Appendix to "An Anatomy of Monopsony: Search Frictions, Amenities and Bargaining in Concentrated Markets," by David Berger, Kyle Herkenhoff, Andreas R. Kostol, and Simon Mongey. In *NBER Macroeconomics Annual 2023*, volume 38, edited by Martin Eichenbaum, Erik Hurst, and Valerie Ramey. Chicago, IL: University of Chicago Press, 2024.

A Data appendix

This section describes the main data sources and key variables and offers some suggestive evidence of the quality of our measures of vacancies and local labor markets.

A.1 Data sources

Linked employer–employee register. Statistics Norway and NAV jointly maintain the Norwegian Matched Employer–Employee Register, a linked employer–employee database (LEED) covering workers' earnings and transitions between employers. The employer reports the data to tax authorities at the end of the year and includes separate identifiers for the firm and its establishments. The data serve multiple purposes, including third-party tax reporting, as the basis for pension contributions and eligibility for safety net programs.

Wages and hours. There were some noteworthy changes to the data structure from 2014 to 2015. The reporting system was automated and based on monthly payments after 2015 and covers a slightly broader set of low-paying jobs and more detailed information on hours, bonuses, overtime, fixed pay, and variable pay. Before 2015, contracts with fewer than 4 hours per week or below an annual NOK10,000 were not reported. Importantly, for the vast majority of jobs, we observe the dates of alterations to the contract and the corresponding wage, industry and occupational codes, geographic location, and tenure at the establishment in both data sources. Hours reported are reasonably well measured in brackets from 4 to 19, 19 to 30, and above 30 hours per week, as pension contributions depend on them. We classify workers as full-time employees if their weekly hours are at least 30.

Occupations and workplace locations. The employment registers report 5-digit occupations. There are about 6,000 different occupations, where some job descriptions have been adjusted from the EU version to meet Norwegian standards and occupational licensing rules. For some positions, the descriptions include information about the rank of the occupation in the hierarchy, e.g., assistants, mid-level managers, top-level management, or members of the executive board[31].

We use the four-digit version. This version combines industry variation in certain occupations, such as code 3114 (machine engineers), which combines machine engineers in shipbuilding and construction. This version is also a natural definition of the career ladder for several skilled occupations, such as the code 2224 (pharmacists), which includes over-the-counter shop assistants, the licensed occupation for handling prescriptions, and senior positions in private pharmacies and clinical and hospital pharmacists.

[31] The first digit gives the skill level: 1: CEO/manager/politician, 2: master's degree, 3: bachelor's degree, 4: customer relations, 5: sales, health care, and service, 6: agriculture, 7: manual vocational occupations, 8: routine vocational, 9: no skill requirement, and 10: military. Occupations 4–8 require 10–12 years of schooling. 1 and 9 and 10 have no formal educational requirements. The second digit gives the field.

The tax registers also include the workplace location of the establishment—the municipality in which the employee must be present to perform her tasks. There are about 400 municipalities and 19 counties in Norway, none of which represent the natural boundaries of a local labor market. We instead use the information about the residence and workplace to define a commuting zone, which is our preferred definition of a local labor market. We follow Bhuller (2009), who aggregate municipalities into 46 regions allowed to cross counties and combine commuting statistics with natural boundaries, such as mountains and fjords.

Population registers. Demographic information on workers comes from longitudinal administrative registers provided by Statistics Norway. These data cover every Norwegian resident from 2006 to 2018 and contain the individual residential location, educational background, and demographic information (including on the worker's sex, age, residential location, spouse, and children).

The national education database (NUDB) includes the highest obtained degree from 1983 to 2018. These files provide information on the field (e.g., plumbing, mechanical engineering, nursing), level (e.g., vocational track in high school, bachelor's, master's, PhD), and years of schooling. There are approximately 5,000 educational codes.

A.2 Combining occupational codes

The International Standard Classification of Occupations (ISCO) underwent a major revision due to a resolution at the Meeting of Experts on Labor Statistics in 2007[32]. Statistics Norway and the Norwegian Labour and Welfare Administration (NAV) jointly adopted the international versions, with 354 and 403 unique four-digit occupations from ISCO88 and ISCO08. The Norwegian versions are named STYRK98 and STYRK08. STYRK98 is currently used in the employer–employee register, available from 2003, and STYRK08 is used in the vacancy data from the employment agency/NAV and after 2011 in the unemployment data. Several occupations, e.g., computer systems designers and computer programmers, were split into smaller groups during the revision. Other codes were collapsed into a larger group; for example, many types of machine operators were made obsolete by technological change. An official version of a two-way correspondence table from STYRK98 to STYRK08 is unavailable.

For our analysis, we need a crosswalk in both directions. Employees look for jobs classified in the new version. Similarly, employers must be able to see potential candidates whose skills are classified by the old version. To create a consistent measure of occupations, we proceed in three steps. First, we identify occupations with an exact match in the two versions. This gives a match of 50 occupations. Second, we identify revised occupations using unemployment periods that overlap with both versions in 2011/2012. To reduce noise from case-worker reporting, we keep 1:1 and 1:many mappings but keep only those with at least 30 percent of each unique STYRK98 code's total. Third, we keep all occupations in the official ISCO correspondence table with 1:1, 1:2, and 2:1 mappings. The remaining occupations are manually identified based on the available descriptions of STYRK codes and the ISCO crosswalk table[33].

[32] See ILO, "ISCO–08 Volume I: International Standard Classification of Occupations, Structure, Group Definitions and Correspondence Tables."

[33] Statistics Norway 1998 (NOS C521); Statistics Norway 2011 (Notater 17/2011).

