Residential Property Price Indexes:
Spatial Coordinates versus Neighbourhood Dummy Variables

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1. Introduction

- It is a difficult task to construct constant quality price indexes for residential (and commercial) properties. Properties with structures on them consist of two main components: **the land component and the structure component**.

- The problem is that each property has a unique location (which affects the price of the land component) and given the fact that **the same property is not sold in every period**, it is difficult to apply the usual matched model methodology when constructing constant quality price indexes.

- **Repeat sales methodology: Bailey, Muth and Nourse (1963) .**
- **Hedonic regression model approach.**

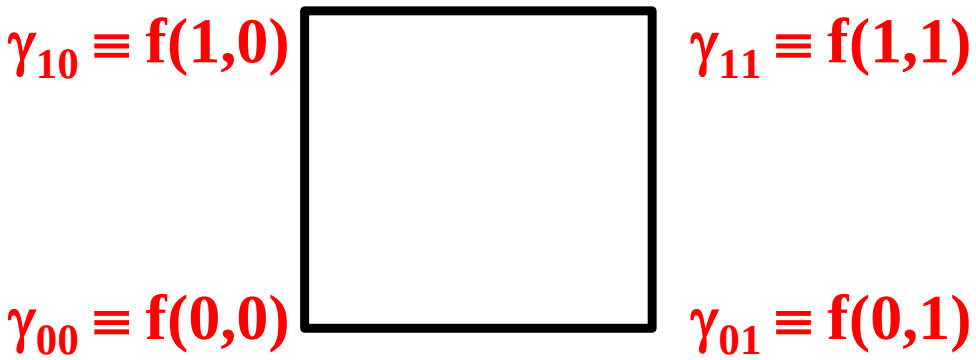
The main question

- The main question that this paper addresses is the following one:
  - Can satisfactory residential property price indexes be constructed using hedonic regression techniques where location effects are modeled using local neighbourhood dummy variables or is it necessary to use spatial coordinates to model location effects.
  - Hill and Scholz (2018) addressed this question and found that it was not necessary to use spatial coordinates to obtain satisfactory property price indexes for Sydney. However, their hedonic regression model did not estimate separate land and structure price indexes for residential properties.
- The present paper addresses the Hill and Scholz question in the context of providing satisfactory residential land price indexes.
  - The spatial coordinate model used in the present paper is a modification of Colwell's (1998) spatial interpolation method. The modification can be viewed as a general nonparametric method for estimating a function of two variables.

2. Bilinear Interpolation on the Unit Square

- Suppose that f(x,y) is a continuous function of two variables, x and y, where $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Suppose that f takes on the values γ_{ij} at the corners of the unit square; i.e., we have:

(1) $\gamma_{00} \equiv f(0,0)$; $\gamma_{10} \equiv f(1,0)$; $\gamma_{01} \equiv f(0,1)$; $\gamma_{11} \equiv f(1,1)$.



  - Assuming that we know (or can estimate) the heights of the function at the corners of the unit square, we look for an approximating continuous function that satisfies counterparts to equations (1) at the corners of the unit square and is a linear function along the four line segments that make up the boundary of the unit square.

Colwell's Model (1989)

- Colwell (1998; 89) showed that the following **quadratic function** of x and y, g(x,y), satisfies these requirements:

(2) $g(x,y) \equiv \gamma_{00}(1-x)(1-y) + \gamma_{10}x(1-y) + \gamma_{01}(1-x)y + \gamma_{11}xy.$

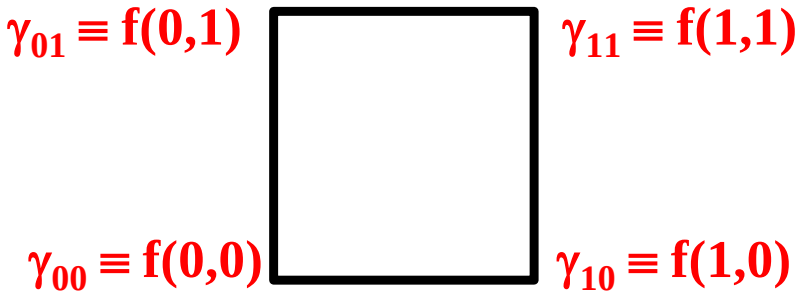
- Colwell (1998; 89) also showed that g(x,y) is a **weighted average of γ_{00}, γ_{10}, γ_{01} and γ_{11} for (x,y)** belonging to the unit square. In order to gain more insight into the properties of g(x,y), rewrite g(x,y) as follows:

(3) $g(x,y) = \gamma_{00} + (\gamma_{10} - \gamma_{00})x + (\gamma_{01} - \gamma_{00})y + [(\gamma_{00} + \gamma_{11}) - (\gamma_{01} + \gamma_{10})]xy.$

Colwell's Model (1989)

(3) $g(x,y) = \gamma_{00} + (\gamma_{10} - \gamma_{00})x + (\gamma_{01} - \gamma_{00})y + [(\gamma_{00} + \gamma_{11}) - (\gamma_{01} + \gamma_{10})]xy.$

- Thus if $\gamma_{00} + \gamma_{11} = \gamma_{01} + \gamma_{10}$, then g(x,y) is a linear function over the unit square.
- However, if $\gamma_{00} + \gamma_{11} \neq \gamma_{01} + \gamma_{10}$, then g(x,y) is a saddle function; i.e., the determinant of the matrix of second order partial derivatives of g(x,y), $\nabla^2 g(x,y)$, is equal to $-[(\gamma_{00} + \gamma_{11}) - (\gamma_{01} + \gamma_{10})]^2 < 0$ and hence $\nabla^2 g(x,y)$ has one positive and one negative eigenvalue.



3. Bilinear Spline Interpolation over a Grid

- In order to explain how Colwell's method works over a grid of squares, we will explain his method for the case of a 3 by 3 grid of squares. The method will be applied to the variables X and Y that are defined over a rectangular region in X,Y space. We assume that X and Y satisfy the following restrictions:

(4) $\mathbf{X_{min} \leq X \leq X_{max}}$ **;** $\mathbf{Y_{min} \leq Y \leq Y_{max}}$

- where $X_{min} < X_{max}$ and $Y_{min} < Y_{max}$.
- We translate and scale X and Y so that the range of the transformed X and Y, x and y, lie in the interval joining **0 and 3**; i.e., define x and y as follows:

(5) $\mathbf{x \equiv 3(X - X_{min})/(X_{max} - X_{min})}$ **;**

$\mathbf{y \equiv 3(Y - Y_{min})/(Y_{max} - Y_{min})}$**.**

- Define the following 3 ***dummy variable*** (or **indicator) *functions*** of x:

(6) $D_1(x) \equiv 1$ if $0 \leq x < 1$; $D_1(x) \equiv 0$ if $x \geq 1$;

$D_2(x) \equiv 1$ if $1 \leq x < 2$; $D_2(x) \equiv 0$ if $x < 1$ or $x \geq 2$;

$D_3(x) \equiv 1$ if $2 \leq x \leq 3$; $D_3(x) \equiv 0$ if $x < 2$.

- Note that if $0 \leq x \leq 3$, then $D_1(x) + D_2(x) + D_3(x) = 1$ so that the 3 dummy variable functions sum to 1 if x lies in the interval between 0 and 3.
- The above definitions can be used to define the 3 *dummy variable functions* of y, $D_1(y)$, $D_2(y)$ and $D_3(y)$, where y replaces x in definitions (6).
- Finally, a set **of 3×3 = 9 *bilateral dummy variable functions***, $D_{ij}(x,y)$, is defined as follows:

(7) $D_{ij}(x,y) \equiv D_i(x)D_j(y)$; i = 1,2,3; j = 1,2,3.

- The domain of definition for the $D_{ij}(x,y)$ is the ***square*** $\mathbf{S_3}$ in two dimensional space with each side of length 3; i.e.,

  $S_3 \equiv \{ (x,y) : 0 \leq x \leq 3; 0 \leq y \leq 3\}$.

- Note that for any (x,y) belonging to S_3, we have $\Sigma_{i=1}^{3} \Sigma_{j=1}^{3} D_{ij}(x,y) = 1$. Thus the bilateral dummy variable functions $D_{ij}(x,y)$ will allocate any $(x,y) \in S_3$ to one of the nine unit square cells that make up S_3.
- Denote the ***cell*** of area 1 that corresponds to x and y such that $D_{ij}(x,y) = 1$ as $\mathbf{C_{ij}}$ for i,j = 1,2,3. Thus the 3 cells in the grid of 9 cells that correspond to y values that satisfy $0 \leq y < 1$ are C_{11}, C_{21} and C_{31}. The 3 cells that correspond to y values such that $1 \leq y < 2$ are C_{12}, C_{22} and C_{32} and the 3 cells that correspond to y values such that $2 \leq y \leq 3$ are C_{13}, C_{23} and C_{33}.

