# The Role of Product Diversification in Skill-biased Technological Change 

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#### Abstract

Since the 1980s, labour demand has shifted toward more educated workers in the US. The most common explanation is that the productivity of skilled workers has risen relative to the unskilled, but it is not easy to explain why aggregate labour productivity was stagnant during the 1980s. This paper suggests an alternative story: introducing new goods involves a fixed labour input, which is biased toward white-collar workers. Hence the transition from Ford-style mass production towards more diversified one has shifted labour demand toward white-collar workers.


Keywords: Skill Demand; product innovation; inequality; productivity JEL classification: E24, E32, J31, L1, O3, O4

[^0]
## 1 Introduction

The wage gap between white-collar workers and blue-collar workers has risen significantly in the US since the 1980s. The UK also experienced a sharp rise in the wage differential during this period. Although this trend is less strong in countries such as Germany and Sweden (Machin and van Reenen, 1998), the shift in labour demand toward white-collar workers is a common finding in many industrialized countries.

The majority of the economic literature (e.g., Autor, Katz and Krueger, 1998; Katz and Murphy, 1992; Autor, Katz and Kearney, 2008) attribute this shift to technological change: that recent technological innovations such as information technology tend to favour skilled workers, a hypothesis referred to as the skillbiased technological change (SBTC) hypothesis. ${ }^{1}$

The most common interpretation is that a certain type of technological innovation enables white-collar workers to produce goods more efficiently than blue collar workers. As a result, both the demand for white-collar workers and their wage, have increased together. This simple framework focuses on process innovation by assuming single representative good production function, but largely ignoring the role of product innovation. It assumes that the rising wage gap is the result of the rising productivity gap between workers, and both white-collar and blue-collar workers constitute variable input.

Alternatively, this paper will present a model which assumes that the demand for white-collar workers rises not because their productivity is growing faster, but because increasing product variety requires white-collar workers as a fixed input. ${ }^{2}$ For example, to develop a new mobile phone, many white-collar workers including engineers, designers, marketing experts, project managers and other administrative support staff are needed regardless of production volume. This view is consistent with the empirical finding that the adjustment of white-collar

[^1]employment is more rigid (Hamermesh, 1993).
This model leads to a new interpretation of skill-biased change, different from the standard theory. In the 1980s in the US, product variety increased dramatically, which has been interpreted as a transition from Ford style standardised production toward more diversified production, the so called "Flexible Manufacturing System" (Milgrom and Roberts, 1990; Mansfield, 1993). We argue that such a change could have increased relative demand for white-collar workers.

It is puzzling, as Card \& DiNardo (2002) point out, that aggregate labour productivity growth was stagnant during the 1980s, while standard SBTC theories would predict that this was the period with the most substantial white-collar labour augmenting technological innovation. As aggregate labour productivity is the weighted average of the productivity of white-collar and blue-collar workers, it could only have been explained as the result of a large decline in the productivity growth of blue-collar workers in 1980s (Acemoglu, 1998; Beaudry, 2003). The implication of such an explanation is that the gains of innovation were not realized in 1980s. This model suggests, however, that the primary contribution of innovation in this period may have been an increase in product variety, rather than an increase in output.

Although white-collar workers are modelled as a fixed input, this does not mean that aggregate labour demand for them is independent of GDP and their wage. White-collar employment is assumed to be fixed per product, but equilibrium product variety increases with GDP growth, increasing the demand for whitecollar workers. ${ }^{3}$ Nevertheless, the adjustment of variety is likely to be more rigid than the adjustment of quantity. If so, during booms, the number of products

[^2]remains below the equilibrium level and more resources are diverted from fixed factors to variable factors in the short run. This can create both pro-cyclicality of productivity and counter-cyclicality of the skill-intensity of employment. Similarly, as the wage for white-collar workers decreases, the equilibrium product variety in the economy increases, increasing the demand for white-collar workers. ${ }^{4}$

The paper is not the first to study the effect of product innovation on skillbiased change: for example, Xiang (2005), Thoenig and Verdier (2003) and Sanders (2002) argued that new goods increase the demand for skilled labour because their production processes are more skill-intensive. They all assume white-collar workers constitutes variable input like in the standard SBTC literature. In contrast, in the paper, an increase in product variety increases the demand for white-collar workers regardless of whether the production processes of the new goods are more skill-intensive or not.

The remainder of the paper is as follows: Section 2 illustrates recent labour market trends. Section 3 explains the role of product innovation in skill-biased technological change. Section 4 presents the model and the simulation results. Section 5 concludes.

## 2 The US labour market trend

The trend in the wage gap between college and non-college educated workers in the US is shown in Figure 1. The wage gap was increasing slowly until early 1970s, and then it began to fall before a dramatic increase in the 1980s, tempering to slower, but still positive growth throughout the 1990-2000s. The dramatic shift in the 1980s drew much attention, and a great deal of literature suspected that the adoption of PCs in the 1980s was the leading explanation for it.

Although the pattern was not identical, such a shift is not confined to the US. Machin and van Reenen (1998) studied the US, the UK, Germany, Japan, France, Denmark, and Sweden, and found that both the employment share and

[^3]Figure 1: College/Highschool graduates wage ratio, 1963-2008


Source: Acemoglu \& Autor (2010)
wage-bill share of non-production workers rose in all of these countries, while the wage gap remained stable, with the exception of the US and the UK. The fact that the employment share rose in all the investigated countries implies that the shift in labour demand toward white-collar workers existed for all of those countries although the wage gap did not increase for most of them. ${ }^{5}$

## SBTC and the Productivity Puzzle

The majority of literature on SBTC has utilized the simple two factor CES function to formulate the skill biased technological change hypothesis. It is assumed that there are two types of labour input - skilled labour and unskilled labour. The functional form is as below: ${ }^{6}$

$$
\begin{equation*}
Q_{t}=\left[\alpha_{t}\left(a_{t} N_{s, t}\right)^{\rho}+\left(1-\alpha_{t}\right)\left(b_{t} N_{u, t}\right)^{\rho}\right]^{\frac{1}{\rho}}, \quad 0<\rho<1 \tag{1}
\end{equation*}
$$

Here, $Q_{t}$ is the output at time $t, N_{s, t}$ is the labour input of skilled workers at

[^4]$t$, which is usually defined as the number of college graduated workers or whitecollar workers. $N_{s, t}$ is the labour input of unskilled workers, defined as the number of workers with lower education or blue-collar workers. $a_{t}$ is the skilled labouraugmenting technology, and $b_{t}$ is the unskilled labour-augmenting technology. $\alpha_{t}$ can be interpreted as the share of production activities assigned to skilled labour. Capital is either non-existent or separable from the composite labour input.

Figure 2: Aggregate labour productivity


Source: Card \& DiNardo (2002), Labour productivity per hour, non-farm business sector

Skill-biased technological change is represented either by an increase in $a_{t}$ relative to $b_{t}$ or by an increase in $\alpha_{t}$. Therefore, skill-biased technological change is supposed to increase aggregate productivity unless the decline in blue-collar labour augmenting technology is large enough to offset the rise in blue-collar labour augmenting technology. However, according to Card and DiNardo (2002), the puzzling fact is that the aggregate labour productivity was stagnant in the 1980s in the US, the period when the shift in labour demand was most dramatic. This can be seen in Figure 2, where between 1979 and 1986, the growth of labour productivity slowed down and its level was below the long term trend.

