Behavioral Macroeconomics Via Sparse Dynamic Programming

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INTRODUCTION

- We economists use simplified models of the world
- Here I develop a model where agents do the same
- Language: "sparsity"
  - A vector $m \in \mathbb{R}^{1,000,000}$ is *sparse* if most entries are 0
  - The agent pays attention to few dimensions of the world (endogenously)
  - His attention vector is sparse
  - He has a low-dimensional (sparse) submodel of the world

Related Literature

- A lot of the behavioral literature is about modeling *tastes* or *beliefs*
- However, there's less on *bounded rationality*
  - O'Donoghue Rabin, $k-$level models, Koszegi and Szeidl, Schwartzstein, Fuster, Hébert and Laibson, Bordalo, Gennaioli and Shleifer...
- Finance / Macro:
  - Inattention: Sims 03, Gabaix and Laibson 02, 06, Mankiw Reis 02, Reis 06, Chetty, Kroft Looney 09, Angeletos La'O 10, Maćkowiak and Wiederholt 10, 16, Masatlioglu and Ok 10, Veldkamp 11, Matejka and Sims 11, Caplin, Dean and Martin 11, Woodford 12, Alvarez Lippi, Paciello 13, Koszegi Szeidl 13, Abel, Eberly and Panageas 13, Greenwood Hanson 14, Croce, Lettau Ludvigson 15
  - Early behavioral models: Campbell Mankiw 89
- I search a very tractable, widely applicable model

Static sparse max: Quick version

"A sparsity-based model of bounded rationality" (QJE 2014)

$$\operatorname*{smax}_{a} u(a,x) \text{ subject to } b(a,x) \geq 0$$

- Form attention-augmented decision utility:

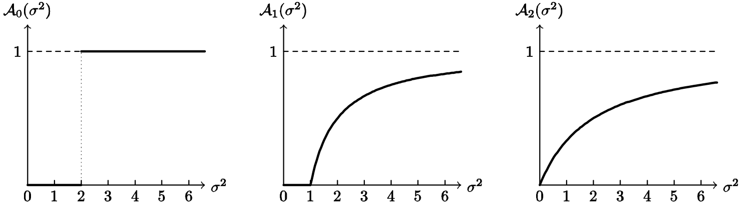
$$u(a,x,m) := u(a, m_1 x_1, ..., m_n x_n)$$

$$a(x,m) := \arg\max_{a} u(a,x,m)$$

- **Proposition**: *Action is* $a^s = a(x, m^*)$ *with optimal attention:*

$$m_i^* = \mathcal{A}_\alpha\left(-\mathbb{E}\left[a'_{m_i} u_{aa} a_{m_i}\right] / \kappa\right)$$

- κ =cognition cost



Application

- Quadratic example: $u(a,x) = -\frac{1}{2}\left(a - \sum_i b_i x_i\right)^2$,

$$
\begin{aligned}
a^r &= \sum_{i=1}^{10^6} b_i x_i : \text{Non-Sparse Action} \\
a^s &= \sum_i b_i m_i x_i : \text{Sparse action} \\
m_i &= \mathcal{A}\left(b_i^2 \sigma_{x_i}^2 / \kappa\right)
\end{aligned}
$$

- Conclusion: we have

$$
\underset{a;m}{\text{smax}}\, u(a,x,m) \text{ subject to } b(a,x,m) \geq 0
$$

  [I didn't explain in these slides how to handle the budget constraint]
- Can write a boundedly rational version of basic chapters of *microeconomics*: consumer theory, competitive equilibrium theory: BR consumer demand, Slutsky matrix, Roy's identity, Edgeworth boxes, BR Arrow-Debreu, welfare theorems...
- Now, let's examine *macroeconomics*.

