# Fiscal Policy and Debt Management with Incomplete Markets 

Anmol Bhandari David Evans Mikhail Golosov Thomas Sargent<br>Minnesota<br>Oregon<br>Princeton<br>NYU

## How should gov't manage its debt over business cycle?

- Is there an "optimal" level of debt?
- How quickly should debt be repaid if away from the optimum?
- How should debt and taxes respond to aggregate shocks?


## What we do

- Our framework
- Ramsey planner with distortionary taxation and incomplete markets
- Existing answers: polar cases
- complete markets: Lucas Stokey (1983)
- risk-free bond only: Barro (1979), AMSS (2002)
$\Longrightarrow$ in both cases: expected level of debt is constant
- This paper
- target level of debt minimizes fiscal risk
- simple formulas to compute that level, speed of reversion to it


## Ramsey problem in a simple environment

- Ramsey problem: $\left\{\tau_{t}, B_{t}\right\}$ that implement the best competitive eqb
- Let $I(s, \tau)$ and $c(s, \tau)$ be the HH's optimal choices given tax rate $\tau$
- Bellman equation
$V\left(B_{-}\right)=\max _{\{\tau(s), B(s)\}} \mathbb{E}\left[c(s, \tau(s))-\frac{1}{1+\gamma} I(s, \tau(s))^{1+\gamma}+\beta V(B(s))\right]$
subject to

$$
B(s)+\tau(s) I(s, \tau(s))=\underbrace{R(s) B_{-}+g(s)}_{E\left(B_{-,}, s\right)} \text { for all } s
$$

## Risk minimization

- Amount of risk depends on debt level: $E(B, s)=R(s) B+g(s)$



## Risk minimization

- Amount of risk depends on debt level: $E(B, s)=R(s) B+g(s)$

- Drift in debt dynamics and variance minimization

$$
\mathbb{E}_{t} B_{t+1} \approx B_{t}-\beta^{2} \times \text { slope of } \operatorname{var}\left[E\left(B_{t}\right)\right]
$$

## Main result

## Risk minimizing debt

Let $B^{*}$ be debt level that minimizes $\operatorname{var}(R(s) B+g(s))$

- Mean of invariant distribution of debt

$$
B^{*}=-\frac{\operatorname{cov}(R, g)}{\operatorname{var}(R)}
$$

- Speed of mean reversion

$$
\frac{\mathbb{E}_{t} B_{t+1}-B^{*}}{B_{t}-B^{*}}=\left(\frac{1}{1+\beta^{2} \operatorname{var}(R)}\right)
$$

## Extensions

- Isoelastic preferences, more general shocks processes
- endogeneous feedback of tax policy on returns
- formulas continue to hold with marginal utility-adjusted returns
- $K \geq 1$ securities
- government chooses portfolio to minimize fiscal risk
- simple applications to optimal maturity management


## Quantitative investigation

- Preferences

$$
\ln c-\frac{1}{3} I^{3}
$$

- 1 security with returns $R_{t}=\frac{p_{t}}{q_{t-1}}$
- 3 shock process:

$$
\begin{aligned}
\mathrm{TFP}: \ln \theta_{t} & =\rho_{\theta} \theta_{t-1}+\sigma_{\theta} \epsilon_{\theta, t} \\
\text { spending: } \ln g_{t} & =\ln \bar{g}+\chi_{g} \epsilon_{\theta, t}+\sigma_{g} \epsilon_{g, t} \\
\text { payoffs: } \ln p_{t} & =\chi_{p} \epsilon_{\theta, t}+\sigma_{p} \epsilon_{p, t}
\end{aligned}
$$

- Benchmark: $p_{t}$ to match real returns on outstanding U.S. debt
- robustness checks: security that matches realized returns on 1-y bill; riskfree security


## Optimal policy: Comparing simulations to formula

## Ergodic Distribution : Effective Debt, $U_{c, t} B_{t}$

| Effective debt: | Using simulation | Using formula |
| :--- | :---: | :---: |
| Mean | -0.07 | -0.06 |
| Half life (years) | 237 | 244 |
| Std. deviation | 0.18 | 0.2 |

- Correlation of returns and output is close to 0 :
- large orthogonal component
- correlation of effective deficits and effective returns is positive
- Variability of effective returns is low
- slow convergence to the mean


## Conclusion

- A comprehensive theory of optimal debt management with incomplete markets
- Tractable approximations of optimal policies
- methods can be extended to other environments
- Key insight for the optimal debt management
- target debt that minimizes risk