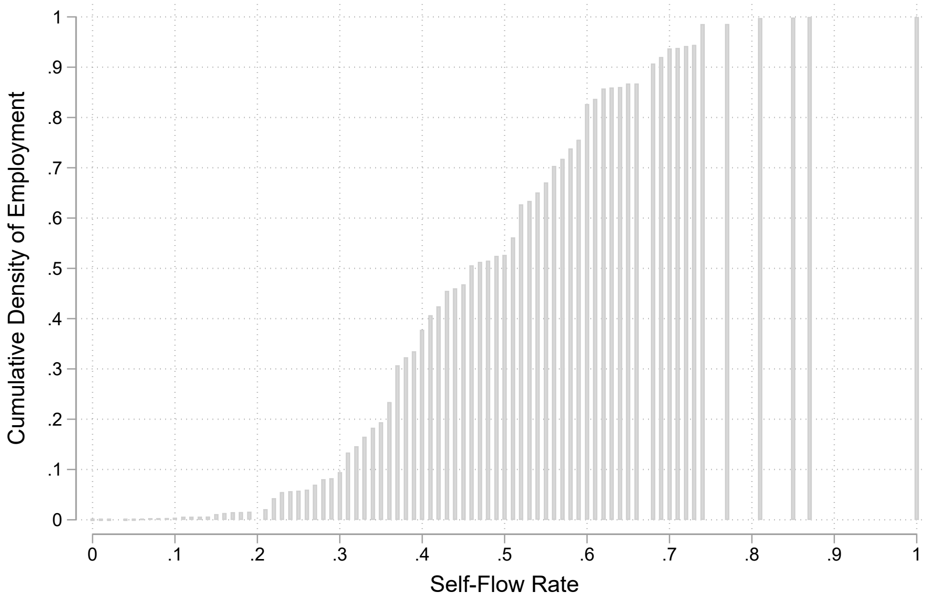

Figure 6: CDF of labor force by self-flow rates

Notes: The self-flow rates are calculated from all EE rates over the period 2006–2016, aggregated by 4-digit occupation.

A final set of consistent occupations depends on the linked sets of occupations with 1:many and many:1 mappings. The resulting number of occupations is reduced from 403 unique versions to 259 consistent occupations. Our crosswalked occupations can be mapped from either STYRK08 or STYRK98 and are available from this link.

A.3 Calculating occupational self-flow rates

The self-flow rates are calculated from the entire dataset of employer-to-employer (EE) transitions as the fraction of EE moves that move from and to the same occupation. We calculate the average self-flow rate of 4-digit occupations to be 43 percent and that of 3-digit occupations to be 45 percent. Next, we calculate the cumulative distribution of employment by the self-flow rate in an occupation. Figure 6 shows that a 50 percent cutoff captures approximately 50 percent of all employment.

B Additional empirical results

Quintiles of firms per market. Table 8 reports our main specifications when we use quintiles of the number of firms in the market as an alternate measure of concentration. Columns (1) through (3) illustrate that greater numbers of firms are associated with greater employer-to-employer transitions, greater wages, and a greater variance of wages, respectively.

Table 8: Alternate measures of concentration in a market: Number of firms

	(1) EE Rate	(2) Log Wage	(3) SD Log Wage
Number of firms, quintile 2	0.000598***	0.0150***	0.0564***
	(3.70e-05)	(0.00344)	(0.00394)
Number of firms, quintile 3	0.00115***	0.0366***	0.0849***
	(4.34e-05)	(0.00386)	(0.00415)
Number of firms, quintile 4	0.00177***	0.0690***	0.0991***
	(4.74e-05)	(0.00410)	(0.00425)
Number of firms, quintile 5	0.00260***	0.114***	0.114***
	(5.52e-05)	(0.00444)	(0.00451)
FE	O-Y	O-Y	O-Y
Controls	Y	Y	Y
Obs.	892,774	892,927	892,176
R^2	0.185	0.821	0.409

Unemployment vs. firms per worker. Table 9 illustrates the relation between the unemployment rate and the number of firms per worker. Column (1) of Table 9 regresses the unemployment rate (at the 4-digit occupation by region by time level) on firms per worker (at the market level). While the bottom quintile of firms per worker has marginally lower unemployment, the two specifications disagree on the relationship. Occupation–year fixed effects suggest higher unemployment in the highest quintile of firms per worker. Occupation–region fixed effects imply a nonmonotonic relationship. Column (2) yields an insignificant difference between the unemployment rate in quintiles 1 and 5 of the firms per worker distribution. Given this nonrobust relationship between unemployment and firms per worker, our vacancy posting cost is designed to deliver a flat relationship between the two variables.

Table 9: Relationship between unemployment and firms per worker in a market

	(1)	(2)
	——Unemployment Rate——	
Firms per worker, quintile 2	0.00330***	0.00323***
	(0.00118)	(0.00122)
Firms per worker, quintile 3	0.00323***	0.00514***
	(0.00125)	(0.00175)
Firms per worker, quintile 4	0.00518***	0.00659***
	(0.00122)	(0.00222)
Firms per worker, quintile 5	0.00640***	0.00452
	(0.00131)	(0.00275)
FE	O-Y	O-R
Controls	N	N
Obs.	964,179	964,132
R^2	0.417	0.514

C Robustness to an alternative definition of labor markets

In this section, we consider an alternative definition of local labor market markets. Rather than clustering occupations by K-means, we instead build on the work of Schmutte (2014) and extract the relevant clusters of occupations within a CZ using modularity maximization.

C.1 The modularity maximization approach

Modularity maximization (MM) is the workhorse model for community detection in network analysis. It identifies groups of nodes (also known as communities) that are densely interconnected within themselves but sparsely connected to nodes outside their group (like a labor market). This is achieved by maximizing a metric called modularity, with a high modularity score indicating that the observed network has more within-group connections than would be expected by chance.

