Making a Difference

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Abstract

Many organizations rely on donations of money, time, and effort to function. Here we consider how such organizations motivate donors concerned with "making a difference". Its tension is between a donor's desire to be marginally important against the firm's desire to make important objectives less precarious. We show the factors that lead firms to undertake unimportant objectives before carrying out more important ones. We show how the need to render donors more marginally important lead organizations (i) to exhibit such a lack of focus that they invest their available resources on a range of their *worst* projects, even when technological considerations suggested focusing on their single best project, (ii) to benefit from being unable to identify projects quality, (iii) to consciously fail to diagnose the needs of their clients, (iv) to suffer from becoming larger, and (iv) to benefit from implementing initiatives that are only of value to a narrow group of people.

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Perhaps the most common way to encourage donations of time, effort, and money is to ask the donor to "make a difference". Such exhortations are ubiquitous, and seen in workplaces, schools, charitable solicitations and a myriad of other settings. The focus of this paper is how organizations can encourage such contributions. The central outcome of the paper is that practices that would normally be deemed inefficient serve to encourage donations. Among them are firms carrying out bad projects before doing good ones, investing only in their worst projects even when those projects have the lowest technological returns, and consciously failing to diagnose the needs of their clients.

The underlying logic is simple. Donors give more if an important endeavor will not be done without their contribution. This requires important initiatives to be precarious. Two problems arise. First, if there is randomness in donations, firms do not like such activities to be precarious, because by definition they are sometimes not done. Instead, they would prefer to prioritize high value activities, yet that reduces donations. Second, donors may not believe that a firm's most important initiatives are precarious. Specifically, if the firm implements its more valuable projects with the first funds that they receive, they don't need much money to do so. Our focus is on how these two issues affect organizational efficiency.

The results depend on three ingredients. First, donors to organizations care about the impact of *their* contribution - this marginal calculation is how we interpret "making a difference". Second, these organizations potentially exhibit diminishing returns.[1] These two assumptions imply that an individual donor may perceive her marginal contribution as being of little value, as the donations of others already fund more valuable activities. Yet all donors are marginal, which implies that aggregate donations can be very sensitive to perceptions of diminishing returns. The final ingredient is a series of inefficient practices used by organizations to mitigate diminishing returns, done to attract more donors.

We model a firm that provides public goods, but cannot fund more public goods than it can raise in donations.[2] These public goods, or projects, vary in their quality (either high or low). Donors care about the marginal impact of their donations on welfare, which depends on their beliefs of the donations of others. In the benchmark model with which we begin the paper, the firm can commit to a priority rule over projects, but where there is also randomness in total donations. So, for example, it could carry out all high value projects

[1] There is considerable empirical evidence in the charitable sector that donors perceive such diminishing returns (Andreoni, 1993, Andreoni and Payne, 2009, Bolton and Katok, 1998, and Robert, 1984).

[2] In the most literal interpretation, donors could be offering money and these public goods could be clients seen by a service agency, grant disbursement by a funding agency, food donations to a food pantry, and so on.

first, which we call triage, or it could delay more valuable opportunities such that they are only done with enough funding. In this section, we focus on the precariousness of outcomes, by assuming that there is randomness in total donations. In that setting, carrying out high value projects first is appealing, as adequate funding may not be attained to carry them out. We characterize optimal priority, and show that only if donors are sufficiently unresponsive to "making a difference" will triage arise. Instead, the firm delays high value projects. With uncertainty in donations characterized by a Normal distribution, there is a single partition in which all high valuation projects are done. The delay in doing high value opportunities depends only on the marginal generosity of donors and the value of a *low* value project. As donors become sufficiently generous, the likelihood of a high value project being completed converges to 50%.

These outcomes occur when donors and the firm have aligned preferences. Yet there are often cases where they are not aligned - for example, donors often like visible manifestations of their contribution, whereas organizations might prefer unrestricted giving. When the objectives of the firm are not aligned with those of donors, we show that triage never occurs and that instead outcomes preferred by donors are more likely to remain undone. We also address the case where firms vary in their inherent efficiency, and show that inefficient firms compound that inefficiency by *choosing* to further distort how they carry out projects.

These results show the central tradeoff with donors who want to make a difference. Yet they rely on the dual assumptions that a firm can commit to turn down high value projects to do less valuable ones, and that they can communicate this to donors. In reality, this may be difficult. As a result, much of the remainder of the paper is concerned with a second issue, namely how such desires to render marginal activities more valuable can be implemented in practice when donors realize that the most important problems will be done first.

When commitment is not possible, the firm carries out projects in descending order of their value. Our emphasis is on a series of practices by firms used to credibly render donors more marginal. To isolate these, we ignore randomness in donations. We begin by considering investment, in a setting where the return to investing in a project is independent of a its quality. The outcome here is a *lack of focus*, where it invests an identical amount in all projects that it does. If the firm could commit, it would only invest in one project, its marginal one, as a way of maximizing donations. However, without commitment it simply does that project first and it is nor longer marginal. Donors then realize that their contributions will allow less valuable opportunities to be carried out. To overcome this, they invest less in more projects.

A more stark outcome arises when better projects have higher marginal return to investments. Usual economic logic suggests focusing investments on only the best opportunities. We show that if marginal donations are sufficiently valuable, the firm does the opposite - it invests nothing in its best projects, and invests only in its *worst* projects - its "orphans". For somewhat lower marginal returns to donation, it invests in both its best and worst projects. This arises as the efficient way to encourage donations.

The firm invests above to flatten the relationship between when a project's priority and its quality. Much of the remainder of the paper concerns other ways to credibly do this. Consider a case where firms vary in their ability to identify high value projects. There is clearly a cost to ignorance - good projects may be missed and not done in favor of worse alternatives. Yet there is a countervailing incentive. This is because firms do their best projects first, and more efficient firms can better identify which ones are best, and hence exhibit greater diminishing returns. We show that if donor generosity is sufficiently high on the margin, the firm benefits from being less informed. Said a different way, donors perceive more efficient firms as "not needing the money".

Mission based organizations choose how to offer access to their services. We extend the logic above to address how they should do so. As an example, consider a setting where the firm serves clients, and a client can be "undeserving", "in need", or "in severe need". Usual economic logic would stress a value to distinguishing between each of these cases. Suppose that the firm can choose how to diagnosis need. As is standard, it rules out the undeserving if it is not too costly to do so. However, for sufficiently generous donors, they never distinguish between the two other cases. An interpretation of this is that agencies simply diagnose clients as "qualified", and serve these clients randomly even if their needs vary considerably. The reason for this practice is not ethical, but rather to encourage donors.

With 1.56 million registered nonprofits in the United States (Urban Institute, 2018), the sector is remarkably fragmented.[3] We offer one reason for this: smaller firms may be more attractive to donors. Consider a sector that can serve N clients in total. Of these N clients, p are in severe need. Clients could be served with one large firm, or many small ones. First consider one very large firm, that gets p severe need clients. The firm will serve these first. Then if the marginal donor serves more than the pth client, funding is harmed. Now consider an alternative where firms are small, and randomly draw clients from the distribution. Because of small scale sampling, the distribution of projects is more random.

[3]The nonprofit sector has 21% of all establishments, but only 5% of GNP. The US Census reports that there are 5.9 million business firms in the US.

For example, a firm with two clients could happen to get two severe cases, or two that are not. This sampling error mitigates diminishing returns and raises donations if donors are sufficiently generous. In this sense, donors give more to small firms because "they need it more". Such motivation for donors has been shown by Borgloh et al., 2013.

The central theme of the paper is that distorted organizational practices can render donors more marginal. As another example, we show how the firm may choose to implement initiatives that are of interest to a narrow group of potential donors, rather than one liked by more. Here donors realize that those who do not like it will not donate, rendering their donation more marginal. Finally, it is often argued that competition can mitigate firm inefficiencies. We show that the impact of competition on firm efficiency is ambiguous. However, with sufficient competition, firms become less efficient and welfare falls.

The model describes the motivation of a generic donor. Its most literal interpretation is an external donor offering cash to a non-profit. By semantics, the donor could alternatively be an employee accepting lower compensation, or exerting more effort, when her contribution matters. As such, the work may apply beyond organizations with obvious social objectives. Instead, what matters is that workers gain utility from the impact of their actions (see Dur and Van Lent, 2018, for recent evidence on this). From this perspective, any organization could be designing tasks to enhance their marginal significance.

Before describing the model, it is worth discussing relevant literature. First, the central point of the paper is that organizations reliant on donors may benefit from practices that would normally be deemed inefficient. There is a small literature on how agency concerns modify the practices of mission based organizations.[4] Interpreting donations as effort aligns this paper with that vein of work. Yet its applications are meant to be broader, if only because the employees of mission based organizations often seem to exhibit an altruistic zealotry that suggests that looking outside worker agency issues may be valuable. Here the contortions by the firm are to attract donors of any stripe. The second relevant literature is empirical, on the charitable sector. The model requires two key assumptions: that donors care about their marginal impact, and that the recipients of these donations exhibit diminishing returns. We describe a large body of empirical work on the charitable sector that supports both assumptions.

Section 1 describes the basic model, and the desire to delay high value opportunities so that they are precarious. In Section 2, we consider how this can be implemented when

[4]See Aghion and Tirole, 1997, Dewatripont et al, 1999, and Prendergast, 2007. Alternatively, entrepreneurs may take non-profit status to reduce their temptation to cheat, as in Glaeser and Shleifer, 2001.

the firm cannot commit but can strategically invest. We identify other ways that firms can credibly render donors more marginal in Section 3. Section 4 addresses the impact of competition. Section 5 concludes.

1 Model

Consider a firm that carries out projects (or equivalently serves clients) subject to the constraint that it can spend no more than the donations it receives. Each project carried out costs c. For much of the paper we will assume that c is small but will be normalized to 1. Donors are small, in the sense that each donor funds one project. Total donations are $D \geq 0$, so the total number of projects carried out is $\frac{D}{c}$.

The projects are of two types, either low value $\underline{v} > 0$, or high value with payoff $\overline{v}$, where $\overline{v} > \underline{v}$. There is an elastic supply of low value projects but an exogenous and known number $\frac{p}{c}$ high value projects. For now, the firm knows each project's type. The firm carries out projects in priority order $v(i)$, $i = 1, 2, ...$, where it carries out project type $v(i) \in \{\underline{v}, \overline{v}\}$ as the ith one done, until its budget is spent. For any realization of D, the objective of the firm is to maximize surplus

$$S = \sum_{i=1}^{\frac{D}{c}} v(i). \tag{1}$$

We assume in this section that the firm can commit to a priority rule. This is observed by donors. The firm chooses its priority ordering before donations are known, and so its objective is to maximize the expected value of S, given by $\overline{S}$.

