The Tail that Keeps the Riskless Rate Low

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Government bond yields have fallen since 2008-09 and have remained low

Our story:

Great Recession

⇓

Change in beliefs about tail risk

⇓

Persistent fall in returns on safe, liquid assets

Key Ingredients

- **Main idea:**
  No one knows the true distribution of aggregate shocks
  → Re-estimate beliefs as new data arrives

- **Estimation of beliefs:**
  → Non-parametric approach: tail risk vs uncertainty
  → Use observed macro data, empirical discipline

- **Tail events:** (e.g. the Great Recession)
  → Large changes in beliefs, in tail probabilities
  → Changes are long-lived, even if the underlying shocks are iid

- **Economic environment:**
  Neoclassical production economy with liquidity constraints

- **Quantitative results:**
  → Large and persistent drop in riskless rates (1.45%)
  → Consistent with evidence from option markets

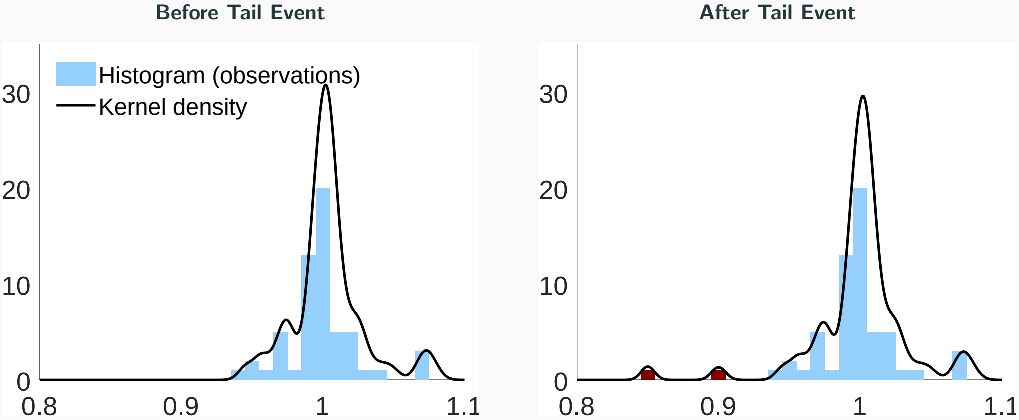
Belief formation

Belief Formation

- Consider an iid shock, ϕ_t, with unknown distribution g
- Information set: finite history of shock realizations $\{\phi_{t-s}\}_{s=0}^{n_t-1}$
- **Goal:** a flexible specification that can capture tail risk
- We use a non-parametric estimator: the Gaussian kernel density

$$\hat{g}_t(x) = \frac{1}{n_t \kappa} \sum_{s=0}^{n_t-1} \Omega\left(\frac{x-\phi_{t-s}}{\kappa}\right)$$

Tail events and beliefs: An example



Tail events → large changes in tail risk (hump on left)

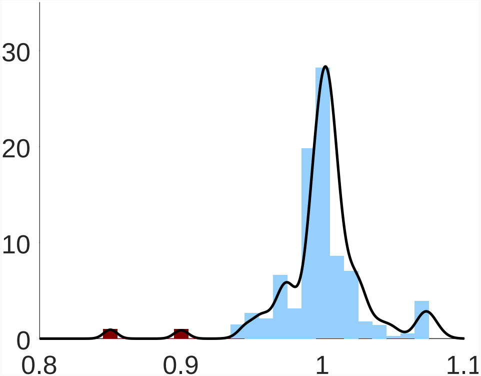
Persistence of belief changes

Exercise I: Simulate future time paths drawing from updated distribution, $\hat{g}_t$

- **Beliefs are martingales:** $\mathbb{E}_t[\hat{g}_{t+j}|\mathcal{I}_t] \approx \hat{g}_t \rightarrow$ **Persistence**

Exercise II: Simulate future time paths without tail events, re-estimate beliefs

- Beliefs eventually revert, but the pace is very slow



Economic Model

Model

- **Production:** $Y_t = K_t^{\alpha} N_t^{1-\alpha}$
  - Aggregate shocks to capital 'quality': $K_t = \phi_t \hat{K}_t$ $\qquad \phi_t \underset{iid}{\sim} g(\cdot)$
  - Law of motion $\hat{K}_{t+1} = K_t(1-\delta) + I_t$
- **Preferences:**
  - Representative HH with stochastic discount factor M_t

