

Behavioral & Experimental Macroeconomics and Policy Analysis: a Complex Systems Approach

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¹**Disclaimer:** the views expressed in this talk are solely those of the author and may differ from official Bank of Canada views

Examples of Complex Systems

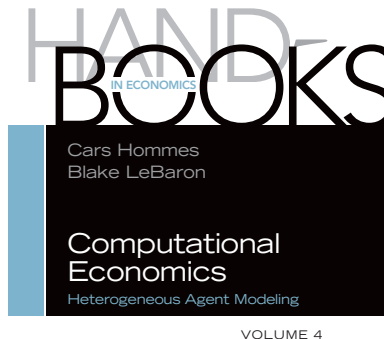


Some Characteristics of Complex Systems

- **interactions** of particles/**heterogeneous agents** at **micro** level create patterns and structure at aggregate level (**emergent macro behaviour**); **More is different**
- **nonlinear** and **critical transitions**: small changes at micro-level may lead to **large and irreversible** changes at macro level
- complex economic systems: “the **particles can think**” agents learn and adapt their behavior, thus changing the laws of motion of the system
How to model **(ir)rationality**?
How to model **expectations in a complex environment**?
Behavioural Theory in this talk: learning of simple, optimal heuristics in a complex, unknown environment

This talk focuses on stylized ‘few types’ models

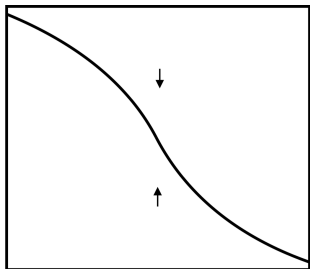
But large literature on detailed **Agent-Based Models (ABMs)**



NORTH-HOLLAND

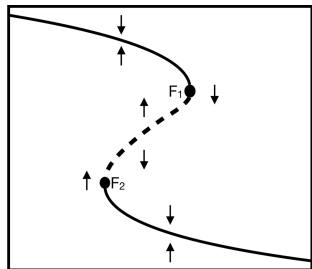
Key Feature Complex Systems: Critical Transitions between Multiple Equilibria; Tipping points

state



parameter

state



parameter

Plan of the Talk

Hommes, JEL 2020, forthcoming

Focus of the survey: boundedly rational expectations in stylized complex systems.

Five behavioural take-aways

- Complex/nonlinear systems exhibit **critical transitions** and **tipping points**
- Simple forecasting heuristics that make us smart
 - learning optimal homogeneous **AR(1)** rule
 - switching between heterogeneous **anchor and adjustment** rules
- Empirical validation of expectations through laboratory macro experiments
- Policy insight: how to **manage** complex economic systems?

Outline

- 1 Introduction Complex Systems
- 2 Learning a simple AR(1) forecasting heuristic**
- 3 Laboratory Experiments on Expectations
- 4 Behavioral Heuristics Switching Model
- 5 GA model with smart heuristic
- 6 Policy insight: managing complex systems
- 7 Conclusions and Discussion

Behavioral Learning Equilibrium (BLE)

Hommes and Zhu, JET 2014

- **simplest/parsimonious misspecification equilibrium**
- for each endogenous variable in the economy
perceived law of motion (PLM) \equiv AR1 process
 \neq actual law of motion (ALM)
- **consistency requirements: fixed point observable statistics**
 - **unconditional mean + autocorrelation of PLM \equiv unconditional mean + autocorrelation of ALM**
- simple learning mechanism for parameters through
sample autocorrelation learning to learn
the **optimal AR(1) heuristic**

Simplest example: asset pricing model with AR(1) driving dividends

1-D linear model driven by autocorrelated shocks/fundamentals

p_t : price

y_t : driving dividends

$$\begin{cases} p_t = \frac{1}{R} [p_{t+1}^e + a + \rho y_t] + \delta_t \\ y_t = a + \rho y_{t-1} + \varepsilon_t, \end{cases} \quad (1)$$

δ_t, ε_t : i.i.d. noise

no noise case: $\delta_t \equiv 0$

Asset pricing model with linear AR(1) forecasts

- **Perceived law of motion (PLM)** of agents:
AR(1) process

$$p_t = \alpha + \beta(p_{t-1} - \alpha) + v_t$$

- α is the **mean**; β is **first-order autocorrelation**
- **Actual law of motion (ALM)**:

$$\begin{cases} p_t = \frac{1}{R} [\alpha + \beta^2(p_{t-1} - \alpha) + a + \rho y_t] + \delta_t \\ y_t = a + \rho y_{t-1} + \varepsilon_t \end{cases}$$

Rational Expectations Equilibrium

$$p_t^* = \frac{aR}{(R-1)(R-\rho)} + \frac{\rho}{R-\rho} y_t. \quad (3)$$

In **special case** when $\{y_t\}$ is i.i.d., i.e. $a = \bar{y}$ and $\rho = 0$, then

$$p_t^* = \frac{a}{R-1} = \frac{\bar{y}}{R-1}$$

first order ACF under rational expectations:

$$\text{Corr}(p_t^*, p_{t-1}^*) = \rho$$

Behavioral Learning Equilibrium (BLE)

Consistency requirements:

Mean and first order autocorrelation of price must satisfy

$$\bar{p} := \frac{\alpha(1 - \beta^2) + \bar{y}}{R - \beta^2} = \alpha,$$

$$F(\beta) := \frac{\beta^2 + R\rho}{\rho\beta^2 + R} = \beta.$$

If $0 < \rho < 1$ and no noise ($\delta_t \equiv 0$) then there exists a **unique** behavioural learning equilibrium (BLE) (α^*, β^*) , given by

$$\alpha^* = \frac{\bar{y}}{R-1} = \bar{p}^* \quad (\text{unbiased})$$

$$\beta^* > \rho \quad (\text{persistence \& volatility amplification})$$

no free parameters; optimal AR(1) rule

Unique BLE in asset pricing model; near unit root

no noise case

$$(\alpha^*, \beta^*) = (1.0, 0.997)$$

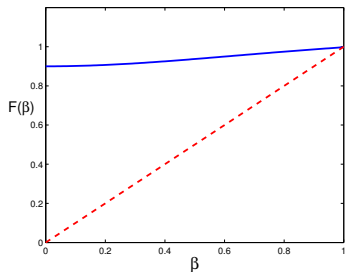
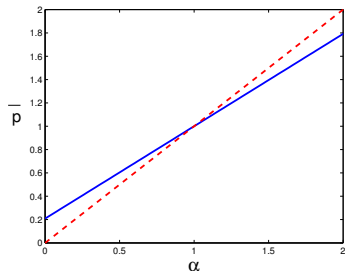


Figure: (a) α^* where mean $\bar{p} = \frac{\alpha(1-\beta^2) + \bar{y}}{R - \beta^2}$ intersects red diagonal α ;
 (b). β^* , where blue $F(\beta) = \frac{\beta^2 + R\rho}{\rho\beta^2 + R}$ intersects red diagonal;
 parameters $R = 1.05$, $\rho = 0.9$, $a = 0.015$.

Sample Autocorrelation Learning (SAC-learning)

- SAC-learning: Hommes and Sorger (1998)

$$\alpha_t = \frac{1}{t+1} \sum_{i=0}^t p_i, \quad \beta_t = \frac{\sum_{i=0}^{t-1} (p_i - \alpha_t)(p_{i+1} - \alpha_t)}{\sum_{i=0}^t (p_i - \alpha_t)^2}$$

- PLM under SAC-learning:

$$p_t = \alpha_{t-1} + \beta_{t-1}(p_{t-1} - \alpha_{t-1}) + v_t$$

- ALM under SAC-learning

$$\begin{cases} p_t = \frac{1}{R} [\alpha_{t-1} + \beta_{t-1}^2 (p_{t-1} - \alpha_{t-1}) + a + \rho y_t], \\ y_t = a + \rho y_{t-1} + \varepsilon_t. \end{cases}$$

unique BLE stable under SAC-learning

$$(\alpha^*, \beta^*) = (1.0, 0.997)$$

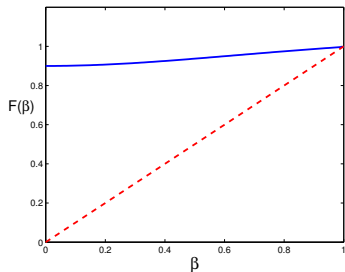
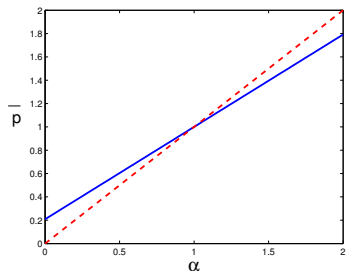


Figure: (a) α^* where mean $\bar{p} = \frac{\alpha(1-\beta^2) + \bar{y}}{R - \beta^2}$ intersects red diagonal α ;

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Simulation of SAC-learning

$$\rho = 0.9; \beta^* = 0.997$$

Learning to believe in near-unit root

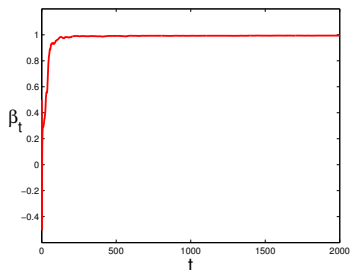
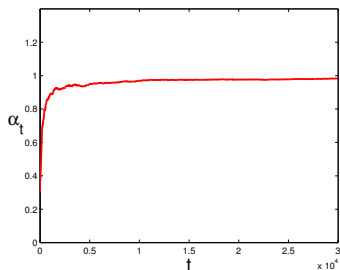


Figure: Time series of α_t and β_t under SAC learning.

- **Converging slowly to (unique) stable SCEE**
 $(\alpha^*, \beta^*) = (1.0, 0.997)$

Behavioral Learning Equilibrium

$\rho = 0.9$; $\beta^* = 0.997$ (no noise case)

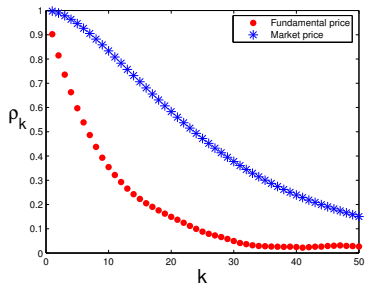
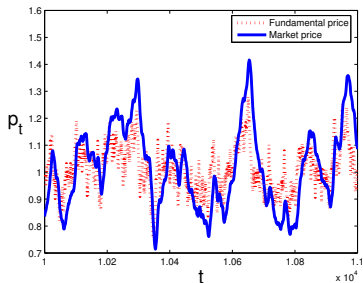


Figure: Time series of fundamental prices (red) and market prices (blue).