There are a large number of potential donors. Donor j has a personal cost of donating of λ^j (net of the cost of the project). Let $\Delta\overline{S}$ be the change in expected surplus from one more donor. Donor j gives if $\Delta\overline{S} \geq \lambda^j$, where $\Delta\overline{S}$ is conditioned on the giving behavior of others.[5] This is the sense in which donors want to make a difference. The distribution of λ is such that there are $\frac{A(\lambda)}{c} \geq 0$ donors with costs below λ, where $A' > 0$. Let $\overline{D}$ be the

[5]For a micro-foundation, assume that potential donors derive utility from a privately consumed good, R, and a public good, S. Donor j's welfare is given by $U_j = R_j + S + g_j I_j$, where R_j is donor j's consumption of the private good, and g_j if 'a 'warm glow" from donating to the public good, where I_j is an indicator variable equal to 1 if she donates. Let S_{-j} be the surplus generated by the donations of all agents other than j, and $\Delta S = S - S_{-j}$. Then $U_j = R_j + S_{-j} + \Delta S + g_j I_j$. Each donor has total resources of 1, and chooses whether to allocate c to the public good. Consumption goods cost r. Taking expectations, consumer j gives to the public good if $g_j \geq \frac{1}{r} - \frac{\Delta\overline{S}}{c}$. Then let $A(\Delta\overline{S})$ be the number of donors where g_j exceeds $\frac{1}{r} - \frac{\Delta\overline{S}}{c}$.

expected number of donors. As a result, expected donations are given by

$$\overline{D} = A(\Delta \overline{S}) \tag{2}$$

There is also randomness in donations.[6] We assume that realized donations D are given by

$$D = \overline{D} + \epsilon. \tag{3}$$

Let F be the distribution of $D \geq 0$, with mean $\overline{D}$ and continuous density f. We assume that F is a truncated Normal distribution (Tobit), $D \sim N(\overline{D}, \sigma^2)$ for $x > 0$, and $f(0) = F(0)$.[7] In order to avoid integer problems, we assume that c is small. This allows us to treat project choice as a continuous variable, so $S = \int_0^{\frac{D}{c}} v(i)di$.[8] Then for any $\overline{D}$, the probability that there are exactly $\frac{x}{c}$ projects done is $f(x)$.

The timing of the model is as follows. The firm first commits to $v(i)$, $i = 1, 2, ...$ Each donor chooses whether to give, where we restrict attention to pure strategies. ϵ is then realized. Projects are then carried out according to the priority rule, and the game ends.

The model has the possibility that total contributions to the firm can be 0. This occurs with probability $F(0)$. In order to focus on cases where the donor believes that her contribution always has some value, we make Assumption 1.[9]

Assumption 1: $F(0)$ is small.

Given Assumption 1, and setting $c = 1$, expected surplus is $\overline{S} = \int_0^{\infty}[1 - F(x)]v(x)dx$. More relevantly, expected marginal surplus is given by

$$\Delta \overline{S} = \int_0^{\infty} v(x)f(x)dx. \tag{4}$$

where f is conditioned on a mean of $\overline{D}$. We use the term *triage* when $v(i) = \overline{v}$ for all $x \leq p$ and $v(i) = \underline{v}$ for all $x > p$. Let Γ be the set where the firm places its p high valuation

[6] This randomness could either be through aggregate randomness in the donation decision or the cost of providing services.

[7] We can think of this as a limited liability constraint where if the firm is to make losses, it can simply liquidate.

[8] Two issues arise with positive discrete project costs. First, there is the familiar public goods problem where more than one donor is needed to implement a project. This can give rise to complementarities across donors, which is not the interest of the paper. Second, with discrete costs and small donors, the points of marginality become the discrete set $f(1), f(2), f(3), ...$ and so on, rather than continuous f below. Such integer complications are not the central issue of the paper, as so we ignore it by assuming c small.

[9] Below, we show an alternative interpretation where this simplification is not necessary. The Assumption is also not relevant in later sections, where we assume that F is degenerate.

projects. Then as $\overline{D} = A(\Delta\overline{S})$, if $\Delta_v = \overline{v} - \underline{v}$,

$$\overline{D} = A(\underline{v} + \int_{\Gamma} \Delta_v f(x)dx). \tag{5}$$

For notational convenience, let $A' = \alpha$.

With Assumption 1, the equilibrium of the game is where (i) Γ is chosen by the firm to maximize $\overline{S} = E_D \int_0^D v(i)di$, subject to (ii) $D \sim F, f$ with mean $\overline{D}$, and (iii) $\overline{D}$ is given by (5).

As a benchmark, note the outcome when F is degenerate. Then there is a single marginal project $A(v^*(A))$. The optimal choice by the firm is then to place a $\overline{v}$ project at location $A(\overline{v})$. It places the other $p-1$ high value projects at any location below $A(\overline{v})$. This indeterminacy arises because with no randomness in demand, all projects up to $A(\overline{v})$ are completed with probability 1. This indeterminacy disappears when F is non-degenerate. Proposition 1, whose proof is in the Appendix, illustrates the outcome.

Proposition 1 1. *The firm places all its $\overline{v}$ projects at the values of $x \in [0, \infty]$ where*

$$\alpha f(x)\underline{v} + (1 - F(x)) \tag{6}$$

is maximized. They are located in a single partition between $x^* - b$ and $x^* + a$, where

$$x^* = \overline{D} - \frac{\sigma}{\alpha \underline{v}}, \tag{7}$$

and $\frac{f(x^*+a)-f(x^*-b)}{F(x^*+a)-F(x^*-b)} = \alpha\underline{v}$. As $\alpha \to \infty$, $x^* \to \overline{D}$ and Γ converges to $(\frac{\overline{D}-p}{2}, \frac{\overline{D}+p}{2})$.

2. *Triage occurs if*

$$\frac{f(p) - f(0)}{F(p)} \leq \alpha\underline{v}. \tag{8}$$

To understand (6), note that the probability of priority x being completed is $1 - F(x)$ and so the direct return to switching from a low to a high value project is $(1 - F(x))\Delta_v$. With no other influences, this would imply triage, as $1 - F(x)$ is decreasing in x. However, from (5), the change in the number of donors from placing a high value project at location x is $\alpha f(x)\Delta_v$. Finally, we show in the Appendix that the marginal value of an additional donor is $[1 - F(0)]\underline{v}$, which given Assumption 1 we approximate by $\underline{v}$.[10] Hence the impact of

[10] The firm can locate its high value projects at any densities of the truncated Normal for which $x > 0$. Consider the strategy of the firm placing a high value project at density $f(x)$ instead of at $f(y)$. It cannot simply switch priorities x and y as expected number of donations may change, say by δ. The firm can resolve this by additionally placing δ low value projects in priorities $1, 2, ..., \delta$. It can do this because there is an elastic supply of low value projects: this is the only role for assuming a large number of $\underline{v}$ projects. Because of this, the marginal value of an additional donor is $(1 - F(0))\underline{v}$, which given Assumption 1, we approximate by $\underline{v}$.

a high value project at priority x through increased donations is $\alpha f(x)\underline{v}\Delta_v$. Summing this term and the direct effect yields (6).

This provides a simple condition for the optimal delay of valuable projects. Now consider the value of x where (6) is maximized, which we denote the *optimal priority*. Maximizing (6) with respect to x yields $\alpha\underline{v} = \frac{f(x^*)}{f'(x^*)}$. As F is normal, this becomes

$$x^* = \overline{D} - \frac{\sigma}{\alpha\underline{v}}. \tag{9}$$

This identifies the single point that maximizes the return to a high value project. The firm needs to allocate not one high value project but $\frac{p}{c}$. In general, these need not be in a single partition. However, for the Normal distribution, all $\overline{v}$ projects are located in a single partition around x^*, between $x^* - b$ and $x^* + a$ given in Proposition 1.[11] As α gets large, x^* converges to $\overline{D}$, $a \to \frac{p}{2}$, and $b \to \frac{p}{2}$. In that case, half of all high value projects remain undone. Finally, triage arises only if there is no $x > p$ that offers a higher return than locating at $x = 0$. This arises if (8) holds.

Consider the implications of Proposition 1. Note that unless (8) holds, the firm delays valuable projects, rendering them more precarious to encourage donations. Note also that optimal delay is such that, on average, high value projects are more likely to be done than not, as $\frac{\sigma}{\alpha\underline{v}} > 0$. Only in the limit as fund raising becomes infinitely important are high value projects completed half the time. (This is because the mode is the mean for the Normal distribution.) Note further that the percentile rank of x^* - its likelihood of that project being completed - is determined by only α and $\underline{v}$. It is intuitively increasing in the marginal willingness to donate, α. Given this, a number of intuitive features lead to triage: (i) as α gets small, x^* hits the boundary of 0, and there is no value to delaying valuable opportunities, and (ii) if $\underline{v}$ is low, as the value of additional funds is not high enough.

The firm here lives hand to mouth - it has no other sources of funds. Yet realistically many firms reliant on donors have endowments, which could increase average donations, or have access to credit markets, which could reduce the variance of consumption. Note, however, from (9) that the probability of the optimal priority project being completed does not depend on either the mean or the variance of F. As x^* increases one for one with $\overline{D}$, the invariance to the mean should be clear. The probability that a project of priority x^* is completed also does not depend on variance, despite the presence of σ in (9). This is because $F(-\frac{\sigma}{\alpha\overline{v}})$ is independent of σ, with the optimal priority always located the $-\frac{1}{\alpha\overline{v}}$ percentile of

[11] This is because for the Normal distribution, $\frac{f(x)}{1-F(x)}$ is hump shaped and single peaked.

a standard Normal.[12] As a result, the outcome here is robust to allowing the firm access to endowment income or to capital markets.[13]

Matching Funds: Note that the model also can be interpreted as the optimal way to design a matching grant. To see this, assume that the firm can assign a financial match of Δ_v to a donation if fund raising is x. Identical analysis would show the value of matching is maximized at x^*.

General Operating Expenses: Assumption 1 rules out the possibility that donors perceive that their donation has no value. Consider a simple extension along these lines where donors only care about high value projects: in reality, charitable institutions find it especially hard to fund raise for General Operating Expenses, as it hardly resonates with most donors. As a result, consider the case where the firm values low value projects at $\underline{v}$ but donors value them at 0. Then, even if we drop Assumption 1, expected donations are given by $\overline{D} = A(\int_\Gamma \Delta_v f(x)dx)$, and all the qualitative results continue to hold.[14]

Archdiocese of Chicago: Another case of interest is where the interests of donors are inversely related those of the firm. As an example, every year the Archdiocese of Chicago runs its annual fund raising campaign, where individual parishes raise funds. These donations are allocated to two "projects". Specifically, some funds are used by the Archdiocese for its own purposes, while the rest is retained by each individual parish. The Archdiocese chooses the rule for how funds are split.

This differs in one important way from above. The two parties do not agree on the ranking of outcomes: parishes prefer the money to stay with them, while the Archdiocese prefers centralized funds. To address this case, let donors values the projects at $\overline{v}$ and $\underline{v}$ as before, but the firm now values the $\underline{v}$ project at $v_f > \overline{v}$. Then the equivalent condition to (6) is given by $\alpha\Delta_v f(x)v_f + (1 - F(x))(\overline{v} - v_f)$, which is optimized at $\frac{f'(x^*)}{f(x^*)} = \frac{\overline{v}-v_f}{\alpha\Delta_v v_f}$, or

$$x^* = \overline{D} - \frac{\sigma(\overline{v} - v_f)}{\alpha\Delta_v v_f}. \tag{10}$$

[12]Note, however, that the *ordering* of projects is affected by variance. As an example, consider a case with little uncertainty and one with more, and assume that the probability of the first high valuation project occurs at the 25th percentile of the distribution. Then with low variance, the 25th percentile occurs at a higher value of x than with a high variance. As a result, the ordering of projects has more low valuation ones done first when the variance is low, whereas the list of low valuation ones is shorted with more variance.

[13]This relies on the assumption of an elastic supply of $\underline{v}$ projects.

[14]The analogous condition to (6) becomes $\alpha f(x)\underline{v} + \frac{1-F(x)}{1-F(0)}$.

As $\overline{v} - v_f < 0$, the optimal priority is greater than the mean, $\overline{D}$. Unlike the case above, the project most preferred by the donor is completed with probability less than one half. Hence, the qualitative results continue to apply, but result in longer delays for the donor's preferred projects.

