# Uncertainty and Business Cycles: Exogenous Impulse or Endogenous Response? 

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## Motivation

- Uncertainty rises sharply in recessions.
- But is uncertainty an exogenous source of business cycles or an endogenous response to them?
- And does the type of uncertainty matter?
- No theoretical consensus on these questions.
- Econometric challenges: "effects"of uncertainty shocks based on recursive schemes in VARs.


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- Any presumed ordering hard to defend on theoretical grounds.
- Recursive structures rule out contemporaneous feedback.


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- $Z_{1 t}$ correlated with $U_{M t}$ and $U_{F t}$, uncorrelated with $Y_{t}$ (shocks).
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- $Z_{2 t}$ correlated with $U_{F t}$, uncorrelated with $U_{M t}$ and $Y_{t}$ (shocks).
- Such instruments have no empirical counterparts. Propose a novel approach: iterative projection IV (IPIV).
- Construct $Z_{1 t}$ and $Z_{2 t}$ from observables using projections.


## Econometric Framework

- Let $\mathbf{X}_{t}$ be a $K \times 1$ vector.
- Consider $p$-th order structural vector autoregressive (SVAR)

$$
\begin{align*}
& \mathbf{X}_{t}=\mathbf{k}+\mathbb{A}_{1} \mathbf{X}_{t-1}+\mathbb{A}_{2} \mathbf{X}_{t-2}+\cdots+\mathbb{A}_{p} \mathbf{X}_{t-p}+\mathbf{H} \mathbf{\Sigma} \mathbf{e}_{t} .  \tag{1}\\
& \mathbf{e}_{t} \sim\left(0, \mathbf{I}_{\mathbf{K}}\right), \quad \boldsymbol{\Sigma}=\left(\begin{array}{cccc}
\sigma_{11} & 0 & \cdot & 0 \\
0 & \sigma_{22} & 0 & 0 \\
0 & \cdot & \cdot & 0 \\
0 & 0 & \cdot & \sigma_{K K}
\end{array}\right) .
\end{align*}
$$

The structural shocks $\mathbf{e}_{t}$ are serially and mutually uncorrelated.

- Unit effect normalization \& restrict admissible parameter space:

$$
\operatorname{diag}(\mathbf{H})=1 \quad \sigma_{j j} \geq 0 \quad \forall j
$$

## Econometric Framework

- The reduced form representation of $\mathbf{X}_{t}$ is a $p$-th order VAR with $M A(\infty)$ representation

$$
\begin{aligned}
\mathbf{X}_{t} & =\boldsymbol{\mu}+\boldsymbol{\Psi}(L) \boldsymbol{\eta}_{t} \\
\boldsymbol{\eta}_{t} & \sim(0, \boldsymbol{\Omega}), \quad \boldsymbol{\Omega}=\boldsymbol{E}\left(\boldsymbol{\eta}_{t} \boldsymbol{\eta}_{t}^{\prime}\right) .
\end{aligned}
$$

- The structural shocks $\mathbf{e}_{t}$ are related to the reduced form innovations by an invertible $K \times K$ matrix $\mathbf{H}$ :

$$
\eta_{t}=\mathbf{H} \Sigma \mathbf{e}_{t} \equiv \mathbf{B} \mathbf{e}_{t}
$$

- Here $K=3$ and $\mathbf{X}_{t}=\left(U_{M t}, Y_{t}, U_{F t}\right)^{\prime}, \mathbf{e}_{t}=\left(e_{M t}, e_{Y t}, e_{F t}\right)^{\prime}$
- Want to identify $\mathbf{e}_{t}=\mathbf{B}^{-1} \boldsymbol{\eta}_{t}$, nine unknown elements in $\mathbf{B} \rightarrow$
- Need nine restrictions for identification.


## Identification

- Covariance structure $\boldsymbol{\eta}_{t}$ provides $K(K+1) / 2=6$ restrictions:

$$
\operatorname{vech}(\boldsymbol{\Omega})=\operatorname{vech}\left(\mathbf{B B}^{\prime}\right)
$$

- Need 3 more for identification
- Suppose we have measures of $Y_{t}, U_{M t}, U_{F t}$, and two generic instruments, $Z_{t}=\left(Z_{1 t}, Z_{2 t}\right)^{\prime}$.

Assumption A: Let $Z_{1 t}$ and $Z_{2 t}$ be two IVs such that
(A.i) $\quad \mathbb{E}\left[Z_{1 t} e_{M t}\right]=\phi_{1 M}, \quad \mathbb{E}\left[Z_{1 t} e_{Y t}\right]=0, \quad \mathbb{E}\left[Z_{1 t} e_{F t}\right]=\phi_{1 F}$
(A.ii) $\quad \mathbb{E}\left[Z_{2 t} e_{M t}\right]=0, \quad \mathbb{E}\left[Z_{2 t} e_{Y t}\right]=0, \quad \mathbb{E}\left[Z_{2 t} e_{F t}\right]=\phi_{2 F}$

- Instrument Exogeneity: $\mathbb{E}\left[Z_{1 t} e_{Y t}\right]=\mathbb{E}\left[Z_{2 t} e_{Y t}\right]=\mathbb{E}\left[Z_{2 t} e_{M t}\right]=0$
- Instrument Relevance: $\phi_{1 M}, \phi_{1 F}, \phi_{2 F} \neq 0$


## Identification

- Let $\mathbf{m}_{1 t}\left(\boldsymbol{\eta}_{t}, Z_{t}\right)=\left(\operatorname{vech}\left(\boldsymbol{\eta}_{t} \boldsymbol{\eta}_{t}^{\prime}\right), \operatorname{vec}\left(Z_{t} \otimes \boldsymbol{\eta}_{t}\right)\right)^{\prime}$ and $\boldsymbol{\beta}_{1}=\operatorname{vec}(\boldsymbol{B})$.
- At the true value of $\boldsymbol{\beta}_{1}$, denoted $\boldsymbol{\beta}_{1}^{0}$, the model satisfies

$$
0=\mathbb{E}\left[\mathbf{g}_{1}\left(\mathbf{m}_{1 t}\left(\boldsymbol{\eta}_{t}, Z_{t}\right) ; \boldsymbol{\beta}_{1}^{0}\right)\right]
$$

- Nonlinear system with nine equations in nine unknowns.


## Identification

Proposition
Under Assumption A with $\phi_{1 M} \neq 0, \phi_{1 F} \neq 0, \phi_{2 F} \neq 0, \operatorname{diag}(\mathbf{H})=1$, and $\sigma_{j j}>0 \forall j, \boldsymbol{\beta}_{1}$ is identified.

In words, identification is achieved by
(1) Use movements in $U_{M t}$ and $U_{F t}$ correlated with $Z_{1 t}$ to identify $U_{M t}$ and $U_{F t}$ shocks, disentangle them from real activity shocks
(2) Use movements in $U_{F t}$ correlated with $Z_{2 t}$ to identify $U_{F t}$ shocks and disentangle them from $U_{M t}$ shocks
(3) Use movements in $Y_{t}$ uncorrelated with both $\mathrm{Z}_{1 t}, \mathrm{Z}_{2 t}$ to identify $Y$ shocks, disentangle them from $U_{M t}$ and $U_{F t}$ shocks

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- Assume $\mathbf{S}_{t}$ driven by $\mathbf{e}_{t}=\left(e_{Y t}, e_{M t} \text { and } e_{F t}\right)^{\prime}$ and idiosyncratic shocks collected into $\mathbf{e}_{S t}$ orthogonal to $\mathbf{e}_{t}$.
- Shocks $\mathbf{e}_{S t}$ presumed not to affect $\mathbf{X}_{t}$. Represent $\mathbf{S}_{j t}, j=1,2$ as

$$
\begin{equation*}
\delta_{S}(L) S_{j t}=\delta_{j 0}+\delta_{j Y} Y_{t}+\delta_{j M} U_{M t}+\delta_{j F} U_{F t}+\delta_{j X}(L)^{\prime} \mathbf{X}_{t-1}+e_{S j t} \tag{2}
\end{equation*}
$$

- Equation (2) motivates two orthogonal decompositions:

$$
\begin{aligned}
d_{1 S}(L) S_{1 t} & =d_{10}+d_{1 Y} e_{Y t}+Z_{1 t} \\
d_{2 S}(L) S_{2 t} & =d_{20}+d_{2 Y} e_{Y t}+d_{2 M} e_{M t}+Z_{2 t}
\end{aligned}
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\end{aligned}
$$

- Problem: projections are infeasible b/c $e_{Y t}, e_{M t}$ are unobserved.
- Solution: generate $Z_{1 t}$ and $Z_{2}$ using iterative approach to jointly solve for $e_{t}$ and $Z_{t}$ that satisfy restrictions for instrument exogeneity \& relevance.


## Iterative Projection IV (IPIV)

$$
\begin{align*}
& d_{1 S}(L) S_{1 t}=d_{10}+d_{1 Y} e_{Y t}+Z_{1 t}  \tag{*}\\
& d_{2 S}(L) S_{2 t}=d_{20}+d_{2 Y} e_{Y t}+d_{2 M} e_{M t}+Z_{2 t} \tag{**}
\end{align*}
$$

Let $T \times 1 \mathbf{e}_{\mathbf{M}}{ }^{(0) k}, \mathbf{e}_{\mathbf{Y}}{ }^{(0) k}$ be the $k^{\text {th }}$ initial guess in a compact set $\mathcal{K}$. Initialize $j=0$.
i Replace $\mathbf{e}_{\mathbf{M}}$ and $\mathbf{e}_{\mathbf{Y}}$ in $(*)$ and $(* *)$ by $\mathbf{e}_{\mathbf{M}}{ }^{(j) k}$ and $\mathbf{e}_{\mathbf{Y}}{ }^{(j) k}$. Obtain $\mathbf{Z}_{1}^{(j) k}$ and $\mathbf{Z}_{2}^{(j) k}$.
ii Use $\mathbf{Z}_{1}^{(j) k}, \mathbf{Z}_{2}^{(j) k}$ to solve $0=\mathbb{E}\left[\mathbf{g}_{1}\left(\mathbf{m}_{1 t}\left(\boldsymbol{\eta}_{t}, Z_{t}\right) ; \boldsymbol{\beta}_{1}^{0}\right)\right]$ for $\boldsymbol{\beta}_{1}$. Form $\mathbf{B}^{(j) k}$ from $\boldsymbol{\beta}_{1}^{(j) k}$.
iii Update shocks $\mathbf{e}^{(j+1) k}=\left(\mathbf{e}_{\mathbf{M}}{ }^{(j+1) k}, \mathbf{e}_{\mathbf{Y}}{ }^{(j+1) k}, \mathbf{e}_{\mathbf{F}}{ }^{(j+1) k}\right)=\left(\mathbf{B}^{(j) k}\right)^{-1} \hat{\boldsymbol{\eta}}$.
iv If $\left\|\mathbf{e}_{\mathbf{M}}{ }^{(j+1) k}-\mathbf{e}_{\mathbf{M}}{ }^{(j) k}\right\| \leq$ tol and $\left\|\mathbf{e}_{\mathbf{Y}}{ }^{(j+1) k}-\mathbf{e}_{\mathbf{Y}}{ }^{(j) k}\right\|<$ tol, stop and let $\mathbf{e}^{k}=\mathbf{e}^{(j) k}, \boldsymbol{\beta}_{1}^{k}=\boldsymbol{\beta}_{1}^{(j) k}$. Else, set $j=j+1$ and return to (i).
v -a Economic constraints: large shock episodes
v-b Econometric constraints: Store $\hat{c}_{1}=\operatorname{corr}\left(Z_{1 t}\left(\beta_{1}^{k}\right), e_{M t}^{k}\right), \hat{c}_{2}=\operatorname{corr}\left(Z_{1 t}\left(\beta_{1}^{k}\right), e_{F t}^{k}\right)$, $\hat{c}_{3}=\operatorname{corr}\left(Z_{2 t}\left(\boldsymbol{\beta}_{1}^{k}\right), e_{F t}^{k}\right), C\left(\boldsymbol{\beta}_{1}^{k}\right)=\frac{1}{3}\left(\left|\hat{c}_{1}\right|+\left|\hat{c}_{2}\right|+\left|\hat{c}_{3}\right|\right)$. Keep $\boldsymbol{\beta}_{1}^{k}$ that satisfy (a) $C\left(\boldsymbol{\beta}_{1}^{k}\right) \geq \bar{C}$, (b), each $\left|\hat{c}_{i}\right| \geq \bar{c}$, and (c) $\operatorname{det}\left(B^{(j) k}\right) \geq \underline{b}$.

