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SPURIOUS TREND AND CYCLE IN THE STATE SPACE DECOMPOSITION OF A TIME SERIES WITH A UNIT ROOT

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ABSTRACT

Recent research has proposed the state space (SS) framework for decomposition of GNP and other economic time series into trend and cycle components, using the Kalman filter. This paper reviews the empirical evidence and suggests that the resulting decomposition may be spurious, just as detrending by linear regression is known to generate spurious trends and cycles in nonstationary time series. A Monte Carlo experiment confirms that when data is generated by a random walk, the SS model tends to indicate (incorrectly) that the series consists of cyclical variations around a smooth trend. The improvement in fit over the true model will typically appear to be statistically significant. These results suggest that caution should be exercised in drawing inferences about the nature of economic processes from the SS decomposition.

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1. Introduction

In a trio of recent papers, Harvey (1985), Watson (1986), and Clark (1987) advocate state space models for extraction of the trend from economic time series. The Kalman filter is used to calculate maximum likelihood estimates of the parameters and the implied estimates of the components themselves. The trend component is viewed as a nonstationary stochastic process, generally a random walk with drift, and the cycle is a stationary process, generally an autoregression. The observed data are the sum of these components so the model allows for long term growth as well as both permanent and transitory stochastic shocks. The empirical case for the appropriateness of the state space model is based by these authors on goodness of fit relative to competing specifications, some of which would have quite different implications for the behavior of trend and cycle.

The primary objective of this paper is to demonstrate that the log likelihood is a misleading criterion for model selection in this context because it is subject to the bias described by Fuller (1976), Dickey (1976), and Dickey and Fuller (1979,1981). Briefly, if the data generating process is stationary in first differences, a model which represents the data as stationary deviations around a smooth trend will show an improvement in fit which appears significant by conventional standards. One needs, therefore, to exercise caution in interpreting estimates of trend and cycle components based on the unobserved model and the Kalman filter components methodology.

2. Modeling GNP

The basic issues are well illustrated by the behavior of real GNP which Harvey, Watson, and Clark all analyze. First differences of real GNP (in logs) exhibit positive autocorrelation in the range 0.3 to 0.4 at lag one. Oddly enough, this characterization of the data serves for both annual and quarterly data! This phenomenon is roughly consistent with GNP being a random walk at some short time interval in which case both quarterly and annual averages would be subject, to about the same degree, to the aggregation effect described by Working (1960) which induces positive autocorrelation. In any case, the growth rate of real GNP is reasonably modeled in the ARIMA framework as an AR(1) or perhaps MA(1) with a coefficient of about 0.4. If we take this to be a correct representation then it poses a dilemma for someone who would like to decompose real GNP into trend and cycle components, as Nelson and Plosser (1981) pointed out.

The trend-cycle model represents x_t as the sum of a trend τ_t and a cycle c_t where c_t is stationary but τ_t is stationary only after differencing. Restrictions need to be put on the SS model if parameters are to be identified. One customary restriction is that τ_t is a random walk with drift (which is itself a random walk in some versions) so that

$$(1) \qquad \tau_t = \tau_{t-1} + d + w_t.$$

The first difference of x, denoted $(1-B)x_t$ using the backshift operation B, is then

$$(2) \qquad (1-B)x_t = d + w_t + (1-B)c_t$$

where w_t is an i.i.d. random shock driving the trend process. Now if $(1-B)x_t$ is autocorrelated at lag one only, then c_t must be serially random or else the term $(1-B)c_t$ would imply longer lag autocorrelations. This leaves us with the following relations between the nonzero second moments of $(1-B)x_t$ and the parameters of (2)

(3) $$\gamma(0) = \sigma_w^2 + 2\sigma_c^2 + \sigma_{wc}$$

(4) $$\gamma(1) = -\sigma_{wc} - \sigma_c^2$$

where $\gamma(0)$ and $\gamma(1)$ are the variance and lag one autocovariance respectively of $(1-B)x_t$ and σ_{wc} is the contemporaneous correlation between w_t and c_t (having ruled out lagged cross-covariances). The model is not identified since we have 3 unknown parameters and only 2 equations. This identification problem is a general one as shown by Nelson and Plosser; regardless of the MA order of $(1-B)x_t$, the model contains an unidentified parameter in the absence of further restrictions.

