Appendix to "Inflation Strikes Back: The Role of Import Competition and the Labor Market," by Mary Amiti, Sebastian Heise, Fatih Karahan, and Ayşegül Şahin, in *NBER Macroeconomics Annual 2023*, volume 38, edited by Martin Eichenbaum, Erik Hurst, and Valerie Ramey, University of Chicago Press 2024.

Appendix

A Theory

In this section we derive the main equations of the theoretical model in Section 3.

A.1 Households

A.1.1 Consumption-Savings Problem

Here, we derive the solution to the household consumption-savings problem. The first-order condition with respect to consumption implies

$$E_0 \mathcal{B}_0^t (C_t - H_t)^{-\sigma} = \lambda_t P_{f,t}. \tag{35}$$

The first-order condition for assets is, for any state,

$$\lambda_t Q_{t+1} = \lambda_{t+1},$$

which can be re-written as

$$\begin{aligned} \frac{\mathcal{B}_0^t (C_t - H_t)^{-\sigma}}{P_{f,t}} Q_{t+1} &= \frac{\mathcal{B}_0^{t+1} (C_{t+1} - H_{t+1})^{-\sigma}}{P_{f,t+1}} \\ Q_{t+1} &= \beta_t \frac{(C_{t+1} - H_{t+1})^{-\sigma}}{(C_t - H_t)^{-\sigma}} \frac{P_{f,t}}{P_{f,t+1}}. \end{aligned}$$

Taking expectations on both sides yields

$$E_t[Q_{t+1}] = \frac{1}{1+R_t} = \beta_t E_t \left[\frac{(C_{t+1} - H_{t+1})^{-\sigma}}{(C_t - H_t)^{-\sigma}} \frac{P_{f,t}}{P_{f,t+1}} \right],$$

which is the Euler equation. Rewriting H_t in terms of previous consumption

$$\frac{1}{1+R_t} = \beta_t E_t \left[\frac{(C_{t+1} - hC_t)^{-\sigma}}{(C_t - hC_{t-1})^{-\sigma}} \frac{P_{f,t}}{P_{f,t+1}} \right]. \tag{36}$$

A.1.2 Labor and Wage Setting

Households supply labor to a labor bundler, whose problem is

$$\max_{\ell_t^s} \{ W_t^s L_t^s - \int_0^1 W_t^{s,\tau} \ell_t^{s,\tau} d\tau \}.$$

This problem implies the standard demand equation shown in the text,

$$\ell_t^{s,\tau} = \left(\frac{W_t^{s,\tau}}{W_t^s}\right)^{-\eta^s} L_t^s,$$

where

$$W_t^s = \left(\int_0^1 (W_t^{s,\tau})^{1-\eta^s}\right)^{1-\eta^s}.$$

Since the labor supply to each sector is additive, we can solve the wage setting problem separately for each sector. Household τ's wage setting problem for sector s is

$$\max_{W_t^{s,\tau}} E_0 \sum_{t=0}^{\infty} \mathcal{B}_0^t \Bigg\{ \frac{(C_t - hC_{t-1})^{-\sigma}}{P_{f,t}} W_t^{s,\tau} \ell_t^{s,\tau} - \frac{\kappa_t^s}{1+\varphi} (\ell_t^{s,\tau})^{1+\varphi} - \frac{\psi_w}{2} \left(\frac{W_t^{s,\tau}}{W_{t-1}^{s,\tau}} - 1\right)^2 L_t^s \Bigg\}, \quad (37)$$

where the first term uses the marginal utility of consumption, $\lambda_t = \mathcal{B}_0^t (C_t - H_t)^{-\sigma} / P_{f,t}$, from the household problem (35) to translate wage income into utility. Plugging in for labor demand $\ell_t^{s,\tau}$ we get

$$\begin{aligned} \max_{W_t^{s,\tau}} E_0 \sum_{t=0}^{\infty} \mathcal{B}_0^t \Bigg\{ & \frac{(C_t - hC_{t-1})^{-\sigma}}{P_{f,t}} (W_t^{s,\tau})^{1-\eta^s} (W_t^s)^{\eta^s} L_t^s \\ & - \frac{\kappa_t^s}{1+\varphi} (W_t^{s,\tau})^{-\eta^s(1+\varphi)} (W_t^s)^{\eta^s(1+\varphi)} (L_t^s)^{1+\varphi} - \frac{\psi_w}{2} \left(\frac{W_t^{s,\tau}}{W_{t-1}^{s,\tau}} - 1\right)^2 L_t^s \Bigg\}. \end{aligned} \quad (38)$$

The first order condition of this problem is

$$\begin{aligned} (\eta^s - 1) \frac{(C_t - hC_{t-1})^{-\sigma}}{P_{f,t}} (W_t^{s,\tau})^{-\eta^s} (W_t^s)^{\eta^s} L_t^s = \kappa_t^s \eta^s (W_t^{s,\tau})^{-\eta^s(1+\varphi)-1} (W_t^s)^{\eta^s(1+\varphi)} (L_t^s)^{1+\varphi} \\ - \psi_w \left(\frac{W_t^{s,\tau}}{W_{t-1}^{s,\tau}} - 1\right) \frac{1}{W_{t-1}^{s,\tau}} L_t^s + E_t \beta_t \psi_w \left(\frac{W_{t+1}^{s,\tau}}{W_t^{s,\tau}} - 1\right) \left(\frac{W_{t+1}^{s,\tau}}{W_t^{s,\tau}}\right) \frac{1}{W_t^{s,\tau}} L_t^s, \end{aligned} \quad (39)$$

Since in equilibrium there is perfect risk sharing, we have $W_t^{s,\tau} = W_t^s$. Defining the real wage as $w_t^s \equiv W_t^s / P_{f,t}$, we obtain

$$(\eta^s - 1)(C_t - hC_{t-1})^{-\sigma} w_t^s L_t^s = \kappa_t^s \eta^s (L_t^s)^{1+\varphi} - \psi_w \pi_t^{s,w} (1 + \pi_t^{s,w}) L_t^s + E_t \beta_t \psi_w \pi_{t+1}^{s,w} (1 + \pi_{t+1}^{s,w}) L_t^s, \quad (40)$$

where $1 + \pi_t^{s,w} = W_t^s / W_{t-1}^s$. Rearranging, we obtain

$$(\eta^s - 1)(C_t - hC_{t-1})^{-\sigma} w_t^s = \kappa_t^s \eta^s (L_t^s)^{\varphi} - \psi_w \pi_t^{s,w} (1 + \pi_t^{s,w}) + E_t \beta_t \psi_w \pi_{t+1}^{s,w} (1 + \pi_{t+1}^{s,w}). \quad (41)$$

If there are no adjustment frictions, then the real wage is a markup over the ratio of the disutility of labor and the marginal utility of consumption.

A.2 Final Output Firm

Profit maximization within each sector implies demand for each differentiated product j of

$$y^s_{f,t}(j) = \left(\frac{P^s_{f,t}(j)}{P^s_{f,t}}\right)^{-\theta} Y^s_{f,t},$$

where the sectoral price index is

$$P^s_{f,t} = \left(\int_0^1 P^s_{f,t}(j)^{1-\theta} dj\right)^{\frac{1}{1-\theta}}.$$

Profit maximization across sectors yields the relative demand for each sector s aggregate

$$Y^s_{f,t} = \frac{\gamma^s_t}{\gamma^{s'}_t}\frac{P^{s'}_{f,t}}{P^s_{f,t}} Y^{s'}_{f,t}. \tag{42}$$

From the production function, $Y_{f,t} = (Y^M_{f,t})^{\gamma^M_t}(Y^S_{f,t})^{\gamma^S_t}$, we can substitute for $Y^S_{f,t}$ from the previous equation and solve for $Y^M_{f,t}$ as a function of total output:

$$Y_{f,t} = Y^M_{f,t}\left(\frac{\gamma^S_t}{\gamma^M_t}\right)^{\gamma^S_t}\left(\frac{P^M_{f,t}}{P^S_{f,t}}\right)^{\gamma^S_t},$$

and hence

$$\begin{aligned} Y^M_{f,t} &= (\gamma^M_t)^{\gamma^S_t}(\gamma^S_t)^{-\gamma^S_t}(P^M_{f,t})^{-\gamma^S_t}(P^S_{f,t})^{\gamma^S_t} Y_{f,t} \\ &= (\gamma^M_t)^{1-\gamma^M_t}(\gamma^S_t)^{-\gamma^S_t}(P^M_{f,t})^{-\gamma^S_t}(P^s_{f,t})^{\gamma^S_t} Y_{f,t}. \end{aligned} \tag{43}$$

This expression gives the demand for the manufacturing output as a function of total final output.

The cost function of the final output firm is

$$C(Y_{f,t}) = P^M_{f,t}Y^M_{f,t} + P^S_{f,t}Y^S_{f,t}. \tag{44}$$

Plugging in for $Y^S_{f,t}$ from (42), we obtain

$$
\begin{aligned}
C(Y_{f,t}) &= P^M_{f,t}Y^M_{f,t} + \frac{\gamma^S_t}{\gamma^M_t} P^M_{f,t}Y^M_{f,t} \\
&= \frac{1}{\gamma^M_t} P^M_{f,t}Y^M_{f,t}.
\end{aligned} \tag{45}
$$

Plugging (43) into the cost function, we get

$$
C(Y_{f,t}) = \left(\frac{1}{\gamma^M_t}\right)^{\gamma^M_t} \left(\frac{1}{\gamma^S_t}\right)^{\gamma^S_t} (P^M_{f,t})^{\gamma^M_t} (P^S_{f,t})^{\gamma^S_t} Y_{f,t}. \tag{46}
$$

Therefore, we can define the aggregate price index as

$$
P_{f,t} = \left(\frac{1}{\gamma^M_t}\right)^{\gamma^M_t} \left(\frac{1}{\gamma^S_t}\right)^{\gamma^S_t} (P^M_{f,t})^{\gamma^M_t} (P^S_{f,t})^{\gamma^S_t}. \tag{47}
$$

We can obtain the aggregate inflation rate as a function of the sectoral inflation rates. Dividing (47) by $P_{f,t-1}$, we get

$$
1 + \pi_t = \Theta_t(\gamma)(1 + \pi^M_t)^{\gamma^M_t} (1 + \pi^S_t)^{\gamma^S_t}, \tag{48}
$$

where $\pi_t = (P_{f,t}/P_{f,t-1}) - 1$ is the inflation rate and

$$
\Theta_t(\gamma) \equiv \frac{(\gamma^M_{t-1})^{\gamma^M_{t-1}} (\gamma^S_{t-1})^{\gamma^S_{t-1}}}{(\gamma^M_t)^{\gamma^M_t} (\gamma^S_t)^{\gamma^S_t}} (P^M_{f,t-1})^{\gamma^M_t - \gamma^M_{t-1}} (P^S_{f,t-1})^{\gamma^S_t - \gamma^S_{t-1}} \tag{49}
$$

is an adjustment factor that takes into account that the shares of goods and services can fluctuate. In steady state, $\Theta_t(\gamma) = 1$. Hence, aggregate inflation is a combination of inflation in the two sectors.

Finally, using equation (43), total spending in sector s is

$$
\begin{aligned}
P^s_{f,t}Y^s_{f,t} &= (\gamma^s_t)^{1-\gamma^s_t} (\gamma^{s'}_t)^{-\gamma^{s'}_t} (P^s_{f,t})^{\gamma^s_t} (P^{s'}_{f,t})^{\gamma^{s'}_t} Y_{f,t} \\
&= \gamma^s_t P_{f,t} Y_{f,t},
\end{aligned} \tag{50}
$$

where the second line follows from (47). Therefore, demand for product j as a function of final output is

$$
y^s_{f,t}(j) = \left(\frac{P^s_{f,t}(j)}{P^s_{f,t}}\right)^{-\theta} Y^s_{f,t} = \gamma^s_t \left(\frac{P^s_{f,t}(j)}{P^s_{f,t}}\right)^{-\theta} \left(\frac{P_{f,t}}{P^s_{f,t}}\right) Y_{f,t}. \tag{51}
$$

A.3 Retailers

Retailers face producer prices $P^s_{x,t}(j,i)$ for their input from industry i. Cost minimization implies that retailers have demand for each industry i of

$$x^s_t(j,i) = \left(\frac{P^s_{x,t}(j,i)}{P^s_{x,t}(j)}\right)^{-\nu} y^s_{f,t}(j), \tag{52}$$

where

$$P^s_{x,t}(j) = \left(\int_0^1 P^s_{x,t}(j,i)^{1-\nu} di\right)^{\frac{1}{1-\nu}} \tag{53}$$

is the producer price index faced by retailer j.

