Social Connections, Strategic Referrals, and On-the-Job Search

Ji-Woong Moon*

Abstract

Workers use their social connections to search for a job. To study its implications on individual and aggregate labor market outcomes, I build a general equilibrium on-the-job search model in which workers have two types of job search methods, formal application and referral, and information on a worker's quality is endogenously transmitted through referral. I first present a novel empirical finding that only workers with a direct job-to-job search experience a significant referral wage premium. The model rationalizes the empirical pattern by a referrer's strategic information policy when a referrer values the hiring more compared to an employer. Despite the incentive misalignment, a referral yields higher match creation rate and ex-ante value of a vacancy compared to those without the referral. I provide a sufficient condition under which the strategic behavior of a referrer is socially more desirable than the transparent information case. Using the general equilibrium model, I can quantify and decompose the referral wage premium into the information effect and the network effect. I also discuss aggregate implications on welfare and inequality.

Keywords: Social connection; Referral; Directed Search; Information Design; Wage Dispersion

JEL Codes: E24, J64, J60, D83

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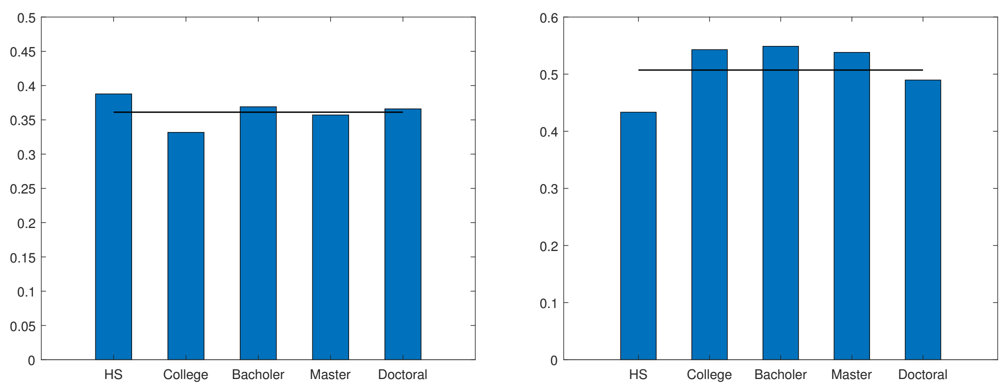
1 Introduction

In the mid-19th century, young Andrew Carnegie emigrated from Scotland to Allegheny, Pennsylvania, and found his first job at a cotton mill owned by a fellow Scottish immigrant. A few years later, through his uncle Thomas Hogan's referral, he moved to a Pittsburgh telegraph office where he became known around Pittsburgh for his skill and attitude.[1] While societies have completely changed since Andrew Carnegie searched for a job, a majority of job searchers in the 21st century still use the same strategy that Andrew Carnegie used more than 150 years ago: Using social connections. Figure 1 from the Survey of Consumer Expectation (SCE) of the Federal Reserve Bank of New York confirms the point. The left panel shows that 36% of employed workers have been referred to the current position, and the right panel shows that more than half of job searchers use their connections when searching for a job. Such prevalent usage of the social connections in the labor market calls for research for understanding how the job search through social connections affects the labor market outcomes such as wages, job-finding rates, and match qualities.



(a) Referred to the current job

(b) Contacted acquaintances for the job search

Figure 1: Referral usage in the labor market. Figure 1(a) is the fraction of workers who have been referred to the current job across educational groups. Figure 1(b) is the fraction of job searchers who use private networks in job search. The horizontal black line represents the population average. Source: Survey of Consumer Expectations,©2013-2018 Federal Reserve Bank of New York (FRBNY)

To address these issues, this paper proposes a general equilibrium on-the-job search model in which workers have two types of job search methods, formal application and referral, and information on a worker's quality is endogenously transmitted through referral. There are two main deviations

[1] Edge (2004)

from previous literature. First, I introduce the incentive misalignment between a referrer and an employer and endogenize the extent to which information is transmitted through referral as an outcome of strategic interactions. Second, I incorporate the referral hirings into a general equilibrium search model with two features: on-the-job search and endogenous wage distribution.

The idea that referral can alleviate incomplete information was discussed since Montgomery (1991). A strand of literature has used the idea to explain the wage premium of referral, but why and how a referrer provides information have been rarely studied. It is a relevant concern, however, because of incentive misalignment between a referrer and an employer. For example, when Thomas Hogan referred his nephew Andrew Carnegie to the telegraph office, Thomas Hogan must have done it for his own (or his nephew's) sake, not for the sake of the firm. Thus, the information transmission through Hogan's referral may not have been transparent as what the firm hoped it to be. Recent experiments add evidence to the possible incentive distortion by identifying strategic responses to changes in referral bonus payment (Bandiera et al., 2009; Beaman and Magruder, 2012). This paper provides a theory to characterize the degree of information transmission, and conditions under which referral yields higher wages and match qualities under the incentive mismatch between a referrer and an employer.

To further motivate the model, I examine whether the referred workers are paid better, and if so, how the wage premium varies across different workers using the SCE data. The heterogeneity that I focus on is whether an individual experiences an unemployment spell between the previous job and the current job because of the following reason. If a referrer chooses how much information delivers, the referrer will choose to provide more information for better jobs than for low-skilled jobs that a referred worker can surely perform. Thus the wage effect of referral, if it exists, would be higher in better jobs than low-level jobs. Although it is impossible to identify the ranking of jobs to an individual, it is true that workers without unemployment spell are more likely in higher positions of the hierarchy of jobs (Burdett and Mortensen, 1998; Shi, 2009). It implies that referral premium would be higher for workers without unemployment spell. I confirm the prediction with the empirical analysis. After controlling for the previous wage and demographics, referral wage premium exists on average, but it is only significant for workers experiencing direct job-to-job transfers.

In the model, I adopt a strategic game between a referrer and an employer from information design literature (Milgrom, 1981; Grossman, 1981), and incorporate it as a piece into a general equilibrium labor model, specifically a directed search model with on-the-job search (Shi, 2009; Menzio and Shi, 2011). In the model, all workers search for a job in the formal market by directly applying to a vacancy. Additionally, a socially connected worker can search in the referral market. Search is directed in both markets, and workers can choose different wage targets in each market. All jobs are identical and face idiosyncratic match-quality shock regardless of their recruiting method. The information that an employer has at the hiring decision is different for a referred and a non-referred worker. While an employer observes a noisy signal on the match-quality for all workers, a referred worker additionally brings a message sent by a referrer. Which message to send is a choice of a referrer so that the informativeness of the message is endogenous. After observing the signal for a non-referred worker and both the signal and the message for a referred worker, an employer makes the hiring decision based on a rational belief. I first define the referrer preferred sequential equilibrium of the information game between a referrer and an employer taking the value of a job as given. The general equilibrium of the model is the equilibrium of information game in addition to the endogenous value of a job, which is determined by workers' on-the-job search. The equilibrium of the information game and the value of a job are jointly determined since the equilibrium strategy of a referrer affects the job-finding rate.

In the equilibrium, the informativeness of the referral message is increasing in the wage of the job. Intuitively, a referrer optimally chooses the uninformative message for low wage jobs because any information can only decrease the hiring probability when a referred is on average over-qualified. For high wage jobs, a referrer voluntarily reveals some information when the match-quality is good, and the amount of information revealed is increasing in the wage as an employer requires more precise information when the wage is high. Since the information difference between a referred and a non-referred is increasing in the wage, the differences in the *ex ante* value of a job and the job-finding rate between a referred and a non-referred are also increasing in the wage. Therefore, the model generates the referral wage premium that is increasing in the current wage, which is consistent with the empirical findings of the paper.

The incentive misalignment between a referrer and an employer limits the amounts of information transmission. The amount of information transferred is indeed minimal in the following sense. The optimal strategy of a referrer only has two message realizations, either 'pass' or 'fail,' and a referrer pools any match quality above a threshold into the 'pass' group. The pooling result is in contrast to the revealing result of Milgrom (1981) and Grossman (1981), and it naturally arises as the employer does not choose the quantity of purchase in the labor market. The pooling result is robust to any incentive provision that is contingent on the true match-quality and events that happen after the hiring decision. In this sense, the result of this paper can be generalized even when a referrer cares about her/his reputation as long as the reputation depends on the true quality.

Although the information transmitted is limited in the equilibrium, it does not mean that the incentive misalignment creates inefficiency. In general, the efficient information transmission is not achievable since an employer cannot commit the selection policy. It makes a difference because an employer makes the hiring decision conditional on the meeting, but the social planner chooses the level of information before the meeting and internalizes the effect of information policy on the market tightness. Nevertheless, I derive that the equilibrium is close to the efficient outcome when the precision of the signal and the elasticity of the matching function are similar. I also derive a sufficient condition under which the equilibrium is more efficient than full-revealing at every wage level. The condition requires that the precision of the signal is weakly greater than the elasticity of the matching function.

Related Literature

Due to its widespread usage in the labor market, there have been many theoretical and empirical analyses that study why referral becomes a popular mean of searching for a job and what are the implications on post-employment wages and match qualities (Holzer, 1987; Ioannides and Datcher Loury, 2004; Topa, 2011).

Montgomery (1991) is a classic paper that rationalizes the use of social connections in the labor market. In his model, workers are heterogeneous in productivity type which is unknown for firms. The referral is useful because workers are more likely to be connected to the same type of workers so it can alleviate incomplete information. Simon and Warner (1992); Galenianos (2013) and Dustmann et al. (2015) also have a similar flavor in the sense that referral is valuable as it provides information about the quality of a worker. This paper adds the strategic concern which is absent in the previous papers. Also, none of the papers analyze referral, endogenous wage distribution and on-the-job search within a unified framework.

