Online appendix to

"Productivity and Potential Output Before, During, and After the Great Recession"

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June 2014

This appendix provides supporting materials and analysis.

- Appendix A discusses the role of data revisions in overstating the strength of productivity growth early in the Great Recession.
- Appendix B describes the data used in the paper.
- Appendix C discusses why, in the context of the Basu, Fernald, Oulton, and Srinivasan (2003) model, one expects the industry slowdown to be most pronounced in IT-intensive industris.
- Appendix D presents variants of the neoclassical growth model in order to derive the link between exogenous technology and endogenous labor-productivity growth. It also discusses the empirical fit of the model over the post-war period, which shows the gains from using the multi-sector model rather than a one-sector model, and to allowing for land growth in the model.

Appendix A: Role of data revisions

Real-time data obscured the slowdown in trend, and overstated productivity's strength early in the recession. Figure A1 shows labor productivity by vintage. The dates correspond to the annual (or, in 2009 and 2013, comprehensive) NIPA revisions; these revisions incorporate additional source data for previous years. Almost every revision since 2005 has lowered the path of labor productivity, with most revision to output (the numerator). These revisions made the slowdown more apparent. Real-time data also overstated the strength of labor productivity growth early in the recession. Until the 2010 revision, productivity appeared to have risen sharply and steadily throughout the recession. The sizeable downward revisions suggest some of the challenges of doing analysis in real time.

Appendix B: Data used in the paper

<u>Fernald (2014) Quarterly Growth-Accounting Data</u>

Data run 1947:Q2-2014:Q1. The vintage used for this paper was from May 7, 2014. Those data include the third (final) release of data for 2013:Q4 and the first (advance) release for 2014:Q1. Many calculations in the paper end in 2013:Q4. Current vintage data are available at http://www.frbsf.org/economics/economists/jfernald/quarterly_tfp.xls.

The dataset includes quarterly growth-accounting measures for the business-sector, including output, hours worked, labor quality (or composition), capital input, and total factor productivity. Output is a geometric average from the income and expenditures sides. Factor utilization follows Basu, Fernald, and Kimball (2006) and Basu, Fernald, Fisher, and Kimball (BFFK, 2013).

Once aggregated to an annual frequency, they are fairly close to the annual BLS multifactor productivity estimates, despite some differences in coverage and implementation.[1] The data are described in greater detail in Fernald (2014).

Key data sources for estimating (unadjusted) quarterly TFP for the U.S. business sector are:

(i) Business output: A geometric average of output as measured from the income and expenditures sides, as recommended by Nalewaik (2011), The expenditure (gross domestic product) side is reported in NIPA tables 1.3.5 and 1.3.6 (gross value added by sector). Nominal business income (the counterpart of gross domestic income) is GDI less nominal non-business output from table 1.3.5. Real business income uses the expenditure-side deflators.

(ii) Hours: From the quarterly BLS productivity and cost release.

(iii) Capital input: Weighted growth in 15 types of disaggregated quarterly capital (5 types of non-residential equipment, 5 types of structures, 3 types of intellectual property, plus inventories and land.) Estimated user costs are used to generate weights in capital input. For equipment, structures, intellectual property, and inventories, the underlying source is the BEA. For land, I interpolate and extrapolate from BLS estimates of land input into the business sector.

(iv) Factor shares: Based on NIPA data on corporate business total business factor costs and payments to labor and capital. Following Jorgenson, Gollop, and Fraumeni (1987) and the BLS, cost equals revenue net of taxes on production and imports (TOPI), plus subsidies, plus the portion of TOPI that is properly allocated to capital (property and motor vehicle taxes). This approach implicitly allocates proprietors' income between labor and capital so that labor's share of non-corporate, non-government businesses matches the share for non-financial corporations.

(v) Labor composition: From 1979:1 on, I use estimates that follow Aaronson and Sullivan (2001), as updated using unpublished code by Bart Hobijn and Joyce Kwok. Prior to 1979, I interpolate and extrapolate annual data from BLS multifactor productivity data.

(vi) Investment versus consumption technology: To decompose aggregate TFP along final demand lines, I create three Tornquist price indices from NIPA data. The first is the price of "equipment," defined as equipment, software, and consumer durables. The second is the price of structures, defined as residential and non-residential structures. The third is the price of non-durable "consumption," defined as everything else—i.e., the price of business output less equipment and structures. I assume the relative price of equipment investment corresponds, quarter-by-quarter, to TFP in consumption relative to equipment investment. (This measure of relative TFP is not, of course, necessarily equal to technology change period by period—see BFFK, 2013.)

To estimate a quarterly series on aggregate utilization, the key data source is the following:

[1] To name six minor differences: (i) BLS covers *private* business, Fernald covers total business. (ii) BLS uses expenditure-side measures of output, whereas Fernald combines income and expenditure-side measures of output. (iii) BLS assumes hyperbolic (rather than geometric) depreciation for capital. (iv) BLS uses more investment categories, available only at an annual frequency. (v) Fernald does not include rental residential capital. (vi) The labor-quality methods are slightly different. Some of these differences reflect what can be done quarterly versus annually. For a review of the methodology and history of the BLS measures, see Dean and Harper (2001).

(vii) Hours-per-worker (H^i / N^i) by industry from the monthly employment report of the BLS. These are used to estimate a series on industry utilization $\Delta \ln U_i = \beta_i \Delta \ln(H^i / N^i)$, where β_i is a coefficient estimated by Basu, Fernald, Fisher, and Kimball (BFFK, 2013). I then calculate an aggregate utilization adjustment as $\Delta \ln U = \sum_i w_i \Delta \ln U_i$, where w_i is the industry weight from BFK (taken as the average value over the full sample).

The resulting utilization-adjusted series differs conceptually from the BFK or BFFK purified technology series along several dimensions. BFK and BFFK use detailed industry data to construct estimates of industry technology change that control for variable factor utilization and deviations from constant returns and perfect competition. They then aggregate these residuals to estimate aggregate technology change. Thus, they do not assume the existence of a constant-returns aggregate production function. The industry data needed to undertake the BFFK estimates are available only annually, not quarterly. As a result, the quarterly series estimated here does not control for deviations from constant returns and perfect competition.[2]

BLS Industry Data. Table A1 provides a list of industries and subgroups used in the paper.

Multifactor productivity (MFP) data by industry cover 60 manufacturing and non-manufacturing industries. MFP is synonymous with TFP. The industry data do not control for labor quality. These data are available at http://www.bls.gov/mfp/mprdload.htm (accessed January 16, 2014).

IT intensity: IT-capital data were also downloaded from http://www.bls.gov/mfp/mprdload.htm (accessed January 16, 2014). To differentiate IT-intensive from non-IT intensive industries, I ranked industries based on the estimated payments for IT as a share of income (that is, the portion of capital's share of income that is attributable to IT, averaged over the full sample period—though using 1987-90 average makes little if any difference). Starting with the most IT-intensive industry, I selected industries until I reached 50 percent of the value-added weight (averaged 1987-2011) for the non-IT-producing "narrow business" economy. The narrow business economy is defined to exclude natural resources, construction, FIRE, and IT producing industries.

