

The Allocation of Socially Responsible Capital

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Abstract

A rapidly increasing share of asset allocation decisions incorporate social values in addition to financial considerations. We argue that the most common strategies for socially motivated investing, which only consider the social value of the firms in an investors' portfolio, are misguided. We develop a tractable framework in which commercial and social investors compete, and identify alternative strategies for social investors that result in higher social welfare and deliver higher financial returns. We discuss several normative implications for socially-motivated investors. From the enterprise perspective, we demonstrate that a focus on increasing profitability can have a greater social impact than a focus on direct social value creation.

1 Introduction

The last several decades have seen an invigoration of socially-motivated investing. Investors who target companies with socially desirable properties, such as their environmental stewardship, social responsibility, and good governance practices (collectively referred to as ESG) argue doing so increases the relative valuation of companies with these properties, incentivizing their broader adoption and thus tilting corporate values toward their own. The market comprising this style of investing is large and rapidly growing. One study finds that a quarter of assets under management of professional investors are allocated with such considerations ([US SIF Foundation, 2018](#)). Sustainability oriented U.S. mutual funds saw over \$20 billion of inflows in 2019, quadruple the level the previous year. In 2019, 483 conventional U.S. mutual funds added ESG consideration language to their prospectuses, with a total of 564 such funds managing \$933 billion in assets ([Morningstar, 2020](#)).

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We argue that traditional socially-motivated investment strategies may be misguided. The most common sustainable investment strategies are guided by the financial return and social value of the companies in an investor's own portfolio. Yet a growing number of critics argue that an investor's contribution to social value should not be judged by the social value of their portfolio companies, but rather by the additional social value created relative to if the investor did not exist at all (e.g. [Brest et al., 2016](#)). The distinction between these two views is driven by the fact that many companies that have high social value could attract investment from traditional investors with only a financial motivation, and therefore socially motivated investors who prioritize these companies may not be contributing to social value. We formalize this critique in an economic model and flesh out its normative implications for the behavior of social investors. To understand the basic logic of why social investors who attend only to the social value of their portfolio companies might achieve sub-optimal outcomes, consider the following highly stylized example.

Suppose there are two investors each of whom holds one unit of capital. One *commercial investor* cares only about financial returns and the other *social investor* cares about both financial returns and social value. Suppose further that there are three enterprises, each of whom needs one unit of capital to survive.

- Firm *A* creates 10 units of profit and 10 units of social value.
- Firm *B* creates 9 units of profit and 0 units of social value.
- Firm *C* creates 8 units of profit and 10 units of social value.

Further suppose that the social investor picks which company to support before the commercial investor does. A social investor who only accounts for the profitability and social value of the company in their portfolio would pick Firm *A*, leaving the commercial investor to pick Firm *B*. As a result, the social investor enjoys 10 units of profit and 10 units of social value are created. If instead the social investor took a holistic view, they would appreciate that Firm *A* could attract the support of the commercial investor. The social investor might then prefer to support Firm *C*, leaving the commercial investor to support Firm *A*. In this case the social investor would enjoy a financial return of 8, and 20 units of social value would be created.

We develop a framework to embed this logic in a competitive financial market, in which many commercial investors and social investors coexist and prices are determined in equilibrium. To highlight the nuances arising from the two approaches to social investing in the previous example, we introduce two types of social investors. In addition to their financial return, *naïve social investors* care about the social welfare created by the companies they support. *Sophisticated social investors* are similar, but care about total social welfare, not just the social value created by the companies in which they invest. Beyond admitting a tractable analysis of equilibrium behavior, our model yields

several normative implications for social investors and entrepreneurs.

First we demonstrate that capital held by different classes of investors has different social values, reflecting the investors' contribution to social welfare. We define the social value of capital for a particular class of investors as the increase in social welfare associated with marginally expanding the pool of capital held by those investors. Both types of social investors have socially valuable capital. However, naïve investors have a lower social value of their capital. Relative to sophisticated investors, naïve investors over-prioritize companies that have both high social impact and high profitability. These companies could attract commercial investment independent of whether social investors support them. Naïve social investors who finance these companies bid up their prices, crowding out commercial investors, who instead finance less profitable and socially impactful firms.

While naïve social investors do lower the cost of capital for socially valuable projects, their displacement of commercial investors also lowers the cost of capital for, and enables financing of, projects that the naïve social investors themselves would not want to support. Sophisticated social investors, by definition, internalize this effect of their investment behavior and do not support projects that could have been financed by commercial investors in their absence.

An important implication of this result is that there are improvements to the investment strategy of naïve social investors. Any capital that naïve investors deploy to fund profitable but also impactful projects that could have been commercially financed should be redeployed to projects with lower profitability (and potentially lower social value). Perhaps surprisingly, this can increase both the social impact *and* financial return to this capital. Why? As the result of naïve investors competing with each other to own shares in certain projects, they are both pushing commercial investors into lower impact projects *and* transferring excessive value to entrepreneurs (or a firm's existing owners). Instead investing directly in projects that are less profitable but more impactful than what a displaced commercial investor would have chosen eliminates both forces, creating higher total social value and delivering higher financial returns to the social investor.

These results highlight the importance of viewing socially aware capital as a scarce resource. Exactly because of this scarcity, enterprises can be more impactful if they reduce their dependence on social capital. The social impact of an enterprise, sometimes called enterprise impact (Brest et al., 2016) depends not only on the amount of capital used by the enterprise, but also on the class of capital used by the enterprise. *All else equal*, enterprises that attract the capital of socially minded investors have lower contribution to social welfare than those that attract the capital of purely commercial investors. Holding fixed the social value created by a firm, it can raise its enterprise impact by reducing its dependence on social capital, freeing social capital to fund another enterprise that is unable to obtain commercial financing. The more profitable is a firm, the less likely it is to rely on scarce, socially valuable capital. Our framework thus provides a new connection between the profitability of

an enterprise and its contribution to social welfare.

Our baseline model considers an environment where all firms have projects of binary scale. We also extend our model to investigate the implications of socially responsible investing when firms have an intensive margin of scale. When there is just a single security for which all investors must pay the same price, social investors can only subsidize a firm's new projects if they are the marginal investor in the security. Enabling new projects therefore requires social investors not only to finance the new projects, but also to displace the firm's existing commercial investors. The social capital used to displace commercial investors generates no marginal social impact. The result is that sophisticated social investors prioritize firms with high social value marginal projects, especially at firms that would attract relatively little commercial capital in the social investors' absence. In this sense, sophisticated social investors participating in standard equity markets prefer supporting small firms. We then demonstrate that the creation of a second security, similar to existing "green bonds," provides more scope for social investors to generate impact. These bonds allow social investors to provide low-cost financing for socially valuable projects without displacing a firm's existing commercial investors, as the second security allows social investors to provide financing at different terms than commercial investors.

Finally we consider an extension in which social investors exhibit varying degrees of altruism. When social investors are naïve we identify a familiar result: social investors and entrepreneurs exhibit positive assortative matching, in that investors who care more about social welfare match to entrepreneurs that create more social value. However when social investors are sophisticated, this result partially reverses. Holding fixed the level of an entrepreneur's profitability, social investors who value social welfare more highly match to entrepreneurs who create less social value. This result arises from the fact that social investors have interdependent utility in that they internalize the social value created by all firms that receive financing. We discuss this more fully in Section 6.

This paper contributes to the literature on investing with social preferences. A number of papers study financing environments where social and commercial investors coexist, and ask how social investors should behave to maximize their impact. [Oehmke and Opp \(2019\)](#) and [Landier and Lovo \(2020\)](#) both study activist social investors who aim to resolve a moral hazard problem amongst entrepreneurs. The two papers investigate how the amount of social and commercial capital influences the bargaining power of social investors and their resulting social impact. [Broccardo et al. \(2020\)](#) highlights that if activist investors all vote as if they are pivotal, well-diversified investors with even a small concern for social welfare will vote in line with the social planner. Like in our analysis, these papers consider social investors who care about social value independently of if it was created by the companies they support. In contrast to these papers, we study passive investors in a complete information environment, whose goal is to enable new projects by offering cheaper capital to firms with socially valuable projects.

Pedersen et al. (2019) and Pastor et al. (2020) study social and commercial investors within a Markowitz framework, derives the optimal portfolios for each class of investors and shows an ESG-adjusted CAPM emerges. The social investors in their framework maximize a combination of financial return and the impact of firms in their portfolio, and hence correspond to our naïve social investors.¹ Heinkel et al. (2001) demonstrates that when socially motivated investors divest from socially unproductive firms their stock price declines, as the remaining investors hold more concentrated portfolios. In contrast to these papers, we study a model without uncertainty, and focus our analysis on the behavior of sophisticated social investors who aim to maximize social welfare rather than the impact of their own portfolio.

Several papers analyze the behavior of individual firms and their prosocial investors. Focusing on the single investor case, Chowdhry et al. (2019) and Roth (2020) analyze when a socially minded investor can have more impact through an investment in a social enterprise than they can through a grant. Hart and Zingales (2017) fleshes out several cases for a stakeholder view of the firm, in which management’s objective encompasses more than profits alone, and considers arrangements by which activist shareholders can induce the firm to take socially efficient actions. Morgan and Tumlinson (2019) argue that corporate social responsibility might be a vehicle to overcome collective action concerns amongst a firm’s prosocial shareholders, and argues that corporate social responsibility might be an efficient channel for prosocial actions when the firm’s production imposes societal externalities. This latter point is also highlighted in Nilsson and Robinson (2017).

On the empirical front, a growing body of research assesses the financial performance of investors with social motives, and has reached mixed conclusions (e.g. Hong and Kacperczyk, 2009, El Ghouli et al., 2011, Eccles et al., 2014, Gray et al., 2015, Kovner and Lerner, 2015, Barber et al., 2017, Baker et al., 2018). In our analysis we will assume that investors are symmetrically informed, and thus our social investors, who maximize a combination of financial and social goals, always enjoy lower financial performance than commercial investors, who maximize financial performance alone.

Our study also relates to the broader economic literature on altruistic motives. Andreoni (1990) highlights the distinction between “pure altruists,” who derive utility from social welfare, and “impure altruists” who derive utility, or “warm glow” from having directly improved social welfare. In this light, our sophisticated social investors can be understood as pure altruists, and our naïve social investors can be understood as impure altruists.

¹Pastor et al. (2020) considers an extension in which investors incorporate the social value of all firms into their objective function, but the analysis assumes that individual investors cannot meaningfully influence the economy and hence their preferences for financial return and the social value of firms within their own portfolio fully determine their investment decisions. In contrast our sophisticated social investors value their own financial return on the same order as their contribution to total social value, and hence the fact that they partially internalize the value created by all firms does influence their decisions.

Our analysis bears a technical resemblance to assignment matching models, commonly employed in trade and labor economics (e.g. Roy, 1951, Becker, 1973, Sattinger, 1979, Costinot and Vogel, 2010). We contribute to this literature by providing a model in which agents sort along two dimensions of heterogeneity, as in Gola (2020), and by studying an environment where one side of the market has interdependent utility in the sense that they care not only about their own match but also the matches of others. As discussed in Section 6, we show that this later feature can partially reverse the classic result of positive assortative matching.

The rest of the paper proceeds as follows. In Section 2 we outline the model for the case where entrepreneurs have binary projects. In Section 3 we analyze the model separately for the case where all social investors are naïve and for the case where all social investors are sophisticated. In Section 4 we study a market in which both types of social investors coexist. In Section 5 we enrich the model such that entrepreneurs have projects with variable scales and in Section 6 we consider investors with varying degrees of concern for social welfare. Section 7 concludes.

2 Model Setup

Players, Technology, and Contracts

There is a mass E of entrepreneurs. Each one is endowed with a project that requires one unit of capital. If entrepreneur i receives the requisite capital, his project returns $\pi_i \in \mathbb{R}^+$ profit and $w_i \in \mathbb{R}^+$ “social value,” where π_i and w_i represent the private and social return of the project respectively.²³ We assume that π_i and w_i have atom-less and full-support distribution with CDF F , and that the features of each project are perfectly observable to all players.