- Let f(x,y) be the function defined over S_3 that we wish to approximate. Define the *heights* γ_{ij} of the function f(x,y) at the 16 vertices of the grid of unit area cells as follows:

(8) $\gamma_{ij} \equiv f(i,j)$; $i = 0,1,2,3$; $j = 0,1,2,3$.

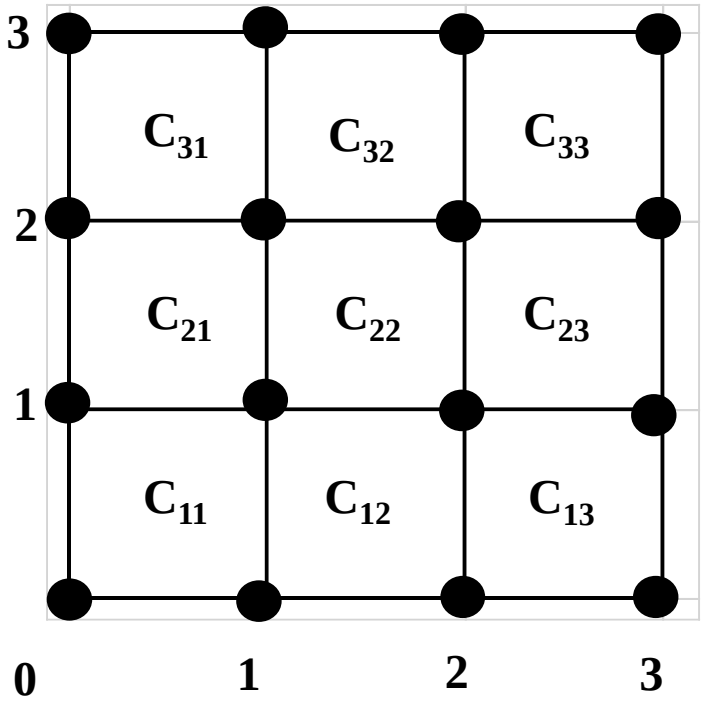
- Define the Colwell (1998; 91-92) ***bilinear spline interpolating approximation*** $g_3(x,y)$ to f(x,y) for any $(x,y) \in S_3$ as follows:

(9) $g_3(x,y) \equiv$

$$
\begin{aligned}
& D_{11}(x,y)[\phi_{00}(1-x)(1-y)+\phi_{10}(x-0)(1-y)+\phi_{01}(1-x)(y-0)+\phi_{11}xy] \\
& + D_{21}(x,y)[\phi_{10}(2-x)(1-y)+\phi_{20}(x-1)(1-y)+\phi_{11}(2-x)(y-0)+\phi_{21}xy] \\
& + D_{31}(x,y)[\phi_{20}(3-x)(1-y)+\phi_{30}(x-2)(1-y)+\phi_{21}(3-x)(y-0)+\phi_{31}xy] \\
& + D_{12}(x,y)[\phi_{01}(1-x)(2-y)+\phi_{11}(x-0)(2-y)+\phi_{02}(1-x)(y-1)+\phi_{12}xy] \\
& + D_{22}(x,y)[\phi_{11}(2-x)(2-y)+\phi_{21}(x-1)(2-y)+\phi_{12}(2-x)(y-1)+\phi_{22}xy] \\
& + D_{32}(x,y)[\phi_{21}(3-x)(2-y)+\phi_{31}(x-2)(2-y)+\phi_{22}(3-x)(y-1)+\phi_{32}xy] \\
& + D_{13}(x,y)[\phi_{02}(1-x)(3-y)+\phi_{12}(x-0)(3-y)+\phi_{03}(1-x)(y-2)+\phi_{13}xy] \\
& + D_{23}(x,y)[\phi_{12}(2-x)(3-y)+\phi_{22}(x-1)(3-y)+\phi_{13}(2-x)(y-2)+\phi_{23}xy] \\
& + D_{33}(x,y)[\phi_{22}(3-x)(3-y)+\phi_{32}(x-2)(3-y)+\phi_{23}(3-x)(y-2)+\phi_{33}xy].
\end{aligned}
$$

- It can be verified that $g_3(x,y)$ is a continuous function of x and y over S_3 and $g_3(x,y)$ is equal to the underlying function f(x,y) when (x,y) is a vertex point of the grid; i.e., we have the following equalities for the 16 vertex points in S_3:

(10) $g_3(i,j) = \gamma_{ij} \equiv f(i,j)$; i = 0,1,2,3; j = 0,1,2,3.



- For each square of unit area in the grid, it can be seen that $g_3(x,y)$ behaves like the bilinear interpolating function $g(x,y)$ that was defined by (2) in the previous section. Thus if (x,y) belongs to the cell C_{ij} where i and j are equal to 1, 2 or 3, then $g_3(x,y)$ is bounded from below by the minimum of the 4 vertex point values $\gamma_{i-1,j-1}$, $\gamma_{i,j-1}$, $\gamma_{i-1,j}$, $\gamma_{i,j}$ and bounded from above by the maximum of the 4 vertex point values $\gamma_{i-1,j-1}$, $\gamma_{i,j-1}$, $\gamma_{i-1,j}$, $\gamma_{i,j}$.

- **Following Poirier (1976; 11-12) and Colwell (1998), we can move from the interpolation model defined by (9) to an econometric estimation model.**

- Thus suppose that we can observe x and y for N observations, say (x_n,y_n) for n = 1,...,N. Suppose also that we can observe $f(x_n,y_n)$ for n = 1,...,N. Finally, suppose that we can approximate the function f(x,y) by $g_3(x,y)$ over S_3.
- Let $\boldsymbol{\gamma \equiv [\gamma_{00}, \gamma_{10},\ldots,\gamma_{33}]}$ be the vector of the 16 $\boldsymbol{\gamma_{ij}}$ which appear in (9) and rewrite $g_3(x,y)$ as $g_3(x,y,\boldsymbol{\gamma})$. Now view $\boldsymbol{\gamma}$ as a vector of parameters which appear in the following linear regression model:

(11) $\boldsymbol{z_n = g_3(x_n,y_n,\gamma) + \varepsilon_n ; \quad n = 1,\ldots,N.}$

- If we are willing to assume that the approximation errors ε_n are independently distributed with 0 means and constant variances, the unknown parameters γ_{ij} in (11) (which are the heights of the "true" function f(x,y) at the vertices in the grid) can be estimated by a least squares regression.
- It can be seen that this method for fitting a two dimensional surface over a bounded set is essentially a nonparametric method.
- If the number of observations N is sufficiently large and the observations are more or less uniformly distributed over the grid, then we can make the grid finer and finer and obtain ever closer approximations to the true underlying function if it is continuous.

4. Colwell's Nonparametric Method versus Penalized Least Squares (the method used by Hill and Scholz).

- Using the notation surrounding (11) above, a simplified version of this approach works as follows: find a function g(x,y) which is a solution to the following ***penalized least squares minimization problem***:

(13) $\min_g \Sigma_{n=1}^{N} [z_n - g(x_n,y_n)]^2 + \lambda J(g)$

- where it is assumed that g(x,y) is twice continuously differentiable and J(g) is some function of the second order partial derivatives of g evaluated at the N observed (x_n,y_n).
- It is difficult to explain how the penalized least squares approach works in the two dimensional case. There are many problems with this method. In the paper, we go into some of the difficulties.

5. The Tokyo Residential Property Sales Data

- There were a total of 5580 observations with structures on the property in our sample of sales of residential property sales in the Tokyo area over the 44 quarters covering 2000-2010. (Diewert and Shimizu (2015)).

- In addition, we had 8493 observations on residential properties with ***no structure on the land plot***.

- Thus there was a total of 14,073 properties in our sample.

- The variables used in our regression analysis to follow and their units of measurement are as follows:
  - **V** = The **value** of the sale of the house in 10,000,000 Yen;
  - **S** = **Structure area** (floor space area) in units of 100 m squared;
  - **L** = **Lot area** in units of 100 meters squared;
  - **A** = Approximate **age** of the structure in years;
  - **NB** = **Number of bed**rooms;
  - **W** = **Width** of the lot in 1/10 meters;
  - **TW** = **Walking time** in minutes to the nearest subway station;
  - **TT** = **Subway running time** in minutes to the Tokyo station from the nearest station during the day (not early morning or night);
  - **X** = **Longitude** of the property; [Or we can use **Ward** or
  - **Y** = **Latitude** of the property; **Postal Code Dummy Variables**]
  - **PS** = **Construction cost** for a new structure in 100,000 Yen per meter squared.

Table 1: Descriptive Statistics for the Variables.