There is another strand of literature focusing on the implications of referral for wage distribution and persistence in labor market outcomes (Mortensen and Vishwanath, 1994; Calvo-Armengol and Jackson, 2004; Galenianos, 2014; Arbex et al., 2018). Most of these papers generate referral wage premium, and it is often linked to network effects. This paper extends the results of this strand of literature by adding endogenous informational transmission, which directly generates the wage premium on top of the indirect network effect. Moreover, no paper studies on-the-job search and endogenous wage distribution.

A large body of empirical literature investigates the effects of social connections on labor market outcomes. Some papers directly study the effect of referral (Simon and Warner, 1992; Burks et al., 2015; Dustmann et al., 2015; Brown et al., 2016), while some other papers focus on more broad concept of social capital using various proxies (Marmaros and Sacerdote, 2002; Cingano and Rosolia, 2012; Schmutte, 2014). One common finding of these papers is that a worker's social connectedness positively affects the worker's labor market outcomes. This paper rationalizes this empirical finding even when a referrer strategically behaves. Moreover, this paper finds and rationalizes the heterogeneous effect of referral across wages.

This paper is also related to a big strand of literature on directed search (Moen, 1997). The closely related papers are models with on-the-job search (Delacroix and Shi, 2006; Shi, 2009; Menzio and Shi, 2011), but none of the papers explicitly model referral and search through social connections. At the expense of introducing referral, the model in this paper loses block-recursivity that reduces computational difficulties for this class of models (Menzio and Shi, 2010). Although the model is not block-recursive, it is still tractable as the value functions depend on the aggregate distribution only through its integration.

Lastly, the strategic game between a referrer and an employer is an application of Milgrom (1981), and it is broadly related to information design literature (Milgrom, 1981; Grossman, 1981; Crawford and Sobel, 1982; Kamenica and Gentzkow, 2011; Lipnowski and Ravid, 2017). A key difference is that, while most papers in this strand of literature take relevant payoffs as a given environment, the payoffs in this paper are endogenously determined in the general equilibrium. Also, this paper shows the optimality of pooling in the labor market context in contrast to the revealing result in previous literature with a similar environment.

2 Empirical Motivation

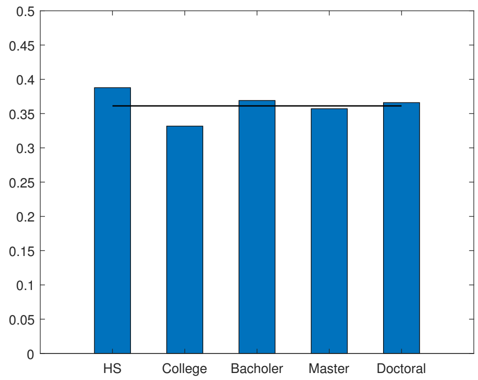
2.1 Data

The data is from the Survey of Consumer Expectations (SCE) by the Federal Reserve Bank New York. The SCE is a monthly survey of around 1,300 individuals. Once a year, the survey asks the respondents a variety of questions about their job search behaviors. The questions include, for example, the job search method through which the current job offer came and the methods that an individual is currently using. Another useful feature of the survey is its detailed information on the previous job, such as wage and occupation, which helps to control unobserved heterogeneity among individuals. This paper uses the job search survey 2013 - 2016, which is the maximum available data at the time of this study. The survey is repeated cross-section, and no individual appears more than once in the sample. The total number of the sample is 4,797.

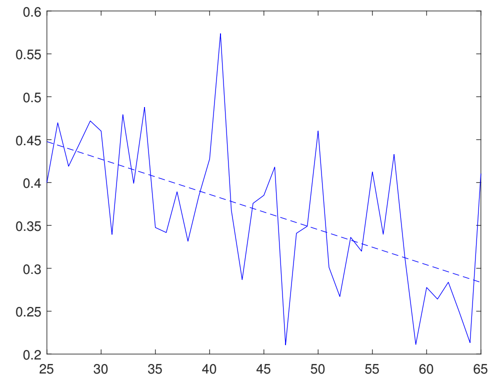
2.2 Pattern of Referral Usage

Figure 2 illustrates the pattern of referral usage across educational attainment and age. Figure 2(a) shows the fraction of workers referred to the current job. The population average is 36%, and the fraction does not vary a lot across education groups, which implies both low and high skilled workers can benefit from referral. Note that 36% is larger than the sum of workers who find the current job through online websites, career center, help ads, and professional registers, which span most of job search methods by oneself. Therefore, although the number is a little smaller than what previous papers suggest, such as Topa (2011), it is safe to conclude that referral is one of the most frequently used job search methods.

Figure 2(b) shows the fraction of referred workers across age. A clear pattern observed is the decreasing trend. The decreasing trend over age fits well with the incomplete information argument of referral. Referral is benefical because it reduces the uncertainty on a worker's quality, but the



(a) Across educational attainment



(b) Across age

Figure 2: Fraction of workers referred to the current job. Figure 2(a) is across educational attainment and Figure 2(b) is across age. The horizontal black line in 2(a) is the population average. The dashed blue line in 2(a) is the non-weighted trend of the solid blue line.
Source: Survey of Consumer Expectations,©2013-2018 Federal Reserve Bank of New York (FRBNY)

benefit decreases over a worker's tenure in the labor market because more information is revealed through the employment history. Although the social learning over life-cycle is beyond the scope of this paper, the model will reproduce the positive relationship between the uncertainty and the referral intensity.

2.3 Effect of Referral

The SCE has detailed information on a variety of facets of job search behaviors and outcomes. The information includes specific methods that an individual uses for job search and the source of the best offer conditional upon receiving more than one offer. If referral is particularly more productive than other job search methods, then it is more likely to deliver the best offer given the usage rate. Two figures in Figure 3 illustrate that it is likely the case. Figure 3(a) is the usage rates of a variety of job search methods conditional on a respondence searching for a job. Figure 3(b) is the percentage from which the best offer arises conditional on more than one offer received. Each bar in the graph corresponds to each choice in Table 1. The SCE classifies referral into three categories according to its sources: from friends or relatives; from co-workers, supervisors, teachers, business associates; current employees at other companies. The referral related choices are marked in bold in Table 1, and blue in Figure 3.

Figure 3(a) illustrates that contacting others is common, but not a dominant job search method

Table 1: Questionaires and Choices

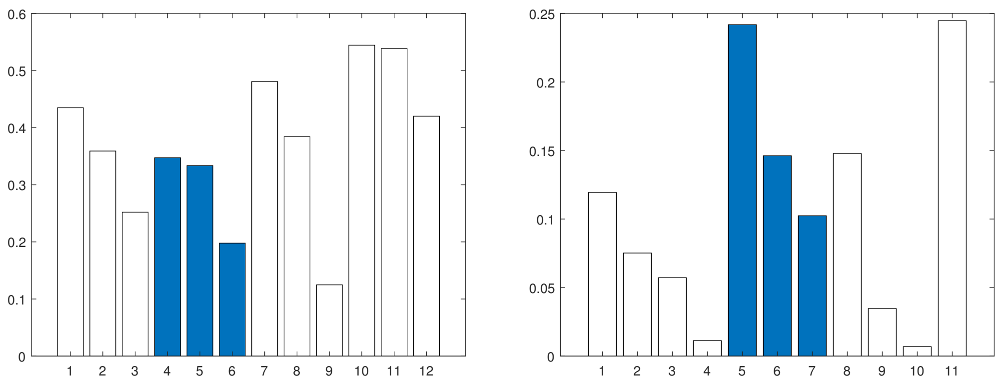
(a). What were ALL the things you have done to look for work during the LAST 4 WEEKS?

1. Contacted an employer directly online or through e-mail
2. Contacted an employer directly through other means, including in-person
3. Contacted an employment agency or career center, including a career center at a school or university
4. **Contacted friends or relatives**
5. **Contacted former co-workers, supervisors, teachers, business associates**
6. **Contacted current employees at other companies**
7. Applied to a job posting online
8. Applied to a job opening found through other means, including help wanted ads
9. Checked union/professional registers
10. Looked at job postings online
11. Looked at job postings elsewhere, including help wanted ads
12. Posted or updated a resume or other employment information, either online or through other means

(b). For this (your best) job offer, how did you learn about this job?

1. Found through the employer's website
2. Inquired with the employer directly through other means, including in-person
3. Found through an employment agency or career center
4. A temporary job was converted to permanent job
5. **Referred by a friend or relative**
6. **Referred by a former co-worker, supervisor, business associate**
7. **Referred by a current employee at the company**
8. Found through an online job search engine
9. Found job opening through other means, including help wanted ads
10. Found through union/professional registers
11. Unsolicited contact by potential employer

Source: Survey of Consumer Expectations,©2013-2018 Federal Reserve Bank of New York (FRBNY)



(a) Job search method

(b) Source of the best offer

Figure 3: Job search method and outcome. Figure 3(a) is the job search method in Table 1(a). Figure 3(b) is the source of the best offer in Table 1(b). The blue colored bars in each graph are referral related choices which are marked in bold in Table 1.
Source: Survey of Consumer Expectations,©2013-2018 Federal Reserve Bank of New York (FRBNY)

in terms of the usage rate. More people search for online job postings or contact an employer directly than ask their friends or co-workers. As expected, it is more common to search for a job by oneself than using connections. Nevertheless, referral produces the best offer for a large fraction of workers. In Figure 3(b), the referral from friends or relatives ranks the second following right after the unsolicited contact by a potential employer, and the referral from co-workers and business associate ranks the fourth with almost identical level to the online job search engine. If all else is equal, the results can be rationalized either if referral produces on average higher offers or if referral produces more offers given time horizon. In any case, a referral is on average more productive than other methods.

In Figure 4, I divide the sample into four groups according to the labor market status, and draw the fraction of workers who use connections and the fraction of workers who get the best offer through a referral. To draw the graph, the three referral related categories in Figure 3 are collected into a single category. There are two apparent patterns observed. First, the fraction of workers who use connections for their job search varies across the labor market status. The full-time workers are less like to use their connections compared to the part-time workers and the unemployed workers. The pattern seems to reflect a fact that contacting others is more costly than searching for a job by oneself. As the part-time workers and the unemployed workers have higher incentive to get a new job, they are more likely to use connections in addition to search by oneself.

