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# TECHNICAL PROGRESS IN U.S. MANUFACTURING SECTORS, 1948-1973: AN APPLICATION OF LIE GROUPS

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## ABSTRACT

The purpose of this paper is to apply the theory of Lie transformation groups as developed by the first author, and derive a testable model of production and technical change. The econometric model is then applied to data derived by F. Gollop and D. Jorgenson for U.S. manufacturing industries for the years 1948-1973.

This is the first empirical work in economics to incorporate the theory of Lie transformation groups, so the results are new, but they are also interesting. Using Zellner's seemingly unrelated regression equations method of generalized least squares produces an estimate of a model for the 21-industry system which has a high degree of explanatory power: The system's weighted-$R^2$ is 0.9675 and all coefficients are statistically significant at the 5% level (on the basis of t-tests). While the "form" of technical change in a given industry of the model is probably new, it is easily characterized within the Lie group structure and the system estimate is statistically significant.

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## 1. INTRODUCTION

Since the seminal studies of Schmookler [1952] and Solow [1957], there have been many studies on technical change for the United States' economy and various industrial sectors.[1] Most of these studies computed rates of technical change for the sectoral or aggregate production function, or sectoral price function.[2] While these studies calculated rates of technical change for the sectoral or aggregate production (price) function under consideration, attempts were not usually made to model technical change.

Although technical change to the purist is the rate of growth of an economy's "stock of technological knowledge", it is clear that this is not a very practical definition, as the stock of knowledge does not lend itself to empirical measurement. Therefore, we concern ourselves with measuring the results of technological advance. Following the established convention, technical change, $\dot{T}/T$, shall be defined under constant returns to scale as that part of the growth of real output, $\dot{Y}/Y$, which is unaccounted for by the weighted sum of the growths of factor inputs, where the weights, $\pi_K$, $\pi_L$, and $\pi_M$, are the distributive shares of capital, labor, and material inputs, respectively:

$$(1) \qquad \dot{T}/T = \dot{Y}/Y - \pi_K(\dot{K}/K) - \pi_L(\dot{L}/L) - \pi_M(\dot{M}/M).$$

Alternatively, technical change could also be defined under constant returns to scale as the negative of the rate of change of the unit price of output unaccounted for by the weighted sum of the rates of change of the factor prices:

$$(1') \qquad \dot{T}/T = -[\dot{P}/P - \pi_K(\dot{r}/r) - \pi_L(\dot{w}/w) - \pi_M(\dot{z}/z)],$$

where $P$, $r$, $w$, and $z$ are the unit prices of output, capital, labor, and materials, respectively. See, for example, Jorgenson and Fraumeni [1980].

While many authors have used various forms of Equations (1) and (1') to calculate rates of technical progress as an end, we shall use it as a means toward the end of "describing" the technical change which has occurred. We will show with the theory of continuous Lie transformation groups that it is possible to estimate particular types of technical progress, free from the restrictions which have been present in the other studies.

## 2. TECHNICAL PROGRESS FUNCTIONS AND THE THEORY OF LIE TRANSFORMATION GROUPS

Our model of production and technical change is a neoclassical framework. We will assume that firms are profit maximizers and that they purchase the services of factors, and sell their output, in markets which are perfectly competitive in the long run. Factors, then, are paid their respective marginal products. Outputs will be assumed to be produced from the employment of three inputs: capital (K), labor (L), and materials or intermediate input (M), according to a general, linearly homogeneous, production function,

$$Y = f(K,L,M) = f(X) \quad \text{with} \quad f(\lambda X) = \lambda f(X), \tag{2}$$

where $f\colon R^3_+ \to R_+$, and $X \equiv [K,L,M]$ is defined as the vector of inputs for notational simplicity. The function $f$ is presumed to satisfy the usual conditions for concavity and differentiability. In addition, we will assume that when technical progress occurs, the mathematical form of the production function is not affected. The effect of technical progress is to cause changes in the "quality" or "effectiveness" of the capital, labor, and material inputs, and therefore, possible substitution among the factors.

More specifically, capital, labor, and material inputs are transformed into "effective capital", $\bar{K}$, "effective labor", $\bar{L}$, and "effective material input", $\overline{M}$, through the technical progress transformation, T,

$$\text{(3)} \qquad T: \begin{cases} \overline{K} = \phi_1(K,L,M,t) = \phi_1(X,t), \\ \overline{L} = \phi_2(K,L,M,t) = \phi_2(X,t), \\ \overline{M} = \phi_3(K,L,M,t) = \phi_3(X,t). \end{cases}$$

The technical progress transformation is composed of the three <u>technical progress functions</u>, $\phi_i: (R_+^3 \times R) \to R_+$ $(i = 1,2,3)$, which in general depend on the values of the three factors, and also the single, real-valued, technical progress parameter, $t$. The functions $\phi_i$, $i = 1,2,3$, are not arbitrary; we will require that they satisfy certain properties. (These properties are discussed in detail in Sato [1980,1981], Mitchell [1984], and Mitchell & Primont [1984], so we will simply state them below.) The assumption above that the form of $f$ is unaffected by technical change implies that the level of output after technical progress, $\overline{Y}$, may be expressed either in terms of the effective inputs, $\overline{X}$, or in terms of the factor inputs and the technical progress parameter, $(X,t)$:

$$\overline{Y} = F(X,t) = F(K,L,M,t) \equiv f(\overline{K},\overline{L},\overline{M}) = f(\overline{X}),$$

where $\overline{X} \equiv [\overline{K},\overline{L},\overline{M}]$, and $F: (R_+^3 \times R) \to R_+$.