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- **Preferences:**
  - Representative HH with stochastic discount factor M_t
- **Role of Liquidity:**
  - Opportunity to invest in an intra-period project: payoff $H(X_t) - X_t$
  - Liquidity constraint: $X_t \leq B_t + \eta \phi_t \hat{K}_t$, where $B_t \equiv$ riskfree bonds

$$\Rightarrow \qquad \frac{1}{R_t^f} = \mathbb{E}_t \left[M_{t+1}(1 + Liq_{t+1}) \right]$$

- **Beliefs:**
  - Distribution g unknown to all agents
  - At each t, observe $\{\phi_1, \ldots, \phi_t\}$
  - Gaussian kernel density estimator $\rightarrow \hat{g}_t$

Quantitative Results

Measurement and Calibration

Aggregate shock:

$$\phi_t = \frac{K_t}{\hat{K}_t} = \frac{\text{Effective capital}}{\text{Yesterday's effective capital} + \text{Investment}}$$

Data: Non-financial assets of US Corporate Business (Flow of Funds)

- Commercial real estate ($\sim 55\%$), equipment and software
- Market value → Effective capital
- Historical cost → Investment

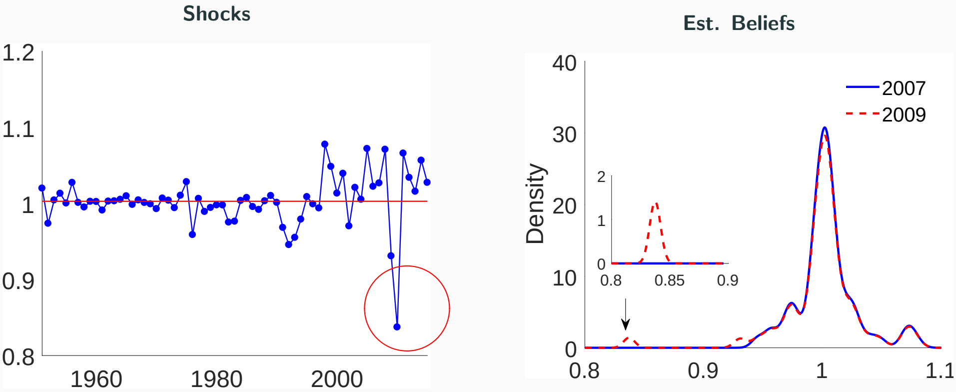
$\Rightarrow$ **Direct measure of ϕ**

$$\phi_t = \frac{K_t}{\hat{K}_t} = \left(\frac{P_t^k K_t}{P_{t-1}^k \hat{K}_t}\right)\left(\frac{PINDX_{t-1}^k}{PINDX_t^k}\right)$$

Calibration:

- Preferences: Risk aversion $= 0.5$, Frisch $= 2$
- Liquidity: $R^f = 0.02$, pledgability of capital $= 0.16$, $H'(X) = \zeta/\sqrt{X}$

Shocks and beliefs



Large negative shocks → Large (and persistent) increase in tail risk

Results: Effect of the Great Recession

Beliefs:

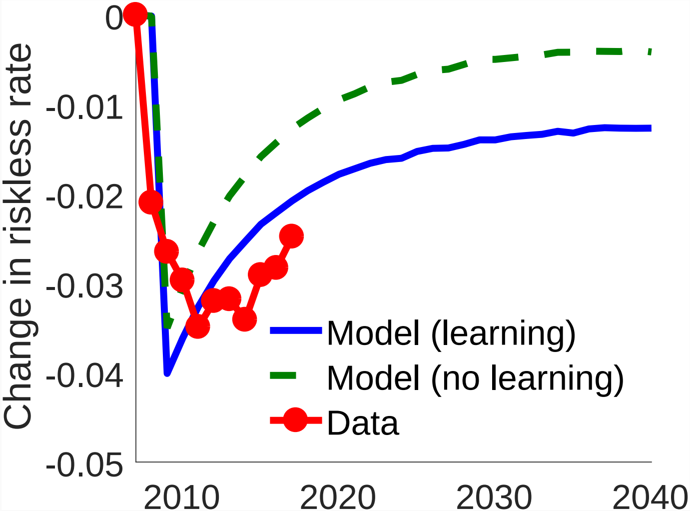
1. Start at 'steady state' of $\hat{g}_{2007}$ (estimated using 1950-2007 data)
2. Feed in the actual shocks from 2008-09 and estimate $\hat{g}_{2009}$

$$(\phi_{2008}, \phi_{2009}) = (0.93, 0.84)$$

Exercises:

1. Baseline: simulate time paths drawing from $\hat{g}_{2009}$, plot mean responses
2. No more crisis: simulate paths drawing from $\hat{g}_{2007}$, plot mean responses