Finance intensity: The BLS produces annual I-O tables at the level of 195 industries/commodities, available at http://www.bls.gov/emp/ep_data_input_output_matrix.htm (accessed January 14, 2014). I aggregated industries 169 private business input-output industries to 60 BLS MFP industries according to NAICS codes. I then measure the finance share for each industry as nominal purchases of intermediate finance and insurance services (there are five such commodities in the underlying I-O tables) relative to total output of the industry. Finance usage was nominal purchases of various financial services as a share of industry gross output. "Finance intensive" is set of industries with the highest shares that constitute 50 percent of the value-added weight of narrow business excluding IT production.

[2] The output data also differ, both in vintage and data source, from the annual data used by BFK.

IT Producing: I define IT-producing industries to be (i) computer and electronic product manufacturing; (ii) publishing (including software); and (iii) computer systems integration and design. These three account for the vast majority of final expenditure on computers, communications, and software. Note that I exclude "information and data processing services" (e.g., cloud storage), since that provides intermediate services rather than final investment in hardware. That is, it is a substitute for direct ownership of IT hardware.

Well-measured industries: Griliches (1994) imagines "a 'degrees of measurability' scale, with wheat production at one end and lawyer services at the other. One can draw a rough dividing line on this scale between what I shall call 'reasonably measurable' sectors and the rest…." Griliches and Nordhaus (2002) draw the dividing line slightly differently. I largely follow Nordhaus, except that (as noted already) I exclude (well-measured) agriculture and mining and (poorly measured) construction and FIRE. I also exclude IT-producing industries. Well-measured thus comprises manufacturing (ex. computers and semiconductors), utilities, transportation, trade, and selected services (broadcasting and telecommunications, and accommodations). Switching trade and the selected services from well-measured to poorly-measured would make the slowdown in well-measured a bit less pronounced. Nevertheless, both well-measured and poorly-measured show a deceleration of more than a percentage point after 2004, so the main takeaway is unaffected by this choice.

State productivity and other data

- BEA GDP by industry and total full time and part time employment by industry were downloaded (February 24, 2014) from https://www.bea.gov/regional. Chain addition and subtraction were used to construct subgroup aggregates to correspond with the BLS industry groupings shown in Appendix Table 1. These data are prior to the 2013 benchmark revision
- State home prices are from Core Logic; and housing elasticity measures are from Saiz (2010). Metropolitan-area elasticities were aggregated to a state level using population weights. (I thank John Krainer and Fred Furlong for providing me with these data).
- For exploratory regressions cited in the text, Liz Laderman provided me with Business Dynamics Statistics data on small job births per capita by state. I measure the creation of firms with 1-5, 5-9, and 10-19 people over the year. I then divide by population to generate small job births per capita by state. (I thank Liz Laderman for providing me with these data).

CBO data

- CBO (2014a) projections for GDP and for (non-farm) business GDP and selected components were accessed via Haver Analytics in February, 2014.
- The non-farm-business labor gap compares unpublished BLS data on hours worked in non-farm business relative to CBO's published potential non-farm business hours. The unpublished BLS productivity-and-cost hours data match the published index values perfectly.3
- CBO publishes projections for GDP and for (non-farm) business GDP. To estimate output of nonprofits and government (i.e., the "non-non-farm" sector) I assume farms grow with other businesses, and ignore the difference between non-farm and total business. Using the

[3] I thank Bob Arnold at the CBO and John Glaser at BLS for help in understanding the data.

NIPA nominal business weights in GDP (0.76, averaged 1995-2007), I can back out an estimated non-business output.

- CBO publishes projections for labor-force growth and for non-farm business hours. In 2024, the potential labor force in CBO (2014a) grows at 0.5 percent, whereas NFB hours grows 0.64 percent. According to CBO staff, the difference primarily reflects continuing decline in government hours (i.e., a shift towards the business sector). So for the total economy, I use the growth in the potential labor force.
- To compare Fernald (2014) and CBO measures of TFP, I convert the Fernald measures from business to non-farm business by assuming that the wedge is the same as the (Bus/NFB) wedge in the BLS MFP data (which were accessed via Haver in April 2014). I also remove the effects of trend labor quality (which is included in the CBO figures but not in the others) from the CBO figures. The estimate uses a biweight filter estimated through 2007 and extended.

Appendix C. Implications of BFOS (2003) model

It is intuitive that, if the story of the mid-2000s productivity slowdown is the waning of the exceptional gains from IT, then the slowdown should be concentrated in IT-intensive industries. After all, that's where the "action" should have been for the speedup and slowdown. This appendix formalizes that idea in the context of the Basu, Fernald, Oulton, and Srinivasan (2003) model.

Formally, BFOS assume an IT user produces market output *Y* and (unobserved) intangible investment *A*, (where the market output can be transformed one-to-one into intangible investment) with a production function:

$$Q_{it} \equiv Y_{it} + A_{it} = F\left(Z_{it} G(K_{it}^{ICT}, C_{it}), K_{it}^{NT}, L_{it}\right), \quad i = 1...N \qquad \text{(A.1)}$$

A accumulates to intangible/organizational capital *C* that, together with IT capital, K^{IT}, produces services. The separability assumption on *G* captures the link between reorganization and IT.

Differentiating, one can show that <u>measured</u> growth in TFP (in terms of observed market output and observed inputs) is:

$$\Delta TFP = \left[\frac{F_C C}{Y}\right]\Delta c - \left[\frac{A}{Y}\right]\Delta a + \frac{F_Z Z}{Y}\Delta z \qquad \text{(A.2)}$$

Measured TFP misses the investment in intangibles, Δa, as well as the service flow from those intangibles, Δc. Other things equal, measured TFP *falls* when growth in unobserved investment, Δa, is faster. It *rises* when growth in complementary/organizational capital, Δc, is faster.

BFOS use the separability assumption for G to express the output elasticity $F_C C / Y$, and the growth rates Δc and Δa, in terms of IT observables and a small number of parameters. Note that, with perfect competition, the first-order conditions imply $F_C C / Y = P_{K,C} C / PY$. Suppose *G* is a CES function (with elasticity of substitution σ) and $\Delta p_{K,j}, j \subset \{ICT, C\}$ be the user cost of the two types of capital.

[4] See BFOS and Basu and Fernald (2008) for further details and derivations.

With the separability assumption, the first-order conditions for *C* and *K* imply that IT-intensive firms—those with a high share of IT in observed market output—are complementary intensive:

$$\frac{P_{K,C}C}{PY} = \left(\frac{1-\alpha}{\alpha}\right)^{\sigma}\left(\frac{P_{K,C}}{P_{K,IT}}\right)^{1-\sigma}\left(\frac{P_{K,IT}K^{IT}}{PY}\right) \equiv \beta\left(\frac{P_{K,C}}{P_{K,IT}}\right)^{1-\sigma} s_{K^{ICT}}$$

Separability also implies a link between Δc and Δk^{ICT} :

$$\Delta c_t = \Delta k_t^{ICT} + \sigma(\Delta p_{K,ICT} - \Delta p_{K,C})_t$$

The remaining challenge in operationalizing (A.2)is to measure unobserved investment. From the perpetual inventory formula $C_{it} = A_{it} + (1-\delta_C)C_{it-1}$, we can express Δa_t in terms of Δc_t and Δc_{t-1}:

$$\Delta a_t = \frac{C}{A}\left[\Delta c_t - \frac{(1-\delta_C)}{(1+g)}\Delta c_{t-1}\right]$$

These allow us to express

$$\rightarrow \Delta TFP = [F_C - 1]\beta\tilde{k}_t + \left[\frac{(1-\delta_C)}{(1+g)}\right]\beta\tilde{k}_{t-1} + s_G\Delta z, \qquad \text{(A.3)}$$

$$\text{where } \tilde{k}_t = s_{K^{ICT}}\left[\Delta k_t^{ICT} + \sigma(\Delta p_{K,ICT} - \Delta p_{K,C})_t\right]\left(P_{K,C}/P_{K,IT}\right)^{1-\sigma}$$

In this expression, the current and lagged growth rate of (suitably transformed) IT capital reflects its assumed link with growth of C and A. The share-weighting reflects the fact that, to have an important effect on measurement, this intangible capital must be sufficiently important. If IT capital has a high share then, other things equal, the model interprets it as implying that intangible capital also has a high share.