Similarly to Schmutte (2014), who uses individual-level mobility data across occupations, we start from the aggregated occupational flows, grouped by 4-digit occupation separately for each commuting zone (CZ). Our algorithm proceeds as follows:

1. First, we isolate single-occupation markets with high self-flow rates (e.g., 50 percent of EE moves).

2. To detect the occupational network among the remaining occupations, we perform modularity maximization, which searches for the grouping of occupations that maximizes the total sum of worker moves within the network relative to a randomly assigned group of occupations. For example, "electrical engineer" is considered a potential network if its inclusion yields a higher modularity score than its combination with a random occupation. We implement the Louvain algorithm, which assigns each occupation i to a network by calculating the change in modularity by moving i into the network of each occupation j to which i is connected (by at least one worker). In each iteration, it picks the j that increases the modularity score the most.

3. This process continues, where the network grows (or stays the same) until there is no change in the modularity score. We use the final network of occupations as the relevant clusters.

C.2 Results

We repeat the main graphical analysis using the MM market definition. Figure 7 provides a graphical presentation of our regression evidence using across-region, within-occupation–year variation, but now using the markets defined based on the MM algorithm. Reassuringly, the pattern remains the same as that for the markets defined based on the K-means clustering algorithm. We repeat this robustness check using the MM market definition with occupation–region fixed effects in Figure 8. Again, the correlations between the Herfindahl index and labor market outcomes remain quantitatively similar and qualitatively the same as in our main empirical analysis.

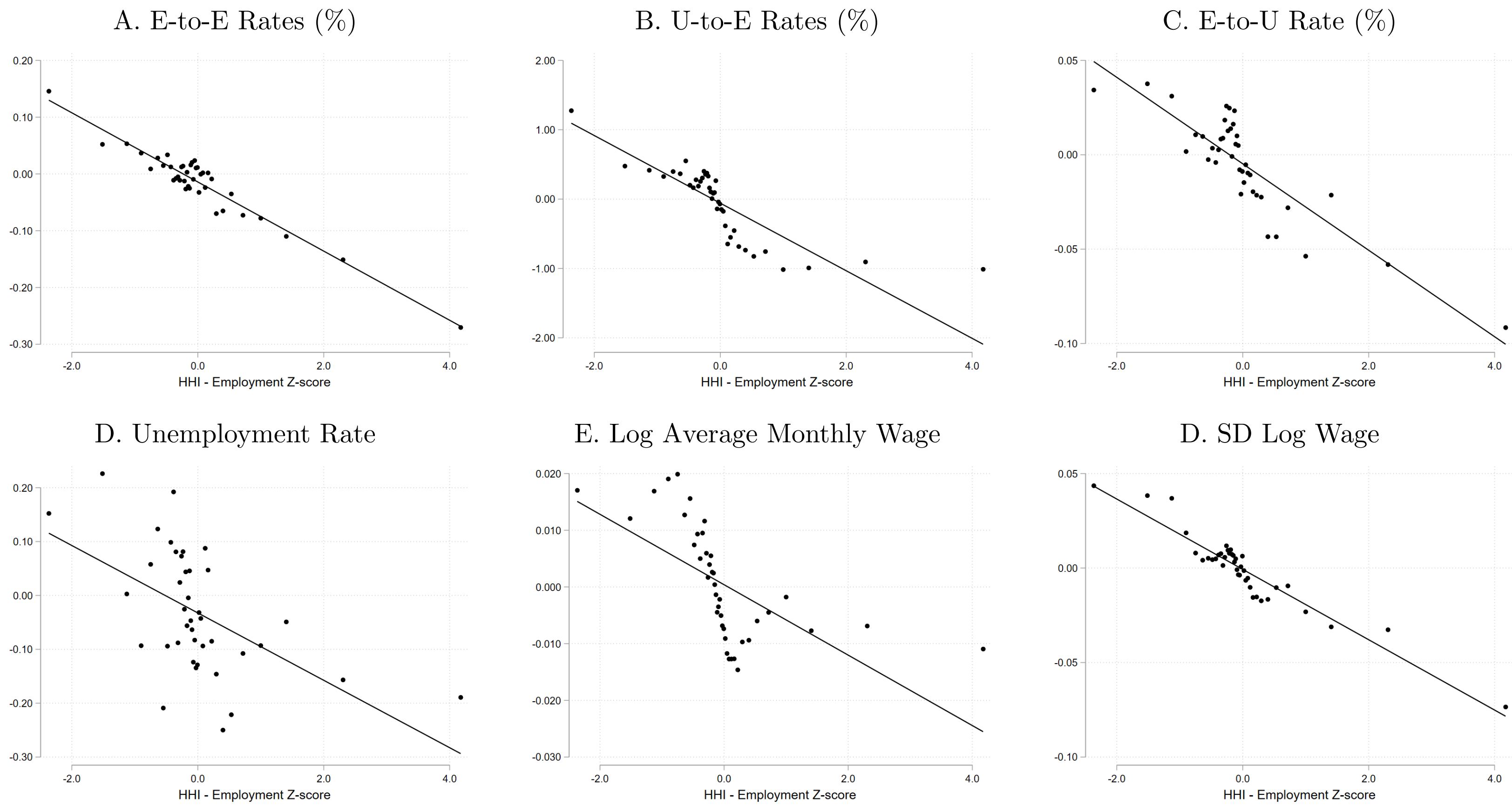

Figure 7: Concentration and labor market outcomes residualized on **occupation–year FEs** – Modularity maximization

Note: For each market (where a market is defined as a cluster of occupations within a commuting zone that maximizes the modularity score), we compute the employment Herfindahl index (HHI). For each 4-digit occupation–commuting zone–year, we compute the average of the dependent variable within 40 centiles of the market HHI, unweighted. We then residualize all x and y variables on occupation–year fixed effects, age composition, gender composition, education composition, lagged firms-per-worker ventiles, lagged labor force growth, and month-of-year dummies. The average NOK/USD in 2021 was 9.