One possible explanation is that the productivity growth of blue-collar workers slowed down during 1980s, and offset the productivity growth of white-collar
workers. This explanation implies that the gains from innovation were not realized in the 1980s.

## 3 The role of product innovation

The literature has largely focused on process innovation and largely ignored the role of product innovation on SBTC. ${ }^{7}$ They assume a single representative good and argue that technological innovation such as the adoption of PC, amplified the productivity of college graduate workers relative to blue-collar workers. There is no place for product innovation in the theoretical framework.

A difficulty in studying the role of product innovation is that it is not easy to measure. Greenwood and Uysal (2005) utilize the trademark registration statistics as a proxy for product variety. Figure 3 shows the trend of trademark registration in the US between 1950 and 2008. The number of trademark registrations has risen steadily since the 1980s, a trend which coincides with the rising wage inequality of the 1980s. ${ }^{8}$

There is some literature which has investigated the role of product innovation on SBTC. Xiang (2005) argued that the introduction of new goods favours skilled labour because new goods are produced with more skill-biased technology than existing goods. This paper shows that the new good's average skilled labour intensity is more than $40 \%$ higher than the old goods in the US manufacturing industries between late 1970s and 1980s.

Thoenig and Verdier (2003) argued that the competitive pressure from southern low-wage countries induces northern countries to adopt skilled-labour intensive technologies because they are harder for southern countries to imitate. It is assumed that the production process of new goods is more skill intensive than old

[^5]Figure 3: The trademark registration


Source: WIPO, "World Intellectual Property Indicators"
goods, which southern countries can also produce. Northern firms are forced to adopt the new technology to avoid competing with southern countries.

Sanders (2002) argued that the development of new goods is skill-biased because production of new goods requires more skilled labour, who can flexibly deal with uncertainty of production, which is higher in the early stage of product life cycle.

However, this literature commonly assumes that introducing new products increases skill demand because the production process of new goods is more skillintensive than old goods. ${ }^{9}$

Nevertheless, this is not necessarily true for every new good, especially for horizontal product differentiation. One recent example is the development of the iPhone 4 white colour version by Apple. It is identical to the black colour version except for the colour, and there is no technological improvement from the black colour version. ${ }^{10}$ This paper will focus on the effect of horizontal product differ-

[^6]entiation, in which case the new goods are not necessarily technologically more sophisticated, and therefore do not necessarily require more skilled workers in the production process.

The difference in this paper is that the introduction of any new goods increases the relative demand for white-collar workers, regardless of the level of technological sophistication, because it requires a fixed labour input which is biased toward white-collar workers.

## The share of fixed cost

It is assumed that the white-collar workers are a fixed input, and blue-collar workers are a variable input. Capital is divided into both fixed and variable parts. Firms can pay for a fixed input only if their variable profit (= revenue - variable cost) is positive. This means that firms can pay for a fixed input, which includes both white-collar labour and a fixed capital, only if price is greater than marginal cost, which implies the mark-up ratio must be greater than 1 . The ratio of total fixed cost to variable cost is defined here:

$$
\begin{equation*}
\frac{W B_{w}+r \cdot k_{f}}{W B_{b}+r \cdot k_{v}}=\widehat{\mu} \tag{2}
\end{equation*}
$$

$W B_{w}$ is the total wage bill for white-collar workers, and $W B_{b}$ is the total wage bill for blue-collar workers. $r \cdot k_{f}$ is the total expenditure on fixed capital, and $r \cdot k_{v}$ is the total expenditure on variable capital. Under the assumption of constant marginal cost and free entry and exit, the ratio $\widehat{\mu}$ must be the same as $\mu=$ $\frac{P-M C}{M C} .{ }^{11}$ Therefore, the values of $\widehat{\mu}$ are constructed using US manufacturing data over 1970-1992 and compared with $\mu$.

The data on the wage bill for both production workers and non-production workers and capital stock comes from the NBER-CES Manufacturing Industry Database, which is based on the ASM (American Survey of Manufacturers). The interest rate used here is the Baa rated corporate bond rate, which comes from

[^7]Figure 4: Mark-up vs Implied Mark-up

the FRB (Federal Reserve Board). The inflation rate is from the U.S. Bureau of Economic Analysis.

The data on the mark-up is from Oliveira Martins et al.(1996). In that paper, the mark-up ratios for 36 manufacturing industries in the US are estimated over 1970-1992 utilizing the method of Roeger (1995), assuming that the mark-up ratio is constant over the period. However, not all industry groups in Oliveira Martins et al.(1996) showed significant estimates for the mark-up ratio, and only the estimates for 26 industry groups among them are used in this paper. The list of mark-up ratios for each industry and the method of estimation is shown in the Appendix.

The rental rate of capital, $r$, is derived following Oliveira Martins et al.(1996):

$$
\begin{equation*}
r=((i-\pi)+\delta) \cdot p_{k} \tag{3}
\end{equation*}
$$

Here, $i$ is nominal interest rate, which is given by the Baa rated corporate bond rate (by Moodies). $\pi$ is the inflation rate, and $\delta$ is the depreciation rate, which is set to $5 \%$ per year. $p_{k}$ is the price index of the investment good.

One problem is that the share of fixed capital in the total capital stock is unobservable. To deal with this, the share of plant and buildings in the total capital stock is used as a proxy for the share of fixed capital. The rationale is
that buildings are usually adjusted more rigidly than equipment or vehicles. For example, at least one head-quarter building and one factory are needed to establish a firm. Then, it is possible to increase equipment without building another factory (upto a certain level). However, this is a crude measure as some part of equipment or vehicles might be fixed capital as well. ${ }^{12}$

The comparison of $\mu$ and $\widehat{\mu}$ is shown in Figure 4 . There is a positive correlation between them. Those industries with a higher share of fixed costs, such as Office \& Computing, Drug \& Medicine and Radio, TV \& Communications, are also shown to have a higher mark-up ratio. Those with a lower share of fixed cost, such as Food Products and Petrol Refineries, are shown to have lower mark-up ratio. However, some industries, especially Tobacco industries, show much higher mark-up ratio than is implied from the ratio of fixed cost to variable cost. This may suggest the existence of excess profit due to market power.

## 4 Model

In this model, people value the variety of consumption as well as the quantity of consumption. People are willing to substitute some consumption quantity for more variety of consumption. In Krugman (1979b), the motivation of technological innovation is not producing the same goods more efficiently but producing new goods to gain more monopoly power. To capture such a "love of variety", this model will utilize Dixit-Stiglitz (1977) style monopolistic competition framework. In this model, it is assumed that only white-collar labour ${ }^{13}$ constitutes the fixed labour input and that only blue-collar labour constitutes the variable labour input. It is a strong assumption, but can be justified if the fixed portion of labour input is relatively biased toward white-collar workers. The result of this assumption is

[^8]that labour demand shifts toward white-collar workers only if the ratio of the fixed labour input to the variable labour input rises.

