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OPTIMAL INCOME REDISTRIBUTION WHEN INDIVIDUAL WELFARE DEPENDS UPON RELATIVE INCOME*

by

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> Our theory ... depends upon the validity of a single hypothesis, viz: that the utility index is a function of relative rather than absolute consumption expenditure.
>
> J. Duesenberry, Income, Saving and the Theory of Consumer Behavior

1. Introduction

The problem of the "appropriate" role of government in redistributing income has a long and interesting history in economics. Dating back in its modern form to Edgeworth, the original argument, based on addition of the welfare of all citizens, diminishing marginal utility of income and no tax disincentive effects was for complete equality, i.e., a tax-transfer system that equalized incomes.

More recently, the rigorous examination of this problem by Mirrlees [1971], Atkinson [1973], Fair [1971], Feldstein [1973], Sheshinski [1971] and Stern [1976], focusing on the tax-induced disincentive to supply labor, has concluded that optimal tax rates are quite modest and that rather little public income redistribution is socially desirable. While the results are fairly sensitive to assumptions about labor supply elasticities and social welfare functions, these results stand in striking

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contrast to both the Edgeworth analysis and the policies and policy proposals of many advanced economies.

One aspect of the debate over income distribution which has not been analyzed in the optimal income tax framework is the popular notion that welfare depends at least in part upon relative income or consumption. We are familiar with the notion that poverty may be a relative, as well as absolute, income phenomenon. For example, Galbraith [1958] argues that "people are poverty striken when their income ... falls markedly behind that of the community." The relative income hypothesis has rarely been analyzed or empirically tested in economics. Probably the most important exception is Duesenberry [1949]. The purpose of the present note is to explore the structure of optimal income taxation/redistribution in an economy where the welfare of individuals depends in part on relative after-tax consumption, i.e., we specify individual welfare as a function of absolute <u>and</u> relative after-tax consumption, with diminishing marginal utility to each. With such a specification, of course, an additional incentive for income redistribution from wealthy to poor citizens is created and the logical impossibility of increasing tax rates to the point where disincentive effects actually reduce tax revenues is potentially removed. The analysis highlights the importance of the marginal valuation placed on upward social mobility in various ranges of the income distribution and its interaction with the elasticity of the marginal utility of consumption; of course, "labor supply" elasticities, the form of the social welfare function, and the skill distribution continue to play an important role.

In Section 2, we introduce the simplest model of optimal income redistribution: linear redistribution plans (credits plus flat rate taxes, i.e., archetypical negative income tax schemes) combined with investment in human capital (a proxy for all forms of increasing or decreasing income). We derive analytically the structure of optimal linear negative income taxes for the case of individual welfare depending upon both the absolute and relative levels of consumption. We demonstrate that increased concern for relative consumption levels leads to higher income guarantees and marginal tax rates.

In Section 3, we examine the extreme "Maxi-min" case (in which the government attempts to maximize the welfare of the worst-off individual). Assuming for analytical convenience a Pareto distribution of skills and logarithmically separable utility, we derive explicit formulas for the optimal marginal tax rate and income guarantee. Of course, these depend upon the usual parameters: the skill distribution, the elasticity of the marginal cost of human investment, and the elasticities of the private and social marginal valuation of absolute and relative consumption. A numerical example reveals that concern for relative status adds very little additional progression to the optimal policy. Indeed, pushing an already extreme case to the extreme-- concern only with the relative income of the worst off individual -- leads to surprisingly little redistribution.

In Section 4, we report the results of some numerical calculations of optimal marginal tax rates and income guarantees for a utilitarian social objective. We examine how these policy parameters respond to variations in concern for relative position as well as the other parameters of the problem.

The optimal tax rate and relative income guarantee in the utilitarian case turn out to be enormously sensitive to concern for relative position. The optimal redistribution schemes range from quite modest to substantial as concern for relative status moves from non-existent to virtually exclusive.

Further, for identical parameters of the skill distribution, marginal cost of human investment elasticity and concern for relative status, we demonstrate that the utilitarian welfare function can lead to substantially more redistribution than the allegedly extremely egalitarian Maxi-min objective. This occurs because the improved welfare of the remainder of the low-income population as their <u>relative</u> income increases suffices to overcome the decline in the absolute level of consumption caused by extremely high tax rates.

Finally, in Section 5, we offer a brief summary and suggestions for future research.