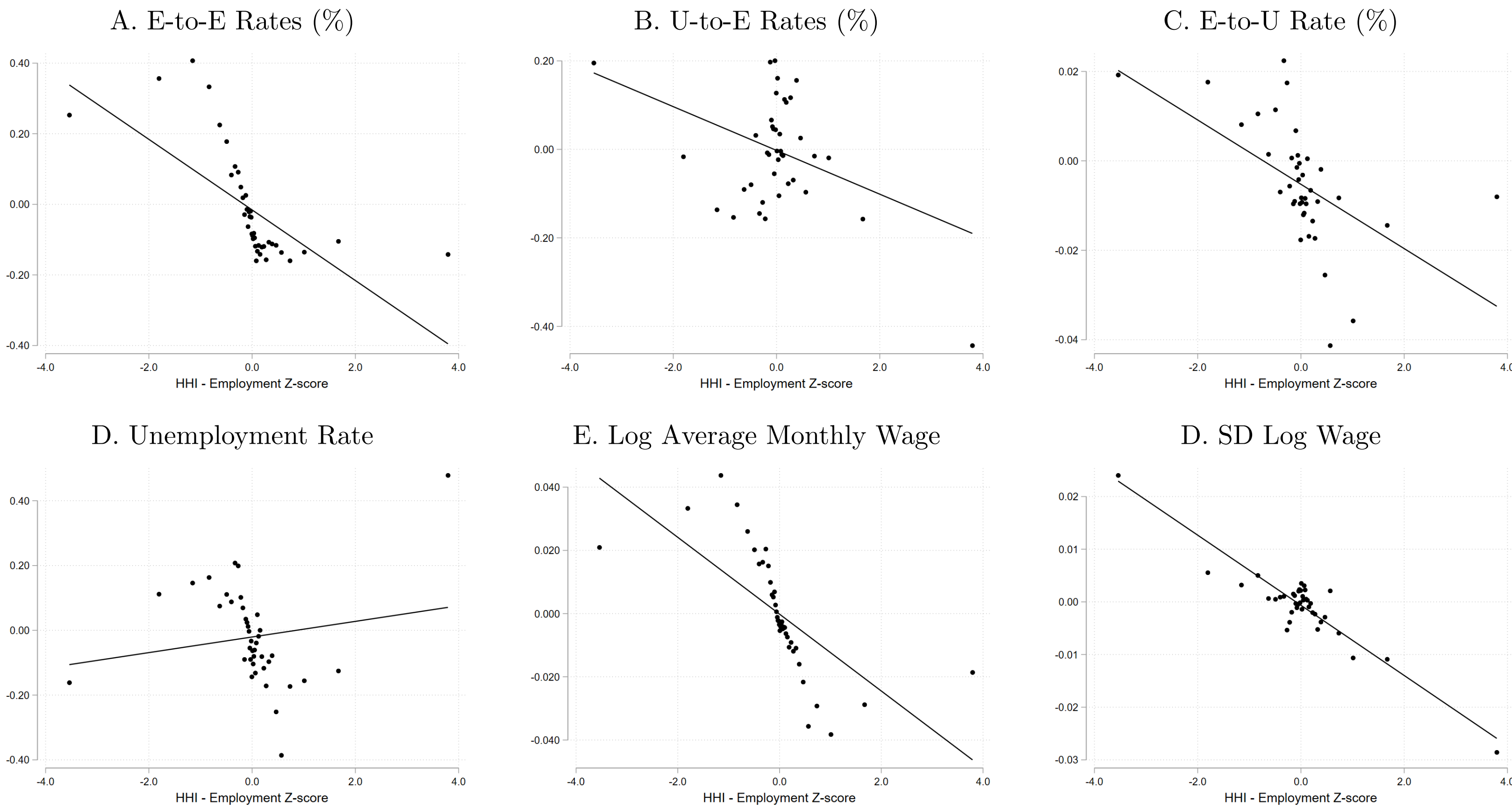

Figure 8: Concentration and labor market outcomes residualized on **occupation–region FEs** – modularity maximization

Note: For each market (where a market is defined as a cluster of occupations within a commuting zone that maximizes the modularity score), we compute the employment Herfindahl index (HHI). For each 4-digit occupation–commuting zone–year, we compute the average of the dependent variable within 40 centiles of the market HHI, unweighted. We then residualize all x and y variables on occupation–region fixed effects, age composition, gender composition, education composition, lagged firms-per-worker ventiles, lagged labor force growth, and month-of-year dummies. The average NOK/USD in 2021 was 9.

C.3 Industry and occupations as markets

Table 10 computes self-flow rates for raw industry and occupation codes. The self-flow rates are quite low using industry definitions. By comparison, markets defined by commuting zone and occupations are higher, especially for the raw 4-digit occupation definition, yielding a self-flow rate of 43 percent. The raw 3-digit occupation definition of a market improves the explained fraction of EE moves by an additional two percentage points, yielding a self-flow rate of 45 percent.

Table 10: Summary Statistics: Within Market EE Shares

	Industry		Occupation			Flow-based
Market definition: CZ×	4	2	7	4	3	
	0.32	0.39	0.33	0.43	0.45	0.52

Notes: Within-market transitions among all employer-employer (EE) transitions defined as a change of the firm with at most 30 days of non-employment between the two spells and at least one year of tenure in the old and the new job. Market is industry/occupation×commuting zone. Outflow from CZ in Norway is 15% (see Bhuller 2009).

D Within-market and all flows

In the main text, we compute all flows using within-market stocks and flows. Table 11 computes flows using all workers regardless of whether they stay or leave the market. The E-to-E rate is higher (mechanically), but the separation and job-finding rates remain very similar.

Table 11: Within-market flows and all flows

	Within market flows	All flows
E-to-E rate (monthly %)	0.65%	1.19%
U-to-E rate (monthly %)	8.08%	8.82%
E-to-U rate (monthly %)	0.35%	0.35%

Figure 9 reports the covariances between the Herfindahl valuse and flows using all workers regardless of whether they stay or leave the market. Panels A through C include occupation–year FEs, and Panels D through F include occupation–region FEs. We find patterns very similar to those in the main text.