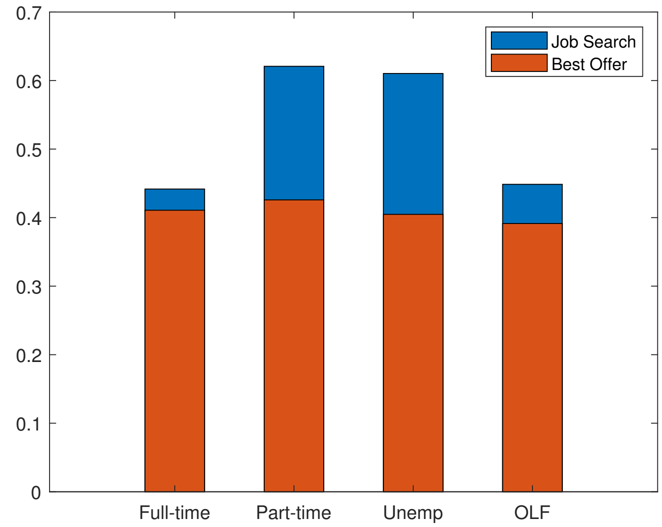


Figure 4: Job search method and sources of the best offer by labor force status

One obvious concern is the selection. If workers who actively use their connections are qualitatively different from workers without using their connections, then Figure 3 may represent the quality difference. To analyze whether referral leads to better labor market outcomes in a more rigorous and standard way, I conduct the following wage regression.

$$\log(w_{it}) = \alpha + \gamma_t + \beta_0 X_i + \beta_1 Ref_i + \beta_2 \log(w_{i,-1}) + \epsilon_{it} \tag{1}$$

where w_{it} is the current wage, X_i is controls, Ref_i is the dummy variable that represents referral status in the current job, and $w_{i,-1}$ is the last wage in the previous position. X_i includes age, gender, educational attainment, and part-time status. The previous wage $w_{i,-1}$ is included to control for unobserved heterogeneity not captured by observable characteristics. The parameter of interest is the coefficient β_1. I run the pooled OLS with the year dummy γ_t capturing inflation and time-varying aggregate shock. To capture the clean effect of referral, I restrict samples by current tenure less than 12 months for the first set of regressions (1) - (3), and 24 months for the second set of regressions (4) - (6).

The model (1) shows that referral wage premium exists after controlling the observable characteristics and the previous wage. The referral wage premium is still significant in the model (4) which includes workers with tenure up to 24 months, although its magnitude becomes smaller. Note that if referral wage premium is the result of the selection by unobserved characteristics of

Table 2: Wage Regression

	(1)	(2)	(3)	(4)	(5)	(6)
prev.wage	0.314***	0.334***	0.277**	0.310***	0.323***	0.303***
	(7.30)	(6.70)	(3.38)	(8.91)	(7.79)	(4.80)
age	0.0119	0.0157	0.00453	-0.00508	-0.0106	0.0477
	(0.60)	(0.75)	(0.10)	(-0.35)	(-0.64)	(1.58)
age*age	-0.000129	-0.000255	0.000161	0.0000989	0.000127	-0.000448
	(-0.57)	(-1.07)	(0.31)	(0.59)	(0.68)	(-1.28)
edu(coll)	0.0834	-0.316**	0.697**	0.0758	-0.117	0.570***
	(0.82)	(-2.67)	(3.17)	(0.99)	(-1.32)	(3.55)
edu(bs)	0.320**	-0.124	1.110***	0.343***	0.129	0.905***
	(2.73)	(-0.93)	(4.56)	(4.08)	(1.32)	(5.35)
edu(ms)	0.568***	0.117	1.261***	0.527***	0.300*	1.076***
	(4.16)	(0.75)	(4.91)	(5.13)	(2.46)	(5.85)
edu(dr)	0.830***	0.409	1.313**	0.896***	0.607**	1.521***
	(3.89)	(1.66)	(3.41)	(5.78)	(3.27)	(5.49)
parttime	-0.726***	-0.594***	-0.804***	-0.695***	-0.546***	-0.727***
	(-8.10)	(-5.73)	(-4.71)	(-9.68)	(-5.98)	(-6.14)
female	-0.339***	-0.308***	-0.102	-0.236***	-0.221**	-0.0865
	(-4.12)	(-3.34)	(-0.60)	(-3.86)	(-3.11)	(-0.73)
referral	0.199*	0.233**	0.0576	0.123*	0.127†	-0.0207
	(2.43)	(2.64)	(0.32)	(2.05)	(1.85)	(-0.18)
N	362	266	96	630	472	158
Sample	All,($\tau \leq 12$)	EE,($\tau \leq 12$)	UE,($\tau \leq 12$)	All,($\tau \leq 24$)	EE,($\tau \leq 24$)	UE,($\tau \leq 24$)

t statistics in parentheses

† $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. τ is tenure (in months) in the current position.

an individual, the effect must be captured by the previous wage as long as the characteristics are persistent. Therefore, the regression results seem to suggest the positive wage effect of referral rather than pure selection. The results are also broadly consistent with previous literature which finds referral wage premium using firm side data (Dustmann et al., 2015; Burks et al., 2015; Brown et al., 2016).

In the model (2) and (3), I divide the sample into two groups according to the presence or the absence of the unemployment spell between the previous and the current jobs. The intuition behind the exercise is that the wage effect will depend on the quality of a job referred if a referrer strategically transmits information. To be specific, a referrer does not necessarily tell a good story about a referred if the referred is already over-qualified for a position. On the other hand, a referrer would like to provide good information and oversell the referred for a relatively good position. Although the quality of a job is unobservable so that it is difficult to test the prediction, whether a worker experiences the unemployment spell between jobs is observable, which can be an indicator for the quality of the next job (Burdett and Mortensen, 1998; Shi, 2009).

The model (2) and (3) confirm the prediction. I conduct the same regression as before for the workers without the unemployment spell in the model (2). The regression results show that the referral premium is positively significant, and the magnitude is larger than that of the entire sample. However, for workers with the unemployment spell, the referral premium is no longer significant.[2] The same patterns are found in the model (5) and (6). So far my knowledge, this is the first paper that shows the heterogeneous effect of referral across a worker's previous labor market status. I rationalize the empirical finding by endogenous information transmission in the theoretical model.

[2]When the result is interpreted as the heterogeneity across wage-ladder, the empirical finding is broadly consistent with Burks et al. (2015) which finds significant referral wage premium in a high-tech firm, but not in call-center firms.

3 Model

3.1 Environment

3.1.1 Workers

Time is continuous. There is a continuum of homogenous workers with measure one. Workers are risk-neutral and discount future with rate $r > 0$. In each point of time, a worker is either employed ($\omega = e$) or unemployed ($\omega = u$). Additionally, a worker is either socially connected ($n = 1$) or not connected ($n = 0$). Thus, the individual state of a worker is summarized by a triple $x \equiv (\omega, w, n)$, where w is a worker's current wage rate.[3] I assume a flow value of home production $b \geq 0$ for the unemployed.

Workers always can search for a job by themselves, but the rate of search is potentially different for the unemployed and employed. The search rate is λ_u when unemployed, and λ_e when employed. In addition to the search by oneself, socially connected workers ($n = 1$) can search for a job through referral with rate λ_f. In either case, workers' search is directed so that the job-finding rate through each job search method is endogenous. Moreover, a worker rationally forms an expectation on the degree of information transmitted when the workers chooses a target wage rate. The detail comes in the later section.

A worker's social connectedness n follows a poisson process. The rate of change can depend on the worker's labor market status ω, which captures an idea that a worker builds social connection with different rate according to the labor market status. I denote the rate of building a connection when the worker's labor market status is ω as ψ_0^ω. Similarly, the rate of losing a connection is ψ_1^ω. Even if the rate does not directly depend on the aggregate state, there can be an externality from unemploymend rate on the aggregate matching efficiency if $\psi_n^u \neq \psi_n^e$.

3.1.2 Production

Productions take place by a pair of an employer and a worker. The productivity of a match has two components, aggregate productivity y that is common for all matches,[4] and match-specific

[3] Throughout the paper, I focus on the fixed wage case.

[4] I will introduce the aggregate shock on y in the quantitative section.

productivity $\phi \in [0,1]$. Given the match-specific productivity ϕ, the output is $y + \epsilon\phi$, where ϵ is a scale parameter. The match-specific productivity ϕ is permanent within the match, but independent across time and matches, thus it is purely idiosyncratic. Moreover, ϕ is realized from a distribution $F(\cdot)$ after the match creation, thus neither the wage nor the hiring decision can directly depend on the realization of ϕ. I assume that ϕ has a bounded support, and the distribution $F(\cdot)$ has a density $f(\cdot)$. Without loss of generality, $F(0) = 0$ and $F(1) = 1$.

The realization of ϕ, however, indirectly affects the hiring decision as the employer observes an informative signal s right before the hiring decision. The signal realization can be interpreted as interview, and it applies to all workers regardless of their referral status. The distribution of s given ϕ is denoted by $F_s(\cdot|\phi)$, which has the following form.

$$F_s(x|\phi) = \begin{cases} (1-\tau)F(x), \quad \text{if} \quad x < \phi \\ (1-\tau)F(x) + \tau, \quad ,\text{if} \quad x \geq \phi \end{cases}$$

In words, a signal s equals ϕ realization with probability τ, and is drawn from $F(\cdot)$ independent to ϕ with probability $1-\tau$. τ is the precision of the signal, and I focus on cases where $\tau \in (0,1)$, which means the signal is neither perfectly informative, nor perfectly uninformative. From the distribution of s, the expected value of ϕ given the signal s is $E(\phi|s) = \tau s + (1-\tau)E(\phi)$, thus the higher the s, the higher the expected ϕ. Note that Menzio and Shi (2011) also use the same information structure while they focus on two extreme cases $\tau = 0$ and $\tau = 1$. Interestingly, interior τ is necessary for the existence of non-trivial information transmission through referral.