By way of an example we now illustrate the so-called "Solow-Stigler controversy". (See Stigler [1961] and Solow [1961].) Recall that $f(X)$ is linearly homogeneous, and suppose that the technical progress functions are given by

$$\overline{K} = \phi_1(X,t) = A(t)K; \ \overline{L} = \phi_2(X,t) = A(t)L; \ \overline{M} = \phi_3(X,t) = A(t)M.$$

(Technical change is of the "Hicks neutral" or "uniform factor augmenting" type.) Then when technical change occurs with K, L, and M fixed, output may be written as

$$\overline{Y} = f(\overline{X}) = f[A(t)X] = A(t)f(X).$$

This correctly indicates that technical change, acting on K, L, and M alone, is completely transformed to a scale effect. In the spirit of Solow and others, all of the growth in output would incorrectly be attributed to a "residual", unexplained by growth in the inputs (which have been fixed), or increases in the quality of the inputs, when in fact the latter has been the true source of the growth in output. In this example, because technical change is completely transformed to a scale effect, we say that the production function is "holothetic" under the given type of technical progress. This example shows that in some situations it may not be possible to estimate technical change and the effects of scale. Fortunately, the theory of continuous Lie transformation groups offers a solution to the holotheticity problem.

It is clear that the situation we want is that in which a particular production function is not "holothetic" under a given type of technical progress. For the purpose of clarity, we formally define holotheticity:

> **Definition. A production function is said to be "holothetic" under a particular type of technical progress transformation if and only if the impact of the technical progress is transformed entirely to a scale effect. Therefore, f is holothetic under T:** $\{\phi_1, \phi_2, \phi_3\}$ **if and only if**

$$\textbf{(4)} \quad \overline{Y} = f(\overline{X}) = f[\phi_1(X,t),\ \phi_2(X,t),\ \phi_3(X,t)] = H[t, f(X)].$$

(See also Mitchell [1984].) It is clear that the isoquants of a production function which is holothetic under a particular type of technical progress will be unchanged with technical progress; they will merely be relabeled.

Let us begin our investigation of the holotheticity problem by imposing some restrictions on the technical progress functions, $\phi_i$.

Property 1: Any two applications of the transformation are equivalent to some single application of the transformation. That is, given two values of $t$, $t_1$ and $t_2$, there exists a third value of $t$, say $t_3$, such that

$$\phi_1[\phi_1(X,t_1),\phi_2(X,t_1),\phi_3(X,t_1),t_2] = \phi_1(X,t_3),$$

$$\phi_2[\phi_1(X,t_1),\phi_2(X,t_1),\phi_3(X,t_1),t_2] = \phi_2(X,t_3),$$

$$\phi_3[\phi_1(X,t_1),\phi_2(X,t_1),\phi_3(X,t_1),t_2] = \phi_3(X,t_3).$$ [3]

(Alternatively, if $\Phi \equiv [\phi_1,\phi_2,\phi_3]$, then the above may be written more compactly as $\Phi[\Phi(X,t_1),t_2] = \Phi(X,t_3)$.)

Property 2: There exists a value of the technical progress parameter, $t_0$, which defines the identity transformation:

$$\phi_1(X,t_0) = K; \quad \phi_2(X,t_0) = L; \quad \phi_3(X,t_0) = M.$$ [4]

(Alternatively, $\Phi(X,t_0) = X$.)

Property 3: For any transformation, $T$, specified by a particular value of the technical progress parameter, $t$, there exists a value, $s$, which defines the inverse transformation:

$$\varphi_1[\varphi_1(X,t), \phi_2(X,t), \varphi_3(X,t),s] = K,$$

$$\phi_2[\phi_1(X,t), \phi_2(X,t), \phi_3(X,t),s] = L,$$

$$\phi_3[\phi_1(X,t), \phi_2(X,t), \phi_3(X,t),s] = M.$$ [5]

(Alternatively, $\Phi[\Phi(X,t),s] = X$.)

A transformation, T: $\{\phi_1, \phi_2, \phi_3\}$, satisfying these properties shall be called a Lie transformation group. (For more on these and related properties, see Mitchell [1984] and Mitchell & Primont [1984].)

The Holotheticity Condition

Given the above restrictions on the technical progress transformation, we now present the condition under which a production function, f, is holothetic under a given technical progress transformation, T.

First, we define the functions $\xi_i(X)$ as

$$\xi_i(X) \equiv \frac{\partial \phi_i(X,t_0)}{\partial t} = \left(\frac{\partial \phi_i}{\partial t}\right)_0, \quad i = 1,2,3.$$

Using these functions, we define a linear operator, U, as

$$(5) \quad U \equiv \xi_1(X)(\partial/\partial K) + \xi_2(X)(\partial/\partial L) + \xi_3(X)(\partial/\partial M).$$

This operator we call the "infinitesimal transformation of technical change", and it can be used to describe the first order measure of technical change:

$$(6) \quad (\partial Y/\partial t)_0 \hat{=} Uf = \xi_1(X)\frac{\partial f}{\partial K} + \xi_2(X)\frac{\partial f}{\partial L} + \xi_3(X)\frac{\partial f}{\partial M}.$$

Having introduced U, the following theorem states the necessary and sufficient condition for holotheticity.

**Theorem 1. The necessary and sufficient condition that a production function, $f$, be holothetic under a technical progress transformation, $T$, is that**

$$(7)\quad Uf = \xi_1 f_K + \xi_2 f_L + \xi_3 f_M = G(f),$$

**where $U$ is given by Equation (5) and $G$ is some arbitrary, nontrivial, function of $f$.**

(For a proof, see Sato [1981].)

