New Evidence on the Determinants of Corporate Hedging

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Abstract

We assemble the most comprehensive data set of oil and gas producers' hedging positions and conduct a systematic study of the determinants of corporate hedging policies. We find that various proxies of tax convexity, managerial incentives, financial constraints, and investment opportunities cannot predict hedging policies. The two most important predictors of the hedging intensity are operating profits and hedging gains, which have negative and positive correlations with the hedging intensity, respectively. Existing theories have difficulty in reconciling these two findings. We extend standard risk management models by incorporating the production-dependent depreciation as a key element in reconciling the theoretical predictions with the empirical patterns. Our model yields additional novel predictions that we test and confirm in the data.

Keywords: Corporate hedging; Risk management; Financial constraints; Profitability; Oil & gas exploration and production; Collateral constraint; Production-dependent depreciation.
JEL Classifications: G23, G30. G32

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1. Introduction

Corporate hedging is an important corporate risk management decisions. For example, independent oil and gas production firms hedge more than a third of their production, with substantial variations across time and firms. Theoretical studies have proposed a variety of determinants of corporate hedging policies, but empirical studies on hedginghave been hamstrung by limited data availability. We overcome this challenge by assembling the most comprehensive data set of oil and gas producers' hedging positions known to the literature. The data consist of hedging, production, reserve, and other financial information of major independent oil and natural gas producers from 2002 to 2016.[1]

These panel data benefit our empirical investigation along several dimensions. First, a long time series enables us to exploit the within-firm time-series variations, which minimizes the contamination due to unobserved time-invariant firm-specific heterogeneity. Previous papers, including Haushalter (2000), Jin and Jorion (2006), Bakke et al. (2016), and Gilje and Taillard (2017), have relatively short time-series.[2] As a result, these studies are cross-sectional in nature and more susceptible to unobserved firm-specific characteristics. Second, our sample covers different market states of the oil and gas markets. It contains both sustained bull and bear periods,[3] and periods with large temporary movement in energy prices. This enables us to examine corporate hedging behaviors across different market states.

Using this dataset, we systematically examine the theoretical determinants of corporate hedging policies. Despite a significantly expanded time series, we do not find statistically significant results supporting the hypothesis in Smith and Stulz (1985) that tax convexity and managerial incentives can predict hedging decisions. Neither do we find a statistically significant relationship between financial constraints and corporate hedging, despite using multiple common proxies of financial constraints such as dividend payer status, cash holdings, and credit ratings. This is inconsistent with the positive relation predicted by risk management models emphasizing the financial constraint (e.g., Froot et al. (1993); Bolton et al. (2011))

[1]The hedging data are from 10-K filings after the establishment of Statement No. 133 of the Financial Accounting Standards Board, "Accounting for Derivative Instruments and Hedging Activities" (SFAS 133), a culmination of FASB's effort to develop a comprehensive framework for derivatives and hedge accounting that which substantially enhances the disclosure requirement of firms' derivatives positions. See Section 3 for more details.

[2]Jin and Jorion (2006) and Bakke et al. (2016) use 10-K filings data from 1998 to 2001 and from 2003 to 2006, respectively; Gilje and Taillard (2017) examine data around 2011.

[3]For example, the period between 2002 and 2007 during which oil price went up by 150% and the period between 2014 and 2016 during which oil price dropped by 80%. (See Figure 1.)

and the negative relation predicted by models emphasizing the collateral constraint (e.g., Rampini and Viswanathan (2010); Rampini et al. (2014)).

The strongest predictor of hedging intensity in our sample is the operating profitability, defined as the operating cash flow scaled by total assets. We find that the operating profitability negatively predicts the hedging intensity. In contrast, we find an insignificantly positive relation between the return on assets (ROA) and the hedging intensity. This difference in coefficients highlights an important difference between the operating cash flow and the net income, which is related to the gains and losses on hedging positions. Using multiple proxies for the unrealized gains and losses on derivatives, we find the hedging gains are positively correlated with the hedging intensity.

The findings that hedging intensity is correlated with the operating profitability and the hedging gains with opposite signs challenge existing theoretical models of risk management. In one strand of these models (Smith and Stulz (1985), Froot et al. (1993), and Bolton et al. (2011)), firms hedge to avoid raising costly external financing or cutting back valuable investment due to low internal funds. Therefore, when the productivity is higher (more valuable investment) or the internal fund is lower, firms hedge more, implying that hedging intensity should be positively correlated with the productivity and negatively correlated with the internal fund. These predictions are opposite to our findings to the extent that our measures of operating profitability and hedging gains are positively correlated with the productivity and internal fund respectively.

Another strand of theories emphasizing the role of the collateral constraint (e.g., Rampini and Viswanathan (2010) and Rampini et al. (2014)) can explain the positive correlation between hedging intensity and hedging gains but not the negative correlation between hedging intensity and operating profitability. In these models, firms need to post collateral for both hedging positions and borrowing. When a firm experiences gains from its existing hedging positions, ceteris paribus the firm can borrow less to execute the same production and investment plans. The reduced debt position frees up the collateral, which the firm can use to increase the hedging position. This generates a positive correlation between hedging gains and hedging intensity, which is consistent with the data. On the other hand, when the operating profitability is low, a financially constrained firm needs to borrow more. The increased borrowing uses up the collateral, forcing the firm to reduce its hedging position, generating a positive correlation between operating profitability and hedging intensity. This prediction is inconsistent with the data.

To explain our empirical findings, we extend the model in Rampini and Viswanathan

(2010); Rampini et al. (2014) by adding the *production-dependent depreciation*, which corresponds to a common depreciation method in the real world called units of production depreciation.[4] Specifically, for oil and gas exploration and production (E&P) firms, the production-dependent depreciation refers to the mechanism that the production of oil and gas depletes firms' capital (i.e., oil and gas reserves).[5] This *production-dependent depreciation* mechanism generates two effects. First, it makes the optimal production less sensitive to the changes in output price. This is because the firm trades off between using the capital for the current production versus the future production. Given a convex production cost function, the firm desires to smooth its production over time. Second, the production-dependent depreciation makes the optimal production dependent on the tightness of the financial constraint in the current period.When the current-period financial constraint is more binding, the shadow price of current profits is higher relative to the present value of future production, making a financially constrained firm produce more than it would under the unconstrained case. These two effects make the dynamics of the production and thus the operating profitability much richer in our model, as compared to the existing models such as Froot et al. (1993), Rampini and Viswanathan (2010), and Bolton et al. (2011), in which the production is determined by the exogenous productivity and the pre-determined capital.

We show that this new mechanism is important in explaining the empirical relation between hedging and a variety of corporate decisions. First, adding this production-dependent depreciation mechanism to the model in Rampini and Viswanathan (2010); Rampini et al. (2014) helps resolve the model's inability to explain the negative correlation between the hedging intensity and the operating profitability. Consider the case when the price increases and thus productivity is higher. Firms' investment increases more than the production, as the production is less sensitive to the changes in price due to the intertemporal production smoothing under the production-dependent depreciation mechanism. Consequently, firms need to borrow more to finance the investment, which crowds out the hedging position via the collateral constraint channel, yielding a negative relation between the operating profitability and the hedging intensity. Such negative relation would not be obtained without the production-dependent depreciation mechanism, as the current production would increase more strongly in response to the higher price and thus generate more cash flows to support the investment, reducing the need for borrowing.

Second, this production-dependent depreciation help reinforces the positive relation be-

[4]Unit-of-production depreciation method is one out of the four common depreciation methods among small and medium-sized enterprises. It is also widely adopted in our sample firms.

[5]See Appendix A for an example of how the production affects the reserve for oil and gas E&P firms.

tween hedging intensity and the current liquidity obtained in the model of Rampini and Viswanathan (2010); Rampini et al. (2014). Similar to the mechanism in Rampini and Viswanathan (2010); Rampini et al. (2014), in our model higher hedging gains lead to higher current liquidity, which relaxes the collateral constraint and leads to more hedging. Furthermore, under the production-dependent depreciation mechanism, a relaxed financial constraint due to hedging gains means firms do not have to produce as much to satisfy their current liquidity needs. The reduced production conserves the capital, further relaxing the collateral constraint and leading to more hedging.

Third, our model predicts that hedging intensity should be positively correlated with the collateral value because a higher collateral value relaxes the collateral constraint and allows more hedging. Empirically, we use a firm's developed reserves as a proxy for the collateral value of the capital. We find firms with more reserve (i.e., more collateral) hedge more, consistent with the model prediction. Finally, in our model, when the collateral constraint is binding, firms need to reduce hedging positions in order to borrow more. In this case, the hedging intensity and the leverage is negatively correlated. Since the tightness of the collateral constraint depends on the availability of the collateral, this negative relation between the hedging intensity and the leverage should be stronger for firms with low reserve (i.e., less collateral). Our further empirical analyses confirm this prediction by showing that the interaction of leverage and reserve positively predicts the hedging intensity.

The rest of the paper proceeds as follows: Section 2 surveys the theoretical predictions and develop testable hypotheses. Section 3 describes the construction of our sample. Section 4 performs detailed empirical analyses. Section 5 presents our model and empirical analyses of model implications. Section 6 concludes.

2. Theoretical Determinants of Hedging

In this section, we review the theoretical determinants of corporate hedging intensity and describe how we construct the corresponding empirical measures.

Tax Benefits. Corporate tax schedule is convex in a firm's pretax earnings due to progressive tax rates and tax shields. The more convex the tax schedule the greater a firm can benefit from hedging by reducing earnings volatility (Smith and Stulz (1985); Graham and Smith (2000).) We construct the tax convexity measure by using the simulated marginal tax rate from John Graham's website.

Managerial Incentives. Manager incentives can affect hedging through two competing channels. On the one hand, Smith and Stulz (1985), Stulz (1990), and Tufano (1996) argue that risk-averse managers have incentives to hedge corporate earnings risks because they are restricted from diversifying corporate cash flow risks through their compensation packages. On the other hand, optimal contract theories (for example, Holmstrom (1979)) suggest that convex compensation packages with respect to firm performance can motivate the manager to exert efforts. Such convex compensation packages can cause managers to to hedge less than the level maximizing firm value. Which effect dominates depends on the compensation sensitivity to earnings volatility, or Vega. To test the effects of managerial incentives on corporate risk hedging, we construct proxies for CEO compensation Delta and Vega following Bakke et al. (2016). For compensation delta, we find the aggregate compensation delta by taking the sum of stock compensation and option delta. For options, we assume the delta factor is 0.5. As a proxy for vega, we use the fraction of options outstanding held by the CEO.While we cannot observe a manager's risk aversion or income effects, such effects can be subsumed by the time-invariant fixed effects due to our panel data structure.

Financial Constraints. A large theoretical literature examines the relationship between hedging and financial constraints. Financial constrained firms are averse to cash flow risks due to a variety of financial distress costs such as external financing costs, fire-sale of assets, and forgone investment opportunities. Hedging such cash flow risks can lower the probability of financial distress and mitigate the underinvestment problem. Smith and Stulz (1985), Bessembinder (1991), Froot et al. (1993), and Bolton et al. (2011) suggest that the more a firm is financially constrained, the more it will hedge. Additionally, these models predict a higher hedging intensity for firms with (1) greater investment needs, and (2) higher profitability.

Another strand of theories predict the opposite relation between financial constraints and hedging intensity. Rampini and Viswanathan (2010); Rampini et al. (2014) predict a negative relation between hedging intensity and financial constraints because hedging requires collateral, which requires financial resources.

Besides these theories, since shareholders of levered firms have a call-option like payoff on the firm value, risk-shifting theories (Jensen and Meckling (1976)) predict that shareholders have incentive to take more risks by hedging less and transferring the cost to creditors. Such risk-shifting (or asset substitution) effects are stronger when the firm is more financially constrained, thus predicting a negative relation between financial constraints and hedging intensity. Purnanandam (2008) and Cheng and Milbradt (2012) document a non-monotonic relationship between hedging and leverage, due to the two counteracting effects mentioned above: the distress costs effects (as in Smith and Stulz (1985) and Froot et al. (1993)) and the risk-shifting effects (as in Jensen and Meckling (1976)).