The retailers are monopolistic competitors, taking price indices as given, and face final demand from (7) of

$$y^s_{f,t}(j) = \gamma^s_t \left(\frac{P^s_{f,t}(j)}{P^s_{f,t}}\right)^{-\theta} \left(\frac{P_{f,t}}{P^s_{f,t}}\right) Y_{f,t}. \tag{54}$$

Retailers face a quadratic adjustment cost of $\gamma^s_t \frac{\psi_p}{2}(\frac{P^s_{f,t}(j)}{P^s_{f,t-1}(j)} - 1)^2 P_{f,t} Y_{f,t}$. Their real profits are

$$\begin{aligned} \Pi^s_t(j) = \gamma^s_t P^s_{f,t}(j)^{1-\theta}(P^s_{f,t})^{\theta-1} Y_{f,t} - p^s_{x,t}(j)\gamma^s_t P^s_{f,t}(j)^{-\theta}(P^s_{f,t})^{\theta-1} P_{f,t} Y_{f,t} \\ - \gamma^s_t \frac{\psi_p}{2}\left(\frac{P^s_{f,t}(j)}{P^s_{f,t-1}(j)} - 1\right)^2 Y_{f,t}, \end{aligned} \tag{55}$$

where $p^s_{x,t}(j) \equiv P^s_{x,t}(j)/P_{f,t}$ are real marginal costs. The firms' maximization problem is

$$\begin{aligned} \max_{P^s_{f,t+k}(j)} E_t\Bigg\{\sum_{k=0}^{\infty} \mathcal{B}^{t+k}_t \left(\frac{U'(C_{t+k})}{U'(C_t)}\right) \Bigg[\left(\frac{P^s_{f,t+k}(j)}{P^s_{f,t+k}} - p^s_{x,t+k}(j)\frac{P_{f,t+k}}{P^s_{f,t+k}}\right)\gamma^s_{t+k} P^s_{f,t+k}(j)^{-\theta}(P^s_{f,t+k})^{\theta} Y_{f,t+k} \\ - \gamma^s_t \frac{\psi_p}{2}\left(\frac{P^s_{f,t+k}(j)}{P^s_{f,t+k-1}(j)} - 1\right)^2 Y_{f,t+k}\Bigg]\Bigg\}. \end{aligned} \tag{56}$$

Under the assumption that all retailers are symmetric, the solution to the maximization problem is

$$\begin{aligned} \gamma^s_t(\theta-1)\frac{Y_{f,t}}{P^s_{f,t}} = \gamma^s_t \theta p^s_{x,t}\left(\frac{P_{f,t}}{P^s_{f,t}}\right)\frac{Y_{f,t}}{P^s_{f,t}} - \gamma^s_t \psi_p \left(\frac{P^s_{f,t}}{P^s_{f,t-1}} - 1\right)\frac{1}{P^s_{f,t-1}} Y_{f,t} \\ + \beta_t \psi_p E_t\left[\gamma^s_{t+1}\frac{(C_{t+1}-H_{t+1})^{-\sigma}}{(C_t - H_t)^{-\sigma}}\left(\frac{P^s_{f,t+1}}{P^s_{f,t}} - 1\right)\left(\frac{P^s_{f,t+1}}{P^s_{f,t}}\right)\frac{1}{P^s_{f,t}} Y_{f,t+1}\right], \end{aligned} \tag{57}$$

which becomes

$$(\theta-1)=\theta\frac{p^s_{x,t}}{p^s_{f,t}}-\psi_p(1+\pi^s_t)\pi^s_t+\beta_t\psi_p E_t\left[\frac{\gamma^s_{t+1}}{\gamma^s_t}\frac{(C_{t+1}-hC_t)^{-\sigma}}{(C_t-hC_{t-1})^{-\sigma}}\frac{Y_{f,t+1}}{Y_{f,t}}(1+\pi^s_{t+1})\pi^s_{t+1}\right], \quad (58)$$

where $\pi^s_t = P^s_{f,t}/P^s_{f,t-1}-1$, and $p^s_{f,t}=P^s_{f,t}/P_{f,t}$.

A.4 Intermediate Goods Firms

A.4.1 Firm and Industry Demand

In this section, we derive the demand faced by producer k. Given price $P^s_{x,t}(j,i,k)$, the first order condition for demand of firm k's output is

$$(N^s)^{\frac{1}{1-\mu}}x^s_t(j,i,k)^{-\frac{1}{\mu}}\left(\sum_{k=1}^{N^s_D}x^s_t(j,i,k)^{\frac{\mu-1}{\mu}}+\sum_{k=1}^{N^s_F}x^s_t(j,i,k)^{\frac{\mu-1}{\mu}}\right)^{\frac{\mu}{\mu-1}-1}=P^s_{x,t}(j,i,k),$$

implying

$$x^s_t(j,i,k)=\left(\frac{P^s_{x,t}(j,i,k)}{P^s_{x,t}(j,i,k')}\right)^{-\mu}x^s_t(j,i,k').$$

Plugging this expression into the aggregator (11) and re-arranging, we get

$$x^s_t(j,i,k)=(N^s)^{\frac{\mu}{\mu-1}}\left(\sum_{k=1}^{N^s_D}P^s_{x,t}(j,i,k)^{1-\mu}+\sum_{k=1}^{N^s_F}P^s_{x,t}(j,i,k)^{1-\mu}\right)^{\frac{\mu}{1-\mu}}P^s_{x,t}(j,i,k)^{-\mu}\frac{x^s_t(j,i)}{N^s}.$$

Thus, the demand faced by firm k is

$$x^s_t(j,i,k)=\left(\frac{P^s_{x,t}(j,i,k)}{P^s_{x,t}(j,i)}\right)^{-\mu}\frac{x^s_t(j,i)}{N^s}, \quad (59)$$

where

$$P^s_{x,t}(j,i)=(N^s)^{\frac{1}{\mu-1}}\left(\sum_{k=1}^{N^s_D}P^s_{x,t}(j,i,k)^{1-\mu}+\sum_{k=1}^{N^s_F}P^s_{x,t}(j,i,k)^{1-\mu}\right)^{\frac{1}{1-\mu}},$$

and $P^s_{x,t}(j,i,k)=P^s_{x,t}(j,i)=P^s_{x,t}(j)$ in a completely symmetric equilibrium.

A.4.2 Roundabout Production Technology

In this section, we describe the roundabout production technology and derive the sectoral demand for domestic intermediates.

The domestic inputs are assembled using all industries' output via a roundabout production technology. The domestic input aggregate $Z_t^s(j,i,k)$ used by firm k in industry i for retailer j in sector s combines inputs from the manufacturing and service sector according to

$$Z_t^s(j,i,k) = (Z_t^{s,M}(j,i,k))^{\gamma_t^M}(Z_t^{s,S}(j,i,k))^{\gamma_t^S}.$$

The sectoral aggregates are in turn combined from all industries using

$$Z_t^{s,s'}(j,i,k) = \left[\int_0^1 z_t^{s,s'}(j,i,k,i')^{\frac{\nu-1}{\nu}} di'\right]^{\frac{\nu}{\nu-1}},$$

where $z_t^{s,s'}(j,i,k,i')$ is the output from intermediate industry i' in sector s' used as input by firm k in industry i in sector s. This output is produced by firms k' in industry i' according to

$$z_t^{s,s'}(j,i,k,i') = (N^s)^{\frac{1}{1-\mu}}\left(\sum_{k=1}^{N_D^s} x_t^s(j,i,k,i',k')^{\frac{\mu-1}{\mu}} + \sum_{k=1}^{N_F^s} x_t^s(j,i,k,i',k')^{\frac{\mu-1}{\mu}}\right)^{\frac{\mu}{\mu-1}}.$$

The demand for producer (k')'s output by industry i' for use as intermediate is, as shown in Appendix A.4.1 for the consumer side

$$z_t^{s,s'}(j,i,k,i',k') = \left(\frac{P_{x,t}^{s'}(j,i',k')}{P_{x,t}^{s'}(j,i')}\right)^{-\mu}\frac{z_t^{s,s'}(j,i,k,i')}{N_t^s},$$

where $P_{x,t}^{s'}(j,i',k')$ is the price charged by firm k'.

The demand for industry i' as input for firm k in industry i in sector s for retailer j is obtained from cost minimization as

$$z_t^{s,s'}(j,i,k,i') = \left(\frac{P_{x,t}^{s'}(j,i')}{P_{x,t}^{s'}(j)}\right)^{-\nu} Z_t^{s,s'}(j,i,k),$$

similar to the demand from retailers derived in (52), where $P_{x,t}^{s'}(j)$ is as before the producer price index, which by symmetry is $P_{x,t}^{s'}$. For the choice of inputs by sector, we have

$$Z_t^{s,s'}(j,i,k) = \gamma_t^{s'}\left(\frac{P_{x,dom,t}}{P_{x,t}^{s'}}\right) Z_t^s(j,i,k), \tag{60}$$

where

$$P_{x,dom,t} = \left(\frac{1}{\gamma_t^M}\right)^{\gamma_t^M}\left(\frac{1}{\gamma_t^S}\right)^{\gamma_t^S}(P_{x,t}^M)^{\gamma_t^M}(P_{x,t}^S)^{\gamma_t^S} \tag{61}$$

is the domestic input price index.

A.4.3 Producers' Marginal Costs

Cost minimization across domestic and foreign intermediates implies

$$M_t^s(j,i,k) = Z_t^s(j,i,k)\left(\frac{P_{x,imp,t}^s}{P_{x,dom,t}}\right)^{-\xi}.$$

Plugging this into the CES aggregator for domestic and foreign inputs, equation (15), yields

$$D_t^s(j,i,k) = Z_t^s(j,i,k)(P_{x,input,t}^s)^{-\xi}(P_{x,dom,t})^{\xi},$$

where

$$P_{x,input,t}^s = [(P_{x,dom,t})^{1-\xi} + (P_{x,imp,t}^s)^{1-\xi}]^{\frac{1}{1-\xi}}$$

is the input price index. It follows that

$$Z_t^s(j,i,k) = \left(\frac{P_{x,dom,t}}{P_{x,input,t}^s}\right)^{-\xi} D_t^s(j,i,k)$$

and

$$M_t^s(j,i,k) = \left(\frac{P_{x,imp,t}^s}{P_{x,input,t}^s}\right)^{-\xi} D_t^s(j,i,k).$$

The expenditure share on imported inputs is

$$\frac{P_{x,imp,t}^s M_t^s(j,i,k)}{P_{x,input,t}^s D_t^s(j,i,k)} = \frac{(P_{x,imp,t}^s)^{1-\xi}}{(P_{x,input,t}^s)^{1-\xi}} = \frac{(P_{x,imp,t}^s)^{1-\xi}}{(P_{x,dom,t})^{1-\xi} + (P_{x,imp,t}^s)^{1-\xi}} \equiv \alpha_s,$$

where α_s is the import share in sector s.