While the same interview technology applies all workers, workers with a referral bring a message m from a referrer into the interview stage so that the expected productivity is conditional on two pieces of information, m and s. The informativeness of m is endogenous, and the referral game between a referrer and an employer determines m as an equilibrium outcome. I will explain the detail in Section 3.1.3.

The match quality ϕ captures uncertainty which is not perfectly observable to an employer *ex ante*. Examples of such uncertainty include a worker's personality, social skills, or firm-specific techniques. A firm potentially benefits by hiring a worker through referral because a referrer may provide some information about ϕ. There can be concerns about the match-specific nature of ϕ in this model. I deliberately ignore the general human capital components for two reasons. Theoretically, I try to isolate the effects of referral from learning. Second, empirical evidence shows that there is no significant difference in both observable and unobservable characteristics[5] between the referred workers and non-referred workers. It suggests that a referral is more than delivering information on a worker's permanent ability.

3.1.3 Referral Game

When a worker meets an employer through a referral, there exists a strategic referrer who provides information on ϕ to an employer. To capture the incentive misalignment, I assume that there exists referral bonus payments z, and the objective of a referrer is to maximize the expected referral bonus payment. As a result, a referrer tries to maximize the hiring probability by persuading an employer to hire everyone while an employer tries to maximize the profit by hiring only good candidiates. I will keep this stylized assumption to clearly deliver key intuitions, and then generalize the intuitions to a case when z is a function of ϕ.

The information transmission is modeled following a persuasion game with verifiable message such as Milgrom (1981) and Grossman (1981). Timing of events is the following. First, a referrer observes ϕ between a referred and an employer, which represents an idea that a referrer possesses better information on the referred worker's quality. After observing ϕ, but before the realization of s, the referrer sends a message $m \in \mathcal{P}[0,1]$ to an employer subject to two conditions: $\phi \in m$, and $m = cl(m)$, where $cl(A)$ is the set closure of A. Lastly, s is realized accroding to $F_s(\cdot|\phi)$. The hiring decision is based on the message m and the realized signal s.

The constraint $\phi \in m$ means the message must be truthful, even though it can be noisy. There are several justifications for the truthful report constraint. Reputation is one reason, and an employer may be able to punish if a referrer actively lies. I put aside such dynamic concerns for future research and directly restrict the referrer's message space to the set of truthful messages. The restriction gives a referrer some credibility which makes information transmission possible.[6]

[5] See Burks et al. (2015) for details.

[6] A referrer cannot deliver any information if the message is purely a cheap-talk. See Lipnowski and Ravid

This type of message space is general enough to cover a variety of information structures as a special case. For instance, a referrer can truthfully deliver the match-specific quality by choosing $m = \{\phi\}$. A referrer can be completely uninformative by sending $m = [0, 1]$ for all ϕ. A particular interest is 'pass/fail' type message.

$$m = \begin{cases} [\bar{\phi}, 1], & \text{if} \quad \phi \geq \bar{\phi} \\ \{\phi\}, & \text{if} \quad \phi < \bar{\phi} \end{cases}$$

Many statements about a worker's quality fall into the pass/fail message. For example, a statement like 'She is proficient with Python and C++' can be viewed as a pass/fail message because it tells that she can handle the programs, but it is not informative enough to infer how good she is.

Let $\sigma(\phi, w) : [0, 1] \times \mathbb{R}_+ \to \mathcal{P}[0, 1]$ be the strategy of a referrer,[7] where w is the wage that the referred worker is searching. An employer forms a posterior belief on ϕ based on the realization of m and s, taking into accounts the strategy of a referrer σ. Denote the posterior belief as $\mu(\phi|m, s, w)$. The employer hires the referred worker if and only if the expected value of a job based on μ exceeds the fixed cost of hiring. Denote the hiring decision of the employer by $h(m, s, w) \in \{0, 1\}$. The relationship between $h(m, w, s)$ and $\mu(m, s, w)$ is straightforward.

$$h(m, w, s) = 1 \iff \int_0^1 J(w, 1, \phi) d\mu(\phi|m, s, w) - z \geq 0$$

where $J(w, 1, \phi)$ is the value of a job, which will be introduced in the next section. Given $\mu(m, s, w)$ and $h(m, s, w)$, the payoff of a referrer who observes ϕ is the following.

$$z \times \int_0^1 h(\sigma(\phi, w), s, w) dF_s(s|\phi)$$

Given the value function of a firm $J(w, n, \phi)$, a sequential equilibrium of the referral game consists of the referrer's strategy $\sigma(\phi, w)$, the employer's belief $\mu(\phi|m, s, w)$ and the hiring decision $h(m, s, w)$ that satisfy the following.

(2017) for details.

[7] I only consider an equilibrium in which the strategy of a referrer does not depend on the referrer's state. It is natural as none of characteristics of a referrer affects the payoffs of the referral game. Also, I only consider the set of pure strategies.

i) (Optimal referral) Given $\mu(m, s, w)$ and $h(m, s, w)$, $\sigma(\phi, w)$ is an optimal message.

$$\forall m, \phi, w \quad \int_0^1 h(\sigma(\phi, w), s, w) dF_s(s|\phi) \geq \int_0^1 h(m, s, w) dF_s(s|\phi) \tag{2}$$

ii) (Optimal hiring) $h(m, s, w) = 1$ if and only if the value of a filled-position is weakly greater than z under $\mu(\phi|m, s, w)$.

$$\forall m, w, s \quad h(m, w, s) = 1 \iff \int_0^1 J(w, 1, \phi) d\mu(\phi|m, s, w) - z \geq 0 \tag{3}$$

iii) (Consistency of belief) $\mu(\phi|m, s, w)$ is consistent with $\sigma(\phi, w)$ for any m in the range of $\sigma(\phi, w)$. $\mu(\phi|m, s, w) = \delta(\min m)$ if m is not in the range of $\sigma(\phi, w)$, where $\delta(\cdot)$ is the dirac-delta function.

The sequential equilibrium definition is self-explanatory, and consistent with the definition adopted in Milgrom (1981). Note that the off-the-path belief is restricted by 'skepticism' in a sense that the employer puts all weights on the worst case $\phi = \min m$. The off-the-path belief, however, does not play an important role.

There are many sequential equilibria that satisfy i) - iii). Among the equilibria, this paper focuses on an equilibrium that maximies the referrer's expected payoff. The referrer preferred equilibrium (σ^*, μ^*, b^*) is an equilibrium with the following property.

iv) (Referrer Optimality) For all w, if (σ, μ, h) satisfies 1 - 3,

$$\underbrace{\int_0^1 \int_0^1 h^*(\sigma^*(\phi, w), s, w) dF_s(s|\phi) dF(\phi)}_{\textit{en ante} \text{ hiring probability in submarket } w} \geq \int_0^1 \int_0^1 h(\sigma(\phi, w), s, w) dF_s(s|\phi) dF(\phi)$$

The refined equilibrium (σ^*, μ^*, h^*) maximizes the referrer's *ex ante* payoff among all sequential equilibria.[8] Qualitative results, however, do not heavily rely on this refinement.

[8] Kamenica and Gentzkow (2011) and Lipnowski and Ravid (2017) also consider the sender preferred equilibrium and payoff.

3.1.4 Search and Matching

Search is directed for both referral hirings and non-referral hirings. I assume that a vacancy must target one form of hiring, either taking workers with a referral only or taking workers without a referral only. While vacancies are separated, workers can simultaneously search in the two markets when socially connected.

There exists a continuum of submarkets indexed by (w, n) in the market for non-referred workers, and a continuum of submarkets indexed by w in the market for referred workers. In both cases, w is fixed wage rate and a commimitment, which means the employer cannot adjust the wage after matching nor voluntarily separate from the match. When posting a vacancy, employers have the rational expectation on the profitability of a vacancy. To be specific, when an employer posts a vacancy for non-referred workers, the employer anticipates that their hiring decision will depend on the signal s and the wage guaranteed w. When an employer posts a vacancy for referred workers, the employer knows that the hiring decision will depend on the equilibrium strategy of a referrer as well as the signal realization. This rational expectation pins down the *ex ante* expected value of a vacancy which shapes the incentive to create vacancies in each submarket.

For each submarket for the non-referred workers, the market tightness is denoted by $\theta(w, n)$, which is the vacancy-worker ratio. Similarly, the market tightness for the referred workers is $\theta_\rho(w)$. Given the market tightness θ, a worker meets an employer with probability $p(\theta)$, and an employer meets a worker with probability $q(\theta) = p(\theta)/\theta$. I impose standard assumptions on p and q. $p : \mathbb{R}_+ \to [0, 1]$ is twice-continuously differentiable, strictly increasing, strictly concave, and $p(0) = 0, p'(0) < \infty$. $q : \mathbb{R}_+ \to [0, 1]$ is twice-continuously differentiable, strictly decreasing, strictly convex, $q(0) = 1, q'(0) < 0$ and $q^{-1}(p(\cdot))$ is concave. Because of the endogenous message m and signal realization s, $p(\theta)$ is not the job-finding probability. Instead, there exists endogenous hiring proability $H(w, n)$ and $H_\rho(w)$ conditional on a meeting, which are the following.

$$
\begin{aligned}
H(w, n) &= \int_0^1 \int_0^1 h_n(s, w) dF_s(s|\phi) dF(\phi) \\
H_\rho(w) &= \int_0^1 \int_0^1 h(\sigma(\phi, w), s, w) dF_s(s|\phi) dF(\phi)
\end{aligned}
$$

where $h_n(s, w) \in \{0, 1\}$ is the indicator function that takes 1 if and only if the expected value of a position given s is weakly positive. As a result, the job-finding probability is $p(\theta(w,n))H(w,n)$, and the vacancy-filling probability is $q(\theta(w,n))H(w,n)$.