## Iterative Projection IV (IPIV)

(1) Instrument exogeneity: holds by construction.
(2) If estimation unconstrained: diverse multiplicity of solutions, esp. if starting values are poor $\Rightarrow$ add restrictions to narrow set:
(3) Additional restrictions for instrument relevance:

- Minimum thresholds for individual and collective instrument strength and $\operatorname{det}(\mathbf{B})>0$ (step (v-b)).
(9) Further winnow solutions using prior economic reasoning: Study estimated shocks in detail check that signs and magnitudes are sensible:
- 1987 crash \& 2007-09 fin. crisis identified as big positive $U_{F t}$ shocks
- Great Recession not identified with big positive Y shock.
(3) Left: handful of credible solutions $(\approx 6)$ all very close and tell same economic story. Results shown for one solution (base case).


## Measuring Uncertainty: Jurado, Ludvigson, Ng (JLN)

- Methodology: DI forecasting plus stochastic volatility model hundreds economic time-series
- One month-ahead uncertainty indexes:
- Macro uncertainty $U_{M t}$ aggregates uncertainty estimates of 134 macro indicators
- Real activity, price, financial
- Financial uncertainty $U_{F t}$ aggregates uncertainty estimates of 147 financial indicators
- Stock, bond returns and risk factors
- Real activity uncertainty $U_{R t}$ aggregates uncertainty estimates of 73 real activity variables


## Measuring Stock Market Returns and Real Activity

- Set $S_{2 t}=r_{S \& P t}$ to generate $Z_{2 t}$
- Set $S_{1 t}=r_{p t} \equiv \alpha_{p} r_{C R S P t}+\left(1-\alpha_{p}\right) r_{s m a l l t}$ to generate $Z_{1 t}$
- Real activity $Y_{t}=$
(1) $\log$ of industrial production $i p_{t}$
(2) $\log$ of total non-farm employment $e m p_{t}$
(3) Real activity factor: $Q_{1 t}$ (cumulative sum of first common factor estimated from large macro dataset).
- Estimation: all parameters by GMM.
- Data: monthly.


## Results

## Time Series of Uncertainty Measures

- Both exhibit large spikes in deep recessions.


Aggregate Financial Uncertainty $U_{F}$


Note: $U_{M}, U_{F}$ are expressed in standardized units. Correlations with the 12 -month moving average of IP growth are reported. The black dots represent months when uncertainty is 1.65 standard deviations above its unconditional mean. The shaded areas correspond to the NBER recession dates. The sample spans the period 1960:07 to 2015:04.

## Time Series of Uncertainty Measures

- $U_{F t}$ less countercyclical than $U_{M t} ; \operatorname{corr}\left(U_{M t}, U_{F t}\right)=0.58$.

Aggregate Macro Uncertainty $U_{M}$


Aggregate Financial Uncertainty $U_{F}$


Note: $U_{M}, U_{F}$ are expressed in standardized units. Correlations with the 12-month moving average of IP growth are reported. The black dots represent months when uncertainty is 1.65 standard deviations above its unconditional mean. The shaded areas correspond to the NBER recession dates. The sample spans the period 1960:07 to 2015:04.

## $\operatorname{IRF}$ for $\operatorname{SVAR}\left(U_{M}, Y, U_{F}\right)^{\prime}$



Note: Bootstrapped 90 percent error bands appear as vertical lines. Responses to positive one standard deviation shocks are reported. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

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## IRF for SVAR $\left(U_{M}, Y, U_{F}\right)^{\prime}$

- Positive $U_{F}$ shocks $\Rightarrow$ sharp, persistent decline in real activity


Note: Bootstrapped 90 percent error bands appear as vertical lines. Responses to positive one standard deviation shocks are reported. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

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## IRF for SVAR $\left(U_{M}, Y, U_{F}\right)^{\prime}$

- Little evidence that $Y$ shocks affect $U_{F}$


Note: Bootstrapped 90 percent error bands appear as vertical lines. Responses to positive one standard deviation shocks are reported. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

## IRF for SVAR $\left(U_{M}, Y, U_{F}\right)^{\prime}$

- Macro uncertainty falls sharply in response to positive $Y$ shocks


Note: Bootstrapped 90 percent error bands appear as vertical lines. Responses to positive one standard deviation shocks are reported. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

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## IRF for SVAR $\left(U_{M}, Y, U_{F}\right)^{\prime}$

- No evidence that positive $U_{M}$ shocks lead to declines in real activity; indeed the opposite.


Note: Bootstrapped 90 percent error bands appear as vertical lines. Responses to positive one standard deviation shocks are reported. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

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## IRF for SVAR $\left(U_{M}, Y, U_{F}\right)^{\prime}$

- Higher macro uncertainty in recessions entirely an endogenous response to lower economic activity.


Note: Bootstrapped 90 percent error bands appear as vertical lines. Responses to positive one standard deviation shocks are reported. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

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## IRF for SVAR $\left(U_{R}, Y, U_{F}\right)^{\prime}$



Note: Bootstrapped 90 percent error bands appear as vertical lines. Responses to one standard deviation shocks are reported. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

## Overidentifying Exclusion Restrictions

- $S_{t}$ assumed external to VAR. This is tantamount to imposing an exclusion restriction on larger VAR that includes $S_{t}$.
- Let $\mathbf{X}_{t}=\left(U_{M t}, Y_{t}, U_{F t}\right)^{\prime}$ and $S_{t}$ stock returns. $\operatorname{VAR}(1)$ with $S_{t}$ :

$$
\underbrace{\left(\begin{array}{cc}
\mathbf{A}_{X X, 0} & \mathbf{A}_{X S, 0} \\
3 \times 3 & 3 \times 2 \\
\mathbf{A}_{S X, 0} & A_{S S, 0} \\
2 \times 3 & 2 \times 2
\end{array}\right)}_{\mathbf{A}_{0} \equiv \mathbf{H}^{-1}}\binom{\mathbf{x}_{t}}{S_{t}}=\left(\begin{array}{cc}
\mathbf{A}_{X X, 1} & \mathbf{A}_{X S, 1} \\
\mathbf{A}_{S X, 1} & A_{S S, 1}
\end{array}\right)\binom{\mathbf{X}_{t-1}}{S_{t-1}}+\left(\begin{array}{cc}
\boldsymbol{\Sigma}_{X} & 0 \\
0 & \Sigma_{S}
\end{array}\right)\binom{\mathbf{e}_{X t}}{\mathbf{e}_{S t}}
$$

- Maintained assumption baseline case: $\mathbf{A}_{X S, 0}=\mathbf{A}_{X S, 1}=\mathbf{0}$.
- Paper: in 4 variable VAR, still need $\mathbf{A}_{X S, 0}=\mathbf{0}$ for identification. But don't need $\mathbf{A}_{X S, 1}=0$.
- Evaluate validity of OID restrictions by comparing IRF for 3 variable $\mathbf{X}_{t}$ with 4 variable $\left(\mathbf{X}_{t}^{\prime}, S_{t}\right)^{\prime}$ where $\mathbf{A}_{X S, 1}$ left unconstrained.


## Evaluating OID Restrictions: Compare IRFs

- IRFs from 3 variable $\mathbf{X}_{t}$ v.s. 4 variable $\left(\mathbf{X}_{t}^{\prime}, S_{t}\right)^{\prime}$ with free $\mathbf{A}_{X S, j} \forall j \geq 1$.


Note: $S_{t}$ is the CRSP value weighted average returns. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

## Evaluating OID Restrictions: Compare IRFs

- Data appear consistent with assumption stock returns can be excluded.



$i p$ Shock







Note: $S_{t}$ is the CRSP value weighted average returns. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

## Test of Recursive Restrictions

- Our SVAR model nests any recursive structure.
- Chi-square test $H_{0}$ : recursive structure is supported by the data.
- Strongly reject lower triangular structure for any possible ordering.
- Inspection of $\hat{\mathbf{A}}_{0}$ reveals non-zero contemporaneous correlations $\rho\left(U_{M}, Y\right), \rho\left(U_{F}, Y\right)$, inconsistent with any recursive ordering.

$$
\hat{\mathbf{A}}_{0}=\left(\begin{array}{rrr}
\mathbf{0 . 5 1 3 0} & \mathbf{0 . 7 8 1 5} & -\mathbf{0 . 0 1 0 6} \\
{[0.0205]} & {[0.0324]} & {[0.0034]} \\
-\mathbf{0 . 3 2 5 1} & \mathbf{0 . 4 4 4 1} & \mathbf{0 . 0 5 9 0} \\
{[0.0135]} & {[0.0184]} & {[0.0024]} \\
-0.0046 & -\mathbf{1 . 0 9 6 9} & \mathbf{0 . 9 3 9 4} \\
{[0.1625]} & {[0.2666]} & {[0.0258]}
\end{array}\right)
$$

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- Our IPIV is a way to isolate those components.


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- ...Uncertainty in financial markets a likely source of business cycles.


## Appendix

## Real Activity Uncertainty $U_{R}$

- Sub-index of $U_{M}$ corresponding to real activity variables.


Note: $U_{R}$ is expressed in standardized units. Correlations with the 12-month moving average of IP growth are reported. The shaded areas correspond to the NBER recession dates. The monthly data span the period 1960:07 to 2015:04.

## Real Activity Uncertainty $U_{R}$

- Special relevance to uncertainty literature, where uncertainty shocks have origins in economic ${\underset{T}{R}}^{\text {fand }}$


Note: $U_{R}$ is expressed in standardized units. Correlations with the 12-month moving average of IP growth are reported. The shaded areas correspond to the NBER recession dates. The monthly data span the period 1960:07 to 2015:04.

## e shock Time series $\left(U_{M}, i p_{,}, U_{F}\right)^{\prime}$



Note: Time series of e shock from SVAR system $\left(U_{M}, i p, U_{F}\right)$. The horizontal line corresponds to 2 standard deviations above/below the unconditional mean of each series. The shocks $e=B^{-1} \eta_{t}$ are reported, where $\eta_{t}$ is the residual from $\operatorname{VAR}(6)$ of ( $U_{M}, i p, U_{F}$ ) and $B=A^{-1} \Sigma$. The shaded areas correspond to the NBER recession dates. The sample spans the period 1960:07 to 2015:04.