The convenient identifying restriction is that the trend and cycle processes are uncorrelated ($\sigma_{wc} = 0$). This uncorrelatedness restriction puts the model into state space (hereafter SS) form, making all the power of the Kalman filter available for parameter estimation and calculation of estimated values of the trend and cycle. However, the SS model also constrains the autocorrelation structure of $(1-B)x_t$. For the heuristic first order case summarized by (3) - (4), it is apparent that $\sigma_{wc} = 0$ implies that $\gamma(1) \leq 0$, so the model is not compatible with the positive autocorrelation observed in growth rates of real GNP and many other economic variables. An AR(1) specification for $(1-B)x_t$ does not remove the dilemma. Watson showed that in the SS model with uncorrelated components the spectral density function of $(1-B)x_t$ has its minimum at frequency zero. The AR(1) process with a positive coefficient instead has its maximum at frequency zero.

If the AR(1) or MA(1) characterization of the growth rate of real GNP is correct then, as we have seen, the SS model is inadmissible. We would then be obliged to conclude that the trend and cycle components are cross-correlated (evidently negatively), or that the trend process is autocorrelated, or both. Certainly the uncorrelatedness of trend and cycle has no compelling economic motivation, and I have suggested in another paper (Nelson, 1987) how a simple macro model might suggest negative correlation between them. Likewise, there is no theory to suggest that the trend component of real GNP, which we could think of as its natural level, should be a random walk. But this does not point us towards a method of detrending the data, it just leaves us with an unidentified model.

A way out of the dilemma was suggested by Harvey who pointed out that positive autocorrelation at lag one can be compatible with the SS model if it is accompanied by some negative autocorrelation at longer lags. This negative autocorrelation in $(1-B)x_t$ need not be large (indeed it might be small enough to be ignored in the specification of an ARIMA model) as long as it is sufficiently persistent. Indeed, sample autocorrelations of growth rates of real GNP both annual and quarterly are persistently negative at longer lags though not individually significant. Harvey showed that this pattern would be produced by a cycle process that is AR(2) with appropriate values for the coefficients, his example being

$$(5) \qquad c_t = 1.27c_{t-1} - .48c_{t-2} + v_t$$

which also implies periodic behavior in the cycle. Combining (5) with the trend (1) we have the model for $(1-B)x_t$

$$(6) \qquad (1-1.27B+.48B^2)(1-B)x_t = (1-1.27B+.48B^2)w_t + (1-B)v_t$$

which corresponds to an ARIMA(2,1,2) univariate representation. The behavior of $(1-B)x_t$ could well be mistaken for ARIMA(1,1,0) if the variance of w is small relative to that of v since we have $(1-1.27B+.48B^2) \approx (1-B)(1-.4B)$ so the terms $(1-B)$ roughly cancel leaving

$$(1-.4B)(1-B)x_t \approx v_t \tag{7}$$

Harvey fitted the SS model to the Nelson/Plosser annual data, imposing a further restriction on the AR(2) that its coefficients imply pseudo–periodic behavior. For the pre- 1948 period Harvey found that the parameters of the cycle process bordered on non-stationarity with an estimated period of 7.0 years. He concluded that an alternative model which allowed cyclical movement in the first differences of the trend, which he dubbed the "cyclical trend" model, gave a better fit to the data. The cyclical trend model would imply that the trend and cycle components cannot be separated. For the 1948-1970 period annual GNP is essentially a random walk, leaving little room for an improved fit using an SS model. The traditional trend plus cycle SS model did not then get much support from Harvey's analysis of GNP data, nor did it from the other time series he examined.