There is a flow cost of vacancy $k > 0$. Also, an employer needs to pay fixed costs z when hiring a worker. It is the referral bonus payment to a referrer for referral hirings, and I assume that the same fixed cost applies to the non-referral hirings to make no cost difference between the two means of hirings. Let the value of a filled-position $J(w,n,\phi)$ be given, which will be introduced in the next section. Then, the free entry condition pins down the market tightness for the non-referred workers as the following.

$$q(\theta(w,n))H(w,n)\int \left(E_\phi[J(w,n,\phi)-z|s]\right)^+ dF(s) \leq k$$
$$q(\theta_\rho(w))H_\rho(w)\int\int \left(E_{\tilde{\phi}}\left[J(w,n,\tilde{\phi})-z|\sigma(\phi,w),s,w\right]-z\right)^+ dF_s(s|\phi)dF(\phi) \leq k$$

where $x^+ \equiv \max\{x,0\}$. Intuitively, a hiring occurs in the market for a non-referred worker only if the net expected value of a position conditional on a signal s is positive. The *ex ante* value of a vacancy is integration over the signal realization whose unconditional distribution is also F. The expected value of a position for a referred worker can be similarly calculated, which involves the equilibrium strategy $\sigma(\phi,w)$.

Submarkets conditioning on n may or may not be realistic. It simplifies the equilibrium analysis significantly, however, by eliminating off-the-path belief restrictions. When submarkets only condition on w, the model requires off-the-path belief restrictions to define an equilibrium, and it often yields multiple equilibria,[9] which are unnecessary complications considering the purpose of this paper. Moreover, at the equilibrium, an employer can perfectly infer n of a worker from the publicly observable previous wage. In this regard, it is a reasonable simplifying assumption to separate submarkets by n.

[9]This is because workers in each submarket are heterogeneous in two dimensions, continuation utility and social connectedness, at the equilibrium. Multi-dimensional heterogeneity usually generates multiple equilibria even with an off-the-path belief restriction. Examples are Chang (2017),Li (2017) and Guerrieri and Shimer (2018).

3.1.5 Value Functions

Let the value of the unemployed workers with wage w as $V_U(n)$. At the steady-state, the below determines $V_U(n)$.

$$rV_U(n) = w + n\lambda_f R_\rho(V_U) + \lambda_u R(V_U, n) + \psi_n^u(V_U(n_-) - V_U(n))$$

where $(R(V, n), R_\rho(V))$ is the return to search.

$$\begin{aligned} R(V_0, n) &\equiv \max_w \{H(w, n)p(\theta(w, n))(V(w, n) - V_0)\} \\ R_\rho(V_0) &\equiv \max_w \{H_\rho(w)p(\theta_\rho(w))(V(w, 1) - V_0)\} \end{aligned}$$

The value of the employed workers $V(w, n)$ is defined similarly.

$$rV(w, n) = w + n\lambda_f R_\rho(V) + \lambda_e R(V, 1) + \delta(V_U(n) - V(w, n)) + \psi_n^e(V(w, n_-) - V(w, n)) + n\Pi$$

The last term Π is the expected referral bonus payment that a worker expects to get. I assume the referral bouns paid in the economy is evenly distributed to all socially connected and employed workers.[10] Therefore, Π depends on the entire distribution of the economy, but it does not affect the equilibrium strategy of a referrer. Thus, I will not specify its determination when qualitative properties of the model are of interests.

Denote the optimal search policy of a worker as $g(w, n)$ and $g_\rho(w)$.

$$g(w, n) = \arg\max_{w'} \left\{H(w', n)p(\theta(w', n))(V(w, n) - V_0)\right\}$$

The optimal search policy determines the endogenous separation probability $p^*(w, n) \equiv H(g(w, n))p(\theta(g(w, n)))$ and $p_\rho^*(w) \equiv H_\rho(g_\rho(w))p(\theta_\rho(w))$ for each w, n. Given the endogenous separation rates, the value of

[10]It implicitly assume a specific matching environment where a referrer is randomly chosen, and only employed workers can make a referral.

a filled-position $J(w, n, \phi)$ is the following.

$$r^*(w,n)J(w,n,\phi) = y + \epsilon\phi - w + \psi_n^e(J(w,n_-,\phi) - J(w,n,\phi)), \quad \text{where} \tag{4}$$

$$r^*(w,n) = r + \delta + \lambda_e p^*(w,n) + n\lambda_f p_\rho^*(w) \tag{5}$$

$r^*(w, n)$ is the effective discount factor including both exogenous and endogenous separation rates.

A realization of ϕ affects the output of the match, but does not affect any other objects including the separation rates. Thus, the value of a filled-position J is an affine function of ϕ, which implies that the expected value of a filled-position conditional on any information set $\mathcal{I}$ is the following.

$$r^*(w,n)E_\phi[J(w,n,\phi)|\mathcal{I}] = r^*(w,n)J(w,n,E(\phi|\mathcal{I})) \tag{6}$$

Equation (6) implies that the hiring rule only depends on the expectation of ϕ given any information set. As $J(w, n, \phi)$ is strictly increasing in ϕ, the hiring rule $h(m, s, w)$ and $h_n(s, w)$ satisfy the following.

$$h(m,s,w) = 1 \iff \int \phi d\mu(\phi|m,s,w) \geq \bar{\phi}_1(w) \tag{7}$$

$$h_n(s,w) = 1 \iff \int \phi dF(\phi|s) \geq \bar{\phi}_n(w) \tag{8}$$

where $\bar{\phi}_n(w)$ is the threshold match quality which is implicitly defined by $J(w, n, \bar{\phi}_n(w)) = z$. Given the signal structure, the conditional expectation of ϕ given s is $\tau s + (1 - \tau)E(\phi)$. Combined with Equation (8), the threshold signal $s_n^*(w)$ is derived as Equation (9).

$$s_n^*(w) = \max\left\{\frac{\bar{\phi}_n(w) - (1-\tau)E(\phi)}{\tau}, 0\right\} \tag{9}$$

The conditional expectation of ϕ given m and s involves the equilibrium strategy σ. Given a message m, the employer assigns a strictly positive posterior probability for $\phi \in m$ only if $\sigma(\phi, w) = m$. Define $\sigma_w^{-1}(m) = \{\phi \in m | \sigma(\phi, w) = m\} \subseteq m$, which is the set of ϕ that has strictly positive posterior probability. The conditional expectation of ϕ given m and s at the equilibrium is the following.

$$\int \phi d\mu = \begin{cases} E(\phi|\phi \in \sigma_w^{-1}(m)), & \text{if} \quad s \notin m \\ \tau' s + (1-\tau')E(\phi|\phi \in \sigma_w^{-1}(m)), & \text{if} \quad s \in m \end{cases} , \quad \text{where} \quad \tau' = \frac{\tau}{\tau + (1-\tau)P(\phi \in \sigma_w^{-1}(m))} \tag{10}$$

where the expectation is taken over the distribution F.[11] Because the message always contains the true quality, the employer immediately knows that s is uninformative whenever $s \notin m$. In this case, the only available information is $\phi \in \sigma_w^{-1}(m)$, thus the conditional expectation of ϕ given m, s is $E(\phi|\phi \in \sigma_w^{-1}(m))$. If $s \in m$, the employer puts the strictly positive weight on s as it can be precise. Note that the weight τ' on the signal conditional on $s \in m$ is larger than the precision of the signal τ because s is less likely to be false conditional on $\phi \in \sigma_w^{-1}(m)$. For instance, if $P(\phi \in \sigma_w^{-1}(m)) = 0$, the employer puts probability 1 on the signal as the probability that the signal is drawn independently to ϕ is 0.

3.2 Equilibrium

3.2.1 Equilibrium Definition

In this paper, I focus on a steady-state equilibrium. The steady-state equilibrium requires a time-invariant joint distribution of wages and social connections. Let $G(w, n)$ be the joint distribution of (w, n) among the employed, and $G_1(w, n) \equiv \frac{\partial}{\partial w} G(w, n)$. With a slightly abuse of notation, let $G(U, n)$ be the measure of the unemployed with the social connection n. For any $(w, 1)$, the outflow consists of the endogenous and the exogenous separations from the job, and the exogenous destruction of the social connection.

$$(\delta + \lambda_f p^*_\rho(w, 1) + \lambda_e p^*(w, 1) + \psi^e_1)G_1(w, 1) \tag{11}$$

[11]This is because the strategy of the referrer is a pure strategy.

The inflow consists of the job to job transfer through referrals and formal hirings, and the exogenous social capital accumulation.

$$\lambda_f p^*_\rho(w',1)G_1(w',1) + \lambda_e p^*(w'',1)G_1(w'',1) + \psi^e_0 G_1(w,0) \tag{12}$$

where w' and w'' are such that $g_\rho(w') = w$ and $g(w'',1) = w$. At the steady-state, the inflow (11) and the outflow (12) must equal to each other for all $(w,1)$. The inflow-outflow equation for the non-connected workers can be derived similarly.

$$(\delta + p^*(w,0) + \psi^e_0)G_1(w,0) = p^*(w'',0)G_1(w'',0) + \psi^e_1 G_1(w,1) \tag{13}$$

The inflow to, and the outflow from the unemployed also must balance.

$$\delta \int dG(w,0) + \psi^u_1 G(U,1) = (\lambda_u p^*_u(0) + \psi^u_0)G(U,0) \tag{14}$$

$$\delta \int dG(w,1) + \psi^u_0 G(U,0) = (\lambda_f p^*_{\rho,u} + \lambda_u p^*_u(1) + \psi^u_1)G(U,1) \tag{15}$$

where $p^*_u(n)$ and $p^*_{\rho,u}$ are the job-finding rate of the unemployed. Lastly, all workers are either employed or unemployed.

$$\int G(w,n)dwdn + G(U,0) + G(U,1) = 1 \tag{16}$$

Given the steady-state distribution, Π must equal to the referral bonus payments paid divided by the total number of employed workers who possess the social connection.

$$\Pi \int dG(w,1) = z \int p^*_\rho(w)dG(w,1) \tag{17}$$

A distribution G is time-invariant if it satisfies all conditions from Equation (11) to (16). Π is time-invariant if G is time-invariant.