Contemporaneously, the coefficient on $\tilde{k}_t$ is negative (since $F_C \approx r + \delta_C < 1$). That reflects that, other things equal, if current IT-capital is growing, the model assumes that that *A* is also growing fast, and the diverted resources/unmeasured investment effect dominates and reduces measured TFP. In contrast, the coefficient on lagged $\tilde{k}_{t-1}$ is positive, since (for given Δc_t), higher $\tilde{k}_{t-1}$ implies fewer diverted resources Δa today.

BFOS used equation (A.3) as a cross-sectional estimating equation. The lags involved are unclear in the stylized model, which omits dynamic considerations such as adjustment costs and time-to-build for reorganization. BFOS consider a “period” to be 5 or more years. BFOS also ignored the relative price terms in operationalizing $\tilde{k}_t$. That is, they took $\tilde{k}_t = s_{K^{ICT}}\Delta k_t^{ICT}$. This is probably not a major problem for the cross-sectional implications, where the relative-price effects are largely common across sectors, so the important cross-industry differences show up in the IT share and IT growth. The relative-price terms are largely soaked up in the coefficients.

In contrast, Oliner, Sichel, and Stiroh (OSS, 2007) focus on the time-series dimension of this model. For those purposes, the relative-price trends are likely to be much more important. They relate the model to the broader literature on measuring intangible investment to calibrate $\sigma = 1.25$ and to measure the trends in relative user costs (in all periods, the user cost of IT falls sharply, so the estimated relative user cost of intangibles to IT rises at 7-10 percent per year).

An alternative way of implementing equation (A.3) is when we can take ICT investment as a direct proxy for unobserved complementary capital investment. To see when that will be the case, we can combine the accumulation equations for complementary capital and ICT capital to find:

$$\frac{A_{it}}{C_{it}} = \frac{I_{it}}{K_{it-1}} + \left(\delta_C - \delta_{ICT}\right) + \sigma \Delta p^{ICT} .$$

If $\delta_C = \delta_{ICT}$ and $\sigma = 0$ (which OSS argue is not the case; it doesn't change the basic point below), then $\Delta a = \Delta i^{ICT}$. This implies that:

$$\begin{aligned} \Delta TFP_{it} &= F_C \tilde{k}_{it} - \left(\frac{P\,A}{P_I I}\right)\left(\frac{P_I I}{PY}\right)\Delta i_{it}^{ICT} + s_G \Delta z_{it} \\ &= F_C \tilde{k}_{it} - b\tilde{i}_{it} + s_G dz_{it}, \text{ where } \tilde{i}_{it} = s_{it}^{I} \Delta i_{it}^{ICT} . \end{aligned} \qquad \text{(A.4)}$$

When will equation (A.4) be preferable to equation (A.3)? The key issue is the lag between ICT investment and complementary investment. For example, suppose a company invested heavily in an expensive enterprise resource management system in the mid-1980s and then spent the next decade learning how best to reorganize to benefit from the improved information availability. Then equation (A.3)—with very long lags—should work well. By contrast, if the reorganization was contemporaneous with the ICT investment, then equation (A.4) should work well (assuming the other conditions involved in deriving it are not too unreasonable) and there might not be long lags.

In the early 2000s, the reduced investment in IT capital was plausibly contemporaneous with reduced investment in intangibles. Hence, (A.4) should work well.

The model above suggests some cross-sectional implications. First, in the model, the proxy for IT use should be the IT income share multiplied by IT growth (as in (A.3)), possibly with a lag, or possibly combined with the IT-investment share multiplied by IT-investment growth (in version (A.4)). It should not merely be the IT income share (i.e., IT intensity). That said, the IT intensity latter is more common in the literature, and the different ways of identifying IT-intensive industries turn out to identify almost the same industries and yield the same results.

Second, in the context of the mid-2000s slowdown, the model implies that the major slowdown should have been in IT-intensive industries (however measured), since that's where the interesting intangible "action" is. The reason, in the model, is the following. Using (A.4), the speedup in the early 2000s in the cross section should have the form:

$$\Delta TFP_{i,00-04} - \Delta TFP_{i,95-00} = F_C\left(\tilde{k}_{i,00-04} - \tilde{k}_{i,95-00}\right) - b\left(\tilde{i}_{i,00-04} - \tilde{i}_{i,95-00}\right) + \varepsilon_{it}$$

This says that industries that sped up in the early 2000s should have been the ones that either (i) had an acceleration in $\tilde{k}_i$ growth or (ii) had a deceleration in $\tilde{i}_{i,t}$. In the cross-section, the IT-share, $s_{K^{ICT}}$ (calculated over full sample), has a weak negative correlation with (i), but a very strong negative correlation with (ii) of -0.73. This does point to the fact that the predictions of this model are not monotonic in the IT-intensity, because of the dynamics. Nevertheless, the strong deceleration in investment is plausibly the important factor.

Rolling this forward to the 2004-07 period,

$$\Delta TFP_{i,04-07} - \Delta TFP_{i,00-04} = F_C\left(\tilde{k}_{i,04-07} - \tilde{k}_{i,00-04}\right) - b\left(\tilde{i}_{i,04-07} - \tilde{i}_{i,00-04}\right) + \varepsilon_{it}$$

In this case, the slowdown should have been in industries that either (i) had a deceleration in $\tilde{k}_i$ or (ii) had an acceleration in $\tilde{i}_{i,t}$. In the cross-sectional data for 2004-07, both of these implications are true: $s_{K^{ICT}}$ has a negative correlation of -0.65 with $\left(\tilde{k}_{i,04-07} - \tilde{k}_{i,00-04}\right)$ and a positive correlation of +0.48 with $\left(\tilde{i}_{i,04-07} - \tilde{i}_{i,00-04}\right)$.

Hence, the model predicts that IT-intensive industries should have seen a larger slowdown after 2004. The story in the model is the following. In the early 2000s, the strength in measured TFP reflected that firms were cutting back on intangible investments, which meant reallocating resources towards measured output, which raised productivity. This effect was more pronounced for IT-intensive industries, and relies on the investment effect dominating the reduced-intangible-capital effect.

After 2004, IT-intensive industries saw a larger deceleration in intangible capital services, but also saw a larger acceleration in IT-intensive investment (reflecting that they had slowed so dramatically in the early 2000s,). So the story is a "return to normal" in the post-2004 period.

Appendix D: Projecting Labor Productivity in Neoclassical Growth Models

This appendix discusses how to estimate steady-state labor productivity growth from estimates of underlying technology growth. It uses a neoclassical model to derive the implications for capital deepening. Section A summarizes the familiar one-sector Solow model. Section B develops a two-sector Solow model, which highlights the key takeaways and intuition for the multi-sector model. Section C derives the (straightforward, but somewhat tedious) extension to the case with consumer durables, land, and inventories.