Watson and Clark both study post-war quarterly real GNP. They find that standard Box-Jenkins identification procedures suggest first order models for first differences of lags, AR(1) and MA(1) fitting about equally well. The AR(1) fitted by Watson is

$$(1-B)x_t = 0.005 + .406(1-B)x_{t-1}; \quad \sigma = 0.0103 \tag{8}$$

$$LogLik = 292.07.$$

This is consistent with the rapid decay of the autocorrelations from a value of .37 at lag one. However, sample autocorrelations at lags longer than three quarters are persistently negative though small (see Clark, Table 1B). Some of this persistent negative sample autocorrelation can be attributed to the negative bias that is inherent in sample autocorrelations (see Kendall, 1973, p. 93) but the bias is not large enough to explain it away. Harvey's SS model with an AR(2) cycle might account for this pattern and that is the specification chosen by both Watson and Clark. Watson's version is

$$x_t = \tau_t + c_t$$

$$(1-B)\tau_t = 0.0008, \qquad \sigma = 0.0057$$

(9)

$$c_t = 1.501c_{t-1} - 0.577c_{t-2}, \sigma = 0.0076$$

$$SE = 0.0099$$

$$LogLik = 294.42$$

σ denotes the estimated standard deviation of the innovation in each component process and SE is the prediction standard error. Similar results are obtained by Clark.

Watson notes that the SS model performs slightly better than the AR model both in terms of standard error and log likelihood. The implied behavior of trend and cycle differs radically, however. Plots by Watson and Clark of the SS model trend component (using the Kalman filter) show it to be very smooth so that short term fluctuations in GNP are predominantly due to cyclical fluctuations. The eye has difficulty distinguishing the trend in these plots from a least squares trend line.

This may be surprising in view of the fact that the σ of the trend process is 75 percent as large as the σ of the cycle process. However, the variance of the cycle component also involves the parameters of the AR(2) which it follows. The variance of the cycle will be very much larger than σ^2 in cases where the sum of the AR coefficients is close to one. To see this note that the formula for the variance of an AR(2) process can be rewritten as

$$var(c) = \frac{(1-\phi_2)}{1+\phi_2} \cdot \frac{1}{d[2(1-\phi_2)-d]} \cdot \sigma^2 \tag{10}$$

where ϕ_1 and ϕ_2 are the AR coefficients, σ^2 is the variance of the innovations, and d is simply $[1-(\phi_1+\phi_2)]$ so that $d=0$ corresponds to the situation where the AR(2) has a unit root and becomes nonstationary. For Watson's model, d is only 0.076 while Clark gets 0.05. The estimated standard deviation of c is therefore .03 (or 3 percent around trend) for the Watson numbers and .036 (or 3.6 percent) for the Clark numbers, while their standard deviations for trend changes are only about .007 (or 0.7 percent).

In contrast, the AR(1) model does not accommodate the usual trend-cycle decomposition, as discussed above. An alternative trend measure used in this case is the definition of the trend as the long term forecast, as proposed by Beveridge and Nelson (1981). However, since there is little autocorrelation in the first differences of GNP, the forecast differs little from the observed value. Thus the ARIMA model suggests that almost all the variation in GNP is due to trend and therefore is permanent, not cyclical. Watson's plot of the Beveridge/Nelson ARIMA trend against actual GNP shows them to be practically the same.

3. The Puzzle

The apparent puzzle posed by the Watson and Clark papers is that widely practiced methodologies seem to lead to very different models of GNP. On the one hand, the Box-Jenkins methodology suggests an ARIMA(1,1,0) model with little evidence of cyclical variation. On the other hand, the SS methodology leads to the characterization of GNP as long and large cyclical movements around a very smooth trend. Likelihood and goodness-of-fit favor the latter.

