	d 2nd	∅ 0th
1	a 9th	d 2nd	(b,c) 1.5th	∅ 0th
0	a 9th	d 2nd	∅ 0th	∅ 0th
Unit of X	1st	2nd	3rd	4th

Table 19

Marginal Use and Marginal Utility of 1 Unit of Y

Unit of Y					
4th	∅ 0th	∅ 0th	∅ 0th	∅ 0th	∅ 0th
3rd	∅ 0th	∅ 0th	e 1st	e 1st	e 1st
2nd	e 1st	e 1st	(b,d) 1.7th	c 3rd	c 3rd
1st	c 3rd	c 3rd	c 3rd	b 6th	b 6th
Units of X	0	1	2	3	4

Table 20

> the good of higher order whose value we wish to determine and the satisfactions that would be attained if we did not have this quantity at our command." (1950, 165)

However, by extending the concept of additivity, we are able to place such "differences" accurately enough for our needs without resorting to cardinality. From Table 16, we have b $\prec$ cd. If we "delete" c from both sides of this relation, we obtain (b,c) $\prec$ d (second utility level). Similarly, since b $\succ$ ce, we must have (b,c) $\succ$ e (first utility level). Therefore (b,c) is intermediate between the first and second positions. In Table 19 we have indicated this by arbitrarily giving it the "1.5th" position. As with a Dewey decimal classification, this is not intended to mean that it is half way between the first and second positions, but merely that it is somewhere in between them.

The marginal use of the second unit of X, given 2Y, is (b,e). This use is not so easy to place on the scale of wants. By comparison to c it can be shown to be higher than the third position. To find an upper bound is more difficult. Because ae $\succ$ bc, we have (a,c) $\succ$ (b,e). Furthermore, cde $\succ$ a implies de $\succ$ (a,c). Therefore, (b,e) $\prec$ de (fourth position). Since (b,e) lies between the third and fourth position, we assign it the "3.5th" position of Table 16.

The marginal utility of the second unit of Y, given 2X, is the importance of replacing the satisfaction of d with that of b, or of (b,d) in our notation. This use presumably has the same relation to (b,c) that bc has to bd. Therefore (b,d) $\succ$ (b,c) (1.5th position). The reader may confirm that (b,d) $\prec$ (c,e) $\prec$ d (second position). We therefore assign (b,d) the "1.7th" utility level.[1]

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[1]The problems that arise when we introduce complementarity indicate that W* does not contain all of the "uses" of interest. We must consider a more complicated set, say W**, the set of all ordered pairs of disjoint subsets of W. Given (P, Q) in W**, P is to be interpreted as the additional wants

(continued)

It is still true that the marginal utility of either good always falls as its own quantity increases. When we have 2 units of Y, the marginal utility of a unit of X falls from 9th to 3.5th to 2nd, and finally to 0th. When we have 2X, the marginal utility of Y falls from 3rd to 1.7th to 1st, and then to 0th.

Furthermore, we note that with complements, we get exactly the opposite of what happens with rival goods: as the quantity of one good increases, the marginal utility of the other tends to increase, instead of decrease as was the case with competitors. For instance, when there are 2X available, the marginal utility of another unit of X rises from 0th to 1.5th to 2nd as Y increases from 0 to 2.

NET RIVALS AND COMPLEMENTS: THE <u>ALEP</u> CRITERION

Rivalness and complementarity are not mutually exclusive concepts. Two goods may be rivals with respect to one want and complements with respect to another. Or using them as complements in one proportion may be rival with using them in another proportion, as in the case of production under variable proportions.

Since when rivalness is the only interaction, the marginal utility of one good falls as the quantity of the other increases, and since the

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1 (cont'd) that are to be satisfied, and Q the wants whose satisfaction is to be omitted. W** then contains all marginal uses, in the broad sense that we need for complementarity (and jointness). It appears that the linear ordering on W*, together with our extended application of additivity, defines a partial ordering on W** which is sufficient to say which of two goods will be valued more highly in any conceivable situation, to prove diminishing marginal utility, and to establish the ALEP criterion, to be discussed below. See McCulloch and Smith (1975) for a proof of the law of the marginal use involving this extended conduct.

the opposite is true when complementarity is the only interaction, we propose that X and Y be designated net rivals in a certain region of the X-Y plane if in that region the marginal utility of the one decreases as the quantity of the other increases holding the quantity of any other goods constant, net complements if the opposite is true, and on net independent if the marginal utility of one is independent of the quantity available of the other. (It can be shown that these concepts are well defined, that is, that X will have qualitatively the same effect on the marginal utility of Y as Y has on the marginal utility of X, even in the Austrian framework of ordinal marginal utility.) This is actually the definition of rival and complementary goods proposed, though in terms of the cross partial derivatives of a smooth cardinal utility function, by Auspitz and Lieben, Edgeworth, and Pareto.[1] We therefore designate it the "ALEP criterion." Note, however, that while these authors used the ALEP criterion as the definition of complements and rivals, the approach of the marginal use theory is to define these concepts in terms of how the goods are used, and then to demonstrate a relationship to the ALEP criterion.

Hicks claims that the "Edgeworth-Pareto definition sins against Pareto's own principle of the immeasurability of utility. If utility is not a quantity,

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[1]Auspitz and Lieben (1889, 482), Edgeworth (1897/1925, 117 n.1), Pareto (1906/1927, 268-9). It is not actually clear that the functions Auspitz and Lieben and Edgeworth differentiate are really what we would call utility functions. For instance, Edgeworth equates his first derivative to a price. Nevertheless the basic idea is definitely there. While Auspitz and Lieben were Austrians by nationality, they are not considered part of the Austrian school. Their approach was closer to that of Walras. The ALEP criterion has recently been rediscovered by Samuelson (1974, 1264-5).

but only an index of the consumer's scale of preferences, his definition of complementary and competitive goods will differ according to the arbitrary measure of utility which is adopted," (1946, 43) However, we have shown that in the Austrian concept of ordinal marginal utility, the criterion does indeed have a precise meaning, so Hick's objection is invalid.