In the motivating example above, the Archdiocese of Chicago has chosen to "delay" outcomes preferred by the donors in the following way. Specifically, it sets a critical level of funding for each parish. All funds below that critical level are retained by the Archdiocese, but all funds beyond that are given to the parish.

Firm Efficiency and Distorted Priorities: So far, we have shown that firms may do seemingly inefficient things to attract donors. Here we extend this to a setting where some firms are inherently more efficient than others. The objective is to see how such efficiency differences affect how they raise funds. The efficiency difference here lies in their ability to identify v. With one parametric restriction, we show that the need to raise funds reinforces these efficiency differences. This is because less efficient firms *choose* even more distorted project choices than do their more efficient counterparts.

To model this, assume that there is a total supply of N projects, where N is large enough to have available low value projects to allow optimal delay. Instead of perfect information on its p high value projects, the firm now receives a signal s on p projects. Receiving this signal means that $v = \overline{v}$ with probability γ, where $1 \geq \gamma \geq \frac{p}{N}$. The metric of exogenous firm efficiency is γ, which is observed by donors.

When the marginal project does not have signal s, its expected quality is $E\underline{v}(\gamma) = \underline{v} + \frac{p(1-\gamma)\Delta_v}{N-p} > \underline{v}$. The analogous condition to (6) then becomes $A' f(x) E\underline{v}(\gamma) + (1 - F(x))$. Consider the case where A is linear. Then as $\frac{E\underline{v}(\gamma)}{d\gamma} < 0$, less efficient firms choose to distort project choice more than more efficient firms. For intuition, inefficient firms value marginal funds more because they are more likely to be spent on an unidentified good project. Firms that are more efficient make such mistakes less often, and so value marginal funds less. As a result, inefficient firms choose to distort project choice more do than their more efficient counterparts.

Investment: The outcomes above arise where the firm is exogenously assigned p high value projects. An alternative is to allow the firm to invest in projects. Here we show that the results are robust to this extension. Specifically, assume that without investment, all projects have return $\underline{v}$ but the firm can invest Δ on a project to increase its return to $\underline{v} + \Delta(x)$ at manpower cost $C(\Delta)$. We assume that $C'(\Delta) > 0, C''(\Delta) \geq 0$, and $C'(0) = k > 0$, so there is a fixed cost of k to investing. The firm has has total (manpower) resources of m to invest, so that $\sum_x C(\Delta(x)) \leq m$. These investments are observed by donors.

The marginal value of Δ at x is $\alpha f(x)\underline{v} + (1 - F(x))$. Then for two points x and y where $\Delta > 0$, $\frac{\Delta^*(x)}{\Delta^*(y)} = \frac{C'^{-1}(\alpha f(x)\underline{v}+(1-F(x)))}{C'^{-1}(\alpha f(y)\underline{v}+(1-F(y)))}$. To see the link to the analysis above, note two familiar cases. The first is where investment has only a fixed cost of k, which increases v from $\underline{v}$ to $\overline{v}$. Then the same results as above arise, where $p \equiv \frac{m}{k}$. The second case is where $C''(\Delta) = 0$. Then the firm invests m on project x^* and nothing on any other project. Hence the results naturally apply to settings where the firm chooses project specific investments.

Empirical Evidence: The model has two essential ingredients: that donors care about their marginal impact, and that institutions suffer from diminishing returns. There is considerable evidence for both.

First consider the evidence on marginal impact. The most natural link is to the literature on matching gifts for charities, where a gift by a donor is matched by funds from elsewhere. Empirical evidence (List, 2011) clearly shows that such matches are an effective way of encouraging donations. Beyond matching grants, the U.S. Trust survey on charitable giving described 94% of high net worth donors as concerned by whether "their gift can make a difference" (U.S. Trust, 2016).[15]

The model also requires diminishing returns. This is typically tested by examining crowd-out in charitable giving. The most common setting to study this is to address whether government support of an organization reduces private donations. Evidence on such crowd out arises in Andreoni, 1993, Andreoni and Payne, 2009, Bolton and Katok, 1998, and Robert, 1984 (though see Riber and Wilhelm, 2002).

Applications: The model most closely resembles a nonprofit institution seeking external donors of money. An alternative setting is employees in mission based organizations taking lower wages based on their perceived contribution to the mission.[16] Such issues are of

[15]Similarly, Grant, 2008, offers experimental evidence to show how a fund raising experiment for student scholarships is enhanced by telling the respondents about the social impact of their donations. The greatest response in fund raising was found for a setting where the respondents were prompted by being told that their donations "made a difference in the lives of others", consistent with a necessary ingredient for this paper's results.

[16]These cases requires one additional component. In order for the same logic to hold with employees, it must be the case that all employees - even those working on low valuation projects - can carry out the counterfactual of imagining what would occur if they were not employed. Specifically they can understand that the musical chairs of reallocated workers would mean that the loss from their departure is the marginal

economic significance, as the nonprofit sector employs 7% of the U.S. workforce.[17] An alternative donation could be effort by employees. Such desires extend beyond the confines of socially oriented organizations. Dur and van Lent, 2018, report that 77% of all workers report that it is "important or very important" for them to have a job that is socially useful. Also relevant is a large management literature, deriving from Hackman and Oldham, 1976, on how firms can motivate though the design of workers' jobs. A major building block of this is what Hackman and Oldham term Task Significance, where workers can see the impact of their actions.[18]

Two Assumptions: It is useful to conclude this section by addressing the purpose of the analysis above. We have made two important assumptions: (i) that the firm can commit to a priority rule, and (ii) that the randomness of donations is Normally distributed. These assumptions have allowed us to focus on two outcomes. First, donors give based on marginal projects executed, and second, firms locate all high value endeavors around the optimal priority project.

First consider the truncated Normal. This is important for three reasons. First, $x^* \neq 0$ requires that $f'(x) > 0$ over some range of the distribution, as arises with the Normal distribution.[19] Second, for the Normal distribution, all high value projects are optimally placed in a single partition. This is not an outcome that need arise with other distributions. Finally, ϵ is independent of $\overline{D}$. As a result, changing $\overline{D}$ does not change $f(x)$ other than at the boundary of 0.[20]

The central feature needed for the paper is that donations depend on marginal projects

project carried out, not necessarily the project that they are engaged in.

[17]Empirical estimates of the reduction in wages in that sector range from 10% to 15% without extensive worker demographic controls (Leete, 2001, Ruhm and Borkowski, 2003, and Salamon and Sokolowski, 2005) to closer to 6% with more controls (Hirsch et al., 2017).

[18]See Fried and Ferris, 1987, and Humphrey, et al., 2007, for empirical evidence.

[19]Consider two other distributions - defined only over the positive line - that are commonly used, namely the Exponential and Rayleigh. The Exponential has a constant hazard function. As a result, triage is always optimal. By contrast, the hazard for the Rayleigh distribution is increasing in x. For α large enough, the outcome is above: the firm places its high value projects in an intermediate range of x's. These outcomes are shown in the Appendix.

[20]This arises because the shape of the distribution of F does not depend on its mean. But for some distributions, densities change with the mean. For example, consider the Weibull, which is defined over the positive line, which has a scale and a shape parameter. When the hazard rate rises with x, as the shape parameter rises, the distribution flattens. As a result, the set of available densities changes as the mean rises, and more relevantly the height of the mode falls.

undertaken. There are other forms of commitment by the firm that may not not result in the marginal project being pivotal.[21] In reality, we believe that it is often difficult for a firm to simply commit to such practices by fiat, and to communicate this to donors. Instead, once donations are received, most firms likely would renege on any promise and carry out their most valuable initiatives first. When the firm cannot commit, the marginal project de facto becomes pivotal as it is the only credible use of marginal funds.

Accordingly, we have focused on the analysis above because it emphasizes the importance of the marginal project, and the cost that a firm will incur to render a valuable project marginal. For the remainder of the paper, we drop both assumptions above. We do not allow the firm to commit to a priority rule and we assume that F is degenerate. As a result, in what follows below, other costs are incurred by firms to render these projects valuable.

2 Commitment and Lack of Focus

Without some commitment mechanism, the only credible outcome is triage. For the remainder of the paper we consider a range of practices that the firm may use to render marginal projects more valuable when they cannot commit. We begin by showing how investment can play such a role.

Consider the extension above where we allowed the firm to invest. It has manpower resources of m. As an initial benchmark, we consider a case where all projects have the same return without investment. It can increase the return on a project from $\underline{v}$ to $\underline{v} + \Delta$ by incurring a manpower cost of $C(\Delta)$, where $C'(x) > 0$ for $x > 0$. In order to emphasize lack of focus caused by our issues, we assume that $C'(0) = 0$, $C'' = 0$, and that F is degenerate. First consider the outcome if the firm can commit. Let $C^{-1}(m)$ be the maximum feasible investment in a single project. When $C'' = 0$ and F is degenerate, the firm assigns all m to its the marginal project, its Dth one. This raises funds of $A(\underline{v} + C^{-1}(m))$.[22] Now assume that the firm cannot commit to a priority rule. Then the outcome of investing $C^{-1}(m)$ in its

[21] An alternative, more extreme, form of commitment is to allow the firm to "burn the money", by doing no projects unless it reaches a certain level of donations. This additional threat would result in more donations than outlined above. Furthermore, as the firm can threaten to do no projects, donors consider the integral of all projects done rather than just the marginal project.

[22] It places one high value project at marginal location $D = A(\underline{v} + C^{-1}(m))$, and places the other $p - 1$ high value projects at any location below D. This is the only outcome that depends on the assumption that $C'' = 0$ above. When $C'' > 0$ in the case where the firm can commit, it may spread out the investment over more than one project. We make the assumption of $C'' = 0$ to render the comparison below more stark. The constrained outcome is identical even when $C'' > 0$.

marginal project is no longer credible, as the firm simply does this project first. Realizing this, donors give less.

Consider the game when the firm cannot commit. Because F is degenerate, there is a single marginal project, D. This project has value $v^*(D) = \underline{v} + \Delta(D)$, where $v(x)$ is non-increasing. Equilibrium now requires that $\Delta(x)$ is chosen by the firm to maximize $\overline{S} = \int_0^D v(i)di$, subject to (i) $A(v^*(D)) \geq D$, (ii) $v(x)$ is non-increasing in x, and (iii) $v^* = v(A(v^*))$. The final condition is that the marginal project on which funding beliefs are based must be the marginal project consistent with that level of funding. The outcome is described in Proposition 2.

Proposition 2 *When the firm cannot commit to the order of projects, it invests an equal amount Δ^* in $A(\underline{v}+\Delta^*)$ projects, where Δ^* is uniquely defined by $A(\underline{v}+\Delta^*)C(\Delta^*) = m$. It does not invest in any other projects.*

Proposition 2 illustrates the strategic response - rather than invest all its resources in a single initiative, the firm spreads its investments across a wide range of projects, even though technologically there is no reason to do so. Two intuitive interpretations arise. First, mission based organizations exhibit a *lack of focus*. Without the commitment problem here, the firm would invest heavily in single project to encourage donations. Here the firm spreads itself thin. This is costly to the firm - rather than raise $A(\underline{v} + C^{-1}(m))$ donations under commitment, it can now only attain $A(\underline{v} + \Delta^*)$. The second interpretation is an emphasis on egalitarianism - that the firm invests an equal amount in a wide range of projects, independent of their quality.