### 4.1 Utility

Consumer utility is increasing with the consumption level of composite good $x$ :

$$
\begin{gather*}
U=u(x)  \tag{4}\\
u^{\prime}>0, u^{\prime \prime}<0
\end{gather*}
$$

The composite good, $x$, is defined by a CES function as below:

$$
x=\left(\int_{0}^{N} q(i)^{\rho} d i\right)^{\frac{1}{\rho}} 0<\rho<1
$$

Here, $i \in[0, N]$ is the index of the product variety, where $N$ represents the maximum level of variety available in the economy ( $N \in R_{++}$). The constant $\rho$ represents the substitutability between different goods. The lower the $\rho$ is, the lower the substitutability is. The elasticity of substitution is $\frac{1}{1-\rho}$. The sum of the consumption quantities of all varieties, $q$, is different from $x$, and it is calculated as below:

$$
q=\int_{0}^{N} q(i) d i
$$

Given $q, x$ increases with N. Therefore, the utility increases with variety given the same total quantity.

### 4.2 Firm's problem

Each variety of good is produced with Cobb-Douglas technology, but production can begin only if the firm employs both fixed labour and fixed capital above a minimum required level $(\bar{l}, \bar{k})$.

$$
q_{i}=A \cdot\left(l_{i}^{b}\right)^{\alpha} \cdot\left(k_{i}^{v}\right)^{1-\alpha} \quad \text { if } \quad l_{i}^{w} \geq \bar{l} \quad \& \quad k_{i}^{f} \geq \bar{k}
$$

Here, $q_{i}$ is the production volume of good $i, l_{i}^{b}$ is the blue-collar labour input for producing good $i$. As it is assumed that only blue-collar workers constitute variable labour input, their employment is equivalent to the variable labour input. $k_{i}^{v}$ is the variable part of the capital input for good $i$. A represents the level of neutral technology, which augments every factor proportionately. ${ }^{14}$

The marginal cost is constant because the Cobb-Douglas production function exhibits constant returns to scale, and each firm is small enough not to influence the overall wage level or interest rate. There is no economy of scope, so every firm produces only one variety. Therefore, the number of goods in the economy increases only if the number of firms increases. ${ }^{15}$ The total variable cost, $c\left(q_{i}\right)$, is:

$$
\begin{aligned}
c\left(q_{i}\right) & =m c \cdot q_{i} \\
& =W_{b} \cdot l_{i}^{b}+r \cdot k_{i}^{v}
\end{aligned}
$$

Here, $m c$ is the marginal cost. $W_{b}$ is the wage for blue-collar workers, and $r$ is the interest rate. The total variable cost is the sum of total wage bill for bluecollar workers and the variable capital cost. It is also assumed that every firm has a symmetric cost structure. However, producing each variety of good incurs a fixed cost, which consists of both a fixed labour input and a fixed capital input:

$$
\text { fixed cost }=W_{w} \cdot \bar{l}+r \cdot \bar{k}
$$

By assumption, the fixed labour input consists of only white-collar workers. $W_{w}$ is the wage for white-collar workers, and $\bar{l}$ is the minimum required level of fixed labour input per variety. As additional employment of fixed labour above that level does not contribute to production at all, the employment of white-collar

[^9]labour for firm i, $l_{i}^{w}$, is always equal to $\bar{l}$. Similarly, the employment of fixed capital is always $\bar{k}$. The interest rate, $r$, is the same for both variable capital and fixed capital. $\bar{l}$ and $\bar{k}$ are assumed to be the same for every firm in the economy. The profit of each firm is:
\[

$$
\begin{aligned}
\pi_{i} & =p_{i} \cdot q_{i}-c\left(q_{i}\right)-\text { fixed cost } \\
& =\left(p_{i}-m c\right) \cdot q_{i}-\text { fixed cost }
\end{aligned}
$$
\]

Firm $i$ 's profit, $\pi_{i}$, is total revenue minus the sum of variable cost and fixed cost. Because every firm has partial monopolistic power, firms set price higher than marginal cost. The lower the substitutability between goods, the higher the mark-up is. All firms set the same price, given the demand curve derived from the CES utility function :

$$
\begin{gathered}
p_{i}^{*}=\frac{m c}{\rho} \\
\operatorname{mark}-u p(=\mu+1)=\frac{p}{m c}=\frac{1}{\rho}
\end{gathered}
$$

## Zero-Profit condition

Free entry is assumed. If any firm earnt positive profit, new firms will enter the market, and production quantity for existing firms will decrease as a result of competition. Therefore, all firms will make zero profit in equilibrium. Hence:

$$
\begin{align*}
& \pi_{i}^{*}=\left(p_{i}^{*}-m c\right) \cdot q_{i}^{*}-\text { fixed cost } \\
& =m c \cdot\left(\frac{1}{\rho}-1\right) \cdot q_{i}-\text { fixed cost }=0 \\
& \frac{W_{w} \cdot \bar{l}+r \cdot \bar{k}(=\text { fixed cost })}{m c \cdot q(=\text { total variable cost })}=\mu=\frac{1}{\rho}-1 \tag{5}
\end{align*}
$$

Equation (5) shows that ratio between the total fixed cost and the total variable cost is determined by the mark-up ratio. Under symmetry, all firms will
produce the same amount of goods with the same amount of input in equilibrium. Therefore, $q_{i}=q, l_{i}=l, k_{i}=k$ for all $i$. Recall that the shift in labour demand toward white-collar workers happens for two reasons in our model:

1. mark-up $\uparrow$ : Total expenditure on fixed factors increases relative to variable factors.
2. fixed capital cost $\downarrow$ : Given a total expenditure for fixed factors, fixed labour cost (the employment of white-collar workers) will constitute a higher share.

The mark-up ratio is unlikely to have risen continuously. However, the fixed capital cost is likely to have declined relative to the fixed labour cost for two reasons. The total fixed capital cost per variety is $r \cdot \bar{k}$, where that of labour is $W_{w} \cdot \bar{l}$. If both the exogenous parameters, $\bar{k}$ and $\bar{l}$, remain constant, the fact that the growth rate of wage is usually higher than that of the interest rate decreases fixed capital cost relative to fixed labour cost. Moreover, the adoption of FMS (Flexible Manufacturing Systems) could have lowered the minimum fixed capital requirement to introduce new variety, $\bar{k}$.

### 4.3 Market clearing condition

The total workforce, $L$, is assumed to be given exogenously, and endogenously allocated between white-collar labour and blue-collar labour:

$$
\begin{gathered}
L_{b}+L_{w}=L \\
L_{w}=N \cdot \bar{l} \\
L_{b}=L-L_{w}=L-N \cdot \bar{l}
\end{gathered}
$$

$L_{b}$ is the total employment of blue-collar workers in the economy, and $L_{w}$ is the total employment of white-collar workers. The labour demand for white-collar workers is proportional to the total number of products in the economy, N. N is endogenously determined in this model, but $\bar{l}$ is exogenous. The employment of
blue-collar workers equals to the remainder of workforce, $L-L_{w}$. Therefore, both $L_{w}$ and $L_{b}$ are determined by $N$. Similarly for capital:

$$
\begin{gathered}
K=K_{v}+K_{f} \\
K_{f}=N \cdot \bar{k} \\
K_{v}=K-N \cdot \bar{k}
\end{gathered}
$$

The total capital stock in the economy, $K$, is exogenously given at each point in time, but endogenously allocated between variable part, $K_{v}$ and fixed part, $K_{f}$. We will however show how capital stock accumulates endogenously over time in section 4.4.