Cost minimization across labor and intermediates implies

$$L_t^s(j,i,k) = \frac{1}{\Lambda_s} A_t^{\rho_s - 1} D_t^s(j,i,k)\left(\frac{W_t^s}{P_{x,input,t}^s}\right)^{-\rho_s}.$$

Plugging this into the CES aggregator for labor and intermediates, equation (14), yields

$$x_t^s(j,i,k) = \frac{1}{\Lambda_s} D_t(j,i,k)(P_{x,input,t}^s)^{\rho_s}(MC_{D,t}^s)^{-\rho_s},$$

where

$$MC_{D,t}^s \equiv \left[\left(\frac{W_t^s}{A_t}\right)^{1-\rho_s} + \Lambda_s (P_{x,input,t}^s)^{1-\rho_s}\right]^{\frac{1}{1-\rho_s}}. \tag{62}$$

It follows that the demand for the intermediate good is

$$D_t^s(j,i,k) = \Lambda_s \left(\frac{P^s_{x,input,t}}{MC^s_{D,t}} \right)^{-\rho_s} x_t^s(j,i,k). \tag{63}$$

Similarly, the demand for labor is

$$L_t^s(j,i,k) = A_t^{\rho_s - 1} \left(\frac{W_t^s}{MC^s_{D,t}} \right)^{-\rho_s} x_t^s(j,i,k). \tag{64}$$

Plugging these two expressions into the firm's cost function yields

$$\begin{aligned} C(x_t^s(j,i,k)) &= W_t L_t^s(j,i,k) + P^s_{x,input,t} D_t^s(j,i,k) \\ &= MC^s_{D,t} \cdot x_t^s(j,i,k). \end{aligned} \tag{65}$$

Thus, $MC^s_{D,t}$ are the firm's marginal costs.

The share of labor in total costs is

$$\begin{aligned} \lambda_t^s &= \frac{A_t^{\rho_s-1}(W_t^s)^{1-\rho_s} MC^{\rho_s}_{D,t}(x_t^s(j,i,k))}{A_t^{\rho_s-1}(W_t^s)^{1-\rho_s} MC^{\rho_s}_{D,t}(x_t^s(j,i,k)) + \Lambda_s (P^s_{x,input,t})^{1-\rho_s} MC^{\rho_s}_{D,t}(x_t^s(j,i,k))} \\ &= \frac{(W_t^s/A_t)^{1-\rho_s}}{(W_t^s/A_t)^{1-\rho_s} + \Lambda_s (P^s_{x,input,t})^{1-\rho_s}}. \end{aligned} \tag{66}$$

This equation links the parameter Λ_s to the labor share in steady state, λ^s.

A.4.4 Price Setting Problem

In this section we find the solution for the firm's profit maximization problem. We first derive the firms' effective elasticity of demand. We then solve the profit maximization problem and obtain firms' prices.

Demand Elasticity

Each producer faces final demand as well as demand for its output as inputs into other industries. Each retailer also demands some output $\gamma_t^s \frac{\psi_p}{2} (\pi_t^s)^2 Y_{f,t}$ to cover its price adjustment cost. From the demand equation (12), each producer thus faces total demand of

$$
\begin{aligned}
x^s_{tot,t}(j,i,k) = &\frac{1}{N^s}\left(\frac{P^s_{x,t}(j,i,k)}{P^s_{x,t}(j,i)}\right)^{-\mu}\left(\frac{P^s_{x,t}(j,i)}{P^s_{x,t}(j)}\right)^{-\nu} \\
&\times\left(y^s_{f,t}(j) + \gamma^s_t\frac{\psi_p}{2}(\pi^s_t)^2 Y_{f,t} + \int_0^1 \sum_{k'\in D} Z^{s,s}_t(j,i',k')di' + \int_0^1 \sum_{k'\in D} Z^{s',s}_t(j,i',k')di'\right),
\end{aligned}
$$

where the first term is the final demand by the associated retailer, the second term are the resources needed for price changes, and the third and fourth terms are the demands for inputs by all other domestic firms in all other industries to produce for the retailer. Plugging in the demand for inputs (60) we get

$$
\begin{aligned}
x^s_{tot,t}(j,i,k) = &\frac{1}{N^s}\left(\frac{P^s_{x,t}(j,i,k)}{P^s_{x,t}(j,i)}\right)^{-\mu}\left(\frac{P^s_{x,t}(j,i)}{P^s_{x,t}(j)}\right)^{-\nu} \times \left(\gamma^s_t\left(\frac{P^s_{f,t}(j)}{P^s_{f,t}}\right)^{-\theta}\left(\frac{P_{f,t}}{P^s_{f,t}}\right)Y_{f,t} + \gamma^s_t\frac{\psi_p}{2}(\pi^s_t)^2 Y_{f,t}\right. \\
&\left. + \gamma^s_t\left(\frac{P_{x,dom,t}}{P^s_{x,t}}\right)\int_0^1 \sum_{k'\in D} Z^s_t(j,i',k')di' + \gamma^s_t\left(\frac{P_{x,dom,t}}{P^s_{x,t}}\right)\int_0^1 \sum_{k'\in D} Z^{s'}_t(j,i',k')\right).
\end{aligned}
$$

We denote by $Z^s_t(j) \equiv \int_0^1 \sum_{k'\in D} Z^s_t(j,i',k')di'$ the demand of inputs to produce for retailer j in sector s to re-write

$$
\begin{aligned}
x^s_{tot,t}(j,i,k) = &\frac{1}{N^s}\left(\frac{P^s_{x,t}(j,i,k)}{P^s_{x,t}(j,i)}\right)^{-\mu}\left(\frac{P^s_{x,t}(j,i)}{P^s_{x,t}(j)}\right)^{-\nu} \times \left(\gamma^s_t\left(\frac{P^s_{f,t}(j)}{P^s_{f,t}}\right)^{-\theta}\left(\frac{P_{f,t}}{P^s_{f,t}}\right)Y_{f,t}\right. \\
&\left. + \gamma^s_t\frac{\psi_p}{2}(\pi^s_t)^2 Y_{f,t} + \gamma^s_t\left(\frac{P_{x,dom,t}}{P^s_{x,t}}\right)Z^s_t(j) + \gamma^s_t\left(\frac{P_{x,dom,t}}{P^s_{x,t}}\right)Z^{s'}_t(j)\right).
\end{aligned} \tag{67}
$$

Each producer faces an effective elasticity of demand of

$$
\mathcal{E}^s_t(j,i,k) \equiv -\frac{d\log x^s_{tot,t}(j,i,k)}{d\log P^s_{x,t}(j,i,k)} = \mu - (\mu-\nu)\frac{\partial \log P^s_{x,t}(j,i)}{\partial \log P^s_{x,t}(j,i,k)}.
$$

From the definition of an industry's price index (13), we have that

$$
\frac{\partial \log P^s_{x,t}(j,i)}{\partial \log P^s_{x,t}(j,i,k)} = \frac{P^s_{x,t}(j,i,k)^{1-\mu}}{\sum_{k=1}^{N^s_D} P^s_{x,t}(j,i,k)^{1-\mu} + \sum_{k=1}^{N^s_F} P^s_{x,t}(j,i,k)^{1-\mu}}. \tag{68}
$$

We can define a firm's market share as

$$\begin{aligned} S_t^s(j,i,k) &\equiv \frac{P_{x,t}^s(j,i,k)x_{tot,t}^s(j,i,k)}{\sum_{k'=1}^{N_D^s} P_{x,t}^s(j,i,k')x_{tot,t}^s(j,i,k') + \sum_{k'=1}^{N_F^s} P_{x,t}^s(j,i,k')x_{tot,t}^s(j,i,k')} \\ &= \left(\frac{1}{N^s}\right)\frac{P_{x,t}^s(j,i,k)^{1-\mu}}{P_{x,t}^s(j,i)^{1-\mu}}. \end{aligned} \tag{69}$$

Using this expression, we can re-express the demand elasticity as

$$\mathcal{E}_t^s(j,i,k) = \mu - (\mu-\nu)S_t^s(j,i,k) = \mu(1 - S_t^s(j,i,k)) + \nu S_t^s(j,i,k). \tag{70}$$

Thus, the firm's demand elasticity is a weighted average of the within-industry and across-industry elasticities of substitution.

Prices

Producer k in industry i in sector s sets prices $P_{x,t}^s(j,i,k)$ to solve

$$\max_{P_{x,t}^s(j,i,k)} [P_{x,t}^s(j,i,k) - MC_{D,t}^s]x_{tot,t}^s(j,i,k),$$

where $x_{tot,t}^s(j,i,k)$ is given by (67). The first-order condition of this problem is

$$\begin{aligned} &[(1-\mu)P_{x,t}^s(j,i,k)^{-\mu} + \mu P_{x,t}^s(j,i,k)^{-\mu-1}MC_{D,t}^s]P_{x,t}^s(j,i)^{\mu-\nu}P_{x,t}^s(j)^{\nu} \\ &+ \left[(\mu-\nu)P_{x,t}^s(j,i,k)^{-\mu}P_{x,t}^s(j,i)^{\mu-\nu-1}P_{x,t}^s(j)^{\nu}\frac{\partial P_{x,t}^s(j,i)}{\partial P_{x,t}^s(j,i,k)}\right][P_{x,t}^s(j,i,k) - MC_{D,t}^s] = 0. \end{aligned}$$

The derivative of the price index is equal to

$$\frac{\partial P_{x,t}^s(j,i)}{\partial P_{x,t}^s(j,i,k)} = \left(\frac{1}{N^s}\right)\left(\frac{P_{x,t}^s(j,i,k)}{P_{x,t}^s(j,i)}\right)^{-\mu} = S_t^s(j,i,k)\frac{P_{x,t}^s(j,i)}{P_{x,t}^s(j,i,k)},$$

where we have used equation (68) and the expression for the market share (69). Plugging in, the first-order condition becomes

$$(1-\mu)P_{x,t}^s(j,i,k) + \mu MC_{D,t}^s + (\mu-\nu)S_t^s(j,i,k)[P_{x,t}^s(j,i,k) - MC_{D,t}^s] = 0,$$

which can be rearranged to

$$P_{x,t}^s(j,i,k) = \frac{\mu - (\mu-\nu)S_t^s(j,i,k)}{(\mu-1) - (\mu-\nu)S_t^s(j,i,k)}MC_{D,t}^s.$$

Using the definition of the demand elasticity, the producer price is thus

$$P^s_{x,t}(j,i,k) = \frac{\mathcal{E}^s_t(j,i,k)}{\mathcal{E}^s_t(j,i,k)-1} MC^s_{D,t}, \tag{71}$$

which can be re-written with real marginal costs by dividing both sides by $P_{f,t}$. We will denote by $P^s_{D,x,t}$ the price of a domestic producer and by $P^s_{F,x,t}$ the price of a foreign producer.

A.5 Aggregation

In this section we derive the aggregate resource constraints. Using equation (67) and symmetry of producers of the same origin, each domestic producer supplying retailer j faces total demand of

$$\begin{aligned} x^s_{tot,t}(j,i,k) = \frac{1}{N^s}\left(\frac{P^s_{D,x,t}}{P^s_{x,t}}\right)^{-\mu} \Bigg(\gamma^s_t \left(\frac{P_{f,t}}{P^s_{f,t}}\right) Y_{f,t} \\ + \gamma^s_t \frac{\psi_p}{2}(\pi^s_t)^2 Y_{f,t} + \gamma^s_t \left(\frac{P_{x,dom,t}}{P^s_{x,t}}\right) Z^s_t(j) + \gamma^s_t \left(\frac{P_{x,dom,t}}{P^s_{x,t}}\right) Z^{s'}_t(j)\Bigg). \end{aligned}$$

We aggregate across domestic producers and integrate across industries and retailers, and use $Y_{f,t} = C_t$, to get gross output by domestic firms in sector s:

$$Y^s_{g,t} = \frac{N^s_D}{N^s}\left(\frac{P_{D,x,t}}{P_{x,t}}\right)^{-\mu}\left(\gamma^s_t\left(\frac{P_{f,t}}{P^s_{f,t}}\right)C_t + \gamma^s_t\frac{\psi_p}{2}(\pi^s_t)^2 C_t + \gamma^s_t\left(\frac{P_{x,dom,t}}{P^s_{x,t}}\right)Z^s_t + \gamma^s_t\left(\frac{P_{x,dom,t}}{P^s_{x,t}}\right)Z^{s'}_t\right).$$

The demand for intermediates by domestic firm k can be derived as

$$\begin{aligned} Z^s_t(j,i,k) &= \left(\frac{P_{x,dom,t}}{P^s_{x,input,t}}\right)^{-\xi} D^s_t(j,i,k) \\ &= \Lambda_s \left(\frac{P_{x,dom,t}}{P^s_{x,input,t}}\right)^{-\xi}\left(\frac{P^s_{x,input,t}}{MC^s_{D,t}}\right)^{-\rho_s} x^s_{tot,t}(j,i,k). \end{aligned}$$

Since only domestic firms demand domestic intermediates, we can obtain the total domestic demand in sector s by summing across domestic firms and using symmetry to obtain

$$Z^s_t = \Lambda_s \left(\frac{P_{x,dom,t}}{P^s_{x,input,t}}\right)^{-\xi}\left(\frac{P^s_{x,input,t}}{MC^s_{D,t}}\right)^{-\rho_s} Y^s_{g,t}.$$

The total demand for labor by firm k is, from (64),

$$L_t^s(j,i,k) = A_t^{\rho_s - 1} \left(\frac{W_t^s}{MC_{D,t}^s} \right)^{-\rho_s} x_{tot,t}^s(j,i,k).$$

Aggregating across firms, industries, and retailers, we obtain

$$L_t^s = A_t^{\rho_s - 1} \left(\frac{W_t^s}{MC_{D,t}^s} \right)^{-\rho_s} Y_{g,t}^s.$$

A.6 Equilibrium Conditions

We now list the equilibrium conditions of the model. We incorporate here our assumption that the services sector consists only of domestic firms, and will write $mc_t^S = mc_{D,t}^S$, and so on.