## e solution fails economics constraint



Note: Time series of e shock from SVAR system $\left(U_{M}, i p, U_{F}\right)$. The horizontal line corresponds to 2 standard deviations above/below the unconditional mean of each series. The shocks $e=B^{-1} \eta_{t}$ are reported, where $\eta_{t}$ is the residual from $\operatorname{VAR}(6)$ of $\left(U_{M}, i p, U_{F}\right)$ and $B=A^{-1} \Sigma$. The shaded areas correspond to the NBER recession dates. The sample spans the period 1960:07 to 2015:04.

## IRF that fails economics constraint



Note: Bootstrapped 90 percent error bands appear as vertical lines. Responses to one standard deviation shocks are reported. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

## Variance Decomposition $\left(U_{M}, Y, U_{F}\right)^{\prime}$

|  | $\operatorname{SVAR}\left(U_{M}, i p, U_{F}\right)^{\prime}$ |  |  | SVAR $\left(U_{M, ~ e m p, ~} U_{F}\right)^{\prime}$ |  |  | $\operatorname{SVAR}\left(U_{M}, Q_{1}, U_{F}\right)^{\prime}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Fraction variation in $U_{M}$ |  |  | Fraction variation in $U_{M}$ |  |  | Fraction variation in $U_{M}$ |  |  |
| $s$ | $U_{M}$ Shock | ip Shock | $U_{F}$ Shock | $U_{M}$ Shock | emp Shock | $U_{F}$ Shock | $U_{M}$ Shock | $Q_{1}$ Shock | $U_{F}$ Shock |
| 1 | 0.371 | 0.527 | 0.102 | 0.531 | 0.376 | 0.093 | 0.390 | 0.497 | 0.113 |
| 12 | 0.419 | 0.409 | 0.172 | 0.601 | 0.249 | 0.150 | 0.434 | 0.371 | 0.195 |
| $\infty$ | 0.420 | 0.368 | 0.212 | 0.619 | 0.220 | 0.161 | 0.478 | 0.322 | 0.200 |
| $s_{\text {max }}$ | 0.511 | 0.528 | 0.215 | 0.664 | 0.384 | 0.161 | 0.572 | 0.498 | 0.203 |
|  | [0.25, 0.79] | [0.22, 0.71 ] | [0.05, 0.57] | [0.34, 0.87] | [0.15, 0.59] | [0.06, 0.46] | [0.30, 0.79] | [0.21, 0.70] | [0.06, 0.53] |
|  | Fraction variation in ip |  |  | Fraction variation in emp |  |  | Fraction variation in $Q_{1}$ |  |  |
| $s$ | $U_{M}$ Shock | ip Shock | $U_{F}$ Shock | $U_{M}$ Shock | emp Shock | $U_{F}$ Shock | $U_{M}$ Shock | $Q_{1}$ Shock | $U_{F}$ Shock |
| 1 | 0.401 | 0.556 | 0.043 | 0.352 | 0.402 | 0.246 | 0.456 | 0.508 | 0.036 |
| 12 | 0.121 | 0.659 | 0.220 | 0.075 | 0.406 | 0.519 | 0.169 | 0.563 | 0.269 |
| $\infty$ | 0.082 | 0.691 | 0.227 | 0.124 | 0.424 | 0.453 | 0.063 | 0.621 | 0.317 |
| $s_{\text {max }}$ | 0.415 | 0.696 | 0.272 | 0.373 | 0.424 | 0.587 | 0.468 | 0.621 | 0.358 |
|  | [0.19, 0.61] | [0.34, 0.94] | [0.04, 0.73 ] | [0.21, 0.63] | [0.16, 0.85] | [0.16, 0.92] | [0.24, 0.62] | [0.33, 0.95] | [0.07, 0.81] |
|  | Fraction variation in $U_{F}$ |  |  | Fraction variation in $U_{F}$ |  |  | Fraction variation in $U_{F}$ |  |  |
| $s$ | $U_{M}$ Shock | ip Shock | $U_{F}$ Shock | $U_{M}$ Shock | emp Shock | $U_{F}$ Shock | $U_{M}$ Shock | $Q_{1}$ Shock | $U_{F}$ Shock |
| 1 | 0.029 | 0.023 | 0.948 | 0.140 | 0.119 | 0.743 | 0.019 | 0.022 | 0.959 |
| 12 | 0.080 | 0.041 | 0.878 | 0.243 | 0.133 | 0.624 | 0.082 | 0.039 | 0.879 |
| $\infty$ | 0.121 | 0.131 | 0.748 | 0.332 | 0.138 | 0.530 | 0.156 | 0.098 | 0.746 |
| $s_{\text {max }}$ | 0.128 | 0.131 | 0.950 | 0.339 | 0.152 | 0.744 | 0.163 | 0.098 | 0.961 |
|  | [0.03, 0.47] | [0.05, 0.52] | [0.53, 0.99] | [0.08, 0.64] | [0.03, 0.58] | [0.33, 0.95] | [0.03, 0.53] | [0.03, 0.48 ] | [0.60, 0.99] |

Note: Each panel shows the fraction of s-step-ahead forecast-error variance of the variable given in the panel title that is explained by the shock named in the column heading. The row denoted " $s=s_{\text {max }}$ "reports the maximum fraction (across all VAR forecast horizons $m$ ) of forecast error variance explained by the shock listed in the column heading. The numbers in parentheses represent the 5th and 95th percentiles of these statistics from bootstrapped samples. The data are monthly and span the period 1960:07 to 2015:04.

## Variance Decomposition $\left(U_{M}, Y, U_{F}\right)^{\prime}$

- Variation in $U_{F}$ driven by its own shocks.

|  | SVAR $\left(U_{M}, i p, U_{F}\right)^{\prime}$ |  |  | SVAR $\left(U_{M, ~ e m p, ~} U_{F}\right)^{\prime}$ |  |  | $\operatorname{SVAR}\left(U_{M}, Q_{1}, U_{F}\right)^{\prime}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Fraction variation in $U_{M}$ |  |  | Fraction variation in $U_{M}$ |  |  | Fraction variation in $U_{M}$ |  |  |
| $s$ | $U_{M}$ Shock | ip Shock | $U_{F}$ Shock | $U_{M}$ Shock | emp Shock | $U_{F}$ Shock | $U_{M}$ Shock | $Q_{1}$ Shock | $U_{F}$ Shock |
| 1 | 0.371 | 0.527 | 0.102 | 0.531 | 0.376 | 0.093 | 0.390 | 0.497 | 0.113 |
| 12 | 0.419 | 0.409 | 0.172 | 0.601 | 0.249 | 0.150 | 0.434 | 0.371 | 0.195 |
| $\infty$ | 0.420 | 0.368 | 0.212 | 0.619 | 0.220 | 0.161 | 0.478 | 0.322 | 0.200 |
| $s_{\text {max }}$ | 0.511 | 0.528 | 0.215 | 0.664 | 0.384 | 0.161 | 0.572 | 0.498 | 0.203 |
|  | [0.25, 0.79] | [0.22, 0.71] | [0.05, 0.57] | [0.34, 0.87] | [0.15, 0.59] | [0.06, 0.46] | [0.30, 0.79] | [0.21, 0.70 ] | [0.06, 0.53] |
|  | Fraction variation in ip |  |  | Fraction variation in emp |  |  | Fraction variation in $Q_{1}$ |  |  |
| $s$ | $U_{M}$ Shock | ip Shock | $U_{F}$ Shock | $U_{M}$ Shock | emp Shock | $U_{F}$ Shock | $U_{M}$ Shock | $Q_{1}$ Shock | $U_{F}$ Shock |
| 1 | 0.401 | 0.556 | 0.043 | 0.352 | 0.402 | 0.246 | 0.456 | 0.508 | 0.036 |
| 12 | 0.121 | 0.659 | 0.220 | 0.075 | 0.406 | 0.519 | 0.169 | 0.563 | 0.269 |
| $\infty$ | 0.082 | 0.691 | 0.227 | 0.124 | 0.424 | 0.453 | 0.063 | 0.621 | 0.317 |
| $s_{\text {max }}$ | 0.415 | 0.696 | 0.272 | 0.373 | 0.424 | 0.587 | 0.468 | 0.621 | 0.358 |
|  | [0.19, 0.61] | [0.34, 0.94] | [0.04, 0.73 ] | [0.21, 0.63 ] | [0.16, 0.85] | [0.16, 0.92] | [0.24, 0.62] | [0.33, 0.95] | [0.07, 0.81 ] |
|  | Fraction variation in $U_{F}$ |  |  | Fraction variation in $U_{F}$ |  |  | Fraction variation in $U_{F}$ |  |  |
| $s$ | $U_{M}$ Shock | ip Shock | $U_{F}$ Shock | $U_{M}$ Shock | emp Shock | $U_{F}$ Shock | $U_{M}$ Shock | $Q_{1}$ Shock | $U_{F}$ Shock |
| 1 | 0.029 | 0.023 | 0.948 | 0.140 | 0.119 | 0.743 | 0.019 | 0.022 | 0.959 |
| 12 | 0.080 | 0.041 | 0.878 | 0.243 | 0.133 | 0.624 | 0.082 | 0.039 | 0.879 |
| $\infty$ | 0.121 | 0.131 | 0.748 | 0.332 | 0.138 | 0.530 | 0.156 | 0.098 | 0.746 |
| $s_{\text {max }}$ | 0.128 | 0.131 | 0.950 | 0.339 | 0.152 | 0.744 | 0.163 | 0.098 | 0.961 |
|  | [0.03, 0.47] | [0.05, 0.52] | [0.53, 0.99] | [0.08, 0.64] | [0.03, 0.58] | [0.33, 0.95] | [0.03,0.53] | [0.03, 0.48] | [0.60,0.99] |

Note: Each panel shows the fraction of $s$-step-ahead forecast-error variance of the variable given in the panel title that is explained by the shock named in the column heading. The row denoted " $s=s_{\text {max }}$ "reports the maximum fraction (across all VAR forecast horizons $m$ ) of forecast error variance explained by the shock listed in the column heading. The numbers in parentheses represent the 5th and 95th percentiles of these statistics from bootstrapped samples. The data are monthly and span the period 1960:07 to 2015:04.