A few equations will be useful as preliminaries. Let hats over a variable represent log changes. As an identity, output growth, $\hat{Y}$, is labor-productivity growth plus growth in hours worked, $\hat{H}$:

$$\hat{Y} = (\hat{Y} - \hat{H}) + \hat{H} .$$

We focus here on full-employment labor productivity, so we abstract from utilization.

Growth in total factor productivity, or the Solow residual, is defined as

$$TFP = \hat{Y} - \alpha\hat{K} - (1-\alpha)\hat{L} \qquad \text{(A.5)}$$

where α is capital's share of income and (1-α) is labor's share. Defining $\hat{L} \equiv \hat{H} + LQ$, where LQ is labor "quality" (composition) growth[5], output per hour growth is:

$$(\hat{Y} - \hat{H}) = TFP + \alpha(\hat{K} - \hat{L}) + LQ . \qquad \text{(A.6)}$$

Growth in output per hour worked reflects TFP growth; the contribution of capital deepening, defined as $\alpha(\hat{K} - \hat{L})$; and increases in labor quality. Economic models suggest mappings between fundamentals and the terms in this identity.

It is sometimes useful to rearrange (A.6) to yield:

$$(\hat{Y} - \hat{H}) = TFP/(1-\alpha) + \alpha(\hat{K} - \hat{Y}) + LQ \qquad \text{(A.7)}$$

We now show how a one-sector and two-sector model map to these equations. Then we allow for a third sector, and for inventories, and land.

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[5] In the BLS multifactor productivity dataset, from 1948 through 2012, hours grew 1.10 percent per year, and labor quality/composition grew 0.32 percent per year. Hence, more than a quarter of labor input growth in the MFP data reflects labor quality. As discussed in the text, labor quality, in turn, reflects the mix of hours across workers with different levels of education, experience, and so forth.

A. The one-sector Solow model

The Solow model provides a particularly simple model that maps exogenous growth in technological progress and the labor force to endogenous capital deepening.

Consider an aggregate production function $Y = K^{\alpha}(AN)^{1-\alpha}$, where labor-augmenting technology *A* grows at rate *g*, and labor input *N* (which captures both raw hours *H* and labor quality *LQ*—henceforth, I do not generally differentiate between the two) grows at rate *n*. Expressing all variables in terms of "effective labor" *AN* yields:

$$y = k^{\alpha}\text{, where } y = Y/AN \text{ and } k = K/AN. \tag{A.8}$$

Capital accumulation takes place according to the perpetual-inventory formula, $\dot{K} = I - \delta K$. Let *s* is the saving rate, so that *sy* is investment per effective worker. In steady-state:

$$sy = (n + \delta + g)k \tag{A.9}$$

Because of diminishing returns to capital, the economy converges to a steady state where *y* and *k* are constant. At that point, investment per effective worker is just enough to offset the effects of depreciation, population growth, and technological change on capital per effective worker. In steady state, the unscaled levels of *Y* and *K* grow at the same rate *g+n*; capital-deepening, *K/N*, grows at rate *g*. Labor productivity *Y/N*, i.e., output per unit of labor input, also grows at rate *g*.

From the production function, measured TFP growth is related to labor-augmenting technology growth by:

$$TFP = \hat{Y} - \alpha\hat{K} - (1-\alpha)\hat{L} = (1-\alpha)g\ .$$

The model maps directly to equations (A.6) and (A.7) above. In steady state, $\hat{K} = \hat{Y}$, and, as in equation(A.7), output per unit of labor grows at $g = TFP/(1-\alpha)$. Alternatively, in terms of equation(A.6), the endogenous contribution of capital deepening to labor-productivity growth is $\alpha(\hat{K} - \hat{L}) = \alpha g = \alpha \cdot TFP/(1-\alpha)$. Thus, we can write growth in output per hour in a form that corresponds closely with the two-sector version below:

$$\hat{Y} - n = TFP + \alpha \cdot TFP/(1-\alpha) \tag{A.10}$$

Growth in output per unit of labor depends on standard TFP growth and induced capital deepening.

B. The two-sector Solow model

In contrast to the predictions of the one-sector model, the capital-output ratio in the data rises steadily after the early 1970s. The literature on investment specific technical change suggests a straightforward fix for this model failure: Capital-deepening doesn't depend on *overall* TFP, but on TFP in the investment sector. A key motivation for this literature is the declining price of business investment goods, especially equipment and software, relative to the price of other goods (such as consumption). The most natural interpretation of the declining relative price is faster technical change in producing investment goods (especially high-tech equipment).[6]

Consider a simple two-sector Solow-type model, where *s* is the share of nominal output that is invested each period.[7] One sector produces investment goods that are used to create capital; the other

[6] On the growth accounting side, see, for example, Jorgenson (2001) or Oliner and Sichel (2000); see also Greenwood, Hercowitz, and Krusell (1997).

[7] This model is a fixed-saving rate version of the two-sector neoclassical growth model in Whelan (2003) and is isomorphic to the one in Greenwood, Hercowitz, and Krusell (1997). Greenwood *et al*. choose a different normalization of the two technology shocks in their model.

produces consumption goods. The two sectors use the same Cobb-Douglas production function, but with potentially different technology levels:

$$I = K_I^{\alpha}(A_I L_I)^{1-\alpha}$$

$$C = QK_C^{\alpha}(A_I L_C)^{1-\alpha}$$

In the consumption equation, we have implicitly defined labor-augmenting technological change as $A_C = Q^{1/(1-\alpha)}A_I$ in order to decompose consumption technology into the product of investment technology A_I and a "consumption specific" piece, $Q^{1/(1-\alpha)}$. Let investment technology A_I grow at rate g_I and the consumption-specific piece Q grow at rate q. Perfect competition and cost-minimization imply that price equals marginal cost. If the sectors face the same factor prices (and the same rate of indirect business taxes), then relative marginal costs depend solely on relative technology:

$$\frac{P_I}{P_C} = \frac{MC^C}{MC^I} = Q$$

The sectors also choose to produce with the same capital-labor ratios, implying that $K_I/A_I L_I = K_C/A_I L_C = K/A_I L$. We can then write the production functions as:

$$I = A_I L_I \left(K/A_I L\right)^{\alpha}$$

$$C = QA_I L_C \left(K/A_I L\right)^{\alpha} \tag{A.11}$$

We can now write the economy's budget constraint in a simple manner:

$$Y^{\text{Inv. Units}} \equiv [I + C/Q] = A_I(L_I + L_C)\left(K/A_I L\right)^{\alpha}, \text{ or}$$

$$y^{\text{Inv. Units}} = k^{\alpha}, \text{ where } y^{\text{Inv. Units}} = Y^{\text{Inv. Units}}/A_I L \text{ and } k = K/A_I L. \tag{A.12}$$

Output here is expressed in investment units, and "effective labor" is in terms of technology in the *investment* sector. The economy mechanically invests a share s of nominal investment, which implies that investment per effective unit of labor is $i = s \cdot y^{\text{Inv. Units}}$.[8]

Capital accumulation turns out to take the same form as in the one-sector model, except that it is only growth in investment technology, g_I, that matters. In particular, in steady state:[9]

$$sy^{\text{Inv. Units}} = (n + \delta + g_I)k \tag{A.13}$$

The production function (A.12) and capital-accumulation equation (A.13) correspond exactly to their one-sector counterparts. Hence, the dynamics of capital in this model reflect technology in the investment sector alone. In steady state, capital per unit of labor, K/L, grows at rate g_I, so the contribution of capital deepening to labor-productivity growth from equation (A.6) is

$$\alpha(\hat{K} - \hat{L}) = \alpha g_I = \alpha \cdot TFP_I/(1-\alpha) \tag{A.14}$$

Consumption technology in this model is "neutral," in that it does not affect investment or capital accumulation; the same result generally carries over to the Ramsey version of this model, with or without variable labor supply. (Basu, Fernald, Fisher, and Kimball, 2013, discuss the idea of consumption-technology neutrality in greater detail.)