2. An Optimal Negative Income Tax Model

There are a variety of ways to parameterize the potential disincentive effects of income taxation (see, for example, Sandmo's [1976] survey.) For illustrative purposes, we consider the educational investment model suggested by Sheshinski [1971].

There is one consumption good, denoted by c, and an educational input (say, school years), denoted by x. Invididuals are distinguished by their ability to produce income from a given educational input. Let this ability be indexed by n, ($n \geq 0$). Income, denoted y, is assumed to depend on n and x: $y = y(n,x)$. For simplicity, this relation is assumed to have a multiplicative form: $y = nx$. The cost of educational investment, in terms of the consumption good, is denoted $g(x)$. We assume that $g'(x) > 0$, $g''(x) > 0$. In the absence of income taxation, the individual's consumption is thus given by $c = y - g(x)$.

Suppose that the government decides to redistribute income via a linear negative income-tax, $t(y)$,

$$t(y) = -\alpha + (1 - \beta)y \quad , \tag{1}$$

where α is the income guarantee, or credit, and $(1 - \beta)$ is the marginal tax rate.

Note that this tax rule, in accord with accepted practices, does not allow a deduction for educational expenses.[1] Accordingly, after tax income, $y - t(y)$, is equal to $\alpha + \beta y$ and consumption is given by[2]

$$c = \alpha + \beta y - g(x) \quad . \tag{2}$$

Each individual is assumed to maximize c with respect to x.[3] The first-order condition is

$$g'(x) = \beta n , \quad \text{or} \quad x = \hat{x}(\beta n) \tag{3}$$

where $\hat{x} = g'^{-1}$. We assume that the solution to (3) exists for all n. Accordingly, the optimum before-tax income is also a function of n and the tax rate, $\hat{y}(n,\hat{x}) = \hat{y}(n,\hat{x}(\beta n))$; optimum consumption is given by $\hat{c} = \alpha + \beta\hat{y} - g(\hat{x})$.

The individual's utility, u, is assumed to depend on two variables: own consumption, c, and the <u>average</u> consumption in the population, denoted $\bar{c}$:

$$u = u(c,\bar{c};\, \theta) \tag{4}$$

where θ is a parameter, $\theta \geq 0$, signifying the degree of concern for the individual's "relative position". In particular, we assume that when $\theta = 0$, $\partial u/\partial c = u_1 > 0$, $\partial u/\partial \bar{c} = u_2 = 0$ and $\partial^2 u/\partial c \partial \bar{c} = u_{12} = 0$. For any $\theta > 0$, we assume that $u_1 > 0$, $u_2 < 0$ and that u is trictly concave and twice differentiable. As a special case we may have $u(c,\bar{c};\, \theta) = v_1(c) + \theta v_2(c/\bar{c})$, where v_i, $i = 1,2$ are increasing, concave functions. The variable $c/\bar{c}$ represents, of course, the individual's relative position in terms of consumption. Clearly, $\theta = 0$ is the standard case when the individual's utility is independent of other individuals' consumption.

Let $f(n)$ be a density function denoting the relative number of individuals with ability n: $f(n) \geq 0$ and $\int f(n)dn = 1$. In choosing the optimum tax parameters (α,β), the government is assumed to follow a utilitarian principle. Thus, its objective is to maximize the social welfare function, W,

(5) $$W = \int u(\hat{c},\bar{c};\ \theta)f(n)dn$$

subject to a balanced budget constraint $\int t(\hat{y})f(n)dn = 0$, or

(6) $$\alpha - (1 - \beta)\bar{y} = 0$$

where $\bar{y} = \hat{y}f(n)dn$ is average income. Accordingly, average consumption is given by

(7) $$\bar{c} = \alpha + \beta\bar{y} - \int g(\hat{x})f(n)dn \quad .$$

The first-order conditions for an interior maximum of (5) subject to (6) and (7) are

(8) $$\int[u_1(\hat{c},\bar{c};\ \theta) + u_2(\hat{c},\bar{c};\ \theta) - \mu]f(n)dn = 0$$

(9) $$\int[u_1(\hat{c},\bar{c};\ \theta)\hat{y} + u_2(\hat{c},\bar{c};\ \theta)\bar{y} - \mu(1 - \frac{1 - \beta}{\beta\lambda})y]f(n)dn = 0$$

where $\mu > 0$ is the shadow price (or Lagrangean-coefficient) of increased income redistribution, u_i, $(i = 1,2)$ denote partial derivatives, and $\lambda = \lambda(x) = g''(x)x/g'(x)$ is the elasticity of the marginal cost of education. We assume that there exists a unique solution to these equations, denoted by (α^*,β^*).