Recalling that $Uf$ represents the first order measure of technical change, Theorem 1 states that the necessary and sufficient condition for holotheticity is that merely the first order measure of technical change be representable as some function of the original production function.

## 3. EXISTENCE AND UNIQUENESS OF A HOLOTHETIC TECHNOLOGY

Given a particular technical progress transformation, we would like to be able to identify all classes of production functions which are holothetic under the given technical progress transformation. Again, from Sato [1980, 1981] we have:

**Theorem 2. Given a technical progress transformation, $T$, which satisfies the conditions of a Lie transformation group, there exists one and only one class of production functions which is holothetic under $T$.**

(For a proof, see Sato [1980,1981].)

Clearly, the implied class of production functions of Theorem 2 must satisfy Equation (7). To solve the partial differential equation in (7) for $f(X)$, John [1971] shows that we should solve the following system of ordinary differential equations:

$$(8)\qquad dK/\xi_1(X) = dL/\xi_2(X) = dM/\xi_3(X) = df/G(f).$$

Example. To illustrate this, consider a specific form of the Hicks neutral type of technical progress used earlier; in particular, let $A(t) = e^{at}$. Then, the system of ordinary differential equations of Equation (8) is $dK/aK = dL/aL = dM/aM = df/G(f)$. Solving this system for the general form of $f(X)$ gives the solution $Y = f(X) = H[KQ(\frac{L}{K}, \frac{M}{K})]$, which is nothing but Shephard's class of homothetic production functions.

## 4. EXISTENCE OF A LIE TECHNICAL PROGRESS TRANSFORMATION

Considering the holotheticity problem from the other point of view, we would like to know what technical progress transformations exist which will result in the holotheticity of a given production function.

**Theorem 3. Given a particular production function, there exists at least one technical progress transformation which satisfies the properties of a Lie transformation group and under which the production function is holothetic.**

It is clear that a Lie type of technical progress transformation could not be unique for there are three functions to be found--$\xi_1(X)$, $\xi_2(X)$, and $\xi_3(X)$--and we need only satisfy the single Equation, (7). Therefore, we are free to choose two of the three infinitesimal transformations; then the third is uniquely determined by Equation (7). The implication for empirical studies is that once the production function, $f$, has been given (if it must be) and a technical progress transformation, $T$, has been chosen, it simply must be verified that $f$ is not holothetic under $T$.

## 5. ESTIMATION OF TECHNICAL CHANGE

Equation (6) indicated that the first order measure of technical change is the infinitesimal transformation, $U$, applied to the production function, $f$. The functions $\xi_1$, $\xi_2$, and $\xi_3$ are the infinitesimal transformations of capital, labor and intermediate inputs, respectively, and the partial derivatives of f with respect to K, L, and M are the marginal products of the respective factors. Noting that $\xi_1(X) = UK$, $\xi_2(X) = UL$, and $\xi_3(X) = UM$, we may interpret the infinitesimal transformations as measuring the technical change which has occurred to the capital, labor, and intermediate inputs. Then we may express the first order measure of technical change as a weighted sum of the three:

(9) Uf = (technical change to capital) MPK

+ (technical change to labor) MPL

+ (technical change to intermediate input) MPM.

(See also Berndt [1980].) If we assume that the inputs are paid their marginal products, and given estimates of technical change, Uf, we can estimate Equation (9)--with an appropriate stochastic term--using different forms for the infinitesimal transformations of capital, labor, and intermediate input.

Recall our assumption that f exhibits constant returns to scale. Then our measure of technical change is exactly as it has traditionally been defined: it is the growth of output unexplained by the weighted sum of the growths of the factor inputs. Then what Kendrick and others have identified as the growth rate of the "total factor productivity index", $\dot{T}/T$, is simply the total measure of technical change:

$$(10) \qquad \dot{T}/T = (1/Y)(\partial Y/\partial t) = \dot{Y}/Y - \pi_K \dot{K}/K - \pi_L \dot{L}/L - \pi_M \dot{M}/M,$$

where, again, $\pi_K$, $\pi_L$, and $\pi_M$ are the relative shares of the respective inputs, and $\pi_K + \pi_L + \pi_M = 1$. If the occurring technical change comprises a Lie transformation group, then near the identity value of t, the change in the total factor productivity index is nothing but the first order measure of technical change, Uf, and the regression equation is

$$(11) \qquad \dot{T}/T \hat{=} (\dot{T}/T)_0 = \pi_K \xi_1/K + \pi_L \xi_2/L + \pi_M \xi_3/M + u,$$

where u is a random disturbance term satisfying the appropriate assumptions. Note what the use of Lie transformation groups has done: The form of the production function need not be specified! Indeed, we do not even have to make any assumptions about the elasticity of substitution! This is the beauty of using Lie transformation groups in productivity analysis, as earlier studies were forced to assume something about the form of the production function and/or the nature of the elasticity of substitution.

The infinitesimal transformations estimated were those corresponding to the familiar factor augmenting types of technical change, the factor additive types, and others to be discussed.

## 6. THE ECONOMETRIC MODEL

Equation (11) was developed as a means of estimating different forms of the infinitesimal transformations of capital, labor, and material inputs in our model. Since the "projective" type technical progress functions have as special cases all of the familiar technical progress functions, let us introduce the projective case.