2.1 Assortative Matching

In the previous section, the return to investing in a project was independent of the project's quality. As such, there is no efficiency distortion per se to spreading out investments. We now turn to the case where investment returns are complementary with project quality. Usual economic logic would suggest that the firm focuses its investments on its best opportunities. We now show how the opposite can arise, where the firm only invests in a range of its *worst* viable projects, those with the lowest technological returns.

To address this, consider the following change to the previous section: instead of surplus being $v_i + \Delta_i$, the return is $v_i(1 + \Delta_i)$. Hence, investment and project quality are complements. Second, to allow closed form solutions, let there be continuous diminishing returns to projects, where if projects are ranked in terms of (uninvested) inherent value, $v_i = v_1 - bi$.

The parameter b measures diminishing returns. Finally, donations are linear in the quality of the marginal project, j, $A(j) = A_0 + \alpha(v_j + \Delta_j)$. To emphasize the starkness of our results, we continue to assume that $C'' = 0$ (so there is no technological reason to spread out investment) and total investment inputs are m.

The firm faces the following tradeoff. Let D^* be aggregate donations in the absence of any investment, where the marginal project has return $v^* = v_1 - bD^* < v_1$. It could follow usual economic logic and assign all its m investment resources to project 1, with return $v_1(1+m)$ on that project. This maximizes the return to assortative matching, but does not increase donations, as the marginal project $v^* > v_1$ remains unaffected. The alternative is to allocate some, or indeed all, of its investment resources to its worst, marginal, projects, namely those around project v^*. Proposition 3 illustrates the outcome.

Proposition 3 *Let surplus from investment be $v_i(1+\Delta_i)$, $A(j) = A_0 + \alpha(v_j + \Delta_j)$, $v_i = v_1 - bi$, and $C''(\Delta) = 0$. Then*

- *If $v_1 > v^* + \frac{\alpha v^{*2}}{\alpha+b}$, the firm invests only in its best project, project 1.*
- *If $v_1 \leq v^* + \frac{\alpha v^{*2}}{\alpha+b}$, then*
    1. *If $\Gamma > 0$, the firm invests only in its worst projects, those with returns between $v^* - \overline{x}$ and $v^* + \overline{x}$,*
    2. *If $\Gamma < 0$, the firm invests in its worst projects, those with returns between $v^* - x$ and $v^* + x$, but also in its best project,*

where $\Gamma = \alpha v^* - \frac{\alpha^2 \overline{x}}{b} - \int_{k^*-b\overline{x}}^{k^*+\alpha\overline{x}} (\frac{v_1 - v(j)}{v(j)})dj + b\left(\frac{v^*+\overline{x}}{v^* - \frac{\alpha\overline{x}}{b}} - 1\right)\left(v_1 - (v^* - \frac{\alpha\overline{x}}{b})\right) - \alpha(\frac{v_1 - v^* + \frac{\alpha}{b}\overline{x}}{v^* + \frac{\alpha}{\beta}\overline{x}}) - b(\frac{v_1 - v^* - \overline{x}}{v^* - \overline{x}})$, and $\int_{k^*-b\overline{x}}^{k^*+a\overline{x}} (\frac{v^*+\overline{x}}{v(j)} - 1)dj = m$. In both cases where the firm invests in marginal projects, the returns to all invested marginal projects are equalized.

Proposition 3 shows how the usual logic of assortative matching is inverted. The firm chooses between investing in its most technologically efficient project, with return v_1, or by investing in its worst viable projects, those with a return close to v^*, to increase donations. The outcome depends on three factors. First, $(v^* + \frac{\alpha v^{*2}}{\alpha+b}) - v_1$ measures the return to diverting a small amount of resources from project 1 to project v^*.[23] Intuitively, it depends on the value of the marginal project, v^*, and donor generosity, α, compared to the most technologically efficient one, v_1. If it is negative, the firm maximizes assortative matching. If it is positive, however, we then need to determine how much the firm wants to divert to increase donations.

[23] The increase in donations is αv^* and the cost in lost benefits of investment is $(\alpha + b)(\frac{v_1}{v^*} - 1)$.

This is determined by Γ, which measures the marginal return to diverting all available investment resources to marginal projects. Doing so increases the marginal project's value by $\overline{x}$. If this is positive, the firm spends all its resources on its worst projects. On the other hand, if $\Gamma < 0$, it spends its optimal amount getting donations, and then uses the residual on its most technologically efficient project.

To summarize, if one was to observe a firm investing all its resources on its least marginally productive activities, this would seem as inefficient a practice as could be imagined. It might even be interpreted as a misguided attempt to create equality within the firm. Yet it arises here not through any desire to equate returns per se, but rather as an effective way of rendering donors more marginally important.

3 Other ways of creating marginal benefits to donors

The central premise of the paper is that organizational inefficiency may render donors more marginally valuable. In this section, we address four other mechanisms to do so - by the firm being less knowledgeable, by minimally diagnosing need, by being small, and by focusing on projects that are of narrow interest. In each of these case, we assume that F is degenerate and that the firm cannot commit to the order of projects.

3.1 Lack of Knowledge

Consider the case above where firms vary in their ability to identify v. Specifically, there are N projects and the firm receives a signal s on p of these, which identify the project as $\overline{v}$ with probability γ.

The revised model is as follows: First, the firm is endowed with type γ, which is observed by the donor. Second, donors give based on equilibrium beliefs of $\Delta\overline{S}$. The firm then carries out projects in order of their expected value, and the game ends.

There are no strategic choices here. Instead, our objective here is simply to show how outcomes vary by γ. The Perfect Bayesian equilibrium of the game is described in the Appendix. The central impact of γ on outcomes is through diminishing returns. To see this, consider two extremes. First, a firm that can perfectly identify v has its first p projects high value, and the remainder low value. If its marginal donor is located at $A(\overline{v}) < p$, this maximizes donations. Yet for generous donors, $A(\underline{v}) > p$, donors realize that their marginal contribution will be spent on low value projects, and will give less. By contrast, now consider a completely uninformed firm, where $\gamma = \frac{p}{N}$. All its projects have value $\underline{v} + \frac{p}{N}\Delta_v$, which lies between $\underline{v}$ and $\overline{v}$. Then if $A(\underline{v}) > p$, uninformed firms raise more funds, as "they need it more".

More generally, consider the case where the marginal project funded does not have the signal s. In that case, the expected quality of the marginal project is $\underline{v} + \frac{p(1-\gamma)\Delta_v}{N-p}$ and $D = A\left(\underline{v} + \frac{p(1-\gamma)\Delta_v}{N-p}\right)$. Here donations are decreasing in firm efficiency: $\frac{dD}{d\gamma} = -A' \frac{p\Delta_v}{N-p} < 0$. While donations are higher for the less efficient, surplus may still be lower as, on average, they carry out worse projects. Expected surplus is given by $S = p(\overline{v} - (1-\gamma)\Delta_v) + \nu(N-p)(\underline{v} + \frac{p(1-\gamma)\Delta_v}{N-p})$, where $\nu = \frac{D(\gamma)-p}{N-p} \leq 1$ is the probability that a project without signal s will be completed. The problem is only interesting if $\nu < 1$. In that case,

$$\frac{dS}{d\gamma} = p\Delta_v(1-\nu) + \frac{dD}{d\gamma}(\underline{v} + \frac{p(1-\gamma)\Delta_v}{N-p}). \tag{11}$$

The first part of this is the usual efficiency gain from better allocation - as $\nu < 1$, identifying high quality projects has value as they are more likely to be completed. However, this must be traded off against reduced funding, given by the second term. The second term dominates for large enough A'. Proposition 4 follows.

Proposition 4 *If $A\left(\underline{v} + \frac{p(1-\gamma)\Delta_v}{N-p}\right) > p$, donations are decreasing in firm efficiency. Furthermore, if $\nu < 1$ and A' is large enough, firm surplus is decreasing in its efficiency.*

For some context in which to evaluate this section, there is a large literature in economics on privately consumed goods showing how efficient firms flourish at the expense of their less efficient counterparts (Jovanovic, 1982, Hopenhayn, 1992, Syverson, 2011). The purpose of the preceding analysis is to show that this may not be true for institutions reliant on donors. The closest relevant evidence on the link between firm efficiency and donations comes from the charitable sector. Here the relevant evidence is on one measure of firm efficiency - the proportion of donations that go to the mission rather than to administrative expenses. A striking feature of this literature is its inconclusiveness, with weak relationships generally being the outcome (Yoruk, 2013, Karlan and Wood, 2014, Parson, 2007, and Buchheit and Parsons, 2006). These are not direct tests of this section, of course, as our measure of efficiency is diagnostic. However, the absence of a notable relationship is consistent with donors imputing lower marginal returns for more efficient agencies.

3.2 Misaligned Preferences

In the last section, the failure to diagnose v flattened the relationship between the firm's no-commitment priority ranking and the quality of the marginal project. Another avenue through which such flattening can be achieved is misalignment of preferences between the donor and firm. To see this, consider a case where the firm values projects at $\underline{v}$ and $\overline{v}$ as above, but where the degree of alignment with the donor preferences is given by $\rho < 1$. Specifically, the donor's valuation is v_d, where $\overline{v}_d = \rho\overline{v} + (1-\rho)\underline{v}$ when the firms valuation is $\overline{v}$ and $\underline{v}_d = (1-\rho)\overline{v}+\rho\underline{v}$ when the firm's valuation is $\underline{v}$. Consider the return to misaligned preferences for donations. Again, it depends on a return to flattening the relationship between firm priority and donor value. If the marginal project carried out is of type $\overline{v}$, misalignment harms donations. However, when the marginal project is of quality $\underline{v}$, $\frac{dD}{d\rho} = -A'\Delta_v < 0$. Once again, donations are increased, though here through misalignment of preferences.

3.3 Optimal Diagnosis and "Qualified"

Mission based organizations must make choices on how to provide access to their services. For example, a social services agency must decide procedures for how applicants qualify for help, or a hospital must choose how to prioritize need. We address this here in a simple extension of the exercise above. Its objective is to show how optimal diagnosis can simply be to exclude cases below a threshold severity, but to make no distinctions beyond that. In more casual terms, optimal diagnosis deems clients "qualified", but makes no more attempt to diagnose severity.

To address this, now consider a setting where firms can choose how they diagnose "severity" of cases. We do so in an extended version of the model of the last section where, in addition to cases $\overline{v}$ and $\underline{v}$, there are cases that are "undeserving", with return $v^- < 0$. Assume that in the population of N "projects", these cases occur in proportions $p_{\overline{v}}, p_{\underline{v}}$, and $1 - p_{\overline{v}} - p_{\underline{v}}$.

A natural way to interpret v is where higher v cases have more severe characteristics than do lower v ones. Accordingly, consider a diagnosis technology where at cost κ_0 the firm can rule out cases of type v^-, and at cost κ_1, case $\underline{v}$ can additionally be ruled out. Without incurring any costs, they cannot distinguish between cases.

The timing of this revised game is as follow. First, the firm commits to its diagnosis technology. After observing this, donors give to the firm, the firm prioritizes case without commitment, and the game ends.