Hicks instead attempted to define the complementarity of X and Y in terms of Allen's "partial" elasticity of substitution σ_{xy} , which is related to the curvature of the indifference surfaces. (Allen 1962/38, 504-5). If it is positive he calls the two goods "substitutes" and if it is negative he calls the two goods "complements". However, it has never been demonstrated that the sign of the substitution elasticity has anything to do with whether X and Y are used in combination with one another or in place of one another. It is about time this question be investigated.

We have demonstrated above that the ALEP criterion <u>is</u> related to whether the goods are rivals or complements. One implication of the ALEP criterion for the structure of commodity preferences, an implication that was not recognized by Hicks, is that if there is a third good, Z, which is completely independent of the first two goods, then the marginal rate of substitution between X and Z will change in one direction as Y increases holding X and Z constant if X and Y are net complements, and will change in the opposite direction as Y increases if X and Y are net rivals. If goods and wants are finely divisible so that the Allen elasticities exist and are well defined, this implies that

$\left.\frac{E(P_x/P_z)}{EY}\right|_{X,Z}$ and $\left.\frac{E(P_y/P_z)}{EX}\right|_{Y,Z}$ will both be positive, negative or zero,

depending on whether X and Y are net complements, rivals, or independents, where E represents the logarithmic differentiation operator:

$$EX = d \log X = dX/X, \text{ etc.,} \tag{1}$$

and P_x, P_y, and P_z represent the prices facing a competitive buyer.

It can be shown that

$$\left.\frac{E(P_x/P_z)}{EY}\right|_{X,Z} = \frac{-k_y \eta_x \sigma_{yy} - (1-k_y \eta_y)\sigma_{xy}}{k_x(\sigma_{xx}\sigma_{yy}-\sigma_{xy}^2)} \tag{2}$$

and

$$\left.\frac{E(P_y/P_z)}{EX}\right|_{Y,Z} = \frac{-k_x \eta_y \sigma_{xx} - (1-k_x \eta_x)\sigma_{xy}}{k_y(\sigma_{xx}\sigma_{yy}-\sigma_{xy}^2)} , \tag{3}$$

where the k's and η's are respectively the budget shares and income elasticities of demand for the three goods.[1] Setting (2) and (3) equal to zero as in the case of independent goods and employing the familiar conditions

$$\sum_{j=1}^{3} k_j \sigma_{ij} = 0 \tag{4}$$

implies that

$$\eta_x \sigma_{yz} = \eta_z \sigma_{xy} = \eta_y \sigma_{xz} . \tag{5}$$

Since independent goods will always have positive income elasticities, equation (5) implies that all three cross substitution elasticities will have the same sign. Since at most one can be negative, it follows that they must all be positive. Therefore if X and Y are independent (and the third good Z is also

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[1]Expressions (2) and (3) do not necessarily have the same sign unless Z is net independent of X and Y. Note, however, that as k_y and k_x go to zero, (2) and (3) respectively take on the sign of $-\sigma_{xy}$. Cp. Hicks (1946, 44).

independent of both X and Y), σ_{xy} will be positive.

It follows that there will be some small amount of ALEP net compliment-arity between X and Y for which σ_{xy} remains positive. Therefore σ_{xy} being negative is not a necessary condition for complementarity.

It is, however, a sufficient condition under not unreasonably restrictive conditions. If σ_{xy} is negative, the numerators of both (2) and (3) will be positive except in unusual cases when some of the income elasticities are negative. It can be shown that the <u>direct</u> elasticity of substitution between X and Y, s_{xy} (which holds the quantity of Z constant), is given by

$$s_{xy} = \frac{k_x k_y (k_x + k_y)(\sigma_{xx}\sigma_{yy} - \sigma_{xy}^2)}{-k_z^2 \sigma_{zz}} \quad (6)$$

Since s_{xy} must be positive and σ_{zz} must be negative, it follows that the denominators of (2) and (3) must also be positive. Therefore if Z is on net independent of both X and Y, negativity of σ_{xy} does indeed imply that X and Y are net complements in the ALEP sense.[1]

Since negativity of σ_{xy} is only a sufficient condition for complementarity, we suggest that the Hicks-Allen definition of the unqualified term "complements" be abandoned, and that instead X and Y be referred to as "positive substitutes" if σ_{xy} is positive, and as "negative substitutes" or "extreme complements" if σ_{xy} is negative. Furthermore, "substitution" in the Hicks-Allen sense should not be confused with rivalness (or "competitivity").

ANIMISTIC ECONOMICS

The dominant position of the Hicks-Allen version of utility theory reflects the popularity of positivist or behavioristic methodology in the

[1]We cannot deduce the full ALEP relation among X, Y, and Z from demand parameters alone. But, what is more useful, we <u>can</u> make inferences about demand relationships from what we know about how X, Y, and Z are used.

social sciences. Positivism attempts to transfer some of the prestige of the natural sciences to the social sciences by recognizing only the methods of the natural sciences as being appropriate for the social sciences.

Natural science made its great forward strides to the extent that it was able to shake off animistic interpretations of nature and substitute empiricism. Animism is the superstitious attribution of human-like motives to natural phenomena, for instance, interpreting earthquakes in terms of punishment by the earth spirits. Science recognizes that natural phenomena have no such meaning, and that all the scientist can do is record observable phenomena and try to generalize to empirically testable propositions. Positivistic economics therefore restricts itself to statements about objectively observable behavior, such as that reflected in the consumer's preference structures, and tries to make empirically testable statements about this behavior.

The Austrian theory of the marginal use boldly flies in the face of positivism by attributing human motives to human beings. It would be a waste of time for a natural scientist to speculate about what he would do in such-and-such a situation if he were an electron. However, it might be very fruitful for an economist to make inferences about the demand for water, gasoline, automobiles and lawns by speculating about whether a consumer will fuel his car with water and sprinkle his lawn with gasoline, or vice-versa. The fact that the economist is one of the subjects of his own study means that he can exploit empathy or "understanding" in Dilthey's sense, a tool that is not available to the natural scientist.[1]

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[1]See Lachmann (1966) and Dilthey (1962).