There is a mass C of commercial investors and a mass S of social investors, each of whom controls one unit of capital. We assume that $C + S < E$ so that some projects go unfinanced.

A contract between some investor and an entrepreneur i specifies the transfer of one unit of capital from the investor to the entrepreneur at a price $p_i \geq 1$. The investor receives financial return p_i on their invested capital, the entrepreneur receives $\pi_i - p_i$, and w_i social value is created. Because we are studying a complete information environment without contracting frictions, this contract can be understood as either debt or equity.

²We assume that w_i encompasses the full social return of the project, including the private return π_i , as well as any consumer and employee surplus and externalities arising from the project.

³We restrict attention to $w_i \geq 0$ for expositional simplicity. In Appendix A.6 we discuss technical nuances that arise from allowing for $w_i < 0$.

Preferences

Index investors and entrepreneurs such that investor i matches with entrepreneur i . Each entrepreneur's utility is their share of the profit, $(\pi_i - p_i)di$ and each commercial investor's utility is their return on capital p_idi . We will separately examine two classes of social investors.

Naïve social investors receive utility from their financial return and from the social value created by the entrepreneur they've financed. That is, naïve social investor i 's utility is

$$(p_i + \theta w_i)di,$$

where θ represents the strength of investor i 's social preference.

Sophisticated social investors receive utility from their income and from the total social value created by all entrepreneurs who receive financing. That is, their utility is

$$p_idi + \theta \int_{j \in \bar{E}} w_j dj = (p_i + \theta w_i)di + \theta \int_{j \in \bar{E} \setminus i} w_j dj,$$

where \bar{E} is the set of entrepreneurs who receive financing.⁴ Thus we can observe that the difference between the utility functions of naïve and sophisticated social investors is that naïve investors do not derive utility from the social output generated by entrepreneurs they do not finance, while sophisticated social investors derive utility equally from all social output regardless of who financed it.

Naïve investors are referred to as such because they do not fully internalize the implications of their investment decision on social welfare. This does not imply that the preferences of naïve investors are incorrect. Naïve social preferences may more closely represent the *behavior* of so-called socially conscious investors, whose investment decisions are guided by measures such as ESG scores. In contrast, sophisticated social preferences may more faithfully represent the *intentions* of real-world social investors to affect social change, and as such our analysis of the behavior of sophisticated social investors might offer normative guidance to real-world investors with social preferences.

Timing of Actions

The central focus of our analysis is to understand how social investors make decisions given that some entrepreneurs can receive commercial financing and some cannot. Therefore we study a model

⁴The objective function of a sophisticated social investor can be formalized by viewing the model as the limit of a sequence of games each with a discrete but increasing number of investors, n . In the discrete version of the game, each sophisticated social investor has utility $\frac{1}{n} \left((p_i + \theta w_i) + \sum_{j \in \bar{E} \setminus i} w_j \right)$, so that the sophisticated social investor treats their profits on the same scale as their contribution to social value.

in which entrepreneurs’ “outside option” is to seek capital on the commercial market if they cannot attract capital from social investors. This is relevant not only for determining the price of equity that social investors must offer, but also for the determination of which projects could attract financing without the intervention of social investors.

Specifically, we study a two-period model. In period 1 each social investor offers a contract to an entrepreneur. In period 2 each commercial investor observes the period 1 actions and then offers a contract to an entrepreneur. Entrepreneurs who have received at least one contract then choose at most one contract to accept, and payoffs are realized.

Equilibrium

The solution concept is Subgame Perfect Equilibrium. All investors choose contracts that are mutual best responses. Among other things this implies that no entrepreneur ever receives offers from more than one investor. Therefore we maintain the convention that entrepreneur i matches with investor i . Further, we adopt the convention that an entrepreneur i who receives no offers for financing has a price of equity $p_i = \pi_i$.

Social Welfare

Our measure of social welfare is $W = \int_{i \in \bar{E}} w_i$, where \bar{E} is the set of entrepreneurs that receive financing. Our interpretation is that w_i is the total social value created by firm i if it receives financing, including the value to the firm’s owners.⁵ Sophisticated social investors can therefore be understood to be maximizing a modified variant of social welfare that increases the weight placed on their own consumption by a multiple of $\frac{1}{1+\theta}$.

Note that we do not adopt a social welfare function that sums over the utility of entrepreneurs and investors. Doing so would induce a standard analysis of public good provision, wherein none of the investors we consider invest sufficiently in businesses that produce high w_i because they do not internalize the benefit accruing to other investors. Instead, we adopt the convention that the social planner cares about value creation w_i , but does not care about the intrinsic “altruistic” utility that social investors derive from the creation of w_i . In this sense we follow [Hart and Zingales \(2017\)](#) and [Broccardo et al. \(2020\)](#).

⁵Under this interpretation, the value accruing to the firm’s owners is determined independently of how ownership is divided, i.e. the welfare weights placed on entrepreneurs and investors are the same.

3 Preliminaries: Equilibrium Structure with Just One Type of Social Investor

To understand the behavior of naïve and sophisticated social investors we first characterize the equilibrium the model in which all investors are either naïve or sophisticated. In Section 4 we present our main results in the model in which both types of social investors coexist.

3.1 Naïve Social Investors

We begin by characterizing the equilibrium of the model where all social investors are naïve.

In the period 2 commercial market, investors' returns must be equalized across all entrepreneurs who receive financing. That is, prices offered to any two entrepreneurs i and j who are both supported by a commercial investor satisfy $p_i = p_j \equiv \bar{\pi}$. Further, for any entrepreneur k who is not, it must be either that $\pi_k \leq \bar{\pi}$, or that they have already received an offer from a social investor with price $p_k \leq \bar{\pi}$. Entrepreneurs who do not receive support in period 2 are those who cannot offer sufficiently high financial return to commercial investors.

In period 1 prices offered to any two entrepreneurs i and j who are both supported by a social investor satisfy $p_i + \theta w_i = p_j + \theta w_j$.⁶ And for an entrepreneur i supported by a social investor and an entrepreneur k who is not, prices must satisfy $p_i + \theta w_i \geq p_k + \theta w_k$.

With the above pricing equations we can now characterize the set of entrepreneurs financed by each type of investor in equilibrium. The equilibrium investment allocation is depicted in Figure 1.

Importantly, commercial investors only invest in projects with social value below some \bar{w} and profits above some $\bar{\pi}$ determined in equilibrium. Marginally profitable projects remain unfinanced, and because commercial investors could finance them in exchange for 100% of their proceeds, commercial investors earn returns of $\bar{\pi}$.

Because commercial investors support all entrepreneurs with profit higher than $\bar{\pi}$ and with social value lower than \bar{w} , $\bar{\pi} + \theta \bar{w}$ is the social investor's effective outside option utility. Prices in equilibrium are set such that social investors achieve this outside option utility. Social investors receive financial return of $\bar{\pi} - \theta(w_i - \bar{w})$. Thus, they are willing to pay (in terms of reduced financial return) for projects that generate high social value. Commercial investors do not find it attractive to invest in

⁶The preceding equality holds so long as prices are finite. In equilibrium a social investor may provide funding in exchange for zero share of the proceeds ($p = 0$) if the project has sufficiently high social impact (akin to philanthropy). In such a case, the above equality need not hold.

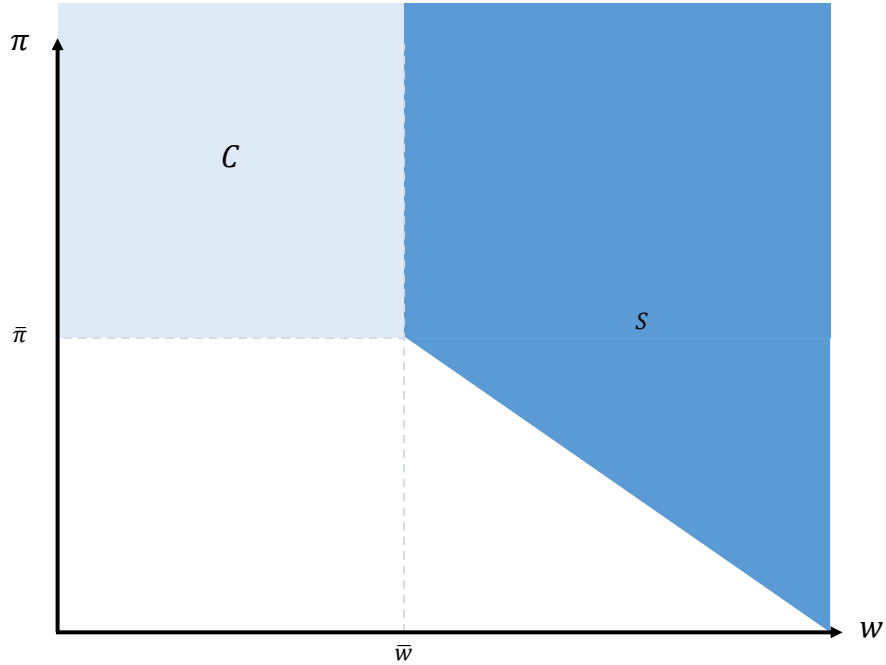


Figure 1: Equilibrium investment with naïve social investors

companies with $\pi_i > \bar{\pi}$ and $w_i > \bar{w}$ precisely because social investors are willing to invest in these companies at higher prices.

A formal characterization of equilibrium investment allocations is relegated to the Appendix Section [A.1](#).

3.2 Sophisticated Social Investors

Next we analyze the equilibrium allocation of capital when social investors are all sophisticated. The equilibrium allocation is depicted in Figure [2](#).

Similar to the case with naïve social investors, there is an equilibrium level of profit $\tilde{\pi}$ below which commercial investors do not fund projects, and all commercial investors receive returns of $\tilde{\pi}$.

In equilibrium, social investors and entrepreneurs in period 1 expect that those entrepreneurs with profits above $\tilde{\pi}$ can receive financing and earn $\pi_i - \tilde{\pi}$ from a commercial offer in period 2.⁷ Therefore, sophisticated social investors recognize that by supporting an entrepreneur i with profits $\pi_i \geq \tilde{\pi}$, their marginal contribution to social welfare is not w_i , but instead the social value corresponding to

⁷Note that this holds even if in period 1 there are more than a mass C of entrepreneurs who meet this criterion, since all players expect that in period 2 a mass of exactly C will meet this criterion.

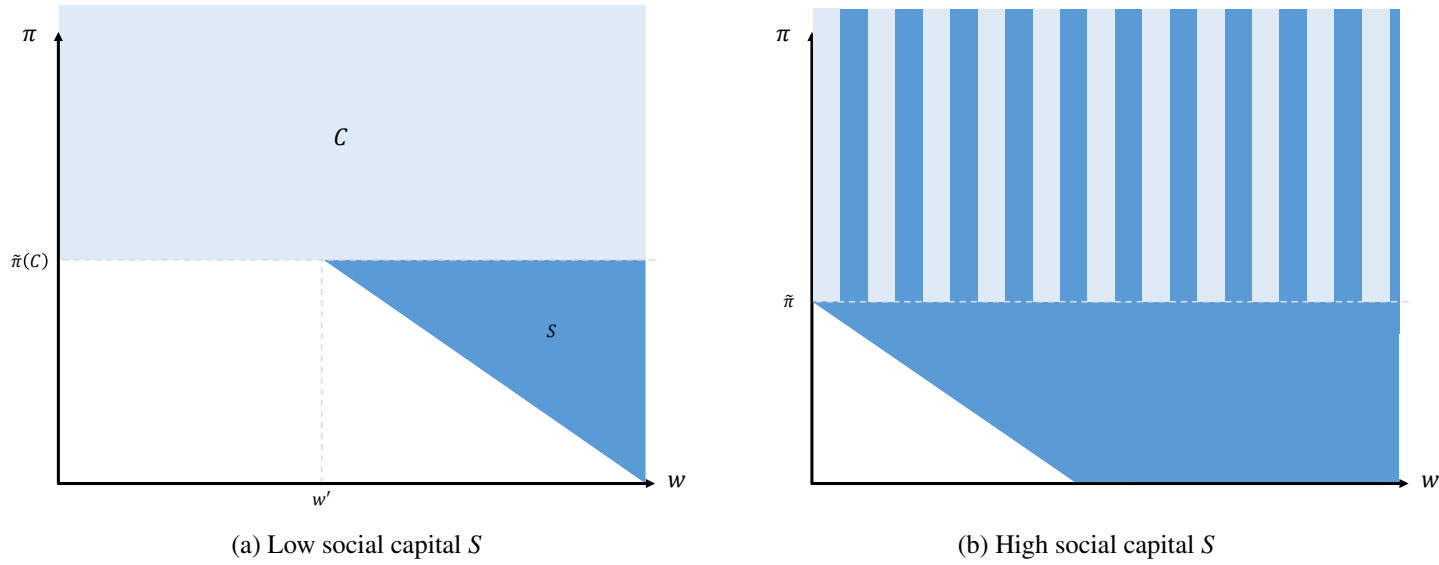


Figure 2: Equilibrium investment with sophisticated social investors

marginally expanding the set of entrepreneurs supported by commercial investors.

