Monetary Policy Analysis when Planning Horizons are Finite

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Finite-Horizon Planning

- Paper proposes to relax a particularly strong assumption of DSGE monetary models: the assumption that agents formulate complete infinite-horizon state-contingent plans that are optimal, under a correct understanding of how the economy will evolve [according to one’s model]

This is surely not feasible in practice, no matter the extent to which one may assume agents are motivated and experienced — for example, even in artificial environments where set of feasible moves from any position is finite (e.g., chess or go), not even the best professional players (human or AI) can solve the game by backward induction, and simply execute the optimal strategy.
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What the best programs (DeepMind, AlphaGo) actually do: each time one must move,

1. **look forward** from one’s current position a finite number of steps, calculating the possible positions that can be reached by finite sequences of moves [under a model of opponent play]
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3. by backward induction from the nodes at which the tree search has been terminated [and value function applied], assign a value to each of the possible initial moves from the current position
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2. evaluate those possible positions, using a **value function** that assigns an estimated probability of winning from that position

3. by backward induction from the nodes at which the tree search has been terminated ([and value function applied]), **assign a value** to each of the possible **initial moves** from the current position

4. **select** the move with highest estimated value
A crucial observation about such a strategy: in practice, the value function that is used cannot be exactly correct — can only evaluate possible positions on the basis of a few features, the average values of which can be estimated from some finite database of prior (or simulated) play — not a complete description of the state.
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- A crucial observation about such a strategy: in practice, the value function that is used cannot be exactly correct — can only evaluate possible positions on the basis of a few features, the average values of which can be estimated from some finite database of prior (or simulated) play — not a complete description of the state.

- This is in fact why forward planning improves the algorithm, even when forward planning is only possible for a modest number of steps ahead.
Goals of the Paper

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- Will illustrate the method in the context of a basic New Keynesian DSGE model.

- Application: consider predicted effects of “forward guidance” about monetary policy years into the future.
Household with \( k \)-Period Planning Horizon

- Household \( i \) problem in period \( t \): choose spending plan \( \{c^i_{\tau}(s_{\tau})\} \) for periods \( t \leq \tau \leq t + k \) to maximize

\[
\hat{E}^i_t \sum_{\tau=t}^{t+k} \beta^{\tau-t} u(c^i_{\tau}; \xi_{\tau}) + \beta^{k+1} v(b^i_{t+k+1}; s_{t+k})
\]

subject to constraints

\[
b^i_{\tau+1} = (1 + i_{\tau}) [b^i_{\tau}(P_{\tau-1}/P_{\tau}) + y_{\tau} - c^i_{\tau}]
\]

for all \( t \leq \tau \leq t + k \),

- Here \( v(b^i_{\tau+1}; s_{\tau}) \) is the value function used to evaluate possible situations in a terminal state \( s_{\tau} \)
Expectations about periods \( t \leq \tau \leq t + k \) used in planning exercise:

- **deduced from structural equations of model** (including monetary policy rule) for periods \( t \) through \( t + k \)
- hence **take account** of any announced changes in policy, over the planning horizon
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Decisions with a Finite Planning Horizon

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  - hence take account of any announced changes in policy, over the planning horizon
  - but no consideration of branches beyond horizon $t + k$ means aggregate conditions in period $t + j$ assumed to be determined by decisions of people who plan only $k - j$ periods ahead

- Just as household models own behavior in future period $t + j$ as if will only have horizon of length $k - j$ then, models all other households and firms as optimizing, but only having horizons of length $k - j$ in period $t + j$
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Value function is learned from past experience
Learning the Household Value Function

As part of its finite-horizon planning exercise in period $t$, each household $i$ computes an estimate of the value of its objective [expected discounted utility from $t$ onward], for any (counterfactual) level of wealth $b^i_t = b$ that it might have brought into the period

— call this function $v_t^{est}(b)$
Learning the Household Value Function

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  — call this function $v^\text{est}_t(b)$

- Let the value function used in its period $t$ planning exercise be $v_t(b)$; this is then updated for the next period’s exercise using the error-correction rule [“constant-gain learning”]

  $$v_{t+1}(b) = v_t(b) + \gamma (v^\text{est}_t(b) - v_t(b))$$

  for all $b$

  for some $0 < \gamma \leq 1$. 
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for all $b$ for some $0 < \gamma \leq 1$.

In a constant environment, this converges to the $v(b)$ that solves the Bellman equation for infinite-horizon optimization [essentially, solution through value-function iteration]
Log-Linearized Dynamics: Aggregate Demand

- Log-linearize household decision rule around perfect foresight steady state with constant inflation $\bar{\Pi}$, and assume value function $v(b)$ that is optimal for that steady state.
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- Let $y^j_t$ be aggregate real spending (and income) in period $t$ if all households have planning horizon $j$ [any $j \geq 0$]; similarly, $\pi^j_t$ overall inflation rate if all firms have planning horizon $j$, $\hat{i}^j_t$ the nominal interest rate if CB reacts to $\pi^j_t$ and $y^j_t$ — then finite-horizon planning implies that

$$y^j_t - g_t = E_t[y^j_{t+1} - g_{t+1}] - \sigma(\hat{i}^j_t - E_t\pi^j_{t+1})$$

for each $j \geq 1$, and

$$y^0_t - g_t = -\sigma\hat{i}^0_t.$$
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for each $j \geq 1$, and