## Blue-collar wage determination

The wage of blue-collar labour is set to equal to the value of marginal revenue product of labour (MRPL). Here, MRPL $=M R \times M P L .{ }^{16}$ Given the above CES-preferences shown in equation (4), $M R=P \cdot \rho$. P, the price of the output, is normalized to 1 . Therefore, $M R=\rho$, and the wage of blue-collar workers is:

$$
\begin{aligned}
W_{b} & =M R \cdot M P L \\
& =\rho \cdot \alpha \cdot A \cdot\left(l^{b}\right)^{\alpha-1} \cdot\left(k^{v}\right)^{1-\alpha} \\
& =\rho \cdot \alpha \cdot A \cdot\left(\frac{K_{v}}{L_{b}}\right)^{1-\alpha} \\
& =\rho \cdot \alpha \cdot A \cdot\left(\frac{K-N \cdot \bar{k}}{L-N \cdot \bar{l}}\right)^{1-\alpha}
\end{aligned}
$$

## White-collar wage determination

The wage determination mechanism for white-collar labour is more complicated since it is impossible to define marginal productivity for a fixed input. However,

[^10]the wage for white-collar can be determined by the labour demand and supply relationship:
$$
L_{w}^{D}=N \cdot \bar{l}
$$

The aggregate demand for white-collar labour is determined by the product of the number of products in the economy and the fixed labour input for each product. The demand for white-collar worker increases with $N$.

As in the model by Caselli (1999), the supply of white-collar labour is endogenously determined by the optimal trade-off of workers between their education cost and the additional wage gained from education. To be a white-collar worker, more education is needed, but is costly. The size of the cost differs among individuals and the wage premium must be high enough to compensate this education cost. ${ }^{17}$

Those with a lower learning cost will decide to go to college and become whitecollar workers for a lower wage premium than those with a higher learning cost, who choose to be blue-collar workers. Following Caselli (1999), it is assumed that each individual's education cost follows uniform distribution $\left[0, \sigma_{e} \cdot W_{b}\right] . \sigma_{e}$ is a parameter which represents both the upper bound and the degree of dispersion of the learning cost. ${ }^{18}$ Average learning cost equals to $\left(\sigma_{e} \cdot W_{b}\right) / 2$. Therefore, the labour supply of white-collar workers (relative to total labour force) is the function of wage premium as below:

$$
\begin{gathered}
L_{w}^{S}=\frac{W_{w}-W_{b}}{\sigma_{e} \cdot W_{b}} L=L_{w}^{D}=N \cdot \bar{l} \\
\therefore W_{w}=W_{b} \cdot\left(1+\sigma_{e} \frac{L_{w}}{L}\right)=W_{b} \cdot\left(1+\sigma_{e} \frac{N \cdot \bar{l}}{L}\right)
\end{gathered}
$$

[^11]An increase in $\sigma_{e}$, which represents higher learning cost, increases the whitecollar wage relative to blue-collar by lowering the supply of white-collar labour. However, an increase in $N$ increases white-collar wage by increasing the demand for them.

## Interest rate determination

The interest rate is determined by the value of marginal revenue product of variable capital (MRPK), and the same interest rate is applied to the rental cost of fixed capital. The interest rate, $r$, is determined by the relative ratio of variable labour and variable capital:

$$
\begin{aligned}
r & =M R \times M P k \\
& =\rho \cdot(1-\alpha) \cdot A \cdot\left(\frac{l_{b}}{k_{v}}\right)^{\alpha} \\
& =\rho \cdot(1-\alpha) \cdot A \cdot\left(\frac{L_{b}}{K_{v}}\right)^{\alpha}
\end{aligned}
$$

## The number of goods in the economy

Substituting the above market clearing conditions into the zero-profit condition of the equation (5) yields an equation as below:

$$
\begin{equation*}
\pi=\rho \cdot \frac{A \cdot L_{b}^{\alpha} \cdot K_{v}^{1-\alpha}}{N}\left(\frac{1}{\rho}-1\right)-\left(W_{w} \cdot \bar{l}+r \cdot \bar{k}\right)=0 \tag{6}
\end{equation*}
$$

Rearrange equation (6) and solve for N to get the equilibrium product variety:

$$
\begin{equation*}
N \cdot\left\{\alpha \cdot \frac{1}{L-N \cdot \bar{l}}\left(1+\sigma_{e} \frac{N \cdot \bar{l}}{L}\right) \cdot \bar{l}+(1-\alpha) \frac{\bar{k}}{K-N \cdot \bar{k}}\right\}=\frac{1}{\rho}-1 \tag{7}
\end{equation*}
$$

The LHS of the equation (7) represents (fixed cost)/(variable cost) ratio, which is continuous and increasing in $N$. In contrast, the RHS of the equation is a constant representing the mark-up ratio. Therefore, there must be a unique solution for N by the intermediate value theorem. Notice that the skill-neutral technology,
$A$ is not included in the equation (7). Therefore, $A$ has no effect on the equilibrium number of goods, $N$.

Increasing the number of goods in the economy, $N$, lowers GDP growth and aggregate labour productivity, ceteris paribus, as it increases the share of fixed inputs, which does not contribute to output growth. That could be one of the reasons why labour productivity was stagnant in the 1980s, while skill-biased technological innovations were supposed to be substantial. The trademark registration statistics imply that there was a surge in product variety in the 1980s, which could have lowered GDP growth and aggregate labour productivity as well as increased the labour demand for white-collar workers.

## Simulation result

Illustrative simulation results are shown after substituting relevant values to the parameters of the model. The list of variables is shown in Table 1.

Table 1: The list of variables

|  | Variables |
| :---: | :---: |
| Exogenous | $\alpha=0.7, A=20, L=1, \rho=0.7, \bar{l}=0.01, \bar{k}=0.05, \sigma_{e}=2$ |
| Endogenous | $N, L_{b}, L_{w}, W_{b}, W_{w}, K, r$ |

$L$ is normalized to 1 , and it is assumed that there is no population growth. The CES utility function is set so that $\rho=0.7$, which implies that the elasticity of substitution between goods eqauls to aproximately 3.33 and the mark-up ratio equals to approximately $1.43 . \bar{l}$ is 0.01 , which means that the fixed labour input per product is $1 \%$ of the total labour endowment of the economy. $\bar{k}=0.05$, which implies that the fixed capital input per product is $5 \%$ of the total capital endowment when $K=1$. However, this share decreases with capital accumulation. $\sigma_{e}=2$, which means that the upper bound of the personal learning cost is twice the blue-collar wage, and the wage of white-collar workers must be twice the blue-collar workers to induce $50 \%$ of workers to choose university education and become white-collar workers.