The only choices for the firm are whether to diagnose no cases, only undeserving cases, or all cases. First consider ruling out undeserving cases. This part is standard, and consists of no more than a comparison of better allocative efficiency (and the higher funding that comes from this) against the cost of diagnosis. Specifically, if

$$(\frac{p_{\overline{v}}\overline{v}+p_{\underline{v}}\underline{v}}{p_{\overline{v}}+p_{\underline{v}}})D_1-(p_{\overline{v}}\overline{v}+p_{\underline{v}}\underline{v}+(1-p_{\overline{v}}-p_{\underline{v}})v^-)D_0\geq\kappa_0, \tag{12}$$

the firm excludes undeserving cases.[24]

Of more conceptual interest is whether to distinguish between $\overline{v}$ and $\underline{v}$ cases. This depends on the return to the marginal case. First, if $A(\overline{v})<Np_{\overline{v}}$, the marginal case is $\overline{v}$, and the same kind of calculus as above continues to hold, where the firm diagnoses if the (positive) cost is not too large. What changes is when funding is sufficiently generous to allow some $\underline{v}$ projects to be done - this requires that $A>Np_{\overline{v}}$. In that case, there is a tradeoff: with better diagnosis, a larger fraction of cases carried out will be of high value, but funding is harmed.

Specifically, if D_2 is the level of donations, surplus with the additional diagnosis is given by $Np_{\overline{v}}\overline{v}+(D_2-Np_{\overline{v}})\underline{v}$. All high value cases are done first, and only the residual will be lower value cases. However, $D_2=A(\underline{v})$, as donors see their marginal dollars being spent on less important cases. By contrast, with no diagnosis of severe cases, funding is $A(\frac{p_{\overline{v}}\overline{v}+p_{\underline{v}}\underline{v}}{p_{\overline{v}}+p_{\underline{v}}})>A(\underline{v})$, but projects are of lower average quality $\frac{p_{\overline{v}}\overline{v}+p_{\underline{v}}\underline{v}}{p_{\overline{v}}+p_{\underline{v}}}$. Diagnosis is only optimal if

$$Np_{\overline{v}}\overline{v}+(A(\underline{v})-Np_{\overline{v}})\underline{v}-\kappa_1\geq A(\frac{p_{\overline{v}}\overline{v}+p_{\underline{v}}\underline{v}}{p_{\overline{v}}+p_{\underline{v}}})(\frac{p_{\overline{v}}\overline{v}+p_{\underline{v}}\underline{v}}{p_{\overline{v}}+p_{\underline{v}}}). \tag{13}$$

Proposition 5 follows.

Proposition 5 *Assume that (12) holds but (13) is violated. Then optimal diagnosis is to distinguish between cases v^- and the other two, but no more. This can arise even if $\kappa_1=0$.*

Proposition 5 offers an alternative view of mission based organizations as egalitarian, where a gatekeeper determines whether a client is "qualified". Once qualified, no distinctions are made between clients, despite variation in severity. Once possible version of this in reality is where clients are treated in a "first come, first served" random way. Yet such egalitarianism is not used here for ethical reasons, but rather as a mechanism to encourage donations.

3.4 "Small is Beautiful"

The nonprofit sector in the United States is remarkably fragmented. There are 1.56 million nonprofits registered with the Internal Revenue Service, for a sector that is little over 5.4% of

[24] Without any diagnosis, the payoff to the firm is $D_0(p_{\overline{v}}\overline{v}+p_{\underline{v}}\underline{v}+(1-p_{\overline{v}}-p_{\underline{v}})v^-)$, where donations are given by $D_0=A(p_{\overline{v}}\overline{v}+p_{\underline{v}}\underline{v}+(1-p_{\overline{v}}-p_{\underline{v}})v^-)$. Surplus is $(p_{\overline{v}}\overline{v}+p_{\underline{v}}\underline{v}+(1-p_{\overline{v}}-p_{\underline{v}})v^-)D_0$. If the firm incurs the cost of κ_0, surplus is $(\frac{p_{\overline{v}}\overline{v}+p_{\underline{v}}\underline{v}}{p_{\overline{v}}+p_{\underline{v}}})D_1$, where $D_1=A(\frac{p_{\overline{v}}\overline{v}+p_{\underline{v}}\underline{v}}{p_{\overline{v}}+p_{\underline{v}}})$. (12) follows.

GNP.[25] Yet there are many reasons to imagine economies of scale, in branding, development, or administration, to would lead to large firms in this sector. Here we illustrate that one reason for the fragmentation of the sector may be that smaller firms may be more attractive to donors, for the reason that donors perceive these smaller institutions as "needing the money more".

To address this, we continue to consider the case where there are N projects, of which p have quality $\overline{v}$, with the remainder of quality $\underline{v}$. Firms cannot commit to a priority rule. However, rather than there being a single firm, there are now Q identical firms who split the N projects. Donors are indifferent between firms, conditional on marginal surplus. Projects are assigned to firms by randomly drawing $\frac{N}{Q}$ projects from the urn of projects of size N. A firm can observe the quality of its draws, but donors cannot - instead, all they observe is the size of the firm.

Specifically, here there are Q firms. Donors choose whether to give and are then equally assigned across the Q firms. The Q firms then receive $\frac{N}{Q}$ draws from the project urn. After observing their quality, they implement projects, and the game ends.

Our objective is to show how firm size affects donations. There are Q firms, with $\frac{A}{Q}$ donors each (modulo integer issues), where A is total donations across all firms. Our objective is to address how A varies with Q. Donors give based on the marginal project. If the marginal project is the k^*th opportunity for a firm, the donor gives based on the k^*th order statistic of its $\frac{N}{Q}$ draws, as the firm does its best projects first. The expected value of the kth order statistic from $\frac{N}{Q}$ draws is $Ev_{kQ} = \nu_{kQ}\overline{v} + (1 - \nu_{kQ})\underline{v}$, where

$$\nu_{kQ} = \sum_{i=1}^{\frac{N}{Q}+1-k} \rho^{\frac{N}{Q}+1-i}(1-\rho)^i. \tag{14}$$

The role of firm size is in how Q affects Ev_{k^*Q} for the marginal project. The outcome is described in Proposition 6.

Proposition 6 *Assume that $A < N$. Then if k^* is the marginal project,*

1. *A necessary and sufficient condition for Q firms to attract more donations than $Q' < Q$ is that $Ev_{k^*(Q)\frac{N}{Q}} > Ev_{k^*(Q')\frac{N}{Q'}}$.*
2. *For donations sufficiently large, $A(\underline{v}) > (max\{p, N-Q\})$, donations are monotonically decreasing in firm size.*

[25] By contrast, the entire business sector has 5.6 million establishments.

3. *For maximum equilibrium donations sufficiently small, $A(v_{11}) < 1$, donations are maximized by the largest possible firm.*

Before describing the Proposition, its intuition can be seen from two extreme cases. First consider the case where N is large and $Q = 1$. Then the empirical distribution of projects at the firm is p high quality projects and $N - p$ low quality ones. Then if donors are generous, $A(\underline{v}) > p$, the marginal project is low quality and funding for the large firm reflects that low marginal value. By contrast, now consider a case where $Q = \frac{N}{2}$, so all firms are of size 2. Each firm has two projects, and carries them out in the order of their value. The expected value of the first and second order statistics of the two draws are given by $Ev_{1\frac{N}{2}} = \underline{v} + (\rho + \rho(1-\rho))(\overline{v} - \underline{v}) < \overline{v}$, and $Ev_{2\frac{N}{2}} = \underline{v} + \rho^2(\overline{v} - \underline{v}) > \underline{v}$ respectively, where $\frac{p}{N} = \rho$. The aggregate distribution for $N = 2$ is flatter than that of the large firm because of sampling error. If the marginal donor is located at a large enough (aggregate) priority, smaller firms attract more donors.

Now consider Proposition 6. The first part characterizes the equilibrium for any Q. If a smaller firm is preferred, it is because its expected v at marginal priority k^* is higher than for a larger firm. But smaller firms have higher v_{k^*} only because of small scale sampling. One case where this arises is when $A(\underline{v}) > (max\{p, N - Q\})$. In that case, the marginal donor for each firm is located at the $\frac{N}{Q}$th order statistic. In that case, donations are determined by the fatness of the tails of the distribution of Ev_k. But

$$Ev_{\frac{N}{Q}Q} = \underline{v} + \rho^{\frac{N}{Q}}(\overline{v} - \underline{v}), \tag{15}$$

which is decreasing in Q. As a result, donations decrease in firm size.[26] Put more simply, in this case small firms sometimes do not have enough money to fund all their high value projects, whereas large firms do. As a result, small firms attract more donors in aggregate. By contrast, for donations sufficiently small, large firms are more effective at raising funds as only the highest valuation projects are completed.

[26]Note this this section has solely considered the link between firm size and total donations. Even if donations rise, it may still be the case that larger firms create more surplus. As an example, consider a setting where less than one half of all projects are high quality. Assume first that one firm has N projects, of which a half are funded. Then all high value projects are done, and the residual are low value. Now consider an alternative where all firms are of size 3, of which 2 are funded. Hence total donations rise. Yet firms with three projects could randomly get three good ones or three bad ones. Those firms with three good ones can only fund two of them, and the firm with three bad ones funds two bad projects. As a result, surplus calculations would need to compare these allocative losses with more funding.

This section has been motivated by the fact that the nonprofit sector is so fragmented. Here we raised the possibility that this could be because smaller firms can raise money more easily than larger firms because they are more "needy". This has been experimentally tested by Borgloh et al, 2013, for the charitable sector. In their setting, potential donors are given information on the income streams of charities, and could choose between a larger and smaller charity. They find that donors are much more likely to give to smaller charities, their interpretation being that "the higher impact of the own donation, and the neediness of the charity organization are decisive for choosing the small organization" (p.1).

3.5 Promoting Narrow Interests

The central idea of the paper is that project choice becomes distorted in order to render donors marginal. Up to now, this has arisen through the order of activities. Here we consider firms investing in projects that are of narrow interest, in the sense of implementing a project that some people like less, but no one likes more, than some alternative. While this reduces surplus from the project, it may make it more likely to be implemented, as interested donors will - correctly - realize that they are more marginal.

The distortion in project choice here is its breath. As a result, we ignore priority by assuming that the firm can only carry out a single project from a set of N. It requires funding of 1. There are A donors who can potentially fund the project. To avoid standard public goods problems, we assume that each potential funder is capable of funding the project. Project N offers benefits of $\overline{v}$ to only $N \leq A$: it is of no value to the other $N - A$. The firm values donations for two ends. First, it values the project for the returns to its N beneficiaries at $V(N)$, where $V'(N) \geq 0$. Second, it carries out activities other than the project. Specifically, if more than one donor gives, the additional funds have marginal surplus $Z > 0$. (Think of these as general operating expenses.) However, each donor value these only at $\underline{v}$. If $\underline{v} \geq 1$, all donors would give. To render the problem interesting, we assume that $\overline{v} > 1 > \underline{v}$.

The timing is as follows. First, the firm chooses $N \leq A$.[27] Each of the N donors then decides whether to donate 1 to the project or not. If no funds are raised, the project is not done. If at least one donor gives a dollar, then the project is done, other funds are spent, and the game ends.

A donor will give if she gets benefits from the project, and sees herself as sufficiently

[27]For example, they could build some initial infrastructure needed for the project - architectural drawings would be a literal example.

marginal. There is clearly no pure strategy equilibrium of this game.[28] Consider a symmetric mixed strategy by donors who value the project. In that equilibrium, each of the N donors who value the project gives 1 with probability b, where $0 < b < 1$, and all other donors give 0. For this to be an equilibrium, it must be that the donor is indifferent about giving. Any donor is marginal only if the other $N-1$ potential donors have not given. This implies that $b(N)$ is defined by

$$\overline{v}(1 - b(N))^{N-1} + (1 - (1 - b(N))^{N-1})\underline{v} = 1, \tag{16}$$

or $b(N) = 1 - (\frac{1-\underline{v}}{\overline{v}-\underline{v}})^{\frac{1}{N-1}}$.