In light of these theories, our empirical tests explore how various empirical proxies of financial constraints are related to hedging intensity. We test the relation between hedging and investment opportunities as better investment opportunities implies a higher cost during financial distress. We also test the relation between profitability and hedging. A higher profitability implies (1) higher internal cash flows and (2) a better investment opportunity. In the first strand of theories following Froot et al. (1993), the first channel predicts lower hedging intensity but the second channel predicts higher hedging intensity, with the second channel typically dominating the first channel. In the other class of models emphasizing the collateral constraint such as Rampini and Viswanathan (2010); Rampini et al. (2014), the first channel is different as higher internal cash flows relax the collateral constraint, suggesting higher hedging intensity, while the second channel via the investment opportunity remains the same. Therefore, these models point to an unambiguous positive relation between profitability and hedging.

3. Data

We follow Jin and Jorion (2006), Bakke et al. (2016), and Gilje and Taillard (2017) and obtain the hedging data for independent oil and natural gas E&P companies from their 10K filings after the establishment of Statement No. 133 of the Financial Accounting Standards Board, "Accounting for Derivative Instruments and Hedging Activities" (SFAS 133).[6] We follow these studies to focus on independent oil and gas producers for three reasons. First, these firms disclose contract-level information about their derivative positions, including the notional volume, the instrument types, and the maturity, some of which such as the maturity and strike price are not explicitly required by SFAS 133 and amendments afterwards. Consequently, we are able to identify contracts that protect firms from downside risks versus

[6] Although firms are required to disclose the face value, contract types, or notional amount of financial instruments with off-balance-sheet risk of accounting loss starting from SFAS 105 in 1990, commodity and other derivatives that involve physical settlement were exempted from disclosure requirements until the implementation of SFAS 133. As a result, early empirical studies on hedging policies have to rely on data from surveys and voluntary disclosures.

other contracts. In contrast, earlier studies use the aggregate notional amount of all financial derivatives, which may include derivatives not intended for downside protection such as short positions in call options. Second, as noted by Jin and Jorion (2006), independent oil and natural gas producers are arguably the best set of firms to study corporate hedging policy because the commodity price risk is the dominant business risk, the risk exposure is easy to estimate, and the financial market to hedge such risks is well developed. Third, using a sample of the firms within the same industry alleviates the concerns that omitted industry characteristics may drive the results.

Our sample period is between 2002 and 2016, substantially longer than those in Jin and Jorion (2006), Bakke et al. (2016), and Gilje and Taillard (2017). We identify independent oil and natural gas producers in the following steps. First, we identify domestic common stocks in the CRSP universe that (1) have a GICS (Global Industry Classification Standard) code of 10102020 (Oil & Gas Exploration & Production) or (2) have a missing GICS code but a Standard Industrial Classification (SIC) code of 1311 for at least one year during our sample period.[7] We then exclude the firms with significant refinery business historically, by excluding those with a GICS code of 10102010 for at least one year during our sample period.[8] If the GICS code is missing for all years, we require the firm to have a SIC code 1311 for the whole sample period. Finally, we exclude microcap stocks by removing firms whose book value of total assets never exceeds $300 million during the sample period. The resulting dataset includes 1,327 firm-year observations for fiscal years ending between 2002 and 2016.[9]

We analyze annual reports, in other words, 10-K filings, of these independent oil and gas companies. In particular, we write a Python program to identify and scrape tables in 10-Ks that report the hedging activities. We then extract their contract-level details: derivative instrument types (put/call/collar options, swaps, futures/ forward, and other contracts), notional volumes, maturities, strike prices (if available) and underlying commodity types (oil, gas and various liquefied gas). We aggregate the volumes of all contracts that protect the downside price risk of future production for each firm year to arrive at the hedging position. These contracts include forward, futures, swap and fixed-price contracts that secure a fixed

[7]We use GICS in priority relative to SIC because cross-checking with Bloomberg reveals that GICS is more reliable than SIC identification approach. For example, SIC identification approach misses some large oil and gas producers, such as EQT, SWN, and UNT.

[8]We also exclude a few firms that have a GICS code of 10102050 (Coal & Consumable Fuels), 55105010 (Independent Power Producers), or 10101020 (Natural gas distribution).

[9]This current version reports results for the independent oil and natural gas producers among the Bloomberg Intelligence (BI) North America Independent Exploration and Production Valuation Peers.

sales price for future production and long positions in put options and collars that set a floor for the sales price.[10] Finally, we scale the total hedged volume, in units of thousands of barrel of oil equivalent per day (MBOEPD), by the actual production volume in the following fiscal year to arrive at our hedge ratio.[11]

We get additional data from standard sources - financial data are from Compustat, CEO compensation data are from ExecuComp and simulated marginal tax rates are from John Graham's website.[12] Table 1 reports how we define and construct our key empirical variables.

Table 2 presents summary statistics on our final sample which has 436 firm-year observations with non-missing hedge ratios. The median hedge ratio is 0.364, which is significantly higher than in prior studies (Haushalter (2000); Jin and Jorion (2006)). The main reason is that we explicitly exclude integrated oil and gas producers that have a much lower propensity to use financial instrument to hedge oil and gas price risk, because their refinery sector uses oil and gas as the inputs and thus provides a natural operational hedge for the exploration and production sector. . As a result, less than 10% of firm-years in our sample have a hedge ratio equal to zero. The median book asset value is $3.6 billion and the market leverage is 29.5%. These companies are profitable with operating profitability of 14.8% for the median firm-year. The median cash holding is only 1.3% and a dividend is paid in about 45% of firm-years.

4. Empirical Results

We conduct a comprehensive study of the determinants of hedging predicted by theories listed in Section 2. Following Haushalter (2000), our empirical approach relies on estimating the following ordinary least-squares multivariate regression using our panel data:

$$HedgeRatio_{it+1} = \alpha + \beta X_{it} + \gamma_i + \delta_t + \varepsilon_{it}$$

[10]The fixed-price contract includes the fixed-price physical delivery contracts, consistent with Almeida et al. (2019). See, for example, ATP Oil & Gas Corporation (ATPG), which has around 50 percent of their hedging positions in fixed-price physical delivery contracts with their customers. Call options are excluded in our analysis as they do not pertain to downside risk management. For robustness checks, we also construct a version that is based on the net position in forward, futures, swap and fixed-price contracts positions as well as put options and collars. The results are similar.

[11]Alternatively, we compute the dollar value of the hedged volume based on the average daily price of the front month futures contract over the current fiscal year and construct a dollar hedge ratio of the dollar value of hedged volume over the dollar value of the production volume.

[12]Available at https://faculty.fuqua.duke.edu/∼jgraham/taxform.html.

The dependent variable of interest is the fraction of year $t+1$ production hedged by firm i at the end of year t. X_{it} is a vector of independent variables of interest that are theorized to influence hedging intensity. γ_i and δ_t are firm and year fixed effects. We compute standard errors that are clustered at both the year and firm levels.

4.1. Tax, Compensation, and Financial Constraint

The estimation results of our baseline multivariate model are in Table 3. In the first column, we include no fixed effects. In columns (2) and (3), we include year and firm fixed effects respectively. In column (4), both year and firm fixed effects are included. The ability to add both year and firm fixed effects due to our long panel is a contribution over existing empirical studies on hedging. As a further robustness check, we run two additional specifications wherein we include the lagged hedge ratio to control for persistence in the dependent variable. We exclude firm fixed effects in these specifications because the identification assumptions for the fixed effect model and the model with the lagged dependent variables are different. The fixed effects and lagged dependent variable estimates are useful for assessing the robustness of our results as they form the bound within which the true coefficient of interest would fall under alternative identification assumptions.[13]

According to theories emphasizing the tax benefits of hedging (Smith and Stulz (1985); Graham and Smith (2000)), higher tax convexity should lead firms to hedge more. Following Haushalter (2000), we measure tax convexity using the simulated marginal tax rate from John Graham's website. We see that the effect of the marginal tax rate on hedging intensity is statistically insignificant in all our specifications. These results are in contrast to Haushalter (2000) who finds a strong positive relationship between marginal tax rates and hedge ratios. Given our expanded data set, it is unlikely that the lack of statistical significance is due to weaker test power. There are two possibilities for the difference. First, the marginal tax rate is a good proxy for tax convexity only in the sample period of Haushalter (2000) between 1992 and 1994, but not in our sample period between 2002 and 2016. Second, Haushalter (2000)'s result is driven by unobserved cross-sectional firm heterogeneity that happens to be correlated with the tax convexity in his short sample period. In contrast, such cross-sectional heterogeneity is absorbed by the fixed effects in our empirical tests.

Theories on managerial compensation (Smith and Stulz (1985) and Stulz (1990)) suggest that compensation delta should increase firm hedging, while compensation vega should re-

[13]See Joshua D. Angrist, Jörn-Steffen Pischke "Mostly Harmless Econometrics: An Empiricist's Companion".

duce it. In Table 3, we see that the linear relationship between the logged compensation and hedging intensity is not strong. The same holds for the relationship between the fraction of options held by the CEO, our proxy for vega, and hedging intensity. Existing studies that examine data over different short sample periods report mixed results. For example, Haushalter (2000) finds that the amount of option-based pay is positively associated with hedging intensity, while Bakke et al. (2016) find that a reduction in option-based pay leads to more hedging. Our results suggest that the actual relationship between compensation and hedging over a longer sample may not be strong in either direction.

As discussed in Section 2, different financial constraint-based theories predict different relations between hedging intensity and financial constraints. We employ the following empirical proxies for financial constraints that are commonly used in the literature: (i) whether the firm pays dividends; (ii) whether the firm has a credit rating; (iii) the firm's size measured by the log of book assets; (iv) the firms' cash holdings divided by its total assets; (v) book leverage. Financial constraints are decreasing in the first four measures and increasing with leverage. Therefore, theories such as Froot et al. (1993); Bolton et al. (2011) will predict hedging intensity to be negatively correlated with the first four and positively with leverage. In contrast, theories such as Rampini et al. (2014), Bolton et al. (2019), and Rampini et al. (2019) predict the opposite patterns, and Purnanandam (2008) and Cheng and Milbradt (2012) predict a nonlinear relation. Th relation between our measures of financial constraints and hedging intensity is reported in Table 3. *Dividend Payer* is negatively and significantly related to hedging intensity under regression specifications in columns (1) and (2). This seems consistent with the theoretical notion that financial constraints and hedging are negatively related. However, when we add firm fixed effects, the relationship between *Dividend Payer* and hedging intensity becomes insignificant in columns (3) and (4). This suggests that the negative relation between firms' payout policy and hedging activity is mostly driven by some time-invariant firm characteristics, highlighting the importance of controlling for the firm fixed effects. Also support the importance of time-invariant firm characteristics, we find that the R^2quadruples from 0.108 in column (1) to 0.419 in column (4). When it comes to the other measures of financial constraints, we find the hedging intensity is not significantly related to either *Credit Rating, Size, Cash Holding* or *Book Leverage* once we control for the time and firm fixed effects.

4.2. Robustness Test Results

The sample size in Table 3 is small due to the inclusion of variables related to tax and compensation, which are more sparsely populated in our data. To overcome this issue, we estimate the regression without the tax and compensation variables. Doing this almost doubles our sample size. These results are presented in Table 4. The coefficient on *Dividend Payer* is insignificant in our preferred specification, consistent with Haushalter (2000). We also do not find significant coefficients when using *Credit Rating* and *Size* as proxies for financial constraints. We find that *Cash Ratio* is negatively correlated with hedging intensity, similar to the finding in Adam and Fernando (2006), but the coefficient is insignificant. Both Haushalter (2000) and Carter et al. (2006) also find an insignificant relation between hedging intensity and the cash ratio. In this larger sample without controlling for tax and CEO compensation, we find a positive and significant coefficient on *Book Leverage* when we use firm and year fixed effects. This is consistent with the finding in Haushalter (2000), supporting theories that posit a positive relationship between financial constraints and hedging intensity.

