# The Gains from Openness: Trade, Multinational Production, and Diffusion* 

*** PRELIMINARY AND INCOMPLETE ***

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#### Abstract

The paper extends the Eaton and Kortum (2002) model of trade by allowing for other ways through which countries gain from openness. We model the simultaneous role of trade, multinational production (MP), and diffusion of ideas, and explore some of the interactions among these different channels. Both trade and MP are substitutes with diffusion, but the relationship among trade and MP is more complex. Trade and MP are alternative ways to serve a foreign market, which makes them substitutes, but we also allow for complementarities by having MP relying on imports of intermediate goods from the home country. The model allows for "bridge" MP or export platforms, creating an additional channel for complementarities between trade and MP. We use trade and MP data to calibrate the model and quantify the gains from openness, trade, MP and diffusion.


[^0]
## 1 Introduction

Our goal in this paper is to quantify the gains from openness, and the role played by trade, multinational production (MP) and diffusion in generating those gains. To do so, we extend the Eaton and Kortum (2002) model of trade by introducing MP and diffusion of ideas. We calibrate the model to match certain key facts of the trade and MP data and use the resulting model to calculate the joint as well as the separate gains from trade, MP and diffusion.

Most attempts to quantify the gains from trade use theories where there is no MP or diffusion (e.g., Eaton and Kortum, 2002; Alvarez and Lucas, 2007; Waugh, 2007), while recent attempts to quantify the gains from MP are based on models that do not allow for trade or diffusion (e.g., Ramondo, 2006; Burstein and Monge-Naranjo, 2007; McGrattan and Precott, 2007). Similarly, studies on the gains from diffusion of ideas typically ignore both trade and MP (Eaton and Kortum, 1999, Klenow and Rodríguez-Clare, 2005). ${ }^{1}$

Considering each of these channels separately, however, may understate or overstate the associated gains depending on the existence of significant sources of complementarity or substitutability among them. Suppose that MP depends on the ability of foreign affiliates to import certain inputs from their home country. In this case, shutting down trade would also decrease MP and generate losses beyond those calculated in models with trade but no MP. Alternatively, trade and MP may behave as substitutes because they are competing ways of serving foreign markets. In this case, shutting down trade would generate smaller losses than in models with only trade because MP would partially replace the lost trade.

Another way to look at this problem is by noting that we do not know whether the gains from trade $(G T)$, the gains from multinational production (GMP), and the gains from diffusion $(G D)$, can be added to compute the overall gains from openness $(G O)$. This depends crucially on the interaction between trade, MP, and diffusion flows: if they behave as substitutes then $G O<G T+G M P+G D$, while if they behave as complements then $G O>G T+G M P+G D$.

The literature has typically modeled trade and "horizontal" FDI as substitutes in the context

[^1]of the "proximity-concentration" trade-off: firms choose to either serve a foreign market by exporting or opening an affiliate there (Brainard, 1997, Markusen and Venables, 1997, Helpman, Melitz, and Yeaple, 2004). On the other hand, the literature has modeled trade and "vertical" FDI as complements: foreign affiliates rely on intermediate goods imported from their parent firms to produce goods that are consumed in other markets (Markusen, 1984, Grossman and Helpman, 1985, Antras, 2003).

The empirical evidence appears consistent with both of these views. Studies using data at the industry, product, or firm level, have concluded that MP and trade flows in intermediate inputs, often conducted within the firm, are complements, while MP and trade flows in final goods are substitutes (Belderbos and Sleuwaegen, 1988, Bloningen, 2001, Head and Ries, 2001, Head, Ries, and Spencer, 2004). Additionally, the empirical evidence points to large intra-firm trade flows related to multinational activities. This is specially true among rich countries, where imported inputs from the home country are large relative to total revenues of foreign affiliates (Bernard, Jensen, and Schott, 2005, Hanson, Mataloni, and Slaughter, 2003, and Alfaro and Charlton, 2007). Furthermore, even among rich countries, foreign subsidiares of multinationals often sell a sizable part of their output outside of the host country. For example, around $30 \%$ of total sales of US affiliates in Europe are not done in the host country (Bloningen, 2005).

This paper presents a general-equilibrium, multi-country, Ricardian model of trade, MP, and technology diffusion. The model has two sectors: tradable intermediate goods, and non-tradable consumption goods. All goods are produced with constant-returns-to-scale technologies which differ across countries, creating incentives for trade and MP. For non-tradable goods, serving a foreign market can only be done through MP, but for tradable goods we have to consider the choice between exports and MP.

Trade flows are affected by iceberg-type costs that may vary across country pairs. To avoid these costs, to benefit from lower costs abroad, or simply to serve a foreign market, firms producing tradable goods may prefer to serve another country through MP rather than exports. But MP entails some efficiency losses or costs. In particular, we assume that MP is subject to two different iceberg-type efficiency costs. First, a country-pair specific cost: irrespective of what good they produce, firms from country $i$ may find it more costly to do MP in country $l$ than in country $j$. Second, a good-specific cost: irrespective of where they do so, firms in country
$i$ may find it more costly to do MP in some goods than in others. ${ }^{2,3}$ Moreover, to introduce complementarity between trade and MP, we assume that affiliates rely at least partially on imported inputs from their home country; in our empirical application we think of this as "intra-firm" trade. Since these imports are affected by trade costs (just as regular trade), this creates an extra cost to MP.

Our set-up allows firms to use a third country as a "bridge" or export platform to serve a particular market; we refer to this as "bridge MP" or simply BMP. ${ }^{4}$ For example, a firm from country $i$ producing a tradable good $u$ can serve country $n$ by doing MP in country $l$. This entails MP costs associated with the pair $\{i, l\}$ and the good $u$, and then also the trade cost associated with the pair $\{l, n\}$.

The multiplicity of choices regarding how to serve a foreign market makes trade and MP substitutes: arm-length trade and MP are alternative ways of serving a foreign market. However, the possibility of BMP creates complementarities between trade and MP: the decision by country $i$ of serving market $n$ producing in a third country $l$ generates a trade flow from $l$ to $n$ associated with MP from $i$ to $l$. Moreover, when country $i$ serves market $n$ through MP, there is an "intrafirm" trade flow in intermediate inputs from country $i$ associated with it. Thus, even in a world without BMP, our model generates complementarities between trade and MP.

We have so far left diffusion out of this brief description of the model. We think of diffusion as happening when a technology from country $i$ gets used by domestic firms anywhere else. This is captured formally by assuming that each technology can be used outside its country of origin by local firms at a cost that varies across goods. Imagine that country $i$ has the best technology worldwide for good $u$, and imagine that the diffusion related efficiency loss for this good is low. Then this good will tend to be produced locally in most countries: there will be little trade and little MP for this good.

We estimate the parameters of the model by matching simulated and observed moments. We use data on bilateral trade and MP flows for a set of OECD countries, as well as data on

[^2]intra-firm trade flows for U.S. multinationals and foreign multinationals operating in the U.S. We follow Eaton and Kortum (2002) in using price data to estimate trade costs. We use the estimated model to compute the joint gains from trade, MP and diffusion; we think of these as the overall gains from openness. We also compute the separate gains from these three channels. Our preliminary results suggest that trade, MP and diffusion behave as substitutes, in the sense that $G T+G M P+G D>G O$. This is mainly because trade and MP are substitutes with diffusion, but also because the complementarities between trade and MP are never strong enough to dominate the overall relationship. This last result comes mainly from the relatively small value of "intra-firm" trade in relation to MP.

The paper is organized as follows. Section 2 presents the model and the equilibrium. Section 3 presents model's calibration and welfare calculations. Section 4 concludes.

## 2 The Model

We extend Eaton and Kortum's (2002) model of trade to incorporate MP, "intra-firm" trade, and diffusion of ideas in a multi-country, general equilibrium set-up. Our model is Ricardian with a continuum of tradable intermediate goods and non-tradable final goods, produced under constant-returns-to-scale. We adopt the probabilistic representation of technologies as first introduced by Eaton and Kortum (2002), but we enrich it to incorporate MP and diffusion.

In the next subsection we present a simple model that has only tradable goods and no diffusion. We use this simplified model to lay out the key ideas regarding substitutability and complementarity between trade and MP. We present the full model in the following subsection.

### 2.1 A Simple Eaton-Kortum Model of Trade and MP

Country $i \in\{1, \ldots, I)$ is endowed with $L_{i}$ units of labor, which is the only factor of production. There is a continuum of tradable goods indexed by $u \in[0,1]$ that enter a representative agent's utility via CES preferences with elasticity of substitution $\sigma$. All production takes place according to constant-returns-to-scale technologies

Following Alvarez and Lucas (2007), we distinguish technologies across goods and countries
by modeling cost rather than productivity parameters. Let $x_{i}(u)$ denote the cost parameter associated with country $i^{\prime} s$ technology to produce good $u$. If there were no MP, then this technology could only be used in country $i$. In this case, the unit cost of good $u$ in country $i$ would be $x_{i}(u)^{\theta} c_{i}$, where $\theta>0$ is a common parameter that amplifies the effect of the variability of cost draws on the pattern of trade and MP and $c_{i}$ is the unit cost of the national input used for producing all goods in country $i$. In the simple model of this section we assume that this national input is produced one-to-one from labor, so $c_{i}$ is equal to the wage in country $i, w_{i}$, but this equivalence is not maintained in the full model presented below. We introduce MP by allowing technologies from one country to be used for production in other countries, as explained next.

It is important to keep track of the countries where technologies originate, where goods are produced, and where goods are consumed. To do so, we will in general use subscript $n$ to denote the country where the good is consumed, $l$ for the country where the good is produced, and $i$ for the country where the technology originates.

Consider an intermediate good $u$ produced in country $l$. This good can be produced using country $l^{\prime} s$ technology at unit $\operatorname{cost} x_{l}(u)^{\theta} c_{l}$. Good $u$ can also be produced in country $l$ using a foreign technology through what we call multinational production (MP), but this entails some efficiency losses that lead to additional costs. In particular, we assume that when country $i^{\prime} s$ technology for good $u$ is used for production in country $l$, the unit production cost is $\left[x_{i}(u) z_{i}(u)\right]^{\theta} c_{l i}$. Here $z_{i}(u) \geq 1$ is a good and country specific parameter that captures how difficult it is to engage in MP for good $u$ by country $i$, irrespective of the country where MP is undertaken. $c_{l i}$ is the unit cost of the multinational input required for production in country $l$ with country $i$ technologies. The unit cost $c_{l i}$ will in general be higher than $c_{l}$, reflecting country-pair specific costs and efficiency losses of MP as opposed to national production (see below).

Trade is subject to "iceberg" transportation costs, with one unit of a good shipped from country $l$ resulting in $k_{n l} \leq 1$ units arriving to country $n$. We assume that $k_{n n}=1$ and that the triangular inequality holds (i.e., $k_{n l} \geq k_{n j} k_{j l}$ for all $n, l, j$ ).

The multinational input combines the national input from the home country (i.e., the country where the technology originates) and the host country (i.e., the country where production takes
place). The home country national input must be shipped to the host country, and this implies paying the corresponding transportation cost: the cost of the home country national input for MP by country $i$ in country $l$ is then $c_{i} / k_{l i}$. In the quantitative section below we will think of MP by $i$ in $l$ as being done by country $i$ multinationals, and we will think of imports of the national input associated with MP as "intra-firm" trade. We envisage this trade being done inside the multinational, which would set up a headquarters in the home country to produce the national input from labor and then export it to the foreign affiliate for final production.

The host country national input has cost $c_{l}$, but MP incurs an "iceberg" type efficiency loss of $h_{l i}<1$ associated with using an idea from $i$ to produce in $l$ : the cost of the host country national input for MP by $i$ in $l$ is then $c_{l} / h_{l i}$. Combining the costs of home and host country national inputs into a CES aggregator, we get the unit cost of the multinational input by $i$ in $l$,

$$
c_{l i}=\left[(1-a)\left(\frac{c_{l}}{h_{l i}}\right)^{1-\rho}+a\left(\frac{c_{i}}{k_{l i}}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}},
$$

where $a \in[0,1]$ and $\rho>1$. (Note that $c_{l l}=c_{l}$.) The parameter $\rho$ indicates the degree of complementarity between the national inputs from the home and host countries. It is a key parameter for our estimated welfare gains.

Since goods are identical except for their cost parameters (i.e., they enter preferences symmetrically), then we follow Alvarez and Lucas (2007) and drop index $u$, labeling goods instead by the vector $(x, z)$ where $x=\left(x_{1}, x_{2}, \ldots, x_{I}\right)$ and $z=\left(z_{1}, z_{2}, \ldots, z_{I}\right)$. The unit cost of an intermediate good ( $x, z$ ) produced in country $l$ with a technology from country $i$ and sold in country $n$ is $\left[x_{i} z_{i}\right]^{\theta} c_{l i} / k_{n l}$. Note that if $l=i$ then the good is exported from $i$ to $n$ at $\operatorname{cost}\left[x_{i} z_{i}\right]^{\theta} c_{i} / k_{n i}$. On the other hand, if $i \neq l=n$ then there is MP and intra-firm trade from $i$ to $n$. Finally, if $i \neq l$ and $l \neq n$, then country $l$ is used as an export platform by country $i$ to serve country $n$. We say that in this case there is "bridge MP" or simply BMP by country $i$ in country $l$. Note also that here there is "intra-firm" trade from $i$ to $l$.