In order to find the effect of "concern for relative status" on the optimum tax scheme, we differentiate (8), (9) and (6) totally w.r.t. θ, evaluating these changes at $\theta = 0$. Assuming for simplicity that λ is constant (i.e. $g(x) = x^{1+\lambda}/(1 + \lambda)$, $\lambda > 0$ a scalar), we have

$$(10)\quad \begin{bmatrix} \int u_{11} f(n)dn & \int u_{11}\hat{y} f(n)dn & -1 \\ \int u_{11}\hat{y} f(n)dn & \int u_{11}\hat{y}^2 f(n)dn - \frac{\mu\bar{y}}{\beta^2\lambda} & -(1-\frac{1-\beta}{\beta\lambda})\bar{y} \\ 1 & (1-\frac{1-\beta}{\beta\lambda})\bar{y} & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial\alpha^*}{\partial\theta} \\ \frac{\partial\beta^*}{\partial\theta} \\ \frac{\partial\mu}{\partial\theta} \end{bmatrix}$$

$$= \begin{bmatrix} -\int u_2 f(n)dn \\ -\int u_2 \bar{y} f(n)dn \\ 0 \end{bmatrix} .$$

Denote the matrix on the L.H.S. of (10) by Δ. By the second-order conditions for maximization, $\Delta < 0$. Solving the system (10) for $\partial\beta^*/\partial\theta$:

$$(11)\quad \frac{\partial\beta^*}{\partial\theta} = \frac{-1}{\Delta}\,\frac{(1-\beta^*)\bar{y}}{\beta^*\lambda}\int u_2 f(n)dn < 0 \quad .$$

Thus , as expected, some <u>concern for relative position, as well as absolute position, always leads to an increase in the optimum marginal tax rate</u>.

It can be shown that $\partial\beta^*/\partial\theta < 0$ implies $\partial\alpha^*/\partial\theta > 0$ and <u>vice versa</u>. Hence, increased concern for relative position will lead both to higher tax rates and to higher credits, or income guarantees.

3. The Maxi-Min Criterion

As an alternative to the utilitarian welfare function, consider the Rawlsian objective criterion (termed Maxi-Min), according to which the government is concerned solely with the welfare of the worst-off individual. Denote this individual's gross income (corresponding to the lowest ability, say, $\underline{n}$) by $\underline{y}$, and his consumption by $\underline{c}$: $\underline{c} = \alpha + \beta\underline{y} - g(\hat{x}(\beta\underline{n}))$.

The government's objective is to maximize

$$(12) \qquad \underline{u} = u(\underline{c},\bar{c};\ \theta)$$

w.r.t. α and β, subject to (6), i.e. to choose the income guarantee and the marginal tax rate to maximize the welfare of the worst-off individual given the disincentive effects of the tax. The first-order conditions now become

$$(13) \qquad u_1 + u_2 - \mu = 0$$

and

$$(14) \qquad u_1\underline{y} + u_2\bar{y} - \mu\left(1 - \frac{1-\beta}{\beta\lambda}\right)\bar{y} = 0 \quad ,$$

where u_1 and u_2 are evaluated at $c = \underline{c}$. One may rewrite (13)-(14), solving for β^*

$$(15) \qquad \beta^* = \frac{1}{1 + \eta\lambda\left(1 - \frac{\underline{y}}{\bar{y}}\right)}$$

where $\eta = u_1/(u_1 + u_2)$, $\eta \geq 1$. Clearly, by our assumptions, $\eta = 1$ when $\theta = 0$, and η increases as θ becomes positive. Consequently, for a given ratio

$\underline{y}/\bar{y}$, $(0 \leq \underline{y}/\bar{y} \leq 1)$, as θ increases from zero, β^* will decrease, i.e. the optimum tax rate increases. Alternatively, for a given θ, as $\underline{y}/\bar{y}$ increases, β^* decreases. These results should be expected: when the ratio of lowest to average before-tax income increases, the optimum tax rate decreases. This reasoning, however, is not completely rigorous, since the ratio $\underline{y}/\bar{y}$ as well as η depend on the chosen β^*. Hence, equation (15) cannot be directly used to calculate the latter. In special cases, however, when $f(n)$ is further specified, an exact solution for β^* is possible. We shall now present such an example.