Name	No. of Obs.	Mean	Std. Dev	Minimum	Maximum
V	14073	6.2491	2.9016	1.8	20
S	14073	0.43464	0.5828	0	2.4789
L	14073	1.0388	0.3986	0.5	2.4977
A	14073	5.8231	9.117	0	49.723
NB	14073	1.5669	2.0412	0	8
W	14073	46.828	12.541	25	90
TW	14073	9.3829	4.3155	1	29
TT	14073	31.244	7.3882	8	48
X	14073	139.67	0.0634	139.56	139.92
Y	14073	35.678	0.0559	35.543	35.816
P_S	14073	1.7733	0.0294	1.73	1.85

6. The Basic Builder's Model using Spatial Coordinates to Model Land Prices

- The ***builder's model*** for valuing a residential property postulates that the value of a residential property is the sum of two components: the value of the land which the structure sits on plus the value of the residential structure.
- This leads to the following ***hedonic regression model*** for period t where the α_t and β_t are the parameters to be estimated in the regression:

(19) $V_{tn} = \alpha_t L_{tn} + \beta_t S_{tn} + \varepsilon_{tn}$; $t = 1,...,44$; $n = 1,...,N(t)$.

- The hedonic regression model defined by (19) applies to new structures. But it is likely that a model that is similar to (19) applies to older structures as well. Older structures will be worth less than newer structures due to the depreciation of the structure. Assuming that we have information on the age of the structure n at time t, say $A_{tn} = A(t,n)$ and assuming a geometric depreciation model, a more realistic hedonic regression model than that defined by (19) above is the following ***basic builder's model***:

$$\textbf{(20)}\quad V_{tn} = \alpha_t L_{tn} + \beta_t(1-\delta)^{A(t,n)} S_{tn} + \varepsilon_{tn}\ ; \qquad t = 1,...,44;\ n = 1,...,N(t)$$

where the parameter δ reflects the ***net depreciation rate*** as the structure ages one additional period.

- Thus equations (20) above could be combined into one big regression and a single depreciation rate δ could be estimated along with 44 land prices α_t and 44 new structure prices β_t so that 89 parameters would have to be estimated. However, experience has shown that it is usually not possible to estimate sensible land and structure prices in a hedonic regression like that defined by (20) due to the multicollinearity between lot size and structure size.
- Thus in order to deal with the multicollinearity problem, we draw on ***exogenous information*** **on new house building costs** from the Japanese Ministry of Land, Infrastructure, Transport and Tourism (MLIT).

- **(21)** $V_{tn} = \alpha_t L_{tn} + P_{St}(1 - \delta)^{A(t,n)} S_{tn} + \varepsilon_{tn}$ **;**

  $t = 1,...,44;\ n = 1,...,N(t).$

- Thus we have **14,073 degrees of freedom** to estimate 44 land price parameters α_t and one annual geometric depreciation rate parameter δ, a total of 45 parameters.
- We estimated the nonlinear regression model defined by (21) for our Tokyo data set using the econometric programming package Shazam; see White (2004). The R^2 for the resulting preliminary nonlinear regression ***Model 0*** was only 0.5545, which is not very satisfactory. However, there are no location variables in Model 0.
- Thus let x_{tn} and y_{tn} equal the **normalized longitude** and **latitude** of property n sold in period t. We will initially approximate the true land price surface f(x,y) by the **4 by 4 Colwell spatial grid function** $g_4(x,y)$ defined above in section 3.

Model 1.

$$(22)\ V_{tn} = \alpha_t\, g_4(x_{tn},y_{tn},\gamma)L_{tn} + P_{St}(1-\delta)^{A(t,n)}S_{tn} + \varepsilon_{tn}\ ;$$
$$t = 1,...,44;\ n = 1,...,N(t).$$

- Note that the γ vector of parameters in $g_4(x_{tn},y_{tn},\gamma)$ consists of the 25 spatial grid parameters γ_{ij} where i, j = 0,1,2,3,4.
- Thus equations (22) contain 44 unknown period t land price parameters α_t, 25 unknown γ_{ij} spatial grid parameters and 1 depreciation rate parameter δ for a total of 70 unknown parameters.

- Our problem now is how exactly should these two value terms be decomposed into ***constant quality price and quantity components*****?**
- Our view is that a suitable constant quality land price index for all houses sold in period t should be α_t and for property n sold in period t, the corresponding constant quality quantity should be $g_4(x_{tn},y_{tn},\gamma)L_{tn}$. Turning to the decomposition of the structure value of property n sold in period t, $P_{St}(1-\delta)^{A(t,n)}S_{tn}$, into price and quantity components, we take P_{St} as the price and $(1-\delta)^{A(t,n)}S_{tn}$ as the corresponding quantity for property n sold in quarter t.
- An alternative way of viewing our land model is that land in each location indexed by the spatial coordinates x_n,y_n can be regarded as a distinct commodity with its own price and quantity. But since our model forces **all land prices in the same location to move proportionally over time, virtually all index number formulae will generate an overall land price series that is proportional to the α_t.**

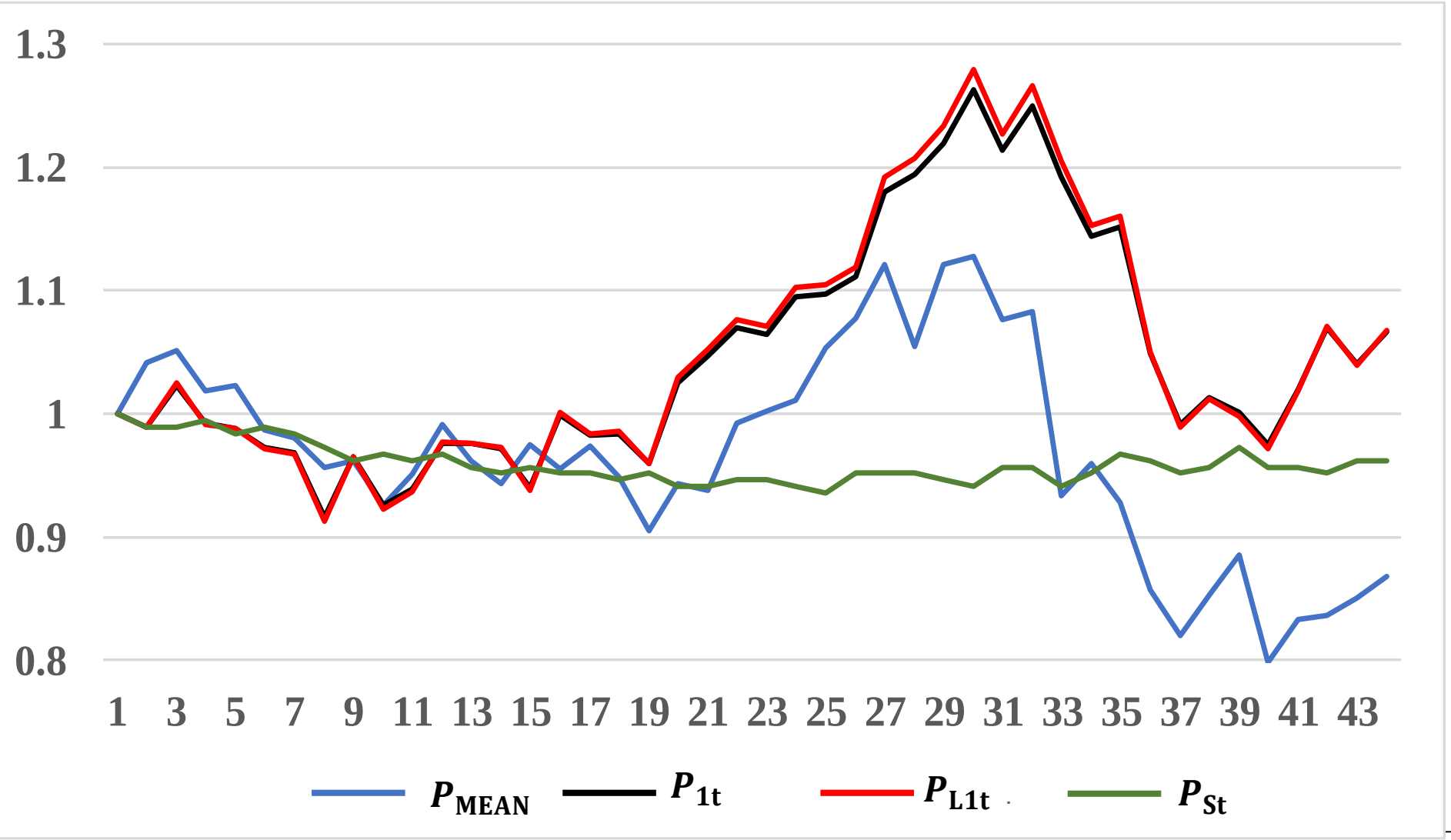
- Note that the above value decompositions of individual property prices sets the price of a square meter of land in quarter t equal to α_t^*, the estimated parameter value for α_t and sets the price of a square meter of structure equal to P_{St}, the official per meter structure cost for quarter t.
- These prices are assumed to be the same across all properties sold in period t and thus **we can set the *aggregate* land and structure price for all residential properties sold in period t equal to P_{Lt} and P_{St} where $P_{Lt} \equiv \alpha_t^*$** for t = 1,...,44. The corresponding ***aggregate* constant quality quantities of land and structures** sold in period t are defined as follows:

$$\textbf{(23)}\quad Q_{Lt} \equiv \sum_{n=1}^{N(t)} g_4(x_{tn},y_{tn},\gamma^*)L_{tn} \; ;$$

$$Q_{St} \equiv \sum_{n=1}^{N(t)} (1-\delta^*)^{A(t,n)}S_{tn} \; ; \qquad t = 1,...,44.$$

- The prices α_t^* and P_{St} and quantities Q_{Lt} and Q_{St} are used to form **chained Fisher overall property price indexes**.