- Market prices fluctuate around fundamental prices
- Persistence & volatility amplification

Persistence & Volatility Amplification in Behavioral Learning Equilibrium

$\rho = 0.9$; $\beta^* = 0.997$ (no noise case)

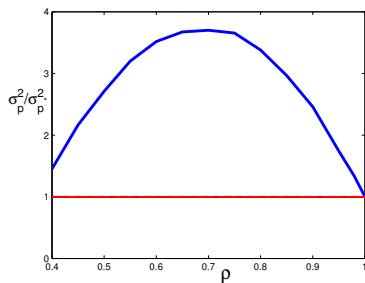
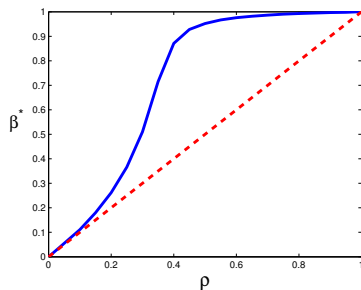


Figure: (a) SCEE β^* as a function of ρ ;
 (b) ratio of variance of market prices and variance of RE fundamental prices as a function of ρ .

Behavioral Equilibria with low and high persistence

co-existence of stable equilibria $\beta^* = 0.3066$ and $\beta^* = 0.9961$ (with noise δ_t)

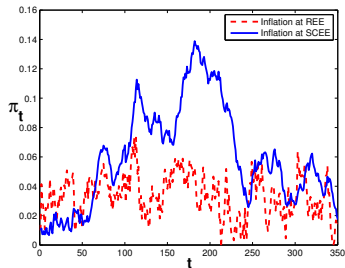
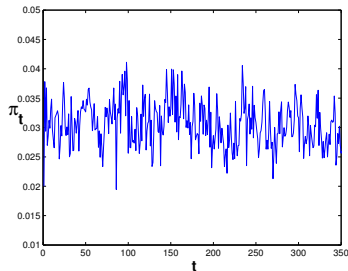


Figure: Convergence to low or high persistence equilibria β^* depending on initial states

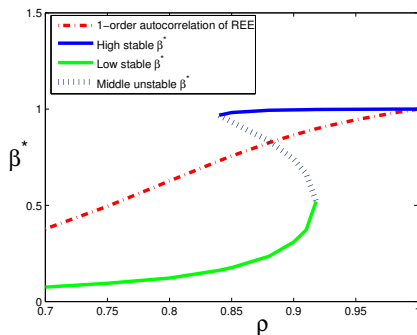
Critical Transitions of Equilibria β^* depending on ρ 

Figure: $\beta^*(1 \rightarrow 2 \rightarrow 3 \rightarrow 2 \rightarrow 1)$ as $\rho \uparrow$, where $\delta = 0.99, \gamma = 0.075, \frac{\sigma_u^2}{\sigma_\epsilon^2} = 0.1$.

High persistence BLE matches US inflation

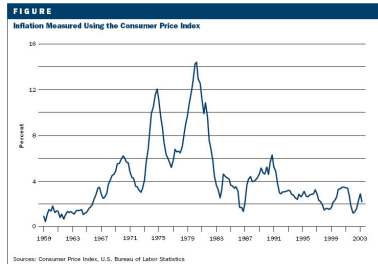
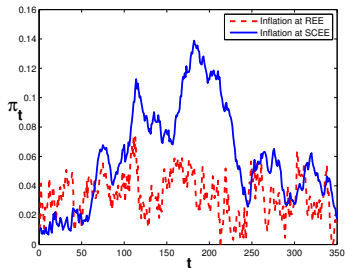


Figure: (a). Time series of inflation at stable SCEE $(\alpha^*, \beta^*) = (0.02, 0.995)$; (b). Empirical time series of inflation: Tallman (Federal Reserve Bank of Atlanta, ECONOMIC REVIEW, Third Quarter 2003).

Recent and ongoing work

Hommes, Mavromatis, Özden and Zhu, (2020)

- application and estimation of BLE in 3-Eq. NK-model
optimal AR(1) rules for both inflation and output
- estimation of BLE in Smets-Wouters DSGE model
- **Relevance: simple behavioral learning equilibria** are important, because **coordination of expectations** may be more likely;
(**different propagation mechanism** of shocks than under RE)
- **future extensions:** optimal AR(2) rule

$$p_t^e = \alpha + \beta_1 p_{t-1} + \beta_2 (p_{t-1} - p_{t-2})$$

Is there **trend-extrapolation** and **mean-reversion**?

Why Macro Experiments?

- If a theory does not work in the lab, why would it work in reality?
- A **macro experiment** studies **group behaviour** in a (simple) **complex system** in the lab, where aggregate behaviour depends on all individual **interactions** and decisions
- A **learning-to-forecast** experiment studies individual expectations and aggregate macro behaviour in simple **expectations feedback systems**
- **Main question:** do agents coordinate on **RE equilibrium** or on **behavioural learning** outcome?