To see the reason for narrowing project interest, note that expected donations are given by $Nb(N)$, where

$$\frac{dNb(N)}{dN} = b(N) + \frac{N}{(N-1)^2}(1 - b(N))log(\frac{1-\underline{v}}{\overline{v}-\underline{v}}). \tag{17}$$

The first term here is positive (there are more potential donors), and the second negative (they are less likely to be marginal), so its sign depends on parameter values.

To show the possibility of an extreme outcome, note that if $\frac{1-z}{v-z}$ high enough, the RHS of (17) is declining in N, for all $N > 1$. If this holds, donations are maximized by implementing projects that are of value to only only person. Now consider the firm's preferences and choice of N. Its expected surplus is given by

$$Nb(N)Z + (1 - (1 - b(N))^N)(V(N) - Z). \tag{18}$$

The first term here is expected donations, $Nb(N)$, times the marginal value of excess dollars, Z, while the second term is the extra utility generated by the project, implemented with probability $1 - (1 - b(N))^N$. The firm chooses N to maximize (18) subject to (16). The outcome reflects the two conflicting issues above, and depends on parameter values.

It is straightforward to show cases where restricting interest is optimal. One immediate case is where $V = 0$, in which case the firm's objective is to maximize expected donations. Yet we showed above that when $\frac{1-\underline{v}}{\overline{v}-\underline{v}}$ is high enough, total donations decline when more potential donors value the project. In that case, the firm choose a project valued by only one person. Once again, project choice is distorted to encourage donors interested in making a difference.

[28] Specifically, if no one donates, there is an incentive for one donor to deviate, as she is always marginal. However, if it is believed that this strategy is followed, no one will donate as they are not marginal. Similarly, given Assumption 1, there is no pure strategy equilibrium where no one donates.

4 Competition

It is often suggested that competition is a remedy to the ailment of firm inefficiency, and many mission-based agencies do compete with each other. Here we address how competition affects outcomes. There are two reasons to do so. The first is simply to understand robustness. A second reason is to address the welfare implications of the model. Specifically, one interpretation of the results above is that while internal distortions arise to attract donations, it is to a good end, namely, to meet the needs of a larger set of clients or opportunities. As such, these distortions may be socially beneficial. Here we add competition to show that this may no longer be the case.

Assume that in the basic model of Section 1, there are now two firms indexed by $k = 1, 2$.[29] Each firm has donations D_k, where as above, D_k has mean $\overline{D}_k$, with distribution F_k, normally distributed for $D_k > 0$ with common variance σ^2. Noise for each firm is independent. The two firms are differentiated in the minds of donors. Specifically, the value that donor j gets from donating to firm k is now given by

$$u_k^j = \int_0^\infty v_k(x) f_k(x) dx - \lambda_k^j, \tag{19}$$

where λ_k^j measures the idiosyncratic preference that donor j has for firm k and v_k is the priority ordering of firm k. The two firms are horizontally differentiated and we normalize the average disutility of donating to zero: $E\lambda_1 = E\lambda_2 = 0$. The distribution of λ_k^j is A, and is independent across k. The total potential donor base is normalized to 1.

The only variation from the basic model above is that now, at the beginning of the game, each firm simultaneously commits to $v_k(x)$. Donors then give. The distributions F_k then generate outcomes for each firm, projects are carried out, and the game ends.[30]

In general, the impact of competition on welfare and the efficiency of the firms is ambiguous. This is for three reasons. First, when firms are horizontally differentiated, some donors will give to firm 2 even if they are unwilling to give to firm 1. This mechanical effect

[29] It is well known that monopolistic competition can give rise to excess entry, and hence potentially harm welfare. This is not our concern here.

[30] In the spirit of the rest of the literature, we have assumed that the objective of the firm is to maximize the surplus generated by its firm. In a setting where ones competitors are also carrying out missions, this may not be appropriate. So, for example, if two institutions are feeding the poor in a particular neighborhood, donors may seem them as easily substitutable. But if this is the case, then one firm may not worry if its competitor gets the donor, as the poor will still be fed. A simple way to implement this would be to assume that the objective of firm i becomes $D_i + \lambda D_j$. If λ is sufficiently high, then while the firm could take its competitors donor by distorting priorities, it would choose not to do so.

increases aggregate donations. Second, without competition the firm gets all the market, and evaluates marginal returns at $A'(\overline{D})$. With symmetric competition, they now evaluate this at half the donor base, and if A is non-linear, these can differ. Finally, and of most interest here, a firm's incentives for efficiency changes. Specifically, donors can become more elastic with respect to the firm's priority choice when there is an alternative potential recipient. Despite this ambiguity overall, we can show that with enough competition, welfare always falls. This is shown in Proposition 7.

Proposition 7 *With undifferentiated Bertrand competition, firm i choose* $x_i^* = \overline{D}_i$*, and competition reduces welfare.*

There are two reasons for welfare falling here. First, firms are undifferentiated, and so there is no mechanical welfare gain from giving donors more choice. Second, when donors are indifferent between firms, they become infinitely elastic to expected returns to their donations in equilibrium. As a result, each firm renders donations maximally desirable to donors by choosing $x_i^* = \overline{D}_i$. However, this means high value projects are completed half the time, a worse outcome than in the model of Section 1. Hence, enough competition reduces welfare.

Consider the evidence on competition among not for profits. Since Rose Ackerman, 1982, the literature has been aware of how competition can generate rent seeking, which in Rose Ackerman takes the form of fund raising expenditures.[31] Empirical evidence corroborates this, with Feigenbaum, 1987, Castenada et al., 2008, and Bose, 2015, all showing how fund raising expenses rise with competition. While fund raising is clearly not the margin on which inefficiency is modeled here, it shows the willingness of non-profits to increasingly distort their activities in competitive settings to attract donors, as required above.

5 Conclusion

The backdrop to this paper is a rich literature showing how more efficient firms tend to thrive.[32] As Jovanovic (p.649) puts it, "the efficient grow and survive, the inefficient decline and fall". This clearly contrasts to our public goods setting, where we have shown a variety of practices that would typically be deemed inefficient, yet raise more donations.

[31] Also see Bilodeau and Slivinski, 1997, and Aldashev and Verdier, 2010.

[32] This occurs both through internal organization of firms, and the fact that efficient, lower cost, firms have larger markets shares (Jovanovic, 1982, Hopenhayn, 1992, Syverson, 2011).

Given this, it is useful to conclude by remembering that the paper posits a specific form of donor motivation. It is not a utility gain that derives from the act of doing something virtuous ("warm glow"), nor does it derive from being associated with an institution that does good. Instead, it is making a difference. Consider an alternative where donors give to organizations that do "most good" rather than "most marginal good". So, for example, a donor compares charitable organizations, and gives to the one that she feels has added most value to society per dollar spent. These are average calculations rather than marginal ones. Consider how this affects the optimal priority calculations in Section 1. A donor no longer gives based on the marginal calculation of $\int_0^\infty v(x)f(x)dx$, but rather the expected average performance of the firm $E_D \int_0^\infty v(x)[1-D(x)]dx$, where the expectation is over total donations D. This has an intuitive maximizer: all high valuation goods are produced before any low valuation ones. Hence our results are sensitive to this form of motivation.

Despite this caveat, we note again how pervasive is the exhortation to "make a difference". Economists have been formally studying motivation for four decades, and its focus is on ways of overcoming various forms of employee malfeasances.[33] Here issues arise because of a "need to be needed". Given the zeal that the employees of many non-profits exhibit and the need for other kinds of donors in this sector, such a lens does not seem out of place.

Studying such a source of motivation leads to a number of novel outcomes. At its most general level, standard measures of efficiency may inhibit donations. We began by show a firm that can commit will fail to prioritize important objectives. In the limit of donor generosity, the most valuable projects remain undone half of the time. Another manifestation of the desire to raise donations is a failure to focus their resources on their most valuable opportunities. Indeed, firms often invest only in their "orphans", their worst projects. It can also lead donors to penalize more efficient or larger organizations, as they (correctly) infer that "they don't need the money". Furthermore, competition may not be a panacea. Finally, while the paper is written most closely linked to a non-profit setting, its implications may be broader. Firms of all stripes rely on the willingness to workers to "volunteer", and this work argues that a myriad of practices may help workers find worth in their actions. As such, these insights may have more general value.

[33] Aghion and Tirole's 1997, Dewatripont et al., 1999, Prendergast, 2007, and Glaeser and Shleifer, 2001.

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Appendix

Proof of Proposition 1: The firm chooses Γ to maximize $\overline{S}$. Consider a possible equilibrium where ranking $i \in \Gamma$. Let surplus from this be $\overline{S}(i)$. Furthermore, let $[\overline{S}(i)|r < (>)k]$ be the surplus with point $i \in \Gamma$ that accrues from all projects with priority below (above) k. The consider another point $j \notin \Gamma$. Then for Γ to be optimal, $\overline{S}(i) \geq \overline{S}(j)$.

We show Proposition 1 in three steps.

1. First consider the case where triage is not carried out in $\Gamma(i)$ so $v(i) = \underline{v}$ for all $i < z(i)$. In that case first consider all $i > z(i)$. If $z(j) > 0$, then $[\overline{S}(i)|r > z] - [\overline{S}(j)|r > z] = (F(j) - F(i))\Delta_v$, where F is evaluated at $i \in \Gamma$. This is because the only difference between them is the likelihood of their completion. The case of $z(j) = 0$ is covered in 3. below. Now consider the impact on $[\overline{S}(i)|r < z(i)]$. This is given by $\int_0^z(i)\underline{v}dF(x)$, where $x \sim N(\Delta\overline{S}(i), \sigma^2)$. However, when $z(j) > 0$, $\overline{D}(j) - \overline{D}(j) = \alpha[f(i) - f(j)]\Delta_v$ and so $[\Delta\overline{S}(i)|r < z(i)] - \Delta\overline{S}(j)|r < z(i)] = \alpha[f(i) - f(j)]\underline{v}\Delta_v$. (The condition for $z(j) > 0$ is $\alpha[f(i) - f(j)]\Delta_v < z(i)$.) As a result the range of projects before the first element of γ is $z(j) = z(i) - \alpha[f(i) - f(j)]\Delta_v$. Adding these shows that he value of switching $i \in \Gamma$ with $j \in \Gamma$ is positive only if

$$\alpha\Delta_v f(j)\underline{v} + 1 - F(j) > \alpha\Delta_v f(i)\underline{v} + 1 - F(i), \tag{20}$$

as required.

2. The analysis above assumes no triage. However, this does not address the issue that changing $\overline{D}$ changes the set of feasible f densities. Specifically, consider when $\overline{D}$ rises from D_0 to D_1, and let Φ be a normal distribution with mean 0 and variance σ^2, with density ϕ. Then all densities ϕ between $-D_0$ and $-D_1$ are now possible elements of Γ whereas they were not before. Similarly, if $\overline{D}$ falls, these densities disappear. As a result, we need to rule out that a change in $\overline{D}$ causes these elements be included in Γ.