The technical progress functions for the projective group are given by

$$(12) \qquad T: \begin{cases} \bar{K} = \frac{1}{D} [Ke^{\beta_4 t} + (\beta_1 + \beta_7 L + \beta_{10} M)t] \\ \bar{L} = \frac{1}{D} [Le^{\beta_8 t} + (\beta_2 + \beta_5 K + \beta_{11} M)t], \\ \bar{M} = \frac{1}{D} [Me^{\beta_{12} t} + (\beta_3 + \beta_6 K + \beta_9 L)t] \end{cases}$$

where $D \equiv 1 - t(\beta_{13}K + \beta_{14}L + \beta_{15}M)$. This group gives rise to infinitesimal transformations for capital, labor, and intermediate inputs of the following form:

$$\begin{aligned} \xi_1(X) &= \beta_1 + \beta_4 K + \beta_7 L + \beta_{10} M + \beta_{13} K^2 + \beta_{14} KL + \beta_{15} KM \\ (13) \quad \xi_2(X) &= \beta_2 + \beta_5 K + \beta_8 L + \beta_{11} M + \beta_{13} KL + \beta_{14} L^2 + \beta_{15} LM \\ \xi_3(X) &= \beta_3 + \beta_6 K + \beta_9 L + \beta_{12} M + \beta_{13} KM + \beta_{14} LM + \beta_{15} M^2 \end{aligned}$$

Substituting (13) into (11), introducing an intercept term, and simplifying gives the regression equation for the projective group,

$$\begin{aligned} (14) \quad \frac{\dot{T}}{T} = {} & \beta_0 + \beta_1 \frac{\pi_K}{K} + \beta_2 \frac{\pi_L}{L} + \beta_3 \frac{\pi_M}{M} + \beta_4 \pi_K + \beta_5 \frac{\pi_L K}{L} \\ & + \beta_6 \frac{\pi_M K}{M} + \beta_7 \frac{\pi_K L}{K} + \beta_8 \pi_L + \beta_9 \frac{\pi_M L}{M} \\ & + \beta_{10} \frac{\pi_K M}{K} + \beta_{11} \frac{\pi_L M}{L} + \beta_{12} \pi_M + \beta_{13} K + \beta_{14} L + \beta_{15} M + u \end{aligned}$$

<u>A Multicollinearity Problem</u>

The above equation is suitable for econometric estimation except for the presence of three terms: $\beta_4 \pi_K$, $\beta_8 \pi_L$, and $\beta_{12} \pi_M$. These terms introduce extreme (perfect) multicollinearity. While multicollinearity is usually a sample problem, here it is not because of the assumption of constant returns to scale, which implies $\pi_K + \pi_L + \pi_M = 1$. This will prohibit the estimation of some kinds of technical change. To see this, suppose we were to hypothesize that technical progress were simultaneously capital- and labor-augmenting. Then the technical progress functions would be given by

$$\bar{K} = e^{\beta_4 t} K, \quad \bar{L} = e^{\beta_8 t} L, \quad \bar{M} = M,$$

and the estimation equation would be $\dot{T}/T = \beta_0 + \beta_4 \pi_K + \beta_8 \pi_L + u$. If we use the assumption of constant returns to scale and substitute $\pi_K = 1 - \pi_L - \pi_M$, then the equation becomes

$$\dot{T}/T = (\beta_0 + \beta_4) + (\beta_8 - \beta_4)\pi_L - \beta_4 \pi_M + u,$$

which is exactly the regression equation one would obtain under technical progress hypothesized to be simultaneously labor- and material-augmenting, which is described by the following technical progress functions:

$$\bar{K} = K, \quad \bar{L} = e^{(\beta_8 - \beta_4)t} L, \quad \bar{M} = e^{-\beta_4 t} M.$$

The upshot of this example is that we will be unable to distinguish between types of technical change which have factor augmenting characteristics for two (or all three) of the factors.

We stated however, that we would estimate factor augmenting types of technical change. This meant that we would consider the technical progress transformations given by the capital-, labor-, and material-augmenting types. These transformations are represented, respectively, as

(15) $$\bar{K} = e^{\beta_4 t} K, \quad \bar{L} = L, \quad \bar{M} = M;$$

(16) $$\bar{K} = K, \quad \bar{L} = e^{\beta_8 t} L, \quad \bar{M} = M;$$

(17) $$\overline{K} = K, \quad \overline{L} = L, \quad \overline{M} = e^{\beta_{12} t} M.$$

For expository purposes, consider the technical progress described by Equation (15). The alternative form of technical change from which this could not be econometrically distinguished is the following:

$$\overline{K} = K, \quad \overline{L} = e^{-\beta_4 t} L, \quad \overline{M} = e^{-\beta_4 t} M.$$

I.e., labor and materials are "augmented" in exactly the same way. Since this is unlikely--although certainly possible--we have more confidence that our separate estimates of the parameters in Equations (15), (16), and (17) are actually estimates of those parameters.

## Data

The data we have employed were compiled by Frank Gollop and Dale Jorgenson [1980]. The documentation of the measures used would be lengthy and appears in their paper, so there is no need to discuss those issues here. Berndt's comments give a discussion of the advantages and disadvantages of Gollop and Jorgenson's methods, but we agree that one feature in particular represents a "substantial contribution": the inclusion of intermediate inputs into the analysis. As Berndt noted (p. 134), "failing to include intermediate inputs in the aggregate American studies involved neglecting a relatively small amount ... of transactions. At the industry level, however, intermediate inputs are quite important." Additionally, the omission of intermediate inputs "would tend to bias upward the measurement of aggregate factor productivity change" (Sato and Suzawa [1983, p. 56]).