Next, we investigate the relation between hedging intensity and investment. In both Tables 3 and 4, we find a negative relation between hedging intensity and investment that is statistically insignificant when we control for the firm and year fixed effects (column (4)). Our results contradict those in Haushalter (2000) and Carter et al. (2006) , both of which find a positive relationship between hedging and capital expenditure, though the effect in Carter et al. (2006) is insignificant. We also study the relation between hedging intensity and Tobin's Q. The relationship is small and insignificant. These results add to the debate in the existing literature. Carter et al. (2006) find a positive relationship between Tobin's Q and hedging while Adam and Fernando (2006) find a negative result. Overall, our results using investment and Q as the explanatory variables for hedging intensities do not provide support for theories predicting that firms hedge to reduce the under-investment problem caused by financial constraints.

4.3. Hedging and Profitability

We first follow the existing empirical literature and use *Return on Assets (ROA)* as our proxy of profitability. The results in the smaller sample of Table 3 indicate an insignificant relationship between ROA and hedging intensity. However, when we use the larger sample in Table 4, we find that ROA has a positive and significant relationship with hedging intensity.

Given that a substantial portion of E&P firms' revenue is protected by hedging derivatives, we further decompose these firms' net income into two variables that have different economic meanings, which are operating profits and gains from hedging positions. The former captures a firm's ability to generate cash flows internally, while the latter is related to a firm's risk management decision with the profit and loss determined by exogenous price movements. To disentangle their relation with firms hedging policies, we separate ROA into operating profitability and the gain/loss on hedging and separately investigate their impacts on hedging intensity.

To sharpen the focus, we conduct univariate regressions to assess the relation between different profitability proxies and the hedging intensity. In Table 5, we verify the positive and significant relationship between ROA and hedging intensity using the univariate regressions of hedging intensity on ROA. In specification (4) with the control of firm fixed effects and specification (6) with lag hedging intensity, the positive relationship is statistically significant at the 5% level. Figure 2 shows the relationship between these two variables in a scatter-plot. We plot the average values of the x and y variables after controlling for firm and year fixed effects in twenty equal-sized bins. The fitted curve has positive slopes, confirming the positive relationship uncovered in the regression analysis. In a sharp contract, Table 6 shows that operating profitability has a strong negative relation with the hedging intensity, while the current hedging gain or loss is positively correlated with the hedging intensity for next-period production.

These novel empirical findings are intriguing. First, the negative relation between the operating profitability and the hedging intensity contrasts with the positive relation between ROA and the hedging intensity, indicating that corporate profits unrelated to operations have distinct effect on the hedging policy from the operating profit. This negative relation also challenges the existing theories of risk management, most of which predict a positive relationship between the operating profit and the hedging intensity. For example, in models similar to Smith and Stulz (1985), Froot et al. (1993), and Bolton et al. (2011), higher profitability implies better investment opportunities and thus firms hedge more to avoid raising costly external financing or cutting back valuable investment due to low internal funds in the future. In models similar to Rampini and Viswanathan (2010) and Rampini et al. (2014), less profitable firms have less internally generated funds and would rely more on external borrowing. As both hedging and borrowing require collateral, when the collateral constraint is binding, borrowing more results in less hedging. Thus these models also predict a positive relation between profitability and the hedging intensity.

Second, the positive effect of current hedging gain or loss on the hedging intensity for next-period production is consistent with the risk management models that emphasize the collateral constraint, such as Rampini and Viswanathan (2010) and Rampini et al. (2014). In these models, hedging gain relaxes the financial constraint, less financially constrained firms can borrow less, leaving more collateral for hedging. This predicts a positive relationship between hedging gains and hedging intensity, which is consistent with our findings. On the other hand, our finding of this positive relation is inconsistent with models such as Smith and Stulz (1985), Froot et al. (1993), and Bolton et al. (2011), in which the collateral constraint is not emphasized. In these models, hedging gains ease the financial constraint, effectively making constrained firms less risk-averse and in turn hedge less.

To conclude, our empirical analysis finds that the operating profitability is the most significant and robust determinant of the hedging intensity among a long list of theoretically motivated predictors. However, the negative relation between the operating profitability and the hedging intensity poses a challenge to the existing risk management models. In the following section, we propose a new economic mechanism to reconcile our empirical findings with the existing theories.

5. The Model

This section presents a model to understand how firms' hedging intensity are correlated with their operating profitability and hedging gains. The model features three important elements: (1) a financial constraint, under which hedging creates value; (2) a collateral constraint, as in Rampini and Viswanathan (2010); Rampini et al. (2014); (3) production-dependent depreciation. Our key innovation is the third feature.

5.1. Setup

In a dynamic economy, a risk-neutral firm (e.g., oil producer) is a price taker and its output price evolves as follows,

$$P_{t+1} = (1+r)\,P_t + \sigma\varepsilon_{t+1}\,. \tag{1}$$

Here, σ is the price volatility. ε_t is a random variable being -1 and 1 with equal probabilities and independent across time. r is the constant risk-free rate.

The firm chooses time-t production volume Q_t, which generates a revenue P_tQ_t and incurs a convex production cost $\frac{c}{2}Q_t^2/K_t$. The operating profits are

$$\Pi_t(P_t, K_t) \equiv P_t Q_t - \frac{c}{2}\frac{Q_t^2}{K_t}. \tag{2}$$

K_t, the capital (e.g., oil reserve) at time t, follows the dynamic

$$K_{t+1} = I_t + K_t - \delta Q_t. \tag{3}$$

Here, I_t is the capital investment, which incurs a convex adjustment cost $\frac{\phi}{2}I_t^2/K_t$. A key innovation of our model is the last term, δQ_t, on the right side of equation (3), which captures the production-dependent depreciation: the higher the production, the more the capital is depreciated. For instance, for oil and gas producers, higher production depletes the reserve faster. See Appendix A for an example. It should be noted that the notion of production-dependent depreciation applies far more widely than just the oil and gas industry. In accounting parlance, this is known as units of production depreciation, and is one of the four most popular depreciation methods for small and medium enterprises. For simplicity, we assume $\delta = 1$ in our benchmark model.

This firm also chooses the hedging position for the next-period, denoted by H_{t+1} (e.g., the number of barrels of oil that is hedged). For simplicity, we assume the firm uses futures contract to hedge. The hedging gain or loss at time $t+1$ depends on the difference between the delivery price of the futures agreed at time t and the realized spot price at time $t+1$. Using no-arbitrage pricing, the delivery price of the futures is $(1+r)P_t$. So the hedging gain at time $t+1$ is

$$\Pi_{t+1}^H(H_{t+1}) = [(1+r)P_t - P_{t+1}]H_{t+1}. \tag{4}$$

Applying equation (1), we know that $\Pi_{t+1}^H(H_{t+1}) = \pm\sigma H_{t+1}$ with equal probabilities. When the spot price P_{t+1} is low, the hedging position H_{t+1} will be profitable and thus protect the firm in the bad state.

The firm is financially constrained and can only borrow risk-free debt.[14] The net debt account (debt minus cash) at the beginning of period t is denoted as B_t,[15] which has an interest rate r^B. When $B > 0$, the firm has positive net borrowing, r^B is equal to the risk-free rate r; when $B < 0$, the firm has net savings, r^B is less than risk-free rate.[16] Net debt's

[14]That is, a firm cannot raise risky debt or new external equity.

[15]In our sample firms, average cash savings are only 3% while the book leverage is in the range of 20%-40% of total assets. So we focus on the net debt.

[16]The assumption that the interest rate on the net debt is lower than risk-free rate when B_t is negative is

dynamics resembles a typical statement of cash flow:

$$B_t - B_{t+1} = \underbrace{\Pi_t}_{\text{oper CF}} \underbrace{-I_t - \Phi\left(I_t, K_t\right)}_{\text{inv CF}} \underbrace{-D_t - r^B B_t}_{\text{fin CF}} + \underbrace{\Pi_t^H\left(H_t\right)}_{\text{hedging gain}}. \tag{5}$$

Here, D_t is dividend, which is restricted to be non-negative, $D_t \geq 0$ due to limited liability. As labeled below the respective terms, the (negative) change in net debt account is the sum of operating cash flow, investing cash flow, financing cash flow, and the hedging gain. Another way to look at equation (5) is that, the difference between cash inflows and outflows (RHS) is equal to the negative change in the net debt account (LHS).

In addition to the financial constraint, the firm is subject to a collateral constraint, as in Rampini and Viswanathan (2010); Rampini et al. (2014):

$$\eta K_{t+1} \geq \left(1 + r^B\right) B_{t+1} + \sigma H_{t+1}. \tag{6}$$

Here, η is a parameter measuring the collateral value of the capital; σ is the maximum loss that the hedging position can incur. Therefore, this equation says that the sum of the total amount of debt and the maximum loss in the hedging position cannot exceed the collateral value, which is ηK_{t+1}.

Finally, to simplify the problem, we assume the firm operates for three dates only, i.e., $t = 1, 2, 3$.[17] At date 3, the firm is liquidated with a constant value$(1 + r)\,\underline{v}$ per unit of capital. This captures the continuation value of the firm in a reduced form.

used to generate equity payout when firms are not financially constrained. If the return on internal savings is the same as the risk-free rate, the firm will never payout. See Riddick and Whited (2009) and Bolton et al. (2011) for more detailed explanations.

[17]For a more general treatment with infinite periods, please refer to the Appendix.

5.2. A firm's problem

At date t, the firm solves the following problem

$$
\begin{aligned}
& V\left(K_t, B_t, H_t, P_t\right) \equiv \max_{H_{t+1}, B_{t+1}, K_{t+1}, Q_t} E\left(\frac{1}{(1+r)^t} \sum_{t=0}^{T} D_t\right) \\
\text{s.t. } & B_t - B_{t+1} = -D_t - r^B B_t + \Pi_t - I_t - \Phi\left(I_t, K_t\right) + \Pi_t^H\left(H_t\right) \\
& K_{t+1} = I_t + (1-\delta) K_t - \delta Q_t \\
& P_{t+1} = (1+r) P_t + \sigma \varepsilon_{t+1} \\
& \eta K_{t+1} \geq \left(1 + r^B\right) B_{t+1} + \sigma H_{t+1} .
\end{aligned}
$$

In this problem, at each date t, a firm is maximizing it shareholders' discounted cash flows given four state variables, the beginning-of-period capital stock K_t, the beginning-of-period debt B_t, the hedging position H_t, and an exogenous state variable, the output price P_t. It has four choice variables, the hedging for next-period production H_{t+1}, the end of period debt B_{t+1}, the current production Q_t, and the current investment I_t.

To simplify the problem, we assume that after Date 2 (i.e., $t = 2$), the firm is liquidated with unit liquidation value of reserve being $(1+r)\underline{v}$. This captures a reduced-form continuation value of the firm at Date 3. Under this assumption of liquidation without production, the debt and hedging position at Date 3 are zero, i.e, $B_3 = H_3 = 0$.

Furthermore, the whole problem is homogeneous of degree one with respect to (K_t, B_t, H_t). Exploiting this feature, we can simplify the problem by dividing the whole problem by K_t. We denote the scaled variables by their corresponding lower cases, such as $h_t \equiv H_t/K_t$ and $b_t \equiv B_t/K_t$. Without loss of generality, we further assume $K_1 = 1$.

5.3. Solutions and implications

We solve this model and summarize the qualitative results in Figure 6. Since our focus is on the collateral constraint, we assume the financial constraint at date 1 is binding and discuss the cases in which the collateral constraint is binding and non-binding, respectively. Here, we present the key predictions in figures and explain the corresponding intuition. Detailed proofs are in Appendix B.

Figure 6 plots the policy functions of the optimization problem. First, Panel (a) shows that the production Q_1 is increasing in the current output price P_1 and decreasing in the current period net liquidity $\hat{w}_1$, which is the current hedging gains, $\Pi_1^H(H_1)$, minus the total payment on the beginning-of-period debt, $\left(1 + r^B\right) B_1$. Q_1 is increasing in P_1 because a higher output price implies a higher marginal benefit of production today and thus firms have incentive to increase the production. Q_1 is decreasing in $\hat{w}_1$ because when the financial constraint is binding, the firm needs to produce *more* than the unconstrained optimal level to generate enough profits to cover its cash outflows. When the current liquidity is high (i.e., a high Π_1^H or a small B_1), there is no such compulsion and henceQ_1 is lower.