Note, however, that the cycle model in (9) an be rearranged in the form

$$c_t = 0.924c_{t-1} + 0.577(c_{t-1} - c_{t-2}) + v_t \tag{11}$$

where 0.924 is the sum of the AR coefficients $(\phi_1+\phi_2)$. The stationarity of the cycle component clearly depends on the inference that $(\phi_1 + \phi_2)$ is less than unity. If the sum is in fact unity, then the cycle becomes a nonstationary process and the SS model becomes effectively the sum of two random walks, one being autocorrelated. The process for the growth rate of GNP would then be

$$(1 - 0.577B)(1 - B)x_t = (1 - 0.577B)(1 - B)w_t + v_t \tag{12}$$

where w and v are the innovations in the trend and cycle process respectively. If the trend is very smooth so that the variance of w is small relative to that of v, then the model will be very similar to the ARIMA(1,1,0) model (8). The distinction between the ARIMA and SS models rests, then, on the inference that $(\phi_1 + \phi_2) < 1$.

Suppose that in fact $(\phi_1 + \phi_2) = 1$ and there is no stationary cycle component. What would be the properties of estimates of $(\phi_1 + \phi_2)$ under the hypothesis? We can get some insight into this from what we know about the case of fitting a strictly linear trend to a process with a unit root. The true process is

(13) $$x_t = x_{t-1} + \phi(x_t - x_{t-2}) + v_t$$

but instead we fit the trend-stationary model

(14) $$x_t = \rho x_{t-1} + \phi(x_{t-1} - x_{t-2}) + dt + v_t$$

or, in the SS framework

$$\tau_t = \tau_{t-1} + d \qquad (\textit{no innovation})$$

(15) $$c_t = \rho \cdot c_{t-1} + \phi \cdot (c_{t-1} - c_{t-2}) + v_t$$

$$x_t = c_t + \tau_t.$$

This is the Dickey-Fuller problem and we know that least squares estimates of the unit root are biased towards zero. The formula given by Nelson and Kang (1981) can be used to approximate the expected value of $\hat{\rho}$. For the number of observations used by Watson ($n = 144$) it gives 0.93 (for large n the formula can also be approximated by $1 - 10/n$). Thus, the apparent stationarity of the cycle is consistent with the bias we expect to get in simple linear detrending if in fact there is no stationary cycle.

The Dickey-Fuller results also tell us that a linear trend model will appear to fit significantly better than the true ARIMA model if one uses conventional testing criteria. The Monte Carlo study in Nelson and Plosser (1982) shows that the t-ratio for $(1 - \hat{\rho})$ is typically about -2, corresponding to a nominal significance level of about 0.025. This in turn corresponds to a difference in log likelihoods of about 2.5, which is about the difference we see in Watson's two models. It would appear that even a constrained linear trend version of the SS model (with zero innovations in the trend process) could be expected to fit better than the ARIMA model even if the latter were the correct model.

As a final comment on the SS models for GNP, I note that the AR parameters for the cycle imply a very long business cycle. Watson's AR(2) is a pseudo periodic process with a cycle period of 40.5 quarters. Clark's model gives 69.7 quarters. A period length of 10 to 17 years is being estimated from data series of lengths 36 and 39 years respectively. Perhaps relevant is that NBER business cycle lengths have been more in the neighborhood of 5 years. Some caution in interpreting these estimates would seem to be in order in view of the finding by Nelson and Kang that linear detrending of an ARIMA(1,1,0) process tends to result in a spurious cycle with a length that is somewhat more than half that of the sample. This again points out the difficulty of inferring the existence of a cycle that is long relative to the length of the available data series.

4. The Dickey-Fuller Phenomenon in the SS Model

The SS model differs from the linear trend model studied by Dickey and Fuller in that it allows for stochastic shocks in the trend process. The objective here is to see whether the resulting model, estimated using the Kalman filter, is subject to the same bias of finding a cycle when there is none. The null hypothesis is that the data generating process is ARIMA, and for this purpose I take the simplest case of a random walk. This defines Model 1 which can be written

$$\tau_t = \tau_{t-1} + d + w_t \qquad (16)$$

$$c_t = 0$$

with parameters d and σ_w^2. An equivalent representation is a linear deterministic trend plus a random walk

$$\tau_t = \tau_{t-1} + d$$

(16a)