Since all technologies are constant returns to scale, the price of the intermediate good ( $x, z$ ) in country $n, p_{n}(x, z)$, is simply the minimum cost at which it can be obtained by $n$, namely

$$
\begin{equation*}
p_{n}(x, z)=\min \left\{\min _{i} x_{i}{ }^{\theta} c_{i} / k_{n i}, \min _{i, l}\left[x_{i} z_{i}\right]^{\theta} c_{l i} / k_{n l}\right\} . \tag{1}
\end{equation*}
$$

The term $\min _{i} x_{i}{ }^{\theta} c_{i} / k_{n i}$ is the minimum cost at which the good can be obtained if there is no MP, taking into account all possible countries from which the good can be imported. We must add the term $\min _{i, l}\left[x_{i} z_{i}\right]^{\theta} c_{l i} / k_{n l}$ to the minimization on the RHS to allow for MP using technologies from $i$ in country $l$ for all $\{i, l\}$ pairs.

We assume that the cost parameters $x_{i}$ and the efficiency loss parameters $z_{i}$ for each good are random variables. Rather than presenting our distributional assumptions for $x_{i}$ and $z_{i}$ directly, however, it proves more convenient to do so indirectly by introducing "latent technology" variables $x_{i}^{N}$ and $x_{i}^{M}$, specifying how these variables are distributed, and then expressing our parameters $x_{i}$ and $z_{i}$ as functions of these variables. We assume that $x_{i}^{N}$ and $x_{i}^{M}$ are drawn independently across goods and countries from an exponential distribution with parameter $\lambda_{i}^{N}$ and $\lambda_{i}^{M}$, respectively. In turn, we assume that $x_{i}=\min \left\{x_{i}^{N}, x_{i}^{M}\right\}$ and $z_{i}=x_{i}^{M} / x .{ }^{5}$

To help the reader in following the derivations below, it is useful to give an economic interpretation to our latent variables $x_{i}^{N}$ and $x_{i}^{M}$. We think of each country having two technologies to produce each good: a "national" technology, which cannot be used for MP, and a "multinational" technology, which can be used for MP. The variables $x_{i}^{N}$ and $x_{i}^{M}$ are the cost parameters associated with the national and multinational technologies, respectively. If the country were closed to MP, then all that would matter would be the best technology available for each good, which has cost parameter $x_{i}=\min \left\{x_{i}^{N}, x_{i}^{M}\right\}$. Note that $x_{i}$ is distributed exponentially with parameter $\lambda_{i} \equiv \lambda_{i}^{N}+\lambda_{i}^{M}$. We refer to $\lambda_{i}$ as the stock of ideas originated in country $i .{ }^{6}$ With MP we also care about $x_{i}^{M}$ separately. If $x_{i}^{N}<x_{i}^{M}$ then doing MP implies using a more inefficient technology, which generates the efficiency loss $z_{i}=x_{i}^{M} / x_{i}^{N}>1$. In spite of this inefficiency, country $i$ may decide to serve country $n$ via MP to save on trade costs and also perhaps to benefit from lower wages there.

### 2.1.1 Equilibrium Analysis

It is easier to characterize the equilibrium trade and MP flows by using the latent technology variables $x_{i}^{N}$ and $x_{i}^{M}$ rather than $x_{i}$ and $z_{i}$. This is because, since $x_{i}^{N}$ and $x_{i}^{M}$ are independent

[^3]and exponentially distributed, then we can use the properties of the exponential distribution to derive sharp and simple results regarding the allocation of countries' purchases across different sources.

We first rewrite the price equation (1) in terms of variables $x_{i}^{N}$ and $x_{i}^{M}$. Using $x^{N}=$ $\left(x_{1}^{N}, x_{2}^{N}, \ldots, x_{I}^{N}\right)$ and $x^{M}=\left(x_{1}^{M}, x_{2}^{M}, \ldots, x_{I}^{M}\right)$, then we have

$$
p_{n}\left(x^{N}, x^{M}\right)=\min \left\{\min _{i}\left(x_{i}^{N}\right)^{\theta} c_{i} / k_{n i}, \min _{i, l}\left(x_{i}^{M}\right)^{\theta} c_{l i} / k_{n l}\right\}
$$

The first term on the RHS minimizes over all possible ways in which country $n$ can procure the good conditional on using national technologies, which precludes MP and implies importing from the country from which the technology originates. In contrast, the second term on the RHS minimizes over all possible ways in which country $n$ can procure the good conditional on using multinational technologies, which allows for MP by $i$ in $l$ for all $\{i, l\}$ combinations.

Using the properties of the exponential distribution, it is easy to show that $p_{n}\left(x^{N}, x^{M}\right)^{1 / \theta}$ is distributed exponentially with parameter

$$
\psi_{n} \equiv \sum_{i}\left(\psi_{n i}^{N}+\psi_{n i}^{M}\right),
$$

where

$$
\psi_{n i}^{N}=\left(c_{i} / k_{n i}\right)^{-1 / \theta} \lambda_{i}^{N} \text { and } \psi_{n i}^{M}=\widetilde{c}_{n i}^{1 / \theta} \lambda_{i}^{M},
$$

and $\widetilde{c}_{n i} \equiv \min _{l}\left\{c_{l i} / k_{n l}\right\}$ is the minimum cost of the multinational input for MP by country $i$ when serving country $n$ (taking into account all possible bridge countries $l$ ).

Let $p_{n}$ be the price index of consumption in country $n$. Given the CES preferences, then we know that

$$
p_{n}^{1-\sigma}=\int p_{n}\left(x^{N}, x^{M}\right)^{1-\sigma} d F\left(x^{N}, x^{M}\right)
$$

where $F\left(x^{N}, x^{M}\right)$ is the joint distribution of $x^{N}$ and $x^{M}$. Using the above results, the price index in country $n$ is then given by

$$
\begin{equation*}
p_{n}=C \psi_{n}^{-\theta} \tag{2}
\end{equation*}
$$

where $C \equiv \Gamma(1+\theta(1-\sigma))^{1 / 1-\sigma}$ is a constant, $\Gamma()$ is the Gamma function, and we restrict parameters such that $1+\theta(1-\sigma)>0$.

As shown by Eaton and Kortum (2002), the average price charged by any country $l$ in country $n$ is the same. Moreover, by the properties of the exponential distribution we know that a share $s_{n l}^{N} \equiv \psi_{n l}^{N} / \psi_{n}$ of goods bought by country $n$ will be produced by country $l$ with national technologies. Thus, letting $X_{n}=w_{n} L_{n}$ denote total spending by country $n$, then

$$
\begin{equation*}
s_{n l}^{N} X_{n} \tag{3}
\end{equation*}
$$

is the value of of goods produced with national technologies in country $l$ that are exported to country $n$.

Similarly, $\frac{\psi_{n i}^{M}}{\psi_{n}} X_{n}$ is the value of goods bought by $n$ that are produced with multinational technologies from $i$. But note that these goods could be produced in any country $l \in \arg \min _{j}\left(\widetilde{c}_{j i} / k_{n j}\right)$. Let $y_{n l i}^{M}$ be the share of total spending by country $n$ on goods produced with country $i$ multinational technologies that are produced in country $l$ (and then shipped to country $n$ ). These are shares over possible bridge countries for the pair $\{n, i\}$, so $\sum_{l} y_{n l i}^{M}=1$. Note that if MP were not feasible then $y_{n i i}^{M}=1$ for all $n, i$, while if BMP was not feasible then $y_{n l i}^{M}=0$ for all $l \neq i, n$. In equilibrium the following "complementary slackness" conditions must hold:

$$
\begin{array}{clc}
c_{l i} / k_{n l}>\tilde{c}_{n i} & \Longrightarrow \quad y_{n l i}^{M}=0 \\
y_{n l i}^{M}>0 & \Longrightarrow c_{l i} / k_{n l}=\widetilde{c}_{n i} . \tag{4}
\end{array}
$$

The value of MP by $i$ in $l$ for $n$ is $s_{n l i}^{M} X_{n}$, where $s_{n l i}^{M} \equiv y_{n l i}^{M} \psi_{n i}^{M} / \psi_{n}$. Summing up over $i$ yields the total imports by country $n$ from $l$ of goods produced with multinational technologies,

$$
\begin{equation*}
\sum_{i} s_{n l i}^{M} X_{n} \tag{5}
\end{equation*}
$$

Adding terms (3) and (5) we obtain imports of individual goods by country $n$ from country $i$. To differentiate this kind of trade from the "intra-firm" trade in the national input associated with multinational activities, we will refer it as "arms-length" trade. Using (3) and (5), total "arms-length" imports from $n$ from $i$ can be written as

$$
\left(s_{n i}^{N}+\sum_{j} s_{n i j}^{M}\right) X_{n}=\left(s_{n i}^{N}+s_{n i i}^{M}\right) X_{n}+\left(\sum_{j \neq i} s_{n i j}^{M}\right) X_{n}
$$

The first term on the right hand side refers to "arms-length" exports from $i$ to $n$ produced with its own (national and multinational) technologies, whereas the second term captures "arms-length" exports from $i$ to $n$ of goods produced by foreign multinationals in $n$.

To calculate the observed imports we need to add "intra-firm" trade. To do so, we first need to get an expression for total MP by $i$ in $l, X_{l i}^{M P}$. Summing up over all destination countries $n$, this is

$$
X_{l i}^{M P}=\sum_{n} s_{n l i}^{M} X_{n}
$$

Let $\omega_{l i}$ be the cost share of the home national input for the production of any good in country $l$ by multinationals from country $i$. This is

$$
\omega_{l i}=\frac{a\left(c_{i} / k_{l i}\right)^{1-\rho}}{(1-a)\left(c_{l} / h_{l i}\right)^{1-\rho}+a\left(c_{i} / k_{l i}\right)^{1-\rho}} .
$$

Imports associated with MP by $i$ in $l$ are then

$$
\begin{equation*}
\omega_{l i} \sum_{n} s_{n l i}^{M} X_{n} \tag{6}
\end{equation*}
$$

Adding up terms in expressions (3), (5), and (6) yields total imports by $n$ from $i \neq n$,

$$
\begin{equation*}
M_{n i} \equiv\left(s_{n i}^{N}+\sum_{j} s_{n i j}^{M}\right) w_{n} L_{n}+\omega_{n i} \sum_{j} s_{j n i}^{M} w_{j} L_{j} . \tag{7}
\end{equation*}
$$

Again, with a slight abuse of notation, we can think of the first term on the right hand side as "arms-length" trade and the second term as "intra-firm" trade. "Arms-length" trade entails exports of individual goods, whereas "intra-firm" trade involves exports of the home national input for MP by "multinationals" producing abroad.

Aggregate imports for country $n$ are simply $M_{n}=\sum_{i \neq n} M_{n i}$. Trade balance conditions close the model, determining equilibrium wages for each country. ${ }^{7}$ Trade balance for country $n$ entails

[^4]total imports equal to total exports, or
\[

$$
\begin{equation*}
\sum_{i \neq n} M_{n i}=\sum_{i \neq n} M_{i n} . \tag{8}
\end{equation*}
$$

\]

### 2.1.2 Some results under symmetry

To gain intuition on the workings of the model, consider the case of symmetric countries ( $L_{i}=L$ ) and symmetric trade costs, $k_{n l}=k$ and $h_{n l}=h$ for all $l \neq n$, with $k<h<1$. This is a case that can be solved analytically, yet the basic intuition carries to the asymmetric case.

Wages, costs, and prices are equalized across countries: $w_{n}=w, c_{n}=c$, and $p_{n}=p$. This implies that the cost of the multinational input collapses to $c_{l i}=c / m$ for all $l \neq i$, where

$$
\begin{equation*}
m \equiv\left[(1-a) h^{\rho-1}+a k^{\rho-1}\right]^{\frac{1}{\rho-1}} \tag{9}
\end{equation*}
$$

It is easy to see that $h>k$ implies that $m>k$, and hence $y_{n l i}^{M}=0$ for all $n \neq l$ : there is no trade in goods produced with multinational technologies. ${ }^{8}$ Thus, in a symmetric world there is no BMP. ${ }^{9}$ Shutting down this source of complementarity allows us to better highlight the complementarity between trade and MP coming from the possibility of using the home country national input when doing MP (i.e. the role of the parameter $\rho$ ).

From (2), the price level in any country is

$$
\begin{equation*}
p=C\left[\lambda+(I-1)\left(k^{1 / \theta} \lambda^{N}+m^{1 / \theta} \lambda^{M}\right)\right]^{-\theta} w \tag{10}
\end{equation*}
$$

Intuitively, the term inside the bracket captures the effective stock of ideas available in any country: $\lambda=\lambda^{N}+\lambda^{M}$ local ideas plus national ideas from other countries discounted by trade costs, $k^{1 / \theta}$, plus multinational ideas from other countries discounted by MP costs, $m^{1 / \theta}$. Note that if $h>k$ then $m^{1 / \theta}>k^{1 / \theta}$, so multinational ideas are discounted by less than national ideas.

[^5]Flows The share that country $n$ will devote to spending on goods produced in country $i \neq n$ with country $i^{\prime} s$ national ideas is simply the contribution of country $i^{\prime} s$ national ideas to the effective stock of ideas available in country $n$. Thus, under symmetry we have

$$
s^{N}=\frac{k^{1 / \theta} \lambda^{N}}{\lambda+(I-1)\left(k^{1 / \theta} \lambda^{N}+m^{1 / \theta} \lambda^{M}\right)}
$$

Similarly, the share that $n$ will spend on goods produced locally with multinational technologies from country $i$ is the contribution of $i^{\prime} s$ multinational ideas to the effective stock of ideas available in country $n,{ }^{10}$

$$
s^{M}=\frac{m^{1 / \theta} \lambda^{M}}{\lambda+(I-1)\left(k^{1 / \theta} \lambda^{N}+m^{1 / \theta} \lambda^{M}\right)}
$$

MP will be accompanied by imports of the home country $i^{\prime} s$ national input. This trade flow is $\omega s^{M}$, so the share of income allocated by any country to imports from any other country is $s^{N}+\omega s^{M}$.