From Gollop and Jorgenson's data, we used their series covering 1948-1973 for labor input, capital input, intermediate input, their prices (to compute shares), and the computed rate of technical change. In light of Kendrick's [1985] analysis of the service sectors of the economy, we chose to focus only on manufacturing sectors. Specifically, we used Gollop and Jorgenson's data for their manufacturing sectors. (See their Table 1.1, pages 30-31, or our tables in the next section.) Therefore, we had 25 observations on rates of technical change, inputs, and shares for each of 21 individual industries. (Data available on request.)

### Methods

Under ideal conditions, we could use ordinary least squares (OLS) regression analysis to estimate an equation for each industry. However, the assumptions of the general linear regression model probably do not apply. In particular, independence of disturbance terms across models is probably an unrealistic assumption. Therefore, we used Zellner's [1962] method of seemingly unrelated regression equations, which is a special case of the generalized least squares (GLS) analysis, to simultaneously estimate all 21 equations. For more on this method, see the Appendix, Zellner [1962], or Theil [1971].

## 7. EMPIRICAL RESULTS

### Factor Augmenting Technical Change

The most frequently used form of technical change is the factor augmenting type, under which an effective factor is the product of an augmenting function of t and the nominal value of the factor. Therefore, capital augmenting (Solow neutral) technical change is given by Equation (15), and the regression equation is $\dot{T}/T = \beta_0 + \beta_4 \pi_K + u$; labor augmenting (Harrod neutral) technical change is given by Equation (16), and the associated regression equation is $\dot{T}/T = \beta_0 + \beta_8 \pi_L + u$; and material augmenting technical change is identified by Equation (17) and the implied regression equation is $\dot{T}/T = \beta_0 + \beta_{12} \pi_M + u$.

Unfortunately, these types of technical change did not test well with Gollop and Jorgenson's data. Using Zellner's method of seemingly unrelated regression equations, none of the models gave a weighted-$R^2$ over 0.5, and none produced more than fourteen significant coefficient estimates, on the basis of the computed t-ratios. We conclude that, as a system applied in all sectors, factor augmenting technical change is not the appropriate type for the data.

Factor Additive Technical Change

One of the distinct advantages of employing Lie transformation groups to analyze technical change is that simple analysis no longer requires the factor augmentation hypothesis. I.e., technical change may be characterized in ways other than those which give constant rates of growth. Consider, for example,

$$\bar{K} = K + \beta_1 t, \quad \bar{L} = L, \quad \bar{M} = M; \tag{18}$$

$$\bar{K} = K, \quad \bar{L} = L + \beta_2 t, \quad \bar{M} = M; \tag{19}$$

$$\bar{K} = K, \quad \bar{L} = L, \quad \bar{M} = M + \beta_3 t. \tag{20}$$

These are more special cases of (12). But in contrast to the cases of Equations (15), (16), and (17), the nonzero rates of growth are not constant, but rather declining with t. E.g. in (18), $\partial\bar{K}/\partial t = \beta_1$ but $\partial \ln\bar{K}/\partial t = \beta_1/(K+\beta_1 t)$, while in (15), $\partial \ln\bar{K}/\partial t = \beta_4$, i.e., constant. From the form of $\phi_1(X,t)$, (18) is called capital additive technical change. Similarly, (19) defines labor additive technical change, and (20) defines material additive technical change. The associated regression equations may be determined directly from (14), or derived through (13) and (11). As in the factor augmenting case, however, the factor additive types did not test well with the data. Under GLS, none of the systems gave a weighted-$R^2$ as high as 0.41 and only the capital additive case produced more than six significant coefficient estimates (only 12). We again conclude that factor additive technical change does not significantly account for U.S. manufacturing technical change.

"Mixed Types" of Technical Change

In the above cases, we estimated systems of equations based on the assumption that technical change was the same across industries. This is clearly restrictive. (In addition to the analyses discussed above, we separately estimated the other nine coefficients in Equation (14), again assuming that technical change was the same in every industry. The results were similar: the weighted-$R^2$ ranged from 0.2356 to only one exceeding 0.5 by a small amount; the number of significant coefficients ranged from only 5, to 15.) As an improvement, however, we attempted to find the single explanatory variable which best described technical change for each industry, and then applied GLS to the resulting system. (Identification of the "best" variable was done with PROC STEPWISE in SAS, with option = MAXR.) The identified variables are given in Table 1 along with the coefficient estimates derived from GLS. While only ten of the coefficients were significant at the 5% level under OLS, 20 were significant under GLS at that level, and the lone exception (furniture) gave $\beta_4$ significant at the 6.6% level. The weighted-$R^2$ for the system, 0.5687, exceeded the values for all 15 systems previously considered.

This line of thought was pursued to the next "level", where technical progress is represented by transformations with two unknown parameters. (We estimated all 102 possible types of technical change under the assumption that technical change was identical in every industry. There are 105 combinations, but we did not estimate those three types which are factor augmenting in two factors. However, none of the models was as good as that which was determined by relaxing the restriction of identical technical change in every industry.) For each industry we identified the two-parameter transformation which best described its technical change. We then applied GLS to the resulting system. For the system, $\bar{R}^2 = 0.8294$, and for 19 industries both t-tests led to rejection of the appropriate null hypothesis at the 5% level. The exceptions were the paper ($\beta_{13}$ significant at 13% level) and furniture ($\beta_1$ significant at 33% level) industries. All other coefficients were significantly different from zero at the 5% level; the results are in Table 2. The most prevalent variables (components) were the $\pi_L M/L$ and $L$ variables, each occurring six times in the system. These components are termed "material-ratio additive to labor" and "inverse-labor additive", respectively. (The names are derived from the forms of the holothetic technologies which are $f = H[(L/M) + Q(K,M)]$ and $f = H[(1/L) + Q(K/L, M/L)]$, respectively. See Sato [1981], Table 1, for two-factor analogues.)