Our equilibrium consists of 38 endogenous variables: C_t, Z_t^M, Z_t^S, π_t, π_t^M, π_t^S, $\pi_t^{M,w}$, $\pi_t^{S,w}$, $p_{f,t}^M$, $p_{f,t}^S$, $p_{x,t}^M$, $p_{x,t}^S$, $p_{D,x,t}^M$, $p_{F,x,t}^M$, $mc_{D,t}^M$, mc_t^S, $mc_{F,t}^M$, $p_{x,input,t}^M$, $p_{x,input,t}^S$, $p_{x,imp,t}^M$, $p_{x,imp,t}^S$, $p_{x,dom,t}$, w_t^M, w_t^S, L_t^M, L_t^S, $Y_{g,t}$, $Y_{g,t}^M$, $Y_{g,t}^S$, A_t, κ_t^M, κ_t^S, R_t, $S_{D,t}^M$, $S_{f,t}^M$, γ_t^M, γ_t^S, and β_t.

We have the following conditions that describe the system:

1. Euler equation:

$$(C_t - hC_{t-1})^{-\sigma} = \beta E_t \left[\frac{1+R_t}{1+\pi_{t+1}} (C_{t+1} - hC_t)^{-\sigma} \right] \tag{72}$$

2. Demand for domestic intermediates:

$$Z_t^s = \Lambda_s (p_{x,dom,t})^{-\xi} (p_{x,input,t}^s)^{\xi - \rho_s} (mc_{D,t}^s)^{\rho_s} Y_{g,t}^s \tag{73}$$

3. Aggregate inflation:

$$1 + \pi_t = \Theta_t(\gamma)(1+\pi_t^M)^{\gamma_t^M} (1+\pi_t^S)^{\gamma_t^S} \tag{74}$$

4. Sectoral inflation:

$$(\theta - 1) = \theta \frac{p_{x,t}^s}{p_{f,t}^s} - \psi_p (1+\pi_t^s)\pi_t^s + \beta \psi_p E_t \left[\frac{\gamma_{t+1}^s}{\gamma_t^s} \frac{(C_{t+1} - hC_t)^{-\sigma}}{(C_t - hC_{t-1})^{-\sigma}} \frac{C_{t+1}}{C_t} (1+\pi_{t+1}^s)\pi_{t+1}^s \right] \tag{75}$$

5. Sectoral wage inflation:

$$1 + \pi_t^{s,w} = \frac{w_t^s}{w_{t-1}^s}(1+\pi_t) \tag{76}$$

6. Sectoral prices:

$$p^s_{f,t} = p^s_{f,t-1}\frac{1+\pi^s_t}{1+\pi_t} \tag{77}$$

7. Retailers' real marginal costs in the goods sector

$$p^M_{x,t} = (N^M)^{\frac{1}{\mu-1}}\left(N^M_D(p^M_{D,x,t})^{1-\mu} + N^M_F(p^M_{F,x,t})^{1-\mu}\right)^{\frac{1}{1-\mu}} \tag{78}$$

8. Retailers' real marginal costs in the services sector

$$p^S_{x,t} = \frac{\mu-(\mu-\nu)S^S_t}{(\mu-1)-(\mu-\nu)S^S_t}mc^S_t \tag{79}$$

9. Domestic manufacturer's price

$$p^M_{D,x,t} = \frac{\mu-(\mu-\nu)S^M_{D,t}}{(\mu-1)-(\mu-\nu)S^M_{D,t}}mc^M_{D,t} \tag{80}$$

10. Foreign manufacturer's price

$$p^M_{F,x,t} = \frac{\mu-(\mu-\nu)S^M_{F,t}}{(\mu-1)-(\mu-\nu)S^M_{F,t}}mc^M_{F,t} \tag{81}$$

11. Domestic producers' real marginal costs:

$$mc^s_{D,t} = \left[\left(\frac{w^s_t}{A_t}\right)^{1-\rho_s} + \Lambda_s(p^s_{x,input,t})^{1-\rho_s}\right]^{\frac{1}{1-\rho_s}} \tag{82}$$

12. Foreign goods producers' real marginal costs:

$$\ln(mc^M_{F,t+1}) = (1-\omega_F)\ln(mc^M_F) + \omega_F\ln(mc^M_{F,t}) + \epsilon^F_{t+1} \tag{83}$$

13. Relative input price index:

$$p^s_{x,input,t} = \left[(p_{x,dom,t})^{1-\xi} + (p^s_{x,imp,t})^{1-\xi}\right]^{\frac{1}{1-\xi}} \tag{84}$$

14. Relative domestic input price index:

$$p_{x,dom,t} = \left(\frac{1}{\gamma^M_t}\right)^{\gamma^M_t}\left(\frac{1}{\gamma^S_t}\right)^{\gamma^S_t}(p^M_{x,t})^{\gamma^M_t}(p^S_{x,t})^{\gamma^S_t} \tag{85}$$

15. Sectoral labor supply:

$$(\eta^s - 1)(C_t - hC_{t-1})^{-\sigma} w_t^s = \kappa_t^s \eta^s (L_t^s)^{\varphi} - \psi_w \pi_t^{s,w}(1 + \pi_t^{s,w}) + \beta_t \psi_w E_t \pi_{t+1}^{s,w}(1 + \pi_{t+1}^{s,w}) \tag{86}$$

16. Sectoral labor demand

$$L_t^s = A_t^{\rho_s - 1} \left(\frac{w_t^s}{mc_{D,t}^s} \right)^{-\rho_s} Y_{g,t}^s \tag{87}$$

17. Goods market clearing:

$$\begin{aligned} Y_{g,t}^M = \frac{N_D^M}{N^M} \left(\frac{p_{D,x,t}^M}{p_{x,t}^M} \right)^{-\mu} \Bigg\{ \gamma_t^M \left(\frac{1}{p_{f,t}^M} \right) C_t \\ + \gamma_t^M \frac{\psi_p}{2} (\pi_t^M)^2 C_t + \gamma_t^M \left(\frac{p_{x,dom,t}}{p_{x,t}^M} \right) Z_t^M + \gamma_t^M \left(\frac{p_{x,dom,t}}{p_{x,t}^M} \right) Z_t^S \Bigg\} \end{aligned} \tag{88}$$

18. Services market clearing:

$$Y_{g,t}^S = \gamma_t^S \left(\frac{1}{p_{f,t}^S} \right) C_t + \gamma_t^S \frac{\psi_p}{2} (\pi_t^S)^2 C_t + \gamma_t^S \left(\frac{p_{x,dom,t}}{p_{x,t}^S} \right) Z_t^M + \gamma_t^S \left(\frac{p_{x,dom,t}}{p_{x,t}^S} \right) Z_t^S \tag{89}$$

19. Aggregate market clearing:

$$Y_{g,t} = Y_{g,t}^M + Y_{g,t}^S \tag{90}$$

20. Domestic firm market shares in goods:

$$S_{D,t}^M = \left(\frac{1}{N^M} \right) \frac{(p_{D,x,t}^M)^{1-\mu}}{(p_{x,t}^M)^{1-\mu}} \tag{91}$$

21. Foreign firm market shares in goods:

$$S_{F,t}^M = \left(\frac{1}{N^M} \right) \frac{(p_{F,x,t}^M)^{1-\mu}}{(p_{x,t}^M)^{1-\mu}} \tag{92}$$

22. Technology process:

$$\ln(A_{t+1}) = \omega_A \ln(A_t) + \epsilon_{t+1}^A \tag{93}$$

23. Relative input price process:

$$\ln(p_{x,imp,t+1}^s) = (1 - \omega_P) \ln(p_{x,imp}^s) + \omega_P \ln(p_{x,imp,t}^s) + \epsilon_{t+1}^{P,s}, \tag{94}$$

24. Labor disutility shocks:

$$\kappa^s_{t+1} = (1-\omega_\kappa)\kappa^s + \omega_\kappa \kappa^s_t + \epsilon^{\kappa,s}_{t+1} \tag{95}$$

25. Monetary policy:

$$R_t = \varrho R_{t-1} + (1-\varrho)R + (1-\varrho)[\Phi_\pi \pi_t + \Phi_y(\ln(Y_{f,t}) - \ln(Y_f))] + \epsilon^M_t \tag{96}$$

26. Discount factor process:

$$\ln(\beta_{t+1}) = (1-\omega_\beta)\ln(\beta) + \omega_\beta \ln(\beta_t) + \epsilon^\beta_{t+1} \tag{97}$$

27. Goods share process:

$$\ln(\gamma^M_{t+1}) = (1-\omega_\gamma)\ln(\gamma^M) + \omega_\gamma \ln(\gamma^M_t) + \epsilon^{\gamma,M}_{t+1} \tag{98}$$

28. Services share process:

$$\gamma^S_t = 1 - \gamma^M_t \tag{99}$$

A.7 Price Change Equation

The change in the markup, $d\ln \mathcal{M}_t(j,i,k)$ is given by

$$\begin{aligned}
d\ln \mathcal{M}_t(j,i,k) &= d\ln\left[\mu - (\mu-\nu)S^s_t(j,i,k)\right] - d\ln\left[(\mu-1) - (\mu-\mu)S^s_t(j,i,k)\right] \\
&= \left[-\frac{\mu-\nu}{\mu-(\mu-\nu)S^s_t(j,i,k)} + \frac{\mu-\nu}{(\mu-1)-(\mu-\nu)S^s_t(j,i,k)}\right] \\
&\times \frac{\partial S^s_t(j,i,k)}{\partial \log S^s_t(j,i,k)} d\ln S^s_t(j,i,k) \\
&= \frac{(\mu-\nu)S^s_t(j,i,k)}{\left[\mu-(\mu-\nu)S^s_t(j,i,k)\right]\left[(\mu-1)-(\mu-\nu)S^s_t(j,i,k)\right]} \\
&\times \left[(1-\mu)d\ln P^s_{x,t}(j,i,k) - (1-\mu)d\ln P^s_{x,t}(j,i)\right] \\
&= \frac{S^s_t(j,i,k)}{\left[\frac{\mu}{\mu-\nu} - S^s_t(j,i,k)\right]\left[1 - \frac{\mu-\nu}{\mu-1}S^s_t(j,i,k)\right]}\left[d\ln P^s_{x,t}(j,i) - d\ln P^s_{x,t}(j,i,k)\right] \\
&= -\Gamma_t(j,i,k)\left[d\ln P^s_{x,t}(j,i,k) - d\ln P^s_{x,t}(j,i)\right],
\end{aligned}$$

where $\Gamma_t(j,i,k) = -(\partial \ln \mathcal{M}_t(j,i,k)/\partial \ln P^s_{x,t}(j,i,k)) \geq 0$ is the elasticity of the markup with respect to a firm's own price. From

$$\Gamma_t(j,i,k) = \frac{S^s_t(j,i,k)}{\left[\frac{\mu}{\mu-\nu} - S^s_t(j,i,k)\right]\left[1 - \frac{\mu-\nu}{\mu-1}S^s_t(j,i,k)\right]}, \tag{100}$$

it follows that $\Gamma_t(j,i,k) = 0$ if $S^s_t(j,i,k) = 0$.