$$c_t = c_{t-1} + v_t$$

with parameters σ_v^2 and d. The linear trend plus cycle model relaxes the unit root for c and introduces an AR parameter ϕ

$$\tau_t = \tau_{t-1} + d$$

(17)

$$c_t = \phi c_{t-1} + v_t.$$

This defines Model 2 with parameters d, ϕ and σ_v^2. The AR(1) specification is appropriate since the second order term in the GNP model is associated, as shown above, with the autocorrelation in first differences of log GNP. Finally, the SS model allows for innovations in the trend process, so Model 3 will be

$$\tau_t = \tau_{t-1} + d + w_t$$

(18)

$$c_t = \phi c_{t-1} + v_t$$

with parameters $d, \phi, \sigma_v^2, \sigma_w^2$. It will be useful to think of Model 2 as Model 3 with the constraint $\sigma_w^2 = 0$, and Model 1 as Model 2 with the constraint $\phi = 1$.

The log likelihood of any of these models can be evaluated using the Kalman filter. The computations are described in detail by Clark for the slightly more complex GNP model having an AR(2) cycle. I have chosen to maximize the log likelihood by grid search over the relevant parameter space, partly to defend against iterating to local minima. The generating process is Model 1 with $d = 0$ and $\sigma = 1$ with initial condition $x_0 = 0$. Initial values of the state variable τ are set at zero and given a variance of 20 so that the initial value gets little weight. The number of observations was set at $n = 50$, 100, and 150. Experimentation suggested that d could be estimated simply as the mean change in x (no further search) without having a meaningful impact on the results. A summary of experiments is given in the following table, where the numbers in parentheses are estimated standard deviations. Each experiment consumed about two hours of cpu time on a VAX 8700.

Note that relaxing the constraint $\phi = 1$ as we go from Model 1 to Model 2 increases the log likelihood by about 2 on average; the amount of the increase rising slightly with the number of observations. Applying a conventional likelihood ratio test in the typical sample one would reject the random walk hypothesis at better than the .05 level. This is the Dickey-Fuller phenomenon. When the constraint that the trend innovation variance is zero ($\sigma_w^2 = 0$) is relaxed in going from Model 2 to Model 3 there is only small further increase in the log likelihood, around 0.2 on average. A conventional likelihood ratio test would not suggest a significant

SUMMARY OF EXPERIMENTAL RESULTS
MEAN AND STANDARD DEVIATION

		Model 1		Model 2			Model 3			
n	# Reps	σ_V^2 or σ_W^2	LogLik	ϕ	σ_V^2	LogLik	ϕ	σ_V^2	σ_W^2	LogLik
50	200	.96 (.20)	-24.51 (5.16)	.90 (.10)	.91 (.20)	-22.73 (5.27)	.85 (.17)	.80 (.29)	.09 (.21)	-22.62 (5.31)
100	100	.98 (.13)	-49.39 (6.72)	.95 (.04)	.95 (.13)	-47.65 (6.76)	.88 (.16)	.77 (.33)	.17 (.29)	-47.41 (6.76)
150	100	1.01 (.12)	-76.18 (8.42)	.96 (.03)	.99 (.12)	-74.12 (8.53)	.92 (.12)	.85 (.29)	.13 (.28)	-73.95 (8.50)

improvement in fit. The combined increase in the log likelihood from Model 1 to Model 3 rises with n and averages 2.23 for $n = 150$, the case most comparable to quarterly post-war U.S. GNP data. This is close to the improvement reported by Watson for the SS model over the ARIMA. If one thought this were a chi- square with two degrees of freedom under the null, then the true model would be rejected at about the .10 level in the typical case.

Note that allowing for stochastic evolution of the trend component in Model 3 tends to reduce the estimated ψ in comparison to the linear trend in Model 2. The stochastic trend tracks the data more closely than a linear trend does, making the apparent cycle less persistent. The mean estimate for ψ with $n = 150$ is very close to the corresponding coefficient for GNP reported by Watson (see equation (11) above).