Given the level of $K \in[0.1,10]$, the equilibrium level of $N$ is jointly derived by solving equation (7) along with the other endogenous variables - $L_{b}, L_{w}, W_{b}, W_{w}$, $r$. The results are shown in Figure 5 to Figure 8.

Figure 5: The share of fixed capital cost in fixed cost


Capital accumulation lowers the interest rate relative to wage. Therefore, the share of fixed capital cost, $r \cdot \bar{k}$, falls and the share of fixed labour cost, $W_{w} \cdot \bar{l}$, rises as capital accumulates, as shown in Figure 5. Given the same level of capital stock, the share of fixed capital cost is lower for lower values of $\bar{k}$. The fall in the interest rate (relative to wage) makes the total fixed cost cheaper, leading to positive profit. Then, new firms enter the market with new product varieties until profit returns to zero, which lowers production quantity per firm. Therefore, capital accumulation increases both product variety and the demand for white-collar workers.

Figure 6: The number of products in the economy



Figure 7: The share of white-collar workers


Figure 8: White-collar to blue-collar wage ratio



Figure 6 shows how the number of products rises as the capital stock grows for different levels of fixed capital input per variety, $\bar{k}$. For smaller values of $\bar{k}$, the number of product varieties is higher at the same level of capital stock because the fixed capital cost, $r \cdot \bar{k}$ decreases with $\bar{k}$.

As the number of products rises, the employment share of white-collar workers also rises, as shown in Figure 7. Because firms need to offer a higher wage to hire more white-collar workers, the relative wage of white-collar workers also rises, as shown in Figure 8.

However, capital accumulation lowers only the capital part of fixed cost, without lowering the labour part of fixed cost. Therefore, capital accumulation increases the share of fixed labour cost in total fixed cost and the growth rates of both the number of products and the demand for white-collar workers approach zero as the share of fixed labour cost in the total fixed cost approaches $100 \%$. This
means that there is an upper bound for skill biased change unless the mark-up ratio rises continuously, which would seem unlikely. Therefore, this model predicts the trend of rising inequality between white-collar and blue-collar workers will slow down in the long-run in spite of continued technological change and capital accumulation. This may be consistent with the empirical findings that skill-biased change has begun to slow down recently (Autor, Katz and Kearney, 2008; Beaudry, 2013).

For a smaller value of $\bar{l}$, the number of products is higher than the case for higher $\bar{l}$ for any level of K. However, both the employment share and relative wage of white-collar workers are lower for smaller values of $\bar{l}$. This is because the rise in the number of products is not large enough to offset the fall in the employment of white-collar labour per variety. In summary, a technology shock which lowers $\bar{l}$, increases product variety, but does not necessarily shift demand toward white-collar workers.

## Recovering Parameters

From the data, the unobservable exogenous parameters, $\bar{k}$ and $\bar{l}$ can be recovered by the model. ${ }^{19}$ The mark-up ratio is taken from Christopoulou \& Vermeulen (2008). It is estimated for whole industries (including service industries as well as manufacturing) in the US for 1981-2004. Data on labour and capital compensation, the number of employees and total capital stock are from EUKLEMS dataset. In EUKLEMS, workers are categorized into 3 groups - high-skilled workers with a university education, middle-skilled workers with highschool or equivalent vocational education and low-skilled workers. I identify the high-skilled workers of the data as the white-collar workers of the model. The number of products, $N$, is defined as the 5 year moving-average of the total trademark registration in the US. ${ }^{20}$ The trend of the parameters, $\bar{k}$ and $\bar{l}$, are then recovered from the data, and shown in Figure 9.

[^12]Figure 9: The trend of parameters $-\bar{k} \& \bar{l}$



The fixed capital input per product, $\bar{k}$, has fallen continuously since early 1980s. This could be due to the introduction of FMS (Flexible Manufacturing System), which enabled the production of another type of good by simply changing the software settings of the machinery. ${ }^{21}$ However, the fixed labour input per variety, $\bar{l}$, remained roughly stable until late 1980s, but began to fall during 1990s. This might be due to the substitution of white-collar workers by IT technology in the workplace since the 1990s.

## The effect of education policy

In this model, the education cost is represented by the parameter $\sigma_{e}$, which represents the upper bound of personal education cost. The effect of a change in $\sigma_{e}$ is shown in Figure 10 and Figure 11. The decline in $\sigma_{e}$ increases product variety as it becomes easier to hire more white-collar workers, but slightly decreases the GDP as increased product variety consumes more fixed input, which could otherwise have been diverted to variable input. The wage gap declines as the fall in the education cost encourages more students to go to university even at the lower expected wage premium following education.

Many people expect the policy of encouraging university education by lowering the private cost of education will contribute to GDP growth. However, it does not necessarily contribute to GDP growth in this model. Increasing the univer-

[^13]Figure 10: The effect on GDP \& variety


Figure 11: The effect on wage \& employment


sity enrolment rate will boost product diversification, which in turn shifts factor inputs from variable toward fixed inputs. As the total variable input in the economy decreases, GDP growth actually slows down as a result of higher university enrolment rate.

However, the utility of consumers may improve due to the increased variety of consumption. Another important effect of public support for education is that it can reduce the wage inequality between white-collar and blue-collar workers as it makes people choose to receive more education at lower expected wage premium.

Moreover, in reality, increased product diversification may also improve export performance. ${ }^{22}$ According to Krugman (1979a), the volume of trade depends on the number of products the country can produce efficiently. The fall in the

[^14]private cost of education from public support will decrease the cost of product diversification, which requires university graduate workers as fixed input, and increase the varieties of the products the country can export to the world market.

### 4.4 Dynamic version

There was no consideration of dynamic optimization behaviour in the model presented above, yet. The level of capital stock was exogenously set. In this section, the level of capital stock is endogenously determined from the dynamic optimization behaviour of agents. To do so, a two-period Overlapping Generations Model is employed.

The agents live two periods. In the first period, they are young and earn labour income, $W_{t}$. They divide it into consumption, $C_{t}$ and saving, $S_{t}$. In the second period, they retire and live on the capital income from the previous period's saving.

$$
\begin{gathered}
\text { young: } C_{t}+S_{t}=W_{t} \\
\text { old: } C_{t+1}=\left(1+r_{t+1}\right) \cdot S_{t}
\end{gathered}
$$

They maximize the inter-temporal utility of the two periods by selecting the optimal level of consumption and saving at time t:

$$
\begin{gather*}
\max _{\left\{C_{t}\right\}} \cdot u\left(C_{t}\right)+\beta \cdot u\left(C_{t+1}\right) \\
u^{\prime}\left(C_{t}\right)=\beta \cdot\left(1+r_{t+1}\right) \cdot u^{\prime}\left(C_{t+1}\right) \tag{8}
\end{gather*}
$$