Sparse Dynamic Programming

Motivating example: Permanent-income problem

- $\max_{(c_t)_{t\geq 0}} \mathbb{E} \sum_t \beta^t c_t^{1-\gamma} / (1-\gamma)$ s.t.:

$$w_{t+1} = (1 + \overline{r} + \widehat{r}_t)(w_t - c_t) + \overline{y} + \widehat{y}_t$$
$$\widehat{r}_{t+1} = \rho_r \widehat{r}_t + \varepsilon^r_{t+1}, \qquad \widehat{y}_{t+1} = \rho_y \widehat{y}_t + \varepsilon^y_{t+1}$$

- What's $c(z_t)$, $z_t := (w_t, \hat{r}_t, \hat{y}_t)$?
- Want to capture: people "do not think" about the interest $\hat{r}_t$.
- We anchor on the default model:

$$w_{t+1} = (1 + \overline{r})(w_t - c_t) + \overline{y}$$

with policy $c_t^d = \frac{\bar{r} w_t + \bar{y}}{R}$.

- Agent has a "simplifiable subjective model":

$$w_{t+1} = F^w\left(c_t, z_t, m\right) = \left(1 + \bar{r} + m_r \widehat{r}_t\right)\left(w_t - c_t\right) + \bar{y} + m_y \widehat{y}_t$$
$$\widehat{y}_{t+1} = F^y\left(c_t, z_t, m\right) = \rho_y \widehat{y}_t + m_{\sigma^y} \varepsilon^y_{t+1},$$

and same for $\widehat{r}_{t+1}$ as for $\widehat{y}_{t+1}$: $F^r := \rho_r \widehat{r}_t + m_{\sigma^r} \varepsilon^r_{t+1}$,

- Attention vector:

$$m = \left(m_y, m_{\sigma_y}, m_r, m_{\sigma_r}\right)$$

- Given true law of motion $z_{t+1} = F^z\left(a_t, z_t\right)$ mental model is:

$$z_{t+1} = F^z\left(a_t, z_t, m\right)$$

- Use (normalizing means to 0)

$$F^{z^i}\left(a, z, m\right) = F^{z^i}\left(a, m^i \odot z\right)$$

- Choose which variables are in the "default model" (so $m_k \equiv 1$): here, wealth.

Sparse Dynamic Programming: General Setup

- Rational problem:

$$V\left(z_0\right)=\max_{\left(a_t\right)_{t\geq 0}}\mathbb{E}\sum_{t=0}^{\infty}\beta^t u\left(a_t,z_t\right)\text{ s.t. }z_{t+1}=F^z\left(a_t,z_t\right)$$

- Take a proxy value function V^p, e.g. V^r or approx. of it.
- Sparse policy $a\left(z,V^p\right)$:

$$a\left(z,V^p\right):=\arg\operatorname*{smax}_{a;m|m^d}\left\{u\left(a,z,m\right)+\beta\mathbb{E}\left[V^p\left(F^z\left(a,z,m\right)\right)\right]\right\}$$

Permanent-Income Example: Solution

- With $c^d(w) = \frac{rw+\bar{y}}{R}$,

$$\ln c_t^{BR} = \ln c^d(w_t) + B_r m_r \widehat{r}_t + B_y m_y \widehat{y}_t + O\left(\|x\|^2\right)$$

$$m_x = \mathcal{A}\left(\sigma_x^2 B_x^2 / \bar{\kappa}^2\right)$$

for $x = r, x$, and closed forms for B_x

- Sensible comparative statics: if σ_r^2 increase, people pay more attention to r_t