We next determine the price offered to any entrepreneur supported by a social investor.

Lemma 1. *For any entrepreneur i supported by a social investor, $p_i = \tilde{\pi}$ if $\pi_i \geq \tilde{\pi}$ and $p_i = \pi_i$ else.*

Lemma 1 dictates that social investors who support entrepreneurs with profit $\pi_i \geq \tilde{\pi}$ must offer contracts that allow the entrepreneur to retain $\pi_i - \tilde{\pi}$ of their profit, as this is the profit the entrepreneur could achieve from a commercial investment. In contrast, social investors who support entrepreneurs with $\pi_i < \tilde{\pi}$ can extract the entrepreneur's full profits. These entrepreneurs expect not to be able to attract commercial financing in period 2. Further, because sophisticated social investors recognize that they cannot contribute to social welfare by undercutting another social investor, and because in equilibrium all social investors prefer their investment to earning $\tilde{\pi}$ and marginally expanding the pool of commercial investments, no social investor will undercut another who is supporting an entrepreneur with profits $\pi_i < \tilde{\pi}$.

In Appendix Section A.2, we formally characterize the allocation of sophisticated social investor's capital; in this section we discuss it informally. Though the equilibrium is unique, we divide the analysis into two cases. When there is relatively little social capital, sophisticated social investors do not invest in any projects that generate at least $\tilde{\pi}$ profit, as they anticipate that all of these projects will receive commercial financing. Instead, they invest in the mass S of entrepreneurs i who have the highest $\pi_i + \theta w_i$, amongst those that generate profits $\pi_i < \tilde{\pi}$ (so that they would not receive commercial financing). This case is depicted in Figure 2a.

When there is relatively more social capital, social investors may allocate some of their capital to projects that could attract commercial financing. This is depicted in Figure 2b. This would be the case when there are not enough enterprises that cannot attract commercial capital but still generate enough social value to offset the loss in financial return.

3.3 Comparison of the Two Equilibria

Before analyzing the model where sophisticated and naïve social investors coexist, we compare social welfare across the regimes with only one type of social investor. Recall, our measure of social welfare is $W = \int_{i \in \bar{E}} w_i$, where \bar{E} is the set of entrepreneurs that receive financing in equilibrium.

Proposition 1. *Social welfare is higher when all social investors are sophisticated than when all social investors are naïve.*

Though all social investors place weight θ on social value, sophisticated social investors create more social value in equilibrium. This is because naïve social investors prioritize investment in entrepreneurs that have high social value w_i even if they could also attract investment in the commercial market. The marginal contribution of these investments to social welfare is not w_i but rather the social value created by the marginal project financed by *commercial* investors. In other words, by supporting an entrepreneur i that could have attracted commercial financing, the investor’s marginal contribution to social welfare is not w_i but rather the social value created by firm that the displaced commercial investor goes on to support. As commercial investors focus single mindedly on financial returns, the *marginal* commercial investment has relatively low social value.⁸

In contrast, sophisticated social investors focus on the set of entrepreneurs who could not attract financing on the commercial market. Relative to naïve social investors, sophisticated social investors may support entrepreneurs with lower contributions to social welfare. But the sophisticated investor’s contribution to social welfare is higher, as they are not displacing a commercial investor who would have supported the firm in their absence.

4 Main Results

In this section we discuss a number of normative results about social investing, in a market in which both sophisticated and naïve social investors coexist. In particular, we demonstrate that there

⁸Note, this does not require any assumption about the joint distribution of profits and social value amongst projects in the population. Instead we are observing that in equilibrium, the marginal projects that commercial investors support have lower social value than the projects that social investors support, as social investors place positive weight on social value when making their investment decision.

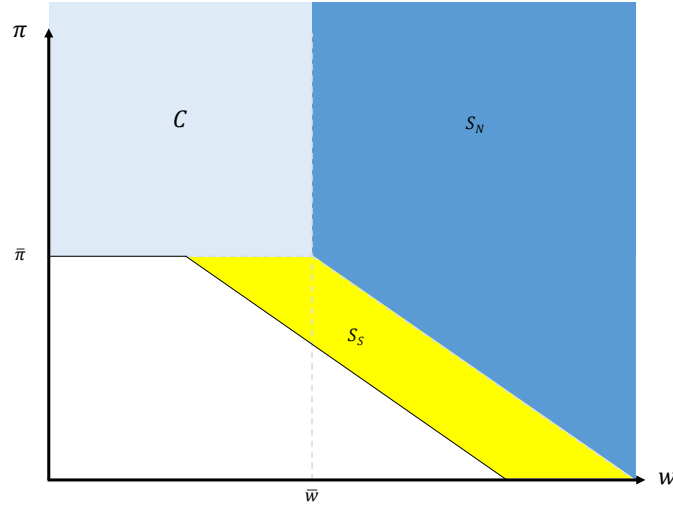


Figure 3: Equilibrium investment with both types of social investors

are investment strategies that deliver higher financial returns and create more social value than those employed by naïve social investors. And we draw a new link between the profitability of a firm and the firm’s social value.

First we characterize the equilibrium in the market with both types of social investors.

4.1 Equilibrium Structure with Both Types of Social Investors

Let there be a mass S_N of naïve social investors, and S_S sophisticated social investors, so that $S \equiv S_N + S_S$.

There is no longer a unique equilibrium in this market. Commercial investors’ behavior is still uniquely pinned down as a function of period 1 actions. But social investors may follow multiple investment profiles in equilibrium leading to different allocations of capital. In Appendix Section A.3 we characterize the full set of equilibria. In this section we focus on the unique, welfare-optimal equilibrium, depicted in Figure 3.

As in Section 3.1 there is some \bar{w} such that naïve social investors only support entrepreneurs with social value greater than \bar{w} and commercial investors only support entrepreneurs with social value less than \bar{w} . Moreover, as in Section 3.1, there exists a $\bar{\pi}$ such that entrepreneurs with profits higher than $\bar{\pi}$ and social value less than \bar{w} are supported by commercial investors.

To characterize the behavior of sophisticated social investors, the following analogue of Lemma 1 holds.

Lemma 2. *For any entrepreneur i supported by a sophisticated social investor,*

if $\pi_i \geq \bar{\pi}$ and $w_i \leq \bar{w}$, $p_i = \bar{\pi}$,

if $\min\{\pi_i, \bar{\pi}\} + \theta w_i \geq \bar{\pi} + \theta \bar{w}$ then p_i solves $p + \theta w_i = \bar{\pi} + \theta \bar{w}$ if such a p exists and $p_i = 0$ if no such p exists

and $p_i = \pi_i$ else.

The price that a sophisticated social investor pays is disciplined by the commercial market if the entrepreneur could have attracted commercial financing, and by the naïve social investors if the entrepreneur could have attracted financing from naïve social investors. Else, a sophisticated social investor can demand an entrepreneur's entire profit, as in Section 3.2.

The above lemmas apply across all equilibria. In the welfare-optimal equilibrium sophisticated social investors do not compete with either commercial investors or naïve social investors, and instead support the set of entrepreneurs i who maximize $\pi_i + \theta w_i$ amongst those who could not attract financing from other investors. We defer formal characterization of this equilibrium to Appendix A.3. For the remainder of our analysis we focus on the welfare-optimal equilibrium to illuminate the model's comparative statics.

4.2 The Social Value of Capital

Next we discuss the social value of capital held by various investors, defined to be the social value created by marginally expanding a given pool of capital. Let $W(C, S_N, S_S)$ be the total social value created in the investor-optimal equilibrium given masses of investors C , S_N , and S_S . Define $r_C \equiv \frac{dW(C, S_N, S_S)}{dC}$ to be the social value of commercial capital, $r_{S_N} \equiv \frac{dW(C, S_N, S_S)}{dS_N}$ to be the social value of naïve social capital, and $r_{S_S} \equiv \frac{dW(C, S_N, S_S)}{dS_S}$ to be the social value of sophisticated social capital. We have the following result.

Lemma 3. *The social value of commercial capital is less than the social value of naïve social capital, which is less than the social value of sophisticated social capital: $r_C \leq r_{S_N} \leq r_{S_S}$.*

Figure 4 depicts how the equilibrium asset allocation changes as each pool of capital is expanded. The grey lines define the equilibrium prior to adding new capital. Each type of marginal capital has a different impact on the equilibrium set of funded projects and which investors finance them. The blue lines denote the shift in the equilibrium induced by expanding a particular set of capital.

Increasing the mass of commercial investors C results in the least social value creation, as commercial investors disregard social welfare entirely. New commercial investors create social value in two ways. First, they fund previously unfinanced marginally profitable projects that have incidental social

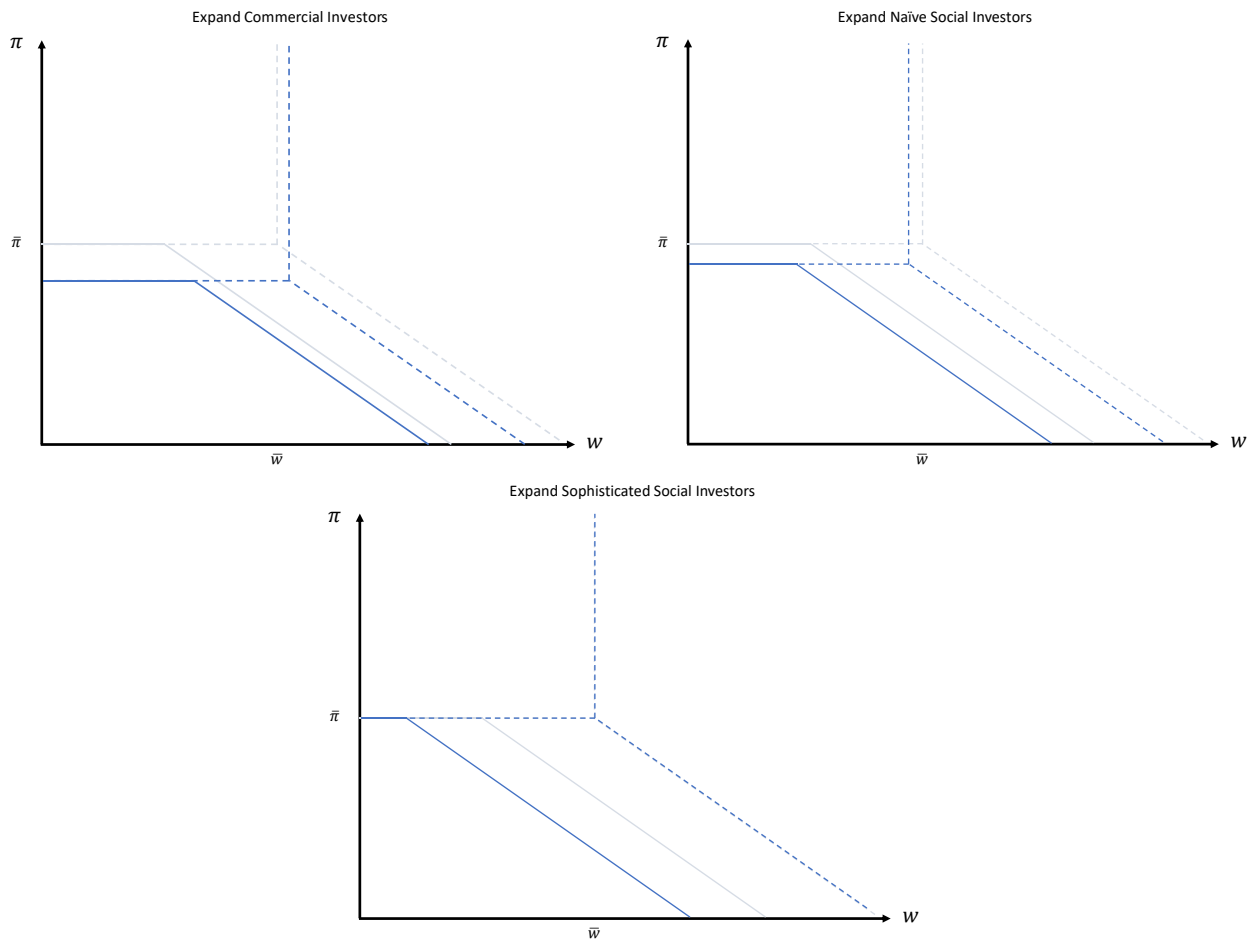


Figure 4: Expanding the pool of Commercial, Naïve Social, and Sophisticated Social Capital

value. Second, they bid up the price of claims on profits and displace social investors, who substitute into higher social value projects.