The distinction between the behavioristic Hicks-Allen approach to consumer preference theory and the empathic Austrian approach may be likened to two men observing a stream. The behaviorist observes a bulge in the surface of the stream (or indifference surface) and declares that nothing more can be said than that the stream bulges in such and such a manner. To him, it is senseless, even unscientific, to inquire what is going on beneath the surface. The Austrian, on the other hand, is not afraid to observe that there is a rock beneath the surface that is deflecting the current. The Austrian looks beneath the surface to investigate and understand the meaning of the superficial phenomenon.[1]

The sterility of the orthodox behavioristic theory of consumer choice has led many economists working independently of the Austrian school to move in a similar direction. Lancaster (1966, 1971) investigates how goods are used to provide "characteristics," similar to Austrian wants, that are the ultimate objects of consumer preference. Becker and his students (e.g. Michael and Becker 1973) have developed a similar model, in which market goods are combined in a "household production function" to create hypothetical "commodities" which are the ultimate preference objects.

However, neither of these approaches insists, as the Austrians do, that preferences on market goods can be broken down in terms of ultimate independent

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[1]Many Austrian school economists have unfortunately overreacted to the behavioristic preoccupation with indifference curves, by denouncing these curves as inadmissible. Kauder (1965, 108) traces this "tabboo" on anything smacking of mathematics back to Hans Mayer. This tradition is in the position of a man who, having noted that a submerged rock is deflecting the current, denies that it is meaningful to say that the stream even has a surface (after all, every point is either in the water or in the air), let alone to discuss the effect of the deflection on the shape of the surface.

preference-objects, nor do they develop the ALEP criterion and its implications, or recognize the ordinal character of marginal utility. Nevertheless, these approaches do belong with the Austrians in the camp of animistic economics.

THE GOSSEN-JEVONS LAW OF DIMINISHING ENJOYMENT VERSUS THE AUSTRIAN LAW OF DIMINISHING MARGINAL UTILITY

Menger gives the following example of the various wants among which an individual might like to allocate a given supply of a good:

> An isolated farmer, after a rich harvest, has more that two hundred bushels of wheat at his disposal. A portion of this secures him the maintenance of his own and his family's lives until the next harvest, and another portion the preservation of health; a third portion assures him seed-grain for the next seeding; a fourth portion may be employed for the production of beer, whiskey, and other luxuries; and a fifth portion may be used for the fattening of his cattle. Several remaining bushels, which he cannot use further for these more important satisfactions, he allots to the feeding of pets in order to make the balance of his grain in some way useful. (1950,129-30).

Only two of these six satisfactions are brought about by successive doses of the commodity in the same activity, i.e., eating bread made from the wheat. Gossen, Jevons, and the other traditional utility theorists had this sort of "dinner table" situation in mind when they stated their laws of diminishing enjoyment. Supposedly the successive doses provide successively less enjoyment, which in turn is to be "measured" by the utility function. Thus, the Gossen-Jevons law of diminishing marginal utility might be called a psycho-physiological law of satiation.

The Austrian law, in the other hand, arises logically from situations in which the individual can choose from among several completely different consumption activities. The Austrian law could be characterized as a mental law of the logic of choice in the face of scarcity. The Austrians do introduce instances in which two different concrete wants are satisfied by successive doses of a good in the same activity, but they do it prinicipally to proliferate the number and variety of importance of the wants they deal with. In fact, these "dinner table" examples potentially provide some embarrassment to the Austrian theory, which always supposes that the consumer is free to choose to satisfy any wants he pleases. In Menger's example, however, there is no way his farmer could choose merely to provide for health and decline to provide for life by consuming just one portion. Fortunately, as a general rule the successive doses <u>are</u> less important to the individual, so that the Austrian theory does not break down.

Von Mises has pointed out this difference between the Gossen-Jevons and the Austrian laws of diminishing marginal utility:

> The [Austrian] law of marginal utility and decreasing marginal value is independent of Gossen's law of the saturation of wants (first law of Gossen). In treating marginal utility we deal neither with sensuous enjoyment nor with saturation and satiety.... Our statement is formal and aprioristic and does not depend on any experience. (1963, 234)

Gaétan Pirou gives a related appraisal of the general approach of the Austrian school:

> Menger's system is sometimes called "psychological economics." On reflection, it appears to me that the term psychological may not be exact.... Menger's work is more an "economic logic" than a "psychological economics." (1945, 64, my trans.)

This unique approach sets the Austrian school methodologically apart from the Jevonian and Lausanne approaches to economics.

JOINT SATISFACTION

Yet another type of technological interrelationship between goods and wants is that of <u>jointness</u>, which arises when one unit of a good can satisfy more than one want simultaneously. This relationship is important when one of the wants can also be satisfied by a second good. The relation between jointness and inferiority (in the sense of having a negative income elasticity) deserves careful analysis. Can inferiority (in some qualified sense applicable to discretely divisible goods) arise in the absence of joint want satisfaction? It would appear not, although I have not been able to demonstrate it.

The assumption of additivity is perfectly natural until it is made explicit. Then it becomes apparent what a restrictive assumption it is. Are we really justified in assuming that the consumer's underlying wants are not interrelated? Lancaster allows his underlying "characteristics," which correspond roughly to Austrian wants, to be highly interrelated. However, I think that if we reflect on the types of interrelationship that are likely to occur, they can be reduced to purely technological interrelationships affecting the satisfaction of unrelated ultimate wants. It would appear that the categories of rivalness, complementarity, and jointness are sufficient to explain any conceivable technological interrelationship.

CONVEXITY OF THE INDIFFERENCE CURVES

Notice that in Figures 2, 3, and 4 we were always able to draw indifference curves that were convex to the origin and were never forced to draw a concave portion. Menger gives an example in which he shows that although a little exchange may be advantageous at a given price ratio, as more and more is traded,

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[1]In this case, the first and second goods would correspond to Menger's goods of "superior" (höher) and "inferior" (minderer or niederer) quality. (1950, 144-5 and 1934, 118-9).

a peak in utility may be reached after which the individual is increasingly worse off. Eventually, he may be worse off than when he started. This phenomenon, which Menger (1950, 187) describes under the heading of the "limits of exchange," is closely related to convexity.