Lucas, JPE, 1986 on Learning and Experiments

“Recent theoretical work is making it increasingly clear that the **multiplicity of equilibria** ... can arise in a wide variety of situations involving sequential trading, in competitive as well as finite agent games. All but a few of these equilibria are, I believe, behaviorally uninteresting: They do not describe **behavior that collections of adaptively behaving people** would ever hit on. I think an appropriate **stability theory** can be useful in weeding out these uninteresting equilibria ... But to be useful, stability theory must be more than simply a fancy way of saying that one does not want to think about certain equilibria. I prefer to view it as an **experimentally testable hypothesis**, as a special instance of the adaptive laws that we believe govern all human behavior.”

Positive versus Negative Feedback Experiments

Heemeijer et al. (JEDC 2009); Bao et al. (JEDC 2012)

- **negative feedback** (strategic substitute environment)

$$p_t = 60 - \frac{20}{21} \left[\sum_{h=1}^6 \frac{1}{6} p_{ht}^e \right] - 60] + \epsilon_t$$

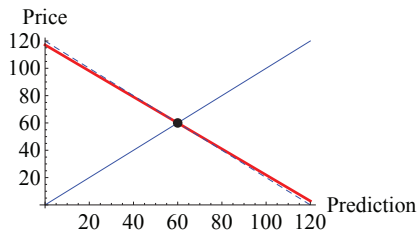
- **positive feedback** (strategic complementarity environment)

$$p_t = 60 + \frac{20}{21} \left[\sum_{h=1}^6 \frac{1}{6} p_{ht}^e - 60 \right] + \epsilon_t$$

- **common feature**: same RE equilibrium 60
- **only difference**: sign in the slope of linear map +0.95 vs -0.95

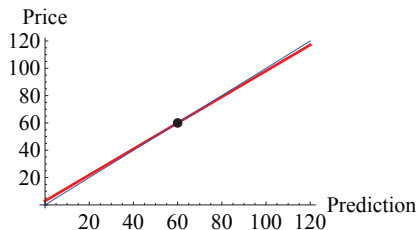
Feedback Mappings in LtFE

negative feedback



$$p_t = 60 - \frac{20}{21} (\bar{p}_t^e - 60) + \varepsilon_t$$

positive feedback



$$p_t = 60 + \frac{20}{21} (\bar{p}_t^e - 60) + \varepsilon_t$$

Concern with macroeconomic theory:

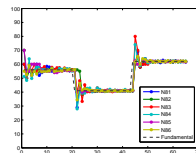
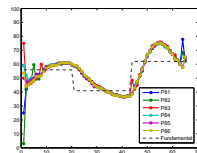
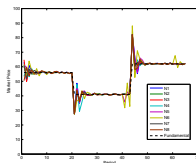
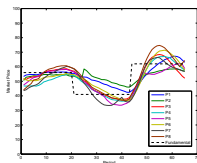
full information rational expectations ignores **almost self-fulfilling equilibria** in (strong) **positive feedback** systems

Positive vs Negative Feedback; Large Shocks

Bao, Hommes, Sonnemans, Tuinstra, JEDC 2012

positive FB (8 groups)
coordination failures

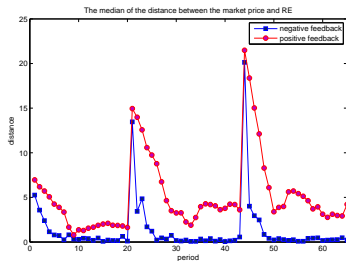
negative FB (8 groups)
coordination on RE



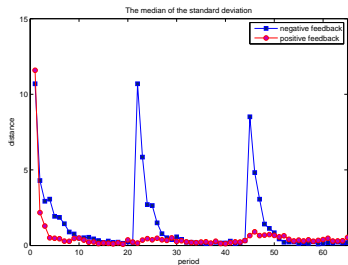
group 8, 6 individuals

Positive/Negative Feedback; Large Shocks

distance to RE price



degree of heterogeneity



positive feedback: quick coordination on ‘wrong’ price

negative feedback: slower coordination on correct RE price

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Heuristics Switching Model

Brock and Hommes, ECMA 1997; Anufriev and Hommes, AEJ:Micro 2012

- agents choose from a number of simple **forecasting heuristics**
- performance based reinforcement learning:**
agents evaluate the **performances** of all heuristics, and tend to **switch** to more successful rules;

fractions of belief types are gradually updated in each period:
(discrete choice model with asynchronous updating)

$$n_{ht} = \delta n_{h,t-1} + (1 - \delta) \frac{e^{\beta U_{h,t-1}}}{Z_{t-1}}$$

where Z_{t-1} is normalization factor.