We can now explore the forces for substitution and complementarity between trade and MP in the model. To do so, consider the effect of a change in the cost of doing MP, captured by the parameter $h$, on trade flows. When $h$ goes up (i.e., MP costs fall), MP increases and "arms-length" trade decreases. This captures the forces of substitution between trade and MP. Simultaneously, there are two effects on "intra-firm" trade. On the one hand, a higher $h$ shifts production towards using more of the host country national input: the higher the elasticity of substitution $\rho$, the stronger the switch towards the this input. On the other hand, since MP increases, both the use of home as well as host country national input increases: the lower $\theta$, the stronger this effect.

More formally, $s^{N}$ decreases with $h$ : "arms-length" trade is a substitute for MP. However, "intra-firm" trade might increase or decrease, because $\omega$ decreases while $s^{M}$ increases with $h .{ }^{11}$ The first effect dominates when $\rho$ is sufficiently high: if $\rho-1>1 / \theta$, then $d \omega s^{M} / d h<0$, and trade and MP are net substitutes. ${ }^{12}$ Notice that even without any "arms-length" trade the

[^6] $s_{n i 1}^{M}=\psi_{n i}^{N} / \psi_{n}$.
$$
\frac{d \log \omega s^{M}}{d \log h}=\left[\frac{1}{\theta}-(\rho-1)\right] \frac{d \log m}{d \log h}-\frac{1}{\theta}(I-1) \frac{m^{1 / \theta} \lambda^{M}}{\widetilde{\lambda}+(I-1)\left(k^{1 / \theta} \lambda^{N}+m^{1 / \theta} \lambda^{M}\right)} \frac{d \log m}{d \log h},
$$
where $d \log m / d \log h>0$.
${ }^{12}$ Note that an increase in $k$ can either increase or decrease $s^{M}$; the condition for $d s^{M} / d k<0$ is stronger than
model can generate substitutability between trade and MP.

Gains We now turn to calculating the gains from openness, trade, and MP. We can compute the gains from openness GO (i.e., the increase in welfare from isolation to benchmark) by comparing the associated real wage levels, $w / p$. Since wages are equal across countries, they can be normalized to one, so we can just compare prices across different scenarios. The price index for the benchmark is given by (10), whereas the analogous result with no trade and no MP is obtained by letting $k \rightarrow 0$ and $h \rightarrow 0$ in (10). This yields

$$
p_{I S O L}=C \lambda^{-\theta} .
$$

The gains from openness $(\widetilde{G O})$ are given by

$$
\begin{equation*}
\widetilde{G O}=\frac{p_{I S O L}}{p}=\left[\frac{\lambda+(I-1)\left(k^{1 / \theta} \lambda^{N}+m^{1 / \theta} \lambda^{M}\right)}{\lambda}\right]^{\theta}, \tag{11}
\end{equation*}
$$

or, $G O=\ln (\widetilde{G O})$. (Below we follow this notation so that expressions for gains with a $\sim$ represent proportional gains.) It is clear that $G O$ increases with $h$ and $k$ : the lower MP or trade costs, the larger the gains from openness.

We calculate gains from trade by computing the gains of moving from isolation to only trade (no MP), GT. Analogously, we calculate gains from MP by computing the gains of moving from isolation to only MP (no trade), GMP. We first derive the price index when there is only trade. From (10), by setting $m^{1 / \theta}=0$, and allowing multinational ideas to be used for domestic production and trade, we get:

$$
p_{T}=C\left[\lambda\left(1+(I-1) k^{1 / \theta}\right)\right]^{-\theta} .
$$

Gains from trade are then given by

$$
\widetilde{G T}=\frac{p_{I S O L}}{p_{T}}=\left[1+(I-1) k^{1 / \theta}\right]^{\theta} .
$$

Not surprisingly, $G T$ increases with $k$. Similarly, the gains from MP (increase in real wage from $\overline{\rho-1>1 / \theta}$.
isolation to only MP) are

$$
\widetilde{G M P}=\frac{p_{I S O L}}{p_{M P}}=\left[\frac{\lambda+(I-1) \widetilde{m}^{1 / \theta} \lambda^{M}}{\lambda}\right]^{\theta}
$$

where $\widetilde{m} \equiv \lim _{k \rightarrow 0} m=(1-a)^{\frac{1}{\rho-1}} h$ is the MP cost adjustment under no trade.
The key role of $\rho$ in generating complementarity between trade and MP can be seen by noting that when $\rho \rightarrow 1$ then $\widetilde{m} \rightarrow 0$. Using the results above, this implies that for low $\rho$ we must have $G O>G T+G M P$ : trade and MP behave as complements. Conversely, when $\rho \rightarrow \infty$, then $m \rightarrow h$ and $\widetilde{m} \rightarrow h$. This implies that for high $\rho$ we have $G O<G T+G M P$ : trade and MP behave as substitutes. More generally, the relationship between $G O$ and $G T+G M P$ depends on the elasticity of substitution $\rho$ and the technology parameter $\theta$. In particular, if $\rho-1>1 / \theta$, then $G O<G T+G M P$, so that trade and MP are net substitutes (see the proof in the Appendix). The intuition is the following. While $\rho-1$ governs the effect of trade costs on trade flows in Armington or Krugman models, $1 / \theta$ has analogous role in Ricardian models. Thus, this condition says that MP and trade are substitutes if the effect of trade costs on "intra-firm" trade flows is larger than their effect on "arms-length" Ricardian trade flows.

Finally, it is useful to calculate the gains from trade given by moving from a situation with only MP to the benchmark, denoted by $G T^{\prime}$. The final goods' price index under no trade is obtained by letting $k \rightarrow 0$ in (10):

$$
p_{k \rightarrow 0}=C\left[\lambda+(I-1) \widetilde{m}^{1 / \theta} \lambda^{M}\right]^{-\theta}
$$

Thus,

$$
\widetilde{G T}^{\prime}=\frac{p_{k \rightarrow 0}}{p}=\left[\frac{\lambda+(I-1)\left(k^{1 / \theta} \lambda^{N}+m^{1 / \theta} \lambda^{M}\right)}{\lambda+(I-1) \widetilde{m}^{1 / \theta} \lambda^{M}}\right]^{\theta} .
$$

We can again think about complementarity and substitutability here by studying the effect of $h$ on $G T^{\prime}$. Again, if $\rho-1>1 / \theta$, then $G T$ unambiguously decreases with $h$, so that one can say that trade and MP are net substitutes (see the proof in the Appendix). Also note that if $\rho \rightarrow 1$ then $\widetilde{m} \rightarrow 0$ and $G T^{\prime} \rightarrow G O$ : when trade and MP are perfect complements, there cannot be MP in the absence of trade, so the gains from trade effectively include also the gains from MP.

### 2.2 Full Model

We now extend the basic model in several dimensions that are important for the quantitative analysis. First, we allow for diffusion by assuming that, just as with MP, a country's technologies can be used elsewhere at an efficiency loss that varies across goods (see below). Second, we allow for the fact that there is a sizable share of goods that are not tradable, but are amenable to MP and diffusion. Third, we take into account that intermediate goods are often used for the production of other intermediate goods, generating an input-output loop that amplifies the gains from openness (see Eaton and Kortum, 2002, and Alvarez and Lucas, 2007).

Formally, we now assume that there is a continuum of non-tradable consumption goods, indexed by $v \in[0,1]$, and a continuum of tradable intermediate goods, indexed by $u \in[0,1]$. The intermediate goods are aggregated into a composite intermediate good via a CES production function,

$$
Q_{m}=\left[\int_{0}^{1} q(u)^{\frac{\sigma-1}{\sigma}} d u\right]^{\frac{\sigma}{\sigma-1}} .
$$

In turn, each intermediate good is produced using this composite intermediate good and labor with a Cobb-Douglas production function with labor share $\beta$. Thus, whereas in the simple model above the national input that is used to produce each of the intermediate goods is produced one-to-one from labor, we now assume that it is produced from labor and the composite intermediate good at $\operatorname{cost} c_{i}^{T}=B w_{i}^{\beta} p_{m i}^{1-\beta}$, where $w_{i}$ is the wage and $p_{m i}$ the price index associated to $Q_{m}$ in country $i$, and $B \equiv \beta^{-\beta}(1-\beta)^{\beta-1}$. The unit cost of intermediate good $u$ produced in country $i$ with its own technology is $x_{i}(u)^{\theta} c_{i}^{T}$.

Similarly to intermediate goods, non-tradable consumption goods are produced from labor and the composite intermediate good $Q_{m}$ with a Cobb-Douglas production function with labor share $\alpha$. The input bundle for consumption goods has unit cost $c_{i}^{N T}=A w_{i}^{\alpha} p_{m i}^{1-\alpha}$ in country $i$, where $A \equiv \alpha^{-\alpha}(1-\alpha)^{\alpha-1}$. The (stochastic) cost parameter associated with the technology for consumption good $v$ is denoted by $\xi_{i}(v)$, and the unit cost of production for good $v$ in country $i$ produced with country $i^{\prime} s$ technology is $\xi_{i}(v)^{\theta} c_{i}^{N T}$. Figure 1 illustrates the cost structure in the closed economy.

We now explain how we capture diffusion and MP for intermediates. As in the simple model presented above, MP implies an efficiency loss that varies across goods, implying that the unit

Figure 1: Cost Structure in the Closed Economy

cost for MP by country $i$ in country $l$ for intermediate good $u$ is $\left[x_{i}(u) z_{i}^{M}(u)\right]^{\theta} c_{l i}^{T}$, where

$$
\begin{equation*}
c_{l i}^{T}=\left[(1-a)\left(\frac{c_{l}^{T}}{h_{l i}^{T}}\right)^{1-\rho}+a\left(\frac{c_{i}^{T}}{k_{l i}}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}} . \tag{12}
\end{equation*}
$$

Thanks to diffusion, country $i$ technologies can also be used in country $l$ for national production at unit cost $\left[x_{i}(u) z_{i}^{G}(u)\right]^{\theta} c_{l}^{T}$. Note that for diffusion there is no need for trade in the national input and no country-pair specific efficiency loss: once an idea diffuses, then it is available everywhere to be used for production as if it was a local idea with cost parameter $x_{i}(u) z_{i}^{G}(u)$.

Diffusion and MP are modeled analogously for consumption goods except that we assume that the unit cost of the multinational input for the production of these goods is

$$
\begin{equation*}
c_{l i}^{N T}=c_{l}^{N T} / h_{n i}^{N T} . \tag{13}
\end{equation*}
$$

In other words, in contrast to MP for intermediates, multinationals do not import some of the national input from their home country, but they do incur in country-pair specific efficiency losses.

We finish this description of the full model by presenting our assumptions regarding the distribution of the cost parameters $x_{i}, z_{i}^{M}$ and $z_{i}^{G}$. Again, we do this indirectly by introducing
latent technology variables $x_{i}^{N}, x_{i}^{M}$ and $x_{i}^{G}$, which are independently drawn from the exponential distribution with parameters $\lambda_{i}^{N}, \lambda_{i}^{M}$ and $\lambda_{i}^{G}$. In turn, we let $x_{i}=\min \left\{x_{i}^{N}, x_{i}^{M}, x_{i}^{G}\right\}$ and $z_{i}^{M}=x_{i}^{M} / x_{i}$ and $z_{i}^{G}=x_{i}^{G} / x_{i}$. Analogously, letting $\zeta_{i}^{M}$ and $\zeta_{i}^{G}$ be the efficiency loss parameters for MP and diffusion in non-tradable goods (just like $z_{i}^{M}$ and $z_{i}^{G}$ for tradable goods), we consider latent technology variables $\xi_{i}^{N}, \xi_{i}^{M}$ and $\xi_{i}^{G}$ that are independently drawn from the exponential distribution with parameters $\lambda_{i}^{N}, \lambda_{i}^{M}$ and $\lambda_{i}^{G}$, and assume that $\xi_{i}=\min \left\{\xi_{i}^{N}, \xi_{i}^{M}, \xi_{i}^{G}\right\}$ and $\zeta_{i}^{M}=\xi_{i}^{M} / \xi_{i}$ and $\zeta_{i}^{G}=\xi_{i}^{G} / \xi_{i}$.

To gain some intuition about the full model in relation to the simple model presented above, consider again the gains from openness in the symmetric case. ${ }^{13}$ It is easy to show that now

$$
\begin{align*}
\widetilde{G O}= & {\left[\frac{\lambda+(I-1)\left(h^{1 / \theta} \lambda^{M}+\lambda^{G}\right)}{\lambda}\right]^{\theta} }  \tag{14}\\
& \cdot\left[\frac{\lambda+(I-1)\left(k^{1 / \theta} \lambda^{N}+m^{1 / \theta} \lambda^{M}+\lambda^{G}\right)}{\lambda}\right]^{\eta \theta},
\end{align*}
$$

where $\eta=(1-\alpha) / \beta$. The first term on the RHS captures the gains from MP and diffusion for non-tradable goods, whereas the second term captures the gains from trade, MP and diffusion for tradable goods. It is interesting to note that the gains for tradable goods are determined by the power $\eta \theta$ rather than simply $\theta$. There are two forces at work here. First, the term $1-\alpha$ in $\eta$ captures the importance of intermediate goods in the production of final goods: a lower $\alpha$ therefore implies stronger gains from openness in intermediates. Second, the term $1 / \beta$ in $\eta$ captures the amplification of the gains from openness thanks to the input-output loop mentioned above: the higher is the share of intermediates in the production of intermediates (i.e., the lower is $\beta$ ), the stronger is this amplification effect.