I conjecture that it can be proven that convex indifference curves may always be found in the Austrian system of utility, subject only to the reservations given in the next section. This has already been proven in the case two independent goods by Jeffrey Smith (McCulloch and Smith, 1975). This issue is of great interest for economic theory. Hicks is not satisfied that he has given adequate justification for his bald assumption of convexity, or what is the same thing, of diminishing marginal rate of substitution:

> Since we know from experience that some points of possible equilibrium do exist on the indifference maps of nearly every one..., it follows that the principle of diminishing marginal rate of substitution must sometimes be true.
>
> However, for us to make progress in economics, it is not enough for us to know that the principle should be true sometimes; we require a more general validity than that. (1946, 22)

Fortunately, the Austrian utility theory leads to a more satisfying development of this important proposition.

Convex indifference curves were first developed in economics as an implication of diminishing marginal utility, provided the goods were on net independent or were net complements. Note, however, that they work out to have the usual curvature even in our example of rival goods.

INSTANCES OF INCREASING MARGINAL UTILITY

Suppose that one unit of X will satisfy want a, but that it takes no less than two units to satisfy want b; one unit cannot "half-way" satisfy b.

Suppose that b is "much greater" than a. If an individual has one unit of X he will use it to satisfy a. If he has two, he will satisfy b. The marginal utility of the first unit is then the importance of a, while the marginal utility of the second is (b,a), the importance of replacing a with b. If b is sufficiently important and a sufficiently unimportant, the marginal utility of the second unit may actually be higher than that of the first unit.[1] Von Mises has recognized that circumstances like these may arise when several units of a good must be used together to provide a given effect, and that they provide exceptions to the general law of diminishing marginal utility. However, as Mises points out, "All this is in perfect agreement with the rightly formulated law of marginal utility..."[2]

If more than one unit of a good must be combined to produce a given effect, either by itself or in a complementary package with another good, we would similarly expect to find instances where we are forced to draw concave segments of our indifference curves. Therefore any proof of convexity arising from the Austrian theory of the marginal use must be qualified to hold only if for each good there is a single quantity in which it enters into the consumption technology.

Nevertheless, we would still expect dinimishing marginal utility and convexity to hold for a given individual as a general rule, if not in every instance. Furthermore, when we look at masses of individuals, we might find that any "lumpiness" in the consumption behavior of any individual becomes

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[1]We may say for certain that the second unit has higher marginal utility than the first if there is a third want c and $b \succ ac \succ c \succ a \succ \emptyset$. By deleting a from both sides of $b \succ ac$, we get $(b,a) \succ c$, whence $(b,a) \succ a$.

[2]Mises (1963, 125). My conjecture is that by "marginal utility" he here has in mind "Grenznutzen," which is less confusingly read as "marginal use."

insignificant in examining the behavior of the group as a whole. Consequently, when describing the reaction of large numbers of individuals to price changes, income transfers, etc., we might expect them to behave, as a general rule, as if for each one decreasing marginal utility and convexity held, even though this may not be exactly true in each individual case.

IS IT REALLY ORDINAL?

It may have occurred to the reader that the easiest way to generate an additive ordering on a set of subsets is to assign a real number, say $m(a_i)$, to each element a_i of W, i = 1, 2, ... n. For each subset P of W define $m(P) = \sum_{a_i \epsilon P} m(a_i)$. Then for each pair P and Q of subsets of W, let $P \succ Q$ whenever $m(P) > m(Q)$. We will call an ordering generated in this manner "essentially cardinal." Clearly such an ordering is additive, for if $P \succ Q$, then $m(P) = m(P-Q)+m(P \cap Q) > m(Q) = m(Q-P) + m(P \cap Q)$, whence $m(P-Q) > m(Q-P)$, so that $P-Q \succ Q-P$. Similarly, $P-Q \succ Q-P$ implies $P \succ Q$, so that <u>any essentially cardinal ordering is also additive</u>.

For example, Tables 21 and 22 show how the additive orderings of Tables 1 and 10 can arise from such a cardinal measure. In Table 21, a, b, c, and d respectively have been given measures 11, 8, 6, and 4. In Table 22, the five wants a, b, c, d, and e have been given measures 9.5, 8.5, 5.7, 5.2, and 2.2. Such numbers will in general not be unique if W is finite.

It seems plausible that <u>all</u> additive orderings, at least on finite sets and sufficiently reasonable infinite sets,must be essentially cardinal. In fact, in 1949 the Italian statistician B. de Finetti conjectured that this is true. If so, it would seem to be mere quibbling to retain an ordinal

A Cardinal Measure
for the Additive Ordering
of Table 1

Set of Wants	"Cardinal Utility"
abcd	29
abc	25
abd	23
acd	21
ab	19
bcd	18
ac	17
ad	15
bc	14
bd	12
a	11
cd	10
b	8
c	6
d	4
∅	0

Table 21

A Cardinal Measure
for the Additive Ordering
of Table 10

Set of Wants	"Cardinal Utility"
abcde	31.1
abcd	28.9
abce	25.9
abde	25.4
abc	23.7
abd	23.2
acde	22.6
bcde	21.6
acd	20.4
abe	20.2
bcd	19.4
ab	18.0
ace	17.4
ade	16.9
bce	16.4
bde	15.9
ac	15.2
ad	14.7
bc	14.2
bd	13.7
cde	13.1
ae	11.7
cd	10.9
be	10.7
a	9.5
b	8.5
ce	7.9
de	7.4
c	5.7
d	5.2
e	2.2
∅	0.0

Table 22

approach once the assumption of additivity is made, for we could then derive all properties of the ordering from a few numbers which we can manipulate in familiar ways. In any case, it could then be argued that the Austrian utility theory is only superficially ordinal, that their assumptions amount to the same thing as cardinality.