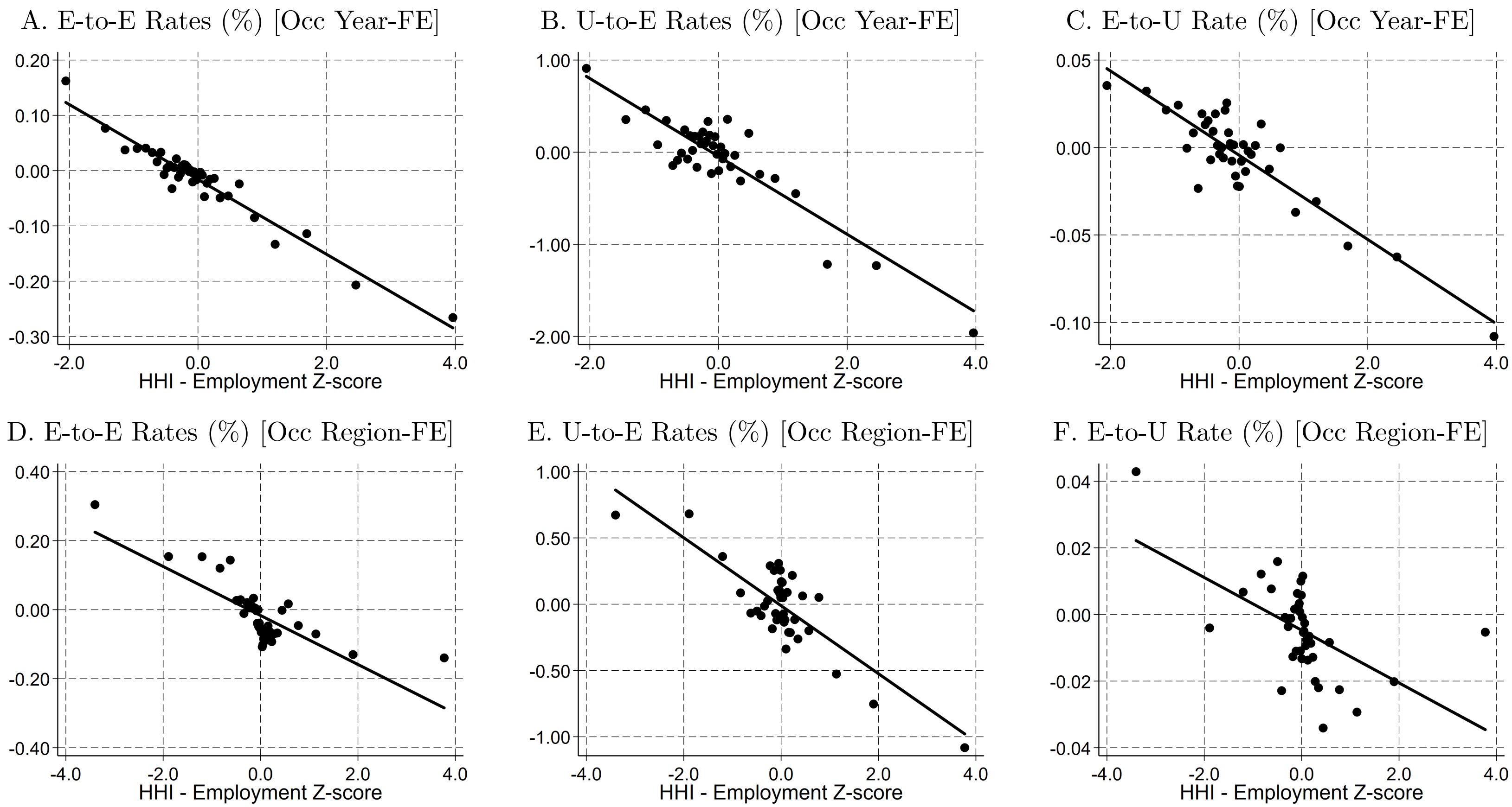

Figure 9: Concentration and job flows using main-text, K-means definitions of markets

Note: For each market (where a market is defined, as in Section 2, as a cluster of occupations within a commuting zone), we compute the employment Herfindahl index (HHI). For each 4-digit occupation–commuting zone–year, we compute the average of the dependent variable within 40 centiles of the market HHI, unweighted. We then residualize all x and y variables on occupation–year fixed effects, age composition, gender composition, education composition, lagged firms-per-worker ventiles, lagged labor force growth, and month-of-year dummies. The average NOK/USD in 2021 was 9.

E Model derivations

In this appendix, we derive the main equations in the text. To keep the algebra simple, we derive the equations without amenities. Adding amenities involves a straightforward modification to these equations. As defined in the text, $P_i := W_i + J_i$ is the joint value of a match, where W_i is the worker continuation value and J_i is the firm continuation value. The surplus is defined as $S_i := W_i + J_i - U_i - V_i = P_i - U_i - V_i$, where we assume that the outside option of the firm is zero, $V_i = 0$ (i.e., the job position is destroyed if not filled by the current worker).

Bargaining and surplus shares. We begin with the Nash bargaining problem of a worker moving from firm i to firm k. This job flow happens only when $P_k > P_i$, and thus, the worker's outside option is $W_i = P_i$ (i.e., to extract the full joint value of the match or, equivalently, to extract the full surplus of the match $W_i - U_i = P_i - U_i = S_i$):

$$\max_{W_k} (P_k - W_k)^{1-\theta} (W_k - P_i)^{\theta},$$

$$\Rightarrow W_k = P_i + \theta [P_k - P_i]. \tag{10}$$

As a convenient accounting device, we write the worker value as the value of unemployment plus some share σ of the match surplus S (see Lise and Postel-Vinay (2020)). Thus, the definition of the worker value $W_k(\sigma)$ is given by a σ share of surplus.