Increasing the mass of naïve social investors S_N causes a displacement of both commercial and sophisticated social entrepreneurs, effectively expanding the pool of each. This results in a larger increase in social welfare than does directly increasing the mass of commercial investors, as it effectively expands the pool of commercial capital by less and the pool of sophisticated capital by more.

Increasing the mass of sophisticated social investors S_S provides more social value than increasing the mass of naïve social investors S_N , as some of the firms that naïve social investors support could have attracted commercial capital and therefore they merely serve to expand the pool of commercial capital. In contrast, expanding sophisticated social capital does not result in the displacement of either of the other two types of investors.

4.3 Reallocating Naïve Social Capital to Improve Social Welfare and Financial Returns

In this section we consider two thought exercises. First, holding fixed the equilibrium behavior of all other investors, we consider the possibility of reallocating the investment of a single naïve social investor. We demonstrate that generically, any naïve social investor who supports a firm with $\pi_i \geq \bar{\pi}$ could reallocate their capital to increase total social welfare and increase their financial return. In this sense naïve investors leave money on the table. We then consider the possibility of converting a naïve social investor into a sophisticated social investor, and show that this always leads to an increase in social welfare and sometimes (but not always) leads to an increase in the investor's financial return.

Proposition 2. *Consider any naïve social investor i that supports a firm with $\pi_i > \bar{\pi}$ in equilibrium. Generically there exists an unfinanced firm j with profits $\pi_j > p_i$, such that if the naïve social investor i were to deviate and offer firm j financing at price π_j , total social welfare would increase as would investor i 's financial return.*

The logic of Proposition 2 can be understood with reference to Figure 5. Fix any naïve social investor i that supports a firm i with profits $\pi_i \geq \bar{\pi}$ and who earns financial return $p_i < \bar{\pi}$ (generically this holds for all naïve investors who support firms with $\pi_i \geq \bar{\pi}$). These investors support the firms highlighted in blue. And consider among the set of unfinanced firms some firm j with profits $\pi_j > p_i$ and with social value $w_j > r_C$. This firm is guaranteed to exist by the assumption that the distribution of firms has full support, and one such firm is highlighted in green.

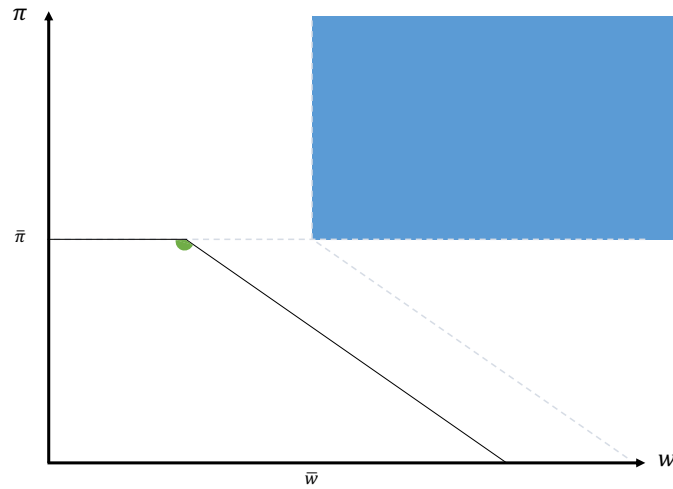


Figure 5: Reallocating Naïve Social Capital Out of Equilibrium

The contribution to social welfare of the equilibrium investment for investor i is low, regardless of the social value w_i of firm i , as investor i is merely displacing commercial investment. Firm j creates less social value than any firm in the blue region of the diagram, but by reallocating investor i 's capital to firm j social welfare increases, as firm j creates more social value than the marginal firm no longer supported by a commercial investor.

Further, by offering firm j a cost of capital $p_j = \pi_j$, investor i can earn higher financial return as well. As with social value, firm j earns lower profits than any firm in the blue region of the diagram. But social investor i can demand the full profit of firm j in exchange for financing, whereas the price that naïve investor i offered to firm i was disciplined by competition with other naïve social investors. Naïve social investors compete up the prices of firms with large contribution to social value even when these firms could have attracted commercial financing. The financial compromise made by naïve investors to support such firms results in a transfer of wealth to the entrepreneur rather than expanding the pool of socially valuable firms. In contrast, the financial compromise made to support a firm that could not attract commercial financing goes entirely toward expanding the pool of socially valuable firms rather than transferring rents to entrepreneurs whose projects would anyway have been feasible.

Proposition 2 demonstrates that naïve social investors leave money on the table in the sense that, relative to the firms these investors support, there are unfinanced firms that could deliver higher financial returns and increase social welfare. However, while it is straightforward to show that converting naïve social investors to sophisticated social investors would result in higher total social welfare, in general we cannot guarantee that this conversion would lead investors to earn higher financial returns. The simple reason is that once naïve investors have been converted to sophisticated social investors,

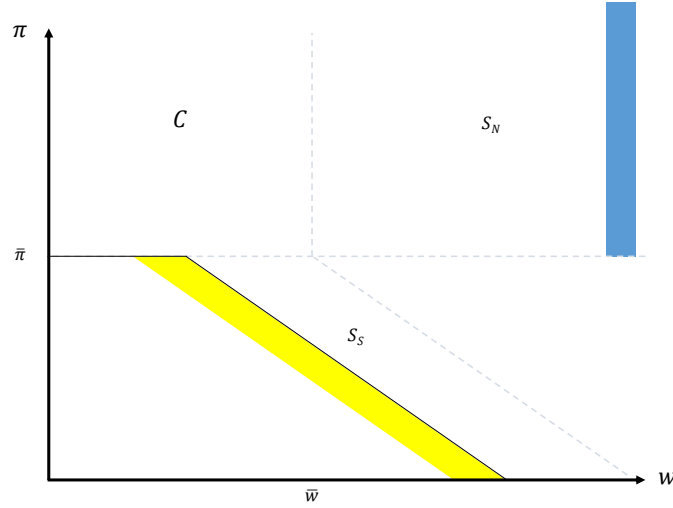


Figure 6: Converting Naïve to Sophisticated Investors

while they would prefer to finance firms in the green region of Figure 5 relative to any firm in the blue region, there may be yet another firm j' which contributes more to social welfare but has lower financial return than firms in the green region. Nevertheless, we can demonstrate the following result.

Proposition 3. *In the welfare-optimal equilibrium, there exists a set of naïve social investors such that were they to be converted to sophisticated social investors they would earn higher returns. Moreover total social welfare would increase.*

The logic of Proposition 3 is depicted in Figure 6. We identify a mass of naïve social investors, shaded in blue, who are supporting entrepreneurs that satisfy two properties.

1. These entrepreneurs could attract commercial capital at market rates, i.e. $\pi_i \geq \bar{\pi}$,
2. These entrepreneurs have very high contribution to social value w_i , so that naïve social investors make a large financial compromise to support them.

By converting these naïve social investors to sophisticated social investors, they instead support the firms shaded in yellow. As above, this results in an increase in social welfare, as the firms in yellow could not have attracted commercial capital, and have contribution to social value higher than r_C . Moreover, by identifying naïve social investors who were making a sufficiently large financial compromise to support firms i with high w_i , we can guarantee that the newly converted sophisticated social investors earn higher profits, as these sophisticated social investors can demand the full profits of the firms they support.

Finally, we note that while converting these naïve social investors to sophisticated social investors increases their profits and total social welfare, it does not increase their utility judged according to

the utility function of naïve social investors. Nevertheless, Proposition 3 offers encouraging news about the prospect of converting naïve social investors to sophisticated social investors in practice. A substantial amount of effort has gone into investigating the hypothesis that ESG investing can increase impact and financial returns (e.g. Eccles et al., 2014), suggesting investors are sensitive to the financial implications of values investing. Our model demonstrates that relative to conventional ESG strategies there is room for improvement in this dimension.

4.4 Enterprise Impact

How should one judge the contribution to social welfare of a particular entrepreneur, sometimes referred to as *enterprise impact* (e.g. Brest et al., 2016)? On first pass it might seem natural for w_i to be the measure of enterprise impact. However, in Section 4.2 we demonstrated that different social value is attributable to different classes of capital, suggesting that the impact of an enterprise should also reflect the social value of the capital it employs. Along these lines, we define the enterprise impact of firm i to be $e_i \equiv w_i - r_i$ where r_i is the social value of capital attributable to the investor who supports entrepreneur i in the welfare-optimal equilibrium. We define the enterprise impact to be 0 for firms that do not receive financing.

This definition of enterprise impact might have practical value for socially motivated investors aiming to quantify the social value of a particular enterprise. Frontier efforts in the impact investing industry often attempt to account for the social value created by the enterprise and the amount of capital employed by the enterprise, such as in the *impact multiple of money* method (Addy et al., 2019). Our analysis highlights that it is also critical to account for the composition of social capital versus commercial capital raised by an enterprise in judging its impact.

This definition of enterprise impact also highlights an alignment between the enterprise impact and profitability of a firm. Firms can increase their enterprise impact by increasing their profitability *even holding fixed their social value* w_i . Increasing the profitability of the firm makes it more likely to attract commercial capital, freeing up capital that is willing to accept lower returns to fund higher social value endeavors. In particular, we have the following result.

Proposition 4. *Suppose firm i attracts financing from some investor in equilibrium. Increasing its profits π_i while holding fixed its social value w_i weakly increases its enterprise impact e_i and total social value created in equilibrium.*

Proposition 4 states that making a firm more profitable increases its enterprise impact *even holding its social value w_i fixed*. Importantly, this result is not driven by an assumption that a firm’s profitability and its social value are correlated. Instead, this result is driven by the observation that the more profitable is a company, the more likely it is to attract a class of investor whose capital has low social

value. Every firm that is successful in attracting financing is effectively crowding out another firm that could have attracted capital in its absence. The more profitable is a firm, the less socially valuable is the firm that it displaces from being funded.

5 Extension: Firms Have Projects with Variable Scales

Up until this point we have assumed that every firm has a binary project, which is completed if and only if it raises a unit of capital. In this section we consider firms that have projects of variable scale. This introduces the possibility that even if a firm could attract commercial financing, social investors that bid up its price might enable the firm to accomplish more. We study the difference in behavior between naïve and sophisticated social investors and draw out further normative implications for investment behavior and security design. Beyond demonstrating that our analysis can be enriched to accommodate this more realistic setting, two new insights emerge from the analysis.