For several years de Finetti's conjecture remained unsolved. In 1959, Kraft, Pratt and Seidenberg finally proved it false by publishing a counter-example. Take for instance the additive ordering of Table 4. It contains the four relations be $\succ$ cd, bc $\succ$ ae, ce $\succ$ b, and ad $\succ$ bce. If the ordering arose from a measure m( ), we would have $m(b) + m(e) > m(c) + m(d)$, $m(b) + m(c) > m(a) + m(e)$, $m(c) + m(e) > m(b)$, and $m(a) + m(d) > m(b) + m(c) + m(e)$. Adding these four numerical inequalities together we get that $m(a) + 2m(b) + 2m(c) + m(d) + 2m(e)$ must be strictly greater than itself, a contradiction. Therefore the additive ordering of Table 4 cannot be essentially cardinal. Similarly, the additive ordering of Table 16 contains the four relations b $\succ$ ce, cd $\succ$ be, ae $\succ$ bc, and bce $\succ$ ad, which would also imply a contradiction if the ordering were essentially cardinal.[1]

Since additivity does not imply measurability, it follows that the Austrian theory of the marginal use is intrinsically ordinal. It admits of situations where no cardinal utility function is possible.

The Austrian literature is full of contradictory statements as to whether utility is expressible cardinally. Wieser tells us,

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[1]Kraft *et al*. (1959) attribute this conjecture to B. de Finetti (1951, 1-10). The ordering of Table 16 is due to Kraft *et al.* That of Table 4, has, to the best of my knowledge, never been published. See Krantz *et al*. (1971, chapt. 5)for theorems relating to additivity. In McCulloch and Smith (1975) it is demonstrated that if W has 5 elements, there are at least 1920 different intrinsically ordinal additive orderings on W*.

> Strictly speaking, we cannot measure [messen] different strength degrees of intensity of interest [Intensitätsgrade des Interesses] against one another or reduce them to a common unit. Nor can we estimate how many times stronger one degree is than another. Since the concepts of unit, of multiple, and of number are not applicable to the degrees of strength of inner phenomena, we may say that we can ascribe to them only intensive, and not extensive or numerical, magnitudes. (1884, 180-1, my trans.)

Yet a few pages later (196), we catch him multiplying a marginal utility by a integer!

When Böhm-Bawerk assigns numbers to the importance of different wants, he is careful to emphasize that they are only an ordinal index:

> To correct misunderstandings which have arisen despite my precautions, I should like to state explicitly that the descending scale represented by the arabic numerals 10 to 1 in this table do not symbolize anything beyond the fact that each concrete want designated by a given number has a lower intensity or importance than any want or wants designated by a higher number or numbers. The series of numbers is not meant to convey the degree to which the importance of a want with a higher index exceeds that of a want with a lower index. It is not by any means my intention to make the statement that a want with an index of 6 is exactly three times as important as one with an index of 2, nor that one with an index of 9 possesses an importance exactly equal to that of wants with indices of 6 and 3 combined. (1959 II, 423, n. 17 to p. 141)

But then, a few sections later, he takes the opposite position:

> We should be permittted to hope for unanimity on the question of man's ability to decide whether one pleasurable emotion is stronger or weaker than another. And it should even be reasonable to suppose that there can be no doubt that we can judge whether one feeling of pleasure is considerably or only negligibly stronger than another. But is it possible for us to determine the degree of difference more exactly, and express it in numerical terms? Can we judge whether pleasurable emotion A is, let us say, three times as great or strong as pleasurable emotion B?
>
> I believe we can really do that or something very much like it. (1959 II, 197-8)

He even devotes one of his "Excurses" to defending the measurability of sensations (1959 III, 124-36).[1]

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[1] See also Machlup's apocryphal debate between Böhm-Bawerk and Čuhel in "Probleme der Wertlehre" (Kaufmann and Machlup 1935).

Hayek (in Menger 1934, xiv, xv) and Stigler (1937, 240) both refer to the following passage in Menger as proving that he had in mind ordinal utlity: "I need hardly point out that the figures in the text are not intended to express numerically the _absolute_ but merely the _relative_ magnitudes of importance of the satisfactions in question." (1950, 183n.) But if we go to the source and read on, Menger's very next words are, "Thus when I designate the importance of two satisfactions with 40 and 20 for example, I am merely saying that the first of the two satisfactions has twice the importance of the second to the economizing individual concerned." Menger is simply espousing the usual, interminate-up-to-a-linear transformation, cardinal utility. In numerous instances we find him adding utilities together.[1]

Only the later Austrian school economists, such as von Mises (above, p. 11). Bilimovič (1934), and Rothbard (1956), can be said to take a consistently ordinal position.

The persistent inconsistency of the older Austrians on the cardinality question is understandable in light of the close relation between measurability and their implicit assumption of additivity. They can hardly be taken to task for being unclear in the nineteenth century about a distinction which mathematicians did not even state until 1949 and did not resolve until 1959. It is natural to draw on cardinal illustrations to force additivity, even if the cardinality has no necessary place in the theory. Perhaps Böhm-Bawerk had this in the back of his mind when he added the proviso "or something very much like it"

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[1]See, _e.g._, Menger (1950, 179). Note also p. 293, n. 1, "...value is a magnitude that can be measured."

(above) to his statement that utilities may be expressed in multiples of one another.

One situation that does lead to essentially cardinal preferences is the hypothetical one in which goods and wants are perfectly divisible. It can be shown that if W* is additively ordered in such a way that W can be partitioned into arbitrarily insignificant subsets, its ordering must be essentially cardinal (Krantz et. al., 1971 I, 206-7).[1] Thus, if a good such as an automobile could be divided into arbitrarily small pieces satisfying arbitrarily trifling wants which when put together would comprise the important wants satisfied by the whole automobile, utility would be essentially cardinal. Such an assumption is not very realistic, to be sure. We would not want to make it a fundamental postulate of all utility theory. Nevertheless in some applications this convenient simplification might be harmless, provided we recognize it as the simplification it is. When we do indulge in it, the additivity of preferences on wants, together with the Austrian logic of choice, will <u>imply</u> as a theorem that the derived cardinal marginal utility diminishes.

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[1]Similarly, Alt (1936) demonstrates that any Bernardelli utility index (1938) that is expressible as a continuous function on commodity space can be monotonically transformed in such a way that Bernardelli's conditional utility is the arithmetic difference of his total utility. If the Bernardelli utility index is not continuous, however, it cannot necessarily be so transformed. As a counterexample, consider the derived commodity preferences that would arise when there are several different indivisible goods, each one of which is capable of satisfying a different basic want, when the ordering on W* happens to be intrinsically ordinal. (There must be five or more goods for this to happen.)