In equilibrium, the Euler equation (8) holds. It is assumed that there are two types of agents in the economy - white-collar and blue-collar workers. They differ in wage income but have the same utility function (and discount rate). They also face the same interest rate. They differ only in personal learning cost, which determines whether they are white-collar or blue-collar workers. Therefore, the same type of workers have the same level of consumption and saving:

$$
\begin{aligned}
K_{t+1} & =S_{t} \\
& =S_{t}^{b} \cdot L_{b, t}+S_{t}^{w} \cdot L_{w, t}
\end{aligned}
$$

$S_{t}^{b}$ is the amount saved by a blue-collar worker and $S_{t}^{w}$ by a white-collar worker. Total saving in the economy, $S_{t}$, is the sum of the saving of blue-collar and whitecollar workers. It is assumed that the capital stock fully depreciates each period. Therefore, the total capital stock in the economy at $t, K_{t}$ is equal to the total saving at $t-1$.

$$
\begin{aligned}
C_{t}^{j} & =\int_{0}^{N} p(i) \cdot c(i) d i \\
& =\int_{0}^{N} c(i) d i \quad \because p=1 \text { for all } i(\text { variety })
\end{aligned}
$$

$C_{t}^{j}$ is the total consumption expenditure of an agent of type $j$ at time $t$. The type $j$ is either $w$ (white-collar) or $b$ (blue-collar). The consumption levels of the agents of the same type are the same. The same variety of good is used both for consumption and investment. ${ }^{23}$ Due to their consumption smoothing behaviour, agents will divert the same portion of every variety of goods into investment goods. Therefore, $S_{t}^{j}=\int_{0}^{N} s(i) d i$, and $s(i)$ is the same for all $i$. Because $c(i)$ is the same for all $i$, the ratio of investment goods to consumption goods, $\frac{s(i)}{c(i)}$ is the same for all i.

## log utility case

Suppose the utility function with respect to the composite consumption bundle, $u$, is a $\log$ function, so that $u(x)=\ln (x)$, then:

[^15]\[

$$
\begin{align*}
u(x) & =\ln \left(\left\{\int_{0}^{N} c(i)^{\rho} d i\right\}^{\frac{1}{\rho}}\right) \\
& =\ln \left(\left\{N \cdot\left(\frac{C_{t}^{j}}{N}\right)^{\rho}\right\}^{\frac{1}{\rho}}\right) \\
& =\ln \left(N^{\left(\frac{1}{\rho}-1\right)} \cdot C_{t}^{j}\right) \\
& \therefore u^{\prime}\left(C_{t}^{j}\right)=\frac{1}{N^{1-\frac{1}{\rho}} \cdot C_{t}^{j}} \tag{9}
\end{align*}
$$
\]

By applying (9) into the Euler equation of (8),

$$
\frac{1}{N_{t}^{\left(1-\frac{1}{\rho}\right)} \cdot\left(W_{t}^{j}-S_{t}^{j}\right)}=\beta\left(1+r_{t+1}\right) \cdot \frac{1}{E_{t}\left[N_{t+1}^{1-\frac{1}{\rho}}\right] \cdot\left(1+r_{t+1}\right) \cdot S_{t}^{j}}
$$

Solving the above equation with respect to $S_{t}^{j}$ :

$$
\begin{aligned}
& S_{t}^{j}=\frac{\beta}{\beta+\left(\frac{N_{t}}{E_{t}\left[N_{t+1}\right]}\right)^{\left(1-\frac{1}{\rho}\right)}} \cdot W_{t}^{j} \\
& \therefore \frac{S_{t}^{j}}{W_{t}^{j}}=\frac{\beta}{\beta+\left(\frac{N_{t}}{E_{t}\left[N_{t+1}\right]}\right)^{\left(1-\frac{1}{\rho}\right)}}
\end{aligned}
$$

The saving rate, $\frac{S_{t}^{j}}{W_{t}^{j}}$, is the same for every agent, and independent from the interest rate as is common in two-period models with log utility. The total capital stock at time $t+1$ is:

$$
K_{t+1}=\frac{\beta}{\beta+\left(I \frac{N_{t}}{\overline{E_{t}\left[N_{t+1}\right]}}\right)^{\left(1-\frac{1}{\rho}\right)}} \cdot\left(W_{t}^{w} \cdot L_{t}^{w}+W_{t}^{b} \cdot L_{t}^{b}\right)
$$

$K_{t+1}$ is determined by the product of the saving rate and the sum of all agents' labour income in the previous period. In the steady state, $K_{t+1}=K_{t}$.

## Expectation formation - self-fulfilling prophecy

A noteworthy point is that the saving rate is also affected by the expectation of the number of products in the next period, $E_{t}\left[N_{t+1}\right]$. If agents expect that the number of products will rise in the future, they will save more because the expected marginal utility of future consumption rises relative to that of today's as the expected number of products increases. As saving increases, the capital stock increases along with the number of products; in the next period. Therefore, the expectation of future product variety leads to a self-fulfilling prophecy in this model: Once agents expect more variety in the future, the variety will actually rise in the future.

Because the expectations of future product variety affect the inter-temporal optimization decision and the path of capital accumulation, we need to formalize expectation formation. One way is a static expectation that $E_{t}\left[N_{t+1}\right]=N_{t}$. This implies that the agents expect that the future variety will be the same as today's. Another way is to assume rational expectations about $N_{t+1}$, so that the expected level equals the actual realization of $N_{t+1}$. However, in the steady state, $E_{t}\left[N_{t+1}\right]=N_{t}$, and both the static expectation and the rational expectation will yield the same result.

## Simulation Result

Figure 12: Policy function


The interval of one generation is assumed to be 30 years, and discount rate in one year is assumed to be $5 \% .^{24}$ Given a level of $K \in[1,10]$, the equilibrium level of $N$ is derived. The expected level of $N_{t+1}$ is set so that the expectation is consistent with the actual realization. Then, the saving rate is derived, along with the future stock of capital. The simulated policy function which shows the relationship between $K_{t}$ and $K_{t+1}$ is shown in Figure 12. The steady state is the point where the policy function intersects with the 45 degree line. Figure 13 shows the relationship between the expected level of future $N$ and the actually realized value of future $N$ around the steady state. The intersection with the 45 degree line is the point where the expectation of future N is rational.

Figure 13: Expected N vs Realized N


### 4.5 The effect on the pro-cyclicality of labour productivity

It is well known that productivity is positively correlated with the business cycle (Basu and Fernald, 2000). However, the positive correlation began to decline in the 1980s (Gali and van Rens, 2014). This model may also help to explain both the pro-cyclicality of labour productivity and its decline since the 1980s. When faced with positive (negative) demand shock, the total output of the economy increases (decreases), but the product variety does not instantaneously increase

[^16](decrease) accordingly as it takes time for new firms to enter the market with new varieties (or for existing firms add new products). Therefore, during a boom (recession), product variety is below (above) the equilibrium level of zero-profit, giving positive (negative) profit.

Because product variety increases (decreases) slower than the production quantity, the share of fixed factors (both labour and capital) of the total factor endowment decreases (increases), which leads to a fall (rise) in the share of white-collar workers in total employment. As fixed factors - including both fixed labour and fixed capital - do not contribute to increasing production quantity, the decrease (increase) in the share of fixed inputs increases (decreases) aggregate labour productivity by increasing the share of variable factors, which directly contributes to output quantity.