Chart 1 Mean Property Price Index and Model 1 Overall and Land Price Indexes and the Official Structure Price Index



Model 2.

- For ***Model 2***, which used $\mathbf{g_5(x_{tn},y_{tn},\gamma)}$ in (22) in place of $\mathbf{g_4(x_{tn},y_{tn},\gamma)}$, the following cells in the **5 by 5** grid of cells had no sales over our sample period: C_{11}, C_{41}, C_{51} and C_{42}. This means that 3 height parameters could not be estimated so we imposed the following restrictions on the parameters of Model 2: $\gamma_{00} = \gamma_{40} = \gamma_{50} = 0$. We also set $\alpha_1 = 1$ so that the remaining land price parameters α_t could be identified. Thus Model 2 had $36 - 3 = 33$ γ_{ij} parameters, 43 land price parameters α_t and 1 depreciation rate parameter δ for a total of **77 parameters**.
- As the grid of squares becomes finer, some of the squares are over Tokyo Bay and so there are no sales for those squares. If these squares are not adjacent to a square which has sales, then the γ_{ij} parameters at the corners cannot be identified.

Model 3.

- For ***Model 3***, which used $\mathbf{g_6(x_{tn},y_{tn},\gamma)}$ in (22) in place of $g_4(x_{tn},y_{tn},\gamma)$, the following 5 cells in the **6 by 6** grid of cells had no sales over our sample period: C_{11}, C_{51}, C_{61}, C_{52} and C_{62}.
- Thus we set the following 5 height parameters equal to 0 in order to identify the remaining height parameters: $\gamma_{00} = \gamma_{50} = \gamma_{60} = \gamma_{51} = \gamma_{61} = 0$.
- We also set $\alpha_1 = 1$ so that the remaining land price parameters α_t could be identified.
- Thus Model 3 had 49 – 5 = **44** γ_{ij} parameters, **43** land price parameters α_t and **1** depreciation rate parameter δ for a total of **88 parameters**.

Model 4.

- ***Model 4*** used $\mathbf{g_7(x_{tn},y_{tn},\gamma)}$ in (22) in place of $\mathbf{g_4(x_{tn},y_{tn},\gamma)}$**.** The following 9 cells in the **7 by 7** grid of cells had no sales over our sample period: C_{11}, C_{21}, C_{51}, C_{61}, C_{71}, C_{52}, C_{62}, C_{72} and C_{17}.
- Thus we set the following 9 height parameters equal to 0 in order to identify the remaining height parameters: $\gamma_{00} = \gamma_{10} = \gamma_{50} = \gamma_{60} = \gamma_{70} = \gamma_{51} = \gamma_{61} = \gamma_{71} = \gamma_{07} = 0$. We also set $\alpha_1 = 1$ so that the remaining land price parameters α_t could be identified.
- Thus Model 4 had 64 – 9 = **55** $\boldsymbol{\gamma_{ij}}$ parameters, **43** land price parameters $\boldsymbol{\alpha_t}$ and **1** depreciation rate parameter $\boldsymbol{\delta}$ for a total of **99 parameters**.

Model 5.

- Finally, ***Model 5*** used $\mathbf{g_8(x_{tn},y_{tn},\gamma)}$ in (22) in place of $g_4(x_{tn},y_{tn},\gamma)$. The following 14 cells in the **8 by 8 grid** of cells had no sales over our sample period: C_{11}, C_{12}, C_{21}, C_{18}, C_{61}, C_{62}, C_{63}, C_{71}, C_{72}, C_{73}, C_{81}, C_{82}, C_{83} and C_{88}.
- **All 4 corner cells were empty** along with many other boundary cells. Thus we set the following 14 height parameters equal to 0 in order to identify the remaining height parameters: $\gamma_{00} = \gamma_{10} = \gamma_{01} = \gamma_{60} = \gamma_{61} = \gamma_{62} = \gamma_{70} = \gamma_{71} = \gamma_{72} = \gamma_{80} = \gamma_{81} = \gamma_{82} = \gamma_{88} = 0$. We also set $\alpha_1 = 1$ so that the remaining land price parameters α_t could be identified.
- Thus Model 5 had 91 – 14 = **77** $\mathbf{\gamma_{ij}}$ parameters, **43** land price parameters $\mathbf{\alpha_t}$ and **1** depreciation rate parameter $\mathbf{\delta}$ for a total of **111 parameters**. **We stopped adding cells at this point**.
- Note: **Model 5 did not fit as well as Model 4**!

The Ward Dummy Model.

- An alternative to using spatial coordinates to measure the influence of location on property prices is to **use postal codes** or **neighbourhoods** as indicators of location.
- There are 23 Wards in Tokyo and each property in our sample belongs to one of these Wards. In order to take into account possible neighbourhood effects on the price of land, **we introduced *ward dummy variables*, $D_{W,tn,j}$, into the hedonic regression (20).**
- These 23 dummy variables are defined as follows: for t = 1,...,44; n = 1,...,N(t); j = 1,...,23:

(24) $D_{W,tn,j} \equiv 1$ if observation n in period t is in Ward j of Tokyo;
$\equiv 0$ if observation n in period t is *not* in Ward j of Tokyo.

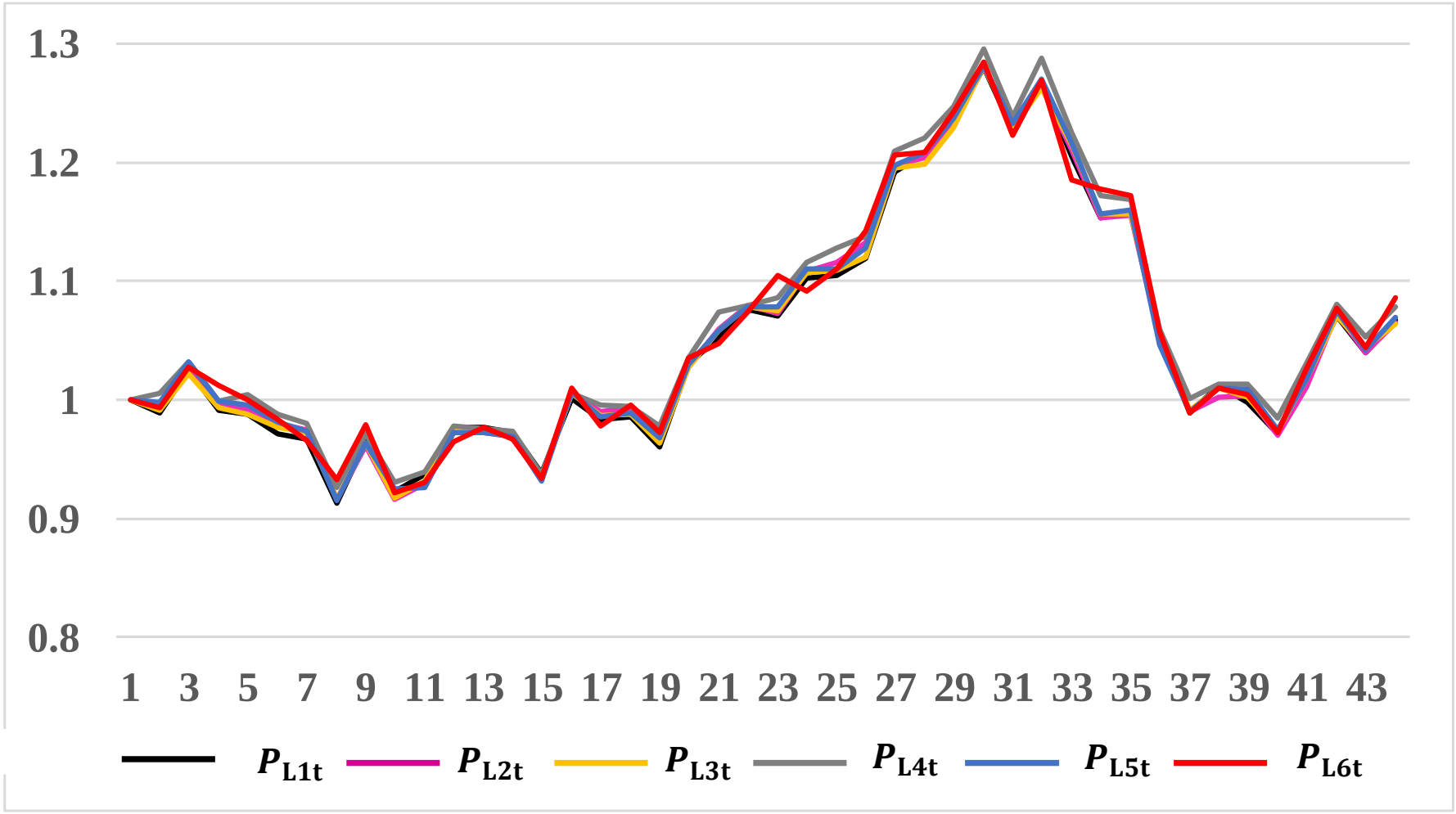
Model 6.