In the data, output is not expressed in investment units but as chained units. Chain GDP growth is defined as share-weighted growth in final expenditure categories:

---

[8] $s \cdot y^{\text{Inv. Units}} = \left[P_I I/(P_I I + P_C C)\right]\left[\left(I + P_C C/P_I\right)/A_I L\right] = I/A_I L$

[9] The time-derivative $\dot{k} = d/dt(K/AL) = (K/AL)(\dot{K}/K - n - g_I)$. Substituting the capital accumulation equation, $\dot{K}/K = I/K - \delta$, yields $\dot{k} = i - (n + g_I + \delta)k$. In steady-state, $\dot{k} = 0$. Substituting for i yields (A.13).

$$\hat{Y} = s\hat{I} + (1-s)\hat{C}$$

From equation (A.12), in steady state, when $k = K / A_I L$ is constant, $\hat{I}$ grows at rate $(n+g_I)$ and $\hat{C}$ grows at rate ($n + g_I + \hat{q}$). Hence,

$$\hat{Y} = n + g_I + (1-s)\hat{q}. \qquad \text{(A.15)}$$

The capital-output ratio grows at $\hat{K} - \hat{Y} = (n+g_I) - (n + g_I + (1-s)\hat{q}) = -(1-s)\hat{q}$ Since consumption TFP growth is generally lower than investment TFP growth, $\hat{q}$ is negative in the data, and the model predicts that the measured capital-output ratio is increasing.

Note that overall TFP growth in chain-units is:

$$\begin{aligned} TFP &= \hat{Y} - \alpha\hat{K} - (1-\alpha)\hat{L} \\ &= n + g_I + (1-s)\hat{q} - \alpha(n+g_I) - (1-\alpha)n \\ &= (1-\alpha)g_I + (1-s)\hat{q} \end{aligned} \qquad \text{(A.16)}$$

Hence, rearranging (A.15) and substituting from (A.16) and (A.14), growth in output per unit of labor can be written:

$$\begin{aligned} \hat{Y} - n &= g_I + (1-s)\hat{q} = \left[(1-\alpha)g_I + (1-s)\hat{q}\right] + \alpha g_I \\ &= TFP + \alpha\frac{TFP_I}{(1-\alpha)} \end{aligned} \qquad \text{(A.17)}$$

This equation takes the same form as (A.10), except that capital deepening is solely in terms of investment-sector TFP growth.

To take this model to the data, we need to decompose aggregate TFP growth (calculated from chained output) into its consumption and investment components. Given the conditions so far, the following two equations hold:

$$TFP = s \cdot TFP_I + (1-s)TFP_C$$

$$P_C - P_I = TFP_C - TFP_I$$

Prices, investment shares, and aggregate TFP are known. Hence, these are two equations in two unknowns—TFP_I and TFP_C.[10]

C. Three sector model

In practice, there are multiple types of capital. The most important distinction is between fast-growing equipment and more slowly growing structures. The argument would naturally extend to more types of capital, as well. Suppose that there's a **D**urable sector that produces equipment, a **B**uilding sector that produces structure, and a **C**onsumption sector: [11]

[10] The calculations in the text use the official price deflators from the national accounts. Gordon (1990) argues that many equipment deflators are not sufficiently adjusted for quality improvements over time. Much of the macroeconomic literature since then has used the Gordon deflators. Of course, as Whelan (2003) points out, much of the discussion of biases in the CPI involve service prices, which also miss a lot of quality improvements, making the overall effect uncertain. Hobijn and McKay (2007) also question these hedonic adjustments.

[11] The mnemonics—**D**urables rather than **E**quipment, for example—is to clearly differentiate the flow output of producing sectors from the accumulated stock of equipment and structures.

$$
\begin{aligned}
D &= \quad (K_D)^{\alpha}(AL_E)^{1-\alpha} \\
B &= Q_B(K_B)^{\alpha}(AL_B)^{1-\alpha} \\
C &= Q_C(K_C)^{\alpha}(AL_C)^{1-\alpha}
\end{aligned}
\tag{A.18}
$$

Some durable goods are consumed as durables. Other durable goods are invested and become equipment capital according to the usual perpetual inventory equation. Similarly, new buildings become gross investment in structures. All three sectors use the same capital aggregate, which uses equipment *E* and structures *S*.

$$
K = E^{c_E} S^{1-c_E} = K_D + K_B + K_C \tag{A.19}
$$

To solve for steady state growth rates, I follow Whelan (2003). In steady state, growth of equipment and structures must be the same in all uses, and labor growth (at rate *n*) is the same in all uses. Let g_X be steady-state growth in variable *X*. In steady-state, the perpetual-inventory formula implies that growth of investment in durables or buildings is equal to growth in the capital stocks of equipment and structures, respectively.[12] That is, $g_E = g_D$ and $g_S = g_B$. In growth rates, then:

$$
\begin{aligned}
g_D &= \alpha(c_E g_D + (1-c_E)g_B) + (1-\alpha)(\hat{a}+n) \\
g_B &= \alpha(c_E g_D + (1-c_E)g_B) + (1-\alpha)(\hat{a}+n) + \hat{q}_S = g_D + \hat{q}_B \\
g_C &= \alpha(c_E g_D + (1-c_E)g_B) + (1-\alpha)(\hat{a}+n) + \hat{q}_C = g_D + \hat{q}_C
\end{aligned}
\tag{A.20}
$$

This is a straightforward system of simultaneous equations that yields:

$$
\begin{aligned}
g_D &= (\hat{a}+n) + \frac{\alpha(1-c_E)}{1-\alpha}\hat{q}_B \\
g_B &= g_D + \hat{q}_B \\
g_C &= g_D + \hat{q}_C
\end{aligned}
\tag{A.21}
$$

Chain GDP growth is share-weighted growth in final expenditure categories. If s_D is the final-expenditure-share of durables and s_B is the final-expenditure-share of buildings, then:

$$
\begin{aligned}
g &= s_D g_D + s_B g_B + (1-s_D-s_B)g_C \\
&= g_D + s_B\hat{q}_B + (1-s_D-s_B)\hat{q}_C \\
&= (\hat{a}+n) + \left[\frac{\alpha(1-c_E)}{1-\alpha} + s_B\right]\hat{q}_B + (1-s_D-s_B)\hat{q}_C
\end{aligned}
\tag{A.22}
$$

Growth in output per unit of labor is then:

$$
g - n = \hat{a} + \left[\frac{\alpha(1-c_E)}{1-\alpha} + s_B\right]\hat{q}_B + (1-s_D-s_B)\hat{q}_C \tag{A.23}
$$

Standard TFP growth for each sector is not in labor-augmenting form, so it equals:

$$
\begin{aligned}
TFP_D &= (1-\alpha)\hat{a} \\
TFP_B &= (1-\alpha)\hat{a} + \hat{q}_B = TFP_D + \hat{q}_B \\
TFP_C &= (1-\alpha)\hat{a} + \hat{q}_C = TFP_D + \hat{q}_C
\end{aligned}
\tag{A.24}
$$

---

[12] In steady-state, $I/K = g + \delta$. Since the right-hand-side is constant, *I* must grow at the same rate as *K*.