In the quantitative section below we will compute $G T$ and $G M P$, which have the same definitions as in Section 2.1.2 and are now the logs of

$$
\widetilde{G T}=\left[1+(I-1) k^{1 / \theta}\right]^{\eta \theta}
$$

[^7]and
$$
\widetilde{G M P}=\left[\frac{\lambda+(I-1) h^{1 / \theta}\left(\lambda^{M}+\lambda^{G}\right)}{\lambda}\right]^{\theta}\left[\frac{\lambda+(I-1) \widetilde{m}^{1 / \theta}\left(\lambda^{M}+\lambda^{G}\right)}{\lambda}\right]^{\theta \eta},
$$
respectively. We will also compute $G D$, which is the $\log$ of
$$
\widetilde{G D}=\left[\frac{\lambda+(I-1) \lambda^{G}}{\lambda}\right]^{\theta(1+\eta)} .
$$

It is important to note that $G O<G D+G T M P$, a reflection of the fact that diffusion and trade/MP are substitutes. This substitutability arises because diffusion, trade and MP are different ways of sharing ideas across countries: once diffusion is available, then trade and MP are less valuable, and once trade and MP are available, then diffusion is less valuable.

We conclude this section with some definitions for the quantitative analysis. Let $X_{n}^{T}$ denote total spending on intermediate (tradable) goods by country $n$ and let $D_{n i} \equiv M_{n i} / X_{n}^{T}$. For the estimation procedure it is convenient to further normalize trade shares by $D_{i i}^{T} \equiv 1-\sum_{n \neq i} D_{n i} .{ }^{14}$ Thus, we focus on the following normalized trade shares

$$
\begin{equation*}
\tau_{n i} \equiv \frac{D_{n i}}{D_{i i}^{T}} \tag{15}
\end{equation*}
$$

It is worth noting that the normalized trade shares $\tau_{n i}$ would be equal to one in a model with no diffusion (i.e., no global technologies) if there were no trade costs (i.e., $k_{n i}=1$ all $n, i$ ). Normalized trade shares will be lower than one in our model both because of trade costs and because of MP and diffusion. Similarly, for $n \neq i$, we let $D_{n i}^{M} \equiv X_{n i}^{M P} / X_{n}$ denote total MP by $i$ in $n$ as a share of total absorption or GDP in $n$, where now $X_{n i}^{M P}$ is equal to the sum of MP in tradable and non-tradable goods, $X_{n i}^{M P}=X_{n i}^{T, M P}+X_{n i}^{N T, M P}$. We further normalize MP shares by $D_{i i} \equiv 1-\eta \sum_{n \neq i} D_{n i},{ }^{15}$

$$
\begin{equation*}
\tau_{n i}^{M} \equiv \frac{D_{n i}^{M}}{D_{i i}} . \tag{16}
\end{equation*}
$$

[^8]
## 3 Model's Calibration

In relating the model to the data, we think of MP from country $i$ in $n$ as the gross value of production in country $n$ generated by affiliate plants of multinationals with home country i. Additionally, we relate imports of the home country national input associated with MP done by $i$ in $n$ in the model with "intra-firm" exports from $i$ to $n$ in the data. We calibrate the model's parameters using data on bilateral trade in manufacturing goods, bilateral sales of foreign affiliates, intra-firm imports by multinational affiliates, manufacturing prices, gross value of production in manufacturing, and a measure of equipped-efficient labor, for nineteen OECD countries. We use the calibrated version of the model to calculate gains from openness.

The main purpose of this calibration exercise is to illustrate the rich implications the model has regarding the interactions among trade, MP and diffusion, across countries, and how these interactions affect the contribution of each of these channels to welfare gains. Further, the calibrated version of the model helps us understanding which parameters are key to evaluate gains from trade, MP, and diffusion, and which others are not.

### 3.1 Data Description

We restrict our analysis to a set of nineteen OECD countries (also considered by Eaton and Kortum, 2002): Australia, Austria, Belgium/Luxemburg, Canada, Denmark, Spain, Finland, France, United Kingdom, Germany, Greece, Italy, Japan, Netherlands, Norway, New Zealand, Portugal, Sweden, United States. For bilateral variables, we have 342 observations, each corresponding to one country-pair. Depending on availability, our observations are for 1990, an average over the period 1990-2002, or for the late 1990s.

We use data on trade flows from country $i$ to country $n$, in the manufacturing sector (our proxy for the tradable sector). These data are from the STAN data set for OECD countries, for both 1990 and an average over 1990-2002 (see below). We take this measure as the empirical counterpart for bilateral trade flows, $M_{n i}$, in the model.

Our measure of bilateral MP flows is gross value of production for multinational affiliates from $i$ in $n .{ }^{16}$ The available data for this variable includes all sectors combined as averages

[^9]over 1990-2002. The main source of these data is UNCTAD (see Ramondo, 2006, for a detailed description). This variable is the empirical counterpart for bilateral MP flows, $X_{n i}^{M P}$, in the model.

We normalize trade flows as indicated by (15). Thus, we need to calculate $X_{i}^{T}$ and $D_{i i}^{T}$, from the data. We compute total expenditure in tradable goods as $X_{i}^{T}=G P M_{i}+I M_{i}-E X_{i}$, where $G P M_{i}$ refers to gross production in manufacturing, $I M_{i}$ refers to imports of manufacturing goods into country $i$ from the remaining 18 OECD countries in the sample, and $E X_{i}$ refers to total manufacturing exports from country $i$ to the rest of the world. We calculate country $i$ 's share of domestic manufacturing sales as $D_{i i}^{T}=\left(G P M_{i}-E X_{i}\right) / X_{i}^{T}$.

Data on manufacturing gross production, exports, and imports are from the STAN database, for each country, for the period 1990-2002. Combining $M_{n i}, X_{n i}^{M P}, X_{i}^{T}$, and $D_{i i}^{T}$, we obtain the empirical counterparts for the normalized bilateral trade shares, $\tau_{n i}$, and the aggregate ratio of MP to trade flows, $\sum_{i, n} X_{n i}^{M P} / \sum_{i, n} M_{n i}$, in the model.

As explained above, we think of intra-firm trade as the empirical counterpart for imports of the home-country national input by multinational affiliates in the model. We only have data on intra-firm trade involving the United States, from the Bureau of Economic Analysis (BEA), from 1999 to 2003. We combine data on intra-firm exports from the United States to affiliates of U.S. multinationals in foreign countries with data on imports done by affiliates of foreign multinationals located in the United States from their parent firms. This is the empirical counterpart for imports of intermediate goods associated with MP for either $n=U S$ or $i=U S$. In the simple model this is $\omega_{n i} \sum_{j} s_{j n i}^{M} w_{j} L_{j}$ (see equation (6)) whereas in the full model (the one we calibrate below) this is $\eta \omega_{n i} \sum_{j} s_{j n i}^{M} w_{j} L_{j}$.

Additionally, from the BEA, for the period 1999-2003, we record bilateral sales of American affiliates abroad and foreign affiliates in the US, only for the manufacturing sector, as share of total sales in the foreign market and US, respectively. This variable is the empirical counterpart for $X_{n i}^{T, M P} / X_{n i}^{M P}$ in the model.

Finally, when the US is one of the trading partners, we are able to compute the empirical counterpart for BMP in the model (i.e. the share of the value of production done by $i$ in $n$ that is sold in a different market $j$ ). The BEA divides total sales of American affiliates abroad into sales to the local market, to the US, and to third foreign markets. Analogous data are available
for foreign affiliates in the US. We use an average over 1999-2003.
As in Eaton and Kortum (2002), we use international price data for 50 manufacturing products from the United Nations International Comparison Program 1990 benchmark study to construct a measure of bilateral trade costs, $k_{n i}$. For each tradable good $u$, and each country pair $i$ and $n$, we compute the logarithm of relative prices, $r_{n i}(u) \equiv \log p_{n}(u)-\log p_{i}(u)$, and pick the second highest (for possible measurement error) as a measure of trade costs. ${ }^{17}$ Our trade cost measure is then given by $\log k_{n i}=-\max _{2 u} r_{n i}(u) .{ }^{18}$

Finally, we need an empirical counterpart for the model variable $L_{i}$. This variable captures the total number of "equipped-efficiency" units available for production, so employment must be adjusted to account for human and physical capital available per worker. We use the measure of equipped-efficient labor constructed by Klenow and Rodriguez-Clare (2005), for OECD(19) countries, as an average over the nineties. Countries with a higher share of equipped- efficient labor are considered larger. We henceforth simply refer to this notion of "equipped-efficiency" units of labor as total labor.

### 3.2 Calibration Procedure

Our procedure is to calibrate some of the model's parameters by targeting moments from the data on trade and MP, for $\operatorname{OECD}(19)$ countries. We reduce the number of parameters to calibrate by assuming that: (i) the stock of ideas relative to the labor force is the same across countries, $\lambda_{i}^{N}+\lambda_{i}^{M}+\lambda_{i}^{G}=\phi L_{i}$; and (ii) $\lambda_{i}^{M}=\delta_{M} \phi L_{i}$ and $\lambda_{i}^{G}=\delta_{G} \phi L_{i}$ for all $i$ for some common parameters $\delta_{M}$ and $\delta_{G}$. These two assumptions imply that we only need to estimate two parameters related to technologies, $\delta_{M}$ and $\delta_{G}$, since $\phi$ will not affect any of the variables of interest for our analysis. ${ }^{19}$

We assume that bilateral MP costs in the tradable sector, $h_{n i}^{T}$, are related to trade costs $k_{n i}$

[^10]according to
\[

$$
\begin{equation*}
h_{n i}^{T}=k_{n i}+\gamma \varepsilon_{n i}\left(1-k_{n i}\right), \tag{17}
\end{equation*}
$$

\]

where $\varepsilon_{n i}$ is independently drawn from the uniform distribution with support $[0,1]$ and $\gamma \in[0,1]$. These assumptions imply that $h_{n i}^{T} \in\left[k_{n i} ; 1\right]$, and that the correlation between $h_{n i}^{T}$ and $k_{n i}$ is regulated by $\gamma$. In particular, higher $\gamma$ implies lower correlation between trade and MP costs, and viceversa. Further, for MP costs in the non-tradable sector, we assume that they are proportional to the ones in the tradable sector:

$$
\begin{equation*}
h_{n i}^{N T}=\mu h_{n i}^{T} \tag{18}
\end{equation*}
$$

The model's parameters to calibrate are $\delta_{M}, \delta_{G}, \theta, \gamma, \rho, a, \mu, \alpha, \beta, \phi$. First, we normalize $\phi=$ 1. Second, for the labor share in the tradable sector $\beta$, and non-tradable sector $\alpha$, we use 0.5 and 0.75 , respectively, as calibrated by Alvarez and Lucas (2007). Unfortunately, the limited data available for intra-firm trade (only with US as origin or destination) is not enough to pin down the elasticity of substitution $\rho$ between home and host country inputs for MP. Hence, we calibrate the model and calculate welfare gains for two values of this elasticity: a reasonable "central" value $\rho=4$ and a "low" value $\rho=1.5$.

We end up with a vector of six model's parameters to calibrate $\left[\delta_{M}, \delta_{G}, \theta, \gamma, a, \mu\right]$, for each $\rho$. These parameters correspond to the share of multinational technologies in the total stock $\delta^{M}$, the share of global ideas in the total stock $\delta^{G}$, the variability of costs draws for tradable and non-tradable goods $\theta$, the importance of the random component of MP costs in (17) given by $\gamma$, the weight of the home-country national input $a$ in the CES cost function for MP in equation (12), and the efficiency loss incurred by MP in the non-tradable sector, $\mu$.

### 3.2.1 Moments

We choose to match six moments of the data that, according to our model, pin down the six parameters of interest. They are:

1. average normalized bilateral trade shares, $\tau_{n i}$, across country pairs;
2. average normalized bilateral MP shares, $\tau_{i n}^{M}$, across country pairs;
3. correlation coefficient between bilateral trade and MP shares, $C O R\left(\tau_{n i} ; \tau_{n i}^{M}\right)$, across country pairs;
4. OLS coefficient on trade costs in the following "gravity" regression:

$$
\begin{equation*}
\log \tau_{n i}=b_{g} \log k_{n i}+S_{i}+H_{n}+v_{n i} \tag{19}
\end{equation*}
$$

where $S_{i}$ and $H_{n}$ are two sets of source and host country fixed effects, respectively;
5. average MP in manufacturing sector by $i$ in $n$ as share of total MP by $i$ in $n, X_{n i}^{T, M P} / X_{n i}^{M P}$, across country-pairs, for $i=U S$ or $n=U S$;
6. average imports of affiliates from $i$ to $n$ as share of total MP by $i$ in $n$ :

$$
\begin{equation*}
\widetilde{\omega}_{n i}=\frac{\eta \omega_{n i} \sum_{j} s_{j n i}^{M} w_{j} L_{j}}{X_{n i}^{M P}} \tag{20}
\end{equation*}
$$

for $n=U S$ or $i=U S$.

Table 2 below summarizes the moments from the data. Normalized trade and MP shares are calculated as an average over the nineties, for each country-pair, for manufacturing goods. Trade costs $k_{n i}$ in (19) are calculated from prices for manufacturing products across OECD countries, for 1990. Consistently, in equation (19), we use data for normalized manufacturing trade flows for 1990. Data on bilateral imports of affiliates needed to calculate (20), as well as bilateral MP in the manufacturing sector are averages over 1999-2003, for country-pairs with $n=U S$ or $i=U S$.