Production Q_t plays a key role in our model. Producing today reduces the stock of future capital. This *production-dependent depreciation* mechanism generates two effects. First, the firm trades off between using the capital to produce now versus in the future. Given the convex production cost function, the firm desires to smooth its production over time. This makes the production less sensitive to the changes in output price. Second, the tightness of the financial constraint in the current period affects the intertemporal tradeoff. When the current-period financial constraint is more binding, the shadow price of current profits is higher relative to the future value of production. Consequently, low current internal funds can force a financially constrained firm to produce more than the optimal level under the unconstrained case. In contrast, in the existing models such as Froot et al. (1993), Rampini and Viswanathan (2010), and Bolton et al. (2011), the production is determined by the exogenous productivity and the pre-determined capital. As we discuss below, this production-dependent depreciation channel helps our model match the empirical patterns better than the existing models.

Panel (b) plots the investment policies I_1. The investment is increasing in both the current net liquidity and the output price. The mechanism is very similar to that in the existing models. When the current net liquidity is high, the firm is less financially constrained and thus can invest more. When the current output price is high, the expected future output price is high as the price follows a random walk,leading to a higher investment. The high current output price also means high current operating profits and less binding financial constraint, which is conducive to a higher investment.

Panel (c) shows that the operating profits Π_1 is increasing with the priceP_1, as both the production $Q_1$and the marginal value of output are high when P_1 is high. Π_1 is decreasing in the net liquidity. The intuition is similar to that of q_1: when liquidity is low, the firm has to produce more than the optimal level to cover current-period cash outflows.

Panel (d) depicts the next-period capital K_2, which is essentially $I_1 - Q_1 + K_1$. Since both $I_1$and Q_1 are increasing in P_1, the former increases K_2 and the latter decreases K_2. Whether K_2 is increasing in P_1 depends on whether the investment or production is more sensitive to

P_1. As discussed above, the existing models only feature the investment channel and thus in these models K_2 is clearly increasing in P_1. Our model adds the production-dependent depreciation mechanism that higher production depletes the capital faster. With this new mechanism, a higher P_1 induces a higher production Q_1 and thus reduces K_2. Thus the benefit of increasing the current production due to an increase in P_1 is offset by the cost of lower future production caused by the lower capital K_2. Therefore, when P_1 increases, firms increase I_1 faster than Q_1, resulting in a K_2 that is increasing in P_1. However, the relationship is not as strong as in a model without production-dependent depreciation. Furthermore, K_2 is increasing in $\hat{w}_1$ because $I_1$is increasing in $\hat{w}_1$ and Q_1 is decreasing in $\hat{w}_1$.

Panel (e) shows that B_2 is increasing in P_1 and decreasing in $\hat{w}_1$. Similar to the discussion of K_2, when the output price P_1 is high, both production and investment are high. The former leads to a higher operating profit and the latter a higher investment cash outflow. Again the investment responds more strongly than the production to the increase in P_1, because of the production-dependent depreciation, and thus the firm needs to borrow more to cover the investment cash outflow in excess of the operating profit, leading to a high B_2. B_2 is decreasing over the current net liquidity $\hat{w}_1$, because ceteris paribus a higher liquidity means the firm can borrow less to execute the same production and investment plans.

Panel (f) plots the hedging intensity, H_2. When the collateral constraint is binding, $H_2 = \frac{1}{\sigma}\left[\eta K_2 - \left(1 + r^B\right) B_2\right]$. The ηK_2 component of H_2 is increasing in P_1 and $\hat{w}_1$, as K_2 is increasing in P_1 and $\hat{w}_1$. The $-\left(1 + r^B\right) B_2$ component of H_2 is decreasing in P_1 and increasing in $\hat{w}_1$, as B_2 is increasing in P_1 and decreasing in $\hat{w}_1$. Therefore, H_2 is clearly increasing in $\hat{w}_1$, consistent with the prediction in models with the collateral constraint (Rampini and Viswanathan (2010) and Rampini et al. (2014)) that firms with higher liquidity hedge more. On the other hand, the relationship between H_2 and P_1 depends on the relative strength of the responses to the changes in P_1between the ηK_2 component and the $-\left(1 + r^B\right) B_2$ component. In our current parameters, the collateral value coefficient η is in a similar magnitude to P_1, which is much higher than $\left(1 + r^B\right)$. Therefore, the increase in ηK_2 due to increased output price P_1 dominates the increase in $\left(1 + r^B\right) B_2$, resulting a positive relationship between H_2 and P_1.

To sum up, our model yields the following four testable predictions. 1) hedging intensity and operating profitability are negatively correlated; 2) hedging intensity and liquidity are positively correlated; 3) hedging intensity and capital (reserve) are positively correlated; 4) hedging intensity and borrowing are negatively correlated.

5.4. Testing model predictions

We proceed to test our model implications mentioned above. Table 6 already shows that hedging intensity is negatively correlated with operating profitability and positively correlated with hedging gains, consistent with our model predictions (1) and (2).

Our model prediction (3) suggests that hedging intensity and capital are positively correlated. For oil and gas companies, capital can be measured by several proxies, including plant, property & equipment (PPE), proved reserves, and developed reserves. We believe developed reserves captures the concept of capital in our model better than the other two, because production of oil and gas directly depletes the developed reserve in the real world. Table 7 shows the empirical results. We find that hedging intensity are positively and significantly correlated with the developed reserves over production ratio across all regression specifications. Furthermore, after controlling for the relation between hedging intensity and the reserve ratio, hedging intensity are still negatively correlated with operating profitability and positively correlated with hedging gains.

Finally, our model prediction (4) suggests a negative relation between hedging intensity and borrowing. Table 7 shows that the book leverage is positively correlated with the hedging intensity, though the correlation is insignificant. This does not contradict our model implications, as in our model and the models of Rampini and Viswanathan (2010) and Rampini et al. (2014) with the collateral constraint, both hedging intensity and borrowing are positively correlated with the amount of collateral.

To further explore the relation between the hedging intensity and borrowing, we conduct a comparative statics analysis by varying the collateral value parameter η. Figure 7 plots the results. When the collateral value η is high ($\eta = 20$ in the blue solid curve), the collateral constraint is not binding, the B_2 and H_2 are only linked through their relation with the capital K_2 and the output price P_1. As a result, the negative relation between these two is relatively flat. When the collateral η is low ($\eta = 10$ in the red dashed curve), the collateral constraint is binding, and the negative relation between the hedging intensity H_2 and borrowing B_2 is much steeper than the previous case. Therefore, whether the collateral constraint is binding affects the strength of the negative correlation between hedging and leverage.

We test this conditional relation between hedging and leverage by adding the interaction term of reserve and leverage in Table 8. The hypothesis is that higher reserve indicates higher collateral value, which leads to a less binding collateral constraint.Thus our model predicts a less negative relation between hedging and leverage for firms with higher reserve, implying a positive coefficient on the interaction term between reserve and leverage. Table 8 confirms this model prediction. In column (4), the interaction term between book leverage and the developed reserve over production is positive and statistically significant at the 10% level.

6. Conclusion

Using the most comprehensive dataset of oil and gas E&P firms' hedging positions known to the literature, we conduct a systematic study of the determinants of corporate hedging policies. Our data covering a long time series between 2002 and 2016 allow us to control for unobserved time-invariant firm-specific heterogeneity and examine the average relation between hedging intensity and a variety of corporate decisions over different commodity market cycles.

Contrary to prior studies, we find no evidence to support the theoretical prediction of a positive relation between tax convexity and hedging intensity nor the link between managerial compensation structure and hedging. We also fail to find robust evidence supporting a positive relation between financial constraints and corporate hedging, as predicted by standard corporate risk management theories. Our empirical proxies for financial constraints include firm size, cash ratio, leverage, dividend payer indicator, credit rating, investment intensity, and Tobin's Q.

Finally, we document two important empirical determinants of firms' hedging policies that are new to the existing literature. We find that when the net income is decomposed into operating profitability and gains/losses on hedging positions, these two components strongly predict hedging intensity with opposite signs. Existing models have difficulty explaining this novel empirical pattern. We extend the existing models by adding a production-dependent depreciation mechanism, in which production consumes capital stock. This feature corresponds to the units of production depreciation method in accounting practice and it realistically captures the tradeoff oil and gas producers face when they decide to convert developed reserves (capital) into oil & gas (output). We show that this production-dependent depreciation mechanism explains why hedging intensity are negatively correlated with operating profitability and positively correlated with hedging gains. Further empirical tests also confirm new predictions generated by the model on the effect of reserve and leverage on hedging intensity.

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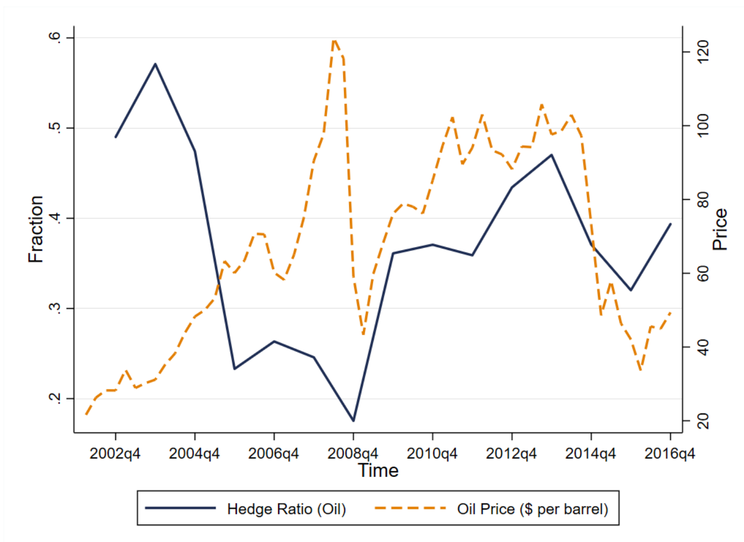
Figure 1: Time Series of Hedging Intensity and Commodity Price

The figure plots the average fraction of next-year oil (natural gas) production hedged and the demeaned average oil (natural gas) price for each year in our sample in the top (bottom) panel.



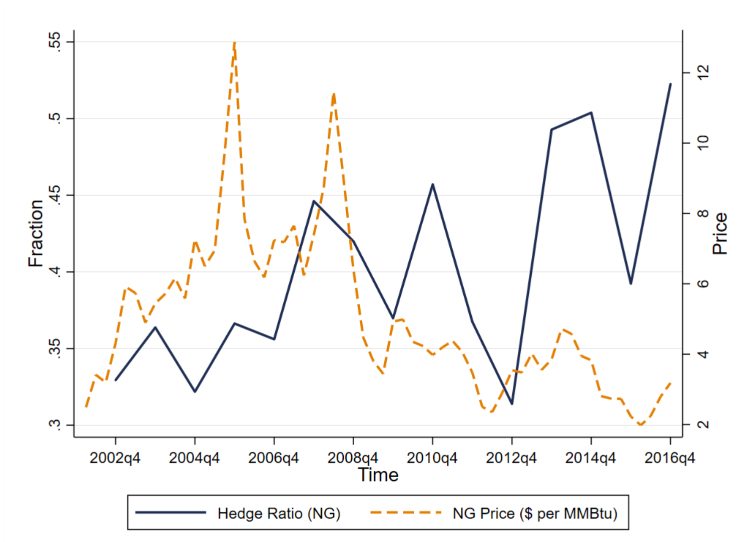


Figure 2: Binned Scatter Plot of Hedging Intensity and Return on Assets

The figure depicts a bin scatter plot of *Hedge Ratio* against *Return on Assets*. The plot is generated after controlling for firm and year fixed effects.