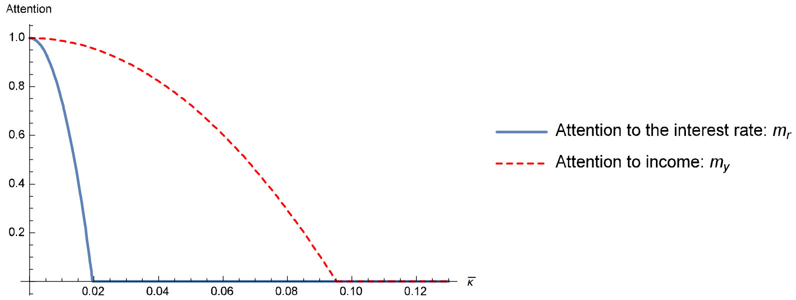
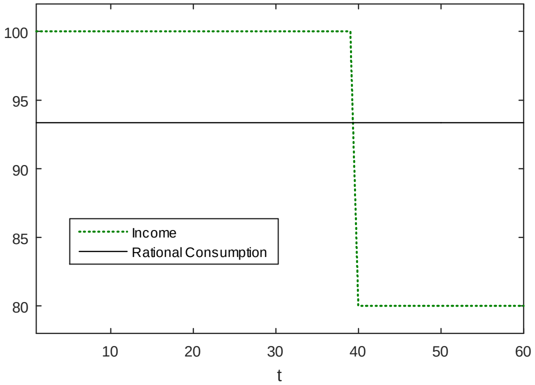


Figure:

LIFE-CYCLE: MODIGLIANI-BRUMBERG (1954)

- Agent works (income $\bar{y}$) for $t \in [0, L)$, and retires (income $y_t = \bar{y} + \widehat{y}$) for $t \in t \in [L, T)$
- No discounting:

$$\max_{(c_t)_{0 \leq t < T}} \sum_{t=0}^{T-1} u\left(c_t\right) \text{ s.t. } \sum_{t=0}^{T-1} c_t \leq \sum_{\tau=0}^{T-1} y_\tau$$



Life-cycle: Behavioral

- Rational agent: $c_t = \frac{w_t - x}{T-t} + \bar{y}$, with $x = -(T-L)\,\widehat{y} > 0 =$ loss of income at retirement

$$c_t = \arg\max_{c_t} v(c_t, w_t, x, t)$$

$$v(c_t, w_t, x, t) := u(c_t) + V^r(w_t + \bar{y} - c_t, x, t+1)$$

$$V^r(w_t, x, t) = (T-t)\, u\left(\frac{w_t - x}{T-t} + \bar{y}\right)$$

- Behavioral agent:

$$c_t = \arg\operatorname*{smax}_{c_t; m_t} v(c_t, w_t, m_t x, t)$$

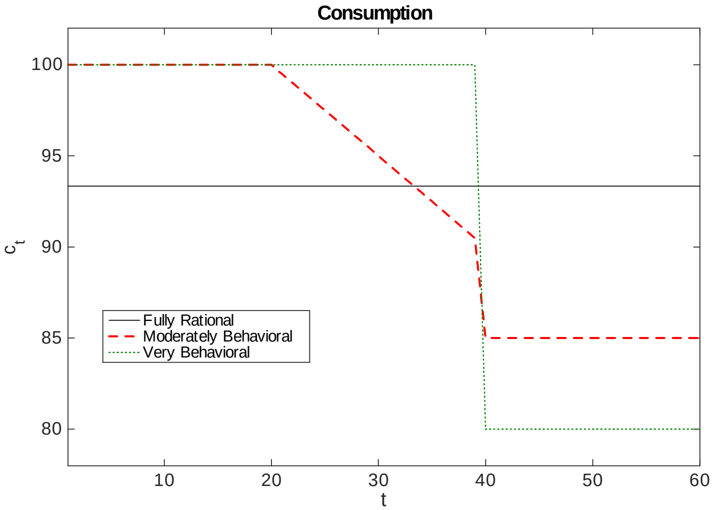
i.e.

$$c_t(m_t) = \frac{w_t - m_t x}{T-t} + \bar{y}$$

$$m_t = \mathcal{A}\left(-\frac{v_{cc}^t}{u_{cc}} \left(\frac{\partial c_t}{\partial m}\right)^2_{|m=0} \frac{1}{\bar{\kappa}^2}\right)$$