- The new ***Model 6*** is defined by the following **nonlinear regression** model:

$$(25)\ V_{tn} = \alpha_t\left(\sum_{j=1}^{23} \omega_j D_{W,tn,j}\right)L_{tn} + P_{St}(1-\delta)^{A(t,n)}S_{tn} + \varepsilon_{tn}\ ;$$
$$t = 1,...,44;\ n = 1,...,N(t).$$

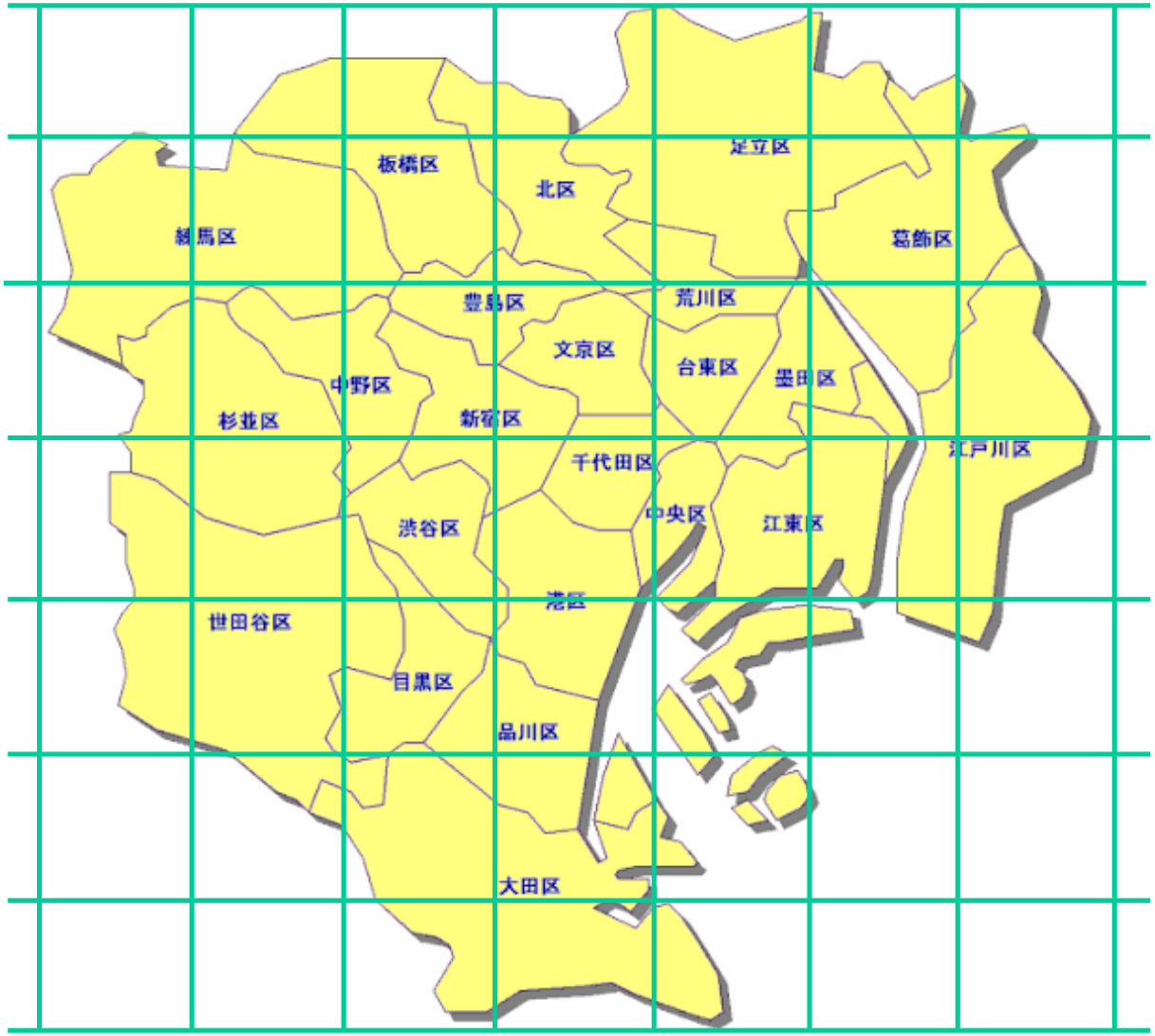
- Comparing the models defined by equations (20) and (25), it can be seen that we have added an additional ***23 ward relative land value parameters,*** $\omega_1,...,\omega_{23}$, to the model defined by (20).
- However, looking at (25), it can be seen that the 44 land price time parameters (the α_t) and the 23 ward parameters (the ω_j) cannot all be identified. Thus we set α_1 equal to 1.
- We compare the land price series from the Ward Model 6 with the spatial Models 1-5 and find practically no difference.

Chart 2 Land Prices for Models 1-6



Comparison in 6 Models.

- The 6 models make use of information on **land plot size**, **structure floor space**, the **age** of the structure (if the property has a structure) and its **location**, either in terms of spatial coordinates or terms of its neighbourhood.
- **These are the most important residential property price determining characteristics in our view**. In the following section, we make use of additional information on housing characteristics and see if this extra information materially changes our estimated land price indexes.
- We will use the spatial coordinate Model 4 as our starting point in the models which follow, since it was *the* ***best fitting model*** studied in this section. This model used the Colwell nonparametric model for modeling the land price surface with the 7×7 = 49 cell grid.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台東区
墨田区
中野区
杉並区
新宿区
千代田区
江戸川区
中央区
渋谷区
江東区
港区
世田谷区
目黒区
品川区
大田区

7. Spatial Coordinate Models that Use Additional Information

- It is likely that property sales that have an older structure on the property will have a different land valuation than a nearby property of the same size that consists of cleared land, since demolition costs are not trivial.
- **Our *Model 7* takes this possibility into account**.
- Define the **land only dummy variable** $D_{L,tn}$ as follows for t = 1,...,44 and n = 1,...,N(t):

(26) $D_{L,tn} \equiv 1$ if observation n in period t is a land only sale;

$\equiv 0$ otherwise.

Model 7.

- Define $D_{S,tn} \equiv 1 - D_{L,tn}$ for t = 1,...,44; n = 1,...,N(t). Thus if property n sold in period t has a structure on it, $D_{S,tn}$ will equal 1. Model 7 estimates the following nonlinear regression:

$$\textbf{(27)}\ V_{tn} = \alpha_t (D_{S,tn} + \phi D_{L,tn}) g_7(x_{tn}, y_{tn}, \gamma) L_{tn} + P_{St}(1 - \delta)^{A(t,n)} S_{tn} + \varepsilon_{tn}\ ; \qquad t = 1,...,44;\ n = 1,...,N(t).$$

- Thus **the parameter ϕ gives the added premium to the property's land price (per meter squared) if the property has no structure on it.** The estimated ϕ was $\phi^* = 1.110$ (t =153)
- We imposed the same restrictions on the γ_{ij} that were imposed in Model 4.
- The R^2 for Model 7 was **0.8175** (the Model 4 R^2 was 0.8156).
- The final log likelihood for Model 7 was **128.75** points higher than the final log likelihood for Model 4 for adding one ϕ.