$$W_k(\sigma) := U + \sigma S_k = U + \sigma [P_k - U]. \tag{11}$$

Equating the solution to the Nash bargaining problem in equation (10) to our accounting definition of the worker continuation value in equation (11) yields the following expression for the surplus share:

$$\begin{aligned} W_k(\sigma) &= P_i + \theta [P_k - P_i], \\ U + \sigma [P_k - U] &= P_i + \theta [P_k - P_i], \\ \sigma (P_i, P_k) &= \frac{\theta [P_k - U] + (1-\theta) [P_i - U]}{[P_k - U]}. \end{aligned} \tag{12}$$

We can write the surplus share in equation (12) equivalently in terms of surplus values:

$$\sigma (S_i, S_k) = \theta + (1-\theta) \frac{S_i}{S_k} \tag{13}$$

We use equation (13) in the law of motion for the surplus share, which we denote σ':

$$\sigma' = \begin{cases} \left(\frac{\theta S_k + (1-\theta) S_i}{S_k} \right) & \text{if } S_k > S_i \\ \max \left\{ \sigma, \frac{S_k}{S_i} \right\} & \text{if } S_k \leq S_i. \end{cases}$$

In the event that the surplus at the new firm k is greater than that at firm i ($S_k > S_i$), then the worker moves and Nash bargains; thus, $\sigma' = \left(\frac{\theta S_k + (1-\theta) S_i}{S_k}\right)$. In the event that the worker meets a firm that offers less surplus, the worker stays at the incumbent firm, and the offer is either matched (with no Nash bargaining) or ignored; thus, $\sigma' = \max\left\{\sigma, \frac{S_k}{S_i}\right\}$ $(S_k \leq S_i)$.

Surplus. To derive an equation for surplus, we begin by describing the continuation value for a worker who is unemployed, U. A worker who finds a job coming from unemployment has an outside option of U, so $W_i = U + \theta\left[P_i - U\right]$, which implies directly that $\sigma = \theta$. We can write the continuation value of unemployment as follows:

$$U = b + \beta\left[U + \sum_k \lambda_{uk} \mathbf{1}\left[W_k(\theta) > U\right]\left[W_k(\theta) - U\right]\right],$$

$$U = b + \beta\left[U + \theta \sum_k \lambda_{uk} \mathbf{1}\left[P_k > U\right]\left[P_k - U\right]\right],$$

$$U = b + \beta\left[U + \theta \sum_k \lambda_{uk} \max\left\{S_k, 0\right\}\right].$$

We can also derive an expression for the joint value of the worker and firm. This joint value takes into account the worker's value from a new match and the worker's value in unemployment:

$$P_i = z_i + \beta\left[P_i + \sum_{k \neq i} \lambda_{ik} \mathbf{1}\left[P_k > P_i\right]\left[W_k\left(\sigma\left(P_i, P_k\right)\right) - P_i\right] + \delta\left(U - P_i\right)\right]. \tag{14}$$

The term $\sum_{k \neq i} \lambda_{ik} \mathbf{1}\left[P_k > P_i\right]\left[W_k\left(\sigma\left(P_i, P_k\right)\right) - P_i\right]$ captures the value of the worker's meeting with a new firm $k \neq i$. The contact rate is λ_{ik}; the worker moves if the surplus in the new firm is greater $\left[P_k > P_i\right]$; and the worker bargains and obtains a surplus share given by equation (12). The term $\delta\left(U - P_i\right)$ captures the value of unemployment if the worker separates. Substituting the definition of the surplus into equation (14) yields

$$P_i = z_i + \beta\left[P_i + \theta \sum_{k \neq i} \lambda_{ik} \max\left\{S_k - S_i, 0\right\} - \delta S_i\right]. \tag{15}$$

By subtracting U from both sides of equation (15), we can derive an expression for the surplus:

$$P_i - U = \left\{z_i + \beta\left[P_i + \theta \sum_{k \neq i} \lambda_{ik} \max\left\{S_k - S_i, 0\right\} - \delta S_i\right]\right\} - \left\{b + \beta\left[U + \theta \sum_k \lambda_{uk} \max\left\{S_k, 0\right\}\right]\right\},$$

$$S_i = z_i - b + \beta\left[S_i + \theta \sum_{k \neq i} \lambda_{ik} \max\left\{S_k - S_i, 0\right\} - \delta S_i - \theta \sum_k \lambda_{uk} \max\left\{S_k, 0\right\}\right]. \tag{16}$$

An important point to note is that with a finite number of firms, the surplus equation can be solved via matrix inversion (see Section E.1). Given contact rates λ_{uk}, the surplus equation can be solved independently of its components W_i, U, and J_i. This remains true with amenities. Moreover, the worker's share of surplus does not alter the size of the surplus—hence its independence of σ. This is a standard result in models with linear utility.

Wage equation. We derive the wage equation in three steps. First, we derive the worker's value from first principles. Second, we use our accounting device whereby worker values are expressed as a share of surplus. Third, we equate these two formulas for the worker's value to solve for the wage. The resulting wage depends only on the surplus.

Step I. The first step is to compute the worker value from first principles. The worker value under the Cahuc, Postel-Vinay, and Robin (2006) (CPVR) bargaining protocol can be written as

$$W_i(\sigma) = w_i(\sigma) + \beta\Big[W_i(\sigma) + \sum_{k\neq i}\lambda_{ik}\left\{\mathbf{1}\left[P_k > P_i\right]\left[\theta P_k + (1-\theta)P_i - W_i(\sigma)\right] + \mathbf{1}\left[W_i(\sigma) < P_k < P_i\right]\left[P_k - W_i(\sigma)\right]\right\} + \delta\left[U_i - W_i(\sigma)\right]\Big].$$

This equation can be written compactly using min and max notation:

$$W_i(\sigma) = w_i(\sigma) + \beta\left[W_i(\sigma) + \sum_{k\neq i}\lambda_{ik}\max\left\{0, \min\left\{\theta P_k + (1-\theta)P_i, P_k\right\} - W_i(\sigma)\right\} + \delta\left[U - W_i(\sigma)\right]\right].$$