However, if the shock lasts long enough, the number of products will gradually adjust to the equilibrium level as new firms enter the market, and the shortrun gain in productivity will diminish due to an increased share of fixed input. ${ }^{25}$ Deregulations in the 1980s and the adoption of flexible manufacturing could have lowered the time lag needed to adjust product variety to the equilibrium level after facing demand shock. This could have lowered the pro-cyclicality of labour productivity.

GDP is determined so that, $Y=A \cdot\left(L^{b}\right)^{\alpha} \cdot\left(K^{v}\right)^{1-\alpha}$. $A$, which represents the level of production technology, is not the same as TFP because it is measured only with variable factor inputs. TFP is measured as the change in output which is not attributed to the change in total factor input. However, if the share of variable input increases during a boom, this increases output more than that is implied by the increase in total factor input (sum of both fixed and variable input) even without any change in $A$. This means that TFP is pro-cyclical.

In this model, TFP and labour productivity are pro-cyclical, which implies that the strong positive correlation of TFP and business-cycle may happen not because a TFP shock creates a business cycle as the RBC model assumes, but

[^17]because a demand shock actually affects TFP.
It is important that this model does not require labour hoarding or underutilisation of capital, unlike the existing explaining the pro-cyclicality of productivity, and does not even require any price (or wage) rigidity. The only condition needed is the rigid adjustment of variety. Given this, the model can generate the procyclicality of TFP even under perfect flexible price and full utilisation of factors.

## 5 Conclusion

Conventional wisdom informs us that the wage for a certain type of labour is determined by its marginal productivity. However, it is unlikely that the wage for fixed labour is determined by its marginal productivity. If fixed labour inputs are biased toward white-collar workers, an increase in the share of the fixed component in factor inputs (which can be driven by product diversification) can increase the wage for white-collar workers relative to blue-collar workers without any increase in relative productivity. Therefore, recent trend of rising wage inequality may not be the result of rising productivity of white-collar workers.

One important implication of this model is that it is possible that the contribution of university education to GDP growth estimated in the existing literature is overstated. For example, if university-graduated white-collar workers are paid $30 \%$ more, it may not necessarily mean that their per-person contribution to GDP growth is $30 \%$ higher. Although increased product diversification (due to the increase in university education) improves consumer welfare, it does not necessarily contribute to GDP growth. However, an increase in public expenditure in education, which lowers private cost of education, can decrease income inequality.

This model also contributes to explanation of pro-cyclicality of labour productivity and its puzzling decline since the 1980s. We show that it is possible that a pure demand shock can increase both labour productivity and TFP. It implies that the strong correlation of TFP and business cycle may occur not because a TFP shock drives the business cycle as the RBC model assumes, but because economic booms increase TFP.

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## Appendix. About mark-up ratio data

The mark-up ratio data comes from Oliveira Martins et al.(1996), who utilized Roeger(1995)'s method. Roeger (1995) utilises the gap between TFPs measured by different methods. Typically, TFP is estimated by calculating Solow residual as below:

$$
\begin{equation*}
S R=\Delta q-\alpha \Delta l-(1-\alpha) \Delta k \tag{10}
\end{equation*}
$$

Here, SR refers to Solow residual, and $\alpha$ is the share of labour income in the output. $\Delta l, \Delta k, \Delta q$ are the differences in the logs of labour input, capital input and output. The contribution of each factor in production is equal to its income share under the assumption of perfect competition.

However, Roeger (1995) showed that TFP can also be estimated using a pricebased Solow residual. It is defined by the difference between the increase in the weighted average of the factor price and the increase in the price of output as below:

$$
\begin{equation*}
S R P=\alpha \Delta w-(1-\alpha) \Delta r-\Delta p \tag{11}
\end{equation*}
$$

Here, SPR refers to price-based Solow residual. $\Delta w, \Delta r, \Delta p$ are the difference in the logs of wage, rental rate of capital and output price. When there is a positive technology shock, the output price rises less than the increase in the factor prices as the factors are consumed less due to the productivity improvement. In theory, under the assumption of perfect competition, TFPs estimated by both methods should be the same in theory. However, they are rarely identical in practice.

The point is that the labour's income share of output is not an accurate measure of labour's contribution to production under imperfect competition. The exact contribution of labour is equal to its income share in the marginal cost, which is lower than the price. Therefore, labour's income share of output underestimates the contribution of labour and overestimates the contribution of capital under imperfect competition. As a result, both Solow residuals are biased, but in different
directions. From the gap between these two types of Solow residuals, the mark-up ratio can be estimated as below:

$$
\begin{gather*}
S R_{t}-S R P_{t}=B \Delta x_{t}+u_{t}  \tag{12}\\
\Delta x_{t}=\left(\Delta y_{t}-\Delta k_{t}\right)+\left(\Delta p_{t}-\Delta r_{t}\right)
\end{gather*}
$$

Here, B is the Learner index defined as $B=\frac{P-M C}{P}$, or $B=1-\frac{1}{\mu}$, where $\mu$ is mark-up ratio. The mark-up ratio is derived by estimating $B$ in equation (12). However, Oliveira Martins et al.(1996) modify Roeger's method to incorporate material inputs in equation (12). The estimation equation used in Oliveira Martins et al.(1996) is:

$$
\begin{equation*}
\Delta y_{t}=B \cdot \Delta x_{t}+\varepsilon_{t} \tag{13}
\end{equation*}
$$

where,

$$
\begin{gathered}
\Delta y_{t}=(\Delta q+\Delta p)-\alpha \cdot(\Delta l+\Delta w)-\beta \cdot\left(\Delta m+\Delta p_{m}\right)-(1-\alpha-\beta) \cdot(\Delta k+\Delta r) \\
\Delta x_{t}=\left(\Delta y_{t}-\Delta k_{t}\right)+\left(\Delta p_{t}-\Delta r_{t}\right)
\end{gathered}
$$

Oliveira Martins et al.(1996) also adjust for the effect of indirect taxes on the estimated mark-up as below:

$$
\mu=\frac{\mu^{e}}{1+\tau}
$$

Here, $\mu^{e}$ is the estimated mark-up, and $\tau$ is indirect tax rate. Estimated mark-up ratios from Oliveira Martins et al.(1996) are shown in Table 2. The industrial classification system they use in Oliveira Martins et al.(1996) is ISIC rev.2. Data on payment, capital stock and material cost are based on NAICS 97 classification in this paper. Therefore, only ISIC rev. 2 industry groups with a clear correspondence to NAICS 97 classifications are used for estimation.