First, when there is only one financial security for which all investors must pay the same price, sophisticated social investors prefer to support smaller firms, all else equal, where size is measured by the amount of capital the firm could attract from commercial investors. As we demonstrate below, for a social investor to increase the social value created by a firm, the investor must offer cheaper capital than the firm could attract from commercial investors. With only one security, this necessarily crowds out commercial investors who then place their capital elsewhere. From the perspective of a sophisticated social investor, this cost is larger for larger firms.

The second insight of this analysis is that when a second security is introduced, social investors need not crowd out commercial investors in order to offer a firm cheap capital, enabling social investors to have a greater impact on social welfare. We discuss the design of this security in Section 5.2.3.

5.1 Model

There is a mass E of entrepreneurs. Each entrepreneur i is endowed with a production function that takes as an input $k \in \mathbb{N}$ capital. If entrepreneur i invests k capital, they produce $\pi_i(k) \in \mathbb{R}^+$ profit and $w_i(k) \in \mathbb{R}^+$ “social value,” where $\pi_i(k)$ and $w_i(k)$ represent the private and social return of the project respectively.⁹ We assume that $\pi_i(\cdot)$ and $w_i(\cdot)$ are perfectly observable to all players. For expositional clarity we assume that $\pi_i(k+1) - \pi_i(k) > \pi_i(k+2) - \pi_i(k+1)$ for all $k > 0$ and i , and

⁹We restrict attention to $w_i(k) \geq 0$ for expositional simplicity. In Appendix A.6 we discuss technical nuances that arise from allowing for $w_i(k) < 0$.

similarly for w_i . Importantly, we do not assume that w_i is strictly increasing, so social investors may desire to reduce the scale of a firm's operations, for example a heavy polluter.

There is a mass C of commercial investors and a mass S of social investors, each of whom controls one unit of capital.

A contract between some investor and an entrepreneur i specifies the transfer of one unit of capital from the investor to the entrepreneur at a price p_i . Therefore, letting \bar{k}_i be the amount of capital invested in entrepreneur i 's project, each of entrepreneur i 's investors receive return p_i on their invested capital, the entrepreneur receives $\pi_i(\bar{k}_i) - p_i\bar{k}_i$ financial return, and $w_i(\bar{k}_i)$ social value is created.

Market Clearing Mechanism

We assume that the market is cleared via uniform price auction. Each entrepreneur i reports an intention to raise k_i units of capital. Each investor then submits a bid $p_i \in \left[0, \frac{\pi_i(k_i)}{k_i}\right]$ to some entrepreneur i . Entrepreneur i then accepts his k_i highest bids.¹⁰ Each investor receives a return \bar{p}_i where \bar{p}_i is the k_i 'th lowest bid that entrepreneur i received.

Preferences

Each entrepreneur i 's utility is their share of the profit, $(\pi_i(\bar{k}_i) - \bar{p}_i\bar{k}_i) di$. Each commercial investor who supports entrepreneur i has utility equal to her return on capital $\bar{p}_i di$. We will separately examine two classes of social investors.

Naïve social investors receive utility from their financial return and from the average social value per unit of capital created by the entrepreneur they've financed. That is, a naïve social investor who supports entrepreneur i receives utility

$$\left(\bar{p}_i + \theta \frac{w_i(\bar{k}_i)}{\bar{k}_i} \right) di,$$

where θ represents the strength of the investor's social preference.

Sophisticated social investors receive utility from their income and from the total social value created by all entrepreneurs who receive financing. That is, a sophisticated social investor who supports entrepreneur i receives utility

$$\bar{p}_i di + \theta \int_j w_j(\bar{k}_j) dj.$$

¹⁰Entrepreneurs that fail to raise their goal of k_i capital instead raise nothing.

As in Sections 3 and 4, the difference between the utility functions of naïve and sophisticated social investors is that naïve investors do not derive utility from the social value generated by entrepreneurs they do not finance.

Timing of Actions

As before, we study a two period model. In period 1 each entrepreneur i proposes to raise k_i capital after which each social investor submits a bid to an entrepreneur. In period 2 each entrepreneur can revise their target k_i , but in doing so relinquishes all bids placed in period 1.¹¹ After observing all prior actions, each commercial investor submits a bid to an entrepreneur. Then the market clears according to the process described above.

Equilibrium

Our solution concept is Subgame Perfect Equilibrium. All investors choose contracts that are mutual best responses. When an entrepreneur is indifferent between two capital targets, they choose the one that maximizes social value.

5.2 Results

For simplicity we focus separately on the case when all social investors are naïve and when all social investors are sophisticated.

5.2.1 Naïve Social Investors

The model with naïve social investors works much the same way as in the single project analysis. There exists a cutoff $\bar{\pi}$ such that all projects receive capital at least to the point where the marginal unit of capital generates profits less than $\bar{\pi}$. Naïve social investors attend to a firm i 's *average* social value per unit of capital, $\frac{w_i(\bar{k}_i)}{\bar{k}_i}$, in equilibrium. Therefore in equilibrium there is some cutoff \bar{w} such that firms i for which $\bar{p}_i + \theta \frac{w_i(\bar{k}_i)}{\bar{k}_i} > \bar{\pi} + \theta \bar{w}$ are entirely financed by social investors, firms that do not clear this hurdle but that yield average profit $\frac{\pi_i(\bar{k}_i)}{\bar{k}_i} \geq \bar{\pi}$ are financed by commercial investors, and

¹¹This is specified for completeness, but in equilibrium no entrepreneur who receives bids from social investors ever revises his target k_i .

firms that cannot clear either hurdle remain unfinanced. Firms i supported by social investors raise the smallest amount of capital \bar{k}_i such that the next unit would yield

$$(\pi_i(\bar{k}_i + 1) + \theta w_i(\bar{k}_i + 1)) - (\pi_i(\bar{k}_i) + \theta w_i(\bar{k}_i)) < \bar{\pi} + \theta \bar{w}$$

and firms i supported by commercial investors raise the smallest amount of capital \bar{k}_i such that the next unit would yield

$$\pi_i(\bar{k}_i + 1) - \pi_i(\bar{k}_i) < \bar{\pi}$$

Therefore, the equilibrium size of a firm is determined by not only the intrinsic properties of the firm but also by the identity of its marginal investor. Firms that attract social capital have scale pinned down by the equilibrium opportunity cost of a social investor, which is different from that of a commercial investor.

There are two important takeaways from the exposition of naïve social investors when projects have variable scale. First, the investment strategy of these investors closely mirrors that of ESG investors, for example in public equities. Such investors purchase shares of companies that they believe will have high financial returns and whose *existing* operations are measured in some way to be socially desirable. In the model, naïve social investors make investments to achieve high financial returns and be associated with companies generating high average impact $\frac{w_k(\bar{k}_i)}{\bar{k}_i}$.

Second, an immediate feature of this model is that naïve social investors never coinvest with commercial investors. This stark pattern arises because social and commercial investors disagree on the relative value of companies with the same profits but different contributions to social value, so there is no price at which both sets of investors would be happy to finance the same investment. Any company financed by social investors is *fully* financed by social investors. While this extreme separation would not arise in a model with, for example, diversification motives, it illustrates an important point. Disagreement about the value of a company among investors implies that to change the scale of the company requires displacing some of its existing investors. This notion of displacement is critical for understanding the difference between the behavior of naïve and sophisticated social investors.

5.2.2 Sophisticated Social Investors

In this section we examine the behavior of sophisticated social investors and draw out several normative implications. The analysis in this section differs qualitatively from the case where projects are binary in Section 3.2, in that now all projects could potentially attract some level of commercial capital. Thus, sophisticated social investors may no longer want to avoid investing in projects that could have attracted a positive amount of commercial capital. We demonstrate that, unlike naïve

social investors, sophisticated social investors have a preference to support small firms.

Given the equilibrium profile of strategies, define the amount of commercial capital that firm i could attract in period 2 to be k_i^C . Let p_i^C be the price of equity that entrepreneur i would receive in the commercial market and let $q_i^C \equiv \pi(k_i^C) - p_i^C k_i^C$ be the entrepreneur's anticipated payoff from commercial investment.

For each firm i and each k define

$$v_i(k) \equiv \left(\frac{\pi_i(k) - q_i^C}{k} + \theta \frac{w_i(k) - w_i(k_i^C)}{k} \right).$$

The first term in $v_i(k)$ represents the financial return that sophisticated social investors draw from investing k capital in firm i . Namely, they can demand the full return from the project minus what entrepreneur i could earn on the commercial market. The second term of $v_i(k)$ represents the sophisticated investor's marginal contribution to social value by investing k capital into firm i . The marginal contribution of the social investors is the $w_i(k)$ social value the firm creates, less the $w_i(k_i^C)$ social value the firm would have created with commercial investment.

In the welfare-optimal equilibrium, sophisticated social investors allocate their capital to maximize

$$\max_{\{k_i\}} \int_{i \in \bar{E}} \left\{ v_i(k_i) \mathbb{I}(k_i > 0) + \theta w_i(k_i^C) \mathbb{I}(k_i = 0) \right\} di$$

such that

$$\int k_i di = S$$

which represents maximizing the social investors' objective function subject to their resource constraint. As in the binary project case, \bar{E} represents the set of entrepreneurs that receive a positive level of financing in equilibrium. Let $\{k_i^S\}$ denote the solution to this maximization problem. Note that in general, k_i^S may be less than k_i^C . In such a case, social investors are effectively paying entrepreneurs to reduce their scale relative to their optimum if financed by commercial investors.

This maximization problem captures two ideas. First, relative to naïve social investors, sophisticated social investors care about their marginal contribution to social value, $w_i(k_i^S) - w_i(k_i^C)$, rather than the firm's total social value $w_i(k_i^S)$, as they recognize that some of this social value would have been financed in their absence (by commercial investors). This parallels the difference in concerns of naïve and sophisticated investors discussed in Section 3.

Second, even though the marginal social value created by sophisticated social investors corresponds only to the output resulting from the final $k_i^S - k_i^C$ invested capital, to achieve this change they have to provide the full amount of capital k_i^S . This has important implications for social investors'

choice of which firms to support.

Consider investments in two firms i and j that would achieve the same marginal contribution to social welfare (for convenience, assume $k_i^S > k_i^C$ and $k_j^S > k_j^C$); i.e.

$$w_i(k_i^S) - w_i(k_i^C) = w_j(k_j^S) - w_j(k_j^C)$$

and

$$k_i^S - k_i^C = k_j^S - k_j^C.$$

Holding fixed financial returns, sophisticated social investors prefer to support the firm that requires displacing less commercial capital, i.e. $v_i(k_i^S) > v_j(k_j^S)$ if and only if $k_i^C < k_j^C$. In this sense sophisticated social investors prefer to support smaller firms. We codify this logic in the following observation.

Observation 1. *When there is just one security for which all investors must pay the same price (i.e. the model specified in Section 5.1, all else equal, sophisticated social investors prefer to support smaller firms.*

Once again, the fact that social investors need to displace commercial investors to generate impact is a source of inefficiency. While in our stylized model the size of the company k_i^C determines the amount of capital that must be displaced to generate impact, in reality there are other factors that may determine the displacement required to induce impact—in general it is easier to induce impact on the intensive margin if the stock of a company is more inelastically demanded. We show in the next section that allowing for project-specific financing alleviates this tension.

5.2.3 Project-Contingent Bonds

So far we have restricted investors to purchase a single asset for each firm, representing an ownership share of the firm's profits. In this section we consider the possibility that firms can issue bonds in addition to equity, and that the issuance of these bonds can be made contingent on the firm producing a specific level of output. That is, in addition to common equity, each firm i can issue a series of bonds b_i^k , where the issuance of such a bond commits firm i to invest k capital, and investors can place bids on these bonds according to the same uniform price auction described above.

The simple observation we make in this section is that social investors who aim to subsidize a firm by providing more generous financing than would a commercial investor never buy common equity. Rather they always buy a bond corresponding to a particular level of investment that would not be supported by commercial investors, while leaving the commercial investors to support output in the range that can attract commercial financing. We have the following result.