In a published comment on Bernadelli's paper, Samuelson (1939) called attention to crucial flaws in a functional example Bernardelli attempted to work out in his mathematical appendix. Samuelson's comments, however, do not reflect on the text of Bernardelli's paper.

PROBABILISTIC CARDINALIZATION OF UTILITY

It is a fairly straightforward exercise to adapt the well-known von Neumann-Morgenstern probabilistic axioms[1] to the Austrian framework and come up with a cardinal utility index for the wants and therefore for commodities. In fact, the additivity concept can be integrated into the traditional von Neumann-Morgenstern axiom sytem in a way that virtually eliminates one of the traditional axioms. What's more, when this is done, the Austrian wants-structure will imply that the resulting cardinal utility index on commodity space will be mathematically concave, and therefore exhibit diminishing marginal utility and indifference curves that are convex toward the origin.

However, doing this rules out intrinsically ordinal rankings on W*. Therefore economists cannot have both the von Neumann-Morgenstern axioms and the possibility of intrinsically ordinal preferences. One or the other has to go. Several economists have questioned the von Neumann-Morgenstern system. Georgescu-Roegen (1954) argues that perhaps preferences are lexocographic and linear, ruling out the possibility of indifference that is crucial to the von Neumann-Morgenstern approach. Taking a different tack, Quandt (1960) and Meginniss (1976)

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[1]See von Neumann and Morgenstern (1953, Appendix) or any advanced text on microeconomics, and Morgenstern (1976, 809). It is a curious inconsistency in the state of economic doctrine that the leaders of the profession acknowledged soon after 1944 that the von Neumann-Morgenstern cardinalization of utility was plausible, yet refused for decades to grant that it meant that the 1934 Hicks-Allen objections to the ALEP criterion were no longer valid. Only thirty years later was Samuelson willing to draw this obvious conclusion (1974, 1264-1265). He was by then even willing to go so far as to acknowledge it as "my von Neumann risk-utility metric" (1974, 1263, emphasis added). Even so, in the same paper he took pains to deny that he was "backsliding" from the Hicks-Allen version of ordinality without marginal utility (1285 n23).

have questioned whether expected utility maximization is necessary for rationality. These authors argue that there is nothing irrational about consumers who instead maximize expected utility plus a term that depends on the standard error of the utility of the gamble (Quandt), or on the entropy of the gamble (Meginniss). Intrinsically ordinal preferences might not be ruled out for consumers like these.

In summary, the issue of probabilistic cardinalization of utility is still up in the air. I personally find intrinsically ordinal preferences and the von Neumann-Morgenstern axiom system about equally plausible. Until this inconsistency is resolved, however, it should be remembered that the purely Austrian approach does admit intrinsically ordinal marginal utility.

TRADUTTORE, TRADITORE

If the Austrian theory of the marginal use is so powerful, why has so little attention been given to it outside of a relatively small circle? We offer two reasons.

First, the Austrian analysis has suffered greatly in translation out of German, since "marginal utility" has traditionally been used as the translation of their technical term "Grenznutzen." The Austrian theory swings on the theorem that a good's "value" or marginal utility is determined by its marginal use. In conventional translations, however, this important theorem comes through as the trivial assertion that a good's marginal utility is determined by its marginal utility. It is small wonder English-speaking economists have been unimpressed.

This confusion was not so much the fault of the translators as of the principals themselves. The source of the problem is the dual meaning of the word "utility" in English. Its common meaning is merely "usefulness." But economists have used it to mean "subjective value."[1] It is understandable that when Wieser and Böhm-Bawerk first became aware of Jevons and Walras, they would assume that their Grenznutzen,or marginal use, corresponded to the English marginal utility. Nutzen is very close to the common English utility, though not to the economist's utility, which in fact corresponds to the Austrians' subjektiver Wert.[2] Unfortunately, after Wieser casually suggested a correspondence between his Grenznutzen and Jevon's "final degree of utility" (above p. 7n), it was accepted as a matter of fact by all parties concerned.[3] This correspondence remained unquestioned for over eighty years, when it was finally challenged by Georgescu-Roegen (1968, 251).

Perpetuating the confusion has been the Anglo-Saxon's provincial belief that everyone writes and thinks in English. One could read H. J. Davenport's 1902 "Proposed Modification in Austrian Theory and Terminology" without ever realizing that Austria is any less English-speaking than Australia. Perhaps he confused the two. He cites two examples in Wieser's Natural Value where

> in the second case [Weiser] employs the word value for seemingly precisely the same meaning as was in the former case expressed by utility, the two statements together being perhaps mostly serviceable as illustrating the confusion of utility with value characteristic of Austrian discussion. (1902, 362n.)

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[1]Thus we find Menger's translator rendering "Nutzlichkeit" as "utility," and then adding a footnote to make it clear that utility is not really utility (1950, 118, n. 6). The translation of Wieser's Natürlicher Werth is equally confusing. For instance, the translator gives Nutzen as "utility" on page 1, and then as "use" a few pages later (1889, 1, 11 and 1893, 1 and 12).

[2]In referring to Jevons and Walras, Wieser identifies "Gebrauchswerth," with their "utility" and "utilité." (1889, 26n.) Jevons confirms that "Wert," as least as used by Gossen, is identical to his "utility." (1931, xxxiv).

[3]"...the English term 'marginal utility'...corresponds exactly to the

(continued)

Examination of the original, however, reveals that in both instances Wieser used "Gebrauchswerth"! (1889, 27n., 43n.) It is the translator who was confused, not Wieser. One would think that before proposing modifications to the Austrian terminology, it would have occurred to Davenport to have ascertained just what their terminology was.

To top it off, by a process of reverse translation the "utility" of Jevons and Hicks has crept back into German as "Nutzen," even in the writings of some of the later Austrians. Already in 1934, Bilimovič thus uses Grenznutzen to mean marginal utility. Today this seems to be the word's standard meaning (Müller-Groeling, 1965). Througout Ludwig von Mises' career spanning six decades, however, he continued to use "Grenznutzen" (and marginal utility" in his English writings after World War II) in Wieser's original sense of "marginal use."