Taking this process one step further produced the best estimate of its kind. We did not estimate all possible three-parameter models. Rather we found the three-parameter model of technical change which was best for each industry, and estimated the resulting system. With $\bar{R}^2 = 0.9771$ under GLS, in 17 industries all three t-tests led to a rejection of the appropriate null hypothesis at the 5% level. On the basis of the available (and approximate) statistical tests, this is a high degree of explanatory power.

As a refinement, and at a slight cost in terms of the weighted-$R^2$ for the system, we dropped the insignificant coefficients from the four industries and re-estimated. The resulting model hypothesized technical progress to be a three-parameter type for 17 industries, a two-parameter type for three industries, and a one-parameter type for miscellaneous manufactures. The generalized least squares estimate ($\bar{R}^2 = 0.9675$) had <u>all</u> remaining coefficients significant at the 5% level; the results are in Table 3.

We continued the analysis to the levels of 4-, 5-, 6-, and 7-parameter types of technical progress transformations. While the weighted-$R^2$ statistics were comparable to the 3-parameter results

Table 1. One-Parameter Technical Change, GLS Results ($\bar{R}^2 = 0.5687$).

| Industry | Variable | Estimated Coefficient |
|---|---|---|
| Food products | $\pi_L K/L$ | $b_5 = -0.46$ |
| Tobacco manufactures | M | $b_{15} = 0.12$ |
| Textile mill products | M | $b_{15} = 0.008$ |
| Apparel | $\pi_K/K$ | $b_1 = -2.25$ |
| Paper & allied products | $\pi_K M/K$ | $b_{10} = -2.00$ |
| Printing & publishing | $\pi_K/K$ | $b_1 = -8.29$ |
| Chemicals | $\pi_K M/K$ | $b_{10} = -0.82$ |
| Petroleum & coal | $\pi_K$ | $b_4 = -0.89$ |
| Rubber | $\pi_M L/M$ | $b_9 = -0.10$ |
| Leather | M | $b_{15} = 0.06$ |
| Lumber (exc. furniture) | $\pi_L M/L$ | $b_{11} = -0.13$ |
| Furniture & fixtures | $\pi_K$ | $b_4 = -0.45$* |
| Stone, clay, & glass | $\pi_M K/M$ | $b_6 = 0.16$ |
| Primary metals | $\pi_K M/K$ | $b_{10} = -0.69$ |
| Fabricated metals | $\pi_K L/K$ | $b_7 = -0.18$ |
| Non-electrical machinery | $\pi_M K/M$ | $b_6 = 0.12$ |
| Electrical machinery | $\pi_L K/L$ | $b_5 = 0.15$ |
| Transportation equipment | $\pi_K$ | $b_4 = -0.76$ |
| Motor vehicles | L | $b_{14} = -0.02$ |
| Professional photographic equipment & watches | $\pi_K L/K$ | $b_7 = -0.33$ |
| Miscellaneous manufactures | $\pi_M K/M$ | $b_6 = 0.19$ |

*Insignificant at 5% level; all others significant at 5% level.

(ranging from 0.9324 to 0.9759), the number of significant models in the system, based on separate t-tests, ranged from only seven to ten. As a last alternative to consider, we relaxed our implicit assumption that the technical change transformations should have the same number of parameters in every industry. Without estimating all possible combinations, we decided on the following method. For each industry we looked at the best 1-parameter, 2-parameter, ..., 12-parameter models, and the 13-parameter model. Among the 13 we chose that model which gave the maximum adjusted-$R^2$ under OLS. After doing this for all 21 industries, we applied GLS to the resulting system. While far more complicated than our "modified 3-parameter model" described above, the results did not justify the added "bulk", and we retained the estimated model in Table 3 as the best estimate. While the estimated model in Table 3, derived from the theory of Lie transformation groups, provides a good statistical "fit", it illustrates a discomforting characteristic. We may not be able to give a simple economic or intuitive interpretation to the technical progress functions. More to the point, the appearance of 19 $\beta_{13}$, $\beta_{14}$, or $\beta_{15}$'s simply defies interpretation, which is unfortunate. However, there is no reason to believe that technical change occurs in simple, intuitive forms.

Table 2

Two-Parameter Technical Change, GLS Results ($\bar{R}^2 = 0.8294$)

| Industry | Estimated Coefficients | |
|---|---|---|
| Food products | $b_{11} = 0.795$ | $b_{13} = -0.021$ |
| Tobacco manufactures | $b_6 = -0.591$ | $b_{10} = -4.08$ |
| Textile mill products | $b_3 = 5.06$ | $b_{15} = 0.025$ |
| Apparel | $b_2 = -5.03$ | $b_{10} = -0.402$ |
| Paper & allied products | $b_{10} = -2.28$ | $b_{13} =$ * |
| Printing & publishing | $b_5 = 0.735$ | $b_6 = -0.293$ |
| Chemicals | $b_2 = 6.65$ | $b_7 = -2.01$ |
| Petroleum & coal | $b_4 = -0.877$ | $b_{14} = 0.074$ |
| Rubber | $b_{14} = -0.088$ | $b_{15} = 0.045$ |
| Leather | $b_2 = -1.55$ | $b_{11} = 0.361$ |
| Lumber (exc. furniture) | $b_9 = -0.346$ | $b_{11} = -0.868$ |
| Furniture & fixtures | $b_1 =$ * | $b_4 = -0.820$ |
| Stone, clay, & glass | $b_3 = 4.91$ | $b_9 = -1.14$ |
| Primary metals | $b_{11} = 0.163$ | $b_{14} = -0.013$ |
| Fabricated metals | $b_{11} = 0.190$ | $b_{14} = -0.007$ |
| Non-electrical machinery | $b_6 = 0.637$ | $b_{10} = 1.07$ |
| Electrical machinery | $b_1 = 5.95$ | $b_7 = -1.22$ |
| Transportation equipment | $b_4 = -0.895$ | $b_{15} = -0.001$ |
| Motor vehicles | $b_{11} = 0.230$ | $b_{14} = -0.027$ |
| Professional photographic equipment & watches | $b_7 = -0.452$ | $b_{14} = -0.009$ |
| Miscellaneous manufactures | $b_4 = -0.263$ | $b_6 = 0.233$ |

*Insignificant at 5% level; all others significant at 5% level.