Overall TFP growth in this economy is output growth less share-weighted input growth:

$$TFP = g - \alpha(c_E g_D + (1-c_E)g_B) - (1-\alpha)n \tag{A.25}$$

Using the second line of (A.22) and then substituting from (A.21), we find:

$$\begin{aligned} TFP &= [g_D + s_B\hat{q}_B + (1-s_D-s_B)\hat{q}_C] - \alpha(g_D + (1-c_E)\hat{q}_B) - (1-\alpha)n \\ &= (1-\alpha)g_D - \alpha(1-c_E)\hat{q}_B + s_S\hat{q}_B + (1-s_D-s_B)\hat{q}_C - (1-\alpha)n \\ &= (1-\alpha)(\hat{a}+n) + \alpha(1-c_E)\hat{q}_B - \alpha(1-c_E)\hat{q}_B + s_B\hat{q}_B + (1-s_D-s_B)\hat{q}_C - (1-\alpha)n \\ &= TFP_D + s_B\hat{q}_B + (1-s_D-s_B)\hat{q}_C \end{aligned} \tag{A.26}$$

Note that aggregate TFP growth is also equal to share-weighted sectoral TFP growth using (A.24).

Define investment TFP growth, TFP_I, in terms of <u>user cost (factor share)</u> weights (rather than expenditure weights):

$$\begin{aligned} TFP_I &= c_E TFP_D + (1-c_E)TFP_B \\ &= TFP_D + (1-c_E)\hat{q}_B \end{aligned} \tag{A.27}$$

We can now write growth in output per unit of labor from (A.23) in terms of overall and investment-sector TFP growth:

$$\begin{aligned} g-n &= \hat{a} + \left[\frac{(1-\alpha)s_B + \alpha(1-c_E)}{1-\alpha}\right]\hat{q}_B + (1-s_D-s_B)\hat{q}_C \\ &= \left[(1-\alpha)\hat{a} + s_B\hat{q}_B + (1-s_D-s_B)\hat{q}_C\right] + \alpha\hat{a} + \left[\frac{(1-\alpha)s_B + \alpha(1-c_E)}{1-\alpha} - s_B\right]\hat{q}_B \\ &= TFP + \left(\alpha\hat{a} + \left[\frac{\alpha(1-c_E)}{1-\alpha}\right]\hat{q}_B\right) \\ &= TFP + \frac{\alpha}{1-\alpha}TFP_I \end{aligned} \tag{A.28}$$

Although the derivation is somewhat involved, this is exactly the same equation as for the two-sector model.

Finally, note that the existence of consumer durables (produced by the durable sector) does not affect this calculation. The weight on durables in final expenditure, s_D, already includes all final uses of durable output (whether for investment or for durable consumption). However, the user cost weight of equipment includes only the portion used for equipment investment.

D. Adding inventories and land

In practice, there are not only multiple types of capital goods, but land. We can derive more general steady-state predictions using the same approach as with the three-sector model above.[13]

Specifically, we assume the same production structure as in (A.18), above:

---

[13] This analysis takes land as exogenous, though not fixed—it can be pulled from other uses, and in the BLS dataset, business use of land grows at about 1-1/2 percent per year. An alternative modeling strategy would be to tie it to the use of structures in some way. That said, the correlation in the BLS dataset between annual changes in structures and land is far from perfect (about 0.4).

$$
\begin{aligned}
D &= \quad (K_D)^{\alpha}(AL_E)^{1-\alpha} \\
B &= Q_B(K_B)^{\alpha}(AL_B)^{1-\alpha} \\
C &= Q_C(K_C)^{\alpha}(AL_C)^{1-\alpha}
\end{aligned} \tag{A.29}
$$

Now, some durable goods are used for consumption (which raises the weight of durables in final output). We also have inventories in capital. Inventories are goods (in the data, roughly half are durable and half are non-durable), but their relative price movements are less pronounced than for equipment. For generality in derivations, we'll allow both the durable and the non-durable sectors to produce inventories.

The capital aggregate now includes inventories, *V*, and land, *T* (for **T**erra), as well as equipment and structures:

$$
K = E^{c_E} S^{1-c_E-c_V-c_T}\left(V_D^{\delta} V_C^{1-\delta}\right)^{c_V} T^{c_T} = K_D + K_B + K_C \tag{A.30}
$$

Using (A.29) and (A.30), we can proceed in the same way as in the three-sector model:

$$
\begin{aligned}
g_D &= \alpha(c_E g_D + (1-c_E-c_V-c_T)g_B + c_V\delta g_D + c_V(1-\delta)g_C + c_T\hat{T}) + (1-\alpha)(\hat{a}+n) \\
g_B &= g_D + \hat{q}_B \\
g_C &= g_D + \hat{q}_C
\end{aligned} \tag{A.31}
$$

TFP growth in each sector is related to the "fundamental shocks" as shown in equation (A.24). TFP growth for "reproducible investment," TFP_I, with user cost (factor share) weights, is then:

$$
\begin{aligned}
TFP_I &= \left(\frac{c_E + c_V\delta}{1-c_T}TFP_D + \frac{(1-c_E-c_V-c_T)}{1-c_T}TFP_B + \frac{c_V(1-\delta)}{1-c_T}TFP_C\right) \\
&= TFP_D + \frac{(1-c_E-c_V-c_T)}{1-c_T}\hat{q}_B + \frac{c_V(1-\delta)}{1-c_T}\hat{q}_C
\end{aligned} \tag{A.32}
$$

Solving the system of equations in (A.31) yields

$$
g_D = \left(\frac{1-\alpha}{1-\alpha(1-c_T)}\right)(\hat{a}+n) + \frac{\alpha c_V(1-\delta)}{1-\alpha(1-c_T)}\hat{q}_C + \frac{\alpha(1-c_E-c_V-c_T)}{1-\alpha(1-c_T)}\hat{q}_B + \left(\frac{\alpha c_T}{1-\alpha(1-c_T)}\right)\hat{T} \tag{A.33}
$$

Adding and subtracting TFP_D, rearranging, and substituting from (A.32), yields:

$$
\begin{aligned}
g_D &= TFP_D + \left[\frac{1}{1-\alpha(1-c_T)} - 1\right]TFP_D + \frac{\alpha c_V(1-\delta)}{1-\alpha(1-c_T)}\hat{q}_C + \frac{\alpha(1-c_E-c_V-c_T)}{1-\alpha(1-c_T)}\hat{q}_B + \left(\frac{\alpha c_T}{1-\alpha(1-c_T)}\right)\hat{T} + \left(\frac{1-\alpha}{1-\alpha(1-c_T)}\right)n \\
&= TFP_D + \left[\frac{\alpha(1-c_T)}{1-\alpha(1-c_T)}\right]\left[TFP_D + \frac{(1-c_E-c_V-c_T)}{1-c_T}\hat{q}_B + \frac{c_V(1-\delta)}{1-c_T}\hat{q}_C\right] + \left(\frac{\alpha c_T}{1-\alpha(1-c_T)}\right)\hat{T} + \left(\frac{1-\alpha}{1-\alpha(1-c_T)}\right)n \\
&= TFP_D + \left[\frac{\alpha(1-c_T)}{1-\alpha(1-c_T)}\right]TFP_I + \left(\frac{\alpha c_T}{1-\alpha(1-c_T)}\right)\hat{T} + \left(\frac{1-\alpha}{1-\alpha(1-c_T)}\right)n
\end{aligned} \tag{A.34}
$$