Finally, the derivative of the markup elasticity with respect to the market share $S(i,j)$ is given by

$$\frac{d\Gamma_t(j,i,k)}{dS^s_t(j,i,k)} =$$
$$\frac{\left[\frac{\mu}{\mu-\nu} - S^s_t(j,i,k)\right]\left[1 - \frac{\mu-\nu}{\mu-1}S^s_t(j,i,k)\right] + \left[1 - \frac{\mu-\nu}{\mu-1}S^s_t(j,i,k)\right] + \frac{\mu-\nu}{\mu-1}\left[\frac{\mu}{\mu-\nu} - S^s_t(j,i,k)\right]}{\left\{\left[\frac{\mu}{\mu-\nu} - S^s_t(j,i,k)\right]\left[1 - \frac{\mu-\nu}{\mu-1}S(i,j)\right]\right\}^2} > 0.$$

B Additional Quantitative Results

B.1 Heterogeneous Labor

In this section, we consider an extension of the baseline model where we allow for two types of labor in both sectors: high-skilled (H) and low-skilled (L). To incorporate heterogeneous labor, we modify the firms' production function (14) to

$$x_t^s(j,i,k) = \left[(A_t L_t^{sH}(j,i,k))^{\frac{\vartheta_s - 1}{\vartheta_s}} + \Lambda_s^{\frac{1}{\vartheta_s}} (\Xi_t^s(j,i,k))^{\frac{\vartheta_s - 1}{\rho_s}} \right]^{\vartheta_s/(\vartheta_s - 1)},$$

where L_t^{sH} is high-skilled labor used in sector s, and Ξ_t^s is a composite of low-skilled labor, L_t^{sL} and intermediates

$$\Xi_t^s(j,i,k) = \left\{ D_t^s(j,i,k)^{\frac{\rho_s - 1}{\rho_s}} + (A_t L_t^{sL}(j,i,k))^{\frac{\rho_s - 1}{\rho_s}} \right\}^{\frac{\rho_s}{\rho_s - 1}}.$$

Here, ρ_s now represents the elasticity of substitution between low-skilled labor and intermediates.

We set the constant Λ_s to match the labor share in each sector in steady state, which is now given by a modified version of (24)

$$\begin{aligned}
\lambda_s &= \frac{W^{sL} L^{sL}(j,i,k) + W^{sH} L^{sH}(j,i,k)}{W^{sL} L^{sL}(j,i,k) + P_{i,input}^s D^s(j,i,k) + W^{sH} L^{sH}(j,i,k)} \\
&= \frac{\Lambda_s (W^{sL}/A)^{1-\rho_s} (P_W^s)^{\rho_s - \vartheta_s} + (W^{sH}/A)^{1-\vartheta_s}}{\Lambda_s (P_W^s)^{1-\vartheta_s} + (W^{sH}/A)^{1-\vartheta_s}},
\end{aligned}$$

where P_W^s is the composite price index of intermediates and low-skilled labor,

$$P_{W,t}^s \equiv \left\{ (P_{i,input,t}^s)^{1-\rho_s} + (W_t^{sL}/A_t)^{1-\rho_s} \right\}^{\frac{1}{1-\rho_s}}.$$

On the household side, the utility function is modified to capture the four types of labor

$$U^\tau = \frac{1}{1-\sigma}(C_t^\tau - H_t)^{1-\sigma} - \sum_{o \in \{L,H\}} \frac{\kappa_t^{Mo}}{1+\varphi} (\ell_t^{Mo,\tau})^{1+\varphi} - \sum_{o \in \{L,H\}} \frac{\kappa_t^{So}}{1+\varphi} (\ell_t^{So,\tau})^{1+\varphi},$$

where the disutility parameter for each sector s and labor type o follows an exogenous process of the form (1). The budget constraint becomes

$$C_t^\tau P_{f,t} + b_t B_t^\tau + Q_{t+1} A_{t+1}^\tau \leq \sum_{o\in\{L,H\}} W_t^{Mo,\tau} \ell_t^{Mo,\tau} + \sum_{o\in\{L,H\}} W_t^{So,\tau} \ell_t^{So,\tau} + B_{t-1}^\tau + A_t^\tau + P_{f,t}\Pi_t^\tau.$$

Households' labor of type o in sector s is combined via a Dixit-Stiglitz aggregator

$$L_t^{so} = \left[\int_0^1 (\ell_t^{so,\tau})^{\frac{\eta^s-1}{\eta^s}} d\tau\right]^{\eta^s/(\eta^s-1)}.$$

The household's wage setting problem now implies the wage setting equation

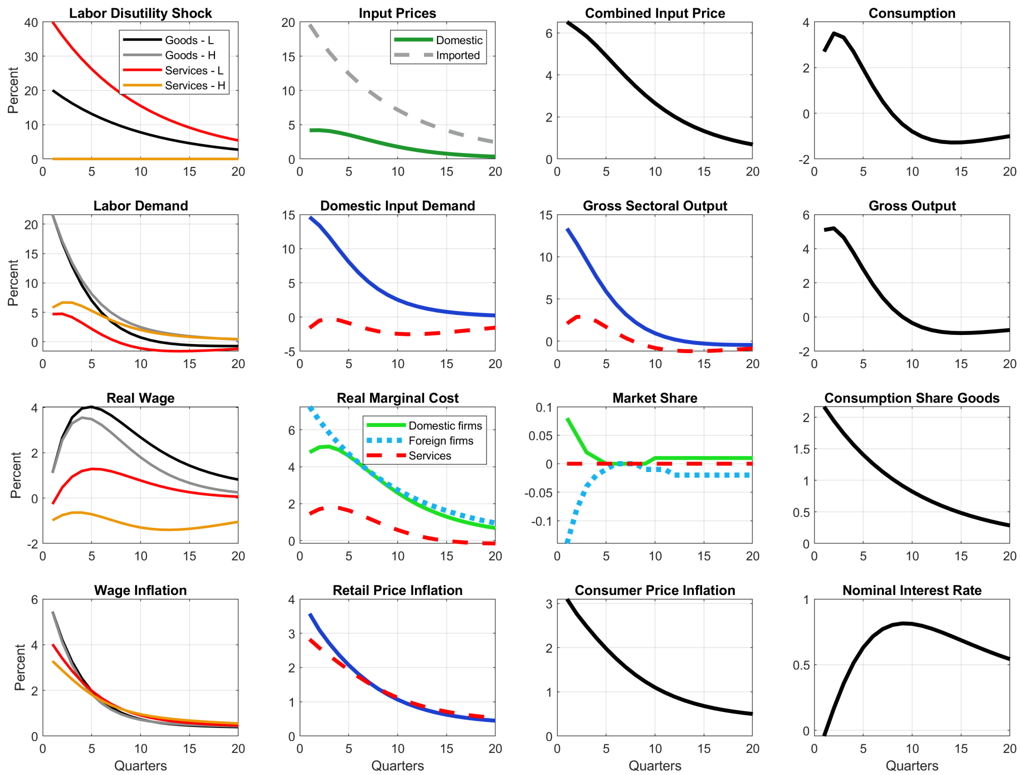
$$(\eta^s - 1)(C_t - hC_{t-1})^{-\sigma} w_t^{so} = \kappa_t^{so}\eta^{so}(L_t^{so})^{(1+\varphi)-1} - \psi_w \pi_t^{so,w}(1+\pi_t^{so,w}) + E_t\beta_t\psi_w\pi_{t+1}^{so,w}(1+\pi_{t+1}^{so,w})$$

for each sector and labor type.

We calibrate ρ_s as in the baseline. We set $\vartheta_S = 1.5$ in services and $\vartheta_M = 2$ in goods, mimicking our calibration for the substitutability between labor and intermediates. We set the disutility parameter $\kappa_{SL} = 1$, and set the other disutilities to match the wage gaps between low- and high-skilled labor and between manufacturing and services from the QWI. We define low-skilled workers as those with at most a high-school degree and high-skilled workers as those with at least some college. The wage gaps are $w^{ML}/w^{SL} = 1.28$, $w^{SH}/w^{SL} = 1.68$, and $w^{MH}/w^{SL} = 1.97$.

We simulate the model with a joint imported input, labor disutility, and foreign competitor shock as in the main text. We assume that the labor disutility shock only affects low-skilled labor, reflecting workers' inability to work remotely in these sectors during the pandemic. The first panel in the first row of Figure A.1 shows the disutility shock to low-skilled goods and services labor. We assume a larger shock than in the baseline since only one type of labor is affected. The second row shows that there is a shift towards domestic labor as in the baseline model, but less so in services due to the shift in demand for goods. In particular, there is only a small increase in demand for low-skilled services, which experienced the largest disutility shock. In the third row, we find that real wages strongly increase in low-skilled services and goods, while real wages in high-skilled services decline. The final row shows that price inflation is around 3 percent in this extended model. Overall, the patterns are qualitatively similar to the baseline, but generate heterogeneous responses of real wages.

Figure A.1: Impulse Responses with Heterogeneous Labor



Source: Author's calculations. Figure shows the effect of a joint disutility, input price, and competitor shock in the extended model with heterogeneous labor and accommodative monetary policy as described in the main text. Each panel shows the percent deviation of a variable from its steady state value against the number of quarters passed since the initial shock. For wage and price inflation, the interest rate, the market share, and the consumption share, we show percentage point changes from steady state. Consumer price inflation is computed as the consumption share weighted average inflation rate across the two sectors.

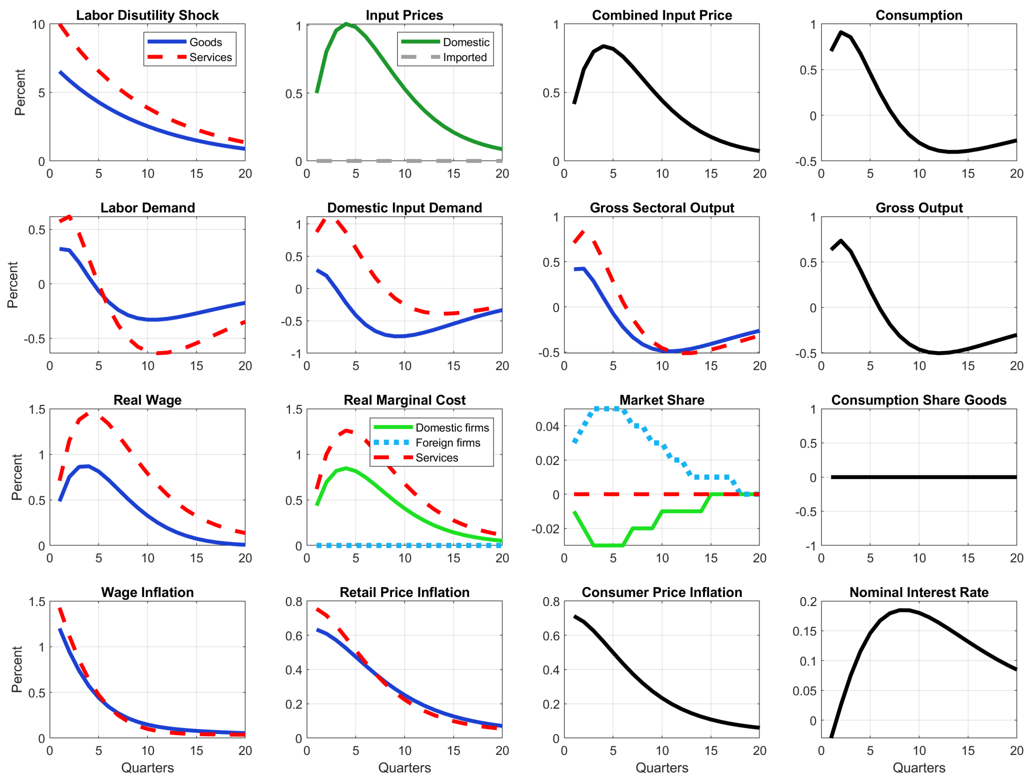
B.2 Additional Results

In this section, we present some additional results from the quantitative analysis.