Definition 1. *A steady-state equilibrium consists of value functions (V_U, V, J), market tightness (θ, θ_ρ), sequential equilibrium of the referral game (σ, μ, h), aggregate variable Π, and aggregate* *distribution G such that*

1. (V_U, V, J) *are proper value functions.*

2. (θ, θ_ρ) *satisfies the free entry conditions.*

3. (σ, μ, h) *is the referrer preferred equilibrium.*

4. Π *is consistent with G.*

5. *G is time-invariant.*

A few comments are necessary. The value functions depend on the aggregate distribution G only through the expected referral payments Π. Thus, the steady-state equilibrium can be easily calculated by iterative method. Moreover, Π cannot be large when parameters are calibrated since observed referral bonus payments are minor compared to labor income. It suggests that the dynamics of the model will be similar to $\Pi = 0$ case. If $\Pi = 0$, the model has a Block-Recursive Equilibrium (Menzio and Shi (2011)) where stochastic shock on y can be incorporated. Motivated by the point, I will impose $\Pi = 0$ on the value functions of workers in the quantitative analysis. It can be interpreted as the limit of the equilibrium as $z \to 0$.

3.2.2 Equilibrium Referral Strategy

In this section, I characterize the referrer preferred equilibrium of the referral game. In order to do so, all the equilibrium objects except (σ, μ, b) are regarded exogenous. First, define $\Gamma(x)$ as the following.

$$\Gamma(x) = \frac{\tau}{\tau + (1-\tau)\bar{F}(x)} x + \frac{(1-\tau)\bar{F}(x)}{\tau + (1-\tau)\bar{F}(x)} E(\phi | \phi \geq x) \tag{18}$$

where $\bar{F}(x) = 1 - F(x)$. The interpretation of $\Gamma(x)$ is the expectation of ϕ conditional on $\phi \geq x$, and the signal realization is x. It is indeed the expectation of ϕ under μ given $m = [x, 1]$ and $s = x$ if the employer believes that all elements in $[x, 1]$ are plausible, i.e, $[x, 1] = \sigma_w^{-1}([x, 1])$. It is immediate that $\Gamma(0) = (1-\tau)E(\phi) = E(\phi | s = 0), \Gamma(1) = 1$ and $\Gamma(x) > x$ for all $x < 1$. The function $\Gamma(x)$ is strictly increasing for all $x \in [0,1]$.

$$\Gamma'(x) = \frac{\tau(\tau + (1-\tau)\bar{F}(x)) + (1-\tau)^2 f(x)\bar{F}(x)\left(E(\phi|\phi \geq x) - x\right)}{\left(\tau + (1-\tau)\bar{F}(x)\right)^2} > 0$$

Note that $\Gamma(x) = \inf_s E(\phi|\phi \geq x, s)$. Thus, $\Gamma(x)$ is the expected match quality that a referrer can guarantee by choosing $m = [x, 1]$. It means that a referrer can induce hiring with certainty by choosing x satisfying $\Gamma(x) = \bar{\phi}_1(w)$. This relation pins down the pass/fail type threshold $\underline{\phi}(w)$ as the inverse of $\Gamma(x)$.

$$\underline{\phi}(w) = \begin{cases} 0, \quad \text{if} \quad \bar{\phi}_1(w) < (1-\tau)E(\phi) \\ \Gamma^{-1}\left(\bar{\phi}_1(w)\right), \quad \text{if} \quad \bar{\phi}_1(w) \in [(1-\tau)E(\phi), 1] \end{cases} \tag{19}$$

Because $\Gamma(x) > x$ and $\Gamma'(x) > 0$, the pass/fail threshold $\underline{\phi}(w)$ is strictly smaller than $\bar{\phi}_1(w)$ for all $\bar{\phi}_1(w) < 1$. It is straightforward to check that there exists a sequential equilibrium in which the referrer's strategy is $\sigma(\phi, w) = [\underline{\phi}(w), 1]$ for $\phi \geq \underline{\phi}(w)$, and $\sigma(\phi, w) = \{\phi\}$ for $\phi < \underline{\phi}(w)$. Note that if a referrer reveals the true quality, only $\phi \geq \bar{\phi}_1(w)$ can induce the hiring. Therefore, the equilibrium payoff of a referrer is strictly greater in the pass/fail equilibrium with $\underline{\phi}(w)$ than in the equilibrium with full-revealing. Furthermore, Proposition 1 tells that the threshold equilibrium with $\underline{\phi}(w)$ is the referrer preferred equilibrium.

Proposition 1. *In any sequential equilibrium, $\phi < \underline{\phi}(w)$ is not hired with probability 1, and $\phi \geq \bar{\phi}_1(w)$ is hired with probability 1.*

The formal proof is in Appendix, and I will only explain the key intuition here. When a referrer observes $\phi \geq \bar{\phi}_1(w)$, the referrer can induce hiring with probability 1 by revealing the true quality. Therefore, in any sequential equilibrium, $\phi \geq \bar{\phi}_1(w)$ must be hired with probability 1. It proves the second part of the proposition. The first part involves some mathmatics, but its intuition is simple. When a referrer pools low $\phi < \bar{\phi}_1(w)$ and high $\phi > \bar{\phi}_1(w)$ into one message, the incentive compatibility condition requires that the probability of hiring under the pooling message must be 1, otherwise a referrer has an incentive to reveal the true quality when ϕ is high. By construction, $\underline{\phi}(w)$ is the threshold match quality that makes the probability of hiring to be 1 under the message

$m = [\underline{\phi}(w), 1]$. It means, for any $\phi < \underline{\phi}(w)$, the message $m = [\phi, 1]$ induces the probability of hiring strictly less than 1. In the appendix, I prove that any set m that contains $\phi < \underline{\phi}(w)$ induces the probability of hiring strictly less than 1, which implies there is no equilibrium in which $\phi < \underline{\phi}(w)$ is hired.

Corollary 1. *The referrer preferred equilibrium strategy $\sigma(\phi, w)$ is sending $[\underline{\phi}(w), 1]$ if $\phi \geq \underline{\phi}(w)$.*

The equilibrium strategy $\sigma(\phi, w)$ is not fully revealing contrary to Milgrom (1981) and Grossman (1981). The pooling result relies on the fact that a referrer's payoff depends on the posterior belief of an employer only through the hiring decision. Therefore, a referrer does not have an incentive to reveal higher ϕ as long as the pooling message leads to hiring, which supports the existence of a pooling equilibrium. Note that in a buyer-seller setting in Milgrom (1981), the seller's payoff is directly increasing in the posterior belief of a buyer. In such case, since the seller can verify the quality, the seller deviates from the pooling whenever the quality is good. It is usual unraveling from the top argument, but it is not applicable to the labor market in which the quantity of purchase is fixed.

Note that m only affects the posterior belief, not the realized ϕ. Therefore, the optimal strategy is the same threshold type under any referral payment $z(\phi)$ such that $z(\phi) \geq 0$ and $z'(\phi) \geq 0$. Similarly, the optimal strategy is identical under any payment scheme that depends on events after the hiring as long as the expected payoffs of a referrer is weakly increasing in ϕ. As most referral payments observed in reality are either lump-sum or two-part that depends on the separation event *ex post*, it is not likely to eliminate the incentive misalignment between a referrer and an employer.

A few properties of $\sigma(\phi, w)$ can be established. First, the threshold type $\underline{\phi}(w)$ is strictly increasing in w because an employer requires strictly higher match quality in a submarket with higher w. Second, as the precision of signal τ increases, the equilibrium strategy $\sigma(\phi, w)$ becomes closer to the truth-telling in a sense that $\underline{\phi}(w) \to \bar{\phi}_1(w)$ for all w as $\tau \to 1$.

3.3 Match Creation Rate and Match Quality

Consider two submarkets with a same wage level w, where one is for the socially connected workers without a referral and the other is for the socially connected workers with a referral. In the market without a referral, a meeting results in a match if and only if $s \geq s_1^*(w)$. The probability of this event is $\bar{F}(s_1^*(w))$ since unconditional distribution of s is F. On the other hand, in the market with a referral, a meeting results in a match if and only if $\phi \geq \underline{\phi}(w)$. The probability of this event is $\bar{F}(\underline{\phi}(w))$. Thus, investigating the magnitude of $\bar{F}(s_1^*(w))$ and $\bar{F}(\underline{\phi}(w))$ answers to which job search method yields higher match creation rate.

Proposition 2. *$\bar{F}(s_1^*(w)) \leq \bar{F}(\underline{\phi}(w))$ for all w. The inequality is strict if and only if $\underline{\phi}(w) > 0$.*

Proposition 2 tells that conditional on a meeting, the meeting is more likely to become a match with a referral. Note that a referrer always can choose uninformative message, therefore $\bar{F}(s_1^*(w)) \leq \bar{F}(\underline{\phi}(w))$ is immediate. The second part of the proposition means that whenever non-trivial information is transmitted through referral, it strictly increases the match creation rate.

Considering that the objective of a referrer is to maximize the probability of hiring, the higher match creation rate with referral may not be surprising. Instead, how the strategic behaviors of a referrer affect the *ex ante* vacancy creation incentive is less straightforward. On the one hand, a referrer provides information about the match quality, which is beneficial for an employer. On the other hand, the information provided is to maximize the probability of hiring, which in turn induces the employer to hire some bad workers as well. Proposition 3 tells that even though a referrer strategically transmits information, a referral is still beneficial for an employer.