Growth in reproducible capital per worker can be expressed as:

$$\hat{K}^{\mathrm{R}} - n = \left(\frac{1}{1-c_T}\right)\left((c_E + c_V\delta)g_D + (1-c_E-c_V-c_T)g_S + c_V(1-\delta)g_S\right) - n$$
$$= g_D + \left(\frac{c_V(1-\delta)}{1-c_T}\right)\hat{q}_C + \left(\frac{1-c_E-c_V-c_T}{1-c_T}\right)\hat{q}_B - n$$

If we substitute for g_D from (A.33), define $\alpha^R = \alpha(1-c_T)$, and rearrange, we find:

$$\hat{K}^{\mathrm{R}} - n = \left[\frac{1}{1-\alpha^R}\right]TFP_I + \left(\frac{\alpha c_T}{1-\alpha^R)}\right)(\hat{T}-n) \tag{A.35}$$

Overall capital deepening is

$$\alpha\left(\hat{K}-n\right) = \alpha(1-c_T)\left(\hat{K}^{\mathrm{R}}-n\right) + \alpha c_T(\hat{T}-n)$$
$$= \left[\frac{\alpha^R}{1-\alpha^R}\right]TFP_I + \left(\frac{\alpha c_T}{1-\alpha^R}\right)(\hat{T}-n) \tag{A.36}$$

From (A.6), output per worker is:

$$g - n = TFP + \alpha\left(\hat{K}-n\right)$$
$$= TFP + \left[\frac{\alpha^R}{1-\alpha^R}\right]TFP_I + \left(\frac{\alpha c_T}{1-\alpha^R}\right)(\hat{T}-n) \tag{A.37}$$

This equation is a natural extension of the one- and two-sector models. If land's share, c_T, is zero, then this equation exactly matches (A.17) and (A.28). If $TFP_I = TFP$, then the equation matches (A.10).

In terms of comparing model projections, land is a complicating factor. Some comparisons are easier, however, since land affects the predictions equally. First, the predictions of the one-sector model with land are the case where $TFP_I = TFP$, so the difference in predictions is just:

$$\left(g^{\text{Multi-Sector}} - n\right) - \left(g^{\text{One Sector}} - n\right) = \left[\frac{\alpha^R}{1-\alpha^R}\right]\left(TFP_I - TFP\right).$$

Second, recall from the second line of equation ?? that, by the definition of chained GDP, that $g = g_D + s_B\hat{q}_B + (1-s_D-s_B)\hat{q}_C$. It follows that components of the capital-output ratio are:

$$g_D - g = s_B\hat{q}_B + (1-s_D-s_B)\hat{q}_C$$
$$g_B - g = (g_D + \hat{q}_B) - g = (1-s_B)\hat{q}_B + (1-s_D-s_B)\hat{q}_C.$$

Third, from equation (A.35) for growth in reproducible capital, and from the chain-GDP equation, it follows that the growth rate of the reproducible-capital-to-output ratio is:

$$\hat{K}^{\mathrm{R}} - g = \left(\frac{1-c_E-c_V-c_T}{1-c_T} - s_B\right)\hat{q}_B + \left(\frac{c_V(1-\delta)}{1-c_T} - s_C\right)\hat{q}_C$$

The inventory share of non-land capital payments, $c_V/(1-c_T)$, is under 10 percent, whereas s_C is about 75 percent. Thus, the second term in brackets is negative. Since $\hat{q}_C$ is also negative in the data, the second piece tends to push growth in the reproducible-capital to output ratio to be positive. On the other side, the weight on building-specific TFP growth is the difference between structure's weight in reproducible capital, $(1-c_E-c_V-c_T)/(1-c_T)$, which averages about 45 percent), and building's share of GDP (which averages 5 percent). Since $\hat{q}_B$ is negative in the data, the building component tends to push this piece negative. In practice, the first positive effect is quantitatively more important.

E. Fit of the Model

Table A2 compares steady-state implications of the model to labor productivity data. Despite its simplifications, the model matches overall and subsample growth closely.

- The model with land (panel B) has a lower effective capital share, which better captures the magnitude of the pickup after 1995.
- Relative to a one-sector model, the multisector version more closely matches subsample variation. The one-sector model especially underpredicts capital deepening after 1973. Because it assumes the capital-output ratio is constant in steady state, it misses the trend increase in the capital-output ratio in the data.

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Table A1
BLS industries, and definitions of sub-groups used in the paper.

		NAICS	IT-prod. (1)	Bus, excl. Nat Res, Con, FIRE (2)	IT-int. (in (2)) (3)	Not-IT-int. (in (2)) (4)	Fin-int. (in (2)) (5)	Not fin. Int (in (2)) (6)	Well (in (2)) (7)	Poor (in (2)) (8)
1	Manufacturing	MN								
2	Nondurable goods	ND								
3	Food, beverage and tobacco product manufacturing	311,312		x		x		x	x	
4	Textile and textile product mills	313,314		x		x		x	x	
5	Apparel, leather, and allied product manufacturing	315,316		x		x		x	x	
6	Paper manufacturing	322		x		x		x	x	
7	Printing and related support activities	323		x		x		x	x	
8	Petroleum and coal products manufacturing	324		x	x			x	x	
9	Chemical manufacturing	325		x	x			x	x	
10	Plastics and rubber products manufacturing	326		x		x		x	x	
11	Durable goods	DM								
12	Wood product manufacturing	321		x		x		x	x	
13	Nonmetallic mineral product manufacturing	327		x		x		x	x	
14	Primary metal manufacturing	331		x		x		x	x	
15	Fabricated metal product manufacturing	332		x		x		x	x	
16	Machinery manufacturing	333		x	x			x	x	
17	Computer and electronic product manufacturing	334	x	x						
18	Electrical equipment, appliance, and component manufacturin	335		x		x		x	x	
19	Transportation equipment manufacturing	336		x		x		x	x	
20	Furniture and related product manufacturing	337		x		x		x	x	
21	Miscellaneous manufacturing	339		x	x			x	x	
22	Agriculture, forestry, fishing, and hunting	11								
23	Farms	111,112								
24	Forestry, fishing, hunting, and related activities	113-115								
25	Mining	21								
26	Oil and gas extraction	211								
27	Mining, except oil and gas	212								
28	Support activities for mining	213								
29	Utilities	22		x	x			x	x	
30	Construction	23								
31	Trade	42,44-45								
32	Wholesale trade	42		x	x		x		x	
33	Retail trade	44,45		x		x	x		x	
34	Transportation and warehousing	48-49								
35	Air transportation	481		x	x			x	x	
36	Rail transportation	482		x		x	x		x	
37	Water transportation	483		x	x		x		x	
38	Truck transportation	484		x		x	x		x	
39	Transit and ground passenger transportation	485		x		x	x		x	
40	Pipeline transportation	486		x	x		x		x	
41	Other transportation and support activities	487,488,492		x		x		x	x	
42	Warehousing and storage	493		x		x		x	x	
43	Information	51								
44	Publishing (incl. software)	511,516	x	x						
45	Motion picture and sound recording industries	512		x	x			x		x
46	Broadcasting and telecommunications	515,517		x	x			x	x	
47	Information and Data Processing Services	518,519		x	x			x		x
48	Finance, Insurance, and Real Estate	52-53								
49	Credit intermed. and related activities	521,522								
50	Securities, commods, and other fin. invest. activities	523								
51	Insurance carriers and related activities	524								
52	Funds, trusts, and other financial vehicles	525								
53	Real estate	531								
54	Rental and leasing services and lessors of intangible assets	532,533								
55	Services	54-81								
56	Legal services	5411		x		x	x			x
57	Computer systems design	5415	x	x						
58	Miscellaneous professional, scientific, and technical services	5412-5414,5416-5419		x	x		x			x
59	Management of companies and enterprises	55		x	x		x			x
60	Administrative and support services	561		x	x		x			x
61	Waste management and remediation services	562		x		x	x			x
62	Education services	61		x		x		x		x
63	Ambulatory health care services	621		x	x		x			x
64	Hospitals and nursing and residential care facilities	622,623		x		x	x			x
65	Social assistance	624		x		x	x			x
66	Performing arts, spectator sports, museums, and related indus	711,712		x		x	x			x
67	Amusement, gambling, and recreation industries	713		x		x	x			x
68	Accommodation	721		x		x	x		x	
69	Food services and drinking places	722		x		x		x		x
70	Other services	81		x		x	x			x