## Variance Decomposition $\left(U_{M}, Y, U_{F}\right)^{\prime}$

- Large fractions of variance in emp driven by $U_{F}$ shocks.

|  | $\operatorname{SVAR}\left(U_{M}, i p, U_{F}\right)^{\prime}$ |  |  | SVAR $\left(U_{M,}, \text { emp, } U_{F}\right)^{\prime}$ |  |  | $\operatorname{SVAR}\left(U_{M}, Q_{1}, U_{F}\right)^{\prime}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Fraction variation in $U_{M}$ |  |  | Fraction variation in $U_{M}$ |  |  | Fraction variation in $U_{M}$ |  |  |
| $s$ | $U_{M}$ Shock | ip Shock | $U_{F}$ Shock | $U_{M}$ Shock | emp Shock | $U_{F}$ Shock | $U_{M}$ Shock | $Q_{1}$ Shock | $U_{F}$ Shock |
| 1 | 0.371 | 0.527 | 0.102 | 0.531 | 0.376 | 0.093 | 0.390 | 0.497 | 0.113 |
| 12 | 0.419 | 0.409 | 0.172 | 0.601 | 0.249 | 0.150 | 0.434 | 0.371 | 0.195 |
| $\infty$ | 0.420 | 0.368 | 0.212 | 0.619 | 0.220 | 0.161 | 0.478 | 0.322 | 0.200 |
| $s_{\text {max }}$ | 0.511 | 0.528 | 0.215 | 0.664 | 0.384 | 0.161 | 0.572 | 0.498 | 0.203 |
|  | [0.25, 0.79] | [0.22,0.71] | [0.05, 0.57] | [0.34, 0.87] | [0.15, 0.59] | [0.06, 0.46] | [0.30, 0.79] | [0.21, 0.70] | [0.06, 0.53 ] |
|  | Fraction variation in ip |  |  | Fraction variation in emp |  |  | Fraction variation in $Q_{1}$ |  |  |
| $s$ | $U_{M}$ Shock | ip Shock | $U_{F}$ Shock | $U_{M}$ Shock | emp Shock | $U_{F}$ Shock | $U_{M}$ Shock | $Q_{1}$ Shock | $U_{F}$ Shock |
| 1 | 0.401 | 0.556 | 0.043 | 0.352 | 0.402 | 0.246 | 0.456 | 0.508 | 0.036 |
| 12 | 0.121 | 0.659 | 0.220 | 0.075 | 0.406 | 0.519 | 0.169 | 0.563 | 0.269 |
| $\infty$ | 0.082 | 0.691 | 0.227 | 0.124 | 0.424 | 0.453 | 0.063 | 0.621 | 0.317 |
| $s_{\text {max }}$ | 0.415 | 0.696 | 0.272 | 0.373 | 0.424 | 0.587 | 0.468 | 0.621 | 0.358 |
|  | [0.19, 0.61] | [0.34, 0.94] | [0.04, 0.73] | [0.21, 0.63] | [0.16, 0.85] | [0.16, 0.92] | [0.24, 0.62] | [0.33, 0.95] | [0.07, 0.81] |
|  | Fraction variation in $U_{F}$ |  |  | Fraction variation in $U_{F}$ |  |  | Fraction variation in $U_{F}$ |  |  |
| $s$ | $U_{M}$ Shock | ip Shock | $U_{F}$ Shock | $U_{M}$ Shock | emp Shock | $U_{F}$ Shock | $U_{M}$ Shock | $Q_{1}$ Shock | $U_{F}$ Shock |
| 1 | 0.029 | 0.023 | 0.948 | 0.140 | 0.119 | 0.743 | 0.019 | 0.022 | 0.959 |
| 12 | 0.080 | 0.041 | 0.878 | 0.243 | 0.133 | 0.624 | 0.082 | 0.039 | 0.879 |
| $\infty$ | 0.121 | 0.131 | 0.748 | 0.332 | 0.138 | 0.530 | 0.156 | 0.098 | 0.746 |
| $s_{\text {max }}$ | 0.128 | 0.131 | 0.950 | 0.339 | 0.152 | 0.744 | 0.163 | 0.098 | 0.961 |
|  | [0.03, 0.47] | [0.05, 0.52] | [0.53, 0.99] | [0.08, 0.64 ] | [0.03, 0.58] | [0.33, 0.95] | [0.03, 0.53] | [0.03, 0.48 ] | [0.60,0.99] |

Note: Each panel shows the fraction of s-step-ahead forecast-error variance of the variable given in the panel title that is explained by the shock named in the column heading. The row denoted " $s=s_{\text {max }}$ "reports the maximum fraction (across all VAR forecast horizons $m$ ) of forecast error variance explained by the shock listed in the column heading. The numbers in parentheses represent the 5th and 95th percentiles of these statistics from bootstrapped samples. The data are monthly and span the period 1960:07 to 2015:04.

## Variance Decomposition $\left(U_{M}, Y, U_{F}\right)^{\prime}$

- Sizable amount variation in $U_{M}$ driven by $Y$ shocks.

|  | $\operatorname{SVAR}\left(U_{M}, i p, U_{F}\right)^{\prime}$ |  |  | SVAR $\left(U_{M, ~ e m p, ~} U_{F}\right)^{\prime}$ |  |  | $\operatorname{SVAR}\left(U_{M}, Q_{1}, U_{F}\right)^{\prime}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Fraction variation in $U_{M}$ |  |  | Fraction variation in $U_{M}$ |  |  | Fraction variation in $U_{M}$ |  |  |
| $s$ | $U_{M}$ Shock | ip Shock | $U_{F}$ Shock | $U_{M}$ Shock | emp Shock | $U_{F}$ Shock | $U_{M}$ Shock | $Q_{1}$ Shock | $U_{F}$ Shock |
| 1 | 0.371 | 0.527 | 0.102 | 0.531 | 0.376 | 0.093 | 0.390 | 0.497 | 0.113 |
| 12 | 0.419 | 0.409 | 0.172 | 0.601 | 0.249 | 0.150 | 0.434 | 0.371 | 0.195 |
| $\infty$ | 0.420 | 0.368 | 0.212 | 0.619 | 0.220 | 0.161 | 0.478 | 0.322 | 0.200 |
| $s_{\text {max }}$ | 0.511 | 0.528 | 0.215 | 0.664 | 0.384 | 0.161 | 0.572 | 0.498 | 0.203 |
|  | [0.25, 0.79] | [0.22, 0.71] | [0.05, 0.57] | [0.34, 0.87] | [0.15, 0.59] | [0.06, 0.46] | [0.30, 0.79] | [0.21, 0.70] | [0.06, 0.53 ] |
|  | Fraction variation in ip |  |  | Fraction variation in emp |  |  | Fraction variation in $Q_{1}$ |  |  |
| $s$ | $U_{M}$ Shock | ip Shock | $U_{F}$ Shock | $U_{M}$ Shock | emp Shock | $U_{F}$ Shock | $U_{M}$ Shock | $Q_{1}$ Shock | $U_{F}$ Shock |
| 1 | 0.401 | 0.556 | 0.043 | 0.352 | 0.402 | 0.246 | 0.456 | 0.508 | 0.036 |
| 12 | 0.121 | 0.659 | 0.220 | 0.075 | 0.406 | 0.519 | 0.169 | 0.563 | 0.269 |
| $\infty$ | 0.082 | 0.691 | 0.227 | 0.124 | 0.424 | 0.453 | 0.063 | 0.621 | 0.317 |
| $s_{\text {max }}$ | 0.415 | 0.696 | 0.272 | 0.373 | 0.424 | 0.587 | 0.468 | 0.621 | 0.358 |
|  | [0.19, 0.61] | [0.34, 0.94] | [0.04, 0.73 ] | [0.21, 0.63] | [0.16, 0.85] | [0.16, 0.92] | [0.24, 0.62] | [0.33, 0.95] | [0.07, 0.81 ] |
|  | Fraction variation in $U_{F}$ |  |  | Fraction variation in $U_{F}$ |  |  | Fraction variation in $U_{F}$ |  |  |
| $s$ | $U_{M}$ Shock | ip Shock | $U_{F}$ Shock | $U_{M}$ Shock | emp Shock | $U_{F}$ Shock | $U_{M}$ Shock | $Q_{1}$ Shock | $U_{F}$ Shock |
| 1 | 0.029 | 0.023 | 0.948 | 0.140 | 0.119 | 0.743 | 0.019 | 0.022 | 0.959 |
| 12 | 0.080 | 0.041 | 0.878 | 0.243 | 0.133 | 0.624 | 0.082 | 0.039 | 0.879 |
| $\infty$ | 0.121 | 0.131 | 0.748 | 0.332 | 0.138 | 0.530 | 0.156 | 0.098 | 0.746 |
| $s_{\text {max }}$ | 0.128 | 0.131 | 0.950 | 0.339 | 0.152 | 0.744 | 0.163 | 0.098 | 0.961 |
|  | [0.03, 0.47] | [0.05, 0.52] | [0.53, 0.99] | [0.08, 0.64] | [0.03, 0.58] | [0.33, 0.95] | [0.03, 0.53] | [0.03, 0.48] | [0.60, 0.99] |

Note: Each panel shows the fraction of s-step-ahead forecast-error variance of the variable given in the panel title that is explained by the shock named in the column heading. The row denoted " $s=s_{\text {max }}$ "reports the maximum fraction (across all VAR forecast horizons $m$ ) of forecast error variance explained by the shock listed in the column heading. The numbers in parentheses represent the 5th and 95th percentiles of these statistics from bootstrapped samples. The data are monthly and span the period 1960:07 to 2015:04.

## IRF for SVAR $\left(U_{M}, i p, U_{F}\right)^{\prime}$ using Baa



Note: $Z_{1}$ is created by using Baa and $Z_{2}$ is generated by using CRSP excess returns. The correlation $\rho\left(Z_{1 t}, \hat{e}_{M t}\right)=0.1988$, $\rho\left(Z_{1 t}, \hat{e}_{F t}\right)=0.1219, \rho\left(Z_{2 t}, \hat{e}_{F t}\right)=-0.1617$ and $\rho\left(Z_{1 t}, Z_{2 t}\right)=-0.20$. The sample is from 1960:07 to 2015:04.

## $\operatorname{IRF}$ for $\operatorname{SVAR}\left(U_{M}, i p, U_{F}\right)^{\prime}$ using noi for $Z_{1}$



Note: $Z_{1}$ is created by using noi and $Z_{2}$ is generated by using CRSP excess returns. The correlation $\rho\left(Z_{1 t}, \hat{e}_{M t}\right)=0.1799$, $\rho\left(Z_{1 t}, \hat{e}_{F t}\right)=-0.0301, \rho\left(Z_{2 t}, \hat{e}_{F t}\right)=-0.1617$ and $\rho\left(Z_{1 t}, Z_{2 t}\right)=0.1612$. One lag of noi is included. The sample is from $1960: 07$ to 2015:04.

## IRF for $\operatorname{SVAR}\left(U_{M}, i p, U_{F}\right)^{\prime}$ using noi for $Z_{2}$



Note: $Z_{1}$ is generated by using CRSP excess returns and $Z_{2}$ is created by using noi. The correlation $\rho\left(Z_{1 t}, \hat{e}_{M t}\right)=-0.1679$, $\rho\left(Z_{1 t}, \hat{e}_{F t}\right)=-0.0702, \rho\left(Z_{2 t}, \hat{e}_{F t}\right)=-0.1536$ and $\rho\left(Z_{1 t}, Z_{2 t}\right)=0.1503$. One lag of noi is included. The sample is from 1960:07 to 2015:04.

## IRF for SVAR $\left(U_{M}, i p, U_{F}\right)^{\prime}$ using $r^{\text {small }}$ Index for $Z_{1}$



Note: $Z_{1}$ is created by using $r^{\text {small }}$ index and $Z_{2}$ is generated by using CRSP excess return. The correlation $\rho\left(Z_{1 t}, \hat{e}_{M t}\right)=-0.0667$, $\rho\left(Z_{1 t}, \hat{e}_{F t}\right)=-0.1840, \rho\left(Z_{2 t}, \hat{e}_{F t}\right)=-0.1617$ and $\rho\left(Z_{1 t}, Z_{2 t}\right)=0.7868$. One lag of $r^{\text {small }}$ is included. The sample is from 1960:07 to 2015:04.

Uncertainty and Business Cycles

## Recursive Identification with Order $\left(U_{F}, U_{M}, i p\right)^{\prime}$

- Under any ordering, $U_{M t}$ shocks, like $U_{F t}$, appear to decrease $Y_{t}$.