Note also that the estimated variance of the trend innovation (σ_w^2) is relatively small. In effect, the SS model divides the variance of the true innovations ($= 1$) between the cycle and trend innovations, giving the former about 0.8 of it and the latter somewhat less than the remainder. The estimated σ_w^2 is not only small on average but is zero in more than half the samples. The limitations of the fineness of the grid do not permit the conclusion that the exact maximum is at zero in these cases, but it is consistent with the boundary solution phenomenon described by Sargan and Bhargava (1983) and discussed in the present context by Harvey and Todd (1983).

Limited experimentation with adding d to the parameter search (rather than setting it to the mean change in x) suggests that it improves the fit of Model 2 over Model 1 by about 0.2 in log likelihood, but not the incremental improvement of Model 3 over Model 2. Estimated ψ in Models 2 and 3 were reduced slightly. This is because by choosing a value of d different from the mean change we get a trend that tracks the data more closely leaving a less autocorrelated cyclical deviation.

5. Summary and Conclusions

The SS model decomposes the observed series into two components: a nonstationary trend which is assumed to be a random walk with drift, and a stationary cycle. The model is identified if these components are assumed to be statistically independent. When the model is fitted to U.S. real GNP it suggests that time series can be represented by a smooth trend process plus a highly autocorrelated cycle which accounts for most of the short term variation in GNP. The SS model is reported in the literature cited above to fit better than the alternative ARIMA representation which does not lend itself to decomposition into independent trend and cycle. The data seem to present a dichotomy: either (1) GNP is a nonstationary ARIMA process in which short term variation is largely or even entirely due to stochastic shocks in its trend (permanent) component, or (2) the trend in GNP is instead very smooth and short term stochastic variation is dominated by an independent cycle component that is also very persistent; and the latter appears to fit the data better.

The choice between these alternatives is not without its political overtones. We tend to think of the trend component of GNP as arising from real factors affecting the aggregate production function. A "real shock" such as OPEC would shift the aggregate production function and therefore the long run path (trend) of output. Monetary and fiscal policy are generally not thought of as having much impact on the long run path of the economy. Cyclical movements in the economy, on the other hand, are thought to be strongly influenced by monetary and fiscal policies, if only to the extent they are unanticipated. Lack of cyclical variation in the economy may suggest that government policy is relatively unimportant in steering the economy, or at least much less important than we had thought. A full discussion of policy implications goes well beyond the scope of this paper, but it is not difficult to see how one's perception of the importance of monetary and fiscal policy might be influenced by which view of the GNP process appeared more likely.

It is not surprising on the basis of intuition that it would be difficult to distinguish empirically between stochastic variation in the trend process and highly autocorrelated cyclical deviations around a very smooth trend, given a finite sample of data. The work of Dickey and Fuller (op. cit.) suggests, however, that conventional statistical tests will tend to favor a smooth trend plus persistent cycle model even when the actual data generating mechanism is entirely nonstationary with no cycle at all. The result is the appearance of trends and cycles that are spurious and not present in the generating process. Dickey and Fuller worked specifically with the case of a linear deterministic trend, while the SS model allows the trend to be a random walk. To see whether spurious trends and cycles may also arise in applications of the SS model, I have generated random walk data so that the generating process is entirely stochastic trend with no cyclical component at all. I then fitted the SS model in repeated samples of various lengths. The results reported in section 4 show that the SS typically fits best with a very smooth trend (small innovation variance), attributing, incorrectly, most of the variation in the data to a highly autocorrelated cyclical component. The typical improvement in likelihood over the true model would be judged significant by conventional criteria (as in the linear trend case) and is comparable to the improvement reported by Watson for actual GNP data.

The results of the experiments reported in this paper suggest that decomposition of GNP and other economic time series in the SS framework using the Kalman filter should be interpreted with caution. The model may be attributing to the cycle component variation that is actually due to the trend, while understating the magnitude of shocks in the trend process. The result could be spurious cycles around a spuriously smooth trend that do not actually portray the characteristics of the underlying process.

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