Tail event + Learning → Persistent Fall R^f



Model vs Data: Long-run changes

Riskless rate	**Change, %**
Model	-1.45
Data:	
1-year real rate	-2.48
5-year real rate, 5 years forward	-1.57
Natural real rate (from Del Negro et al. ’17)	-0.66

Model: Average in stochastic steady states under $\hat{g}_{2009}$ minus the one under $\hat{g}_{2007}$

Data: Average in 2013-2017 minus average in 2005-2007

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Role of Liquidity:

Liquidity premium	**Change, %**
Model	-1.43
Data (from Del Negro et al. ’17)	-0.52

Almost all of the drop in R^f comes from the interaction of tail risk and liquidity

Increase in tail risk ⇒ Liquidity from capital lower and riskier ⇒ bonds become more valuable

What about equity valuations and options prices?

Interpret equity as a levered claim on the value of the representative firm

Returns and valuations:

Changes in	**Model**	**Data**
Expected return on equity, $\mathbb{E}(R^e)$ (%)	-0.07	-0.18
Equity premium, $\mathbb{E}(R^e - R^f)$ (%)	1.39	3.83
ln Equity/Capital	0.01	0.22

Higher tail risk does not imply a large fall in equity valuations

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Tail risk indicators:

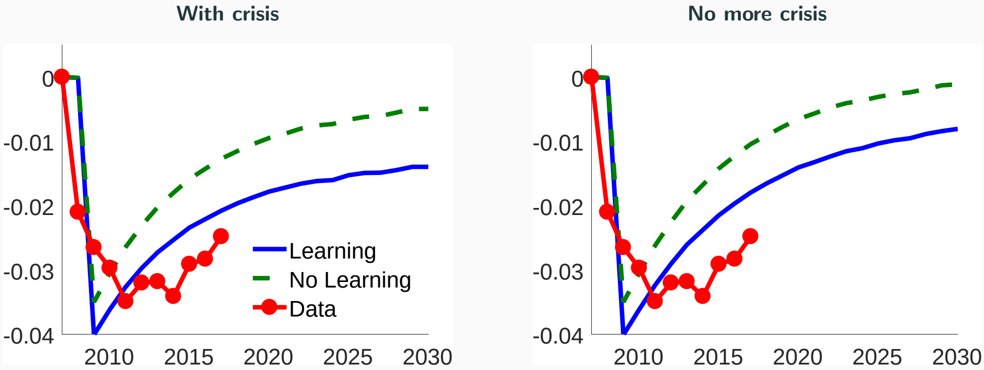
Changes in	**Model**	**Data**
Third moment $\mathbb{E}^Q(R^e - \bar{R}^e)^3$	-0.002	-0.002
$\Pr^Q\left(R^e - \bar{R}^e \leq -0.30\right)$	0.022	0.015

Expectations and probabilities are under the risk-neutral measure.

Option prices show increase in tail risk

What if there are no more crisis?

- With crises: Draw future shocks from $\hat{g}_{2009}$ (benchmark)
- No more crises: Draw future shocks from $\hat{g}_{2007}$



Long-lived effects even if crises never occur again

Conclusion

- Obviously, no one knows the true distribution of shocks
- New data permanently reshapes our assessment of macro risks
- Tail events have long-lived effects on beliefs as data on tail events is scarce
- A new perspective on the persistent drop of riskless rates

Appendix

Contribution to the Literature

Low interest rates:

- Hall (2017), Barro et al. (2014), Bernanke et al. (2011), Carvalho et al. (2016), Caballero et al. (2016), Bigio (2015) and Del Negro et al. (2017)
  - *We add : new mechanism, acting through belief revisions*

Belief-driven business cycles

- Tail risk: Kozlowski, Veldkamp and Venkateswaran (2017)
  - *We add: riskless rate, liquidity*
- Belief shocks: Gourio (2012), Angeletos and La'O (2013), Bloom (2009)...
  - *We add: endogenous belief revisions, persistence*
- Learning models: Johannes et. al. (2012), Cogley and Sargent (2005)...
  - *We add: production, non-parametric learning*
- Endogenous uncertainty: Fajgelbaum et.al. (2014), Straub and Ulbricht (2013)...
  - *We add: empirical discipline on beliefs, larger effects*