- U_{ht} **fitness measure** (e.g. utility, forecasting errors, etc.)
- β is **intensity of choice**.
- δ **asynchronous** updating

Heuristic Switching Model: four forecasting heuristics

Anufriev and Hommes, AEJ:Micro 2012

- **adaptive expectations** rule, [$w = 0.65$]

$$\text{ADA} \quad p_{1,t+1}^e = 0.65 p_{t-1} + 0.35 p_{1,t}^e$$

- **weak trend-following** rule, [$\gamma = 0.4$]

$$\text{WTR} \quad p_{2,t+1}^e = p_{t-1} + 0.4(p_{t-1} - p_{t-2})$$

- **strong trend-following** rule, [$\gamma = 1.3$]

$$\text{STR} \quad p_{3,t+1}^e = p_{t-1} + 1.3(p_{t-1} - p_{t-2})$$

- **anchoring and adjustment heuristic** with learnable anchor

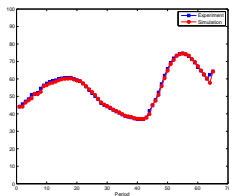
$$\text{LAA} \quad p_{4,t+1}^e = \frac{1}{2}(p_{t-1}^{av} + p_{t-1}) + (p_{t-1} - p_{t-2})$$

Problem: but where do these 4 rules and their coefficients come from?

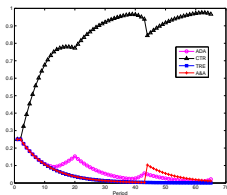
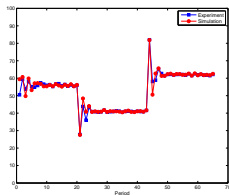
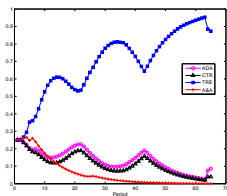
Positive vs Negative Feedback; Large Shocks

Heuristics Switching Model Simulations

prices



strategy frequencies



positive feedback: trend-followers amplify fluctuations

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Learning First Order Forecasting Heuristic

Simple heuristics that make us smart (Anufriev et al., 2019)

Agents learn two **parameters** of linear heuristic?

- Agent i uses a **first order forecasting heuristic** h to predict p_t :
anchor and adjustment rule

$$p_{i,h,t}^e = \alpha_{i,h,t} p_{t-1} + (1 - \alpha_{i,h,t}) p_{i,t-1}^e + \beta_{i,h,t} (p_{t-1} - p_{t-2}).$$

- The rule h requires **two** parameters: an **anchor** $\alpha_{i,h,t}$ and a **trend** $\beta_{i,h,t}$
- General constraint: $\alpha \in [0, 1]$, $\beta \in [-1.1, 1.1]$.
- The rule generalizes popular HSM heuristics: naive, adaptive expectations and trend extrapolation.
- **RE**: $\alpha = 0$, $\beta = 0$, $p_{i,t-1}^e = p^f$.

Learning by GA's through simple heuristics

Simple heuristics that make us smart (Anufriev et al., 2019)

- Every agent has a list of $H = 20$ different heuristics (α, β) .
- When agent i learns the last realized price p_{t-1} , she tries to re-optimize the rules with GA evolutionary operators:
 - ① sample (with replacement) 20 new heuristics from the old depending on their *hypothetical forecasting performance* (**reproduction**); (**survival of the fittest**)
 - ② **mutation**: with some small probability “mutate” them (modify (α, β) of each heuristic);
 - ③ **election**: compare the new and the old heuristics in terms of their hypothetical forecasting performance – pick the better ones.

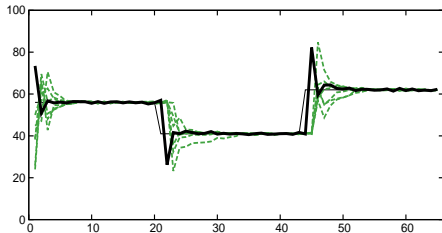
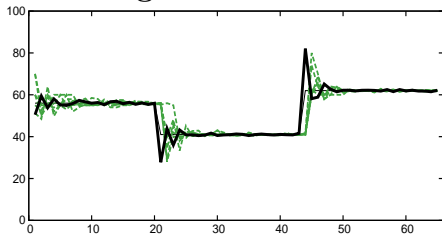
Process mimics natural selection: worse forecasting heuristics are likely to be *replaced* by better; *inefficient experimentation* screened out.

Remark: agents learn *independently*.

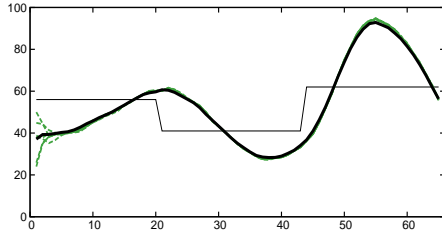
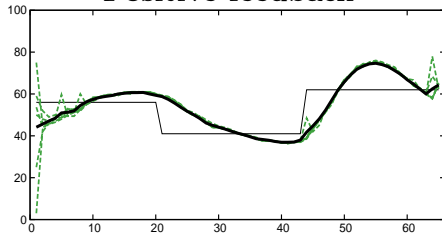
Lab experiment (top) and 65-period simulations (bottom)

experimental data Bao et al. (2012)

Negative feedback



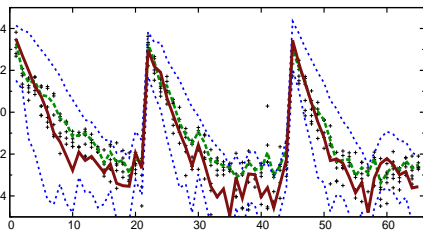
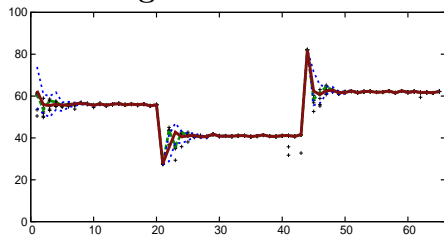
Positive feedback