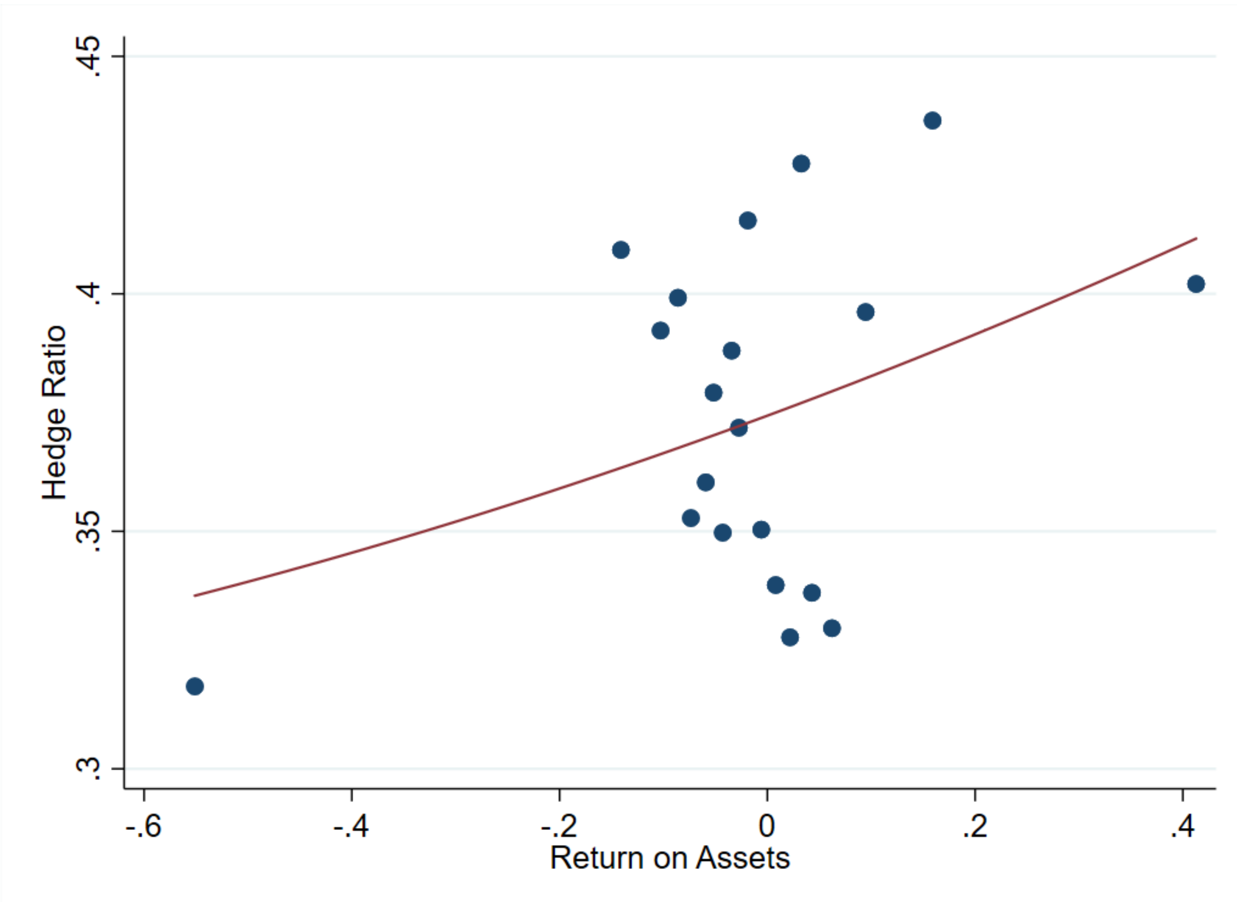


Figure 3: Binned Scatter Plot of Hedging Intensity and Operating Profitability

The figure depicts a bin scatter plot of *Hedge Ratio* against *Operating Profitability*. The plot is generated after controlling for firm and year fixed effects.

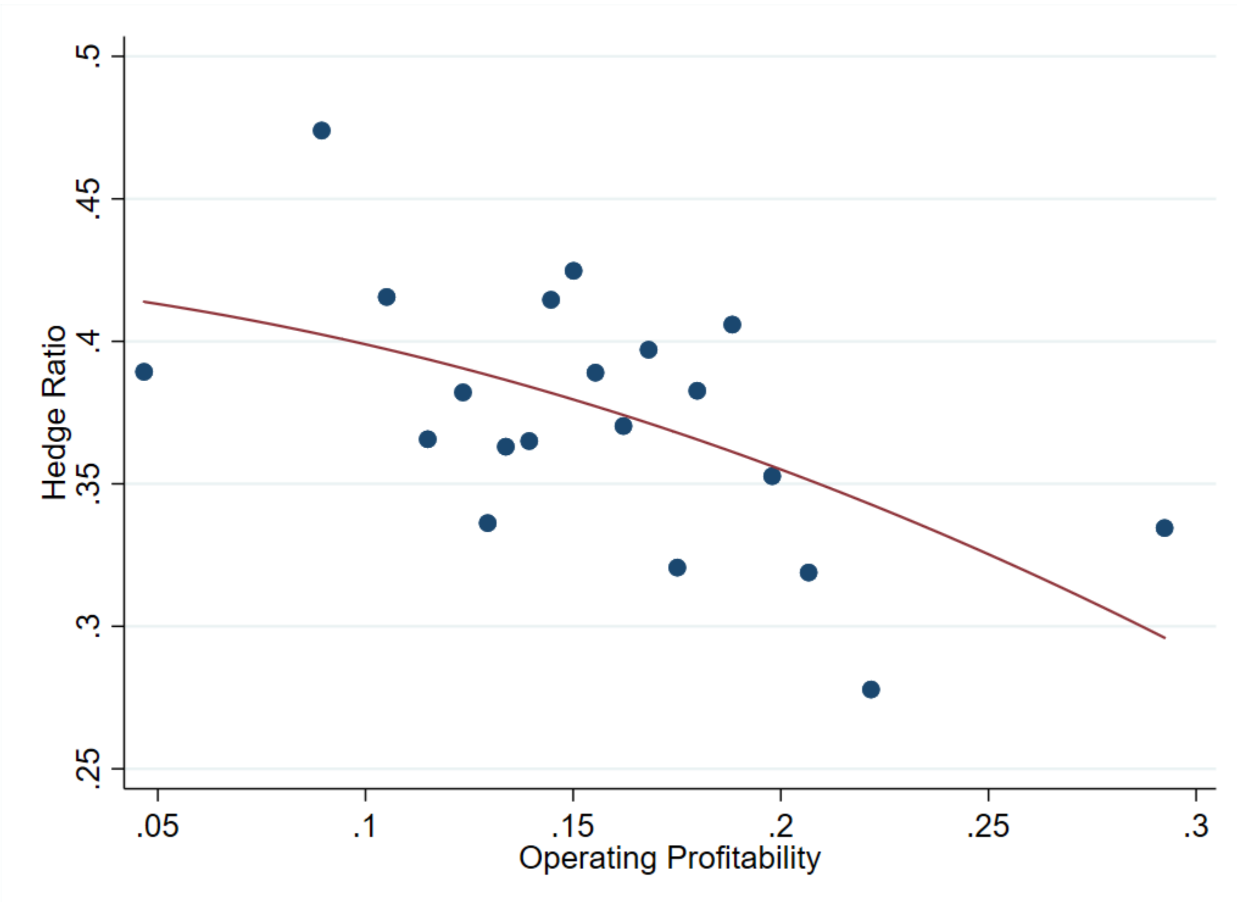


Figure 4: Binned Scatter Plot of Hedging Intensity and Gain/Loss on Hedging

The figure depicts a bin scatter plot of *Hedge Ratio* against *Gain/Loss on Hedging*. The plot is generated after controlling for firm and year fixed effects.

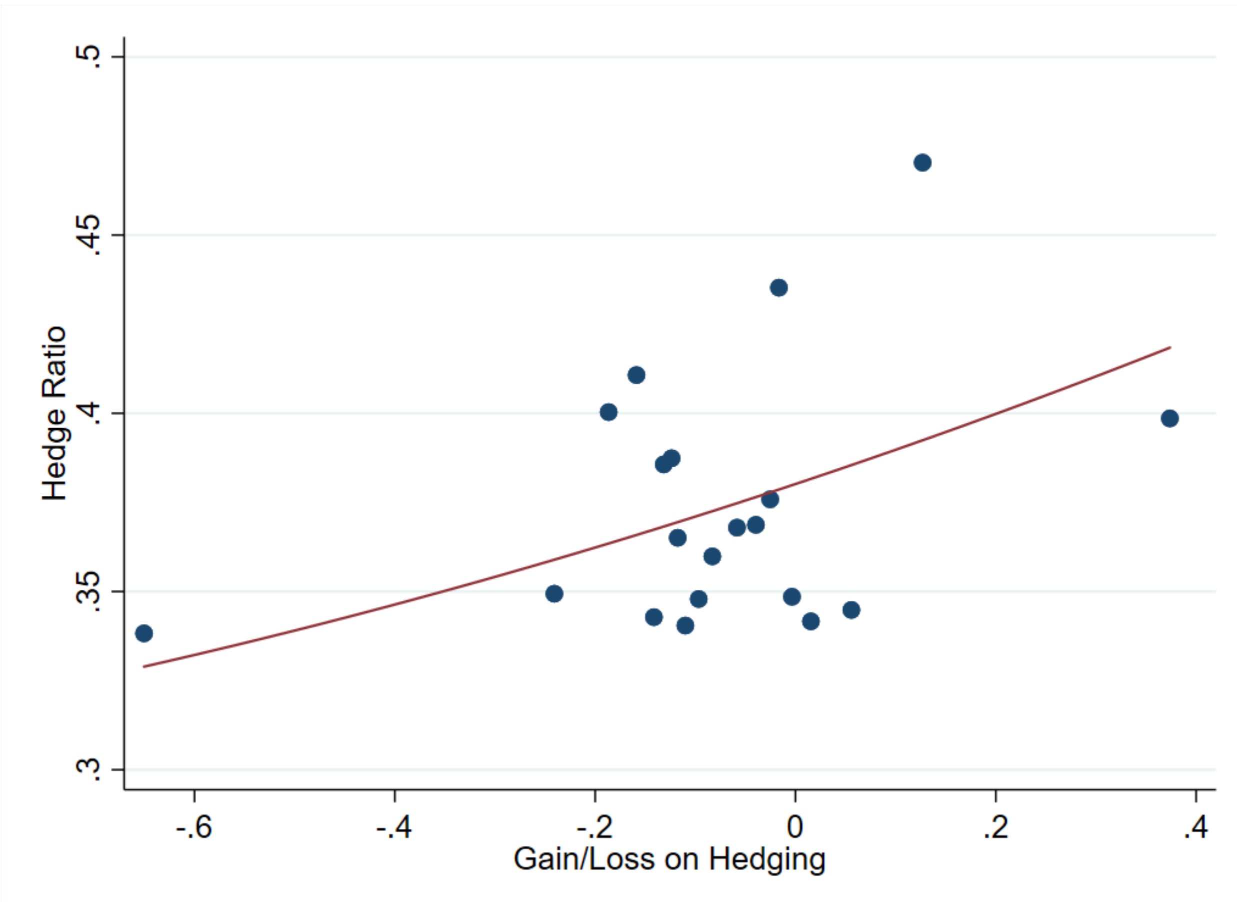


Figure 5: Binned Scatter Plot of Hedging Intensity and Reserves

The figure depicts a bin scatter plot of *Hedge Ratio* against *Developed Reserves/Production*. The plot is generated after controlling for firm and year fixed effects.