The firm's problem

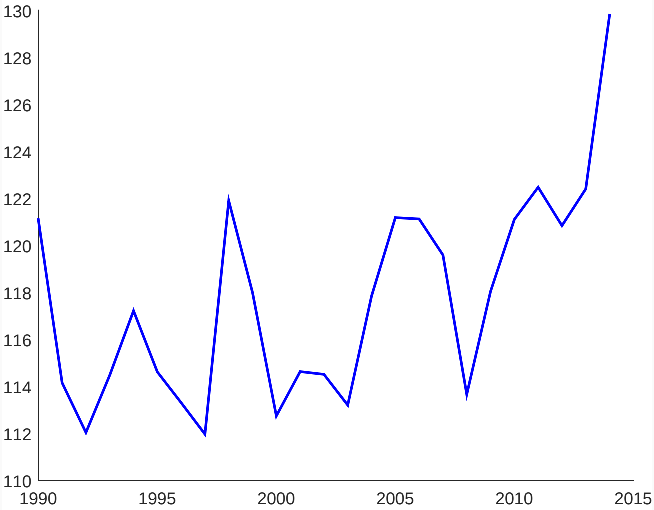
$$
\begin{aligned}
V\left(K_t, B_t, S_t\right) = \max_{X_t, N_t, B_{t+1}, \hat{K}_{t+1}} \quad & H\left(X_t\right) - X_t + F\left(K_t, N_t\right) - W_t N_t + K_t\left(1-\delta\right) \\
& + B_t - P_t B_{t+1} - \hat{K}_{t+1} + \beta \mathbb{E}_t M_{t+1} V\left(K_{t+1}, B_{t+1}, S_{t+1}\right)
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t.} \quad X_t &\leq B_t + \eta K_t, \\
K_{t+1} &= \phi_{t+1} \hat{K}_{t+1}
\end{aligned}
$$

Optimality conditions:

$$
\begin{aligned}
1 &= \beta \mathbb{E}_t \left\{M_{t+1} \phi_{t+1} \left[F_1\left(K_{t+1}, N_{t+1}\right) + 1 - \delta + \eta \mu_{t+1}\right]\right\} \\
P_t &= \beta \mathbb{E}_t \left\{M_{t+1}\left(1 + \mu_{t+1}\right)\right\} \\
\mu_t &= H'\left(X_t\right) - 1
\end{aligned}
$$

The SKEW Index



Source: CBOE. Constructed from out-of-the-money put options on S&P 500. A level $>$ 100 indicates negative skewness.

Long-run analysis

	$\hat{g}_{2007}$	$\hat{g}_{2009}$	**Change**
Model with liquidity ($\eta > 0$)			
$\ln F(K, N)$	2.39	2.36	-0.03
$\ln X$	2.68	2.65	-0.03
$\ln K$	4.10	4.06	-0.04
Riskless rate (R^f), in %	2.31	0.86	-1.45
Return on capital (R^v) in %	5.30	5.29	-0.01
Premium ($R^v - R^f$) in %	2.99	4.43	1.44
Model without liquidity ($\eta = 0$)			
$\ln F(K, N)$	2.27	2.19	-0.09
$\ln X$	1.29	1.29	0.00
$\ln K$	3.93	3.80	-0.13
Riskless rate (R^f) in %	2.31	2.29	-0.02
Risky return (R^v) in %	5.28	5.27	-0.01
Risk premium ($R^v - R^f$) in %	2.97	2.98	0.01

Risk Aversion

Interest rates in the long-run, without liquidity effects:

σ	$\hat{g}_{2007}$	$\hat{g}_{2009}$	**Change**
0.5	2.31	2.29	-0.02
2	2.31	2.23	-0.08
10	2.31	1.67	-0.64

Calibration

Parameter	Value	Description	Target	Value
Preferences:				
β	0.95	Discount factor		
γ	0.50	1/Frisch elasticity		
π	1	Labor disutility		
σ	0.5	Risk aversion		
Technology:				
α	0.40	Capital share		
δ	0.06	Depreciation rate		
Liquidity: $H(X) = 2\zeta\sqrt{X} - \xi$				
η	0.16	Pledgability of capital	Short term obligations	16%
$\bar{B}$	4.93	Supply of liquid assets	Liquid assets	9%
ζ	3.93	Investment technology	Riskless rate	2%
ξ	9.00	Investment fixed cost	Capital-output ratio	3.5

Data

- **5-year rate, 5 years forward**:
  Nominal 5y rate, 5 years forward from Treasury yield curve
  Expected 5y inflation, 5y forward, from Cleveland Fed 5y and 10y exp inflation
- **Expected returns** $\mathbb{E}(R^e)$: Follow Cochrane (2011) and Hall (2015)
  Regress 1y S&P return to log of the ratio of the S&P to its dividends and log of the ratio of consumption to disposable income forecast model
- **Third moment**:

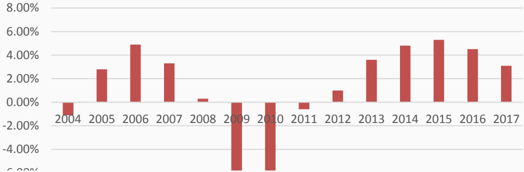
$$SKEW_t = 100 - 10\frac{\mathbb{E}(R^e - \bar{R}^e)^3}{(VIX_t/100)^3}\ .$$

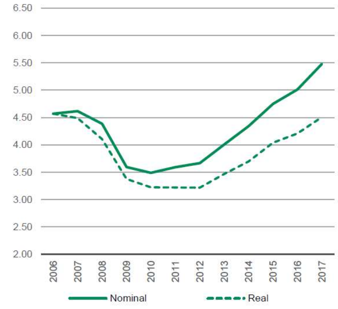
- **Tail probabilities**: Approximate distribution for $\omega = \frac{x-\mu}{\sigma}$:

$$f\left(\omega\right) = \varphi\left(\omega\right)\left[1 - \gamma\frac{\left(3\omega - \omega^3\right)}{6}\right] \qquad \text{where} \qquad \gamma = E\left[\frac{x-\mu}{\sigma}\right]^3$$

Why shocks to capital 'quality'?

- Most direct way to generate large, negative capital returns + transparent measurement
- Price changes tied to productive value → without persistence, countercyclical investment
  - E.g. discount factor induced price changes ruled out

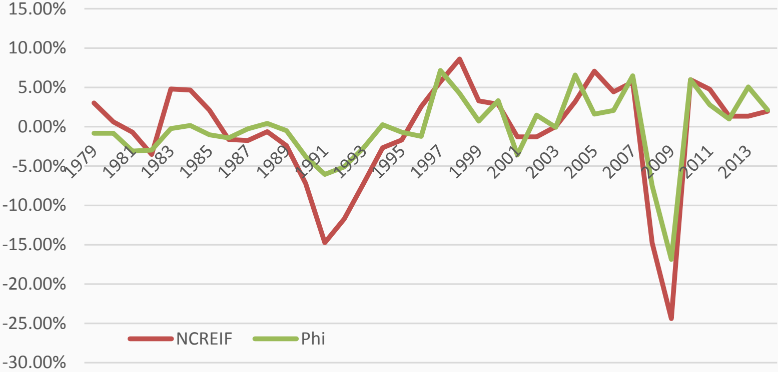




Source: Prologis Research

Measurement

- Concern: Methodological changes in the FoF for valuing non-financial assets
- Issue: Consistently measured data series available only for shorter samples
- Strategy: Use NCREIF Property Index for comparison



What if the learning sample includes pre-1950 data?

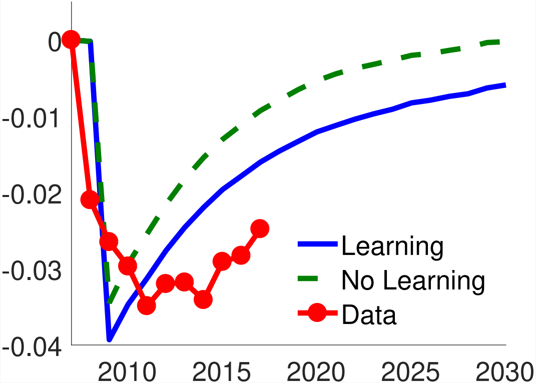
- **Concern:** Effect of new observations with a longer sample? Great Depression?
- **Issues:** Data availability? Discounting of old data?
- **Strategy:** Use the 1950-2009 sample as a proxy for 1890-1949
    - Great Depression: $\{\phi_{1929}, \phi_{1930}\} = \{\phi_{2008}, \phi_{2009}\}^{\varepsilon}$, $\varepsilon \in \{1, 2\}$
    - Weights: Observation in $t - s$ is given a weight λ^s, $\lambda \leq 1$
- Exercise I: Simulate by drawing from $\hat{g}_{2009}$

Parameters		**Long-run Average**		
ϵ	λ	$\hat{g}_{2007}$	$\hat{g}_{2009}$	Chg
1	1	1.68	0.87	-0.81
1	0.99	2.35	0.90	-1.44
2	1	1.22	0.37	-0.85
2	0.99	2.08	0.63	-1.45

More data (+ modest discounting) yields similar results

What if the learning sample includes pre-1950 data? (contd..)

- Exercise II: Simulate by drawing from $\hat{g}_{2007}$



Similar patterns even with discounting and no more tail events