Note: Bootstrapped 90 percent error bands appear as dashed lines. Responses to one standard deviation shocks are reported. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

## Recursive Identification with Order $\left(U_{F}, U_{M}, i p\right)^{\prime}$

- Inspection of $\hat{\mathbf{A}}_{0}$ reveals non-zero contemporaneous correlations $\rho\left(U_{M}, Y\right), \rho\left(U_{F}, Y\right)$, inconsistent with any recursive ordering.

$$
\hat{\mathbf{A}}_{0}=\left(\begin{array}{rrr}
1 & \mathbf{1 . 5 2 3 3} & -0.0206 \\
& {[0.2110]} & {[0.0583]} \\
-\mathbf{0 . 7 3 2 1} & 1 & 0.1328 \\
{[0.1563]} & & {[0.0702]} \\
-0.0049 & -\mathbf{1 . 1 6 7 6} & 1 \\
{[0.6933]} & {[0.5902]} &
\end{array}\right)
$$

## IRF for SVAR $\left(U_{M}, i p\right)^{\prime}$



Note: Bootstrapped 90 percent error bands appear as dashed lines. Responses to one standard deviation shocks are reported. Response units are reported in percentage points. $Z_{1}$ is generated by using CRSP excess returns. The sample is from 1960:07 to 2015:04.

## Variance Decomposition for $\left(U_{R}, Y, U_{F}\right)^{\prime}$

|  | SVAR $\left(U_{R}, i p, U_{F}\right)^{\prime}$ |  |  | $\operatorname{SVAR}\left(U_{R}, e m p, U_{F}\right)^{\prime}$ |  |  | $\operatorname{SVAR}\left(U_{R}, Q_{1}, U_{F}\right)^{\prime}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Fraction variation in $U_{R}$ |  |  | Fraction variation in $U_{R}$ |  |  | Fraction variation in $U_{R}$ |  |  |
| $s$ | $U_{R}$ Shock | ip Shock | $U_{F}$ Shock | $U_{R}$ Shock | emp Shock | $U_{F}$ Shock | $U_{R}$ Shock | $Q_{1}$ Shock | $U_{F}$ Shock |
| $s=1$ | 0.359 | 0.513 | 0.128 | 0.483 | 0.405 | 0.112 | 0.391 | 0.482 | 0.127 |
| $s=12$ | 0.253 | 0.463 | 0.285 | 0.409 | 0.292 | 0.299 | 0.263 | 0.440 | 0.297 |
| $s=\infty$ | 0.302 | 0.407 | 0.291 | 0.419 | 0.263 | 0.318 | 0.327 | 0.379 | 0.294 |
| $s=s_{\text {max }}$ | 0.302 | 0.407 | 0.291 | 0.519 | 0.405 | 0.318 | 0.437 | 0.515 | 0.305 |
|  | [0.16,0.72] | [0.18, 0.80 ] | [0.07,0.63] | [0.23, 0.80 ] | [0.13, 0.69 ] | [0.07, 0.62] | [0.19, 0.70] | [0.22,0.75] | [0.06, 0.62] |
|  | Fraction variation in $i p$ |  |  | Fraction variation in emp |  |  | Fraction variation in $Q_{1}$ |  |  |
| $s$ | $U_{R}$ Shock | ip Shock | $U_{F}$ Shock | $U_{R}$ Shock | emp Shock | $U_{F}$ Shock | $U_{R}$ Shock | $Q_{1}$ Shock | $U_{F}$ Shock |
| $s=1$ | 0.391 | 0.577 | 0.032 | 0.378 | 0.392 | 0.230 | 0.439 | 0.532 | 0.029 |
| $s=12$ | 0.295 | 0.456 | 0.249 | 0.220 | 0.217 | 0.563 | 0.362 | 0.371 | 0.267 |
| $s=\infty$ | 0.211 | 0.326 | 0.463 | 0.092 | 0.064 | 0.845 | 0.265 | 0.233 | 0.502 |
| $s=s_{\text {max }}$ | 0.397 | 0.580 | 0.463 | 0.392 | 0.395 | 0.845 | 0.442 | 0.534 | 0.502 |
|  | [0.10,0.73] | [0.22, 0.89 ] | [0.08, 0.84$]$ | [0.13,0.68] | [0.14, 0.74] | [0.32, 0.96 ] | [0.19, 0.72] | [0.27, 0.81 ] | [0.09, 0.87 ] |
|  | Fraction variation in $U_{F}$ |  |  | Fraction variation in $U_{F}$ |  |  | Fraction variation in $U_{F}$ |  |  |
| - | $U_{R}$ Shock | ip Shock | $U_{F}$ Shock | $U_{R}$ Shock | emp Shock | $U_{F}$ Shock | $U_{R}$ Shock | $Q_{1}$ Shock | $U_{F}$ Shock |
| $s=1$ | 0.010 | 0.059 | 0.941 | 0.050 | 0.182 | 0.768 | 0.001 | 0.055 | 0.944 |
| $s=12$ | 0.011 | 0.083 | 0.906 | 0.094 | 0.200 | 0.707 | 0.015 | 0.079 | 0.906 |
| $s=\infty$ | 0.117 | 0.093 | 0.790 | 0.214 | 0.167 | 0.619 | 0.150 | 0.082 | 0.768 |
| $s=s_{\text {max }}$ | 0.117 | 0.093 | 0.943 | 0.217 | 0.216 | 0.774 | 0.150 | 0.082 | 0.947 |
|  | [0.04, 0.35] | [0.03, 0.52 ] | [0.56, 0.99] | [0.06, 0.49] | [0.04, 0.64 ] | [0.37, 0.97] | [0.04, 0.39] | [0.02,0.53] | [0.59,0.99] |

Note: Each panel shows the fraction of $s$-step-ahead forecast-error variance of the variable given in the panel title that is explained by the shock named in the column heading. The row denoted " $s=s_{m a x}$ "reports the maximum fraction (across all VAR forecast horizons $m$ ) of forecast error variance explained by the shock listed in the column heading. The numbers in parentheses represent the 5th and 95th percentiles of these statistics from bootstrapped samples. The data are monthly and span the period 1960:07 to 2015:04.

## Recursive $\operatorname{IRF}\left(U_{F}, U_{M}, i p\right)^{\prime}$

- Recursive IRF $\left(U_{F}, U_{M}, i p\right)^{\prime}$


Note: Bootstrapped 90 percent error bands appear as dashed lines. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

## Time Series of Price Uncertainty

- Time Series of Price Uncertainty.


Positive $e_{\pi}$ exceeding 2 standard deviations


Positive $e_{\pi}^{x}$ exceeding 2 standard deviations


Note: The upper panel plots $U_{\pi}$ and $U_{\pi}^{x}$ where the latter excludes uncertainties for 5 volatile sub-series defined in the text, expressed in standardized units. The middle and lower panel exhibit shocks that are at least 2 standard deviations above the unconditional mean for $U_{\pi}$ and $U_{\pi}^{x}$. The shaded areas correspond to the NBER recession dates. The data are monthly and span the period 1960:07 to 2015:04.

## SVAR IRF $\left(U_{\pi}, e m p, U_{F}\right)^{\prime}$

- SVAR IRF $\left(U_{\pi}, e m p, U_{F}\right)^{\prime}$


Note: Bootstrapped 90 percent error bands appear as dashed lines. Responses to one standard deviation shocks are reported. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

## SVAR $\operatorname{IRF}\left(U_{M}(12), e m p, U_{F}(12)\right)^{\prime}$

- SVAR $\operatorname{IRF}\left(U_{M}(12), e m p, U_{F}(12)\right)^{\prime}$


Note: Bootstrapped 90 percent error bands appear as dashed lines. Responses to one standard deviation shocks are reported. Response units are reported in percentage points. The sample spans the period 1960:07 to 2015:04.

## Test of Recursive Restrictions, Real Uncertainty

- Test of Recursive Restrictions, Real Uncertainty

| Ordering: | $\left(U_{\left.R, i p, U_{F}\right)^{\prime}}\right.$ | $\left(U_{R}(12), i p, U_{F}(12)\right)^{\prime}$ |
| :---: | :---: | :---: |
| $H_{0}: B_{R Y}=B_{R F}=B_{Y F}=0$ | $\mathbf{1 3 3 . 6 9}$ | $\mathbf{3 0 3 . 2 4}$ |
|  | $[71.23]$ | $[77.88]$ |
| $H_{0}: B_{Y R}=B_{Y F}=B_{R F}=0$ | $\mathbf{2 9 . 1 1}$ | $\mathbf{1 6 7 . 5 7}$ |
|  | $[35.83]$ | $[52.54]$ |
| $H_{0}: B_{R Y}=B_{R F}=B_{F Y}=0$ | $\mathbf{1 3 0 . 4 1}$ | $\mathbf{3 0 6 . 3 4}$ |
|  | $[77.34]$ | $[72.79]$ |
| $\chi_{5 \%}^{2}(3)$ | 7.81 | 7.81 |
|  | $\left(U_{R}, \text { emp, } U_{F}\right)^{\prime}$ | $\left(U_{R}(12), \text { emp, } U_{F}(12)\right)^{\prime}$ |
| $H_{0}: B_{R Y}=B_{R F}=B_{Y F}=0$ | $\mathbf{1 7 8 . 6 8}$ | $\mathbf{3 2 7 . 9 1}$ |
|  | $[62.11]$ | $[76.35]$ |
| $H_{0}: B_{Y R}=B_{Y F}=B_{R F}=0$ | $\mathbf{8 5 . 5 8}$ | $\mathbf{2 4 4 . 8 5}$ |
|  | $[46.43]$ | $[67.50]$ |
| $H_{0}: B_{R Y}=B_{R F}=B_{F Y}=0$ | $\mathbf{1 5 4 . 7 6}$ | $\mathbf{3 1 0 . 6 6}$ |
|  | $[76.22]$ | $[78.04]$ |
| $\chi_{5 \%}^{2}(3)$ | 7.81 | 7.81 |

Note: The table reports the Wald test statistic for testing the null hypothesis given in the column. The bold indicates that Wald test rejects the null at 95 percent level according to $\chi^{2}(3)$ distribution. The SVAR system is solved using GMM and delta method is used for computing the standard error. Estimates of $\mathbf{B}$ are based on the SVAR identified with external instruments described in the text. The mean of bootstrap Wald statistics is reported in parenthesis. The sample size spans 1960:07 to 2015:04.

## SVAR IRF $\left(U_{M}(1), \text { emp, } U_{F}(1)\right)^{\prime}$ with 1987 Dummies

- SVAR IRF $\left(U_{M}(1) \text {, emp }, U_{F}(1)\right)^{\prime}$ using 1987 Crash Dummies


Note: The red line exhibits the 90 percent robust confidence set defined in the appendix. The sample spans the period 1962:07 to 2015:04.

## Pre-2008 SVAR IRF $\left(U_{M}(1), \text { emp, } U_{F}(1)\right)^{\prime}$

- Pre-2008 SVAR IRF $\left(U_{M}(1), e m p, U_{F}(1)\right)^{\prime}$


Note: The red line exhibits the 90 percent robust confidence set defined in the appendix. The sample spans the period 1962:07 to 2015:04.