Table 3. "Modified" Three-Parameter Technical Change, GLS Results ($\bar{R}^2 = 0.9675$)

| Industry | Estimated Coefficients* | | |
|---|---|---|---|
| Food products | $b_{11} = 0.910$ | $b_{13} = -0.024$ | $b_{14} = 0.018$ |
| Tobacco manufactures | $b_3 = -1.87$ | $b_{10} = -4.06$ | $b_{13} = -0.170$ |
| Textile mill products | $b_2 = -19.67$ | $b_3 = 11.27$ | $b_{11} = 1.21$ |
| Apparel | $b_5 = -2.36$ | $b_{13} = 0.102$ | $b_{14} = -0.056$ |
| Paper & allied products | $b_3 = 1.05$ | $b_7 = -2.95$ | $b_{15} = -0.006$ |
| Printing & publishing | $b_3 = 3.09$ | $b_5 = 1.19$ | $b_6 = -0.797$ |
| Chemicals | $b_2 = 24.27$ | $b_3 = -4.72$ | $b_{14} = 0.025$ |
| Petroleum & coal | $b_3 = -4.67$ | $b_7 = -12.33$ | $b_{11} = -0.954$ |
| Rubber | $b_2 = -4.16$ | $b_{13} = 0.018$ | $b_{14} = -0.091$ |
| Leather | $b_2 = -10.51$ | $b_{11} = 3.03$ | $b_{15} = -0.385$ |
| Lumber (exc. furniture) | $b_1 = -3.75$ | $b_9 = 0.157$ | $b_{13} = -0.055$ |
| Furniture & fixtures | $b_1 = 4.68$ | $b_4 = -2.33$ | $b_{15} = 0.019$ |
| Stone, clay, & glass | $b_3 = 5.64$ | $b_9 = -1.32$ | $b_{10} = -0.754$ |
| Primary metals | $b_{11} = 0.486$ | $b_{13} = -0.004$ | $b_{14} = -0.015$ |
| Fabricated metals | $b_1 = -11.00$ | $b_4 = 0.909$ | $b_{14} = -0.012$ |
| Non-electrical machinery | $b_1 = 4.53$ | $b_{11} = 0.628$ | $b_{14} = -0.009$ |
| Electrical machinery | $b_1 = 6.10$ | $b_4 = 0.286$ | $b_7 = -1.26$ |
| Transportation equipment | $b_4 = -0.866$ | $b_{15} = -0.001$ | |
| Motor vehicles | $b_{11} = 0.214$ | $b_{14} = -0.031$ | |
| Professional photographic equipment & watches | $b_7 = -0.475$ | $b_{14} = -0.0078$ | |
| Miscellaneous manufactures | $b_6 = 0.204$ | | |

*All coefficients statistically significant at the 5% level.

## 8. CONCLUSIONS

In the foregoing sections we have developed the theory of Lie transformation groups and applied it to a model of [production and] technical change. We then developed an econometric model which would statistically described some time series for rates of technical change, and applied the model to data derived by Gollop and Jorgenson for U.S. manufacturing industries over the sample period 1948-1973.

The econometric results were reported in Section 7, and the results are both new and interesting. As far as we know, this work is the first empirical study in economics to incorporate the theory of Lie transformation groups. This would be an empty claim were the results uninteresting, but they were not, at least from a statistical point of view. Using Zellner's seemingly unrelated regression equations approach to generalized least squares produced an estimate of a set of models of technical change which had a high degree of explanatory power as measured by the system's weighted-$R^2$ statistic (0.9675), and gave all individual coefficients to be statistically different from zero at the 5% level.

APPENDIX

Equation (14) is the basic form of our regression model. The full model would be represented by

$$\text{(A1)} \qquad y_t^s = \beta_o^s + \sum_{j \varepsilon I^s} z_{t,j}^s \beta_j^s + u_t^s, \qquad \begin{array}{l} s = 1,2, \ldots, 21; \\ t = 1,2, \ldots, 25; \end{array}$$

where $y_t^s$ is the rate of technical change for sector s in period t; $I^s$ is the set of indexes for the unknown parameters in the infinitesimal transformation for sector s; $k_s$ is the number of elements in $I^s$; $z_{t,j}^s$ is the value of the $j^{th}$ variable for sector s in period t; and $u_t^s$ summarizes stochastic and neglected variables for sector s in period t. (More specifically with regard to $z_{t,j}^s$, it is the variable having the coefficient $\beta_j$ in Equation (14), where the sector and period indexes have been omitted.) Note that $\sum_j z_{t,j}^s \beta_j^s = U^s f^s / f^s$, the infinitesimal transformation for sector s applied to the sector's production function as a fraction of the sector's output.