65-period ahead Monte Carlo simulations (1000)

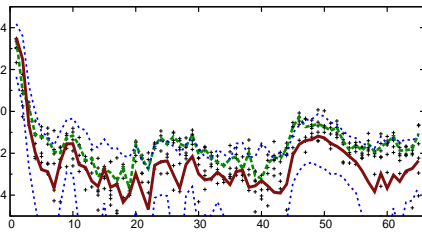
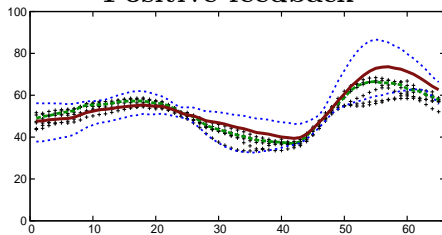
experimental data Bao et al. (2012)

Negative feedback



SD individual predictions

Positive feedback

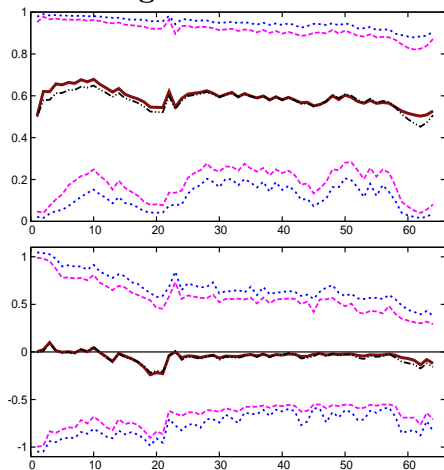


SD individual predictions

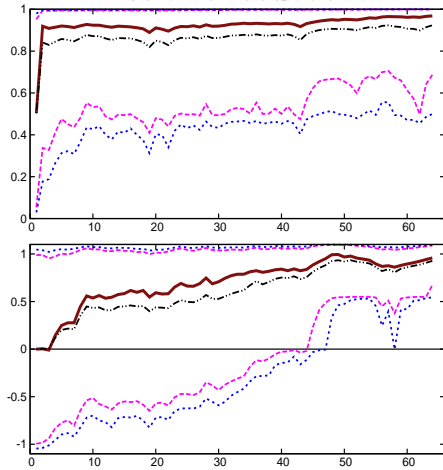
Anchor α_t (top) and trend β_t (bottom) parameters

experimental data Bao et al. (2012)

Negative feedback



Positive feedback



Average Heuristics

Under **negative feedback** agents learn to use **adaptive expectations**:

$$p_{i,t}^e \approx 0.5p_{t-1} + 0.5p_{i,t-1}^e$$

Under **positive feedback** agents learn to become **trend-follower**:

$$p_{i,t}^e \approx 0.95p_{t-1} + 0.05p_{i,t-1}^e + 0.9(p_{t-1} - p_{t-2})$$

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Housing market experiment with **positive** versus **negative feedback**

Bao and Hommes, 2019

$$p_t = \frac{1}{1+r} [(1-c)\bar{p}_{t+1}^e + \bar{y}] + \nu_t, \quad \lambda = \frac{1-c}{1+r}$$

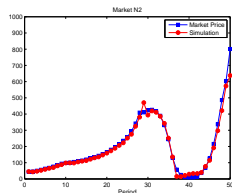
Behavioural intuition: an increase of housing supply (parameter c) adds **negative feedback** to the system, **weakening the overall positive feedback** (through speculators) making the system more stable

Managing Positive Feedback through Negative FB Policy

Housing Market Experiments, Bao and Hommes, 2019

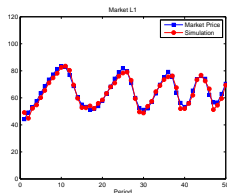
no FB policy
large bubble

($\lambda = 0.95$; $r = 5\%$)



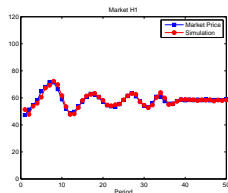
weak negative
FB policy
oscillations

($\lambda = 0.85$; $r = 18\%$)



strong negative
FB policy
stable

($\lambda = 0.71$; $r = 40\%$)



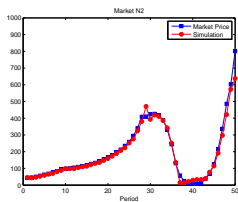
adding negative FB **stabilizes** complex positive FB system