Even though these six moments jointly identify the six parameters to calibrate, given $\rho$, some moments are more responsive to some parameters than others. Intuitively, one can think that the share of multinational technologies and global technologies, $\delta_{M}$ and $\delta_{G}$, are pinned down by average trade and MP flows, $\tau_{n i}$ and $\tau_{n i}^{M}$ (moments 1 and 2). Higher $\delta_{M}$ implies more MP (and more "intra-firm" trade), while higher $\delta_{G}$ implies less trade and MP. On the other hand, the correlation between trade and MP flows (moment 3) is determined in the model both by the correlation between trade and MP costs, which is linked to $\gamma$, and by the complementarity between trade and MP costs, which is linked to the elasticity of substitution $\rho$.

To understand the role of the OLS coefficient $b_{g}$ in (19), recall that Eaton and Kortum (2002) run a regression like the one in (19) without including source and host country fixed effects; their resulting coefficient is an unbiased estimate of $1 / \theta$ in their model. We add source and host country fixed effects to the regression as mandated by the model given the presence of MP and diffusion. But since total trade flows are the sum of arms-length and intra-firm trade, the estimated coefficient $b_{g}$ is now affected by the way in which intra-firm trade responds to trade costs that is determined by $\rho$ in our model. We also have to consider that MP costs $h_{n i}^{T}$ indirectly affect trade flows. Since $h_{n i}^{T}$ are part of the residual $\nu_{n i}$ in (19), the positive correlation between $k_{n i}$ and $h_{n i}^{T}$ lowers $b_{g}$. All this implies that $b_{g}$ (moment 4) helps to pin down several parameters: $\theta, \rho$, and $\gamma$. The share of MP in tradable goods (moment 5) pins down the efficiency loss of doing MP in the non-tradable sector, $\mu$. Finally, the share of intra-firm imports in total MP $\widetilde{\omega}_{n i}$ in (20)(moment 6) helps to pin down the CES parameter $a$ in the cost function for MP.

For a given $\rho$ and a set of parameter values $\Delta$, matrix of trade costs $k_{n i}$, matrix of random draws $\varepsilon_{n i}$ ( $\varepsilon$ matrix), and vector of country sizes $L_{n}$, we compute the equilibrium of the model and generate a simulated data set with 361 observations (one for each country-pair, including the domestic pairs) for each of the following variables: MP costs $h_{n i}^{T}$ and $h_{n i}^{N T}$, normalized trade shares $\tau_{n i}$, normalized MP shares $\tau_{n i}^{M}$, intra-firm trade shares $\widetilde{\omega}_{n i}$, and MP in tradable goods by $i$ in $n$ as share of total MP by $i$ in $n$. Additionally, the model equilibrium generates "bridge" MP between country-pairs (i.e. the share of the value of production done by $i$ in $n$ that is sold in a different market $j$ ).

The algorithm used to compute the equilibrium builds on the one in Rodríguez-Clare (2007), which in turn extends the one in Alvarez and Lucas (2007) (see the Appendix for a description). For the simulated data, we compute the six moments enumerated above. For a given $\rho$, a set of parameter values $\Delta$, and a $\varepsilon$ matrix, we can compute a vector of simulated moments, denoted by $\operatorname{MOM}_{s}(\rho, \Delta, \varepsilon)$. We use a simulated method of moments procedure that minimizes

$$
\Delta^{*}(\rho)=\underset{\Delta}{\arg \min }\left[M O M_{d}-\sum_{\varepsilon \in \Omega} \operatorname{MOM}_{s}(\rho, \Delta, \varepsilon)\right]^{\prime} I\left[M O M_{d}-\sum_{\varepsilon \in \Omega} \operatorname{MOM}_{s}(\rho, \Delta, \varepsilon)\right] .
$$

The set $\Omega$ includes the $\varepsilon$ matrices used for different simulations, $I$ is the identity matrix, and $M O M_{d}$ is the vector of moments from the data. ${ }^{20,21}$ Table 2 below reports the targeted data moments.

### 3.3 Results

The calibrated parameters are reported in Table 1, and the targeted moments in Table $2 .{ }^{22}$ We refer to calibration (I), with $\rho=4$, as the "benchmark". The remaining columns recalibrate the parameters of the model under different assumptions: (II) lowers $\rho$ to 1.5; (III) forces no diffusion $\left(\delta_{G}=0\right)$; and (IV) shows how the calibration would change if the intra-firm trade share, $\widetilde{\omega}_{n i}$, were double the one we observe in the data ( 0.15 rather than 0.074 ), and $\rho=1.5$. Additionally, the last two rows of this table show the implied statistics for MP costs, $h_{n i}^{T}$, and its correlation with trade costs, $k_{n i}$, for each calibration.

There are several things to note about these results. First, the estimate of $\theta$ does not vary significantly with $\rho$. Its value is higher than Eaton and Kortum's (2002) central result of $\theta=0.12$, but within the range of their estimates, $[0.08,0.28]$. The difference between our results and Eaton and Kortum's is due to the presence of MP and diffusion, which leads to an intercept in the gravity equation that affects the estimated OLS coefficient $b_{g}$ in (19). ${ }^{23}$

Second, the results in Table 1 also show that lower values of $\rho$ correspond to higher values of $\gamma$. As we increase the complementarity between home and host country inputs in MP (lower $\rho$ ), we need a lower correlation between trade and MP costs (higher $\gamma$ ) to match the observed positive correlation between trade and MP flows across country pairs. However, the more dramatic change in this parameter occurs when we not only decrease $\rho$, but also target a much higher intra-firm trade share, doubling the one observed in the data (from 0.074 to 0.15 ); as column IV shows, $\gamma$ increases from 0.83 to 0.96 . As higher bilateral intra-firm trade increases the correlation

[^11]| Parameter | (I) | (II) | (III) | (IV) | Definition |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\delta_{G}$ | 0.025 | 0.019 | 0 | 0.0275 | share of global technologies |
| $\delta_{M}$ | 0.23 | 0.22 | 0.17 | 0.25 | share of multinational technologies |
| $\theta$ | 0.25 | 0.24 | 0.17 | 0.22 | variability of cost draws |
| $\gamma$ | 0.79 | 0.83 | 0.54 | 0.96 | $h_{n i}^{T}=k_{n i}+\gamma \epsilon_{n i}\left(1-k_{n i}\right)$ |
| $a$ | 0.34 | 0.17 | 0.31 | 0.36 | weight of Home intermediate input bundle in (12) |
| $\mu$ | 0.68 | 0.67 | 0.75 | 0.66 | MP cost in non-tradable goods: $h_{n i}^{N T}=\mu h_{n i}^{T}$ |
| $\rho$ | 4 | 1.5 | 4 | 1.5 | elasticity of substitution in MP (12) |
| $E_{h}$ | $\begin{gathered} 0.77 \\ (0.18) \end{gathered}$ | $\begin{gathered} \hline 0.78 \\ (0.18) \end{gathered}$ | $\begin{gathered} 0.78 \\ (0.19) \end{gathered}$ | $\begin{gathered} 0.80 \\ (0.19) \end{gathered}$ | average MP cost |
| $C O R_{k h}$ | 0.70 | 0.67 | 0.67 | 0.56 | correlation trade and MP costs |

Table 1: Parameters' Estimates.
between bilateral trade and MP, in order to much the observed correlation, the model requires a higher value of $\gamma$, that is, less correlation between trade and MP costs. In fact, the last row of Table (1) shows that the correlation between $k$ and $h$ drops from 0.67 to 0.56 when we double the average intra-firm trade share.

Third, the parameter $a$, which regulates intra-firm trade, changes across calibrations with $\rho$. When the elasticity of substitution between source and host country input bundles decreases (lower $\rho$ ), the model generates more intra-firm trade shares (higher $\widetilde{\omega}_{n i}$ ). Hence, the weight of home inputs in the multinational production function has to decrease in order to match the observed intra-firm share. While in the benchmark calibration (I) with $\rho=4, a=0.35$, with $\rho=1.5$ in (II), the parameter $a$ decreases to $0.17 .{ }^{24}$

[^12]In the Appendix, we show the Jacobian matrix for the benchmark calibration. This matrix numerically computes $d \log M_{j} / d \log P_{i}$, where $M_{j}$ denotes the j-moment, and $P_{i}$ the i-parameter. Our analysis of this Jacobian confirms our priors regarding the identification of the main parameters: $\mu$ is identified by the average of $X_{n i}^{T, M P} / X_{n i}^{M P}, a$ is identified by the average of $\widetilde{\omega}_{n i}, \gamma$ is identified by $\operatorname{COR}\left(\tau_{n i} ; \tau_{n i}^{M}\right), \delta_{M}$ and $\delta_{G}$ are jointly identified by the average $\tau_{n i}$ and $\tau_{n i}^{M}$, and $\theta$ is identified by $b_{g}$.

| Moments | Data | (I) | (II) | (III) | (IV) |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Average normalized trade share $\tau_{n i}$ | 0.033 | 0.032 | 0.033 | 0.032 | 0.031 |
| Average normalized MP share $\tau_{n i}^{M}$ | $(0.06)$ |  |  |  |  |
|  | 0.025 | 0.025 | 0.027 | 0.021 | 0.025 |
| Correlation $\left(\tau_{n i} ; \tau_{n i}^{M}\right)$ | $(0.05)$ |  |  |  |  |
|  | 0.70 | 0.66 | 0.67 | 0.63 | 0.73 |
| OLS coefficient $b_{g}^{\dagger}$ |  |  |  |  |  |
|  | 4.70 | 4.40 | 4.40 | 5.85 | 4.47 |
| Average share of MP in tradable goods (for US) | $(0.36)$ |  |  |  |  |
|  | 0.48 | 0.49 | 0.51 | 0.52 | 0.49 |
| Average imported inputs' share (US) $\widetilde{\omega}_{n i}$ | $(0.12)$ |  |  |  |  |
|  | 0.074 | 0.077 | 0.076 | 0.09 | 0.5 |
| Average trade costs $k_{n i}$ | $(0.07)$ |  |  |  |  |
| Average equipped-efficient labor $L_{i}$ (in millions) | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 |
|  | 1.74 | 1.74 | 1.74 | 1.74 | 1.74 |

$\left(^{\dagger}\right)$ : Equation 19; S.E. for data moments in parenthesis.

Table 2: Moments: Data and Model.
One important question is whether we need diffusion to match the data. The inclusion of global ideas in the model deters both trade and MP: when more ideas are available at not extra cost to be used everywhere, certain goods stop being imported or produced through MP. ${ }^{25}$ As diffusion is not directly observed in the data, we ask whether a model without diffusion is able to match the moments in Table 2. We recalibrate the model parameters assuming that $\delta_{G}=0$, and we leave the OLS coefficient $b_{g}$ in 19 as an out-of-sample moment to compare with the data.

[^13]|  | Data | (I) | (II) | (III) |
| :--- | :---: | :---: | :---: | :--- |
| Variation Coef. for $\tau_{n i}$ | 1.85 | 1.14 | 1.13 | 1.16 |
| Variation Coef. for $\tau_{n i}^{M}$ | 1.90 | 1.83 | 1.89 | 1.84 |
| "Bridge" MP (from US) | 0.30 | 0.005 | 0.005 | 0.003 |
| "Bridge" MP (all) | $\mathrm{N} / \mathrm{A}$ | 0.005 | 0.0067 | 0.0011 |

Table 3: Out-of-sample Moments: Data and Model

Column (III) in Table 1 shows that the calibrated $\theta$ drops from 0.25 to 0.17 . The resulting OLS coefficient $b_{g}$ in Table 2 is much higher than the one observed in the data. Intuitively, without diffusion, normalized trade shares are higher than in the data, and the only parameter left in the model to decrease trade is $\theta$. However, a lower $\theta$ implies a weaker pattern of comparative advantage, and hence variation in trade costs have a higher effect on trade flows, implying a higher $b_{g} .{ }^{26}$ Thus, we conclude that diffusion represented by these global technologies, that in the model is a (costless) competing alternative to trade and MP, does have a role beyond the technological diffusion entailed by MP. As we show below, it is an important contributor to overall gains from openness.

We next present some tables and figures that illustrate features of the models that we did not target in our calibrations. The goal is understanding dimensions in which the model succeed, and the ones in which it fails to pick patterns of the data.

Table 3 shows statistics generated by the model that are not included in the calibration. Across calibrations, the model does well in predicting the variation in normalized MP shares across country pairs. However, the implied variation in normalized trade shares is consistently lower than in the data. One failure of the model is that it generates a very low share of BMP. While the data for affiliates from the US in OECD countries shows that $30 \%$ of the value of production is sold in countries other than the country of production, in the model this is only $0.6 \%$. Similar numbers are obtained if all country-pairs are considered (we do not have data

[^14]to compare with). In future work we plan to allow for $\lambda_{i} / L_{i}$ to differ across countries, so that countries with low $\lambda_{i} / L_{i}$ would naturally become "export platforms". This would naturally increase BMP.

The remaining moments in Table 3 just show that, for all country-country, the model generates very similar averages of intra-firm trade shares, and share of MP in tradable goods, to the ones observed for the US.

|  | Levels | GDP shares |
| :--- | :---: | :--- |
| Exports | 0.94 | 0.67 |
| Imports | 0.94 | 0.67 |
| Outward MP | 0.95 | 0.26 |
| Inward MP | 0.86 | -0.05 |

Table 4: Correlations between model and data (benchmark calibration I)

Table 4 shows the model's fit with the data regarding aggregate flows by country: exports, imports, outward MP and inward MP. The predictions of the model correspond to the benchmark calibration with $\rho=4$. The first column shows the correlation between the model and the data in levels, while the second column shows this correlation as shares of GDP. Not surprisingly, the model performs well in terms of levels and it also does quite well regarding exports and imports relative to size. However, it does poorly in terms of aggregate MP shares: while the correlation for outward MP adjusted by size is lower than for trade but still positive, the correlation for inward MP relative to recipient size is virtually zero. Why is the model failing on this dimension? We further explore this issue in Figures 2 and 3 below.