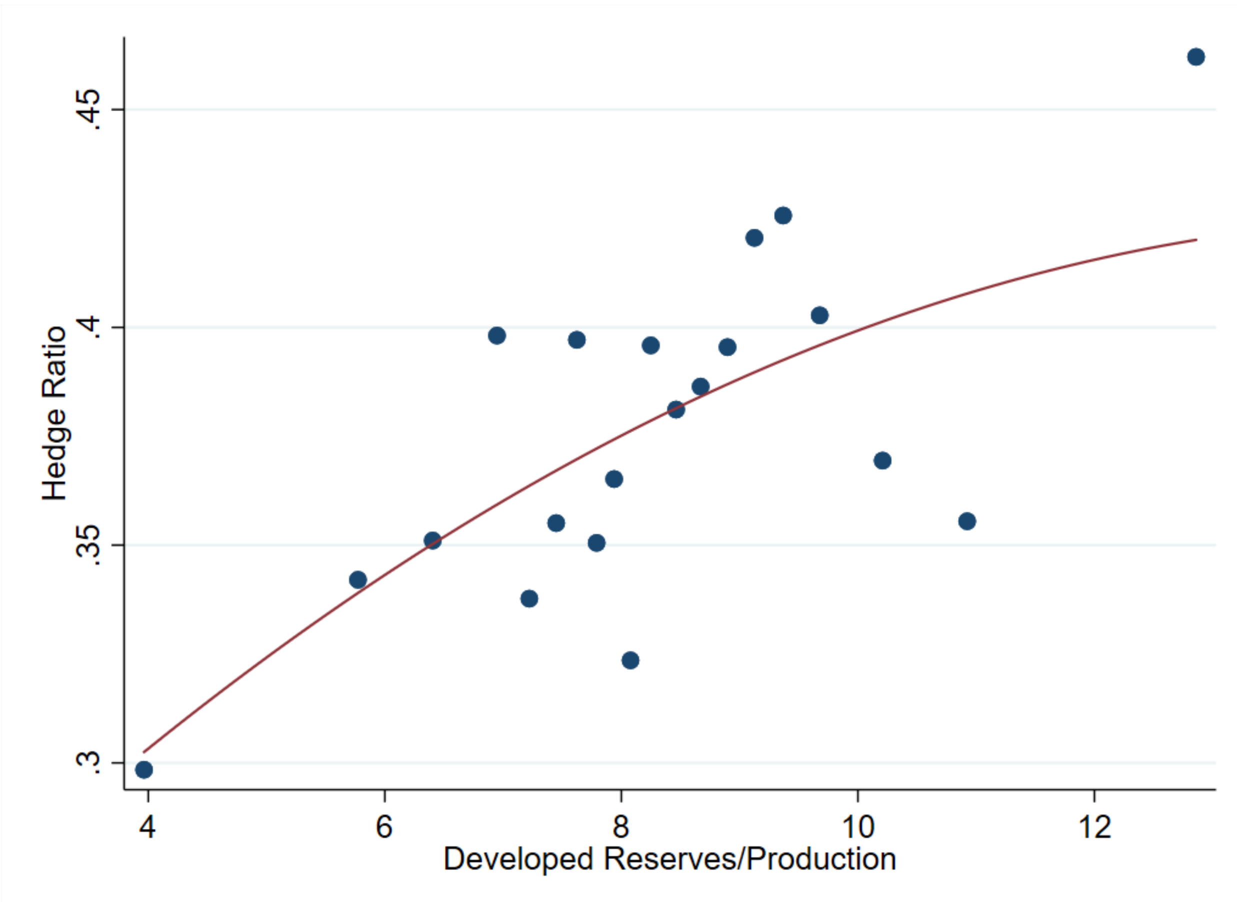
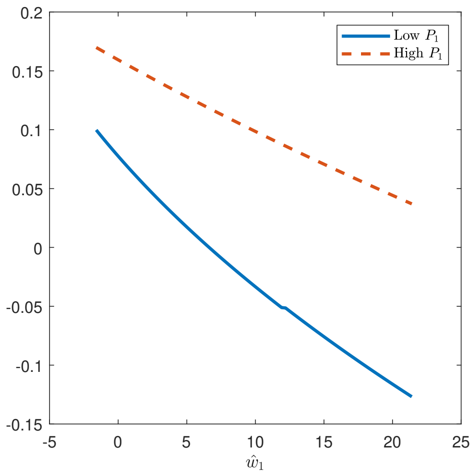


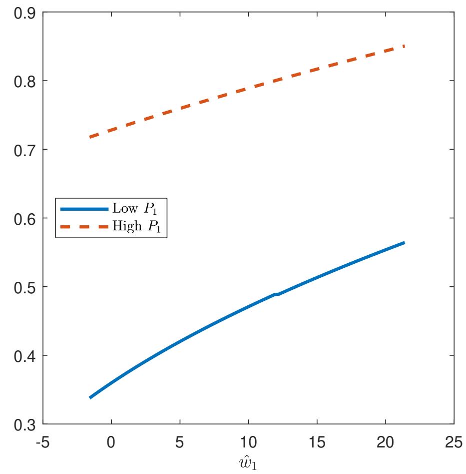
Figure 6: Model Solution: Policy Functions

The figure presents the policy functions of the model. The horizontal axis is $\hat{w}_1 \equiv -\sigma\varepsilon_1 h_1 - (1 + r^B)\, b_1$, the net liquidity at date 1, which is the sum of hedging gain or loss minus the total payment on debt. Benchmark parameters: $r = 4\%$, $r^B\,(B < 0) = 2\%$, $\sigma = 24$, $c = \phi = 80$, $\eta = 20$, $\underline{v} = 55$, $K_1 = 1$. $P_1 = (36, 72)$.

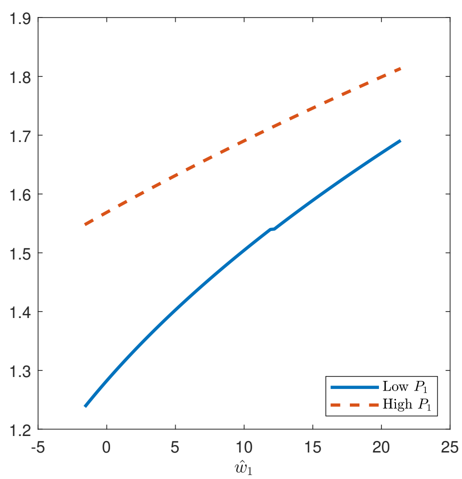
(a) Q_1



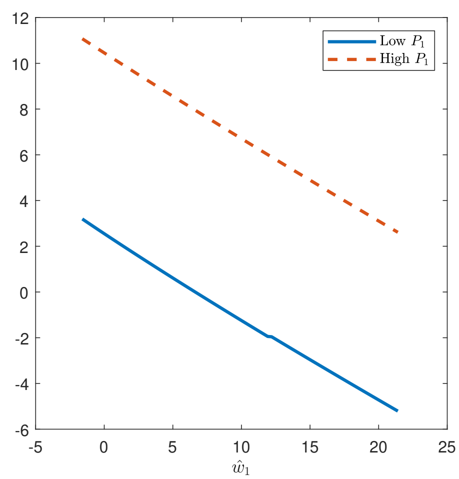
(b) I_1



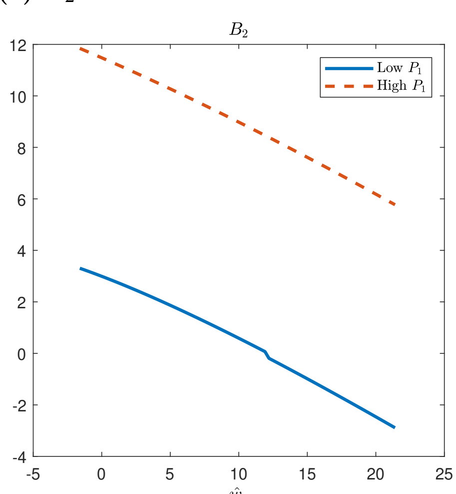
(c) K_2/K_1



(d) $\pi_1 = P_1 q_1 - \frac{c}{2} q_1^2$



(e) B_2



(f) H_2

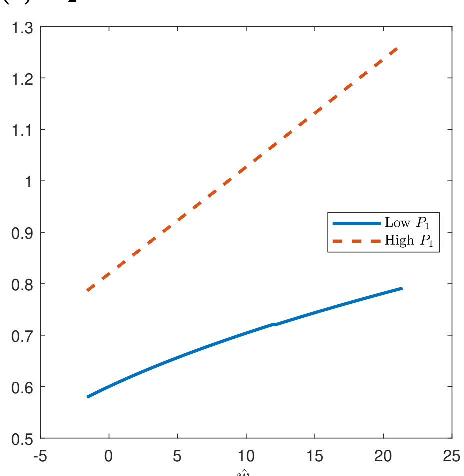


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Figure 7: Hedging Intensity and Borrowings

The figure explores how the relation between borrowing and hedging is affected by the collateral value η.

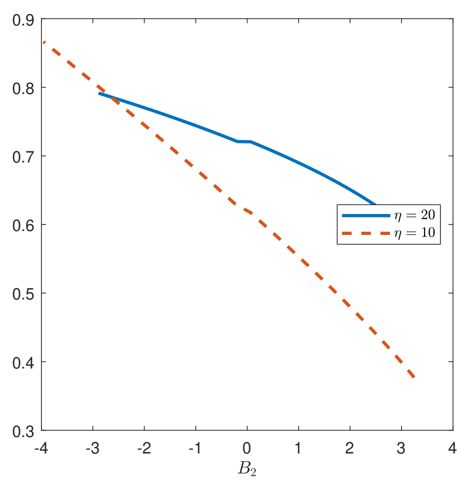

Table 1: Variable Definitions

This table defines the key variables used in the analysis and describes how they are constructed from data in financial statements.

Variable	Definition
Hedge Ratio	The fraction of next year's production that is hedged through a financial instrument
Book Value of Assets	Book value of assets (*at*)
Book Leverage	Ratio of the sum of long-term debt and debt in current liabilities (*dltt+dlc*) to the book value of assets
Cash Ratio	Ratio of cash and short-term investments (*che*) to the book value of assets
Investment intensity	Ratio of capital expenditures (*capx*) to the book value of assets
Operating Profitability	Ratio of cash flow from operating activities (*oancf*) to the book value of assets
Return on Assets	Ratio of net income (*ni*) to the book value of assets
Gain/Loss from Hedging	*-fopo/at*; If missing then use *-(oancf-ni-dpc-txdc-esubc-sppiv–recch-invch-apalch-txach)/at*
Tobin's Q	Ratio of market value of assets less deferred taxes and investment tax credits ($at + prcc\_f \times csho - ceq - txditc$) to the book value of assets
Dividend Payer	An indicator taking the value 1 if the firm paid a dividend in a given year ($dvc > 0$), and 0 otherwise
Credit Rating (numerical)	We translate the letter rating from rating agencies into a numerical value. For each higher notch, the value increases by 1. For unrated firms, we use the value 0.
Log (Compensation Delta)	The log of the CEO's compensation delta
Options Held (as frac)	Fraction of options outstanding held by the CEO
Marginal Tax Rate	The simulated marginal tax rate (before interest) from John Graham's website
Developed Reserves /Production	The ratio of the developed reserves at the end of this year to next year's production

Table 2: Summary Statistics

This table presents the summary statistics of the key variables used in the paper. Variable definitions are in Table 1. The sample period is 2002 to 2016.

	N	Mean	SD	P5	P25	P50	P75	P95
Hedge Ratio	436	0.374	0.242	0.000	0.185	0.377	0.549	0.794
Book Value of Assets ($ mm)	436	8792.03	12593.81	266.09	1259.54	3664.47	9524.24	41611.00
Cash Ratio	436	0.037	0.054	0.000	0.003	0.013	0.054	0.145
Book Leverage	436	0.329	0.203	0.091	0.223	0.295	0.390	0.658
Return on Assets	436	-0.019	0.271	-0.374	-0.011	0.036	0.068	0.119
Operating Profitability	436	0.156	0.075	0.057	0.109	0.148	0.190	0.284
Gain/Loss on Hedging	436	-0.082	0.278	-0.499	-0.054	-0.011	-0.001	0.036
Tobin's Q	436	1.480	0.591	0.770	1.087	1.347	1.706	2.610
Investment intensity	436	0.247	0.118	0.087	0.161	0.221	0.320	0.469
Dividend Payer	436	0.656	0.476	0	0	1	1	1
Credit Rating (numerical)	436	8.862	5.257	0	8	10	13	16
Developed Reserves/Production	434	8.27	3.36	3.81	6.21	7.83	9.63	13.79

Table 3: Determinants of Hedging Intensity

The table reports results from estimating the following ordinary least squares regression:

$$HedgeRatio_{it+1} = \alpha + \beta X_{it} + \gamma_i + \delta_t + \varepsilon_{it}$$

X_{it} is a vector of theoretically motivated determinants of hedging intensity. Fixed effects and the lag of the dependent variable are included as indicated. T denotes time fixed effects and F denotes firm fixed effects. Standard errors, reported below coefficients, are clustered at the firm and year levels. ***, **, * denote 1%, 5%, and 10% statistical significance, respectively.