Figure A.2 shows the impulse responses to our calibrated labor disutility shock in isolation. We adjust the monetary policy shock so that the nominal interest rate in the first period is approximately zero by setting $\epsilon_t^M = -0.0008$. The first row of panels shows that the shock raises the domestic input price. The second row illustrates that labor demand falls in both goods and in services by about 0.3 percent and 0.5 percent at its peak, respectively, as described in the calibration. The increase in labor demand raises the real wage in particular in services, where the disutility shock is larger, as shown in the third row. The rise in real wages leads to a relative shift from labor towards domestic intermediates in services. Foreign firms gain market share as a result of the higher domestic costs and gross output contracts. The last row shows that the shock raises wage inflation by 1.2 percentage point and price inflation by about 0.8 percentage point.

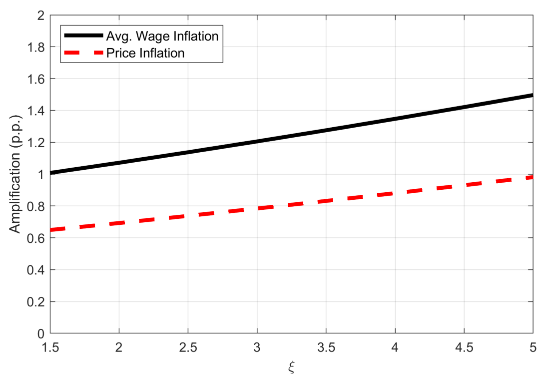
Figure A.3 shows the amplification as a function of the elasticity of substitution between domestic and imported intermediate inputs, ξ, similar to Figure 10 in the main text. This elasticity increases amplification from 0.6 to 1 percentage point for price inflation and for 1 to 1.6 percentage point for wage inflation.

Figure A.2: Effect of Labor Disutility Shock Only



Source: Author's calculations. Figure shows the effect of a shock to labor disutility under an accommodative monetary policy with monetary policy shock discussed in the main text on key variables. Each panel shows the percent deviation of a variable from its steady state value against the number of quarters passed since the initial shock. For wage and price inflation, the interest rate, the market share, and the consumption share of goods, we show percentage point changes from steady state. Consumer price inflation is computed as the consumption share weighted average inflation rate across the two sectors.

Figure A.3: Amplification on Impact: Sensitivity to ξ



Source: Author's calculations. Figure plots the difference between the impulse responses of consumer price inflation and average wage inflation for the joint shock relative to the impulse response of the summed separate shocks on impact (the difference between the red dashed and the black solid line from Figure 9 in quarter one) as a function of ξ.

C IV-LP Pass-Through Regressions

We construct impulse response functions of a change in wages or input prices on prices using an instrumental variables local projection (IV-LP) approach following Ramey (2016). Specifically, we estimate

$$\sum_{j=1}^{k}(\ln(P_{i,t+j})-\ln(P_{it}))=\beta_k\sum_{j=1}^{k}(\ln(W_{i,t+j})-\ln(W_{it}))+\alpha X_{it}+\delta_i+\psi_t+\epsilon_{it} \tag{101}$$

for wages and

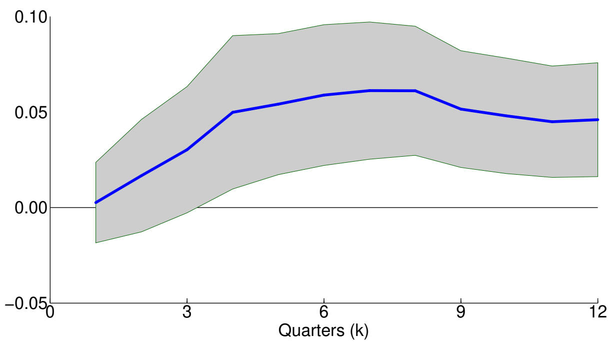
$$\sum_{j=1}^{k}(\ln(P_{i,t+j})-\ln(P_{it}))=\gamma_k\sum_{j=1}^{k}(\ln(P_{i,t+j,input})-\ln(P_{it,input}))+\alpha X_{it}+\delta_i+\psi_t+\epsilon_{it} \tag{102}$$

for input prices, where P_{it} is the producer price index in industry i and quarter t, W_{it} is the industry's wage index, and $P_{it,input}$ is the industry's input price as constructed in the main text. We instrument for both the cumulative wage term $\sum_{j=1}^{k}(\ln(W_{i,t+j})-\ln(W_{it}))$ and for the cumulative input price term $\sum_{j=1}^{k}(\ln(P_{i,t+j,input})-\ln(P_{it,input})$ with two instruments: the contemporaneous, 12-quarter change in wages, $\ln(W_{it})-\ln(W_{i,t-12})$ and the contemporaneous, 12-quarter change in input prices, $\ln(P_{it,input})-\ln(P_{i,t-12,input})$ to obtain the impact of a contemporaneous shock. The choice of these instruments is driven by the first stage of the regression. We found that 12-quarter changes have a better first stage than 4-quarter changes.

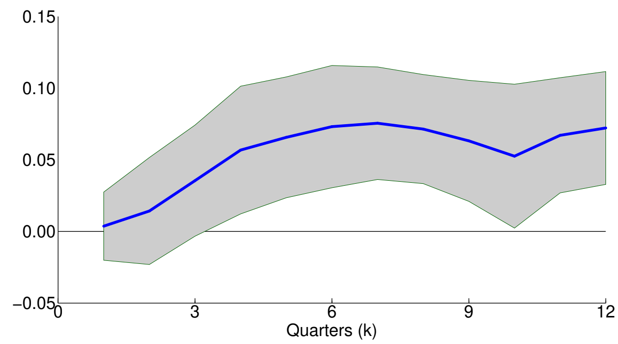
The top left panel of Figure A.4 shows the estimated IV coefficients on the cumulative wage term, β_k, for $k=1,...,12$ using all industries in our dataset for the period 2013:q1 to 2021:q3. Pass-through of wage shocks increases steadily over time until about 8 quarters, peaking at about 6%. The right panel of Figure A.4 presents results for the goods sector only, and the bottom panel presents results for the services sector. Pass-through is relatively similar in both sectors. We note that our findings differ quantitatively from the results in Heise et al. (2022) since we use 6-digit NAICS industries as opposed to 5-digit in our earlier paper and due to the different time period considered.

Figure A.5 shows the estimated IV coefficients on the cumulative input price term, γ_k. Pass-through is significantly stronger for input prices than for wages. Moreover, pass-through is higher in goods than in services.

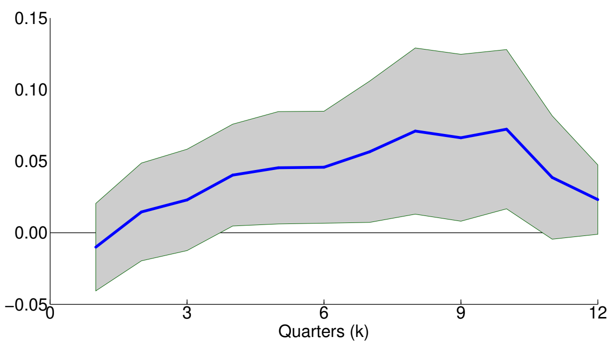
Figure A.4: Impulse Response Functions of Wages using the IV-LP Approach



(a) All Industries



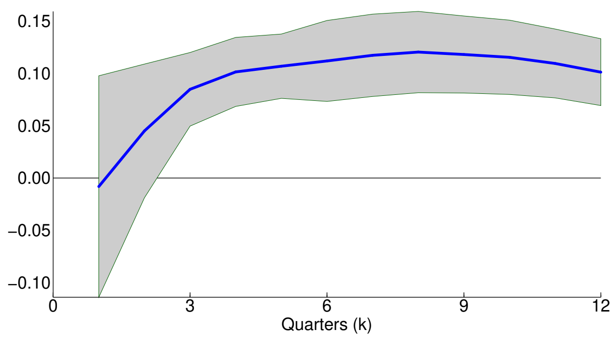
(b) Goods



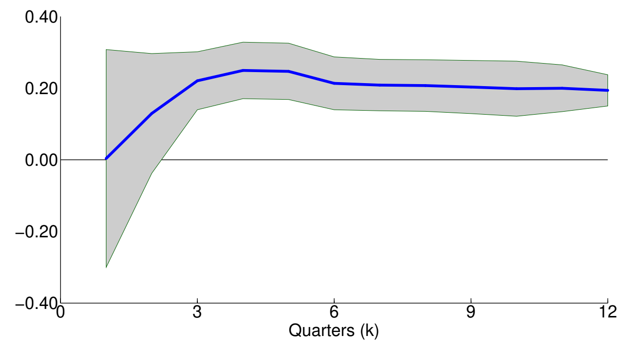
(c) Services

Source: BLS, Census Bureau Quarterly Census of Employment and Wages, authors' calculations. Note: The figure presents the estimated coefficients β_k from specification (101) and their 90 percent confidence intervals for $k = 1, ..., 20$ quarters. Prices are the seasonally-adjusted producer price indices and wages are the seasonally-adjusted average weekly wages of 6-digit NAICS industries. All data are at the quarterly frequency. Controls in the regression are employment shares of middle-aged and older workers, employment shares of those with high school, associate's, or bachelor's degrees, and employment share of female workers. Panel (a) presents the estimated coefficients β_k based on a regression using all industries in our sample. Panels (b) uses only goods industries and Panel (c) uses only service industries.

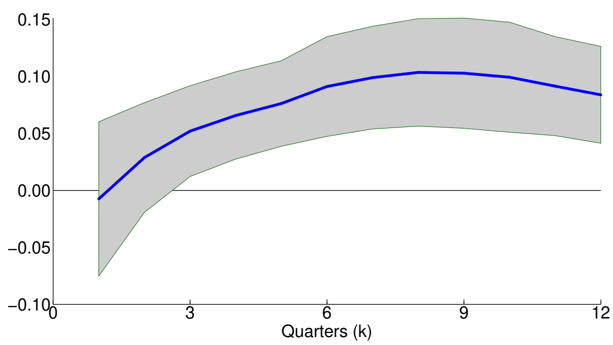
Figure A.5: Impulse Response Functions of Input Prices using the IV-LP Approach



(a) All Industries



(b) Goods



(c) Services

Source: BLS, Census Bureau Quarterly Census of Employment and Wages, authors' calculations. Note: The figure presents the estimated coefficients γ_k from specification (102) and their 90 percent confidence intervals for $k = 1, ..., 20$ quarters. Prices are the seasonally-adjusted producer price indices and wages are the seasonally-adjusted average weekly wages of 6-digit NAICS industries. All data are at the quarterly frequency. Controls in the regression are employment shares of middle-aged and older workers, employment shares of those with high school, associate's, or bachelor's degrees, and employment share of female workers. Panel (a) presents the estimated coefficients γ_k based on a regression using all industries in our sample. Panels (b) uses only goods industries and Panel (c) uses only service industries.

D Additional Empirical Results

In this section, we present some additional empirical results using our industry-level data.

D.1 Summary Statistics

We recompute the summary statistics in Table 3, but residualize all wage and input price changes with industry fixed effects. The results are in Table A.1. Here, the fourth row of each panel corresponds to the fifth row of each panel in Table 3. As in the text, the correlation between wage and input price changes in both goods and services rises in 2020.