The left panel of Figure 2 shows outward MP as a share of GDP for the model and the data, against the model's GDP, $w_{i} L_{i} .{ }^{27}$ The model correctly captures large countries as the United States, Japan, and Germany, but fails in picking some small countries that either have very high (The Netherlands) or very low (e.g., Spain and New Zealand) outward MP relative to size. The

[^15]right panel shows the analogous scatter for exports. Generally, the model overestimates export shares, but particularly so, for small countries.

Figure 3 is analogous to Figure 2 for inward MP and import shares. While the ratios of inward MP to GDP have a clear negative relationship with size in the model, the data displays a much weaker correlation among small countries. Consequently, the model fails in capturing aggregate inward MP for small countries. Meanwhile, the model captures accurately the relationship between import shares and recipient's size, except for small countries with very high or very low import shares.

Some of the failures of the model in capturing these aggregate patterns might be caused by the way we calibrate technologies. Dropping the proportionality assumption between the stock of technologies $\lambda_{i}$ and size $L_{i}$ may improve the model along this dimension.

Finally, it is interesting to consider the growth implications of the quantitative model. Given the economies of scale associated with the stock of non-rival ideas being proportional to the population level, this model entails quasi-endogenous growth as in Jones (1995), and Kortum (1997). In fact, we can easily get that the growth rate for real wages is given by ${ }^{28}$

$$
g=\theta(1+\eta) g_{L},
$$

where $g_{L}$ is the rate of growth of the labor force in the model. We set $g_{L}$ equal to the rate of growth of people employed in $\mathrm{R} \& \mathrm{D}$, which is $4.8 \%$ over the last decades in the five top $\mathrm{R} \& \mathrm{D}$ countries (see Jones, 2002). Using $\eta=(1-\alpha) / \beta=0.5$ and our estimate for $\theta$ then $g=1.8 \%$, which is just a bit higher than the rate of TFP growth rate observed in the OECD over the last four decades ( $1.5 \%$ according to Klenow and Rodríguez-Clare, 2005).

## 4 Gains from Openness

Gains from openness, trade, MP and diffusion are given by changes in real wages in terms of the final consumption good: $w_{i} / p_{i}$. We calculate real wages under five counterfactual scenarios: (1) isolation, (2) trade but no MP and no diffusion, (3) MP but no trade and no diffusion,

[^16](4) diffusion but no trade and no MP, (5) trade and MP but no diffusion, and (6) MP and diffusion but no trade. The increase in the real wage as we move from counterfactual (1) to the benchmark yields the gains from openness, $G O$. Similarly, the increase in the real wage as we move from (1) to (2) yields the gains from trade, $G T$; from (1) to (3) yields the gains from MP, $G M P$; from (1) to (4) yields the gains from diffusion, GD; from (1) to (5) the joint gains from trade and MP, GTMP. Finally, the increase in the real wage from (6) to the benchmark yields our alternative measure of the gains from trade in the presence of diffusion and MP, $G T^{\prime}$.

We present gains from openness, trade, MP, and diffusion, for the benchmark values of trade costs, MP costs, and shares $\delta_{M}$ and $\delta_{G}$ estimated above, for nineteen OECD countries. Table 5 shows these calculations for the three values of $\rho$ and the corresponding values of the parameters estimated above (see Table 1). The implied gains from openness are large: log gains of around 0.5 imply percentage gains of $65 \%$ on average for the 19 countries in our sample. Of course, these gains will be much larger for the smaller countries, as we show below when we report gains for individual countries.

Interestingly, the gains from trade implied by the model are smaller than the gains from MP, which in turn are smaller than the gains from diffusion. The reason is that MP flows are actually higher than trade flows. For example, total inward MP flows are more than double the total imports in the data. This could seem contradictory with the finding of a small share of multinational technologies (i.e., $\delta_{M}<23 \%$ ). But there are two forces that make MP larger than trade (in the model) in spite of the low share of technologies that allow MP: first, MP costs are lower than trade costs $\left(E_{h}=0.74>E_{k}=0.6\right)$, and second, MP is feasible for non-tradable goods. Similarly, the gains from diffusion are large in spite of a low share of global technologies (i.e., $\delta_{G}<2.5 \%$ ) because of the absence of any costs of diffusion and because of the presence of diffusion in both tradable and non-tradable goods.

In all cases, trade and diffusion behave as substitutes with diffusion: $G O<G D+G M P T$. The difference can be big. In our bechmark calibration the percentage gains from openness are $73 \%$ whereas the added percentage gains from diffusion, trade and MP $(\exp (G D+G M P T))$ are $116 \%$. Results are similar if we instead use the calibration with $\rho=1.5$.

Turning to the relationship between trade and MP, Table 5 shows that they behave as substitutes for the benchmark calibration in the sense that $G M P T<G T+G M P$ : whereas
the joint gains from trade and MP are $54 \%$, the sum of the separate gains from trade and MP is $73 \%$. The difference is smaller for $\rho=1.5$, but even in this case trade and MP behave as substitutes.

It is interesting to ask why it is that even for $\rho=1.5$ we find that trade and MP behave as substitutes. The reason is the relatively small levels of intra-firm trade, which we are using to discipline the parameter $a$ for each $\rho$. In particular, when $\rho$ falls from 4 to 1.5 we have to decrease $a$ from 0.34 to 0.17 , and this weakens the higher complementarity associated with a lower $\rho$. To explore this idea further we recalibrated the model with $\rho=1.5$ to match a value of intra-firm trade that is twice as large as the one in the data, i.e. we used an average $\widetilde{\omega}_{n i}$ of 0.148 rather than 0.074 . We find that now trade and MP behave as complements, although only weakly: $G T+G M P=0.33<0.36=G T M P$. These results suggest that it is "difficult" to get trade and MP to behave as complements: we need a rather low value of $\rho$ and to consider a level of intra-firm trade that is twice what we see in the data.

If we consider all three channels simultaneously, they behave as substitutes (i.e., $G O<$ $G D+G T+G M P)$ for all calibrations in which we allow for diffusion (I, II, and IV). In the benchmark calibration, the difference between the joint gains and the sum of the separate gains is quite high: whereas $G O$ are $73 \%$, the added gains from diffusion, trade and MP are $141 \%$.

Turning to the gains from trade given the presence of MP and diffusion, $G T^{\prime}$, we see that as one would expect - it increases with the degree of complementarity between trade and MP. In particular, it increases from 0.06 to 0.11 as $\rho$ falls from 4 to 1.5. It is interesting to compare this measure of gains to the gains from trade in Eaton and Kortum (2002), which we associate with $G T$ under $\theta=0.12$ and label $G T_{E K}$. Table 5 shows that $G T_{E K}=0.021$ of $2.1 \%{ }^{29}$ There are three sources of differences between $G T^{\prime}$ and $G T_{E K}$. First, the fact that there is diffusion and MP in our model, and that in general these flows behave as substitutes with trade, implies that $G T^{\prime}$ will tend to be lower than $G T$ and $G T_{E K}$. Second, the higher value of $\theta=0.25$ that we estimate in comparison with Eaton and Kortum's $\theta=0.12$ will increase $G T^{\prime}$ and $G T$ over $G T_{E K}$. We see that the latter effect dominates, so that $G T^{\prime}>G T_{E K}$. For calibration (II) we see that $G T^{\prime}$ is more than 5 times higher than $G T_{E K}$.

Table 5 also shows the gains of moving from the benchmark to a case of frictionless trade,

[^17]|  | $\log (w / p)-\log \left(w^{i s o} / p^{i s o}\right)$ |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| From isolation to: | (I) | $(\mathrm{II})$ | $(\mathrm{III})$ | $(\mathrm{IV})$ |
| trade, MP, and diffusion (GO) | 0.55 | 0.53 | 0.14 | 0.49 |
| only trade (GT) | 0.19 | 0.18 | 0.07 | 0.15 |
| only MP (GMP) | 0.35 | 0.24 | 0.09 | 0.18 |
| only diffusion (GD) | 0.34 | 0.33 | 0 | 0.32 |
| only trade and MP (GTMP) | 0.43 | 0.39 | 0.14 | 0.36 |
| only trade with $\theta=0.12\left(G T_{E K}\right)$ | 0.02 | 0.02 | 0.02 | 0.02 |
| trade given MP and diffusion (GT') | 0.06 | 0.11 | 0.05 | 0.11 |
| From benchmark to frictionless trade and MP | 0.75 | 0.75 | 0.69 | 0.70 |
| From benchmark to frictionless trade | 0.36 | 0.33 | 0.29 | 0.34 |
| From benchmark to frictionless MP | 0.43 | 0.39 | 0.41 | 0.34 |

Table 5: Gains from Openness: benchmark (average OECD)
frictionless MP, or frictionless trade and MP. The results suggest significant gains in further reducing trade and MP barriers.

Table 6 shows $G O, G T, G M P, G D, G T M P, G T^{\prime}$ and $G T_{E K}$ for each country in our sample under $\rho=4$. Countries are ordered by size (according to total equipped labor). Indeed, gains from openness decrease with size. Moreover, for all countries diffusion, trade and MP behaves as substitutes in the sense that $G D+G T+G M P>G O$. Notice that a country like Belgium, which represents around $1 \%$ of total worldwide equipped labor, has $G O=0.82$, which imply percentage gains of $129 \%$. This is less than half of the sum of the separate gains from diffusion, trade and MP are $285 \%$. Of course, this difference between $G O$ and $G D+G T+G M P$ is lower for lower values of $\rho$.

|  | GO | GT | GMP |  | $\left[\log \left(w^{\prime} / p^{\prime i s o} / p^{i s o}\right)\right] * 100$ |  |  | $G T_{E K}$ | $\overline{G_{Z} G}$ | $\text { L/ } \sum_{(\%)} L$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| New Zealand | 110 | 39 | 75 | 81 | 85 | 8 | 13 | 5 | 48 | 0.3 |
| Portugal | 82 | 27 | 36 | 64 | 49 | 8 | 6 | 3 | 55 | 0.5 |
| Greece | 79 | 19 | 49 | 59 | 54 | 4 | 13 | 1 | 50 | 0.5 |
| Finland | 95 | 36 | 57 | 69 | 71 | 10 | 10 | 5 | 48 | 0.6 |
| Norway | 98 | 27 | 74 | 67 | 80 | 5 | 20 | 2 | 42 | 0.7 |
| Denmark | 92 | 32 | 56 | 66 | 68 | 10 | 12 | 4 | 47 | 0.8 |
| Austria | 86 | 31 | 56 | 59 | 66 | 8 | 13 | 3 | 45 | 0.9 |
| Belgium | 82 | 33 | 49 | 53 | 62 | 11 | 11 | 5 | 44 | 1.1 |
| Sweden | 83 | 29 | 59 | 50 | 68 | 8 | 17 | 3 | 38 | 1.1 |
| Australia | 58 | 22 | 34 | 33 | 44 | 8 | 11 | 2 | 39 | 1.72 |
| Netherlands | 59 | 18 | 31 | 40 | 39 | 6 | 9 | 1 | 46 | 1.73 |
| Spain | 42 | 12 | 21 | 26 | 27 | 5 | 8 | 1 | 41 | 2.7 |
| Canada | 37 | 14 | 14 | 22 | 23 | 7 | 5 | 1 | 39 | 2.9 |
| Italy | 34 | 14 | 16 | 18 | 23 | 6 | 6 | 1 | 35 | 5.3 |
| United Kingdom | 25 | 8 | 8 | 15 | 14 | 4 | 40 | 36 | 5.8 |  |
| France | 30 | 12 | 13 | 16 | 21 | 6 | 6 | 1 | 33 | 6.5 |
| Germany | 19 | 8 | 6 | 10 | 12 | 5 | 3 | 1 | 28 | 10 |
| Japan | 11 | 3 | 5 | 5 | 8 | 2 | 4 | 0 | 17 | 20 |
| United States | 6 | 2 | 3 | 2 | 5 | 2 | 2 | 0 | 8 | 37 |

Countries sorted by R\&D employment.

Table 6: Gains from Openness, by country.