   This is ruled out by showing that for the Normal distribution, if under $\Gamma(i)$, there exists $v(i) \notin \Gamma(i)$ for $i < z(i)$, with densities $\phi(x)$ from $x \in \{-\overline{D}(i), -\overline{D}(i) + z\}$, then the return to including j for $\phi(x) < \phi(-\overline{D}(i))$ is negative. To show this, consider the return above with the Normal distribution but where the return to increasing $\overline{D}$ is not necessarily $\underline{v}$ but any $\nu \geq 0$. The objective of the firm is then to choose the ρ values of x where $\alpha f(x)\nu + 1 - F(x)$ is maximized. Differentiating this with respect to x yields $\alpha f'(x)\nu - f(x)$, and so the firm's welfare from a high value project is maximized at

$\frac{f'(x^*)}{f(x^*)} = \frac{1}{\alpha\nu}$. For a Normal distribution with zero mean, $-\frac{z}{\sigma}f'(z) = f(z)$. As a result, $\frac{f'(x^*)}{f(x^*)} = \frac{1}{\alpha\nu}$ simplifies to

$$x^* = \overline{D} - \frac{\sigma}{\alpha\nu}. \tag{21}$$

Now consider the location of the ρ high value projects. For the Normal distribution, for $x < \overline{D}$, $\alpha f'(x)\underline{v} - f(x) = (\alpha\nu - \frac{x}{\sigma})f'(x)$, which is increasing until $x = x^*$ and continuously declining after. As a result, all high value projects are located in a single partition around x^*, between $x^* - b$ and $x^* + a$, $(a, b) > 0$, with $a - b = p$, where

$$f(x^* + a) - f(x^* - b) = \frac{F(x^* + a) - F(x^* - b)}{\alpha\nu} \tag{22}$$

This shows that the optimal choice of location for Γ is an interval. Therefore for all $i < z(i)$, $i \notin \Gamma$, then it is also the case that under j, all densities ϕ below those in i will also not be in Γ. Hence the firm does not want to locate $\overline{v}$ projects in this region. This implies that $\nu = \underline{v}$.

3. The final case is $z(i) = 0$ so that all $i \in \Gamma$ for all $i \leq p$. As the firm chooses an interval where all $i \leq p$ are in Γ, we can consider the return to replacing point 0 with $p + \epsilon$ for ϵ small. As $F(0)$ is assumed small, this has positive return only if

$$f(p) - f(0) > \frac{F(p)}{\alpha\underline{v}}, \tag{23}$$

as required.

More general distributions: Rewrite (6) as $[\alpha\underline{v}\eta(x) + 1](1 - F(x))\Delta_v$, where $\eta(x)$ is the hazard rate evaluated at x. First consider the Exponential distribution $f(x) = \lambda e^{-\lambda x}$, where the hazard rate is given by $\frac{f(x)}{S(x)} = \lambda$ where $S(x)$ is the survival rate. As the mean of the exponential is $\frac{1}{\lambda}$, this implies that λ is defined by $\lambda = \frac{1}{\overline{D}}$. The analog to (6) is $[\alpha\lambda\underline{v} + 1]e^{-\lambda x}\Delta_v$. This is always decreasing in x, and triage is always used.

An alternative is the Rayleigh distribution, $F(x) = 1 - e^{-\frac{x^2}{2\sigma^2}}$, for $x > 0$. This implies that $\overline{D} = \sqrt{\frac{\pi}{2}}\sigma$, as this is the expected value of the Rayleigh. Then the hazard rate is $\frac{f(x)}{S(x)} = \frac{x}{\sigma}$, which is linearly increasing in x. Here the analog to (6) is $[\alpha\underline{v}\frac{x}{\sigma} + 1]e^{-\frac{x^2}{2\sigma^2}}\Delta_v$. Triage may now no longer be optimal. Specifically, if $\Omega(x) = [\alpha\underline{v}\frac{x}{\sigma} + 1]e^{-\frac{x^2}{2\sigma^2}}$, then $\Omega'(x) > 0$ iff $\alpha\sigma\underline{v} > \frac{x}{\alpha} + \alpha x^2\underline{v}$. This is positive for small x but decreasing for larger x. This implies that the firm places its high value projects in an intermediate range of x's.

Proof of Proposition 2: Assume first that the firm chooses $\Delta(i)$ such that $D = A(\underline{v} + \Delta(A(.))$. Below we will show that this is always the case. The firm then chooses $\Delta(i)$ to max $A(\underline{v}+\Delta(A(.))$. As $\Delta' \leq 0$, this is maximized by choosing $\Delta(i) = \Delta^*$ for all $i \in [0, A(\underline{v}+\Delta^*)]$. (The firm never invest less than Δ^* as any additional projects beyond $A(\underline{v} + \Delta)$ are never funded.) Δ^* maximizes $\underline{v} + \Delta^*$ subject to $\frac{\Delta^*}{D} \geq m$. The budget constraint binds when $m = C(\Delta^*)A(\underline{v} + \Delta^*)$. As $C' > 0$ and $A' > 0$, Δ^* is uniquely defined.

This assumes that $D = A(v^*)$. To show that this is the case, consider an alternative where $\Delta = \tilde{\Delta} > \Delta^*$ for the first $\frac{m}{\tilde{\Delta}}$ projects. Then $A(\underline{v} + \tilde{\Delta}) > \underline{v} + \tilde{\Delta}$, and $A(\underline{v}) < \underline{v} + \tilde{\Delta}$ as Δ^* above maximizes $A(.)$ conditional on $A = D$. In this case, there exists no v for which $A = D$, as $v(D) \neq A$. Instead, the pure strategy equilibrium that maximizes donations where $v^*(D) = D$ is where $D = \underline{v} + \frac{m}{\tilde{\Delta}}$. This arises as an outcome where (i) the firm invests $\tilde{\Delta}$ on $\frac{m}{\tilde{\Delta}}$ projects, (ii) of the $A(\underline{v} + \tilde{\Delta})$ donors willing to give at that value, $\underline{v} + \frac{m}{\tilde{\Delta}}$ of them donate, and the remainder do not. To implement this, all donors who have $\lambda_j \geq -\underline{v}$ give, any subset of those whose $\underline{v}+\tilde{\Delta} < \lambda_j < -\underline{v}$ where this subset is of size $\underline{v}+\frac{m}{\tilde{\Delta}} - A(\underline{v})$. This is an equilibrium as no donors beyond $\underline{v} + \frac{m}{\tilde{\Delta}}$ are willing to donate, as $A(\underline{v}) < \underline{v} + \tilde{\Delta}$. However, the firm prefers to set $A = D$ as above, as $\underline{v} + \frac{m}{\tilde{\Delta}} < \underline{v} + \frac{m}{\Delta^*}$. The Proposition follows.

Proof of Proposition 3: Let I_i be the investment in project i, so that surplus from investment is $v_i(1 + I_i)$. Note that $A(v(k^*)) = A_0 + \alpha(v_{k^*}(1 + I_{k^*}))$, $v_k = v_1 - bk$, and $C''(\Delta) = 0$. The firm chooses I_i to maximize $\int_0^{A(v^*)}(v_i + v_iI_i)di$ subject to $v_{k^*}(1 + I_{k^*}) = v(A(v_{k^*}(1 + I_{k^*})))$, $v_k + \Delta_k \geq v_{k'} + \Delta_{k'}$ for $k < k'$, and $\int_0^{\infty} \Delta_j dj \leq m$.

Let k^* be the priority of the marginal project without investment, where $v_{k^*} \equiv v^*$, and $v(A(v^*)) = v^*$. Note that investments will be allocated to at most two parts of the priority distribution. First, investment may be allocated to project 1. To see this, consider two priorities x and y, $x < y$, where transferring investments between them results in an unchanged A. But as surplus from project i is $v_i + v_iI_i$, surplus is higher from x than y $v_x > v_y$. As a result, for any I_i where $\frac{dA}{dI_i} = 0$, $I_1 > 0$, and $I_i = 0$, $i \neq 1$.

This implies that for any $I_i > 0$, for $i > 1$, it must be that $\frac{dA}{dI_i} > 0$. But if $\frac{dA}{dI_i} > 0$, because $v_k(1+I_k) \geq v_{k'}(1+I_{k')}$ for $k > k'$, it must be the case that $v_k(1+I_k) = v_{k'}(1+I_{k'})$ for some partition. If $v_k(1 + I_k) - v_{k'}(1 + I_{k'}) = \delta > 0$, then surplus can be increased by transferring δ_k from project k to project 1. Hence for some range of projects, $v_k(1 + I_k) = v_{k'}(1 + I_{k'})$.

Now consider the optimal partition where returns are equalized. Without investment, $A(v^*) = k^*$. Now assume that the firm choose investments such that the marginal project has value $v^* + x$. The priority where $A(z) = v^* + x$ is given by $z = k^* + \alpha x$. To generate a return of $v^* + x$ for this priority $k^* + \alpha x$ requires $I_{k^*+\alpha x} = \frac{v^*+x}{v^*-\frac{\alpha x}{b}} - 1 > 0$. However, diminishing returns requires that $v_k + \Delta_k = v_{k'} + \Delta_{k'}$ for all $k \in [k^* - bx, k^* + \alpha x]$. These priorities have a range of valuations from $v^* + x$ to $v^* - \frac{\alpha}{b}x$. To implement this, the firm invests $I(j) = \frac{v^*+x}{v(j)} - 1 > 0$ in priority j.

Then consider the value of increasing the marginal project from value v^* to $v^* + x$. This implies donations rise from k^* to $k^* + \alpha x$. The marginal project $k^* + \alpha x$ has value $v^* - \frac{\alpha}{b}x$. This increase in donations then offers additional surplus of $\alpha x v^* - \frac{\alpha x}{2}(\frac{\alpha x}{b}) = \alpha x v^* - \frac{\alpha^2 x^2}{2b}$ to the principal. However, the firm loses surplus of $v_1 - j$ for any marginal investment in project j, $j > 1$. Given this, the firm chooses x^* to maximize

$$\alpha x v^* - \frac{\alpha^2 x^2}{2b} - \int_{k^*-bx}^{k^*+ax} (\frac{v^* + x}{v(j)} - 1)(v_1 - v(j))dj. \tag{24}$$

Differentiating (24) with respect to x yields

$$\alpha v^* - \frac{\alpha^2 x}{b} - \int_{k^*-bx}^{k^*+\alpha x} (\frac{v_1 - v(j)}{v(j)})dj + b\left(\frac{v^* + x}{v^* - \frac{\alpha x}{b}} - 1\right)\left(v_1 - (v^* - \frac{\alpha x}{b})\right)$$
$$- \alpha(\frac{v_1 - v^* + \frac{\alpha}{b}x}{v^* + \frac{\alpha}{\beta}x}) - b(\frac{v_1 - v^* - x}{v^* - x}). \tag{25}$$

This is decreasing in x so the second order condition holds.

In order to determine outcomes, we first consider (25) at $x = 0$ and then where x is at its maximum level, where m is exhausted. First at $x = 0$, (25) becomes

$$\Omega = \alpha v^* - (\alpha + b)(\frac{v_1 - v^*}{v^*}). \tag{26}$$

If this is positive, then the firm will divert at least some investment resources to marginal projects. Ω positive if $v_1 < v^* + \frac{\alpha v^{*2}}{\alpha+\beta}$.