D.2 Longer Time Horizons

To check whether our results hold over longer time horizons, we re-run our baseline analysis in the goods and services sectors using eight and twelve quarter differences for all variables. Column 1 of Table A.2 shows that we still see a positive correlation of prices in the goods sector with input prices, wages, and foreign competitors' prices. In Column 2, we see similar results as in the baseline specification, with stronger correlations between foreign competitors' prices, input prices and wage changes in 2021. Column 3 includes additional interactions and continues to find that the interaction of wages and input prices completely explains the increase in the pass-through of costs in 2021. The final column includes additional interactions for 2020 and whether an industry was in the top quartile of the wage and input price change distribution. In contrast to our baseline specification, we find that the coefficient on the quadruple interaction becomes insignificant.

Table A.3 repeats the analysis using twelve quarter differences. Column 2 shows that, in contrast to our baseline regression, we do not see evidence that the correlation between competitors' prices and producer prices increased in 2021. The remaining results are similar to the eight quarter analysis.

In Tables A.4 and A.5, we do the same for services and find similar results to our baseline regression.

D.3 Constrained Regression

One concern with our findings in the main text is that we did not impose the restriction that the coefficient on the wage and the coefficient on labor productivity are of equal and opposite signs, as implied by the theory. We therefore re-run our baseline regression (34), but impose the restriction $\beta_1 + \beta_2 = 0$. The results, in column 1 of Table A.6, are similar to those in the main text. In the second column, we additionally include interactions with 2021.

We also interact productivity with a 2021 dummy, and impose the additional constraint that the coefficients on the wage and productivity terms interacted with 2021 are of equal and opposite signs. We still find that the correlation of wages with prices increases in 2021, as in the baseline. In column 3, we additionally include the interaction between wage changes and input price changes. While we still find a positive interaction effect in 2021, this effect is no longer significant once we impose the constraint.

D.4 Domestic Competitors

One issue with our findings in the main text is that we do not control for domestic competitors. Some of the correlation of prices with input costs and wages could be due to a response to domestic competitors' price changes. While our industry-level data do not permit us to take into account within-industry competition, we construct a measure of domestic competition using the price index at the more aggregated 4-digit NAICS industry level. Specifically, we compute for each 6-digit industry a 4-quarter producer price change of the associated 4-digit industry in each quarter by taking a weighted average across the 4-quarter PPI changes of all associated 6-digit industries, using total shipments in 2021 as weight. We include the resulting variable $\Delta \ln(p_{it}^{PPI4})$ in the regression, interacted with industry i's domestic share, $1 - s_i$. To be consistent, we construct the foreign competitors' price change analogously.

The results in column 1 of Table A.7 still indicate a positive correlation of prices in the goods sector with input prices, wages, and foreign competitors' prices. In addition to that, we also find a positive correlation with our proxy for domestic competitors. In column 2, we further add interactions with 2021 and find that pass-through of input prices and wages increased in that year, as before. While we do find a strengthening correlation of producer prices with foreign competitors' prices, we do not find a similar strengthening of the correlation with domestic competitors' prices. Column 3 presents our non-linear specification results. As before, we find a positive and significant interaction between wage changes and input price changes in 2021.

D.5 Regression With Shift in Demand

A concern with our analysis is that while we focus on changes in input costs, demand factors could also be responsible for our findings. To examine whether an increase in demand could be behind our results, we re-run our baseline analysis in the goods sector with time-by-3-digit NAICS industry fixed effects. These fixed effects sweep out any variation that occurs at the broad 3-digit industry level. If demand shocks affect all industries that are part of a broader 3-digit aggregate equally, then the remaining variation is due to supply factors. Since the productivity measure is at most at the 3-digit level, this regression does not separately identify a productivity effect.

Column 1 of Table A.8 shows pass-through coefficients very similar to those in our baseline regression. Thus, most of the variation we pick up is due to variation within 3-digit industries. Column 2 shows that as before we find a significant pick-up in the correlation between domestic prices, wages, and input prices in 2021. The final column shows the results from the non-linear specification. As in the baseline, we find a significant and positive interaction effect in 2021.

Table A.1: Changes in Input Prices and Wages in Goods and Services, Residualized by Industry Fixed Effects

	Goods		Services	
2013:Q1 - 2019:Q4	$\Delta \ln(P_{it,input})$	$\Delta \ln(Wage_{it})$	$\Delta \ln(P_{it,input})$	$\Delta \ln(Wage_{it})$
Mean	-0.011	-0.004	-0.009	-0.006
P50	-0.007	-0.004	-0.003	-0.004
Mean of 4th quartile	0.050	0.051	0.077	0.037
Correlation	0.046		0.076	
2020				
Mean	-0.035	0.008	-0.068	0.016
P50	-0.027	0.003	-0.036	0.007
Mean of 4th quartile	0.014	0.089	-0.009	0.090
Correlation	0.312		0.205	
2021				
Mean	0.136	0.022	0.166	0.027
P50	0.112	0.025	0.111	0.022
Mean of 4th quartile	0.283	0.099	0.399	0.110
Correlation	0.229		0.192	

Notes: The table shows summary statistics on the average four-quarter change in wages and input prices for goods (first two columns) and services (last two columns), where these changes are residualized by industry fixed effects. Each panel focuses on changes in a specific time period. The first row shows the mean of the four-quarter change. The second row presents the median, and the third row the average over industries in the 4th quartile. The fourth row shows the correlation between wage and industry price changes.

Table A.2: Pass-Through for Goods with 8Q Differences, 2013:Q1 - 2021:Q3

	(1) $\Delta \ln(p_{it})$	(2) $\Delta \ln(p_{it})$	(3) $\Delta \ln(p_{it})$	(4) $\Delta \ln(p_{it})$
$s_i \times \Delta \ln(p_{it,imp})$	0.279***	0.256***	0.257***	0.242***
	(0.038)	(0.031)	(0.031)	(0.035)
$s_i \times \Delta \ln(p_{it,imp}) \times$ Year=20				0.068
				(0.041)
$s_i \times \Delta \ln(p_{it,imp}) \times$ Year=21		0.202***	0.187***	0.203***
		(0.043)	(0.049)	(0.049)
$\Delta \ln(p_{it,input})$	0.334***	0.290***	0.303***	0.312***
	(0.041)	(0.023)	(0.025)	(0.032)
$\Delta \ln(p_{it,input}) \times$ Year=20				-0.003
				(0.029)
$\Delta \ln(p_{it,input}) \times$ Year=21		0.228***	0.070	0.050
		(0.042)	(0.052)	(0.064)
$\Delta \ln(Wage_{it})$	0.022**	0.008	0.007	-0.004
	(0.010)	(0.014)	(0.013)	(0.013)
$\Delta \ln(Wage_{it}) \times$ Year=20				0.023
				(0.029)
$\Delta \ln(Wage_{it}) \times$ Year=21		0.092**	-0.082**	-0.072**
		(0.044)	(0.033)	(0.033)
$\Delta \ln(Wage_{it}) \times \Delta \ln(p_{it,input})$			-0.083	0.063
			(0.174)	(0.182)
$\Delta \ln(Wage_{it}) \times \Delta \ln(p_{it,input}) \times$ Year=20				-0.970
				(0.961)
$\Delta \ln(Wage_{it}) \times \Delta \ln(p_{it,input}) \times$ Year=21			1.775***	1.633***
			(0.271)	(0.301)
$\Delta \ln(Wage_{it}) \times \Delta \ln(p_{it,input}) \times \times$HH $\times$ Year=20				0.192
				(1.015)
$\Delta \ln(A_{it})$	-0.168***	-0.168***	-0.183***	-0.179***
	(0.022)	(0.022)	(0.022)	(0.026)
Time Fixed Effects	Yes	Yes	Yes	Yes
Industry Fixed Effects	Yes	Yes	Yes	Yes
Worker Composition	Yes	Yes	Yes	Yes
Nonlinear Effects	No	No	Yes	Yes
R2	0.152	0.160	0.165	0.166
Observations	8,240	8,240	8,240	8,240

Notes: The table shows the results from running the baseline regression (34) for goods, but replacing 4-quarter differences with 8-quarter differences. The first column shows the results for the baseline regression. The second column includes interactions for 2021. The third column includes an interaction between wage changes and intermediate input price changes. The fourth column includes additional interactions for 2020. All regressions include time and industry fixed effects and controls for the log share of workers 25-54, log share of workers 55+, log share of women, and log shares of workers with a high-school degree, associates degree, and bachelors degree or higher. The last two columns additionally include non-linear terms from (31), i.e., an interaction term between wage changes and input price changes, an interaction between productivity and input price changes, and squared wage and input price terms. We interact all terms with dummies for 2021. The last column contains additionally interactions of competitors' prices, input prices, wages, and productivity with a dummy for 2020, as well as interactions between wages, input price changes, a dummy for 2020, and dummies for whether both wage and input price change were above median (HH), the wage change was below median and the input price change above median (LH), and the wage change was above median and the input price change below median (HL). We only report in the table the main coefficients of interest.

Table A.3: Pass-Through for Goods with 12Q Differences, 2013:Q1 - 2021:Q3

	(1) $\Delta \ln(p_{it})$	(2) $\Delta \ln(p_{it})$	(3) $\Delta \ln(p_{it})$	(4) $\Delta \ln(p_{it})$
$s_i \times \Delta \ln(p_{it,imp})$	0.226***	0.232***	0.231***	0.235***
	(0.022)	(0.023)	(0.021)	(0.023)
$s_i \times \Delta \ln(p_{it,imp}) \times$ Year=20				-0.023
				(0.027)
$s_i \times \Delta \ln(p_{it,imp}) \times$ Year=21		-0.025	-0.037	-0.043
		(0.049)	(0.044)	(0.049)
$\Delta \ln(p_{it,input})$	0.337***	0.306***	0.306***	0.310***
	(0.040)	(0.025)	(0.031)	(0.036)
$\Delta \ln(p_{it,input}) \times$ Year=20				-0.000
				(0.047)
$\Delta \ln(p_{it,input}) \times$ Year=21		0.204***	0.085	0.077
		(0.050)	(0.072)	(0.084)
$\Delta \ln(Wage_{it})$	0.008	-0.017	-0.011	-0.017
	(0.013)	(0.015)	(0.013)	(0.014)
$\Delta \ln(Wage_{it}) \times$ Year=20				0.047**
				(0.019)
$\Delta \ln(Wage_{it}) \times$ Year=21		0.138***	0.019	0.025
		(0.036)	(0.024)	(0.024)
$\Delta \ln(Wage_{it}) \times \Delta \ln(p_{it,input})$			0.055	0.079
			(0.110)	(0.110)
$\Delta \ln(Wage_{it}) \times \Delta \ln(p_{it,input}) \times$ Year=20				-0.816
				(0.714)
$\Delta \ln(Wage_{it}) \times \Delta \ln(p_{it,input}) \times$ Year=21			1.083***	1.063***
			(0.135)	(0.143)
$\Delta \ln(Wage_{it}) \times \Delta \ln(p_{it,input}) \times \times$HH $\times$ Year=20				0.218
				(0.688)
$\Delta \ln(A_{it})$	-0.136***	-0.136***	-0.162***	-0.185***
	(0.032)	(0.033)	(0.026)	(0.025)
Time Fixed Effects	Yes	Yes	Yes	Yes
Industry Fixed Effects	Yes	Yes	Yes	Yes
Worker Composition	Yes	Yes	Yes	Yes
Nonlinear Effects	No	No	Yes	Yes
R2	0.147	0.154	0.160	0.162
Observations	6,960	6,960	6,960	6,960

Notes: The table shows the results from running the baseline regression (34) for goods, but replacing 4-quarter differences with 12-quarter differences. The first column shows the results for the baseline regression. The second column includes interactions for 2021. The third column includes an interaction between wage changes and intermediate input price changes. The fourth column includes additional interactions for 2020. All regressions include time and industry fixed effects and controls for the log share of workers 25-54, log share of workers 55+, log share of women, and log shares of workers with a high-school degree, associates degree, and bachelors degree or higher. The last two columns additionally include non-linear terms from (31), i.e., an interaction term between wage changes and input price changes, an interaction between productivity and input price changes, and squared wage and input price terms. We interact all terms with dummies for 2021. The last column contains additionally interactions of competitors' prices, input prices, wages, and productivity with a dummy for 2020, as well as interactions between wages, input price changes, a dummy for 2020, and dummies for whether both wage and input price change were above median (HH), the wage change was below median and the input price change above median (LH), and the wage change was above median and the input price change below median (HL). We only report in the table the main coefficients of interest.