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## A Equilibrium analysis of the full model

We conduct this analysis with the latent variables $\left\{x_{i}^{N}, x_{i}^{M}, x_{i}^{G}\right\}$ and $\left\{\xi_{i}^{N}, \xi_{i}^{M}, \xi_{i}^{G}\right\}$. Intermediate good prices satisfy

$$
p_{n}\left(x^{N}, x^{M}, x^{G}\right)=\min \left\{\min _{i}\left\{\left(x_{i}^{N}\right)^{\theta} \frac{c_{i}^{T}}{k_{n i}}\right\},\left(x^{G}\right)^{\theta} \widetilde{c}_{n}^{T}, \min _{i}\left\{\left(x_{i}^{M}\right)^{\theta} \widetilde{c}_{n i}^{T}\right\}\right\}
$$

while for consumption goods we have

$$
p_{n}^{N T}\left(\xi^{N}, \xi^{M}, \xi^{G}\right)=\min \left\{\left(\xi_{n}^{N}\right)^{\theta} c_{n}^{N T},\left(\xi^{G}\right)^{\theta} c_{n}^{N T}, \min _{i}\left\{\left(\xi_{i}^{M}\right)^{\theta} c_{n i}^{N T}\right\}\right\}
$$

where $\widetilde{c}_{n}^{T} \equiv \min _{l}\left\{c_{l}^{T} / k_{n l}\right\}, \widetilde{c}_{n i}^{T} \equiv \min _{l}\left\{c_{l i}^{T} / k_{n l}\right\}, x^{G}=\min _{i} x_{i}^{G}$ and $\xi^{G}=\min _{i} \xi_{i}^{G}$.
The variable $p_{n}\left(x^{N}, x^{M}, x^{G}\right)^{1 / \theta}$ is still distributed exponentially with parameter $\psi_{n}$ but now

$$
\psi_{n} \equiv \sum_{i}\left(\psi_{n i}^{N}+\psi_{n i}^{M}\right)+\psi_{n}^{G}
$$

where

$$
\psi_{n i}^{N}=\left(c_{i}^{T} / k_{n i}\right)^{-1 / \theta} \lambda_{i}^{N}, \quad \psi_{n i}^{M}=\left(\widetilde{c}_{n i}^{T}\right)^{-1 / \theta} \lambda_{i}^{M}, \quad \psi_{n}^{G}=\left(\widetilde{c}_{n}^{T}\right)^{-1 / \theta} \lambda^{G},
$$

and

$$
\lambda^{G}=\sum_{i} \lambda_{i}^{G} .
$$

The price index for intermediates, $p_{m n}$, is now given by (2).
Similarly, the price index of the consumption CES aggregate in country $n$ is given by

$$
p_{n}=C \chi_{n}^{-\theta}
$$

where $\chi_{n}$ plays the same role for consumption goods as $\psi_{n}$ for intermediate goods, with

$$
\chi_{n} \equiv \chi_{n n}^{N}+\sum_{n i}^{M} \chi+\chi_{n}^{G}
$$

where

$$
\chi_{n n}=\left(c_{n}^{N T}\right)^{-1 / \theta}\left(\lambda_{n}^{N}+\lambda^{G}\right), \quad \chi_{n i}^{M}=\left(c_{n}^{N T}\right)^{-1 / \theta} \lambda_{i}^{M}, \quad \chi_{n}^{G}=\left(c_{n}^{N T}\right)^{-1 / \theta} \lambda^{G} .
$$

The analysis in Section 2.1.1 to compute total imports by $n$ from $i$ is still valid except for three changes. First, the value of of intermediate goods produced with national technologies in country $l$ that are exported to country $n$ is no longer $s_{n l}^{N} X_{n}$ but $s_{n l}^{N} X_{n}^{T}$, where $X_{n}^{T}$ is total spending on intermediates by country $n$. Similarly, total imports by country $n$ from $l$ of intermediate goods produced with multinational technologies are now $\sum_{i} s_{n l i}^{M} X_{n}^{T}$, while $\sum_{n} s_{n l i}^{M} X_{n}^{T}$ is now the total MP in intermediates by $i$ in $l, X_{l i}^{T, M P}$, and intra-firm exports from $i$ to $l$ are $\omega_{l i} X_{l i}^{T, M P}$.

Second, we now have to take into account trade in intermediate goods that are produced with global technologies. The value of intermediate goods bought by $n$ that are produced with global technologies is $\frac{\chi_{n}^{G}}{\chi_{n}} X_{n}^{T}$. These goods could be produced domestically or imported from any country $l \in \arg \min _{j}\left\{c_{j} / k_{n j}\right\}$. Let $y_{n l}^{G}$ be the share of total spending by country $n$ on goods produced with global technologies that are produced in country $l$ (and then shipped to country $n)$. Clearly, $\sum_{l} y_{n l}^{G}=1$. In equilibrium, the following "complementary slackness" conditions must hold:

$$
\begin{aligned}
c_{l} / k_{n l}>\widetilde{c}_{n} & \Longrightarrow \quad y_{n l}^{G}=0, \\
y_{n l}^{G}>0 \quad & \Longrightarrow \quad c_{l} / k_{n l}=\widetilde{c}_{n} .
\end{aligned}
$$

Letting $s_{n l}^{G} \equiv y_{n l}^{G} \frac{\psi_{n}^{G}}{\psi_{n}}$, then imports by country $n$ of goods produced in country $l$ with global technologies are

$$
s_{n l}^{G} X_{n}^{T} .
$$

Total imports by $n$ from $i \neq n$ are now

$$
M_{n i}=s_{n i}^{N} X_{n}^{T}+s_{n i}^{G} X_{n}^{T}+\sum_{j} s_{n i j}^{M} X_{n}^{T}+\omega_{n i} \sum_{j} s_{j n i}^{M} X_{j}^{T}
$$

Total spending on final goods by country $n$ is $X_{n}=w_{n} L_{n}$, while it can be shown that total spending on tradable intermediate goods is $X_{n}^{T}=\eta X_{n}$. This result follows from assuming CobbDouglas production functions for both intermediate and final goods and is proved in the next
section of the Appendix. Total imports by $n$ from $i$ are then

$$
M_{n i}=\eta\left(s_{n i}^{N}+s_{n i}^{G}+\sum_{j} s_{n i j}^{M}\right) w_{n} L_{n}+\eta \omega_{n i} \sum_{j} s_{j n i}^{M} w_{j} L_{j} .
$$

Finally, the total value of MP by $i$ in $n$ is now given by the value of MP for intermediate goods, $X_{n i}^{T, M P}$, plus the corresponding value for consumption goods, $X_{n i}^{N T, M P}$. Since these goods are non-tradable, we simply need to derive an expression for the share of goods $v \in[0,1]$ bought by country $n$ that are produced with multinational technologies from country $i$. Again, from the properties of the exponential distribution, this is given by $s_{n i}^{N T, M} \equiv \chi_{n i}^{M} / \chi_{n}$. Thus, $X_{n i}^{N T, M P} \equiv s_{n i}^{N T} X_{n}$, and the total value of MP by country $i$ in country $n$ is

$$
X_{n i}^{M P} \equiv X_{n i}^{T, M P}+X_{n i}^{N T, M P}=\sum_{j} s_{j n i}^{M} X_{j}^{T}+s_{n i}^{N T, M} X_{n}
$$

or

$$
X_{n i}^{M P}=\eta \sum_{j} s_{j n i}^{M} w_{j} L_{j}+s_{n i}^{N T} w_{n} L_{n}
$$

## B Proofs

First we prove that for the symmetric example analyzed in Section 2.1.2, if $\rho-1>1 / \theta$, and $h>k$, then $G T M P<G T+G M P$.

Proof: Let $\widetilde{\lambda}=\lambda+\lambda^{M}+\lambda^{G}$. Recall that $\widetilde{G T M P}, \widetilde{G T}$, and $\widetilde{G M P}$ are given by:

$$
\begin{aligned}
\widetilde{G T M P} & =\frac{p_{I S O L}}{p_{T M P}}=\left[\frac{\widetilde{\lambda}+(I-1) h^{1 / \theta}\left(\lambda^{M}+\lambda^{G}\right)}{\widetilde{\lambda}}\right]^{\theta} \cdot\left[\frac{\widetilde{\lambda}+(I-1)\left(k^{1 / \theta} \lambda^{N}+m^{1 / \theta}\left(\lambda^{M}+\lambda^{G}\right)\right)}{\widetilde{\lambda}}\right]^{\eta \theta}, \\
\widetilde{G T} & =\frac{p_{I S O L}}{p_{T}}=\left[1+(I-1) k^{1 / \theta}\right]^{\eta \theta}, \\
\widetilde{G M P} & =\frac{p_{I S O L}}{p_{M P}}=\left[\frac{\widetilde{\lambda}+(I-1) h^{1 / \theta}\left(\lambda^{M}+\lambda^{G}\right)}{\widetilde{\lambda}}\right]^{\theta}\left[\frac{\widetilde{\lambda}+(I-1) \widetilde{m^{1 / \theta}\left(\lambda^{M}+\lambda^{G}\right)}}{\widetilde{\lambda}}\right]^{\theta \eta} .
\end{aligned}
$$

We find sufficient conditions under which $\widetilde{G T} \cdot \widetilde{G M P}>\widetilde{G T M P}$.

$$
\begin{aligned}
& {\left[1+(I-1) k^{1 / \theta}\right] \cdot } {\left[\widetilde{\lambda}+(I-1) \widetilde{m}^{1 / \theta}\left(\lambda^{M}+\lambda^{G}\right)\right] } \\
&>\widetilde{\lambda}+(I-1)\left(k^{1 / \theta} \lambda^{N}+m^{1 / \theta}\left(\lambda^{M}+\lambda^{G}\right)\right) \\
& \widetilde{m}^{1 / \theta}+k^{1 / \theta}+(I-1)(k \widetilde{m})^{1 / \theta}>m^{1 / \theta} .
\end{aligned}
$$

For the above inequality to hold it is sufficient that:

$$
\widetilde{m}^{1 / \theta}+k^{1 / \theta}>m^{1 / \theta} .
$$

Recall that $m \equiv\left[(1-a) h^{\rho-1}+a k^{\rho-1}\right]^{\frac{1}{\rho-1}}$, and $\widetilde{m} \equiv(1-a)^{\frac{1}{\rho-1}} h$. Thus, replacing these expressions in the inequality above, and rearranging we get:

$$
\left[\left((1-a)^{\frac{1}{\rho-1}} h\right)^{1 / \theta}+k^{1 / \theta}\right]^{\theta}>\left[\left((1-a)^{\frac{1}{\rho-1}} h\right)^{\rho-1}+a k^{\rho-1}\right]^{\frac{1}{\rho-1}} .
$$

For $k \leq 1, h \leq 1$, and $a \leq 1$, if $1 / \theta<\rho-1$, then the inequality above holds because the function $f(x)=\left(c^{x}+d^{x}\right)^{1 / x}$ is decreasing in $x$ for $x>1$ and $\left.c, d \in\right] 0,1[$. This implies that $\widetilde{G T M P}<\widetilde{G T} \cdot \widetilde{G M P}$.

Second, we prove that for the symmetric example analyzed in Section 2.1.2, if $\rho-1>1 / \theta$, and $h>k$, then $d \log G T^{\prime} / d \log h<0$.

Proof: Let $\widetilde{\lambda}=\lambda+\lambda^{M}+\lambda^{G}, x=\widetilde{m}^{1 / \theta}$, and $y=m^{1 / \theta}$. Note that since $\widetilde{m}>m, x<m$. We can rewrite $G T^{\prime}$ as

$$
G T^{\prime}=\left(\frac{\widetilde{\lambda}+(I-1)\left(k^{1 / \theta} \lambda^{N}+y \lambda^{M}+\lambda^{G}\right)}{\widetilde{\lambda}+(I-1) x\left(\lambda^{M}+\lambda^{G}\right)}\right)^{\eta \theta}
$$

Letting $y^{\prime}=d y / d \log h, x^{\prime}=d x / d \log h, H \equiv \widetilde{\lambda}+(I-1) x\left(\lambda^{M}+\lambda^{G}\right)$, and $M \equiv \widetilde{\lambda}+(I-$ 1) $\left(k^{1 / \theta} \lambda^{N}+y \lambda^{M}+\lambda^{G}\right)$, then

$$
\frac{d \log G T^{\prime}}{d \log h}=\frac{\eta \theta(I-1) \lambda^{M}\left(y^{\prime} H-x^{\prime} M\right)}{M H} .
$$

Hence,

$$
\left(\frac{H}{\eta \theta(I-1) \lambda^{M}}\right) \frac{d \log G T^{\prime}}{d \log h}=\frac{y^{\prime} H-x^{\prime} M}{M} .
$$

As $M>H>0, G T^{\prime}$ is decreasing in $h$ if $x^{\prime}>y^{\prime}>0$. Recall that $x=\widetilde{m}^{1 / \theta}$ and $y=m^{1 / \theta}$,
where $\widetilde{m}^{1 / \theta}=(1-a)^{\frac{1}{\rho-1}} h$, and $m=\left[(1-a) h^{\rho-1}+a k^{\rho-1}\right]^{\frac{1}{\rho-1}}$. Thus,

$$
d \ln x / d \ln h=(1 / \theta) d \ln \widetilde{m} / d \ln h=1 / \theta,
$$

and

$$
\begin{aligned}
d \ln y / d \ln h & =(1 / \theta) d \ln m / d \ln h \\
& =(1 / \theta)\left(\frac{1}{\rho-1}\right) \frac{(1-a)(\rho-1) h^{\rho-1}}{(1-a) h^{\rho-1}+a k^{\rho-1}} \\
& =(1 / \theta) \frac{(1-a) h^{\rho-1}}{(1-a) h^{\rho-1}+a k^{\rho-1}} .
\end{aligned}
$$

But $x^{\prime}>y$ if and only if $d \ln x / d \ln h>(d \ln y / d \ln h)(y / x)$, and this is equivalent to

$$
\begin{aligned}
& 1>\frac{(1-a) h^{\rho-1}}{(1-a) h^{\rho-1}+a k^{\rho-1}}\left(\frac{\left[(1-a) h^{\rho-1}+a k^{\rho-1}\right]^{\frac{1}{\rho-1}}}{(1-a)^{\frac{1}{\rho-1}} h}\right)^{1 / \theta} \\
& 1>\left(\frac{(1-a) h^{\rho-1}}{(1-a) h^{\rho-1}+a k^{\rho-1}}\right)^{1+1 / \theta(1-\rho)} .
\end{aligned}
$$

This is true as long as $1+1 / \theta(1-\rho)>0$, or $\theta(\rho-1)>1$, which implies $\rho>1+1 / \theta$. This establishes that if $\rho>1+\frac{1}{\theta}$, then $\frac{d \log G T^{\prime}}{d \log h}<0$.