What's to be done? Menger's Principles and Böhm-Bawerk's Capital and Interest are still well worth reading in their present English translations, provided the reader is wary of the word "utility." In Menger, the underlying word is often "Nutzlichkeit," and the reader can substitute "usefulness." In Böhm-Bawerk, it may be "Nutzen" and the reader should try substituting "Use," especially in the expression "marginal utility." The same substitution usually makes sense in von Mises' Human Action. Unfortunately, simple substitution is not so easy in the important section of von Mises' Theory of Money and Credit

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3 (cont'd) German Grenzutzen..." (Böhm-Bawerk 1959 II, 423-24, n. 18). "The term marginal utility is in correspondence with the German term Grenz-nutz [sic]." (Marshall 1890, 142). "Le Grenznutzen de M. de Boehm-Bawek n'est autre chose que...ma rareté." (Walrus 1886, vii). See also Walras' letter to Menger, dated 2 February 1887, especially the paragraph written but struck out admitting his weakness in German (Antonelli 1953, 285-6). Bell (1953, 423, 431) first introduced the term "marginal use," though even he did not directly question the traditional equation of "Grenznutzen" with "marginal utility." In a letter to me dated March 28, 1975, Böhm-Bawerk's translator, Hans Sennholz, now concurs with the distinction between Grenznutzen and marginal utility.

relating to the value of money (1924/1953, 97-154).

The second reason we offer for the neglect of the Austrian theory is that they are not well known even in German speaking countries. Two World Wars disturbed the continuity of the school. Their own country, Austria-Hungary, was dismembered after World War I. The Austrian fragment became uninhabitable around 1935; von Mises' exile to Switzerland in that year marked the end of Vienna as an intellectual center for the Austrian school.[1] And Menger's methodological disputes with Schmoller, the head of the official German historical school, effectively prevented adherents of the Austrian theories from obtaining academic positions in Germany.[2]

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[1] See Kaufmann and Machlup (1935), especially "Abschied von Professor Mises" and "Klagelied des Kreises."

[2] See Hayek in Menger (1934), xxii-xxiii, and Kaufmann and Machlup (1935), especially "Die Grenznutzenschule."

CONCLUSION

The Austrian theory of the marginal use raises almost as many problems as it has solved. We list here a few of these unsolved problems.

Complementarity and rivalness do lead to the ALEP criterion in the examples we worked out above, but we have made no attempt to formalize this rule into a general theorem. Intuitively, the ALEP condition must appear when the complementary or rival relationships are somehow active in the inner or outer marginal uses, but it is not clear exactly what the circumstances are under which this holds.

Although the theory leads to quasi-concavity of commodity preferences over goods in the particular cases we worked out, even when rival or complementary interactions are present, it has only been proven that this must be generally true when there are two goods, and then only in the case when the two goods are independent. Perhaps preferences do not really have to be quasi-concave after all.

And finally, it must be resolved whether the possibility of intrinsically ordinal preferences nullifies the von Neumann-Morgenstern axiom system, or if instead the validity of those axioms rules out intrinsically ordinal preferences.

After over a century, the Austrian theory is still in its youth. Perhaps the day has come for Felix Kaufmann's young Grenznutzler to return from the netherworld of economic doctrine:

> There I will quietly lie in wait,
> Amid my neglected writings,
> Until I hear the trumpet call
> of Complementary Goods.

Then through the sky will gallop Böhm-Bawerk,
Polemics will thunder and flash!
Then armed with a quill I'll rise up from the grave,
To fight for the Grenznutzen school![1]

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[1] Kaufmann and Machlup (1935), "Die Grenznutzenschule" (my translation).

REFERENCES

Allen, R.G.D. Mathematical Analysis for Economists (New London: Macmillan, 1962; originally 1938).

Alt, Franz. "Über die Meßbarkeit des Nutzens," Zeitschrift für Nationalökonomie 7 (1936), 161-9.

Antonelli, Étienne. "Léon Walras et Carl Menger à travers leur correspondance," Économie Appliquée 6 (April-Sept. 1953) 269-287.

Auspitz, Rudolf and Richard Lieben. Untersuchungen über die Theorie des Preises, (Leipzig: Dunckler and Humbolt, 1889).

Bell, John Fred. A History of Economic Thought. (New York: Ronald Press, 1953).

Bernardelli, Harro. "The End of Marginal Utility Theory?" Economica N.S. 5 (May 1938), 192-212.

Bilimovič, Alexander. "Wie können unmeßbare psychische Größen in das Gleichungssystem des wirtschaftlichen Gleichgewichts eingeführt werden?" Zeitschrift für Nationalökonomie 5 (1934), 145-184.

Böhm-Bawerk, Eugen. Kapital und Kapitalzins. (Innsbruck: Wagner'schen Universitäts-Buchhandlung, 1909).
1909 A - First half-volume, "Positive Theorie des Kapitales."
1909 B - Second half-volume, "Exkurse zur Positiven Theorie des Kapitales."

__________. Capital and Interest. (South Holland, Ill.: Libertarian Press, 1959). Translated by George D. Huncke and Hans F. Sennholz.
1959 II - Second volume, "Positive Theory of Capital," Corresponds to 1909 A.
1959 III - Third volume, "Further Essays on Capital and Interest," Corresponds to 1909 B.

Davenport, H. J. "Proposed Modifications in Austrian Theory and Terminology," Quarterly Journal of Economics 16 (May 1902), 355-84.

De Finetti, B. "La 'logica del plausible' secondo la concezione di Polya," Atti della XLII Riunione della Societa Italiana per il Progresso delle Scienze, 1949. (1951). (Not consulted by present author.)

Dilthey, Wilhelm. Pattern and Meaning in History: Thoughts on History and Society. Edited with an introduction by H. P. Rickman. New York: Harper Torchbooks, 1962. (Translated from various writings, 1882-1910.)

Edgeworth, F. Y. "The Pure Theory of Monopoly," in Edgeworth, Papers Relating to Political Economy. (London: Macmillan, 1925). Vol. I, 111-142. (Retranslated from the Italian translation, published 1897 in the Giornale degli Economisti, of the lost English original.)