Let $f(n)$ be the Pareto distribution:

$$f(n) = \delta \underline{n}^{\delta} n^{-\delta-1} \quad , \quad n \geq \underline{n} \tag{16}$$

where $\delta > 1$ and $\underline{n} > 0$ are parameters, the latter denoting the lowest ability level.

Suppose that the utility function is logarithmic

$$u(c,\bar{c};\,\theta) = \log c + \theta \log \left(\frac{c}{\bar{c}}\right) \quad . \tag{17}$$

We continue to assume that $g(x) = (1/1+\lambda)x^{1+\lambda}$, where $\lambda > 0$, is the elasticity of the marginal cost of education.

Using the individual's first-order conditions we find that

$$1 - \frac{\underline{y}}{\bar{y}} = \frac{1+\lambda}{\lambda\delta} \quad . \tag{18}$$

For $\bar{y}$ to be finite we have to assume that $1 + \lambda < \lambda\delta$;

(19)
$$\eta = \frac{u_1}{u_1 + u_2}$$
$$= \frac{1 + \theta}{1 + \theta(\frac{1 + \lambda}{\lambda\delta})} \quad .$$

Condition (15) thus becomes

(20)
$$\beta^* = \frac{1}{1 + \frac{(1 + \theta)\lambda(1 + \lambda)}{\lambda\delta + \theta(1 + \lambda)}}$$

Now, when $\theta = 0$ we have

(21)
$$\beta^* = \frac{1}{1 + \frac{1 + \lambda}{\delta}}$$

Notice that the optimum marginal tax rate, as expected, is positively correlated with the elasticity of marginal cost, λ, and with the dispersion of ability, represented by (the inverse of) the parameter, δ.

From (20) we note that

(22)
$$\frac{\partial\beta^*}{\partial\theta} = -\frac{\lambda(1 + \lambda)[\lambda\delta - (1 + \lambda)]}{\beta^{*2}[\lambda\delta + \theta(1 + \lambda)]^2} < 0$$

since, by assumption, $\lambda\delta - (1 + \lambda) > 0$. Thus, the optimum marginal tax rate and income guarantee increase as concern for "relative position" increases.[4]

Furthermore, it can be seen from (21) that as $\theta \to \infty$ (i.e. the individual becomes exclusively concerned with relative consumption),

(23) $$\beta^* \to \frac{1}{1 + \lambda} \quad ,$$

and hence the **dispersion** in the skill distribution no longer affects the optimal policy. For the range of reasonable values of λ[5] and δ, the difference between the optimum tax rate at $\theta = 0$ and at $\theta = \infty$ is quite small. Table 1 presents some sample calculations.

Nóte also that

(24) $$\frac{\alpha^*}{\bar{y}} = 1 - \beta^* \quad ,$$

the "relative income guarantee" equals the marginal tax rate. From Table 1 we note that even this extreme egalitarianism combined with enormous concern by the worst-off individual over relative status leads to surprisingly little income redistribution relative to the Edgeworth analysis.[6]

Finally, while the amount of redistribution is nontrivial for the **Maxi-min criterion**, concern for relative position does not seem to provide **much by way of additional progressive taxation.**

We turn now to a consideration of whether concern for relative position becomes important when the government follows a somewhat less egalitarian social objective.

Table 1

Optimal Marginal Tax Rate and Relative Income Guarantee for Alternative Parameter Values (Maxi-min Objective)

		θ				
		0.0	1.0	5.0	10.0	∞
δ	λ					
2.5:	0.75	0.41	0.42	0.43	0.43	0.43
	1.0	0.44	0.47	0.49	0.49	0.50
	1.25	0.47	0.51	0.54	0.55	0.56
	0.60	0.35	0.36	0.37	0.37	0.38
3.0:	0.75	0.37	0.40	0.42	0.42	0.43
	1.0	0.40	0.44	0.46	0.49	0.50
	1.25	0.43	0.48	0.53	0.54	0.56

4. A Utilitarian Social Objective

Returning to the utilitarian social objective discussed in Section 2, we may solve the system of equations (10) explicitly for the case of logarithmically separable utility and a Pareto skill distribution. This yields the following:

$$(1+\theta)\int \frac{1}{c} f(n)dn - \frac{\theta}{c} = \mu \tag{25}$$

$$(1+\theta)\int \frac{y}{c} f(n)dn - \theta\frac{\bar{y}}{c} = \mu\left(1 - \frac{1-\beta}{\beta\lambda}\right)\bar{y} \quad . \tag{26}$$

Substituting from the individuals' first-order conditions for x, we derive the expressions for y, c, $\bar{y}$, $\bar{c}$, and α. This leaves us with two nonlinear equations in the two unknowns μ and β. We have solved these equations numerically for certain values of the parameters δ, λ, and θ. The results are reported in Table 2. Several characteristics of the results are worth noting.