Proposition 5. *In the unique equilibrium of the model with output-contingent bonds, total social value is higher than in the model where firms can only issue common equity.*

The reason that social investors can create more value when investing in output-contingent bonds is that they can subsidize the cost of capital for high-value marginal projects without also furnishing the capital for the inframarginal projects that would have attracted commercial financing regardless. Therefore, in the market with output-contingent bonds, social investors do not factor a firm’s size into the decision of whether to subsidize its marginal projects (i.e. those that would not be completed with only commercial investors).

The output-contingent bonds resemble green bonds, in that they are a means to offer subsidized capital to firms earmarked for projects with high social or environmental value. The primary distinction is that social investors support levels of output that would not have been financed by commercial investors. Therefore, a practical implementation of this market would verify that each project has negative net present value at its commercial cost of capital in addition to verifying that the project has high social or environmental value.

6 Extension: Heterogeneous Investor Altruism θ

In this section we consider an extension of the model in which we allow the altruism parameter θ to vary across investors. Our aim is to explore the model’s implications with regards to assortative matching. A classic exercise in the assignment matching literature is to identify conditions under which agents exhibit positive assortative matching (e.g. Roy, 1951, Becker, 1973, Sattinger, 1979, Costinot and Vogel, 2010, Gola, 2020) — i.e. when do agents with higher “types” match with one another? We demonstrate when social investors in our model are naïve, investors with higher altruism match with entrepreneurs with higher social value for familiar reasons. In contrast, when investors are sophisticated, they exhibit a variant of *negative* assortative matching. This latter result arises from the fact that sophisticated social investors have interdependent utility; their utility depends not only on the terms of their own match but also on the matches of other investors.

6.1 Model

The model is the same as in Section 2 with the exception that for the set of social investors we now index their altruism parameter θ_i by i , and let it vary across investors. Specifically we assume that there is a finite set $\Theta \equiv \{\theta^1, \dots, \theta^n\}$ of potential levels of altruism, with $\theta^j < \theta^k$ for $1 \leq j < k \leq n$. We make no assumption about the distribution of θ_i . As before we let C be the mass of commercial

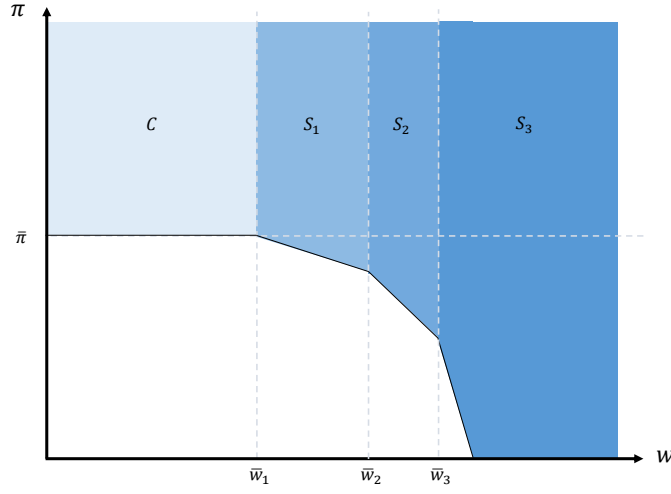


Figure 7: Equilibrium sorting with naïve social investors

investors and we now let S^l be the mass of social investors with altruism parameter θ^l . In both the model with naïve social investors and the model with sophisticated social investors, commercial investors can be understood as having an altruism parameter of 0. We maintain all other assumptions of the model in Section 2.

6.2 Naïve Social Investors

We now characterize the equilibrium of the model where all social investors are naïve and demonstrate that social investors and entrepreneurs exhibit positive assortative matching on θ and w .

The period 2 commercial market operates in exactly the same manner as in Section 3.1; there is a level $\bar{\pi}$ such that firms that generate profits $\pi_i \geq \bar{\pi}$ and who do not already have investment offers with $p_i \leq \bar{\pi}$ receive commercial financing at price $p_i = \bar{\pi}$.

In period 1 prices offered to any two entrepreneurs i and j who are both supported by a social investor with type θ^l satisfy $p_i + \theta^l w_i = p_j + \theta^l w_j$.¹² And for an entrepreneur i supported by a social investor of type θ^l and an entrepreneur k who is not, prices must satisfy $p_i + \theta^l w_i \geq p_k + \theta^l w_k$.

With the above pricing equations we can now characterize the set of entrepreneurs financed by each type of investor in equilibrium. The equilibrium investment allocation is depicted in Figure 7.

Relative to Section 3.1 the principle novelty is that we can now establish assortative matching

¹²The preceding equality holds so long as prices are finite. In equilibrium a social investor may provide funding in exchange for zero share of the proceeds ($p = 0$) if the project has sufficiently high social impact (akin to philanthropy). In such a case, the above equality need not hold.

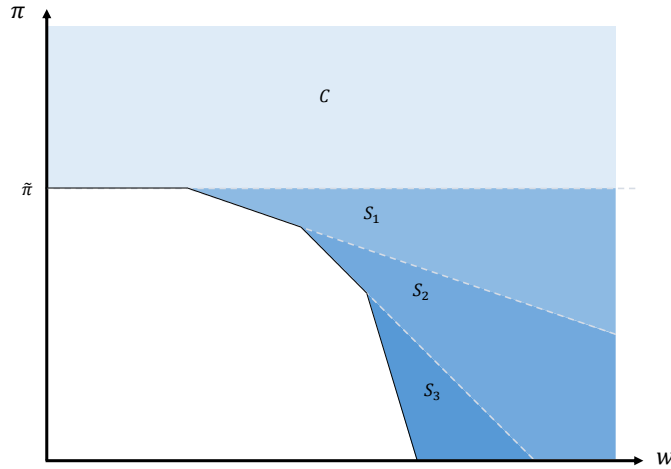


Figure 8: Equilibrium sorting with sophisticated social investors

in equilibrium. Namely, investors partition the set of entrepreneurs who receive financing such that investors with higher θ_i match with entrepreneurs who have higher w_i . This stems from the fact that the utility of investor i is supermodular in θ_i and w_i , and hence social investors with higher altruism have a higher willingness to pay for projects with high social value. This positive assortative matching echoes many results in the assignment matching literature cited above. As we will see in the following section, this result breaks down, and partially reverses when social investors are sophisticated.

6.3 Sophisticated Social Investors

When social investors are sophisticated there is a multiplicity of equilibria; Figure 8 depicts the investment allocation in the welfare-optimal equilibrium. Appendix Section A.5 offers a formal characterization of the welfare-optimal equilibrium.

Relative to when social investors are naïve, the equilibrium allocation features two important differences. First, as in Section 3.2, so long as social capital is sufficiently scarce, sophisticated social investors exclusively support firms that could not attract commercial financing. Second, and novel to this section, positive assortative matching breaks down, *even among the set of firms supported by social investors*. In fact, holding fixed a level of profits π' , social investors exhibit negative assortative matching; the higher is the social investor's altruism parameter θ_i , the lower is the social value w_i of the firm they support. This negative assortative matching holds despite the fact that the utility of sophisticated social investors is still supermodular in their altruism parameter θ_i and the social value w_i of the firm they support.¹³

¹³All equilibria with the same investment frontier depicted in Figure 8 are welfare-optimal. Therefore, formally, there

In equilibrium, in order for a sophisticated social investor i not to deviate and support a firm that could have attracted commercial investment it must be that

$$\pi_i + \theta_i w_i \geq \tilde{\pi} + \theta_i w'$$

where $\tilde{\pi}$ is the equilibrium level of commercial returns and w' is the social value created by the firm financed by the displaced commercial investor. This incentive compatibility condition is easier to satisfy for social investors with higher altruism parameters. Therefore in the welfare optimal equilibrium, it is the sophisticated social investors who care the least about social welfare that match to the most impactful entrepreneurs for a given level of profitability, as these are the entrepreneurs who are most able to entice social investors away from commercial markets. In contrast, sophisticated social investors with higher altruism parameters are willing to forgo commercial returns to support entrepreneurs with lower contribution to social welfare for a given profit level. And because sophisticated social investors derive utility from the social value created by all firms supported in equilibrium, social investors with high altruism parameters do not compete with social investors with low altruism parameters, as they recognize that doing so would not expand social value. Therefore, that positive assortative matching breaks down, and partially reverses when social investors are sophisticated arises from the fact that sophisticated social investors have interdependent utilities in the sense that their utility depends not only on the firm they match to and the price they reach but also on the matching of other investors and entrepreneurs.

We close this section with one final remark. We view this result as an interesting contribution to the literature on assignment matching models but are cautious in interpreting it as a normative result for social investors, as it relies on a feature of the model that may be our biggest departure from the real world. Namely, while sophisticated social investors recognize that firms they do not finance can still search for financing on the commercial market, sophisticated social investors who support firms that are not eligible for commercial financing are all pivotal. Consider an equilibrium in which social investor i with low altruism parameter is assigned to support a high social value firm j that cannot attract commercial investment. Because sophisticated social investors all choose their investment decisions at the same time, if social investor i deviated to invest in a higher profit firm, then firm j would go unfinanced. However, in a richer model in which social investors make their decisions dynamically, social investor i might expect that another social investor with higher altruism parameter would replace them if they were to deviate and leave firm j unfinanced. In such a case, this negative assortative matching result would break down.

The above notwithstanding, we conjecture that some variant of negative assortative matching

exists a welfare-optimal equilibrium such that holding the level of entrepreneur profit fixed, social investors engage in negative assortative matching. But there may be other equilibria with equivalent allocations that do not feature negative assortative matching.

might survive other extensions of the model that more closely align with reality. Perhaps most plausibly, if social investors' utility was some combination of naïve and sophisticated — i.e. social investors derive utility from both total social value and also especially from the social value of the firm they support — then social investors with low altruism who were assigned to support the most impactful firms might not be tempted to deviate even if they anticipated that another social investor would willingly replace them. And social investors with higher altruism assigned to support lower social value firms would now face an increased temptation to undercut their competitors financing higher social value firms, but the knowledge that low altruism investors will only support high social value firms might still deter this kind of competition.

7 Conclusion

This paper provides a new framework understand to how values-based investing generates impact. We abstract from activism, instead considering a model in which investors influence social outcomes by pursuing passive investment strategies that affect the set of companies or projects that are financed in equilibrium. Precisely how socially conscious investors think about social value when assembling their portfolios matters. If investors only care about the social value generated by companies in their portfolio, values-based investing has limited impact on total social value creation and generates low financial returns.

Investors following these strategies, which closely resemble the construction of conventional ESG portfolios, are making inefficient use of their social capital because they displace commercial investors who do not care about social value creation but would have supported some socially valuable companies anyway. The impact of socially conscious investors is therefore in part determined by the preferences of the investors displaced by the arrival of socially conscious capital.

Focusing on the the displacement of investors induced by the preferences of others generates several additional insights, and our framework admits a rich graphical analysis to illustrate these points. First, not only is there room for improvement in the social value created by conventional ESG investing strategies, but this improvement can also come with increased financial returns. Second, the logic of the displacement principle also works in reverse. By investing in their own profitability, companies are able to reduce their reliance on social capital by attracting commercial capital, thereby contributing to social value creation. Reducing dependence on social capital frees it up to be deployed to other projects that could not be supported by commercial investors. In this light, companies can do good by doing well.

Further, modifying our framework to incorporate an intensive margin of company scale highlights

an important limitation of traditional socially responsible investing. Disagreement about the value of a company between investors of different types means that to induce change in a company requires displacing other investors, which is an inefficient use of resources. Investors who recognize the inefficiency of displacement will prefer to invest in companies with smaller scale, all else equal. We show that this friction can be alleviated by introducing project specific financing, allowing social investors to provide financing for projects that existing commercial investors would view as value destroying, without displacing existing investors on inframarginal projects.