The size of the land plot:

- We group the observations into 4 groups, depending on the size of the land plot. The **cutoff sizes of land plot** are $\mathbf{L_0, L_1, L_2}$ **and** $\mathbf{L_3}$.
- For each observation n in period t, we define the four ***land dummy variables***, $D_{L,tn,k}$, for k = 1,2,3,4 as follows:

(28) $\mathbf{D_{L,tn,k} \equiv 1}$ **if observation tn has land area that belongs to group k;**

$\mathbf{\equiv 0}$ **if observation tn has land area that does not belong to group k.**

- These dummy variables are used in the definition of the following **piecewise linear function** of L_{tn}, $\mathbf{f_L(L_{tn})}$, defined as follows:

$$\mathbf{(29)\ f_L(L_{tn},\lambda) \equiv D_{L,tn,1}[\lambda_0 L_0 + \lambda_1(L_{tn} - L_0)] + D_{L,tn,2}[\lambda_0 L_1 + \lambda_1(L_1 - L_0) + \lambda_2(L_{tn} - L_1)] + D_{L,tn,3}[\lambda_0 L_0 + \lambda_1(L_1 - L_0) + \lambda_2(L_2 - L_1) + \lambda_3(L_{tn} - L_2)] + D_{L,tn,4}[\lambda_0 L_0 + \lambda_1(L_1 - L_0) + \lambda_2(L_2 - L_1) + \lambda_3(L_3 - L_2) + \lambda_4(L_{tn} - L_3)]}$$

- where $\boldsymbol{\lambda \equiv [\lambda_0,\lambda_1,\lambda_2,\lambda_3,\lambda_4]}$ and the λ_k are 5 unknown parameters and $\mathbf{L_0 \equiv 0.5, L_1 \equiv 1, L_2 \equiv 1.5}$ and $\mathbf{L_3 \equiv 2}$.

Model 8: Splines for the Land Plot Area

- Thus we are allowing the per meter squared price of land to vary as the size of the land plot increases. We expect the marginal price of land to decrease as lot size becomes very large. ***Model 8*** is the following nonlinear regression:

$$\textbf{(30)}\ V_{tn} = \alpha_t (D_{S,tn} + \phi D_{L,tn}) g_7(x_{tn}, y_{tn}, \gamma) f_L(L_{tn}, \lambda) + P_{St}(1-\delta)^{A(t,n)} S_{tn} + \varepsilon_{tn}\ ;\quad t = 1,\ldots,44;\ n = 1,\ldots,N(t).$$

- where the function f_L is defined above by (29) and ε_{tn} is an error term. There are 43 unknown land price parameters α_t, (we set $\alpha_1 = 1$), **1** land only premium parameter ϕ, **55** land price height parameters γ_{ij}, **4** marginal price of land parameters λ_k (we set $\lambda_1 = 1$) and **1** depreciation rate δ to estimate or 104 unknown parameters in all.
- The R^2 for Model 8 was **0.8222**, increase in LL was **328.27**.

Model 9: Splines for the Structure Size.

- In our next model, we allow the per square meter price **of a square meter of structure to vary as the floor space of the structure increases.** The rational for this model is that **bigger houses** are likely to be of **higher quality**.
- For each observation n in period t, we define the **3** structure dummy variables, $D_{S,tn,m}$, for m = 1,2,3 as follows:

(31) $D_{S,tn,m}$ **≡ 1 if observation tn has structure area that belongs to group m;**
≡ 0 if observation tn has structure area that does not belong to group m.

- These dummy variables are used in the definition of the following piecewise linear function of S_{tn}, $f_S(S_{tn})$, defined as follows:

$$\textbf{(32)}\ f_S(S_{tn},\mu) \equiv D_{S,tn,1}[\mu_0 S_0 + \mu_1(S_{tn} - S_0)] + D_{S,tn,2}[\mu_0 S_1 + \mu_1(S_1 - S_0) + \mu_2(S_{tn} - S_1)] + D_{S,tn,3}[\mu_0 S_0 + \mu_1(S_1 - S_0) + \mu_2(S_2 - S_1) + \mu_3(S_{tn} - S_2)].$$

Model 9. Piecewise Linear Splines for Structure Size

- The exogenous break points are $S_0 \equiv 0.5$, $S_1 \equiv 1$ and $S_2 \equiv 1.5$.
- ***Model 9*** is the following nonlinear regression:

$$(33)\ V_{tn} = \alpha_t (\mathbf{D_{S,tn}} + \phi D_{L,tn}) g_7(x_{tn},y_{tn},\gamma) f_L(L_{tn},\lambda) + P_{St}(1-\delta)^{A(t,n)} \mathbf{f_S(S_{tn},\mu)} + \varepsilon_{tn}; \quad t = 1,...,44;\ n = 1,...,N(t);$$

- where $\mu \equiv [\mu_0,\mu_1,\mu_2,\mu_3]$ and we set $\mu_1 = 1$.
- The function $\mathbf{f_L}$ is defined above by (29), the function $\mathbf{f_S}$ is defined by (32) and ε_{tn} is an error term. There are **43** unknown land price parameters α_t, **1** land only premium parameter ϕ, **55** land price height parameters γ_{ij}, **4** marginal price of land parameters λ_k, **3** marginal price of structure parameters μ_m and **1** depreciation rate δ to estimate or **107** unknown parameters to estimate.
- The R^2 for Model 9 was **0.8256**, increase in LL was **136.32**.

Adding the Subway Time Variables: TW and TT.

- Our next model, we make use of the two subway variables: **TW**, the walking time in minutes to the nearest subway station, and **TT**, the subway running time in minutes to the Tokyo central station.
- The sample minimum time for TW was 1 minute and the minimum time for TT was 8 minutes.
- Our next model allows the price of land to decrease as these two subway time variables increase.
- These variables have proven to be highly significant in other studies of Tokyo property prices.

Model 10: Adding the Subway Time Variables

- Thus ***Model 10*** is the following nonlinear regression:

$$(34)\ V_{tn} = \alpha_t [D_{S,tn} + \phi D_{L,tn}] g_7(x_{tn},y_{tn},\gamma) f_L(L_{tn},\lambda)$$
$$x[1+\tau(TW_{tn}-1)][1+\rho(TT_{tn}-8)] + P_{St}(1-\delta)^{A(t,n)} f_S(S_{tn},\mu) + \varepsilon_{tn};$$
$$t = 1,...,44;\ n = 1,...,N(t)$$

- where the function f_L is defined above by (29), the function f_S is defined by (32), τ is the percentage change in the price of land due to a one minute increase in walking time, ρ is the percentage change in the price of land due to a one minute increase in subway running time to Tokyo central station and ε_{tn} is an error term.
- There are **109** unknown parameters in Model 10.
- The R^2 for Model 10 was **0.8383**, increase in LL was **531.13**.

Adding the Number of Bedrooms

- In our next model, we introduce the **number of bedrooms NB_{tn}** as a property characteristic that can affect structure value if the property n in quarter t has a structure on it.
- For the properties in our sample, the number of bedrooms ranged from 2 to 8. Since there were relatively few observations with 6, 7 or 8 bedrooms, we grouped these last 3 categories into a single category.
- Define the bedroom dummy variables $D_{NB,tn,i}$ for observation tn as follows for i = 2,3,4,5; t = 1,...,44 and n = 1,...,N(t):

(35) $D_{NB,tn,i}$ ≡ 1 if observation tn has a structure on it with i bedrooms;
≡ 0 elsewhere.

Model 11: Adding the Number of Bedrooms

- ***Model 11*** is the following nonlinear regression:

$$(36)\ V_{tn} = \alpha_t [D_{S,tn} + \phi D_{L,tn}] g_7(x_{tn},y_{tn},\gamma) f_L(L_{tn},\lambda)$$
$$x[1+\tau(TW_{tn}-1)][1+\rho(TT_{tn}-8)]$$
$$+ P_{St}(1-\delta)^{A(t,n)} f_S(S_{tn},\mu)[\Sigma_{i=2}^{6} \kappa_i D_{NB,tn,i}] + \varepsilon_{tn};$$
$$t = 1,...,44;\ n = 1,...,N(t)$$

- where the all of the functions and parameters which appear in (36) were defined in the previous model except that we have now added 5 bedroom variables, κ_2, κ_3, κ_4, κ_5 and κ_6.
- We make the same normalizations as we made in Model 10 and in addition, we set $\kappa_2 = 1$.
- Model 11 has a total **113** unknown parameters.
- The R^2 for Model 11 was **0.8400**, increase in LL was **75.03**.

Adding the Width of the Land Plot.

- The final additional variable that we introduced into our property nonlinear regression model was **the width of the land plot**, W_{tn} for property sale n in period t.
- Recall that W_{tn} is measured in 10ths of a meter and **the range of this property width variable was 25 to 90**.
- Other residential property hedonic regression models for Tokyo have shown that this variable is a very significant one: the greater is the lot width, the more valuable is the land plot.
- We assume that the width variable affects the land value component of property value and does not affect the structure value.
- We modeled the width variable as a single continuous variable rather than using splines or step functions on W_{tn}.