Let $\overline{x}$ defined by $\int_{k^*-b\overline{x}}^{k^*+a\overline{x}} (\frac{v^*+\overline{x}}{v(j)} - 1)dj = m$. This is the maximum increase feasible in x with resources of m. Then evaluate (25) at $\overline{x}$. This is given by

$$\alpha v^* - \frac{\alpha^2 \overline{x}}{b} - \int_{k^*-b\overline{x}}^{k^*+\alpha\overline{x}} (\frac{v_1 - v(j)}{v(j)})dj + b\left(\frac{v^* + \overline{x}}{v^* - \frac{\alpha\overline{x}}{b}} - 1\right)\left(v_1 - (v^* - \frac{\alpha\overline{x}}{b})\right)$$
$$- \alpha(\frac{v_1 - v^* + \frac{\alpha}{b}\overline{x}}{v^* + \frac{\alpha}{\beta}\overline{x}}) - b(\frac{v_1 - v^* - \overline{x}}{v^* - \overline{x}}). \tag{27}$$

If this is positive, the marginal return to investing in marginal projects is positive when evaluating at investment capacity, and hence there is no investment in project 1. If, however, $\Omega > 0$ but $\Gamma < 0$, there exists a value of x^* where optimal investment in marginal projects is less than m. As a result, the firm invests the optimal x^* in these projects and the remainder in project 1.

Proof of Proposition 4: For a given γ, with donors $D(\gamma)$, equilibrium requires that $v(D(v^*(\gamma))) = v^*(\gamma)$, where $v^*(\gamma)$ is the expected value of the marginal project. The level of donations must feasible, $D(\gamma) \leq A(v^*(\gamma))$ and, finally, that there is an allocation rule on λ_j for who donates if $D(\gamma) < A(v^*(\gamma))$ that is consistent with the incentives of donors. We first calculate $D(\gamma)$. There are three cases:

- If $A(\gamma\overline{v} + (1-\gamma)\underline{v}) \leq p$, then the unique value where $v(A(v^*)) = v^*$ is where the marginal project has signal s. In that case $D = A(\gamma\overline{v} + (1-\gamma)\underline{v})$, where $\frac{dD}{d\gamma} > 0$.

- The expected value of projects without signal s is $\underline{v} + \frac{p(1-\gamma)\Delta_v}{N-p}$. Then if $A(\underline{v} + \frac{p(1-\gamma)\Delta_v}{N-p}) > p$, the unique value where $v(A(v^*)) = v^*$ is where the marginal project does not have signal s. Then donations are $D_1(\gamma) = A\left(\underline{v} + \frac{p(1-\gamma)\Delta_v}{N-p}\right)$. Here donations are decreasing in firm efficiency: $D_1' = -A' \frac{p\Delta_v}{N-p} < 0$. Expected surplus is then given by

$$S = p(\overline{v} - (1-\gamma)\Delta_v) + \nu(N-p)(\underline{v} + \frac{p(1-\gamma)\Delta_v}{N-p}), \tag{28}$$

  where $\nu = \frac{D_1(\gamma)-p}{N-p} \leq 1$ is the probability that a project without signal s will be completed. If $\rho < 1$,

$$\frac{dS}{d\gamma} = p\Delta_v(1-\nu) + A'(\gamma)(\underline{v} + \frac{p(1-\gamma)\Delta_v}{N-p}), \tag{29}$$

  as required. The second term dominates for A' large enough.

- The final case is where $A(\gamma\overline{v} + (1-\gamma)\underline{v}) > p > A(\underline{v} + \frac{p(1-\gamma)\Delta_v}{N-p})$. Here there is no value of v such that $v(A(V^*)) = v^*$, and so there is no pure strategy equilibrium where $D = A$. Instead, the maximum number of projects where $v(D) = v$ is given by p.

  We now construct an equilibrium where $D = p < A(\gamma\overline{v} + (1-\gamma)\underline{v})$. To do this, let $\lambda^*(\underline{v} + \frac{p(1-\gamma)\Delta_v}{N-p}))$ be the donor who is indifferent when the marginal project does not have signal s and $\lambda^*(\gamma\overline{v} + (1-\gamma)\underline{v})$ be the marginal donor when it does have signal s. Then the equilibrium is where (i) all donors with $\lambda < \lambda^*(\gamma\overline{v} + (1-\gamma)\underline{v})$ give, and (ii) for donors between $\lambda^*(\underline{v} + \frac{p(1-\gamma)\Delta_v}{N-p}))$ and $\lambda^*(\gamma\overline{v} + (1-\gamma)\underline{v})$, a subset of size $p - A(\underline{v} + \frac{p(1-\gamma)\Delta_v}{N-p})$ also donate. As $A(\gamma\overline{v} + (1-\gamma)\underline{v}) > p$, such a subset always exists. This is an equilibrium because any additional donor has marginal valuation $\underline{v} + \frac{p(1-\gamma)\Delta_v}{N-p}$, but no donor between $\lambda^*(\underline{v} + \frac{p(1-\gamma)\Delta_v}{N-p}))$ and $\lambda^*(\gamma\overline{v} + (1-\gamma)\underline{v})$ will donate with this return.

Proof of Proposition 6: The objective here is to identify how total donations A vary with Q. For any Q, each firm obtains $\frac{A}{Q}$ donors if $\frac{A}{Q}$ is an integer. If not, the residual projects are assigned randomly up to a maximum of one additional project per firm. Consider equilibrium aggregate donations $A(Q)$. Donor j gives if $\Delta S \geq \lambda_j$, where ΔS is conditioned on the equilibrium choices of other donors.

Now consider ΔS. There are N potential projects, and each firm receives a random draw of $\frac{N}{Q}$ of these. After receiving its draw, the firm can identify the quality of the projects. However, donors give without knowledge of the realized valuations, and so compute order statistics to determine the expected value of the marginal project. The expected value of the kth order statistic from $\frac{N}{Q}$ draws is $Ev_{kQ} = \nu_{kQ}\overline{v} + (1-\nu_{kQ})\underline{v}$, where $\nu_{kQ} = \sum_{i=1}^{\frac{N}{Q}+1-k} \rho^{\frac{N}{Q}+1-i}(1-\rho)^i$.

To compute aggregate donations, first compute aggregate expected valuations, ordered as $V_Q = \{Ev_{11}, Ev_{12,}, Ev_{13}, ...Ev_{1Q}, Ev_{21}, Ev_{22,}, ...Ev_{\frac{N}{Q}Q}\}$. Equilibrium donations D for a given Q is characterized by (i) $D \leq A(Ev_{k^*Q})$, and (ii) $Ev_{k^*Q} = V_Q(D(Ev_{k^*Q}))$. First note that as $A' > 0$, there can be at most one k for which $v(A(Ev_{k\frac{N}{Q}})) = Ev_{k\frac{N}{Q}}$. Then there are two cases:

- If there exists a k^* such that $v(A(Ev_{k\frac{N}{Q}})) = Ev_{k\frac{N}{Q}}$, then $\Delta S = Ev_{k^*\frac{N}{Q}}$, and so $D = A(Ev_{k^*\frac{N}{Q}})$.
- If there is no k for which $v(A(Ev_{k\frac{N}{Q}})) = Ev_{k\frac{N}{Q}}$, then $A < D$ implies that for some k^*, $A(Ev_{k^*\frac{N}{Q}}) > k^*Q$, but $A(Ev_{(k^*+1)\frac{N}{Q}}) < k^*Q$. In this case, total donations are k^*Q and the marginal project has return $Ev_{k^*\frac{N}{Q}}$.

The first part is immediate. To see the second part, note that as $A(Ev_{k^*\frac{N}{Q}}) > k^*Q$, but $A(Ev_{(k^*+1)\frac{N}{Q}}) < k^*Q$, the maximum total donations consistent $D \leq A(v_{k'\frac{N}{Q}})$ is $D = k^*Q$. The remaining issue is to show an equilibrium with k^*Q donations that is incentive compatible. Because $A(Ev_{k\frac{N}{Q}}) > k^*Q$, the equilibrium requires that k^*Q donors give and $A(Ev_{k^*\frac{N}{Q}}) - kQ$ do not. First note that all donors with $\lambda < \lambda^*(Ev_{(k+1)\frac{N}{Q}})$ give with probability 1. As a result, for donors between $\lambda^*((Ev_{(k)\frac{N}{Q}}))$ and $\lambda^*((Ev_{(k+1)\frac{N}{Q}}))$, any subset of size $kQ - A(Ev_{(k+1)\frac{N}{Q}})$ also donates. As $A((Ev_{(k)\frac{N}{Q}})) > kQ$, such a group always exists. Note that it is an equilibrium where any subset of size $kQ - A(Ev_{(k+1)\frac{N}{Q}})$ also donates. With those beliefs, no additional donors give as the marginal return becomes $Ev_{(k^*+1)\frac{N}{Q}}$ and no donor with γ between $\lambda^*((Ev_{(k)\frac{N}{Q}}))$ and $\lambda^*((Ev_{(k+1)\frac{N}{Q}}))$ will donate with this return.

This identifies the equilibrium outcome for a given Q. Let $k^*(Q)$ be the equilibrium for any Q. Now consider the impact of changing firm size from Q' to $Q < Q'$. This increases donations if $A(Ev_{k^*(Q)\frac{N}{Q}}) > A(Ev_{k^*(Q')\frac{N}{Q'}})$. Hence only if $Ev_{k^*(Q)\frac{N}{Q}} > Ev_{k^*(Q')\frac{N}{Q'}}$ is the smaller firm chosen. This completes the first part of the Proposition.

Now consider the case where $A(\underline{v}) > (max\{p, N-Q\})$. As $A < N$, this implies that $k^* = \frac{N}{Q}$. In that case, the marginal donor is located at the $\frac{N}{Q}$th order statistic, and so total donations are given by $A(Ev_{\frac{N}{Q}Q})$. But

$$Ev_{\frac{N}{Q}Q} = \underline{v} + \rho^{\frac{N}{Q}}(\overline{v} - \underline{v}), \tag{30}$$

which is decreasing in Q. As a result, donations are decreasing in firm size, as required.

Finally, for donations sufficiently small, donations are maximized by the largest possible firm. As a sufficient condition, note that $Ev_{11} > Ev_{1Q}$ for any $Q > 1$. As a result, if $A(Ev_{11}) < 1$, donations are maximized with one firm, as required.

Proof of Proposition 7: First consider its effect on total donations. Consider a scenario first where firm 1 does not have a competitor. Then total expected donors are $A(\Delta\overline{S}_1)$. The addition of firm 2 raises total expected donations to $\overline{D}_c = A(\Delta\overline{S}_1) + A(\Delta\overline{S}_2) - A(\Delta\overline{S}_1)A(\Delta\overline{S}_2)) \geq A(\Delta\overline{S}_1)$. Now consider $v)j(i)$. To address this, note that donor j gives to firm 1 if $\lambda_1^j + \Delta\overline{S}_1 \geq max\{0, \lambda_2^j + \Delta\overline{S}_2\}$. Then if G is the CDF of $\lambda_1^j - \lambda_2^j$, the probability of a donation for firm 1 is $A(\Delta\overline{S}_1)G(\Delta\overline{S}_2 - \Delta\overline{S}_1)$. In equilibrium, both firms choose the same strategy and so $G(\Delta\overline{S}_2 - \Delta\overline{S}_1) = \frac{1}{2}$. Then the analogous condition to (6) for firm 1 is

$$f(x)\underline{v}[\frac{A'}{2} + A(\overline{S}_1)g(0)] + (1 - F(x)), \tag{31}$$

where g is the density of G. However, consider the case where γ_k^j is close to 0 for all j. Then as $g(0) \to \infty$, $A(\Delta\overline{S}_1)$ is close to 0 for $\Delta\overline{S}_1 < \Delta\overline{S}_2$, and close to 1 for $\Delta\overline{S}_1 > \Delta\overline{S}_2$. In case both firms choose $x^* = \overline{D}_i$, where high value projects are completed with probability $\frac{1}{2}$. Note further that welfare falls here, because as $\gamma_i^j \to 0$, total donations converge to $A(\Delta\overline{S}_1)$. Proposition 7 follows.