Table 2: The mark-up ratio in the US manufacturing, 1970-1992

| Sector name | (ISIC rev.2) | Sector (Naics 97) | mark-up |
| :---: | :---: | :---: | :---: |
| Food Products | 3112~ | $311000 \sim 312000$ | 1.05 |
| Beverages | 3130~ | - | - |
| Tobacco products | 3140~ | $312200 \sim 313000$ | 1.56 |
| Textiles | 3210~ | $313000 \sim 313000$ | 1.08 |
| Wearing apparel | $3220 \sim$ | $315000 \sim 316000$ | 1.10 |
| Leather products | $3230 \sim$ | 316000~321000 | 1.08 |
| Wood products | 3310~ | 321000~322000 | 1.22 |
| Furniture | 3320~ | 337000~339000 | 1.06 |
| Paper products \& Pulp | 3410~ | 322000~323000 | 1.13 |
| Printing \& Publishing | $3420 \sim$ | $323000 \sim 324000$ | 1.19 |
| Industrial chemicals | 3510~ | 325130~325400 | 1.18 |
| Drugs \& Medicines | 3522~ | 325400~325500 | 1.44 |
| Chemical products | 3529~ | $325500 \sim 326000$ | 1.26 |
| Petroleum refineries | 3530~ | 324110 | 1.03 |
| Petroleum \& Coal products | 3540~ | 324121~324199 | 1.11 |
| Rubber products | 3550~ | - | - |
| Plastic products | 3560~ | 326000~326200 | 1.07 |
| Pottery \& China | $3610 \sim$ | 327000~327200 | 1.09 |
| Glass products | 3620~ | 327200~327300 | 1.17 |
| Non-metal products | 3690~ | 327300~331000 | 1.18 |
| Iron \& Steel | $3710 \sim$ | $331000 \sim 331300$ | 1.10 |
| Non-ferrous metals | 3720~ | 331300~332000 | 1.14 |
| Metal products | 3810~ | 332000~333000 | 1.09 |
| Office \& Computing mach. | 3825~ | 334000~334200 | 1.54 |
| Machinery \& Equipment | 3829~ | 333000~334000 | 1.06 |
| Radio, TV \& Comm. equip. | 3832~ | 334200~334300 | 1.40 |
| Electrical apparatus | 3839~ | - | - |
| Shipbuilding \& Repair | 3841~ | - | - |
| Railroad equipment | 3842~ | - | - |
| Motor vehicles | 3843~ | 336000~336400 | 1.09 |
| Motorcycles \& Bicycles | 3844~ | 336991 | 1.13 |
| Aircraft | 3845~ | - | - |
| Other transport equipment | 3849~ | - | - |
| Professional goods | 3850~ | - | - |
| Other manufacturing | 3900~ | $339000 \sim 340000$ | 1.08 |


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[^1]:    ${ }^{1}$ Autor, Katz and Krueger (1998), for example, found that the share of college-graduate workers had risen faster in more computer-intensive industries.
    ${ }^{2}$ There is literature which assumed that nonproduction workers are more likely to be overhead labour or quasi-fixed (Ramey 1991; Nekarda and Ramey 2013; Gujarati and Lewis, 1972)

[^2]:    ${ }^{3}$ This is in line with Gujarati and Lewis (1972), who said "It is assumed that wages paid to production workers are essentially variable costs of production, whereas those paid to nonproduction workers are mostly in the nature of overhead or fixed costs, at least in the short-run." Our model predicts that a short-run expansion of output, which does not involve an increase in product variety, does not increase the demand for white-collar workers, while long-run growth of output, which accompanies the increase in the product variety, increases the demand for white-collar workers.

[^3]:    ${ }^{4}$ This implies the elasticity of substitution between white-collar and blue-collar workers increases with the degree of aggregation.

[^4]:    ${ }^{5}$ The wage differential between non-production workers and production workers in Sweden declined slightly from 1.549 in 1977 to 1.509 in 1989, but the employment share of non-production workers rose from 0.288 to 0.303 .
    ${ }^{6}$ Acemoglu and Autor (2010) called it as the 'canonical' model

[^5]:    ${ }^{7}$ However, the product innovation accounts for very significant part of R\&D activities. For example, according to Petrin and Warzynski (2012), $74 \%$ of total R\&D expenditure is for product innovation in Denmark.
    ${ }^{8}$ Xiang (2005) attributes the surge in inequality in the 1980 s to the burst of new products, "such as fiber optic cables, Windows series software, VCRs and soft contact lens."

[^6]:    ${ }^{9}$ This contrasts with Nelson and Phelps (1966) who argue that more educated workers are needed to adopt the latest vintage of production technology more quickly.
    ${ }^{10}$ However, Apple spent a significant amount on R\&D (because making it whiter involves some technological difficulties - such as the UV protection issue) simply to make it white.

[^7]:    ${ }^{11}$ While mark-up is the ratio between the price and the cost, $\mu$ is the ratio between variable profit and marginal cost. $\mu=$ mark-up -1

[^8]:    ${ }^{12}$ As we discuss long-run equilibrium, the term "fixed capital" means the capital which does not adjust as the output level varies in the long run as well as in the short run.
    ${ }^{13}$ In this model, we define 'white-collar workers' to be the same as 'non-production workers', and assume that they have a higher education level than production workers. Similarly, 'bluecollar workers' is synonymous with 'production workers'.

[^9]:    ${ }^{14}$ TFP is defined as the change in output which is not caused by the change in input. However, $A$ differs from typical TFP in that it only accounts for the change in variable input excluding fixed input while typical TFP accounts for both fixed and variable input.
    ${ }^{15}$ The Dixit-Stiglitz sytle monopolistic competition model, which implies single product firm, is adopted for simplicity although in reality most firms are multi-product firms. Within this frameworks, however, multi-product firms can be understood as different divisions within the same firm, independently producing different goods.

[^10]:    ${ }^{16}$ In a monopolistic competition market, $M R<P$, unlike a perfect competitive market where $M R=P$.

[^11]:    ${ }^{17}$ The education cost is defined in broader terms, and includes not only tuition fee but also any opportunity cost of lost labour income, lost leisure, personal effort and other obstacles to education such as credit constraints.
    ${ }^{18}$ Unlike Caselli (1999), where the learning cost is independent of the wage, it is assumed that the learning cost is proportional to the wage level of blue-collar workers, as it is likely that the opportunity cost of education increases with the wage level of unskilled labour.

[^12]:    ${ }^{19} \bar{k}$ and $\bar{l}$ are calibrated to replicate the levels of employment and the wages of both white-collar and blue-collar workers given observed $N, L, K, r$ and mark-up ratio.
    ${ }^{20}$ One interpretation is that a product survives for 5 years before being replaced by another.

[^13]:    ${ }^{21}$ According to Mansfield (1993), "the average year of first use of flexible manufacturing systems by major firms" is 1977.

[^14]:    ${ }^{22}$ Although this model assumes closed economy, it can be extended to an open economy, which remains to be further studied.

[^15]:    ${ }^{23}$ In this model, the investment good is not inherently different from consumption good.

[^16]:    ${ }^{24}$ Therefore, the discount factor, $\beta=0.95^{30} \simeq 0.215$. There is no population or TFP growth. Parameters are set as $L=1, A=40, \alpha=0.7, \rho=0.7, \bar{l}=0.01$ and $\bar{k}=0.05$.

[^17]:    ${ }^{25}$ The output expansion does not affect productivity if the variety and the quantity grow at the same rate, leaving the share of fixed inputs unchanged.