Table A2
Historical Predictions of Growth Models

A. No Land

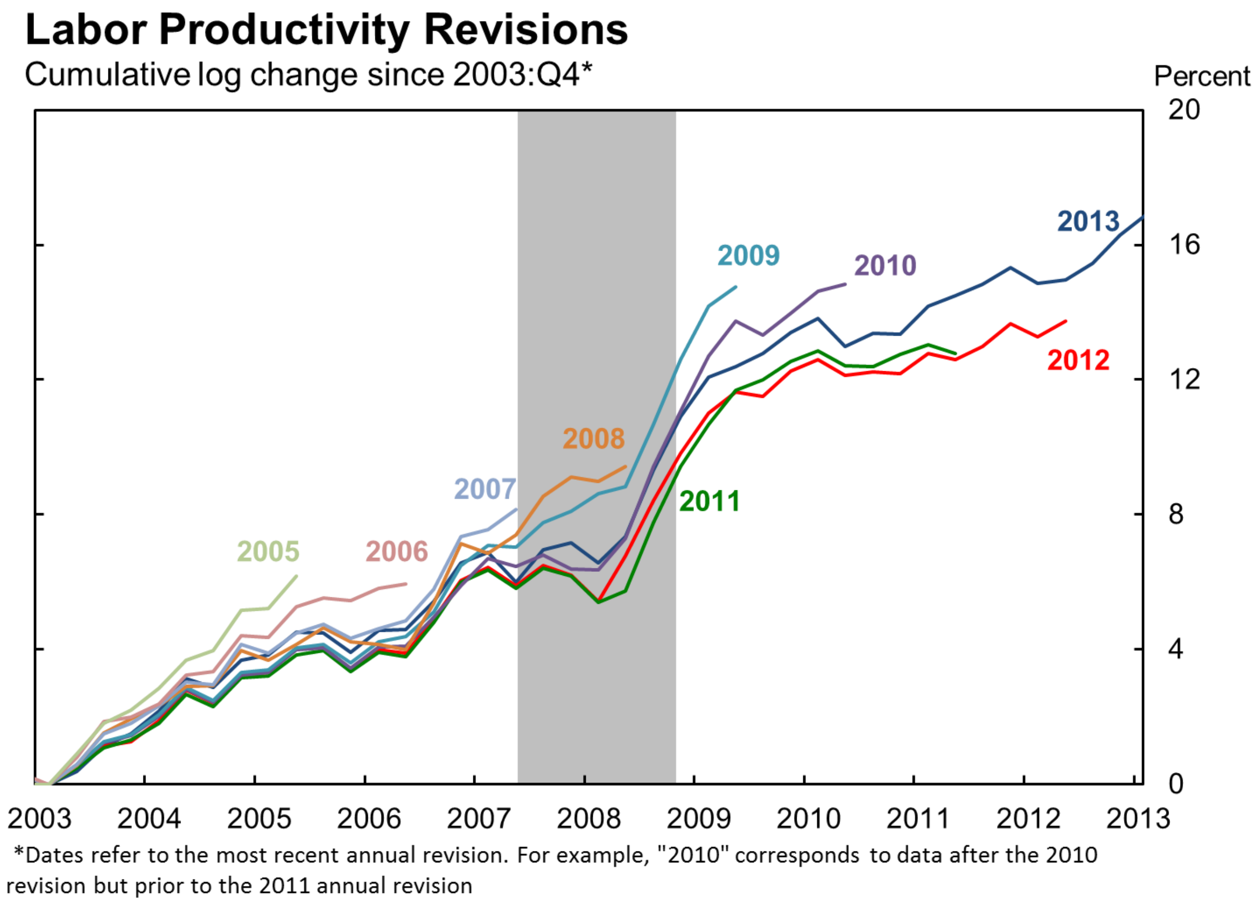
	Overall TFP	Invest. TFP	One-Sector Predicted Y/L	Multi-Sector Predicted Y/L	Actual Output per Unit Labor	*Memo: Labor Quality*	*Memo: Actual Output/Hour (5)+(6)*
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Full Sample	1.3	1.8	1.9	2.2	**2.0**	*0.4*	*2.4*
pre-1973Q2	2.1	2.2	3.2	3.2	**2.9**	*0.3*	*3.2*
1973Q2-1995Q4	0.4	1.0	0.7	0.9	**1.0**	*0.4*	*1.4*
1995:Q4-2007:Q4	1.4	2.9	2.1	2.8	**2.4**	*0.4*	*2.4*

B. Adding Land as a Factor of Production

	Overall TFP	Invest. TFP	One-Sector Predicted Y/L	Multi-Sector Predicted Y/L	Actual Output per Unit Labor	*Memo: Labor Quality*	*Memo: Actual Output/Hour (5)+(6)*
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Full Sample	1.3	1.8	1.9	2.1	**2.0**	*0.4*	*2.4*
pre-1973Q2	2.1	2.2	3.1	3.1	**2.9**	*0.3*	*3.2*
1973Q2-1995Q4	0.4	1.0	0.6	0.8	**1.0**	*0.4*	*1.4*
1995:Q4-2007:Q4	1.4	2.9	2.0	2.6	**2.4**	*0.4*	*2.8*

Notes: Column (3) shows predictions of one-sector growth model for output per unit of (quality-adjusted) labor. In panel A, that prediction depends on column (1) according to $TFP/(1-\alpha)$. Column (4) shows predictions of multi-sector growth model. In top panel, that depends on columns (1) and (2) according to $TFP + \alpha \cdot TFP_I / (1-\alpha)$. See text for how land is incorporated as a factor of production in bottom panel. The predictions are compared with actual output per unit of quality-adjusted labor in Column (5). The more typical output per hour is shown in Column (7). All calculations take capital's share α=0.33, which is the full-sample average in the Fernald dataset. Investment TFP averages equipment TFP and structures TFP, where the weight on equipment includes the weight of inventories.

Figure A1
Labor productivity revisions



Source: BLS Productivity and Cost releases, and Haver. Output in these series correspond to the expenditure side of the national accounts rather than the average of the expenditure and income sides.