First, unlike the Maxi-min case, the optimal marginal tax rates and relative income guarantees are extremely sensitive to variations in θ, our parameter reflecting concern for relative versus absolute consumption. The optimal marginal tax rates and income guarantees range from a quite modest 27% to a very substantial 67%. For a given λ and δ, the increase in θ from zero (no concern for relative position) to five (predominant concern for relative position) approximately doubles the optimal marginal tax rate and income guarantee. It is thus obvious that a strong relative income effect *may* overcome somewhat the tendency of disincentive effects to hold down marginal tax rates and income guarantees.[6]

Table 2

Optimal Marginal Tax Rates and Relative Income Guarantees, Alternative Parameter Values (Utilitarian Case)

	θ		
	0.0	1.0	5.0
$\delta = 2.5$			
$\lambda = 0.75$	0.37	0.48	0.67
$= 1.0$	0.35	0.46	0.64
$= 1.25$	0.34	0.46	0.64
$\delta = 3.0$			
$\lambda = 0.6$	0.29	0.39	0.57
$= 0.75$	0.27	0.37	0.55
$= 1.0$	0.27	0.37	0.55
$= 1.25$	0.27	0.37	0.55

Second, as in the Maxi-min case, the results are only slightly affected by modest variations in λ, the (constant) elasticity of marginal cost of human investment, and only modestly affected by variations in δ, the skill dispersion parameter.

Finally, we note a surprising result: For the case of extreme concern for relative position, the optimal marginal tax rates and relative income guarantees for the utilitarian case actually exceed -- in some cases substantially exceed -- those for the corresponding Maxi-min case! With concern for relative position paramount, the additional welfare of the balance of the low skill population from higher relative income guarantees certainly may outweigh the welfare loss due to lower total income in society. While the absolute consumption of the lowest skill group declines in this case, its relative position improves enough to more than overcome the value of the decline in their absolute consumption and that of the higher skill groups.

For example, we note that when $\theta = 5$ and $\lambda = 1$, the ratio of per capita consumption in the utilitarian case to per capita consumption in the Maxi-min case equals 0.88 when $\delta = 3.0$ and 0.76 when $\delta = 2.5$. The higher tax rates in the utilitarian case lead to dramatically lower average levels of consumption.

Thus, the concern for relative income may make the Maxi-min criterion less egalitarian than a utilitarian social objective!

5. Conclusion.

In considering the conjecture that individual welfare depends in part upon relative consumption, we have explored some of the implications for public policies which redistribute income via progressive income taxation. Briefly, these implications include the following:

1) The class of linear negative income taxes exhibits increasing marginal tax rates and relative income guarantees as concern for relative position increases;

2) The Maxi-min criterion yields relatively modest redistribution schemes which are made only slightly more progressive with concern for relative position (recall the specific utility function and skill distribution);

3) Concern for relative consumption may substantially increase the optimal marginal tax rate and relative income guarantee with a utilitarian objective. Indeed, strong concern for relative consumption may induce substantially more redistribution than with a Rawlsian Maxi-min social objective.

We conclude with a word of warning lest we be misconstrued. Our purpose here is to explore the potential implications of individual concern for relative consumption on the design of optimal redistribution schemes. While it is clear that such concern can be extremely important, it seems to be so in cases where concern for relative consumption is extremely strong. Too often, such a concern is considered "obvious." Evidence that this is indeed the case is virtually nonexistent, let alone convincing. We hope that by demonstrating the potential policy relevance of empirical information on the "relative consumption effect," we will encourage much additional empirical research on the subject by economists and other social scientists.

FOOTNOTES

[1] See Boskin [1976] for a discussion of the tax treatment of human investment.

[2] We think of c and y as lifetime consumption and income, respectively.

[3] If we also include the tax disincentives to hours of work and saving (see Boskin [1977])), the results reported below would be strengthened.

[4] If there is a substantial negative (positive) relative income effect on labor supply, it would weaken (strengthen) the conclusions reported here.

[5] See Lazear [1976] for an estimate of $1/\lambda$.

[6] Indeed, such redistribution schemes are no more progressive than those actually in practice in several countries.

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