Last, we use the accounting definition of the worker value $W_i(\sigma) = U + \sigma\left[P_i - U\right]$, and by the definition of surplus $W_i(\sigma) - U = \sigma\left[P_i - U\right]$, we arrive at the following expression for the worker value:

$$W_i(\sigma) = w_i(\sigma) + \beta\left[W_i(\sigma) + \sum_{k\neq i}\lambda_{ik}\max\left\{0, \min\left\{\theta\left[P_k - P_i\right], P_k - P_i\right\} + (1-\sigma)\left[P_i - U\right]\right\} - \delta\sigma S_i\right].$$

Continuing to simplify, we can write the worker continuation value in terms of surplus:

$$W_i(\sigma) = w_i(\sigma) + \beta\left[W_i(\sigma) + \sum_{k\neq i}\lambda_{ik}\max\left\{0, \min\left\{\theta\left[S_k - S_i\right], S_k - S_i\right\} + (1-\sigma)S_i\right\} - \delta\sigma S_i\right]. \quad (17)$$

Step II. Our second step is to derive the worker's wage value from our accounting definition of the worker value in equation (11). Starting from equation (11), we have

$$W_i(\sigma) = U + \sigma S_i,$$
$$(1-\beta)W_i(\sigma) = (1-\beta)U + \sigma(1-\beta)S_i.$$

We then substitute the expression for surplus $(1-\beta)S_i$ by subtracting βS_i from both sides of the surplus equation (16) to arrive at an alternate expression for the worker's continuation value:

$$(1-\beta)\,W_i(\sigma)=\sigma z_i+(1-\sigma)\,b+\beta\left[\sigma\theta\sum_{k\neq i}\lambda_{ik}\max\{S_k-S_i,0\}-\sigma\delta S_i+(1-\sigma)\,\theta\sum_{k}\lambda_{uk}\max\{S_k,0\}\right]. \tag{18}$$

<u>*Step III.*</u> Equating the first expression for the worker's value in equation (18) to the second expression for the worker's value in equation (18), we obtain an expression that can be solved to obtain the wages:

$$\begin{aligned}
&w_i(\sigma)+\beta\left[\sum_{k\neq i}\lambda_{ik}\max\{0,\min\{\theta[S_k-S_i],S_k-S_i\}+(1-\sigma)\,S_i\}-\delta\sigma S_i\right]\\
&=\sigma z_i+(1-\sigma)\,b+\beta\left[\sigma\theta\sum_{k\neq i}\lambda_{ik}\max\{S_k-S_i,0\}-\sigma\delta S_i+(1-\sigma)\,\theta\sum_{k}\lambda_{uk}\max\{S_k,0\}\right].
\end{aligned}$$

Rearranging the above expression, we can derive an expression for the wages of a worker in terms of surplus:

$$\begin{aligned}
w_i(\sigma)=\sigma z_i+(1-\sigma)\,b+\beta\Bigg[&(1-\sigma)\,\theta\sum_{k}\lambda_{uk}\max\{S_k,0\}\\
&-\sum_{k\neq i}\lambda_{ik}\max\{0,\min\{(1-\sigma)\,\theta\,(S_k-S_i)\,,(S_k-S_i)\}+(1-\sigma)\,S_i\}\Bigg].
\end{aligned} \tag{19}$$

Similar to the baseline model with amenities, this simpler wage expression includes three terms: (i) workers obtain σ of production, (ii) workers obtain (1 - σ) of their outside option, and last, (iii) there is backloading since firms that offer greater future pay prospects can initially pay less.

E.1 Solving for surplus and wages via matrix inversion

Because there is a finite number of firms, the solution for surplus and wages (conditional on contact rates) can be reduced to a series of matrix inversions. We briefly describe how we vectorize the model to make it easier to solve computationally. We first define $\tilde{z}_i=z_i-b$ and rearrange (16) to obtain the following expression for surplus:

$$(1-\beta\,(1-\delta))\,S_i=\tilde{z}_i+\beta\theta\sum_{k\neq i}\lambda_{ik}\max\{S_k-S_i,0\}-\beta\theta\sum_{k}\lambda_{uk}\max\{S_k,0\}\,. \tag{20}$$

For pedagogical purposes, we work through the vectorization for the case of $N=3$ firms. We first define the matrix Ψ to be the lower triangular matrix of contact rates, where firms are ordered by their productivity. We let $i=1$ correspond to the highest productivity value, and since there are no amenities, no workers ever leave the highest-productivity firm. Thus, Ψ describes the optimal policy of workers and their flow rates across employers:

$$\Psi := \begin{bmatrix} 0 & 0 & 0 \\ \lambda_{21} & 0 & 0 \\ \lambda_{31} & \lambda_{32} & 0 \end{bmatrix}.$$

With amenities, the ranking of firms in terms of (z, ε)-tuples must be guessed and then iterated on until convergence. However, a similar vectorization is easily derived.

We must introduce some additional notation to vectorize the surplus equation. Let $\boldsymbol{I}_N$ denote the identity matrix, $\boldsymbol{i}_N$ be an $N \times 1$ column vector of ones, $\boldsymbol{\lambda}_u$ denote the column vector of unemployed worker contact rates, $\boldsymbol{S}$ denote the column vector of surpluses, and $\otimes$ denote the Kronecker product.