Table A.4: Pass Through for Services with 8Q Differences, 2013:Q1 - 2021:Q3

	(1) $\Delta \ln(p_{it})$	(2) $\Delta \ln(p_{it})$	(3) $\Delta \ln(p_{it})$
$\Delta \ln(p_{it,input})$	0.089***	0.084***	0.108***
	(0.009)	(0.011)	(0.018)
$\Delta \ln(p_{it,input}) \times$ Year=21		0.140***	0.218***
		(0.029)	(0.048)
$\Delta \ln(Wage_{it})$	0.084**	0.055*	0.035
	(0.032)	(0.029)	(0.030)
$\Delta \ln(Wage_{it}) \times$ Year=21		0.144***	0.238
		(0.049)	(0.147)
$\Delta \ln(Wage_{it}) \times \Delta \ln(p_{it,input})$			-0.108*
			(0.061)
$\Delta \ln(Wage_{it}) \times \Delta \ln(p_{it,input}) \times$ Year=21			-1.198*
			(0.650)
$\Delta \ln(A_{it})$	0.003	0.005	0.007
	(0.014)	(0.012)	(0.014)
Time Fixed Effects	Yes	Yes	Yes
Industry Fixed Effects	Yes	Yes	Yes
Worker Composition	Yes	Yes	Yes
Nonlinear Effects	No	No	Yes
R2	0.059	0.064	0.068
Observations	4,522	4,522	4,522

Notes: The table shows the results from running the baseline regression (34) for services but replacing 4-quarter differences with 8-quarter differences. The first column shows the results for the baseline regression. The second column includes interactions for 2021. The third column includes an interaction between wage changes and intermediate input price changes. All regressions include time and industry fixed effects and controls for the log share of workers 25-54, log share of workers 55+, log share of women, and log shares of workers with a high-school degree, associates degree, and bachelors degree or higher. The last column additionally includes non-linear terms from (31), i.e., an interaction term between wage changes and input price changes, an interaction between productivity and input price changes, and squared wage and input price terms. We interact all terms with dummies for 2021. We only report in the table the main coefficients of interest.

Table A.5: Pass Through for Services with 12Q Differences, 2013:Q1 - 2021:Q3

	(1) $\Delta\ln(p_{it})$	(2) $\Delta\ln(p_{it})$	(3) $\Delta\ln(p_{it})$
$\Delta\ln(p_{it,input})$	0.095***	0.094***	0.136***
	(0.008)	(0.008)	(0.018)
$\Delta\ln(p_{it,input}) \times$ Year=21		0.075	0.106
		(0.076)	(0.166)
$\Delta\ln(Wage_{it})$	0.067**	0.042	0.022
	(0.030)	(0.029)	(0.034)
$\Delta\ln(Wage_{it}) \times$ Year=21		0.106**	0.214
		(0.042)	(0.155)
$\Delta\ln(Wage_{it}) \times \Delta\ln(p_{it,input})$			-0.100*
			(0.050)
$\Delta\ln(Wage_{it}) \times \Delta\ln(p_{it,input}) \times$ Year=21			-1.212
			(1.232)
$\Delta\ln(A_{it})$	0.013	0.012	0.012
	(0.017)	(0.017)	(0.016)
Time Fixed Effects	Yes	Yes	Yes
Industry Fixed Effects	Yes	Yes	Yes
Worker Composition	Yes	Yes	Yes
Nonlinear Effects	No	No	Yes
R2	0.072	0.074	0.079
Observations	3,802	3,802	3,802

Notes: The table shows the results from running the baseline regression (34) for services but replacing 4-quarter differences with 12-quarter differences. The first column shows the results for the baseline regression. The second column includes interactions for 2021. The third column includes an interaction between wage changes and intermediate input price changes. All regressions include time and industry fixed effects and controls for the log share of workers 25-54, log share of workers 55+, log share of women, and log shares of workers with a high-school degree, associates degree, and bachelors degree or higher. The last column additionally includes non-linear terms from (31), i.e., an interaction term between wage changes and input price changes, an interaction between productivity and input price changes, and squared wage and input price terms. We interact all terms with dummies for 2021. We only report in the table the main coefficients of interest.

Table A.6: Pass Through for Traded Industries with Constraints, 2013:Q1 - 2021:Q3

	(1) $\Delta\ln(p_{it})$	(2) $\Delta\ln(p_{it})$	(3) $\Delta\ln(p_{it})$
$s_i \cdot \Delta\ln(p_{it,imp})$	0.241***	0.191***	0.192***
	(0.029)	(0.026)	(0.026)
$s_i \cdot \Delta\ln(p_{it,imp}) \times$ Year=21		0.484***	0.488***
		(0.119)	(0.121)
$\Delta\ln(p_{it,input})$	0.360***	0.286***	0.315***
	(0.025)	(0.019)	(0.022)
$\Delta\ln(p_{it,input}) \times$ Year=21		0.143**	-0.034
		(0.057)	(0.085)
$\Delta\ln(Wage_{it})$	0.103***	0.087***	0.084***
	(0.010)	(0.009)	(0.009)
$\Delta\ln(Wage_{it}) \times$ Year=21		0.093*	0.084
		(0.047)	(0.055)
$\Delta\ln(Wage_{it}) \times \Delta\ln(p_{it,input})$			-0.310
			(0.272)
$\Delta\ln(Wage_{it}) \times \Delta\ln(p_{it,input}) \times$ Year=21			0.760
			(0.650)
$\Delta\ln(A_{it})$	-0.103***	-0.087***	-0.084***
	(0.010)	(0.009)	(0.009)
$\Delta\ln(A_{it}) \times$ Year=21		-0.093*	-0.084
		(0.047)	(0.055)
Time Fixed Effects	Yes	Yes	Yes
Industry Fixed Effects	Yes	Yes	Yes
Worker Composition	Yes	Yes	Yes
Nonlinear Effects	No	No	Yes
Observations	9,549	9,549	9,549

Notes: The table shows the results from running the baseline regression (34) for goods but imposing that $\beta_1 + \beta_2 = 0$. The first column shows the results for the baseline regression. The second column includes interactions for 2021, where we impose for these terms as well that the coefficients on wage changes and productivity changes add to zero. The third column includes an interaction between wage changes and intermediate input price changes. All regressions include time and industry fixed effects and controls for the log share of workers 25-54, log share of workers 55+, log share of women, and log shares of workers with a high-school degree, associates degree, and bachelors degree or higher. The last column additionally includes non-linear terms from (31), i.e., an interaction term between wage changes and input price changes, an interaction between productivity and input price changes, and squared wage and input price terms. We interact all terms with dummies for 2021. We only report in the table the main coefficients of interest.

Table A.7: Pass Through for Domestic Competitors, 2013:Q1 - 2021:Q3

	(1) $\Delta \ln(p_{it})$	(2) $\Delta \ln(p_{it})$	(3) $\Delta \ln(p_{it})$
$s_i \times \Delta \ln(p_{it,imp}^{PPI4})$	0.242***	0.179***	0.175***
	(0.050)	(0.021)	(0.019)
$s_i \times \Delta \ln(p_{it,imp}^{PPI4}) \times$ Year=21		0.537***	0.551***
		(0.114)	(0.119)
$(1 - s_i) \times \Delta \ln(p_{it}^{PPI4})$	0.531***	0.491***	0.491***
	(0.022)	(0.043)	(0.043)
$(1 - s_i) \times \Delta \ln(p_{it}^{PPI4}) \times$ Year=21		0.036	0.030
		(0.045)	(0.046)
$\Delta \ln(p_{it,input})$	0.155***	0.119***	0.145***
	(0.023)	(0.023)	(0.024)
$\Delta \ln(p_{it,input}) \times$ Year=21		0.097***	-0.073
		(0.022)	(0.064)
$\Delta \ln(Wage_{it})$	0.040**	0.019	0.015
	(0.016)	(0.013)	(0.010)
$\Delta \ln(Wage_{it}) \times$ Year=21		0.160***	0.071
		(0.020)	(0.049)
$\Delta \ln(Wage_{it}) \times \Delta \ln(p_{it,input})$			-0.358
			(0.258)
$\Delta \ln(Wage_{it}) \times \Delta \ln(p_{it,input}) \times$ Year=21			1.058***
			(0.310)
$\Delta \ln(A_{it})$	-0.096***	-0.099***	-0.093***
	(0.013)	(0.015)	(0.016)
$\Delta \ln(A_{it}) \times$ Year=21		0.019	-0.036
		(0.032)	(0.044)
Time Fixed Effects	Yes	Yes	Yes
Industry Fixed Effects	Yes	Yes	Yes
Worker Composition	Yes	Yes	Yes
Nonlinear Effects	No	No	Yes
R2	0.231	0.241	0.243
Observations	9,857	9,857	9,857

Notes: The table shows the results from running the baseline regression (34) for goods but incorporating an additional control for the price change of domestic competitors, $\Delta \ln(p_{it}^{PPI4})$, constructed as the weighted average PPI change of the corresponding 4-digit NAICS industry. We interact this price change with the domestic share, $1 - s_i$. The first column shows the results for the baseline regression. The second column includes interactions for 2021. The third column includes an interaction between wage changes and intermediate input price changes. All regressions include time and industry fixed effects and controls for the log share of workers 25-54, log share of workers 55+, log share of women, and log shares of workers with a high-school degree, associates degree, and bachelors degree or higher. The last column additionally includes non-linear terms from (31), i.e., an interaction term between wage changes and input price changes, an interaction between productivity and input price changes, and squared wage and input price terms. We interact all terms with dummies for 2021. We only report in the table the main coefficients of interest.

Table A.8: Pass Through with Time-by-Industry Fixed Effects, 2013:Q1 - 2021:Q3

	(1) $\Delta\ln(p_{it})$	(2) $\Delta\ln(p_{it})$	(3) $\Delta\ln(p_{it})$
$s_i \cdot \Delta\ln(p_{it,imp})$	0.215***	0.173***	0.174***
	(0.041)	(0.029)	(0.029)
$s_i \cdot \Delta\ln(p_{it,imp}) \times$ Year=21		0.407***	0.422***
		(0.130)	(0.139)
$\Delta\ln(p_{it,input})$	0.308***	0.234***	0.276***
	(0.054)	(0.042)	(0.046)
$\Delta\ln(p_{it,input}) \times$ Year=21		0.166***	-0.044
		(0.060)	(0.091)
$\Delta\ln(Wage_{it})$	0.032*	0.014	0.007
	(0.018)	(0.015)	(0.012)
$\Delta\ln(Wage_{it}) \times$ Year=21		0.100***	-0.030
		(0.018)	(0.056)
$\Delta\ln(Wage_{it}) \times \ln(p_{it,input})$			-0.662*
			(0.355)
$\Delta\ln(Wage_{it}) \times \Delta\ln(p_{it,input}) \times$ Year=21			1.844***
			(0.435)
Time by 3-digit Industry Fixed Effects	Yes	Yes	Yes
Worker Composition	Yes	Yes	Yes
Nonlinear Effects	No	No	Yes
R2	0.065	0.078	0.081
Observations	9,549	9,549	9,549

Notes: The table shows the results from running the baseline regression (34) for goods but replacing quarter and 6-digit NAICS industry fixed effects by quarter-by-3-digit NAICS fixed effects. The first column shows the results for the baseline regression. The second column includes interactions for 2021. The third column includes an interaction between wage changes and intermediate input price changes. All regressions include controls for the log share of workers 25-54, log share of workers 55+, log share of women, and log shares of workers with a high-school degree, associates degree, and bachelors degree or higher. The last column additionally includes non-linear terms from (31), i.e., an interaction term between wage changes and input price changes, an interaction between productivity and input price changes, and squared wage and input price terms. We interact all terms with dummies for 2021. We only report in the table the main coefficients of interest.