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Compare to prediction with infinite-horizon optimization:

$$y_t - g_t = E_t[y_{t+1} - g_{t+1}] - \sigma(\hat{i}_t - E_t\pi_{t+1})$$
Price Setting

Can apply a similar analysis to the decisions of price-setting firms:

- Assume a Calvo-Yun model of staggered price adjustment by monopolistic competitors

- But suppose that a firm $f$ that reoptimizes its price has only a $k$-period planning horizon
  
  — assigns value $\tilde{v}(P_t^f / P_{t+k})$ to continuation profits, if newly chosen price $P_t^f$ still applies in period $t + k + 1$

  — and again this value function is learned using an error-correction rule
Linearizing optimal finite-horizon pricing rule, and aggregating prices of all firms, yields

\[ \pi^j_t = \kappa (y^j_t - y^*_t) + \beta E_t \pi^{j-1}_{t+1} \]

for all \( j \geq 1 \), and

\[ \pi^0_t = \kappa (y^0_t - y^*_t) \]
Log-Linearized Dynamics: Aggregate Supply

Linearizing optimal finite-horizon pricing rule, and aggregating prices of all firms, yields

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Compare to prediction with infinite-horizon optimization:

\[ \pi_t = \kappa (y_t - y^*_t) + \beta E_t \pi_{t+1} \]
Closing the model: assume monetary policy specified by a reaction function

\[ \hat{i}_t = i^*_t + \phi_{\pi,t}\pi_t + \phi_{y,t}y_t \]

with possibly time-varying coefficients \( \phi_{\pi,t}, \phi_{y,t} \geq 0 \).

Hence for any planning horizon \( j \geq 0 \),

\[ \hat{i}^j_t = i^*_t + \phi_{\pi,t}\pi^j_t + \phi_{y,t}y^j_t \]
A Forward Guidance Experiment

Suppose that for some time prior to the policy experiment, economy has been in steady state with inflation rate $\bar{\Pi}$, and value functions optimal for that environment have been learned.
A Forward Guidance Experiment

- Suppose that for some time prior to the policy experiment, economy has been in steady state with inflation rate $\bar{\Pi}$, and value functions optimal for that environment have been learned.

- But suppose that at some date $t_0$, it is announced that monetary policy will be determined by a different reaction function (consistent with a different inflation target, and possibly different response coefficients as well) until some date $T$.

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— from date $T$ onward, policy will instead revert to “normal” reaction function, a Taylor rule consistent with inflation rate $\bar{\Pi}$ once again.
For any assumed planning horizon $k$, we can uniquely solve for model’s predictions for dynamics

— horizon $j = 0$ variables involve no forward planning, so uniquely determined [by backward-looking value function]

— horizon $j \geq 1$ variables uniquely determined if horizon $j - 1$ variables are uniquely determined
Equilibria with Forward Guidance

- For any assumed planning horizon $k$, we can **uniquely** solve for model’s predictions for dynamics:
  - horizon $j = 0$ variables involve no forward planning, so uniquely determined [by backward-looking value function]
  - horizon $j \geq 1$ variables uniquely determined if horizon $j - 1$ variables are uniquely determined

- If the planning horizons of all households and firms satisfy $k \geq T - t_0 - 1$, then model predictions **coincide** with a rational expectations equilibrium
  - the **specific** RE solution in which economy returns to steady state from date $T$ onward
If the newly announced reaction function conforms to the “Taylor Principle” \[ \phi_\pi + (1 - \beta/\kappa)\phi_y > 1 \], then this RE solution converges for \( T \to \infty \)

\( \Rightarrow \) even in the case of a permanent change in policy, solution with finite-horizon planning will approximate the RE solution, if sufficiently many have sufficiently long horizons
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- and solves the equilibrium selection problem that bedevils RE analysis
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But the situation is very different if Taylor Principle not satisfied, as in case of an interest-rate peg
RE results, for temporary peg [e.g., commit to keep interest rate at lower bound], if select equilibrium which returns to steady state from \( t = T \) onward:

- effects on output and inflation predicted to grow explosively as \( T \to \infty \Rightarrow \) forward guidance should be extremely effective (if credible)

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Instead, with any finite horizon \( k \), commitment to a low-rate peg is predicted to be stimulative, but effects remain bounded no matter how long the commitment

— and may be quite modest, if \( k \) is not too large
Equilibria with an Interest-Rate Peg

- RE results, for a **permanent** peg: all non-explosive RE solutions converge in long run to inflation rate consistent with **Fisher Equation**

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Equilibria with an Interest-Rate Peg

- RE results, for a **permanent** peg: all non-explosive RE solutions converge in long run to inflation rate consistent with **Fisher Equation**
  - implying that commitment to maintaining a lower nominal interest rate should **lower** inflation, at least eventually

- Instead, with any finite horizon \( k \), commitment to lower rate (even if permanent) must **increase** inflation
  - **none** of the RE solutions are similar to the finite-horizon solution, for **any** distribution of planning horizons
  - suggesting that RE analysis may be quite misleading in this case
Conclusions

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- In some cases, RE outcome (with suitable equilibrium selection) is a decent approximation to what a model of finite-horizon forward planning would imply.

- But in other cases (e.g., commitment to maintain a fixed nominal interest rate for a long time), conclusions are very different, even if one assumes highly sophisticated forward planning for a long distance into future.

- Hence checking the robustness of conclusions to modest departures from perfect rationality is important for monetary policy analysis.