Under ideal conditions, there will be independence between disturbances of the same equation but different periods, between disturbances of different equations and different periods, and between disturbances of different equations but the same period. (These conditions are in addition to the usual conditions for the general linear model.) If these hold, then (A1) may be estimated with OLS sector-by-sector, s = 1,2, ..., 21. However, these conditions of independence probably do not hold. While we will maintain that $u_r^s$ and $u_t^s$ $(r \neq t)$ are independent,[7] and that $u_r^q$ and $u_t^s$ $(q \neq s, r \neq t)$ are independent, it would be unreasonable to expect $u_t^q$ and $u_t^s$ $(q \neq s)$ to be independent. Economic conditions affecting technical change in sector q will surely have an effect in sector s. To the extent that this is true, $u_t^q$ and $u_t^s$ will be correlated. In that case, sector-by-sector OLS estimation of (A1) will give inefficient estimators.

Given the presence of these correlations, we used Zellner's method for two-stage Aitken estimators for the generalized least squares model (GLS), which estimators are asymptotically equivalent to their corresponding maximum likelihood estimators.

Let us define the $25 \times (k_s + 1)$ matrix $Z^s \equiv [z^s_{t,j}]$ $(s = 1,2, \ldots, 21;\ t = 1,2, \ldots, 25;\ j = 0,1, \ldots, k_s)$, where $z^s_{t,j}$ is as before except we now have $z^s_{t,0} \equiv 1$ for all $s$ and $t$. Also, let $y^s \equiv [y^s_1, y^s_2, \ldots, y^s_{25}]'$, $\beta^s \equiv [\beta^s_0, \beta^s_1, \ldots, \beta^s_{k_s}]'$, and $u^s = [u^s_1, u^s_2, \ldots, u^s_{25}]'$. Then our full model can be written as one equation:

$$\text{(A2)} \quad y = \begin{bmatrix} y^1 \\ y^2 \\ \vdots \\ y^{21} \end{bmatrix} = \begin{bmatrix} Z^1 & 0 & \ldots & 0 \\ 0 & Z^2 & \ldots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \ldots & Z^{21} \end{bmatrix} \begin{bmatrix} \beta^1 \\ \beta^2 \\ \vdots \\ \beta^{21} \end{bmatrix} + \begin{bmatrix} u^1 \\ u^2 \\ \vdots \\ u^{21} \end{bmatrix} = Z\beta + u,$$

where the null matrix in the $i^{th}$ "column" of the partitioned matrix, $Z$, is $25 \times (k_i + 1)$. In (A2), $y$ and $u$ are column vectors of 525 elements, $\beta$ is a column vector with

$K \equiv \sum_{s=1}^{21} (k_s + 1)$ elements, and $Z$ is a $525 \times K$ matrix.

If all disturbances are independent, then (A2) may be estimated by OLS and the least squares estimate of $\beta$ would be $b = (Z'Z)^{-1}Z'y$. Furthermore, if the disturbances were distributed according to $N(0,\Sigma)$, where $0 \in R^{525}$ and $\Sigma = \sigma^2 I_{25} \otimes I_{21}$ $= \sigma^2 I_{525}$ ($\sigma^2 = \text{Var}(u^s_t)$ for all $s,t$), then $b$ is distributed according to $N(\beta, \sigma^2(Z'Z)^{-1})$. However, when we permit $u^q_t$ and $u^s_t$ $(q \neq s)$ to be correlated, then $b$ will no longer be an efficient estimator of $\beta$, since the disturbances are no longer distributed according to $N(0,\sigma^2 I_{525})$.

If we permit correlation only across sectors for each period, then $Var(u) = V$, where $V$ is block diagonal with $\sum = \sigma^2 I_{25}$ along the diagonal. Under these conditions the Aitken estimator for $\beta$ is $b^* = (Z'V^{-1}Z)^{-1}Z'V^{-1}y$, and if $u$ comes from $N(0,V)$ then $b^*$ comes from $N(\beta,\sigma^2(Z'V^{-1}Z)^{-1})$. Unfortunately, $V$ is not known. Zellner's [1962] method for these seemingly unrelated regression equations is to replace $\sum$ with the matrix of mean squares and products of the estimated least squares disturbance vectors, $\hat{u}^s$: $\hat{\sum} = S = \frac{1}{n}[s_{ij}]$ where $s_{ij} = \hat{u}^i \cdot \hat{u}^j$, and $n$ is the number of observations. This matrix $S$ is then put on the diagonal of $V$ to give $\hat{V}$ and we have our estimator $b^{**} = (Z'\hat{V}^{-1}Z)^{-1}Z'\hat{V}^{-1}y$ which is distributed according to $N(\beta,\sigma^2(Z'\hat{V}^{-1}Z)^{-1})$ when $u$ comes from $N(0,\hat{V})$. This is the estimator we have used. (See also Theil [1971].)

# NOTES

[1]Kendrick [1961, 1973], Kendrick and Sato [1963], Sato [1970], Christenson and Jorgenson [1973], Gollop and Jorgenson [1980], Jorgenson and Fraumeni [1980], and Kendrick and Grossman [1980], to name a few.

[2]Jorgenson and Fraumeni [1980] took the dual approach and estimated sectoral price functions with disaggregated data.

[3]If $t$ represents time, then a reasonable specification for $t_3$ would be $t_3 = t_1 + t_2$.

[4]If $t$ represents time, the obvious value for $t_0$ would be zero.

[5]Again, if $s$ and $t$ are measures of time, $s = -t$ and Property 3 is analogous to Fisher's "time-reversal test" in index number theory.

[6]See Mitchell [1984].

[7]This implies that the disturbances for sector $s$ are serially uncorrelated. On the basis of Durbin-Watson statistics, this assumption was justified in all our models.

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