## Monte Carlo Procedure

1 For each MC replication $i=1, \ldots, I$, draw $T \times 1$ vectors $\mathbf{e}_{F}^{(i)}, \mathbf{e}_{Y}^{(i)}, \mathbf{e}_{M}^{(i)}$ independently from $N(0,1)$.
2 Generate true data for $\left(U_{M}^{(i)}, Y^{(i)}, U_{F}^{(i)}\right)$ from the trivariate VAR

$$
\underbrace{\left(\begin{array}{ccc}
A_{M M}(0) & A_{M Y}(0) & A_{M F}(0) \\
A_{Y M}(0) & A_{Y Y}(0) & A_{Y F}(0) \\
A_{F M}(0) & A_{F Y}(0) & A_{F F}(0)
\end{array}\right)}_{\mathbf{A}_{0}}\left(\begin{array}{c}
U_{M}^{(i)} \\
Y_{t}^{(i)} \\
U_{F t}^{(i)}
\end{array}\right)=\underbrace{\left(\begin{array}{lll}
A_{M M}(1) & A_{M Y}(1) & A_{M F}(1) \\
A_{Y M}(1) & A_{Y Y}(1) & A_{Y F}(1) \\
A_{F M}(1) & A_{F Y}(1) & A_{F F}(1)
\end{array}\right)}_{\mathbf{A}_{1}}\left(\begin{array}{c}
U_{M t-1}^{(i)} \\
Y_{t-1}^{(i)} \\
U_{F t-1}^{(i)}
\end{array}\right)+\left(\begin{array}{c}
e_{M t}^{(i)} \\
e_{Y t}^{(i)} \\
e_{F t}^{(i)}
\end{array}\right)
$$

3 Generate data for $S_{1 t}$ and $S_{2 t}$ by drawing $T \times 1$ vectors $e_{S 1 t}^{(i)}, e_{S 2 t}^{(i)}$ independently from $N(0,1)$ distributions, where

$$
\begin{aligned}
& S_{1 t}^{(i)}=d_{10}+d_{11} S 1_{t-1}^{(i)}+d_{12} e_{M t}^{(i)}+d_{13} e_{Y t}^{(i)}+d_{14} e_{F t}^{(i)}+d_{15} e_{S 1 t}^{(i)}+d_{16} e_{S 2 t}^{(i)} \\
& S_{2 t}^{(i)}=d_{20}+d_{21} S_{2 t-1}^{(i)}+d_{22} e_{M t}^{(i)}+d_{23} e_{Y t}^{(i)}+d_{24} e_{F t}^{(i)}+d_{25} e_{S 1 t}^{(i)}
\end{aligned}
$$

4 Initialize $j=0$ and $\left(\hat{\mathbf{e}}_{Y}^{(i),[0]}, \hat{\mathbf{e}}_{M}^{(i),[0]}\right)^{\prime}=\left(Y^{(i)}, U_{M}^{(i)}\right)^{\prime}$.
4.1 Given $\left(\hat{\mathbf{e}}_{Y}^{(i),[j]}, \hat{\mathbf{e}}_{M}^{(i),[j]}\right)$, calculate the $\mathbf{Z}$ by running the following regressions.

$$
S_{1 t}^{(i)}=\beta_{1}^{\prime} x_{1 t}^{(i),[j]}+Z_{1 t}^{(i),[j]} \text { and } S_{2 t}^{(i)}=\beta_{2}^{\prime} x_{2 t}^{(i),[j]}+Z_{2 t}^{(i),[j]}
$$

where $x_{1 t}^{(i)}=\left(1, S 1_{t-1}^{(i)}, e_{Y}^{(i),[j]}\right)^{\prime}$ and $x_{2 t}^{(i)}=\left(1, S_{2 t-1}^{(i)}, e_{Y}^{(i),[j]}, e_{M}^{(i),[j]}\right)^{\prime}$,
4.2 Use $Z_{1}^{(i),[j]}$ and $Z_{2}^{(i),[j]}$ and estimates vech $\left(\hat{\boldsymbol{\eta}}_{t}^{(i)} \hat{\boldsymbol{\eta}}_{t}^{(i) \prime}\right)$ and $\operatorname{vec}\left(Z_{t}^{(i),[j]} \otimes \hat{\boldsymbol{\eta}}_{t}^{(i)}\right)$ to impose Assumption A of the paper and solve for $\mathbf{B}$. We obtain $\hat{e}_{Y}^{(i),[j+1]}, \hat{e}_{M}^{(i),[j+1]}, \hat{\boldsymbol{e}}_{F}^{(i),[j+1]}$ from $\hat{\mathbf{e}}^{(i),[j+1]}=\left(\mathbf{B}^{(i),[j]}\right)^{-1} \hat{\boldsymbol{\eta}}_{t}^{(i)}$
4.3 If $\left\|\hat{\mathbf{e}}^{(i),[j+1]}-\hat{\mathbf{e}}^{(i),[j]}\right\|<\epsilon$ (where $\epsilon$ is an arbitrarily small number), then set $\hat{\mathbf{e}}^{(i)}=\hat{\mathbf{e}}^{(i),[j]}$ and $\mathbf{Z}^{(i)}=\mathbf{Z}^{(i),[j]}$.

Otherwise, set $j=j+1$ and return to step 4.1.
5 Store $\hat{c}_{1}=\operatorname{corr}\left(\mathbf{Z}_{\mathbf{1 t}}{ }^{(i)}, \hat{\mathbf{e}}_{\mathbf{M t}}^{(i)}\right), \hat{c}_{2}=\operatorname{corr}\left(\mathbf{Z}_{\mathbf{1 t}}{ }^{(i)}, \hat{\mathbf{e}}_{\mathbf{F t}}^{(i)}\right), \hat{c}_{3}=\operatorname{corr}\left(\mathbf{Z}_{\mathbf{2} \mathbf{t}}{ }^{(i)}, \hat{\mathbf{e}}_{\mathbf{F t}}^{(i)}\right), C\left(\beta_{1}\right)=\frac{1}{3}\left(\left|\hat{c}_{1}\right|+\left|\hat{c}_{2}\right|+\left|\hat{c}_{3}\right|\right)$. Keep replication $i$ that satisfies (a) $C\left(\beta_{1}\right) \geq \bar{C}$, (b), each $\hat{c}_{i} \geq \bar{c}$, and (c) $\operatorname{det}\left(B^{(j)}\right) \geq \bar{b}$.

## Iterative Monte Carlo

$$
Y_{t}=A_{1} Y_{t-1}+H \Sigma e_{t}, B \equiv H \Sigma
$$

| True |  |  | Estimated |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | $=$ | $\left(\begin{array}{ccc}0.660 & -0.710 & 0.270 \\ 0.420 & 0.470 & -0.140 \\ 0.490 & 0.500 & 2.600\end{array}\right)_{\times 10^{-2}}$ | B | $=$ | $\left(\begin{array}{ccc}0.646 & -0.710 & 0.288 \\ 0.424 & 0.470 & -0.117 \\ 0.379 & 0.471 & 2.611\end{array}\right)$ | $)_{\times 10^{-2}}$ |
| $A_{1}$ |  | $\left(\begin{array}{ccc}0.996 & 0.027 & 0.010 \\ -0.023 & 0.983 & -0.002 \\ -0.045 & 0.040 & 0.978\end{array}\right)$ | $\hat{A}_{1}$ | $=$ | $\left(\begin{array}{ccc}0.996 & 0.029 & 0.010 \\ -0.023 & 0.983 & -0.002 \\ -0.046 & 0.041 & 0.978\end{array}\right)$ |  |
| $\operatorname{diag}(\boldsymbol{\Sigma})$ | $=$ | $[0.660,0.470,2.600]_{\times 10^{-2}}$ | $\operatorname{diag}(\hat{\mathbf{\Sigma}})$ | $=$ | $[0.646,0.470,2.611]_{\times 10^{-2}}$ |  |
| RMSE( ${ }_{\text {B }}$ ) |  | $\left(\begin{array}{lll}0.023 & 0.022 & 0.034 \\ 0.012 & 0.007 & 0.030 \\ 0.138 & 0.094 & 0.023\end{array}\right)_{\times 10^{-2}}$ | $\operatorname{RMSE}\left(\hat{A}_{1}\right)$ |  | $\left(\begin{array}{lll}0.001 & 0.001 & 0.001 \\ 0.000 & 0.001 & 0.000 \\ 0.002 & 0.003 & 0.002\end{array}\right)$ |  |
| $\operatorname{corr}\left(Z_{1 t}, e_{t}\right)$ | $=$ | [-0.077, 0.000, -0.118] | $\begin{aligned} & \operatorname{corr}\left(Z_{1 t}(\hat{\beta}), e_{t}\right) \\ & \operatorname{corr}\left(Z_{1 t}(\hat{\beta}), \hat{e}_{t}\right) \end{aligned}$ | $=$ | $\begin{aligned} & {[-0.073,0.000,-0.119]} \\ & {[-0.073,0.000,-0.124]} \end{aligned}$ |  |
| $\operatorname{corr}\left(Z_{2 t}, e_{t}\right)$ | $=$ | [0.000, 0.000, -0.166] | $\begin{aligned} & \operatorname{corr}\left(Z_{2 t}(\hat{\beta}), e_{t}\right) \\ & \operatorname{corr}\left(Z_{2 t}(\hat{\beta}), \hat{e}_{t}\right) \end{aligned}$ | $=$ | $\begin{aligned} & {[-0.002,0.002,-0.165]} \\ & {[0.000,0.000,-0.169]} \end{aligned}$ |  |
| $\operatorname{corr}\left(e_{t}, \hat{e}_{t}\right)$ |  | [0.995, 0.996, 0.995] |  |  |  |  |

Reported are the average of estimates over 5000 replications. IPIV initial guess: $\left(\mathbf{e}_{1}^{[i](0)}, \mathbf{e}_{2}^{[i](0)}\right)^{\prime}=\left(\mathbf{X}_{1}^{[i]}, \mathbf{X}_{2}^{[i]}\right)^{\prime}$. The sample size $T=500$.

## Returns Uncertainty with and without Jumps

- Returns with and without Jumps

$$
\begin{gathered}
\hline \text { Model without Jump } \\
r_{t}^{N J} \sim N\left(\kappa_{1}, \kappa_{2}\right)
\end{gathered}
$$

$$
\begin{gathered}
\hline \hline \text { Model with Jump } \\
r_{t}^{J}=w_{t}+z_{t} \\
w_{t} \sim N\left(\mu, \sigma^{2}\right) \\
z_{t} \mid j \sim N\left(j \theta, j \delta^{2}\right) \\
j \sim \text { Poission }(\omega) \\
E\left(r_{t}^{J}\right)=\kappa_{1}=\mu+\omega \theta \\
\operatorname{Var}\left(r_{t}^{J}\right)=\kappa_{2}=\sigma^{2}+\omega\left(\theta^{2}+\delta^{2}\right)
\end{gathered}
$$

$\quad E\left(\mathcal{U}_{t}^{r}\right)$
$\sqrt{\operatorname{Var}\left(\mathcal{U}_{t}^{r}\right)}$
Skewness
Kurtosis

$$
\begin{gathered}
0.0335 \\
6.62 \times 10^{-5} \\
1.3831 \\
6.6707
\end{gathered}
$$

$\sqrt{\operatorname{Var}\left(\mathcal{U}_{t}^{r}\right)}$
Skewness Kurtosis
Number > 3 std
Note: The table reports the mean, standard deviation, skewness and kurtosis of the uncertainty measure of returns with and without jumps. The model is specified in each column and both model has the same unconditional mean $\kappa_{1}$ and variance $\kappa_{2}$. We calibrate the mean $(\mu)$, volatility $(\sigma)$, jump intensity $(\omega)$, mean jump size $(\theta)$ and volatility of jumps ( $\delta$ ) according to the true distribution of the aggregate stock returns as in Table 2 in Backus, Chernov and Martin (2011). Last row reports the number of samples that exceed 3 standard deviation above its mean. The monte carlo sample size is 20,000 .