	Hedge Ratio					
	(1)	(2)	(3)	(4)	(5)	(6)
Marginal Tax Rate	-0.168	-0.062	0.005	-0.017	-0.101	-0.077
	(0.184)	(0.160)	(0.138)	(0.115)	(0.123)	(0.103)
Log (Compensation Delta)	0.083***	0.099***	-0.003	0.007	0.043*	0.048**
	(0.028)	(0.027)	(0.038)	(0.041)	(0.023)	(0.021)
Options held (as frac)	-6.550	-4.676	14.775	13.306	-5.037	-2.586
	(9.981)	(9.524)	(8.925)	(8.178)	(6.457)	(5.685)
Log Assets (BV)	-0.063	-0.109**	0.102	0.111	-0.024	-0.051
	(0.038)	(0.037)	(0.078)	(0.090)	(0.035)	(0.030)
Cash Ratio	-0.259	-0.630	0.244	0.180	0.115	-0.308
	(0.431)	(0.451)	(0.475)	(0.490)	(0.361)	(0.343)
Book Leverage	0.225	0.103	0.195	0.071	0.185*	0.048
	(0.157)	(0.135)	(0.146)	(0.110)	(0.107)	(0.090)
Dividend Payer	-0.105	-0.106	-0.057	-0.080	-0.073	-0.063
	(0.073)	(0.064)	(0.080)	(0.076)	(0.054)	(0.049)
Credit Rating (numerical)	0.000	0.007	0.010	0.006	-0.002	0.003
	(0.005)	(0.005)	(0.016)	(0.015)	(.)	(0.002)
Return on Assets	0.222	0.101	0.229**	0.072	0.210	0.050
	(0.156)	(0.125)	(0.095)	(0.061)	(0.135)	(0.090)
Investment intensity	-0.009	-0.242	0.162	0.132	-0.032	-0.127
	(0.244)	(0.351)	(0.218)	(0.293)	(0.243)	(0.237)
Tobin's Q	-0.044	-0.051	0.028	0.026	-0.020	-0.045
	(0.069)	(0.073)	(0.040)	(0.036)	(0.044)	(0.047)
Lag Hedge Ratio					0.563***	0.568***
					(0.091)	(0.088)
Fixed Effects	-	T	F	T,F	-	T
Observations	254	254	253	253	240	240
Adj. R^2	0.077	0.136	0.450	0.479	0.379	0.420

Table 4: Determinants of Hedging Intensity (Excluding tax and compensation)

The table reports results from estimating the following ordinary least squares regression:

$$HedgeRatio_{it+1} = \alpha + \beta X_{it} + \gamma_i + \delta_t + \varepsilon_{it}$$

X_{it} is a vector of theoretically motivated determinants of hedging intensity. Variables related to tax convexity and managerial compensation are excluded. Fixed effects and the lag of the dependent variable are included as indicated. T denotes time fixed effects and F denotes firm fixed effects. Standard errors, reported below coefficients, are clustered at the firm and year levels. ***, **, * denote 1%, 5%, and 10% statistical significance, respectively.

	Hedge Ratio					
	(1)	(2)	(3)	(4)	(5)	(6)
Log Assets (BV)	0.034*	0.032	0.028	0.050	0.013	0.010
	(0.020)	(0.023)	(0.026)	(0.036)	(0.011)	(0.012)
Cash Ratio	-0.590***	-0.625***	-0.265*	-0.251*	-0.227**	-0.261**
	(0.156)	(0.172)	(0.127)	(0.135)	(0.107)	(0.102)
Book Leverage	0.191*	0.162	-0.083	-0.066	0.115***	0.082*
	(0.104)	(0.107)	(0.069)	(0.075)	(0.038)	(0.041)
Dividend Payer	-0.036	-0.034	-0.036	-0.031	-0.018	-0.016
	(0.044)	(0.045)	(0.037)	(0.038)	(0.023)	(0.023)
Credit Rating (numerical)	-0.004	-0.005	0.008	0.006	-0.002	-0.001
	(0.005)	(0.006)	(0.006)	(0.006)	(0.002)	(0.002)
Return on Assets	0.122	0.088	0.022	-0.026	0.100**	0.039
	(0.086)	(0.086)	(0.050)	(0.025)	(0.041)	(0.027)
Investment intensity	0.361**	0.386**	0.005	0.016	-0.027	0.003
	(0.149)	(0.180)	(0.087)	(0.105)	(0.076)	(0.100)
Tobin's Q	0.049	0.052	0.042**	0.044**	0.053***	0.035
	(0.033)	(0.041)	(0.019)	(0.018)	(0.015)	(0.021)
Lag Hedge Ratio					0.618***	0.634***
					(0.045)	(0.045)
Fixed Effects	-	T	F	T,F	-	T
Observations	647	647	642	642	568	568
Adj.R^2	0.088	0.098	0.541	0.555	0.492	0.508

Table 5: Hedging Intensity and Gross Profitability

The table reports results from estimating the following ordinary least squares regression:

$$HedgeRatio_{it+1} = \alpha + \beta X_{it} + \gamma_i + \delta_t + \varepsilon_{it}$$

The independent variable is the *Return on Assets*. Fixed effects and the lag of the dependent variable are included as indicated. T denotes time fixed effects and F denotes firm fixed effects. Standard errors, reported below coefficients, are clustered at the firm and year levels. ***, **, * denote 1%, 5%, and 10% statistical significance, respectively.

	Hedge Ratio					
	(1)	(2)	(3)	(4)	(5)	(6)
Return on Assets	0.065	0.067	0.078***	0.066**	0.063***	0.058**
	(0.047)	(0.059)	(0.014)	(0.023)	(0.019)	(0.021)
Lag Hedge Ratio					0.636***	0.647***
					(0.062)	(0.056)
Constant	0.375***	0.375***	0.375***	0.375***	0.144***	0.140***
	(0.030)	(0.028)	(0.004)	(0.000)	(0.028)	(0.024)
Fixed Effects	-	T	F	T,F	-	T
Observations	436	436	436	436	389	389
Adj.R^2	0.003	0.004	0.463	0.465	0.404	0.421

Table 6: Hedging Intensity and Profitability: Dissection

The table reports results from estimating the following ordinary least squares regression:

$$HedgeRatio_{it+1} = \alpha + \beta X_{it} + \gamma_i + \delta_t + \varepsilon_{it}$$

The independent variable in the top panel is *Operating Profitability* while in the bottom panel it is the *Gain/Loss on Hedging*. Fixed effects and the lag of the dependent variable are included as indicated. T denotes time fixed effects and F denotes firm fixed effects. Standard errors, reported below coefficients, are clustered at the firm and year levels. ***, **, * denote 1%, 5%, and 10% statistical significance, respectively.

	Hedge Ratio					
	(1)	(2)	(3)	(4)	(5)	(6)
Operating Profitability	-0.987***	-1.118***	-0.470***	-0.506**	-0.660***	-0.689***
	(0.202)	(0.250)	(0.137)	(0.208)	(0.129)	(0.187)
Lag Hedge Ratio					0.596***	0.606***
					(0.060)	(0.056)
Constant	0.528***	0.548***	0.447***	0.453***	0.261***	0.262***
	(0.046)	(0.048)	(0.021)	(0.030)	(0.039)	(0.046)
Fixed Effects	-	T	F	T,F	-	T
Observations	436	436	436	436	389	389
Adj.R^2	0.091	0.094	0.472	0.476	0.438	0.451

	Hedge Ratio					
	(1)	(2)	(3)	(4)	(5)	(6)
Gain/Loss on Hedging	0.083*	0.073	0.098***	0.079***	0.087***	0.076***
	(0.041)	(0.052)	(0.012)	(0.020)	(0.002)	(0.006)
Lag Hedge Ratio					0.637***	0.649***
					(0.062)	(0.056)
Constant	0.381***	0.380***	0.382***	0.380***	0.149***	0.144***
	(0.030)	(0.028)	(0.004)	(0.000)	(0.027)	(0.025)
Fixed Effects	-	T	F	T,F	-	T
Observations	436	436	436	436	389	389
Adj.R^2	0.007	0.005	0.468	0.467	0.409	0.425

Table 7: Model Implications for Hedging Intensity: Interaction

The table reports results from estimating the following ordinary least squares regression:

$$HedgeRatio_{it+1} = \alpha + \beta_1 X_{it} + \gamma_i + \delta_t + \varepsilon_{it}$$

X_{it} is a vector of the key variables from the model. These are *Operating Profitability, Gain/Loss from Hedging, Book Leverage,* and *Developed Reserves/Production.* Fixed effects and the lag of the dependent variable are included as indicated. T denotes time fixed effects and F denotes firm fixed effects. Standard errors, reported below coefficients, are clustered at the firm and year levels. ***, **, * denote 1%, 5%, and 10% statistical significance, respectively.

	Hedge Ratio					
	(1)	(2)	(3)	(4)	(5)	(6)
Operating Profitability	-0.664***	-0.688**	-0.331**	-0.349	-0.491***	-0.481**
	(0.136)	(0.245)	(0.143)	(0.211)	(0.123)	(0.201)
Gain/Loss on Hedging	0.070	0.040	0.111***	0.082***	0.086**	0.057*
	(0.054)	(0.059)	(0.024)	(0.023)	(0.032)	(0.026)
Book Leverage	0.169	0.173	0.137**	0.134*	0.111**	0.102**
	(0.108)	(0.120)	(0.063)	(0.066)	(0.038)	(0.039)
Developed Reserves/Production	0.018***	0.019***	0.009	0.010**	0.007***	0.008***
	(0.004)	(0.005)	(0.005)	(0.004)	(0.002)	(0.002)
Lag Hedge Ratio					0.569***	0.577***
					(0.064)	(0.056)
Constant	0.283***	0.273***	0.319***	0.310***	0.154***	0.146**
	(0.050)	(0.070)	(0.030)	(0.042)	(0.033)	(0.054)
Fixed Effects	-	T	F	T,F	-	T
Observations	434	434	434	434	387	387
Adj.R^2	0.145	0.148	0.486	0.484	0.450	0.457

Table 8: Model Implications for Hedging Intensity: Interaction

The table reports results from estimating the following ordinary least squares regression:

$$HedgeRatio_{it+1} = \alpha + \beta_1 X_{it} + \beta_2 Resv_{it} \times Lev_{it} + \gamma_i + \delta_t + \varepsilon_{it}$$

X_{it} is a vector of the key variables from the model. These are *Operating Profitability, Gain/Loss from Hedging, Book Leverage,* and *Developed Reserves/Production.*The independent variable of interest, $Resv_{it} \times Lev_{it}$, is the interaction of *Developed Reserves/Production* and *Book Leverage.* Fixed effects and the lag of the dependent variable are included as indicated. T denotes time fixed effects and F denotes firm fixed effects. Standard errors, reported below coefficients, are clustered at the firm and year levels. ***, **, * denote 1%, 5%, and 10% statistical significance, respectively.

	Hedge Ratio					
	(1)	(2)	(3)	(4)	(5)	(6)
Operating Profitability	-0.669***	-0.720**	-0.340**	-0.400*	-0.476***	-0.479*
	(0.130)	(0.243)	(0.129)	(0.208)	(0.127)	(0.202
Gain/Loss on Hedging	0.056	0.028	0.099***	0.070**	0.076*	0.048
	(0.056)	(0.061)	(0.024)	(0.027)	(0.036)	(0.030
Book Leverage	-0.235	-0.229	-0.232*	-0.227*	-0.271	-0.259
	(0.226)	(0.225)	(0.121)	(0.120)	(0.162)	(0.160
Developed Reserves/Production	-0.001	-0.000	-0.009	-0.009	-0.010	-0.009
	(0.009)	(0.011)	(0.007)	(0.007)	(0.008)	(0.008
Book Leverage × Developed Reserves/Production	0.060*	0.061*	0.055**	0.056**	0.058**	0.056*
	(0.032)	(0.033)	(0.019)	(0.019)	(0.025)	(0.026
Lag Hedge Ratio					0.555***	0.564**
					(0.066)	(0.058
Constant	0.413***	0.410***	0.443***	0.445***	0.273***	0.262**
	(0.077)	(0.101)	(0.049)	(0.059)	(0.079)	(0.087
Fixed Effects	-	T	F	T,F	-	T
Observations	434	434	434	434	387	387
Adj.R^2	0.160	0.164	0.497	0.495	0.461	0.468

Appendix

A. An Example of Oil/Gas Reserve Dynamics

We provide a real example of natural gas reserves, using the 2006 10-K filings of Chesapeake Energy Corporation .[18] The example illustrates how production directly reduces the stock of reserves.