Proposition 3. *Whenever $\underline{\phi}(w) > 0$, the expected value of a job conditional on a meeting is higher with a referral.*

$$\bar{F}(\underline{\phi}(w)) \left(J(w, 1, E[\phi|\phi \geq \underline{\phi}(w)]) - z\right) > \bar{F}(s_1^*(w)) \left(J(w, 1, E[\phi|s \geq s_1^*(w)]) - z\right) \tag{20}$$

The proof is in the appendix. The intuition behind the result of Proposition 3 is that the hiring rule directly depends on ϕ with a referral, but the hiring rule depends only on s without a referral. Thus, more hiring is more efficient with a referral. This efficiency gains allow both the expected value of a job and the match creation rate to be higher. Therefore, an employer is also better off with a referral even if a referrer strategically behaves.

Proposition 3 has several implications. First, since the expected value of a job conditional on a meeting is higher with a referral, the market tightness $\theta_\rho(w)$ is also higher than $\theta(w, 1)$.

As a result, the job-finding probability is higher in the referral market given a wage level w, $p(\theta_\rho(w))H_\rho(w) > p(\theta(w,1))H(w,1)$. Also, while a socially connected worker searches for the same wage level in both formal and referral markets when the current wage is low, the worker starts to search different wages in both markets when the current wage exceeds a threshold level. In such case, the worker searches for a higher wage in the referral market, $g_\rho(w) > g(w,1)$.

The *ex post* average match quality with a referral is $E(\phi|\phi \geq \underline{\phi}(w))$, and without a referral is $E(\phi|s \geq s_1^*(w))$. As the quality through a referral directly depends on ϕ rather than the noisy signal s, the match quality through referral is more likely to be higher. Indeed, regardless of the underlying distribution F, the *ex post* average match quality with a referral is strictly higher if w is large enough. Whether it is true for all wages depends on F. In most numerical calculations, $E(\phi|\phi \geq \underline{\phi}(w)) > E(\phi|s \geq s_1^*(w))$ for all w except when F is U-shaped and $f(0) = \infty$. Even such cases, the range of w that the inequality is reversed is a small region near the threshold. Thus, it is safe to argue that a referral increases *ex post* match quality.

3.4 Efficiency

The strategic motive of a referrer determines the extent to which information is transmitted in a referral. Then, a natural question is whether the incomplete information transmission is creating inefficiency. To answer the question, I define the efficient level of information transmission $\phi_e(w)$ as the solution of Equation (21).

$$\max_{\theta,\phi} p(\theta)\bar{F}(\phi) \quad \text{s.t} \quad q(\theta)\bar{F}(\phi)\Big(J(w,1,E(\tilde{\phi}|\tilde{\phi} \geq \phi) - z\Big) = k \tag{21}$$

In words, $\phi_e(w)$ is the cutoff match quality that maximizes the hiring probability subject to the free entry condition. When the value of a job is calculated, the endogenous separation rates $p^*(w,n), p_\rho^*(w)$ are taken as given. $\theta_e(w)$ is efficient in the following sense. If all other firms and workers follow the equilibrium strategy, and the social planner certainty can choose the threshold match quality in a submarket w, $\phi_e(w)$ is the threshold level that the social planner will choose.

In general, the efficient level $\phi_e(w)$, the equilibrium threshold $\underline{\phi}(w)$, and the full-revealing threshold $\bar{\phi}_1(w)$ are all different. It is easily seen because $\phi_e(w)$ depends on the matching function, but $\underline{\phi}(w)$ and $\bar{\phi}_1(w)$ are independent of the matching function since both objects are chosen after a worker and an employer meet each other. Also, $\underline{\phi}(w)$ depends on the precision of signal τ as it restricts a referrer's ability to conceal information, but $\phi_e(w)$ and $\bar{\phi}_1(w)$ are independent of τ.

As the trade-off that the social planner faces is governed by the elasticity of the matching function, a simpler characterization is available when the matching function is the Cobb-Douglas. Proposition 4 characterize a sufficient condition under which the equilibrium outcome is more efficient than full-revealing.

Proposition 4. *Suppose the matching function is the Cobb-Douglas $p(\theta) = \theta^\alpha, q(\theta) = \theta^{\alpha-1}$. If $\tau \geq \alpha$, then the equilibrium is more efficient than full-revealing in the sense that $\phi_e(w) \leq \underline{\phi}(w) \leq \bar{\phi}_1(w)$ for all w. The inequality is strict whenever $\underline{\phi}(w) \in (0,1)$.*

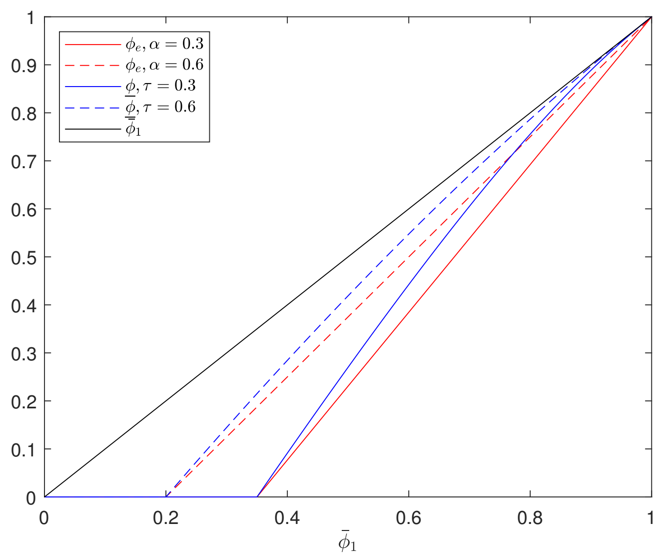


Figure 5: Efficiency Comparison: Full-revealing and Equilibrium

The formal proof is in Appendix. Figure 5 graphically illustrates Proposition 4. The horizontal axis is the required level of productivity $\bar{\phi}_1$, and I draw ϕ_e for two different values of α (red) and $\underline{\phi}$ for two different values of τ (blue) as a function of $\bar{\phi}_1$. By construction, the full-revealing threshold is 45-degree line. An immediate observation is that the full-revealing threshold is too high. This is because the hiring rule under the full-revealing only depends on the *ex post* incentive of firms, but the welfare depends on both *ex ante* and *ex post* incentive of firms through $p(\theta)$ and $\bar{F}(\phi)$. Because $\bar{F}(\phi)$ appears in both the firm and the worker values, but firms do not consider it under the full information, $\bar{\phi}_1$ is too high compared to the efficient level. Then, it is intuitive to see how the equilibrium can lead a higher welfare. Since the equilibrium threshold $\underline{\phi}(w)$ is always lower than $\bar{\phi}_1(w)$ and converges to $\bar{\phi}_1(w)$ as $\tau \to 1$, $\underline{\phi}(w)$ is closer to the efficient level unless the precision of signal is not too low. A sufficient condition that guarantees $\underline{\phi}(w) \geq \phi_e(w)$ is $\tau \geq \alpha$.

4 Quantitative Analysis

5 Conclusion

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A Appendix

A.1 Construction of the optimal strategy

Throughout the appendix, I consider a wage level w in which the hiring probability through referral is in $(0,1)$. The necessary and sufficient condition is the following.

$$\int_0^1 J(w,1,\phi)dF_\phi(\phi) < z, \quad J(w,1,1) > z \tag{22}$$

I will omit w in every notation, but all strategies and beliefs depend on w. Denote ϕ^* as the required productivity at this wage w. For a set $A \subseteq [0,1]$, P_A denote the probability of $\phi \in A$, i.e, $P_A = \int_A dF_\phi(\phi)$.

Lemma 1. *Let (σ, b, h) be a sequantial equilibrium. If $\int_0^1 h(m_0, s)dF_s(s) > 0$, then $h(m_0, s) = 1$ for all s.*

Proof. If m_0 is a singleton set, then the proof is trivial because the belief b is independent of s given m_0. Assume m_0 has more than two elements. Let $h(m_0, s_0) = 1$ for some $s_0 \in m$.

$$\phi^* \leq E(\phi|m_0, s_0) \leq E(\phi|m_0, \sup m_0) \leq E(\phi|\{\sup m_0\}, \sup m_0) = \sup m_0$$

Therefore, a message $m' = \{\sup m_0\}$ always induces hiring. It means $\int_0^1 h(m_0, s)dF_s(s) = 1$ unless a referrer deviates when $\phi = \sup m_0$. □

Lemma 2. *At any sequantial equilibrium, $h(m, s)$ is independent of s.*

Lemma 2 is a direct result from Lemma 1. From the lemma, the notation $h(m)$ is well-defined. Define the hiring set $\mathcal{H} \subseteq [0,1]$ associated with a sequantial equilibrium (σ, b, h) by the following.

$$\mathcal{H} = \{\phi \in [0,1] : h(\sigma(\phi)) = 1\}$$

Lemma 3. *The hiring set $\mathcal{H}$ is closed.*

Proof. Let h be the limit of a sequance $h_n \in \mathcal{H}$. First, observe that $[\phi^*, 1] \subseteq \mathcal{H}$ in any sequantial equilibrium as the truthful report always leads to hiring for $\phi \geq \phi^*$. Thus if $h \geq \phi^*$, then $h \in \mathcal{H}$.