Model 12: Adding the Property Width Variable

- ***Model 12*** is the following nonlinear regression:

$$(37)\ V_{tn} = \alpha_t [D_{S,tn}+\phi D_{L,tn}]g_7(x_{tn},y_{tn},\gamma)$$
$$xf_L(L_{tn},\lambda)[1+\tau(TW_{tn}-1)]\ [1+\rho(TT_{tn}-8)]\ [1+\sigma(W_{tn}-25)]$$
$$+ P_{St}(1-\delta)^{A(t,n)} f_S(S_{tn},\mu)[\Sigma_{i=2}^{6} \kappa_i D_{NB,tn,i}]\ + \varepsilon_{tn}\ ;$$
$$t = 1,...,44;\ n = 1,...,N(t)$$

- where the all of the functions and parameters which appear in (37) were defined in the previous model except σ.
- Thus we have added 1 additional unknown parameter to Model 11 so Model 12 has a total **114** unknown parameters.
- σ^* **was 0.00402** (t = 27.4) so an extra meter of lot width adds about 4% to the per meter squared price of the land plot.
- The R^2 for Model 11 was **0.8488**, increase in LL was **401.54**.

The Problem of Negative Predicted Land Prices

- Although the fact that Model 12 generated 4 negative estimated γ_{ij}^* did not lead to any negative predicted prices for land for the properties in our sample, these negative estimates could lead to negative land prices for properties not in our sample.
- Hence, it may be useful to perform a final regression where we restrict the γ_{ij} to be nonnegative. **This can be done by replacing γ_{01}, γ_{67}, γ_{77} and γ_{52} in the function $g_7(x_{tn},y_{tn},\gamma)$ by the squares of these parameters and then rerunning the model defined by (37).**
- ***Model 13*** is the resulting model.
- The reduction in LL for Model 13 over Model 12 was **1.19**.
- The R^2 for Model 13 was **0.8488**, the same as for Model 12.

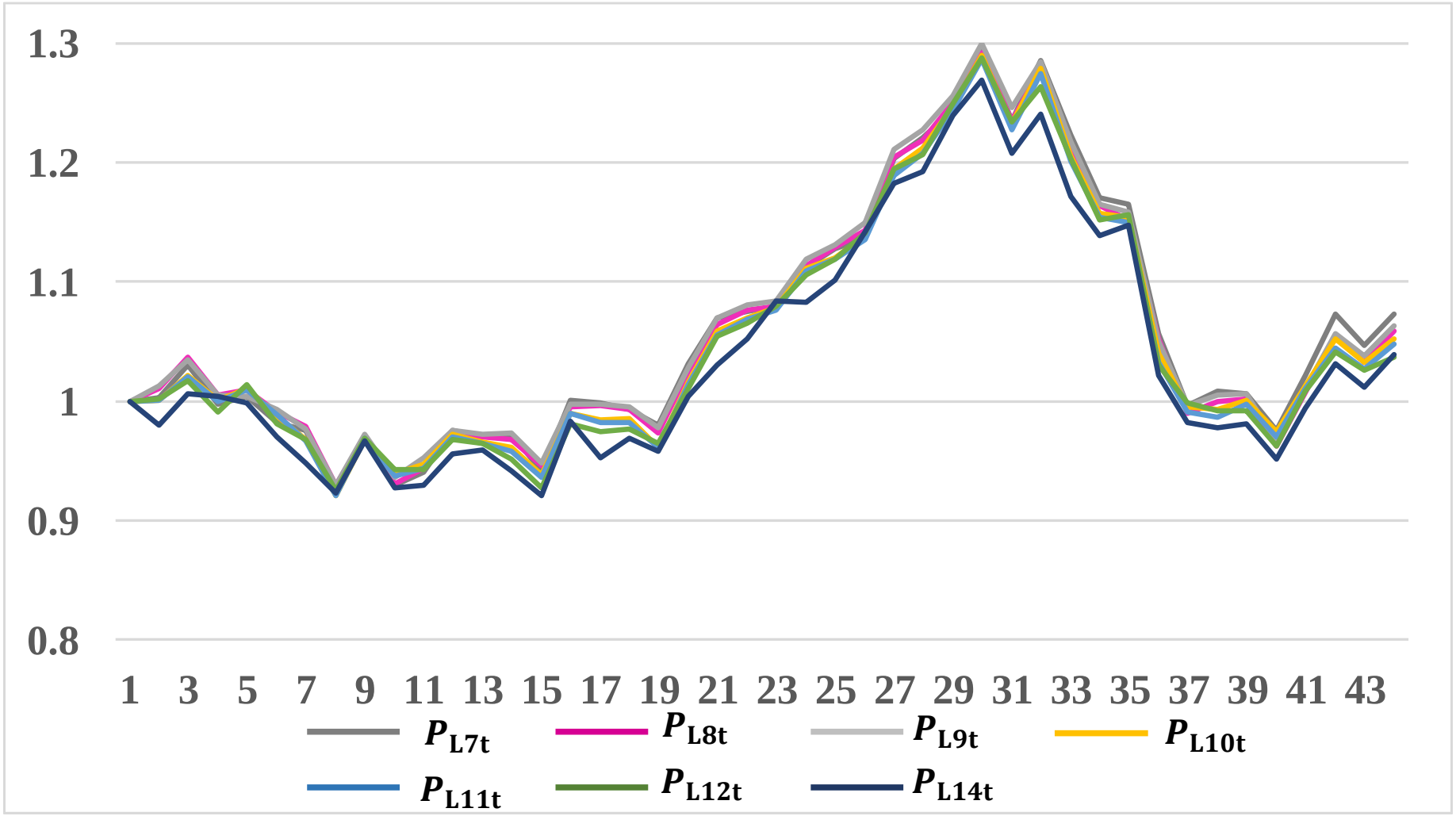
Model 14: The Ward Dummy Variable Model Revisited

- Our final model in this section is a **Ward dummy variable model** that **adds more explanatory property characteristics** to the **Ward Dummy Model 6** defined by equations (25).
- ***Model 14*** is defined by the following nonlinear regression model:

$$(38)\ V_{tn} = \alpha_t [D_{S,tn} + \phi D_{L,tn}][\sum_{j=1}^{23} \omega_j D_{W,tn,j}]$$
$$x f_L(L_{tn},\lambda)[1+\tau(TW_{tn}-1)][1+\rho(TT_{tn}-8)][1+\sigma(W_{tn}-25)]$$
$$+ P_{St}(1-\delta)^{A(t,n)} f_S(S_{tn},\mu)[\sum_{i=2}^{6} \kappa_i D_{NB,tn,i}] + \varepsilon_{tn} ;$$

- Thus Model 14 is **basically the same** as Model 12 and 13 except that the Ward dummy variable terms, $\sum_{j=1}^{23} \omega_j D_{W,tn,j}$, replace the Colwell locational grid function, $g_7(x_{tn},y_{tn},\gamma)$.
- The R^2 for Model 14 was **0.8300**, increase in LL over Model 6 was 478.6 .
- ~~We compare land prices for Models 7-14 in the next slide.~~

Chart 3 Land Price Indexes for Models 7-12 and 14



8. Overall Residential Property Price Indexes

- There is one additional overall property price index that we calculate in this section and that is an index that is based on a **"traditional" hedonic property price regression** that uses the **logarithm of the selling price** as the dependent variable and has **time dummy variables**.
- Define the **kth time dummy variable** $D_{T,tn,k}$ for property n sold in period t as follows:

(39) $D_{T,tn,k} \equiv 1$ **if** $t = k$; $D_{T,tn,k} \equiv 0$ **if** $t \neq k$.