We can continue to work with (20) in the case of three firms to derive the factorized surplus formula:

$$\left(\boldsymbol{I}_N - \beta(1-\delta)\boldsymbol{I}_N\right)\begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} \tilde{z}_1 \\ \tilde{z}_2 \\ \tilde{z}_3 \end{bmatrix} + \beta\theta \begin{bmatrix} 0 & 0 & 0 \\ \lambda_{21}(S_1 - S_2) & 0 & 0 \\ \lambda_{31}(S_1 - S_3) & \lambda_{32}(S_2 - S_3) & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \beta\theta \begin{bmatrix} \lambda_{u1} & \lambda_{u2} & \lambda_{u3} \\ \lambda_{u1} & \lambda_{u2} & \lambda_{u3} \\ \lambda_{u1} & \lambda_{u2} & \lambda_{u3} \\ \lambda_{u1} & \lambda_{u2} & \lambda_{u3} \end{bmatrix}\begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix},$$

$$\left(\boldsymbol{I}_N - \beta(1-\delta)\boldsymbol{I}_N\right)\boldsymbol{S} = \tilde{\boldsymbol{z}} + \beta\theta \begin{bmatrix} 0 \\ \lambda_{21}(S_1 - S_2) \\ \lambda_{31}(S_1 - S_3) + \lambda_{32}(S_2 - S_3) \end{bmatrix} - \beta\theta\left(\boldsymbol{i}_N \otimes \boldsymbol{\lambda}_u'\right)\boldsymbol{S},$$

$$\left(\boldsymbol{I}_N - \beta(1-\delta)\boldsymbol{I}_N + \beta\theta\left(\boldsymbol{i}_N \otimes \boldsymbol{\lambda}_u'\right)\right)\boldsymbol{S} = \tilde{\boldsymbol{z}} + \beta\theta\left(\begin{bmatrix} 0 & 0 & 0 \\ \lambda_{21} & 0 & 0 \\ \lambda_{31} & \lambda_{32} & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & \lambda_{21} & 0 \\ 0 & 0 & \lambda_{31} + \lambda_{32} \end{bmatrix}\right)\begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix}. \tag{21}$$

The first matrix is Ψ, and the second matrix is $diag\left(\Psi \times \boldsymbol{i}_N\right)$:

$$\Psi \times \boldsymbol{i}_N = \begin{bmatrix} 0 & 0 & 0 \\ \lambda_{21} & 0 & 0 \\ \lambda_{31} & \lambda_{32} & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda_{21} \\ \lambda_{31} + \lambda_{32} \end{bmatrix}$$

Inspecting (21), it becomes clear that we can generalize the expression to N firms as follows:

$$\left(\boldsymbol{I}_N - \beta(1-\delta)\boldsymbol{I}_N + \beta\theta\left(\boldsymbol{i}_N \otimes \boldsymbol{\lambda}_u'\right)\right)\boldsymbol{S} = \tilde{\boldsymbol{z}} + \beta\theta\left(\Psi - diag\left(\Psi \times \boldsymbol{i}_N\right)\right)\boldsymbol{S}$$

$$\left(\boldsymbol{I}_N - \beta(1-\delta)\boldsymbol{I}_N + \beta\theta\left(\boldsymbol{i}_N \otimes \boldsymbol{\lambda}_u' - \Psi + diag\left(\Psi \times \boldsymbol{i}_N\right)\right)\right)\boldsymbol{S} = \tilde{\boldsymbol{z}}$$

We can then invert the resulting matrix to solve for the vector of surpluses, $\boldsymbol{S}$:

$$\boldsymbol{S} = \left(\boldsymbol{I}_N - \beta(1-\delta)\boldsymbol{I}_N + \beta\theta\left(\boldsymbol{i}_N \otimes \boldsymbol{\lambda}_u' - \Psi + diag\left(\Psi \times \boldsymbol{i}_N\right)\right)\right)^{-1} \times \tilde{\boldsymbol{z}} \tag{22}$$

With the surpluses given by (22), we can then solve for wages using equation (19). Importantly, our accounting device in which we express worker continuation values as a share of surplus allows us to solve for wages in terms of surplus values *alone*. These features of our model make it fast to solve and readily tractable to the incorporation of amenities. The framework can be easily modified to include rich worker heterogeneity, as well.

E.2 Solution method with amenities

We describe the solution method for the full model with amenities. Within each market j, perform the following steps to solve for the market equilibrium:

i. Guess a vector of vacancies $\{v_i\}$.

ii. Guess a vector of employment $\{n_i(\varepsilon)\}$.

iii. Combine vacancies and employment to compute contact rates for workers using equation (2) and for firms using equation (3) (note that this computation of contact rates can be easily vectorized).

iv. Guess a ranking of firms in the (z, ε) space.

v. Solve for the vector of surplus values $\boldsymbol{S}$ via matrix inversion.

vi. Iterate on the ranking of firms until it agrees with $\boldsymbol{S}$.

vii. Solve for the stationary distribution of workers across firms using (8) (note that this law of motion can be easily vectorized).

viii. Solve the firm's objective value for a new vector of vacancies using equation (7) (note that the update for vacancies can be easily vectorized).

ix. Iterate until the vacancies converge.

x. Recover wages using (19), and simulate individual worker/wage histories as required to compute moments.

F Identification

Table 12 illustrates the identification argument for $\overline{\varepsilon}$, θ, and γ. These parameters primarily govern labor market power, in addition to the number of firms per market M_j. The first row is our baseline economy. The second row holds all parameters fixed, except $\overline{\varepsilon}$, which is set to approximately zero. Notably the fraction of EE moves down the poach rank ladder is approximately zero. The third row raises θ. Wage growth falls by a factor of 10 as initial wages of job finders are now relatively higher. Lastly, the fourth row lowers the curvature of the vacancy posting cost γ significantly. As a result, market concentration increases. High z firms expand and post disproportionately more vacancies than low z firms. Thus Table 12 demonstrates that $\overline{\varepsilon}$, θ, and γ are intuitively identified by job flows down the job ladder, wage growth, and concentration.

Table 12: Partial Jacobian

	HHI	Log wage growth	Frac. EE moves down ladder
Baseline ($\overline{\varepsilon} = 0.76, \theta = 0.18, \gamma = 1.16$)	0.0925	0.0106	0.2051
Baseline except $\overline{\varepsilon} = 1e^{-4}$	0.7790	0.0017	0.0004
Baseline except $\theta = 0.5$	0.1748	0.0015	0.1731
Baseline except $\gamma = 0.5$	0.2727	**	0.1895

*Notes. ** negative wages imply log wage growth is not well defined in this economy.*