## Measuring Uncertainty: Jurado, Ludvigson, Ng (JLN)

- Methodology: DI forecasting plus stochastic volatility model
- Let $y_{j t}^{C} \in Y_{t}^{C}=\left(y_{1 t}^{C}, \ldots, y_{N_{C} t}^{C}\right)^{\prime}$ be a variable in category $C$. JLN estimate its $h$-period ahead uncertainty, $\mathcal{U}_{j t}^{C}(h)$, defined

$$
\mathcal{U}_{j t}^{C}(h) \equiv \sqrt{\mathbb{E}\left[\left(y_{j t+h}^{C}-\mathbb{E}\left[y_{j t+h}^{C} \mid I_{t}\right]\right)^{2} \mid I_{t}\right]}
$$

- Aggregate uncertainty in category $C$ :

$$
\mathcal{U}_{C t}(h) \equiv \operatorname{plim}_{N_{C} \rightarrow \infty} \sum_{j=1}^{N_{C}} \frac{1}{N_{C}} \mathcal{U}_{j t}^{C}(h) \equiv \mathbb{E}_{C}\left[\mathcal{U}_{j t}^{C}(h)\right]
$$

- Focus on $h=1$ month-ahead uncertainty in three categories:

| Category $(C)$ | $Y_{t}^{C}$ | $N_{C}$ |
| :--- | :--- | :--- |
| $(\mathrm{M}):$ Macro | all variables in $\chi^{M}(\mathrm{JLN})$ | 134 |
| (F): Financial | all variables in $\chi^{F}($ new $)$ | 147 |
| (R): Real activity | real activity variables in $\chi^{M}$ (new) | 73 |

## Econometric Model

- For each $y_{j t}, j=1, \ldots, N_{y}$, we specify:

$$
\begin{aligned}
y_{j, t+1} & =\underbrace{E\left[y_{j t+1} \mid I_{t}\right]}_{\text {forecastable }}+\underbrace{v_{j, t+1}^{y}}_{\text {unforecastable }} \\
v_{j, t+1}^{y} & =\underbrace{\sigma_{j, t+1}^{y}}_{\text {stochastic vol }} \varepsilon_{j, t+1}^{y} \\
\log \left[\left(\sigma_{j, t+1}^{y}\right)^{2}\right] & =\alpha_{j}^{y}+\beta_{j}^{y} \log \left(\sigma_{j t}^{y}\right)^{2}+\tau_{j}^{y} \eta_{j, t+1},
\end{aligned}
$$

where $\varepsilon_{j, t+1}$ and $\eta_{j, t+1}$ are iid $N(0,1)$ random variables.

## Econometric Model

- For each $y_{j t}, j=1, \ldots, N_{y}$, we specify:

$$
\begin{aligned}
y_{j, t+1} & =\underbrace{E\left[y_{j t+1} \mid I_{t}\right]}_{\text {forecastable }}+\underbrace{v_{j, t+1}^{y}}_{\text {unforecastable }} \\
v_{j, t+1}^{y} & =\underbrace{\sigma_{j, t+1}^{y}}_{\text {stochastic vol }} \varepsilon_{j, t+1}^{y} \\
\log \left[\left(\sigma_{j, t+1}^{y}\right)^{2}\right] & =\alpha_{j}^{y}+\beta_{j}^{y} \log \left(\sigma_{j t}^{y}\right)^{2}+\tau_{j}^{y} \eta_{j, t+1},
\end{aligned}
$$

where $\varepsilon_{j, t+1}$ and $\eta_{j, t+1}$ are iid $N(0,1)$ random variables.

- Estimation:
(1) $\hat{E}\left[y_{j t+1} \mid I_{t}\right]$ using diffusion index forecasts.
(2) $\log \left(\widehat{\sigma}_{j t}^{y}\right)^{2}$ stochastic volatility estimates, improved version of Kim, Shephard, and Chib (1998, RES) algorithm.


## Stochastic Volatility Estimates

- From the model, $v_{j, t+1}^{y}=\sigma_{j, t+1}^{y} \varepsilon_{j, t+1}^{y}$. Take logs:

$$
\begin{aligned}
& \log \left[\left(v_{j, t+1}^{y}\right)^{2}\right]=\log \left[\left(\sigma_{j, t+1}^{y}\right)^{2}\right]+\log \left[\left(\varepsilon_{j, t+1}^{y}\right)^{2}\right] \\
& \log \left[\left(\sigma_{j, t+1}^{y}\right)^{2}\right]=\alpha_{j}^{y}+\beta_{j}^{y} \log \left[\left(\sigma_{j t}^{y}\right)^{2}\right]+\left(\tau_{j}^{y}\right) \eta_{j, t+1} .
\end{aligned}
$$

- Has the state-space representation

$$
\begin{aligned}
& z_{j t}=x_{j t}+\epsilon_{j t} \quad \text { observation equation } \\
& x_{j t}=\alpha_{j}+\beta_{j} x_{j t-1}+\tau_{j} \eta_{j t} \quad \text { state equation }
\end{aligned}
$$

- Difficulty: $\epsilon_{j, t} \equiv \log \left(\varepsilon_{j, t}^{y}\right)^{2} \sim \log \chi^{2}(1)$.
- Solution: Kim, Shephard, and Chib (1998, RES) MCMC mixture of normals approximation:

$$
p(\epsilon)=\sum_{k=1}^{K} \pi_{k} \phi\left(\epsilon ; m_{k}, s_{k}^{2}\right) .
$$

- Interweaving: Kastner-Fruhwrith-Schnattner (2013).


## Computing Individual Uncertainty $(h=1)$

- Using definition of forecast variance :

$$
\begin{aligned}
\Omega_{j t}^{y}(1) & =E\left[\left(\sigma_{j, t+1}^{y}\right)^{2}\left(\varepsilon_{j, t+1}^{y}\right)^{2} \mid I_{t}\right] \\
& =E\left[\left(\sigma_{j, t+1}^{y}\right)^{2} \mid I_{t}\right] \\
& =\exp \left\{\alpha_{j}^{y}+\beta_{j}^{y} \log \left(\sigma_{j t}^{y}\right)^{2}+\frac{1}{2}\left(\tau_{j}^{y}\right)^{2}\right\} .
\end{aligned}
$$

- The last equality follows from the $\operatorname{AR}(1)$ law of motion for $\log \left(\sigma_{j, t+1}^{y}\right)^{2}$, and the normality of $\eta_{j, t+1}$.
- Given estimates: $\hat{\alpha}_{j}^{y}, \hat{\beta}_{j}^{y},\left(\hat{\tau}_{j}^{y}\right)^{2}$, and $\left\{\widehat{\log }\left(\sigma_{j t}^{y}\right)^{2}\right\}_{t=1}^{T}$, compute $\hat{\Omega}_{j t}^{y}(1)$ using this expression.


## Computing Individual Uncertainty ( $h \geq 1$ )

- Define $q=\max \left(\right.$ lags $_{y}$, lags $_{F}$, lags $\left._{w}, h\right)$
- Let $\mathcal{Z}_{t} \equiv\left(\hat{F}_{t}^{\prime}, W_{t}^{\prime}\right)^{\prime}$ and define $\mathcal{F}_{t} \equiv\left(\mathcal{Z}_{t}, \ldots, \mathcal{Z}_{t-q+1}\right)^{\prime}$ and $Y_{j t} \equiv\left(y_{j t}, \ldots, y_{j, t-q+1}\right)^{\prime}:$

$$
\begin{aligned}
\binom{\mathcal{F}_{t}}{Y_{j t}} & =\left(\begin{array}{cc}
\Phi^{F} & 0 \\
\Lambda_{j}^{\prime} & \Phi_{j}^{Y}
\end{array}\right)\binom{\mathcal{F}_{t-1}}{Y_{j, t-1}}+\binom{V_{t}^{\mathcal{F}}}{V_{j t}^{Y}} \\
\mathcal{Y}_{j t} & =\Phi_{j}^{\mathcal{Y}} \mathcal{Y}_{j, t-1}+V_{j t}^{\mathcal{Y}}
\end{aligned}
$$

## Computing Individual Uncertainty $(h \geq 1)$

- Define $q=\max \left(\operatorname{lags}_{y}\right.$, lags $_{F}$, lags $\left._{w}, h\right)$
- Let $\mathcal{Z}_{t} \equiv\left(\hat{F}_{t}^{\prime}, W_{t}^{\prime}\right)^{\prime}$ and define $\mathcal{F}_{t} \equiv\left(\mathcal{Z}_{t}, \ldots, \mathcal{Z}_{t-q+1}\right)^{\prime}$ and $Y_{j t} \equiv\left(y_{j t}, \ldots, y_{j, t-q+1}\right)^{\prime}:$

$$
\begin{aligned}
\binom{\mathcal{F}_{t}}{Y_{j t}} & =\left(\begin{array}{cc}
\Phi^{F} & 0 \\
\Lambda_{j}^{\prime} & \Phi_{j}^{Y}
\end{array}\right)\binom{\mathcal{F}_{t-1}}{Y_{j, t-1}}+\binom{V_{t}^{\mathcal{F}}}{V_{j t}^{Y}} \\
\mathcal{Y}_{j t} & =\Phi_{j}^{\mathcal{Y}} \mathcal{Y}_{j, t-1}+V_{j t}^{\mathcal{Y}} .
\end{aligned}
$$

- Forecast Error Variance $\Omega_{j t}^{\mathcal{Y}}(h) \equiv E_{t}\left[\left(\mathcal{Y}_{j, t+h}-E_{t}\left[\mathcal{Y}_{j, t+h}\right]\right)^{2}\right]$. The following recursion holds (with $\Omega_{j t}^{\mathcal{Y}}(0) \equiv 0$ ):

$$
\Omega_{j t}^{\mathcal{Y}}(h)=\Phi_{j}^{\mathcal{Y}} \Omega_{j t}^{\mathcal{Y}}(h-1) \Phi_{j}^{\mathcal{Y} \prime}+E_{t}\left[V_{j, t+h}^{\mathcal{Y}} V_{j, t+h}^{\mathcal{Y} \prime}\right],
$$

- Then $h$-period ahead uncertainty in $y_{j t}$ is

$$
\mathcal{U}_{j t}^{y}(h)=\sqrt{1_{j}^{\prime} \mathcal{U}_{j t}^{\mathcal{Y}}(h) 1_{j}} .
$$

$1_{i}$ a selection vector picks out the element for uncertainty in $y_{i, t}$.