Consider the other case $h < \phi^*$. Then for sufficiently small ϵ, $h_n < \phi^* - \epsilon$ for all $n \geq N$. Since all h_n leads to the hiring,[12]

$$\frac{\tau}{P_{\sigma(h_n)}(1-\tau)+\tau}h_n + \frac{P_{\sigma(h_n)}(1-\tau)}{P_{\sigma(h_n)}(1-\tau)+\tau}\int_{\sigma(h_n)} \phi \frac{dF_\phi(\phi)}{P_{\sigma(h_n)}} \geq \phi^*$$

Note that if $P_{\sigma(h_n)} = 0$, the LHS of the equation is $h_n < \phi^* - \epsilon$, thus $P_{\sigma(h_n)} > 0$. Moreover, $P_{\sigma(h_n)} \geq \delta_\epsilon > 0$ is satisfied for all n, where δ_ϵ is given by the following.

$$\frac{\tau}{\bar{F}(\delta_\epsilon)(1-\tau)+\tau}(\phi^* - \epsilon) + \frac{\bar{F}(\delta_\epsilon)(1-\tau)}{\bar{F}(\delta_\epsilon)(1-\tau)+\tau}\int_{1-\delta_\epsilon}^{1} \phi \frac{dF_\phi(\phi)}{\bar{F}(\delta_\epsilon)} = \phi^*, \quad \bar{F}(x) \equiv 1 - F_\phi(x)$$

Therefore, $\sigma(h_n)$ can have only finitely many images. Thus, there exists a subsequance h_{k_n} such that $h_{k_n} \to h$ and $\sigma(h_{k_n}) = m$ for all n. Since m is closed, $h \in m$. As m leads to the hiring, $h \in \mathcal{H}$. □

Lemma 4. *For any x and a set $A \subseteq [x, 1]$ with $0 < P_A < \bar{F}(x)$, the following holds.*

$$\begin{aligned} &\frac{\tau}{\tau + P_A(1-\tau)}x + \frac{P_A(1-\tau)}{\tau + P_A(1-\tau)}E(\phi|\phi \geq x, \phi \in A) \\ < \ &\frac{\tau}{\tau + \bar{F}(x)(1-\tau)}x + \frac{\bar{F}(x)(1-\tau)}{\tau + \bar{F}(x)(1-\tau)}E(\phi|\phi \geq x) \end{aligned}$$

Proof. Observe that

$$E(\phi|\phi \geq x) = \frac{\bar{F}(x) - P_A}{\bar{F}(x)}E(\phi|\phi \geq x, \phi \notin A) + \frac{P_A}{\bar{F}(x)}E(\phi|\phi \geq x, \phi \in A) > \frac{P_A}{\bar{F}(x)}E(\phi|\phi \geq x, \phi \in A)$$

Therefore,

$$\begin{aligned} &\frac{\tau}{\tau + P_A(1-\tau)}x + \frac{P_A(1-\tau)}{\tau + P_A(1-\tau)}E(\phi|\phi \geq x, \phi \in A) \\ < \ &\frac{\tau}{\tau + P_A(1-\tau)}x + \frac{\bar{F}(x)(1-\tau)}{\tau + P_A(1-\tau)}E(\phi|\phi \geq x) \\ < \ &\frac{\tau}{\tau + \bar{F}(x)(1-\tau)}x + \frac{\bar{F}(x)(1-\tau)}{\tau + \bar{F}(x)(1-\tau)}E(\phi|\phi \geq x) \end{aligned}$$

□

[12] In the equation, I implicitly assume that b puts strictly positive probability on all $\phi \in \sigma(h_n)$. If not, changing $\sigma(h_n)$ to $\hat{\sigma}(h_n) = \{x \in \sigma(h_n) : b(x|\sigma(h_n)) > 0\}$ works the same manner.

Proposition 5. *There is no equilibrium under which $x < \underline{\phi}$ is hired.*

Proof. Suppose by way of contradiction, $x < \underline{\phi}$ is hired in a sequantial equilibrium (σ, b, h). Let $\phi_0 = \inf \mathcal{H}$. By definition, $\phi_0 < \underline{\phi}$, and ϕ_0 is hired by Lemma 3. Define a set A by the following.

$$A = \left\{ \phi \in \sigma(\phi_0) : \int b(\phi|\sigma(\phi_0), s) dF_s > 0 \right\}$$

In other words, A is the set of ϕ that the employer puts strictly positive posterior belief given $\sigma(\phi_0)$. Since ϕ_0 is hired, all elements of $\sigma(\phi_0)$ must be hired, which means $A \subseteq [\phi_0, 1]$. Moreover, $P_A > 0$ because ϕ_0 can be hired only through pooling as $\phi_0 < \underline{\phi} < \phi^*$. On the other hand, $P_A < \bar{F}(\phi_0)$ as $[\phi_0, 1]$ does not lead to hiring by the definition of $\underline{\phi}$. Therefore,

$$\begin{aligned} \phi^* &> \frac{\tau}{\tau + \bar{F}(\phi_0)(1-\tau)} \phi_0 + \frac{\bar{F}(\phi_0)(1-\tau)}{\tau + \bar{F}(\phi_0)(1-\tau)} E(\phi|\phi \geq \phi_0) \\ &> \frac{\tau}{\tau + P_A(1-\tau)} \phi_0 + \frac{P_A(1-\tau)}{\tau + P_A(1-\tau)} E(\phi|\phi \in A) \end{aligned}$$

The second inequality comes from Lemma 4. It contradicts to ϕ_0 being hired in the equilibrium. □

A.2 Match creation rate and match quality

Proposition 6. *Conditional on a meeting, the match creation probability and expected value of a job are higher under the pooling than no information.*

Proof. The match creation probability is $Pr(\phi \geq \underline{\phi})$ under the pooling, and $Pr(s \geq s^*)$ without any information. Note that the marginal distribution of s equals to the marginal distribution of ϕ, thus the first claim holds if and only if $\underline{\phi} < s^*$. To show it, define the following function.

$$g(x) \equiv \frac{\tau}{\tau + \bar{F}(x)(1-\tau)} x + \frac{\bar{F}(x)(1-\tau)}{\tau + \bar{F}(x)(1-\tau)} E(\phi|\phi \geq x)$$

Clearly, $g(x)$ is strictly increasing in x and $g(\underline{\phi}) = \phi^*$ by definition. Moreover, $g(s^*) > \phi^*$ since

$$g(s^*) > \tau s^* + (1-\tau)E(\phi|\phi \geq s^*) > \tau s^* + (1-\tau)E(\phi) = \phi^*$$

Therefore, $\underline{\phi} < s^*$. The expected value of a job under pooling EV_{pool} is higher than no information EV_{no} by the following.

$$
\begin{aligned}
EV_{no} &= \int_{s^*}^{1} \left[J(w,1,\tau s + (1-\tau)E(\phi)) - z\right] dF_s(s) \\
&< \int_{s^*}^{1} \left[J(w,1,\tau s + (1-\tau)E(\phi|\phi \geq \underline{\phi})) - z\right] dF_s(s) \\
&< \int_{\underline{\phi}}^{1} \left[J(w,1,\tau s + (1-\tau)E(\phi|\phi \geq \underline{\phi}) - z\right] dF_s(s) \\
&= (1 - F_\phi(\underline{\phi})) \left[J(w,1,E(\phi|\phi \geq \underline{\phi})) - z\right] = EV_{pool}
\end{aligned}
$$

The third line holds because

$$
\tau\underline{\phi} + (1-\tau)E(\phi|\phi \geq \underline{\phi}) > \frac{\tau}{\tau + \bar{F}(\underline{\phi})(1-\tau)}\underline{\phi} + \frac{\bar{F}(\underline{\phi})(1-\tau)}{\tau + \bar{F}(\underline{\phi})(1-\tau)}E(\phi|\phi \geq \underline{\phi}) = \phi^*
$$

and $J(w,1,\phi^*) = z$ by the definition of ϕ^*. □

A.3 Efficiency: A special case $p(\theta) = \theta^\alpha$

The value of a filled position $J(w,1,\phi)$ is an affine function of ϕ, thus it can be expressed by the following form.

$$
J(w,1,\phi) = -a + b\phi
$$

By the definition, the full-revealing threshold ϕ^* satisfies $J(w,1,\phi^*) = z$, thus $\phi^* = \frac{a+z}{b}$. As we can always define $\tilde{a} = a + z$, it is without loss of generality to assume $z = 0$. When $p(\theta) = \theta^\alpha$, the efficient allocation (θ_e, ϕ_e) maximizes the following problem.

$$
\max_{\theta,\phi} \theta^\alpha (1 - F_\phi(\phi)) \quad \text{s.t} \quad \theta^{\alpha-1}(1 - F_\phi(\phi)) \left(-a + b\int_\phi^1 t \frac{dF_\phi(t)}{1 - F_\phi(\phi)}\right)
$$

From the first order conditions,

$$\begin{aligned}
\partial_\theta &: \quad \alpha\theta = -\lambda(1-\alpha)\left(-a + b\int_{\phi}^{1} t\frac{dF_\phi(t)}{1-F_\phi(\phi)}\right) \\
\partial_\phi &: \quad \theta = \lambda(-a + b\phi)
\end{aligned}$$

Combining two conditions yields the following condition that pins down ϕ_e.

$$\alpha\phi_e + (1-\alpha)\int_{\phi_e}^{1} t\frac{dF_\phi(t)}{1-F_\phi(\phi)} = \phi^*$$

The left-hand side of the equation is strictly greater than ϕ^* when ϕ_e is changed to ϕ^*, which means $\phi_e < \phi^*$ unless $\phi^* = 1$. Therefore, regardless of α, the efficient level of information transmission is lower than full-revealing.

Recall that the pooling threshold $\underline{\phi}$ satisfies the following condition.

$$\frac{\tau}{\tau + \bar{F}(\underline{\phi})(1-\tau)}\underline{\phi} + \frac{\bar{F}(\underline{\phi})(1-\tau)}{\tau + \bar{F}(\underline{\phi})(1-\tau)}\int_{\underline{\phi}}^{1} t\frac{dF_\phi(t)}{1-F_\phi(\phi)} = \phi^*$$

Thus, $\alpha \leq \tau$ is a sufficient condition to ϕ_e being lower than $\underline{\phi}$. In this case, as $\phi_e < \underline{\phi} < \phi^*$, the pooling is informationally more efficient than the full-revealing.