- Just one free parameter for a lifetime, $\bar{\kappa}$

BOUNDEDLY RATIONAL LIFE-CYCLE



Evidence

1. Expenditure declines after the age of 45 (Aguiar and Hurst 2013).
2. There is a fall in expenditure
    2.1 At retirement (Bernheim, Skinner and Weinberg 01).
    2.2 At end of unemployment benefits (Ganong and Noel 16): very hard to reconcile with rational model

- Basic Hyperbolic: gives a smooth shape, no drop of consumption at retirement

Computational implementation

$$V\left(z_0\right)=\max_{\left(a_t\right)}\sum_{t\geq 0}\beta^t u\left(a_t,z_t\right)\ \text{s.t.}\ z_{t+1}=F^z\left(a_t,z_t\right)$$

- 1. Calculate some proxy value function $V^p\left(z\right)$ – e.g. Taylor expansion of $V^r\left(z\right)$ around default (it's already in your computers)
  2. Choose "default" model, where many variables are set to constants (e.g. all variables but wealth w_t)
  3. Put "m's" in front of stochastic variables (except default variables): $F^{z^i}\left(a,z,m\right)$, $u\left(a,z,m\right)$
  4. Do
     $$a\left(z,V^p\right):=\arg\ \underset{a;m|m^d}{\text{smax}}\left\{u\left(a,z,m\right)+\beta\mathbb{E}\left[V^p\left(F^z\left(a,z,m\right)\right)\right]\right\}$$
  5. This way, you can simulate the whole policy / life forward.
- Example:
  - Investment (Q-theory) with boundedly rational firms, dynamic portfolio choice.
  - Paper handles general equilibrium too (Cass Koopmans for now; Kydland-Prescott in the works).

Conclusions: Micro behavior for Macro

1. Agents react more to near, rather than distant, shocks.
2. Very low sensitivity to interest rates (Hall 88) (so rat. IES should be low). However, people seem OK with non-smooth consumption profiles (so rat. IES should be high).
3. Failure of the rational high-frequency Euler equation.
4. Agents have a too small buffer of savings
5. Agents start saving “too late” for retirement (controversial)
6. High MPC to tax rebates (Parker 15, Kueng 15).
7. We don’t solve the full macro equilibrium in our head

Conclusions: Aggregate Macro

1. The Lucas critique has zero (or only partial) bite
2. Macro policy ("A behavioral New Keynesian model")
   - 2.1 Forward guidance by central bank is less powerful with behavioral agents
   - 2.2 Fiscal policy is more powerful, as they're non Ricardian
   - 2.3 Economy is more stable, even at the ZLB
3. The agent is a hybrid of Lucas neoclassical agent and a present-looking old Keynesian agent.
   - 3.1 Like Lucas agent: Has general methodology + sensitivity to important parameters
   - 3.2 Like old Keynesian agent: Has more common-sense behavior

   So this may be a useful synthesis.

Some things ready to do

1. Estimate data sets with behaviorally-enriched models (Ganong and Noel 16).
2. Revisit basic macro models to see where boundedly rational features matters for outcomes and policy (ongoing work)
3. Explore new models (e.g. high-dim.) with those easier-to-use, "sparsely rational" agents that can decide before solving the details of the GE in their heads

Program: seeking unified bounded rationality in economics

1. Micro: "A sparsity-based model of bounded rationality" (2014): Fairly general and simple device,

   $$\operatorname*{smax}_a u(a, x) \text{ subject to } b(a, x) \geq 0$$

   Basic consumer and competitive equilibrium theory: BR consumer demand, Slutsky matrix, Edgeworth boxes, BR Arrow-Debreu, ...
2. Macro: "Behavioral Macroeconomics via Sparse Dynamic programming"
3. Macro policy: "A Behavioral New Keynesian model"
4. Taxation: "Optimal taxation with behavioral agents" (with E. Farhi)
5. Game theory: "Some game theory with sparsity-based bounded rationality". Sparse *Nash equilibrium*
6. Finance: in the works.