Georgescu-Roegen, Nicholas. "Choice, Expectations and Measurability," Quarterly Journal of Economics 68 (1954), 503-34. Reprinted in Georgescu-Roegen, Analytical Economics: Issues and Problems. (Cambridge, Mass.: Harvard Univ. Press, 1966).

____________. "Utility," International Encyclopedia of the Social Sciences, edited by D. S. Sills. (Macmillan and The Free Press, 1968).

Hicks, J. R. Value and Capital, 2nd ed. (Oxford: Clarendon Press, 1946).

Jevons, W. Stanley. The Theory of Political Economy, 4th ed. (London: Macmillan, 1931).

Kauder, Emil. A History of Marginal Utility Theory. (Princeton: Princeton University Press, 1965).

Kaufmann, Felix, and Fritz Machlup, "Ausgewählte Miseskreislieder," mimeo, Vienna, 1935. Copies available from present author on request.

Kraft, Charles H., John W. Pratt, and A. Seidenberg. "Intuitive Probability on Finite Sets." The Annals of Mathematical Statistics 30 (1959), 408-19.

Krantz, David H., R. Duncan Luce, Patrick Supes, and Amos Tversky. Foundations of Measurement. Vol. I: "Additive and Polynomial Representations." (New York: Academic Press 1971).

Lachmann, Ludwig M. "Die geistesgeschichtliche Bedeutung der österreichischen Schule in der Volkswirtschaftslehre." Zeitschrift für Nationalökonomie 26 (1966), 152-67. Translation in preparation.

Lancaster, Kelvin J. "A New Approach to Consumer Theory," Journal of Political Economy 75 (April 1966), 132-57.

____________. Consumer Demand: A New Approach. (New York: Columbia University Press, 1971).

Lindgren, J. Ralph. "Adam Smith's Solution to the Paradox of Value," Mimeo, Lehigh University, 1976.

Marshall, Alfred. Principles of Economics, 1st ed. (London: Macmillan, 1890).

McCulloch, J. Huston, and Jeffrey D. Smith. "An Austrian Proof of Quasi-Concave Preferences," Boston College Discussion Paper #70, 1975, submitted to Econometrica.

Meginniss, James R. "A New Class of Symmetric Utility Rules for Gambles, Subjective Marginal Probability Functions, and a Generalized Bayes' Rule," Proceedings of the Business and Economics Section, American Statistical Association (1976), 471-6.

Menger, Carl. Grundsätze der Volkswirthschaftslehre: Erster, Allgemeiner Theil, (Vienna: Wilhelm Braumüller, 1871). Projected subsequent parts did not appear. Later reproduced (London: London School of Economics, 1934), with English introduction by F. A. Hayek.

__________. Principles of Economics. (Glencoe, Ill.: The Free Press, 1950) Translation by James Dingwall and Bert F. Hoselitz of Menger [1934]. Introduction by Frank H. Knight.

Michael, Robert T., and Gary S. Becker. "On the New Theory of Consumer Behavior," Swedish Journal of Economics 75 (1973), 378-96.

Morgenstern, Oskar. "Thirteen Critical Points in Contemporary Economic Theory: An Interpretation," Journal of Economic Literature 10 (December 1972) 1163-1189.

__________. "The Collaboration between Oskar Morgenstern and John von Neumann on the Theory of Games," Journal of Economic Literature 14 (Sept. 1976), 805-816.

Müller-Groeling, Dr. Hubertus. Maximierung des Sozialen Gesamtnutzens und Einkommens Gleichheit. (Cologne: Carl Heymanns Verlag K. G., 1965).

Pareto, Vilfredo. Manuel d'Économie Politique, 2nd ed. (Paris: Marcel Giard, 1927). Translated from the Italian by Alfred Bonnet. First Italian edition, 1906.

Pirou, Gaëtan. L'Utilité marginale de C. Menger à J.-B. Clark, 3rd ed. (Paris: Les Éditions Domat-Monchrestien, 1945).

Rothbard, Murray N. "Towards a Reconstruction of Utility and Welfare Economics," in Mary Sennholz, ed., On Freedom and Free Enterprise. (New York: Van Nostrand, 1956.) Von Mises Festschrift.

Samuelson, Paul A. "The End of Marginal Utility: A Note on Dr. Bernardelli's Article," Economica N.S.6 (February 1939), 86-87. Reply by Bernardelli, pp. 88-89.

__________. "Complementarity: An Essay on the 40th Anniversary of the Hicks-Allen Revolution in Demand Theory," Journal of Economic Literature 12 (December, 1974), 1255-1289.

Smith, Adam. The Wealth of Nations. (New York: Modern Library, 1937; originally 1776).

Stigler, George J. "The Economics of Carl Menger," Journal of Political Economy 45 (April 1937), 229-250.

__________. "The Development of Utility Theory," Chapter 5 of Essays in the History of Economics. Chicago: University of Chicago Press, 1965. Reprinted from Journal of Political Economy 58 (August, October, 1950), 307-27, 373-96.

Strotz, Robert H. "The Empirical Implications of a Utility Tree," Econometrica 25 (April 1957), 269-80. Correction by W. M Gorman, 27 (July 1959), and reply.

von Mises, Ludwig. The Theory of Money and Credit. (New York: Yale University Press, 1953). Parts 1-3 translated by E. H. Batson from the second German edition of 1924.

__________. Human Action, 2nd ed. (New Haven: Yale, 1963.)

von Neumann, John, and Oskar Morgenstern. Theory of Games and Economic Behavior, 3rd ed. (Princeton: Princeton University Press, 1953; 1st ed. 1944).

Walras, Léon. Théorie de la Monnaie. (Lausanne: Corbaz & Cie., 1886.)

Wieser, Friedrich von. Über den Ursprung und die Hauptgesetze des Wirthschaftlichen Werthes. (Vienna: Alfred Hölder, 1884). No translation.

__________. Der Natürlicher Werth. (Vienna: Alfred Hölder, 1889).

__________. Natural Value. (London: Macmillan, 1893). A translation of Wieser [1889] by Christian A. Malloch.

Young, Henry A. "Preference Scales and Utility Functions," Mimeo, University Rochester, September, 1969.