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A Equilibrium Characterizations and Proofs

A.1 Characterization of the Equilibrium with Naïve Investors in Section 3.1

Lemma 4. *Projects financed by social investors have higher social value than projects financed by commercial investors. Formally, if entrepreneur i is financed by a social investor and entrepreneur j is financed by a commercial investor, $w_i \geq w_j$.*

Lemma 4 implies that there is some \bar{w} such that social investors only support entrepreneurs with social impact greater than \bar{w} and commercial investors only support entrepreneurs with impact less than \bar{w} . Let $\bar{\pi}$ be defined such that

$$|\{i \in E : w_i \leq \bar{w}, \pi_i \geq \bar{\pi}\}| = C.$$

Lemma 5. *For entrepreneurs with $w_i \leq \bar{w}$, commercial investors support the mass C of them with highest profits, i.e. those with profit $\pi_i \geq \bar{\pi}$. Prices are set such that $p_i = \bar{\pi}$.*

Proof. This is an immediate implication of the commercial investor pricing equations. \square

Lemmas 4 and 5 imply that commercial investors support entrepreneurs with profit higher than $\bar{\pi}$ and with social impact lower than \bar{w} . Entrepreneurs with profit lower than $\bar{\pi}$ cannot generate sufficient profits to attract commercial support, and entrepreneurs with social impact higher than \bar{w} are priced by social investors and therefore yield returns too low to attract commercial support.

Lemma 6. *For entrepreneurs with $w_i \geq \bar{w}$, social investors support those with $\min\{\pi_i, \bar{\pi}\} + \theta w_i \geq \bar{\pi} + \theta \bar{w}$ and the price they pay is such that either $p_i + \theta w_i = \bar{\pi} + \theta \bar{w}$ if such a $p_i \geq 0$ exists and $p_i = 0$ else.*

Because commercial investors support all entrepreneurs with profit higher than $\bar{\pi}$ and with social value lower than \bar{w} , $\bar{\pi} + \theta \bar{w}$ is the social investor's effective outside option utility. Prices in equilibrium are set such that investors achieve this outside option utility.

Social investors receive financial return $\bar{\pi} - \theta(w_i - \bar{w})$. Thus, they are willing to pay (in terms of reduced financial return) for projects that generate high social value. Commercial investors do not find it attractive to invest in companies with $\pi_i > \bar{\pi}$ and $w_i > \bar{w}$ precisely because social investors are willing to invest in these companies at higher valuations.

Lemmas 5 and 6 allow for the graphical characterization of the equilibrium, depicted in Figure 1. That $\bar{\pi}$ and \bar{w} are uniquely pinned down is implied by the resource constraints, i.e. that the commercial investors have mass C and social investors have mass S .

A.2 Characterization of the Equilibrium with Sophisticated Investors in Section 3.2

As sophisticated social investors care about total social welfare rather than the social value of the entrepreneur they support, their expectations about which entrepreneurs will be financed in the period 2 commercial market will determine whom they finance in period 1. Therefore we analyze this model starting from period 2.

As in the previous section, the equilibrium pricing condition for the commercial investors dictates that for all entrepreneurs i, j that are supported by a commercial investor

$$p_i = p_j$$

and for any entrepreneur i supported by a commercial investor and entrepreneur k who is not,

$$p_i \geq p_k$$

These lead to the following lemma. Let E_2 be the set of entrepreneurs still looking for financing in period 2 in equilibrium, and let $\tilde{\pi}$ be defined so that

$$|\{i \in E_2 : \pi_i \geq \tilde{\pi}\}| = C.$$

Lemma 7. *In equilibrium, every entrepreneur in period 2 with profits $\pi_i \geq \tilde{\pi}$ is financed and has price p_i such that $p_i = \tilde{\pi}$.*

In equilibrium, social investors and entrepreneurs in period 1 expect that those entrepreneurs with profits above $\tilde{\pi}$ can receive financing and earn $\pi_i - \tilde{\pi}$ by waiting until period 2.¹⁴

Using the equilibrium prices from Lemma 1, we can now characterize the set of entrepreneurs supported by social investors in period 1, through the following algorithm. Let $\tilde{\pi}(M)$ be defined such that

$$|\{i \in E : \pi_i \geq \tilde{\pi}(M)\}| = M.$$

- For each w , define the set $s_w \equiv \{i \in E : \pi_i \leq \tilde{\pi}(C), \pi_i + \theta w_i \geq \tilde{\pi}(C) + \theta w\}$.
- If there exists a w such that $|s_w| = S$, then in equilibrium the social investors support entrepreneurs in s_w and have a strict preference for these firms over supporting any firm with $\pi_i > \tilde{\pi}(C)$

¹⁴Note that this holds even if in period 1 there are more than a mass C of entrepreneurs who meet this criterion, since all players expect that in period 2 a mass of exactly C will meet this criterion.

- Else, a fraction $\beta > 0$ of social investors will support firms that could have attracted commercial investment.

- Define $s_0(\beta) = \{i \in E : \pi_i \leq \tilde{\pi}(\beta S + C), \pi_i + \theta w_i \geq \tilde{\pi}(\beta S + C)\}$

- β will be chosen such that $|s_0(\beta)| = (1 - \beta)S$.

Lemma 8. *If there exists a w such that $|s_w| = S$, then in the unique equilibrium, sophisticated social investors support entrepreneurs in S_w and commercial investors support entrepreneurs with $\pi_i \geq \tilde{\pi}(C)$. Else, in the unique equilibrium $(1 - \beta)S$ social investors support entrepreneurs in $s_0(\beta)$. The remaining social investors and all commercial investors support entrepreneurs with $\pi_i \geq \tilde{\pi}(\beta S + C)$, and prices are such that all of these investors earn $\tilde{\pi}(\beta S + C)$ profit.*

Proof. First consider the case where there exists a w such that $|s_w| = S$. In this case each entrepreneur $i \in S_w$ is supported by a sophisticated social investor at price $p_i = \pi_i$. Therefore each social investor earns π_i profit. There are three types of deviations we must consider – supporting an entrepreneur j with profit $\pi_j > \tilde{\pi}(C)$, who will get commercial financing in period 2, supporting an entrepreneur $k \in S_w$ who already receives support from another social investor, or supporting an entrepreneur l who does not receive any investment in equilibrium. In the first case, the social investor cannot earn more than $\tilde{\pi}(C)$ profit, as this is what the commercial investor earns. Moreover, she recognizes that by undercutting a commercial investor, her contribution to social welfare is not w_j , but the social value of the new entrepreneur supported by the commercial investor whose capital she has displaced. By definition this new entrepreneur has social value less than w , and so by definition of S_w this is not a profitable deviation.

In the second case entrepreneur k produces $\pi_k \leq \tilde{\pi}(C)$ profit and the social investor recognizes that by undercutting another social investor she is not contributing to the total social welfare created because entrepreneur k will be financed independently of her actions, and the displaced social investor cannot reallocate her capital. Therefore by definition of S_w these are not profitable deviations. In the final case, the most attractive entrepreneur to support is the one with profit $\pi_l = \tilde{\pi}(C)$ and social impact $w_l = w$, but once again by definition of S_w this is not a profitable deviation.

The case where no w exists such that $|s_w| = S$ is similar. □

Proof of Proposition 1

The proof that social welfare is higher when social investors are sophisticated rather than naïve has two steps.

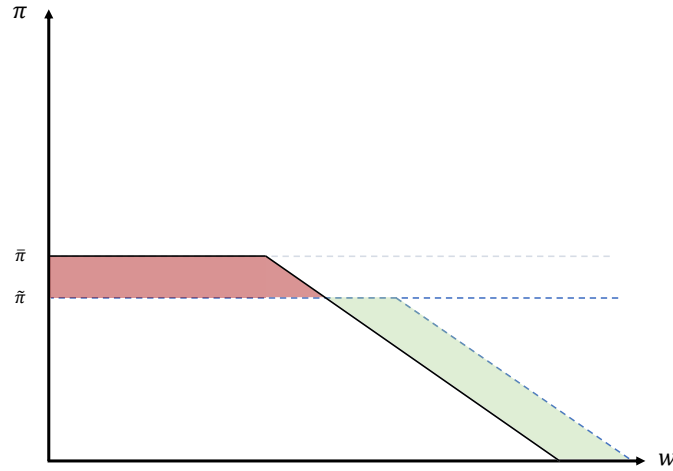


Figure 9: Comparing equilibrium investment for each type of social investors

Let α be the fraction of naïve social investors who support firms with profits above $\bar{\pi}$ in equilibrium, and let β be the fraction of sophisticated social investors who support firms with profits above $\tilde{\pi}$ in equilibrium. In other words, α is the proportion of naïve social investors who support firms that could have achieved commercial financing, and similarly β is the proportion of sophisticated social investors that support firms that could have attracted commercial financing. For the case where $\beta = 0$ this is trivially satisfied, as α is always positive. For the case where $\beta > 0$, the diagonal line that delineates the investments of sophisticated social investors meets the horizontal line that delineates the investments of commercial investors at $w = 0$. This was proved in Section A.2 and is depicted in Figure 2b.

In contrast the diagonal line that delineates the investments of naïve investors always meets the horizontal line that delineates the investments of commercial investors at a $w > 0$. This was proved in Section A.1 and is depicted in Figure 1. Noting that the mass of naïve social investors equals the mass of sophisticated social investors, and the mass of commercial investors is held constant, this implies that $\alpha > \beta$.

Next we show that the fact that $\alpha > \beta$ implies that social welfare is higher when social investors are sophisticated. That $\alpha > \beta$ implies that $\bar{\pi} > \tilde{\pi}$ (i.e. that the horizontal line delineating commercial investments is higher when social investors are naïve than when they are sophisticated). Figure 9 depicts the difference in equilibrium investments when moving the horizontal line from $\bar{\pi}$ to $\tilde{\pi}$. Relative to the equilibrium investment allocation with naïve social investors, the equilibrium allocation with sophisticated social investors loses the projects shaded in red and gains the (equal mass of) projects shaded in green. Every project i in the green region has higher w_i than does every project j in the red region. Therefore social welfare is higher when social investors are sophisticated.

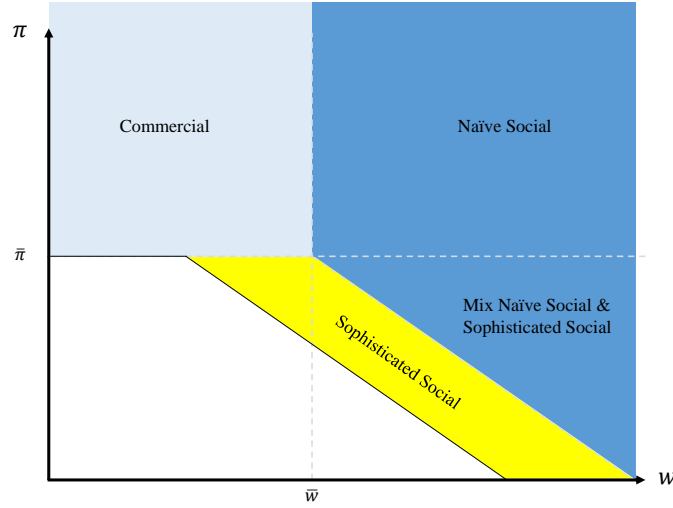


Figure 10: General Equilibrium With Both Types of Social Investors

A.3 Characterization of the Equilibrium with Both Types of Social Investors in Section 4

Figure 10 depicts the general equilibrium structure. To characterize this structure we use the following three lemmas.