## Sources of Uncertainty

- Forecast error variance is not equal to stochastic volatility in residuals $v_{j t}^{y}$ unless $h=1$.

$$
\begin{aligned}
\Omega_{j t}^{Y}(h)= & \underbrace{\Phi_{j}^{Y} \Omega_{j t}^{Y}(h-1) \Phi_{j}^{Y \prime}}_{\text {autoregressive }}+\underbrace{\Omega_{j t}^{Z}(h-1)}_{\text {Factor }}+\underbrace{E_{t}\left(V_{j t+h}^{Y} V_{j t+h}^{Y \prime}\right)}_{\text {stochastic volatility } Y} \\
& +\underbrace{2 \Phi_{j}^{Y} \Omega_{j t}^{Y Z}(h-1)}_{\text {covariance }}
\end{aligned}
$$

## Sources of Uncertainty

- Forecast error variance is not equal to stochastic volatility in residuals $v_{j t}^{y}$ unless $h=1$.

$$
\begin{aligned}
\Omega_{j t}^{Y}(h)= & \underbrace{\Phi_{j}^{Y} \Omega_{j t}^{Y}(h-1) \Phi_{j}^{Y \prime}}_{\text {autoregressive }}+\underbrace{\Omega_{j t}^{Z}(h-1)}_{\text {Factor }}+\underbrace{E_{t}\left(V_{j t+h}^{Y} V_{j t+h}^{Y \prime}\right)}_{\text {stochastic volatility } Y} \\
& +\underbrace{2 \Phi_{j}^{Y} \Omega_{j t}^{Y Z}(h-1)}_{\text {covariance }}
\end{aligned}
$$

- Autoregressive component when $h>1$


## Sources of Uncertainty

- Forecast error variance is not equal to stochastic volatility in residuals $v_{j t}^{y}$ unless $h=1$.

$$
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\end{aligned}
$$

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- Predictor Uncertainty: error in forecasting $F_{t}$ and $W_{t}$ contribute to uncertainty when $h>1$


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\end{aligned}
$$

- Autoregressive component when $h>1$
- Predictor Uncertainty: error in forecasting $F_{t}$ and $W_{t}$ contribute to uncertainty when $h>1$
- Covariance component: $\operatorname{cov}\left(y_{t+h}-y_{t+h \mid t}, F_{t+h}-F_{t+h \mid t}\right)$, non-zero when $h>2$.


## Variance Decomposition with $U_{M}(12)$ and $U_{F}(12)$

- Variance Decomposition with $U_{M}(12)$ and $U_{F}(12)$

|  | SVAR $\left(U_{M}(12), i p, U_{F}(12)\right)^{\prime}$ |  |  | SVAR $\left(U_{M}(12), e m p, U_{F}(12)\right)^{\prime}$ |  |  | SVAR $\left(U_{M}(12), Q_{1}, U_{F}(12)\right)^{\prime}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Fraction variation in $U_{M}(12)$ |  |  | Fraction variation in $U_{M}(12)$ |  |  | Fraction variation in $U_{M}(12)$ |  |  |
| $s$ | $U_{M}(12)$ Shock | ip Shock | $U_{F}(12)$ Shock | $U_{M}(12)$ Shock | emp Shock | $U_{F}(12)$ Shock | $U_{M}(12)$ Shock | $Q_{1}$ Shock | $U_{F}(12)$ Shock |
| 1 | 0.548 | 0.432 | 0.020 | 0.621 | 0.360 | 0.019 | 0.590 | 0.381 | 0.029 |
| 12 | 0.763 | 0.219 | 0.018 | 0.776 | 0.212 | 0.012 | 0.801 | 0.168 | 0.031 |
| $\infty$ | 0.635 | 0.206 | 0.159 | 0.682 | 0.135 | 0.183 | 0.692 | 0.202 | 0.106 |
| $s_{\text {max }}$ | 0.813 | 0.432 | 0.165 | 0.682 | 0.135 | 0.183 | 0.868 | 0.388 | 0.107 |
| max | [0.48, 0.94 ] | [0.17,0.66] | [0.06, 0.51 ] | [0.37,0.96] | [0.10, 0.62] | [0.09, 0.52] | [0.48,0.95] | [0.17, 0.61] | [0.04, 0.49] |
|  | Fraction variation in ip |  |  | Fraction variation in emp |  |  | Fraction variation in $Q_{1}$ |  |  |
| $s$ | $U_{M}(12)$ Shock | ip Shock | $U_{F}(12)$ Shock | $U_{M}(12)$ Shock | emp Shock | $U_{F}(12)$ Shock | $U_{M}(12)$ Shock | $Q_{1}$ Shock | $U_{F}(12)$ Shock |
| 1 | 0.379 | 0.591 | 0.030 | 0.342 | 0.355 | 0.303 | 0.384 | 0.602 | 0.014 |
| 12 | 0.124 | 0.757 | 0.119 | 0.076 | 0.433 | 0.491 | 0.099 | 0.748 | 0.154 |
| $\infty$ | 0.202 | 0.697 | 0.101 | 0.269 | 0.482 | 0.250 | 0.256 | 0.623 | 0.121 |
| $s_{\text {max }}$ | $0.382$ | $0.772$ | $0.145$ | $0.342$ | $0.482$ | $0.519$ | $0.388$ | $0.751$ | $0.210$ |
|  | $[0.20,0.71]$ | [0.42,0.93] | $[0.04,0.59]$ | $[0.23,0.76]$ | $[0.17,0.86]$ | $[0.18,0.88]$ | $[0.23,0.75]$ | [0.41, 0.96] | $[0.05,0.66]$ |
|  | Fraction variation in $U_{F}(12)$ |  |  | Fraction variation in $U_{F}(12)$ |  |  | Fraction variation in $U_{F}(12)$ |  |  |
| $s$ | $U_{M}(12)$ Shock | ip Shock | $U_{F}(12)$ Shock | $U_{M}(12)$ Shock | emp Shock | $U_{F}(12)$ Shock | $U_{M}(12)$ Shock | $Q_{1}$ Shock | $U_{F}(12)$ Shock |
| 1 | 0.091 | 0.002 | 0.907 | 0.273 | 0.090 | 0.637 | 0.059 | 0.001 | 0.940 |
| 12 | 0.165 | 0.017 | 0.819 | 0.389 | 0.108 | 0.503 | 0.127 | 0.016 | 0.858 |
| $\infty$ | 0.200 | 0.162 | 0.638 | 0.448 | 0.165 | 0.387 | 0.178 | 0.151 | 0.671 |
| $s_{\text {max }}$ | 0.206 | 0.162 | 0.907 | 0.464 | 0.165 | 0.637 | 0.178 | 0.151 | 0.945 |
|  | [0.04, 0.71] | [0.05, 0.46] | [0.37,0.99] | [0.09, 0.76] | [0.04, 0.59] | [0.20,0.94] | [0.04, 0.69] | [0.05, 0.48] | [0.40, 0.99] |

Note: Each panel shows the fraction of $s$-step-ahead forecast-error variance of the variable given in the panel title that is explained by the shock named in the column heading. The row denoted " $s=s_{\text {max }}$ "reports the maximum fraction (across all VAR forecast horizons $m$ ) of forecast error variance explained by the shock listed in the column heading. The numbers in parentheses represent the 5th and 95th percentiles of these statistics from bootstrapped samples. The data are monthly and span the period 1960:07 to 2015:04.

## IRF for SVAR $\left(U_{M}, i p, U_{F}\right)^{\prime}$ using $V X O$ in $Z_{1}$



Note: Bootstrapped 90 percent error bands appear as dashed lines. Responses to one standard deviation shocks are reported. Response units are reported in percentage points. $Z_{1}$ is created by using $V X O$ and $Z_{2}$ is generated by using $r_{p}, \alpha=0.94$. The correlation $\rho\left(Z_{1 t}, \hat{e}_{M t}\right)=0.1650, \rho\left(Z_{1 t}, \hat{e}_{F t}\right)=0.1299$ and $\rho\left(Z_{2 t}, \hat{e}_{F t}\right)=-0.1662$. The sample is from 1962:07 to 2015:04.

## IRF for $\operatorname{SVAR}\left(U_{M}, i p, U_{F}\right)^{\prime}$ using $V X O$ in $Z_{2}$



Note: Bootstrapped 90 percent error bands appear as dashed lines. Responses to one standard deviation shocks are reported. Response units are reported in percentage points. $Z_{1}$ is generated by using $r_{p}, \alpha=0.94$ and $Z_{2}$ is created by using $V X O$. The correlation $\rho\left(Z_{1 t}, \hat{e}_{M t}\right)=-0.1115, \rho\left(Z_{1 t}, \hat{e}_{F t}\right)=-0.1491$ and $\rho\left(Z_{2 t}, \hat{e}_{F t}\right)=0.1969$. The sample is from 1962:07 to 2015:04.

## 6 Survived Solutions for System $\left(U_{M}, i p, U_{F}\right)^{\prime}$

- 6 Survived Solutions for System $\left(U_{M}, i p, U_{F}\right)^{\prime}$


## Panel A: Summary of Results from 6 Solutions

|  | Summary Statistics of $\hat{e}=\left(e_{M,}, e_{Y}, e_{F}\right)$ |  |  | Instrument Relevance |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case | Skewness | Kurtosis |  | $\hat{c}_{1}$ | $\hat{c}_{2}$ | $\hat{c}_{3}$ | $C$ |
| Baseline | $(0.48,-0.42,-1.70)$ | $(5.87,6.05,19.65)$ |  | -0.07 | -0.17 | -0.16 | 0.134 |
| Sol \#1 | $(0.33,-0.46,-1.82)$ | $(5.71,6.54,20.68)$ |  | -0.07 | -0.17 | -0.16 | 0.135 |
| Sol \#2 | $(0.50,-0.43,-1.77)$ | $(5.88,6.28,20.08)$ |  | -0.08 | -0.17 | -0.16 | 0.135 |
| Sol \#3 | $(0.36,-0.46,-1.81)$ | $(5.74,6.49,20.59)$ |  | -0.07 | -0.17 | -0.16 | 0.135 |
| Sol \#4 | $(0.32,-0.45,-1.78)$ | $(5.69,6.39,20.41)$ |  | -0.07 | -0.17 | -0.16 | 0.134 |
| Sol \#5 | $(0.30,-0.44,-1.74)$ | $(5.66,6.37,19.86)$ |  | -0.08 | -0.17 | -0.15 | 0.134 |

Panel B: Correlation Matrix of $\hat{e}$
$\left[\begin{array}{ccccccc}1 & 0.991 & 0.998 & 0.994 & 0.989 & 0.987 \\ & 1 & 0.985 & 0.999 & 0.999 & 0.999 \\ & & 1 & 0.988 & 0.982 & 0.983 \\ & & & 1 & 0.999 & 0.998 \\ & & & & 1 & 0.998 \\ & & & & & 1\end{array}\right) \quad\left(\begin{array}{cccccccc}1 & 0.985 & 0.995 & 0.988 & 0.987 & 0.987 \\ & 1 & 0.986 & 0.999 & 0.999 & 0.998 \\ & & & 1 & 0.990 & 0.984 & 0.982 \\ & & & & & 1 & 0.999 & 0.998 \\ 1 & 0.993 & 0.994 & 0.995 & 0.998 & 0.996 \\ & 1 & 0.998 & 0.999 & 0.999 & 0.997 \\ & & 1 & 0.999 & 0.997 & 0.999 \\ & & & 1 & 0.999 & 0.998 \\ & & & 1 & 0.998\end{array}\right)$

Note: Panel A reports the skewness and kurtosis of $\hat{e}$ and instrument relevance for 6 survived solutions in system $\left(U_{M}, i p, U_{F}\right)^{\prime}$. Panel B reports the matrix of correlation $\hat{e}$ across 6 solutions. The monthly data span the period 1960:07 to 2015:04.