Note: policy under RE: **do not interfere**

Simulated 1-period ahead forecasts HSM

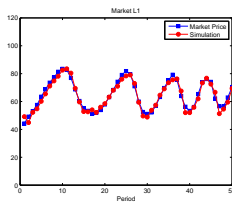
strong positive FB

$$\lambda = 0.95$$



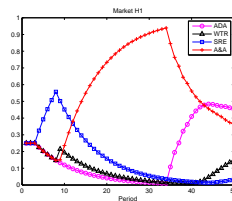
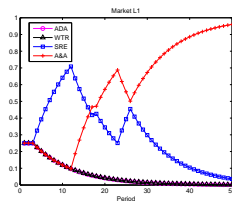
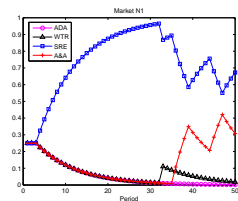
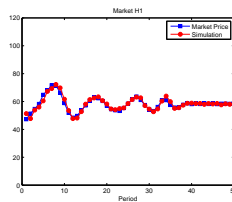
medium positive FB

$$\lambda = 0.85$$



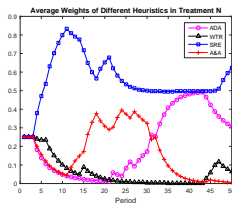
weak positive FB

$$\lambda = 0.7$$

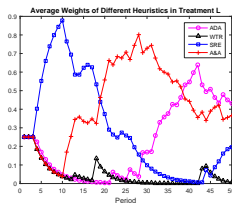


Average simulated 1-period ahead forecasts HSM

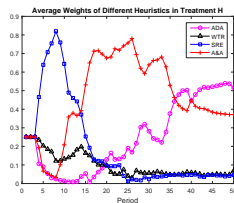
strong positive FB
 $\lambda = 0.95$



medium positive FB
 $\lambda = 0.85$



weak positive FB
 $\lambda = 0.7$



Policy Implication: negative FB policies that weaken the overall positive feedback may **stabilize** markets by preventing coordination on trend-following behaviour

Outline

- 1 Introduction Complex Systems
- 2 Learning a simple AR(1) forecasting heuristic
- 3 Laboratory Experiments on Expectations
- 4 Behavioral Heuristics Switching Model
- 5 GA model with smart heuristic
- 6 Policy insight: managing complex systems
- 7 Conclusions and Discussion

Five Behavioral Take-aways

- complex systems (non-linearity, heterogeneity, etc.) exhibit **critical transitions** between **multiple equilibria**
- adaptive learning of optimal AR(1) rule generates **near-unit root** and **excess volatility** and **persistence amplification**
- parsimonious **heterogeneous expectations** switching model based on relative performance
- heuristics switching between anchor and adjustment rules **fits experimental & empirical** data well
- ‘negative feedback’ policies can affect the self-organisation process, **prevent coordination on trend-following behaviour** and stabilize complex markets

Open Questions

- optimal AR(1) versus optimal **AR(2)**
short-run **trend-extrapolation** versus average **mean-reversion**
Which data are better explained by learning AR(2)?
- **homogeneous** versus **heterogeneous** expectations
homogeneous AR(2) versus heterogeneous switching
between mean-reverting rule and trend-extrapolating rule
Are **bubbles and crashes** better explained by heterogeneous agents model?
- **optimal policy** under simple behavioral **forecasting heuristics**

Thank you very much!

Good luck with your thesis on behavioral macro

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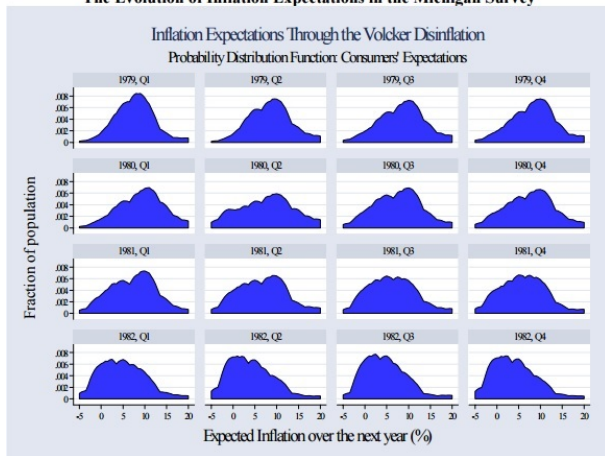
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Survey of Professional Forecasters: bimodal distribution

Mankiw, Reis and Wolfers, 2003

**Figure 12: The Volcker Disinflation:
The Evolution of Inflation Expectations in the Michigan Survey**



NK model with fundamentalists versus naive

Cornea-Madeira, Hommes and Massaro, JBES 2017

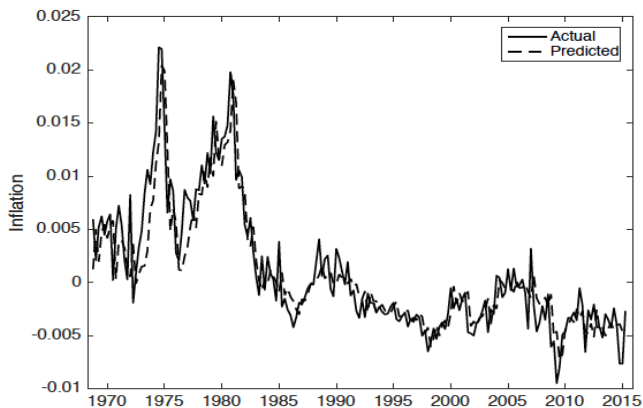
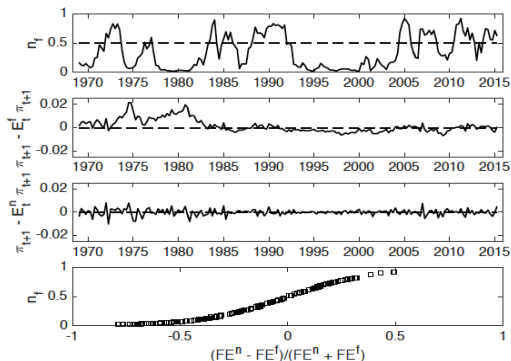