Analogous to Section 3.1 and Lemma 5, there exists a $\bar{\pi}$ and \bar{w} such that

Lemma 9. *For entrepreneurs with $w_i \leq \bar{w}$, commercial investors support the mass C of them with highest profits, i.e. those with profit $\pi_i \geq \bar{\pi}$. Prices are set such that $p_i = \bar{\pi}$.*

Analogous to Lemma 6 we have

Lemma 10. *Naïve investors support entrepreneurs with $\min\{\pi_i, \bar{\pi}\} + \theta w_i \geq \bar{\pi} + \theta \bar{w}$.*

Finally, analogous to the equilibrium characterized in Section A.2, there exists a \tilde{w} , weakly larger w_i for the firm i that represents the marginal firm supported by commercial investors, such that

Lemma 11. *Sophisticated social investors support entrepreneurs with profits $\pi_i < \bar{\pi}$ and with $\pi_i + \theta w_i > \bar{\pi} + \tilde{w}$. If $\tilde{w} > 0$ then the former inequalities completely characterize the set of entrepreneurs supported by sophisticated social investors. If $\tilde{w} = 0$ then some sophisticated social investors support firms with $\pi_i > \bar{\pi}$.*

A few features of the equilibrium are of note. First, both sophisticated and naïve investors may support firms in the region below the horizontal line $\pi = \bar{\pi}$ and to the right of the diagonal line $\pi_i + \theta w_i \geq \bar{\pi} + \theta \bar{w}$. This is because in equilibrium, sophisticated investors know they are pivotal for the firms they support in this region, since no commercial investor would support them if the sophisticated investor were to deviate.

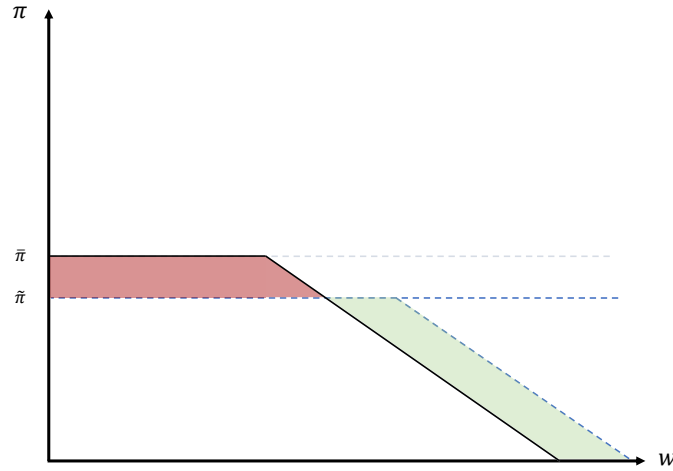


Figure 11: Comparing Welfare-Optimal Equilibrium to a Generic Other One

Second, only naïve social investors support firms in the region above $\pi = \bar{\pi}$ and to the right of $w = \bar{w}$ because these are firms that could attract commercial support if only a naïve investor did not support them, and so by the same logic as in Section 3.2 these firms cannot attract the support of sophisticated social investors.

Next we argue that the equilibrium depicted in Figure 3 is welfare-optimal. This is the equilibrium in which only naïve social investors support firms in the region below the horizontal line $\pi = \bar{\pi}$ and to the right of the diagonal line $\pi_i + \theta w_i \geq \bar{\pi} + \theta \bar{w}$. Of all equilibria, this is the one in which the horizontal line $\pi = \bar{\pi}$ is the highest, as the fewest commercial investors are displaced by naïve social investors. By the same logic as in the proof of Proposition 1, the equilibrium in which this line is the highest has the highest social welfare.

A comparison between the welfare-optimal equilibrium and a generic other one is depicted in Figure 11. Compared to the generic other equilibrium, in the welfare optimal equilibrium the marginally supported firms are shaded in green and the marginally unsupported firms are shaded in red. All firms shaded in green have higher social value than all firms shaded in red. Hence the welfare-optimal equilibrium indeed features the highest social welfare.

A.4 Omitted Proofs From Sections 4 and 5

Proof of Lemma 3

Figures 4a, b, and c depict how expanding each pool of capital changes the investment allocation.

Figure 4a corresponds to expanding commercial capital, Figure 4b corresponds to expanding the pool of naïve social capital, and Figure 4c corresponds to expanding the pool of sophisticated social capital.

First, it is immediate that the value of marginally expanding commercial capital, r_C is less than the value of marginally expanding naïve social capital, r_{S_N} , as the latter case involves adding an investor who maximizes a combination of financial return and social value while the former involves adding an investor who single-mindedly maximizes financial return.

Second, to see that the value of marginally expanding sophisticated social capital, r_{S_S} is larger than the value than marginally expanding naïve social capital, r_{S_N} , we use the same argument as in the proof of Proposition 1. The investment allocations in Figures 4b and c have the same mass, but the horizontal line is lower in Figure 4b than it is in Figure 4c. Every entrepreneur funded in Figure 4c and not in Figure 4a has higher w_i than every entrepreneur funded in Figure 4a and not in Figure 4c.

Proof of Propositions 2 and 3

We sketch the proof of Proposition 2 with reference to Figure 5, where the mass of blue naïve social investors are moved to instead support the green mass of entrepreneurs. Because we have assumed that the set of projects has full support, for any level of financial compromise that a naïve social investor makes to support a firm that could have attracted commercial financing, there exists a firm that is not supported by any investor but that could offer a higher return than the naïve social investor is earning, and such that if the naïve social investor were to reallocate her capital to the new firm total social welfare would increase. This new firm has lower w_i than the one that the naïve social investor supported, but the naïve social investor's contribution to social welfare is higher when supporting the new firm because it could not attract commercial financing.

We sketch this proof of Proposition 3 with reference to Figure 6. First we identify a set of naïve social investors who are earning 0 return in equilibrium. The assumption that the distribution of projects has full support guarantees that this set of investors has positive mass. We convert these naïve social investors to sophisticated social investors, noting that this strictly expands the set of firms supported by sophisticated social investors. We make no assumption about which of the firms supported by sophisticated social investors are supported by our “converted” investors. Instead we note that in equilibrium all sophisticated social investors (save for a set of measure 0) earn positive profits, as the price for each of these firms is $p_i = \pi_i$. Therefore our converted investors earn higher profits than prior to their conversion. That social welfare is higher is a straightforward consequence of Proposition 1.

Proof of Proposition 4

This is a straightforward implication of the equilibrium depicted in Figure 3. Fix any entrepreneur i who is being supported by a sophisticated social investor. Raising his profit π_i to $\pi_j < \pi'$ does not change the type of capital he attracts. But raising his profit to $\pi_j > \bar{\pi}$ causes him to instead be supported by a commercial investor and his enterprise impact increases. Social welfare increases by $(r_C - r_{S_S}) di$ because the sophisticated social investor can now support another entrepreneur.

Proof of Proposition 5

This is a straightforward consequence of the fact that social investors could replicate the common equity outcome in the output-contingent bonds model.

A.5 Characterization of the Welfare-Optimal Equilibrium with Sophisticated Investors in Section 6.3 and Proof Sketch of Negative Assortative Matching

The welfare-optimal equilibrium can be established following the construction in Section A.2, where the process is first followed for social investors with $\theta_i = \theta^1$ and then is repeated for each group of social investors with progressively higher levels of altruism. However, to establish that the equilibrium depicted in Figure 8 is indeed welfare-optimal, it is instructive to outline a different method for constructing an equilibrium in this model.

Consider the following process. Define $\bar{\pi}(x)$ to be the level of profits of the entrepreneur who earns higher profits than all but a mass x of all other entrepreneurs. Now, conjecture an equilibrium in which a mass of s social investors support firms that could attract commercial financing. In this equilibrium, there is a unique level of returns $\bar{\pi}(C+s)$ investors get from supporting firms that could have attracted commercial financing. And there is a unique social value $w'(C+s)$ that represents the social value created by commercial investors if a small mass of commercial investors were displaced by social investors.

Define $\sigma(i)$ to be any ordering over all sophisticated social investors $i \in S^1 \cup \dots \cup S^n$.

At step 0, social investor $\sigma^{-1}(0)$ is called to support an entrepreneur. If she chooses an entrepreneur i with profits $\pi_i \geq \bar{\pi}(C+s)$, assign her a price of $p_i = \bar{\pi}(C+s)$, and a social value of $w'_i = w'(C+s)$. Else assign her a price of $p_i = \pi_i$ and a social value of $w'_i = w_i$. Social investor $\sigma^{-1}(0)$ chooses the entrepreneur i that maximizes $p_i + \theta_{\sigma^{-1}(0)} w'_i$.

At step $\sigma^{-1}(k)$, social investor k is called on to support an entrepreneur. She chooses an entrepreneur to support according to the same process, excluding any entrepreneur that has been chosen in a previous step.

At the culmination of this process, if exactly a mass s of social investors have chosen entrepreneurs with profits above $\bar{\pi}(C+s)$, then it is straightforward to show that this process has identified an equilibrium investment allocation. Moreover, it is straightforward to show that all equilibrium investment allocations can be identified by conjecturing a mass s of social investors who support entrepreneurs with profits above $\bar{\pi}(C+s)$ and identifying an ordering σ that rationalizes it.

Now, assign every social investor an index that is increasing in their altruism parameter θ_i , so that the least altruistic social investors have the lowest indices. Then the equilibrium depicted in Figure 8 corresponds to the ordering $\sigma(i) = i$. We will demonstrate that the equilibrium arising from any other ordering σ' that results in a different allocation produces lower social welfare than σ .

Take some σ' . Identify two social investors, j and k such that

- $\theta_j > \theta_k$
- $\sigma'(j) < \sigma'(k)$
- If social investor k were to choose an entrepreneur at step $\sigma'(j)$, she would choose the same entrepreneur as did social investor j
- If social investor j were to choose an entrepreneur at step $\sigma'(k)$, she would choose a different entrepreneur as did social investor k

The first two bullet points imply that σ' orders social investors j and k differently than does σ . If the third and fourth bullet points cannot be satisfied for any two such investors, then the equilibrium investment allocation is the same as in σ . Now consider an alternative ordering σ'' , which is the same as σ' except that $\sigma''(j) = \sigma'(k)$ and $\sigma''(k) = \sigma'(j)$ (i.e. j and k are reordered but everything else is preserved). Then the two orderings result in the same allocation up until $\sigma'(k)$. At $\sigma'(k)$, the new ordering σ'' calls on investor j instead of investor k . Because $\theta_j > \theta_k$, investor j now chooses an entrepreneur with strictly higher social value. It is straightforward to demonstrate that σ'' results in an allocation with weakly higher social welfare than the one arising from σ' , and if the allocation resulting from σ'' is different than that resulting from σ' then the one resulting from σ'' results in weakly higher welfare.

Now take σ'' and repeat the above process. Continue to do so until σ'' results in the same allocation as σ . So long as the allocation arising from σ' and from σ are different, it is straightforward to show that at least one transformation resulted in a strict welfare improvement. Therefore σ induces the

welfare-optimal equilibrium.

A.6 Discussion of the Equilibrium when Social Value Can Be Negative

In this section we discuss how the analysis differs when we drop the assumption that $w_i \geq 0$ for all firms. We restrict the discussion to the binary project case, but the case where projects have multiple sizes is similar. The first difference in the analysis, which applies to both naïve and sophisticated social investors in Sections 3.1 and 3.2, is that social investors may not deploy all of their capital in equilibrium. This would be the case when they have exhausted the projects with positive social value, and the remaining projects have sufficiently negative social value that their social value is not offset by the accompanying financial returns.

The second and final difference in the analysis applies only to the sophisticated investors in Section 3.2. There may now be multiple equilibria across which sophisticated social investors may deploy different amounts of capital. This is because the sophisticated social investor who invests in firms with profits above $\tilde{\pi}$ cares about the contribution to social value of the marginal firm supported by commercial investors. This social value depends on the mass of social investors who invest in firms with profits greater than $\tilde{\pi}$. Call this mass βS , to remain consistent with the notation in Section A.2. It may be that for some small $\beta_1 S$, the social value of the marginal firm supported by commercial investors is sufficiently negative that sophisticated social investors decide not to deploy all of their capital. However, if all (or more) social investors were to deploy their capital, such that the mass of social investors supporting firms with profits greater than $\tilde{\pi}$ is $\beta_2 S > \beta_1 S$, then it is possible that the social value of the marginal firm supported by commercial investors would now be positive. In such a case both investment profiles could be supported in equilibrium.

When there are multiple equilibria, all of our propositions would hold when applied to the welfare-optimal equilibrium.