- Our **best time dummy variable hedonic regression model** is the following ***Model 15***:

$$\ln V_{tn} = \sum_{k=2}^{44} \alpha_k D_{T,tn,k} + \sum_{j=1}^{23} \omega_j D_{W,tn,j} + \lambda \ln L_{tn} + \mu S_{tn} + \delta A_{tn} + \tau TW_{tn} + \rho TT_{tn} + \sigma W_{tn} + \sum_{i=3}^{6} \kappa_i D_{NB,tn,i} + \varepsilon_{tn}; \quad t = 1,...,44; \; n = 1,...,N(t). \tag{40}$$

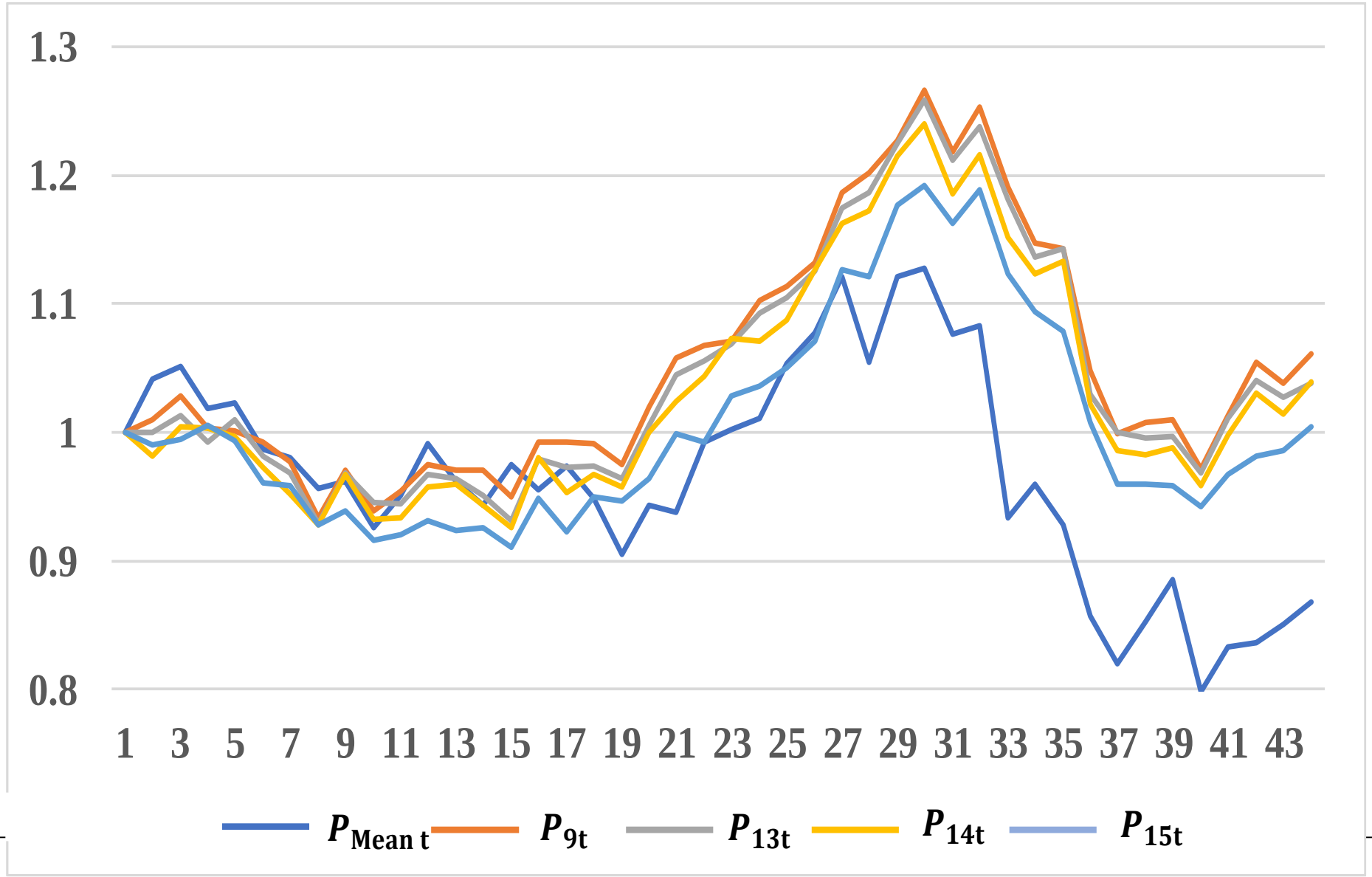
Model 15: The Traditional Time Dummy Model

- **$\ln V_{tn}$** and **$\ln L_{tn}$** denote the natural logarithms of property value **V_{tn}** and property lot size **L_{tn}** respectively, the **$D_{T,tn,k}$** are **time dummy variables**, the **$D_{W,tn,j}$** are **Ward dummy variables**, **S_{tn}** is the floor space area of the property.
- We ran an initial linear regression using **L_{tn}** as an independent variable in place of ***lnL_{tn}***.
- However, this regression had a log likelihood which was 204.99 points lower than our final linear regression defined by (40). The **R^2** for this preliminary regression was **0.8274**.
- Note that we could not use **$\ln S_{tn}$** as an independent variable because many observations had no structure on them and hence **S_{tn} is equal to 0** for these properties and thus we could not take the logarithm of 0.

Model 15: The Traditional Time Dummy Model

- The log likelihood of this model **cannot be compared with other models** because the dependent variable is now the **logarithm of the property price** instead of the **property price**.
- There are **75** unknown parameters in the model defined by equations (40).
- The R^2 for Model 15 was **0.8323**. (Not bad!).
- We set $\alpha_1^* = 0$. The sequence of **overall property price indexes** $\mathbf{P_{15t}}$ generated by this model are the exponentials of the estimated α_t^*; i.e., define $\equiv \exp[\alpha_t^*]$ for t = 1,...,44.
- The next slide compares the **mean property price index** $\mathbf{P_{Mean\ t}}$, $\mathbf{P_{9t}}$ (based on Model 9, a minimal Colwell model), $\mathbf{P_{13t}}$ (our best Colwell spatial coordinates model), $\mathbf{P_{14t}}$ (our best Ward dummy variable model) and $\mathbf{P_{15t}}$ (our best log price time dummy hedonic model).

Chart 4 Land Price Indexes for Models 7-12 and 14



- The mean index, $P_{Mean\ t}$, has a **large downward bias** as compared to the other 4 indexes which is due to its **neglect of age effects**. However, the movements in this index are similar to the movements in the other indexes.
- The property price index P_{15t} generated by a traditional log price time dummy hedonic regression model has a **downward bias** (**due to its imperfect specification of age effects)** but it is not large.
- The Model 9 property price index, a **Colwell spatial coordinates model that used only the 4 fundamental characteristics of a residential property** (land plot area, structure floor space area, the age of the structure and some locational variable) generated an overall property price index P_{9t} that is **quite close to our best Colwell spatial model**, Model 14, which generated the overall property price index P_{14t}.

- Thus it is probably not necessary for national statistical agencies to collect a great deal of information on housing characteristics in order to produce a decent overall property price index (as well as decent land and structure subindexes).
- The Model 14 property price index, P_{14t}, that used **local neighbourhood information about properties** instead of spatial coordinate information turned out to be fairly close to our best Colwell spatial index, P_{13t}. Thus following the advice of Hill and Scholz (2018), **it is probably not necessary to utilize spatial coordinate information in order to construct a satisfactory overall residential property price index**.
- Diewert (2010) also observed a similar result.
- In addition to these **four fundamental variables**, we need an **exogenous building cost measure** in order to implement our basic models.

9. Conclusion

- Satisfactory residential land price indexes and overall residential property price indexes can be constructed using local neighbourhood dummy variables as explanatory variables in residential property regression models. **It is not necessary to use spatial coordinates to model location effects on property prices**.
- However, the use of **spatial coordinates to model location effects does lead to better fitting regression models**.
- The most important housing characteristics information that is needed in order to construct satisfactory residential land and overall property price indexes is information on **lot size, floor space area** of the property structure (if there is a structure on the property), the **age of the structure** and some information on the **location** of the property. In order to obtain a satisfactory land price index, our method requires the use of **exogenous information** on residential **construction costs**.

- However, additional information on the characteristics of the property will improve the fit of our hedonic regressions but **the effects of the additional information on the resulting land and structure price indexes was minimal** for our application to Tokyo residential property price indexes.
- Having land only sales of residential properties should help improve the accuracy of the land price index that is generated by a property regression model. However, for our Japanese data, **we found that the value of the land component of a land only property earned a 10-15% premium** over the land value of a neighbouring property of the same size but with a structure on the property. We attribute this premium to the **costs of demolishing an older structure**.

- Our models that used spatial coordinates to account for locational effects on the value of land used **Colwell's nonparametric method** for fitting a surface. This nonparametric method is much easier to implement than the penalized least squares approach used by Hill and Scholz (2018) to model locational effects on property prices. In section 4 of the paper, we pointed out some of the **theoretical advantages of Colwell's method**.
- The potential bias in using property price indexes that are based on taking **mean** or **median averages** of property prices in a period can be very large. Typically, these methods will have a downward bias due to their **neglect of structure depreciation**.

- A traditional log price time dummy hedonic regression model that has structure age as an explanatory variable will typically reduce the bias that is inherent in an index based on taking averages of property prices. For our Tokyo data, we found that the traditional hedonic regression model led to an index which had a small downward bias; see Chart 4 in the previous section.
- Our emphasis in this paper has been to **develop reliable methods for the construction of the land component of residential property price indexes**. This task is important for national statistical agencies because the Balance Sheet Accounts in the System of National Accounts requires estimates for the price and volume of land used in production and consumption. In particular, this information is required in order to obtain more accurate estimates of national (and sectoral) Total Factor Productivity growth but for the vast majority of countries, this information is simply not available.