Table A1: An Example of Oil/Gas Reserve Changes

This table presents the summary of changes in estimated reserves of Chesapeake for the fiscal year 2006. Its original 10-K filings are available here. Here, mbbl is the one thousand barrels of crude oil; mmcf is one thousand cubic feet of natural gas; mmcfe measures the total energy measuring in mmcf, where one mbbl of oil is considered equivalent to six mmcf of gas.

	Oil (mbbl)	Gas (mmcf)	Total (mmcfe)
December 31, 2006			
Proved reserves, beginning of period	103,323	6,900,754	7,520,690
Extensions, discoveries and other additions	8,456	777,858	828,594
Revisions of previous estimates	(3,822)	539,606	516,676
Production	(8,654)	(526,459)	(578,383)
Sale of reserves-in-place	(3)	(123)	(141)
Purchase of reserves-in-place	6,730	627,798	668,178
Proved reserves, end of period	106,030	8,319,434	8,955,614
Proved developed reserves:			
Beginning of period	76,238	4,442,270	4,899,694
End of period	76,705	5,113,211	5,573,441

[18] The link is here.

B. Details of Model Solutions

First notice that the problem is homogeneous of degree one with respect to (K_t, B_t, H_t). Therefore, we can denominate everything by the capital K_t, and $V(K_t, B_t, P_t) = K_t v(b_t, P_t)$.

$$v(b_t, h_t, P_t) = \max_{h_{t+1}, b_{t+1}, i_t, q_t} \left\{ d_t + \frac{i_t - \xi q_t + 1 - \delta}{1+r} E\left(v\left(b_{t+1}, h_{t+1}, P_{t+1}\right)\right) \right\}$$

$$\begin{aligned}
\text{s.t. } & d_t = (i_t - \xi q_t + 1 - \delta) b_{t+1} - \left(1 + r^B\right) b_t + P_t q_t - \frac{c}{2} q_t^2 - i_t - \frac{\phi}{2} i_t^2 + \left[(1+r) P_{t-1} - P_t\right] h_t \\
& P_{t+1} = (1+r) P_t + \sigma \varepsilon_{t+1}\,, \\
& \eta \geq \left(1 + r^B\right) b_{t+1} + \sigma h_{t+1}\,,\ [\mu_{t+1}] \\
& d_t \geq 0\,.\ [\lambda_t]
\end{aligned}$$

Here the corresponding Lagrangian multipliers are in the squared brackets on the same line.

First of all, the first order conditions (FOCs) with respect to i_t and q_t are

$$\begin{aligned}
0 &= (1 + \lambda_t)(b_{t+1} - 1 - \phi i_t) + \frac{1}{1+r} E(v_{t+1}) \\
0 &= (1 + \lambda_t)(-\delta b_{t+1} + P_t - c q_t) - \frac{\xi}{1+r} E(v_{t+1})
\end{aligned}$$

which gives

$$\frac{P_t - c q_t}{\xi} = 1 + \phi i_t = \frac{1}{1+\lambda_t} \frac{\partial E(V_{t+1})}{\partial K_{t+1}} = \frac{1}{1+\lambda_t} \frac{1}{1+r} E(v_{t+1}) + b_{t+1}\,.$$

Here, the marginal value of capital is divided into two parts, first through the scaled value function v, second through its impact on the leverage ratio b_{t+1}.

Second, the FOCs with respect to h_2 and b_2 are

$$\frac{i_t - \delta q_t + 1}{1+r} \frac{\partial E(v_{t+1})}{\partial h_{t+1}} - \mu_2 \sigma = 0\,, \tag{7}$$

$$(1 + \lambda_t)(i_t - \delta q_t + 1) + \frac{i_1 - \delta q_1 + 1}{1+r} \frac{\partial E(v_{t+1})}{\partial b_{t+1}} - \mu_2 \left(1 + r^B\right) = 0\,. \tag{8}$$

Therefore, optimal hedging without collateral constraint ($\mu_{t+1} = 0$) gives perfect hedge across states,

$$0 = \frac{\partial E(v_{t+1})}{\partial h_{t+1}} = E\left(\frac{\partial v_{t+1}}{\partial h_{t+1}}\right)\,.$$

And if the collateral constraint is binding, $\mu_{t+1} > 0$, we have

$$0 = 1 + \lambda_t + \frac{1}{1+r}\left(\frac{\partial E\left(v_{t+1}\right)}{\partial b_{t+1}} - \frac{1+r^B}{\sigma}\frac{\partial E\left(v_{t+1}\right)}{\partial h_{t+1}}\right). \tag{9}$$

That is, when a firm is choosing its optimal leverage, it considers the gross shadow price at date 1 $(1+\lambda_t)$, its marginal costs for future values, $\frac{\partial E(v_{t+1})}{\partial b_{t+1}}$, as well as its externality on continuation value through its effects on hedging due to the collateral constraint.

We then solve the problem backward. At date 3, because the company is liquidated, it has no debt or hedging positions. $B_3 = H_3 = 0$. Collateral constraint is satisfied.

Date 2 optimal problem. At date 2, given (P_2, B_2, H_2, K_2), the firm solves the following problem

$$\begin{aligned}
& v_2\left(P_2, b_2\right) \equiv \max_{i_2, q_2}\left[d_2 + \underline{v}\left(i_2 - \delta q_2 + 1\right)\right] \\
\text{s.t. } & d_2 = -\left(1+r^B\right) b_2 + \left[P_2 q_2 - \frac{1}{2} c q_2^2 - i - \frac{1}{2}\phi i_2^2\right] + \left(\left(1+r\right) P_1 - P_2\right) h_2 \geq 0 \\
& i_2 - \delta q_2 + 1 \geq 0\,.
\end{aligned}$$

For convenience, we use superscript $+$ to denote the case when the price is high, i.e, $P_2 = P_2^+ \equiv (1+r) P_1 + \sigma$; similarly, when price is low, $P_2 = P_2 = P_2^- \equiv (1+r) P_1 - \sigma$, we use superscript "$-$". Denote the maximum liquidity the firm can attain at date 2,

$$w_2 \equiv \left(\left(1+r\right) P_1 - P_2\right) h_2 - \left(1+r^B\right) b_2 + \frac{1}{2c} P_2^2 + \frac{1}{2\phi}\,. \tag{10}$$

w_2 can be understood as the net liquidity $\left(\left(1+r\right) P_1 - P_2\right) h_2 - \left(1+r^B\right) b_2$ plus two other terms, $\frac{1}{2c}P_2^2 + \frac{1}{2\phi}$. The two other terms denote the maximum liquidity that can be achieved from production and investment to liquidities.

When the financial constraint $d_2 \geq 0$ is not binding, i.e., $\lambda_2 = 0$, we have

$$\begin{aligned}
q_2^* =& \frac{P_2 - \delta \underline{v}}{c}, \\
i_2^* =& \frac{\underline{v} - 1}{\phi}, \\
v_2^u =& w_2 + \frac{1}{2}\left(\frac{2\delta - \delta^2}{c} + \frac{1}{\phi}\right)\underline{v}^2 + \left(1 - \frac{1}{\phi} - \frac{P_2}{c}\right)\underline{v}\,.
\end{aligned}$$

And this case can happen if and only if $w_2 > \frac{1}{2}\left(\frac{\delta^2}{c}+\frac{1}{\phi}\right)\underline{v}^2$. Note that v_2^u is linear in both hedging gain $((1+r)P_1 - P_2)h_2$ and borrowing b_2 here.

When the financial constraint is binding, i.e., $\lambda_2 > 0$, we have

$$q_2^* = \frac{P_2 - \xi\sqrt{2w_2/\left(\frac{\delta^2}{c}+\frac{1}{\phi}\right)}}{c}$$

$$i_2^* = \frac{\sqrt{2w_2/\left(\frac{\delta^2}{c}+\frac{1}{\phi}\right)}-1}{\phi}$$

$$v_2^c = \underline{v}\left(i_2^* - q_2^* + 1\right) = \underline{v}\left[\sqrt{2w_2\left(\frac{\delta^2}{c}+\frac{1}{\phi}\right)}+1-\frac{1}{\phi}-\frac{P_2}{c}\right].$$

Note that v_2^c is increasing and concave in hedging gain $((1+r)P_1 - P_2)h_2$ but decreasing and convex in the borrowing b_2.

The financial constraint must be binding in at least one state at date 2 if the collateral constraint is not binding. If it was not binding in both states, then the expected marginal value of net debt is $\frac{\partial E(v_{t+1})}{\partial b_{t+1}} = -\left(1+r^B\right)$. After discounted, it is $-\frac{1+r^B}{1+r} \geq -1$. So the firm can pay out one more dollar at date 1 by borrowing one more dollar, and then pays back $1+r^B$ dollar at date 2. This increases the date 1 value function by $1-\frac{1+r^B}{1+r} \geq 0$, which suggest the case is not optimal or at least not uniquely optimal. To make the problem realistic, we assume that the financial constraint at date 2 when price is low, i.e., $P_2 = P_2^- = (1+r)P_1 - \sigma$, is always binding. In this case, we can use equation (7)-(9) to solve the relation between hedging h_2 and borrowing b_2:

$$\begin{aligned}
&\hat{h}_2(b_2, P_1) \\
&\equiv \arg\max_{h_2} \frac{1}{2}\left[v_2^+(b_2,P_1) + v_2^-(b_2,P_1)\right] \\
&= \begin{cases}
\frac{1}{\sigma}\left\{\frac{1}{2}\left(\frac{\delta^2}{c}+\frac{1}{\phi}\right)\underline{v}^2 + \left(1+r^B\right)b_2 - \frac{1}{2c}\left[(1+r)P_1-\sigma\right]^2 - \frac{1}{2\phi}\right\} & \text{if } \mu_2 = 0,\ \lambda_2^+ \geq 0 \geq \lambda_2^-; \\
\frac{1}{c}(1+r)P_1 & \text{if } \mu_2 = 0,\ \lambda_2^+, \lambda_2^- > 0; \\
\frac{1}{\sigma}\left[\eta - \left(1+r^B\right)b_2\right] & \text{if } \mu_2 > 0
\end{cases}
\end{aligned}$$

Date 1 optimal problem. Given the value function at date 2, date 1 firm maximizes

$$v\left(w_1, P_1\right) = \max_{h_2, b_2, i_1, q_1} \left\{ d_1 + \frac{i_1 - \delta q_1 + 1}{1+r} \left[\frac{1}{2} \left(v_2^+ + v_2^- \right) \right] \right\}$$

$$\text{s.t. } d_1 = w_1 + \left(i_1 - \delta q_1 + 1\right) b_2 - \frac{1}{2} c \left(q_1 - \frac{P_1}{c} \right)^2 - \frac{1}{2} \phi \left(i_1 + \frac{1}{\phi} \right)^2 .$$

where $w_1 \equiv -\sigma \varepsilon_1 h_1 - \left(1 + r^B\right) b_1 + \frac{1}{2c} P_1^2 + \frac{1}{2\phi}$ is the maximum liquidity without borrowing at date 1.

We focus on the case when the budget constraint is binding, which gives

$$q_1^* = \frac{P_1 - b_2 - \frac{\hat{v}_2}{\lambda_1 (1+r)}}{c}$$

$$i_1^* = \frac{b_2 + \frac{\hat{v}_2}{\lambda_1 (1+r)} - 1}{\phi}$$

where the Lagrangian multiplier, $\lambda_1 = \frac{E(v_{t+1})}{1+r} \left[\frac{w_1 + \left(1 - \frac{P_1}{c} - \frac{1}{\phi}\right) b_2}{\frac{1}{2}\left(\frac{1}{c} + \frac{1}{\phi}\right)} + b_2^2 \right]^{-\frac{1}{2}}$ in the benchmark case $\delta = 1$. And the optimal b_2 is given by the solution of (9). We solve all scenarios, where b_2 has explicit solutions as roots of quadratic equations with P_1 and w_1. Instead of writing them explicitly, we present the solutions in figures, as in section 5.