Figure: Actual vs. predicted inflation

NK model with fundamentalists versus naive

Cornea-Madeira, Hommes and Massaro, JBES 2017

Evolution of weight of fundamentalists $n_{f,t}$



on average more **back-ward looking** agents

Mean	0.353
Median	0.276
Maximum	0.924
Minimum	0.019
Std. Dev.	0.282
Skewness	0.418
Kurtosis	1.720
Auto-corr. Q(-1)	0.887

Top panel: Time series of the fraction of fundamentalists $n_{f,t}$

Second panel: Distance between actual and fundamental inflation

Third panel: Distance between inflation and naive forecast

Bottom: Scatter plot $n_{f,t}$ vs relative forecast error naive rule

NK model with fundamentalists versus naive expectations estimated on survey data professional forecasters

Cornea-Madeira, Hommes and Massaro, JBES 2017

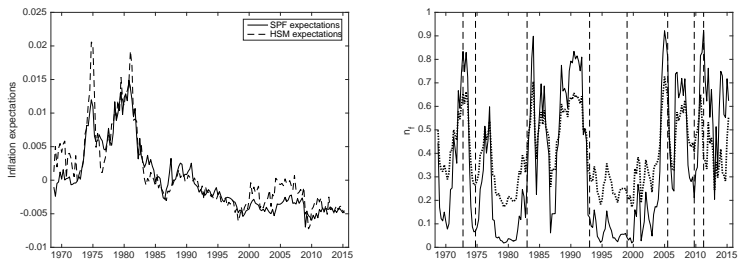
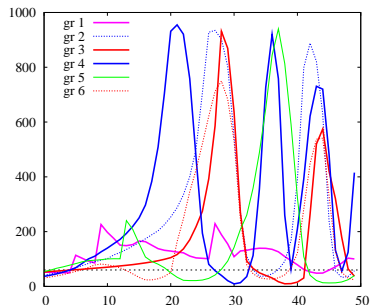


Figure: SPF forecasts vs. HSM expectations and estimated structural breaks with fractions of fundamentalists for inflation and SPF. **SPF switch slower than inflation expectations**

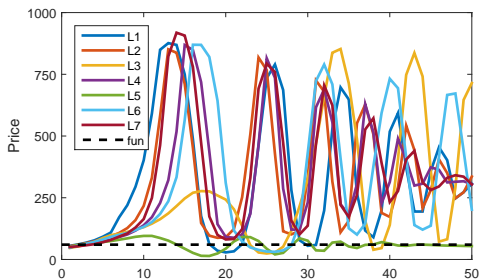
What happens without fundamental robot traders?

Hommes, Sonnemans, Tuinstra, vd Velden, JEBO 2008; Bao et al, 2016

groups of 6 subjects



groups of 25-30 subjects

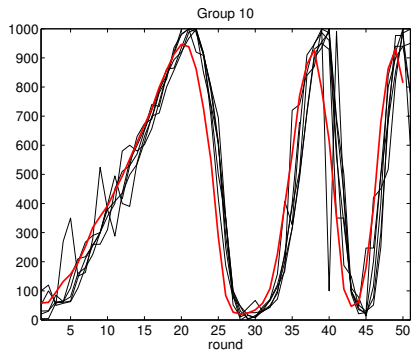


Does **positive feedback** cause **instability**?

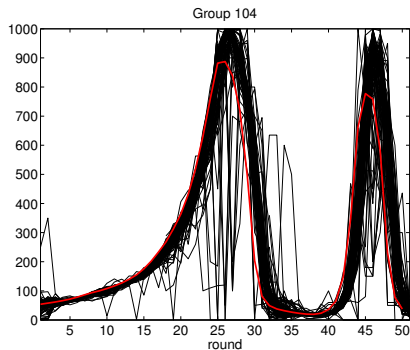
Coordination on bubbles in even larger groups

IBSEN Horizon 2020; Hommes, Kopanyi-Peuker, Sonnemans, 2018

group of 6 subjects



group of 100 subjects



Are bubbles caused by (strong) positive feedback?

Switching model estimated on housing markets

Bolt et al., JEDC 2019

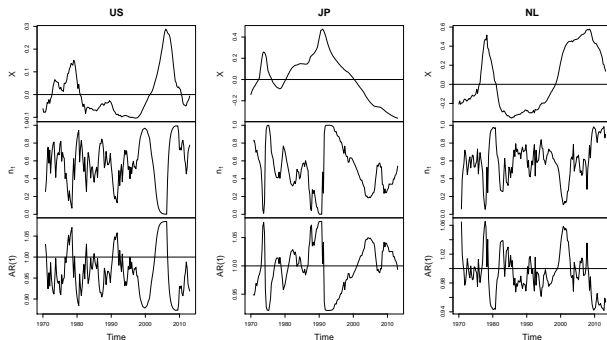


Figure: **Top panels:** relative house price deviations X_t from fundamentals;
Middle panels: time-varying fractions of mean-reverting fundamentalists;
Bottom: estimated market sentiment as time-varying AR(1) coefficient.