Third, we prove that $X_{n}^{T}=\eta X_{n}$.
Proof: Let $Z_{n}$ be total quantity of the input bundle produced in country $n .{ }^{30}$ Let $Q_{m n}$ be the total quantity of the composite intermediate good used to produce $Z_{n}, Q_{f n}$ the total quantity of the composite intermediate good used to produce consumption goods, and $Q_{n}=Q_{m n}+Q_{f n}$ the total quantity of the composite intermediate good produced in $n$. Let $L_{m n}$ be the total quantity of labor used to produce intermediate goods, and $L_{f n}$ the total quantity of labor used to produce final (consumption) goods. It must be that $L_{n}=L_{m n}+L_{f n}$. Note that $p_{m n} Q_{n}$ is the total cost of the intermediate goods used in production in country $n$, so $p_{m n} Q_{n}=X_{n}^{T}$. We first calculate the total cost of the intermediate goods produced in country $n$. This includes the

[^18]total cost of the domestic input bundle for intermediates,
$$
w_{n} L_{m n}+p_{m n} Q_{m n}=c_{n} Z_{n},
$$
plus the intra-firm imports of foreign multinationals located in $n$,
$$
\sum_{i \neq n} \omega_{n i} \sum_{j} s_{j n i}^{M} p_{m j} Q_{j},
$$
minus the exports of the domestic input bundle for intermediates to country $n^{\prime} s$ subsidiaries abroad,
$$
\sum_{i \neq n} \omega_{i n} \sum_{j} s_{j i n}^{M} p_{m j} Q_{j} .
$$

Hence, the total cost of intermediate goods produced in country $n$ is

$$
w_{n} L_{m n}+p_{m n} Q_{m n}+\sum_{i \neq n} \omega_{n i} \sum_{j} s_{j n i}^{M} p_{m j} Q_{j}-\sum_{i \neq n} \omega_{i n} \sum_{j} s_{j i n}^{M} p_{m j} Q_{j} .
$$

Second, we calculate the total value of intermediate goods produced in country $n$. This is composed of the value of sales (domestic plus exports) using national technologies, $\sum_{j} s_{j n}^{T} p_{m j} Q_{j}$, plus the value of sales (domestic plus exports through VMP) using domestic and foreign multinational technologies, $\sum_{i} \sum_{j} s_{j n i}^{M} p_{m j} Q_{j}$,

$$
\sum_{j} s_{j n}^{T} p_{m j} Q_{j}+\sum_{i} \sum_{j} s_{j n i}^{M} p_{m j} Q_{j} .
$$

In equilibrium, we must have these two things equal, hence

$$
\begin{aligned}
& w_{n} L_{m n}+p_{m n} Q_{m n}+\sum_{i \neq n} \omega_{n i} \sum_{j} s_{j n i}^{M} p_{m j} Q_{j}-\sum_{i \neq n} \omega_{i n} \sum_{j} s_{j i n}^{M} p_{m j} Q_{j} \\
= & \sum_{j} s_{j n}^{T} p_{m j} Q_{j}+\sum_{i} \sum_{j} s_{j n i}^{M} p_{m j} Q_{j} .
\end{aligned}
$$

The trade balance condition is imports equal exports, or

$$
\sum_{i \neq n} M_{n i}=\sum_{i \neq n} M_{i n},
$$

with

$$
\begin{aligned}
M_{n i} & =\left(s_{n i}^{T}+\sum_{j} s_{n i j}^{M}\right) p_{m n} Q_{n}+\omega_{n i} \sum_{j} s_{j n i}^{M} p_{m j} Q_{j}, \\
M_{i n} & =\left(s_{i n}^{T}+\sum_{j} s_{i n j}^{M}\right) p_{m i} Q_{i}+\omega_{i n} \sum_{j} s_{j i n}^{M} p_{m j} Q_{j} .
\end{aligned}
$$

We have

$$
\begin{aligned}
w_{n} L_{m n}+p_{m n} Q_{m n}+\sum_{i \neq n} \omega_{n i} \sum_{j} s_{j n i}^{M} p_{m j} Q_{j}-\sum_{i \neq n} \omega_{i n} \sum_{j} s_{j i n}^{M} p_{m j} Q_{j} & = \\
\left(s_{n n}^{T}+\sum_{i} s_{n n i}^{M}\right) p_{m n} Q_{n} & +\sum_{j \neq n}\left(s_{j n} p_{m j} Q_{j}+\sum_{i} s_{j n i}^{M} p_{m j} Q_{j}\right) \\
w_{n} L_{m n}+p_{m n} Q_{m n}+\sum_{i \neq n} \omega_{n i} \sum_{j} s_{j n i}^{M} p_{m j} Q_{j}-\sum_{i \neq n} \omega_{i n} \sum_{j} s_{j i n}^{M} p_{m j} Q_{j} & = \\
\left(s_{n n}^{T}+\sum_{i} s_{n n i}^{M}\right) p_{m n} Q_{n} & +\sum_{j \neq n}\left(M_{j n}-\omega_{j n} \sum_{l} s_{l j n}^{M} p_{m l} Q_{l}\right) \\
w_{n} L_{m n}+p_{m n} Q_{m n}+\sum_{i \neq n} \omega_{n i} \sum_{j} s_{j n i}^{M} p_{m j} Q_{j} & = \\
\left(s_{n n}^{T}+\sum_{i} s_{n n i}^{M}\right) p_{m n} Q_{n} & +\sum_{j \neq n} M_{j n} .
\end{aligned}
$$

From the trade balance condition, we then have

$$
\begin{aligned}
w_{n} L_{m n}+p_{m n} Q_{m n}+\sum_{i \neq n} \omega_{n i} \sum_{j} s_{j n i}^{M} p_{m j} Q_{j} & =\left(s_{n n}^{T}+\sum_{i} s_{n n i}^{M}\right) p_{m n} Q_{n}+\sum_{i \neq n} M_{n i} \\
w_{n} L_{m n}+p_{m n} Q_{n} & =\left(s_{n n}^{T}+\sum_{i} s_{n n i}^{M}\right) p_{m n} Q_{n}+\sum_{i \neq n}\left(M_{n i}-\omega_{n i} \sum_{j} s_{j n i}^{M} p_{m j} Q_{j}\right) \\
w_{n} L_{m n}+p_{m n} Q_{n} & =\left(s_{n n}^{T}+\sum_{i} s_{n n i}^{M}\right) p_{m n} Q_{n}+\sum_{i \neq n}\left(s_{n i}^{T}+\sum_{j} s_{n i j}^{M}\right) p_{m n} Q_{n} \\
w_{n} L_{m n}+p_{m n} Q_{n} & =\left(\sum_{i} s_{n i}^{T}+\sum_{i} \sum_{j} s_{n i j}^{M}\right) p_{m n} Q_{n} .
\end{aligned}
$$

But, we know that $s_{n l}^{T} \equiv \psi_{n l} / \psi_{n}$, and $s_{n l i}^{M} \equiv y_{n l i} \psi_{n i}^{M} / \psi_{n}$. Hence,

$$
\sum_{i} s_{n i}^{T}+\sum_{i} \sum_{j} s_{n i j}^{M}=\frac{\sum_{i} \psi_{n i}+\sum_{i} \sum_{j} y_{n i j} \psi_{n j}^{M}}{\psi_{n}}=\frac{\sum_{i} \psi_{n i}+\sum_{j}\left(\sum_{i} y_{n i j}\right) \psi_{n j}^{M}}{\psi_{n}}
$$

Given $\sum_{i} y_{n i j}=1$, we have

$$
\sum_{i} s_{n i}^{T}+\sum_{i} \sum_{j} s_{n i j}^{M}=\frac{\sum_{i} \psi_{n i}+\sum_{j} \psi_{n j}^{M}}{\psi_{n}}=\frac{\sum_{i}\left(\psi_{n i}+\psi_{n i}^{M}\right)}{\psi_{n}}=1
$$

where the last equality follows from $\psi_{n} \equiv \sum_{i}\left(\psi_{n i}+\psi_{n i}^{M}\right)$. Thus,

$$
\begin{equation*}
w_{n} L_{m n}+p_{m n} Q_{m n}=p_{m n} Q_{n} . \tag{21}
\end{equation*}
$$

We know that

$$
\begin{equation*}
\frac{L_{f n}}{Q_{f n}}=\left(\frac{\alpha}{1-\alpha}\right) \frac{p_{m n}}{w_{n}} \tag{22}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{L_{m n}}{Q_{m n}}=\left(\frac{\beta}{1-\beta}\right) \frac{p_{m n}}{w_{n}} . \tag{23}
\end{equation*}
$$

Plugging 23 into 21 we get

$$
\left(\frac{\beta}{1-\beta}\right) p_{m n} Q_{m n}+p_{m n} Q_{m n}=p_{m n} Q_{n}
$$

from which we get

$$
Q_{m n}=(1-\beta) Q_{n} .
$$

Using $Q_{f m}+Q_{m n}=Q_{n}$, we then get

$$
\begin{equation*}
Q_{f n}=\beta Q_{n} \tag{24}
\end{equation*}
$$

Plugging $Q_{m n}=(1-\beta) Q_{n}$ back into (21), we get

$$
w_{n} L_{m n}=\beta p_{m n} Q_{n} .
$$

Using $L_{m n}+L_{f n}=L_{n}$, we then get

$$
\begin{equation*}
w_{n}\left(L_{n}-L_{f n}\right)=\beta p_{m n} Q_{n} . \tag{25}
\end{equation*}
$$

From (22) and (24), we get

$$
w_{n} L_{f n}=\left(\frac{\alpha}{1-\alpha}\right) \beta p_{m n} Q_{n} .
$$

Using (25), we then have

$$
L_{f n}=\left(\frac{\alpha}{1-\alpha}\right)\left(L_{n}-L_{f n}\right)
$$

and hence

$$
L_{f n}=\alpha L_{n}
$$

Plugging into (25), we get

$$
(1-\alpha) w_{n} L_{n}=\beta p_{m n} Q_{n}
$$

or

$$
X_{n}^{T}=\left(\frac{1-\alpha}{\beta}\right) w_{n} L_{n}
$$

## C Algorithm

We now explain the algorithm to solve for the equilibrium. Given a matrix $Y$ with elements $y_{n i}$ then one can solve the system forgetting about the complementary slackness conditions in (4) by following an extension of the algorithm in Alvarez and Lucas (2007). This is as follows: first, there is a function $p_{m}(w)$ that solves for the vector of prices $p_{m}$ given the vector of wages $w$. Second, there is a mapping $w^{\prime}=T(w ; Y)$ whose fixed point, $w=F(Y)$, gives the equilibrium wages given $Y$.

The final step is to solve for the equilibrium $Y$. Let $C_{T}(Y)$ be matrix with typical element $c_{i} / k_{n i}$ associated with $Y$ and let $C_{M P}(Y)$ be the matrix with typical element $c_{n i}$ associated with $Y$. Let $M(Y)$ be a matrix with typical element given by $\chi\left(c_{i}(Y) / k_{n i} \leq c_{n i}(Y)\right.$ ) (where $\chi(A)=1$ if the statement $A$ is true and $\chi(A)=0$ otherwise). Finally, let $\Gamma(Y)$ be a matrix with typical
element given by

$$
\gamma_{n i}(Y)=\frac{\min \left\{c_{i}(Y) / k_{n i}, c_{n i}(Y)\right\}}{y_{n i} c_{i}(Y) / k_{n i}+\left(1-y_{n i}\right) c_{n i}} .
$$

We use a mapping $Y^{\prime}=H(Y)=Y \cdot \Gamma(Y)+M(Y) \cdot(I-\Gamma(Y))$, where $I$ is a $N x N$ matrix of ones and where the operation $A \cdot B$ is the entry-wise or Hadamard matrix multiplication,

Note that if $\widetilde{Y}$ is a fixed point of $H(Y)$ then $\Gamma(Y)=I$, which implies that $Y$ satisfies the complementary slackness conditions in (4). The algorithm to find the equilibrium $Y$ is to start with $y_{n i}=0$ for all $n, i$ and then iterate on $Y^{\prime}=H(Y)$ until all the elements of $\Gamma(x)$ are sufficiently close to one.

## D Calibration: Jacobian

$$
J=\left(\begin{array}{ccccccc}
d \log M_{i} / d \log P_{j} & \delta_{M} & \theta & \delta_{G} & \gamma & a & 1 / \mu \\
\tau & -0.11 & 0.37 & -0.01 & -0.3 & 0.1 & 0 \\
\tau^{M} & 0.17 & 0.32 & -0.01 & 0.75 & -0.03 & -2.6 \\
C O R\left(\tau, \tau^{M}\right) & 0.006 & 0.09 & 0.002 & -0.45 & 0.04 & -0.91 \\
\widetilde{\omega} & -0.02 & -0.23 & 0.002 & -0.64 & 0.41 & 2.45 \\
X^{T, M P} / X^{M P} & -0.02 & -0.22 & 0.002 & -0.12 & -0.04 & 2.47 \\
1 / b_{g} & -0.01 & 0.21 & -0.00 & -0.16 & 0.02 & 0
\end{array}\right)
$$

where $M_{i}$ is the moment in the i-row, and $P_{j}$ is the parameter in the j -column. Each cell shows the numerical $\log$ derivative of moment $j$ with respect to parameter $i$.


Figure 2: The figure shows two scatter plots, by OECD(19) country. The left (right) panel shows outward MP (exports) as share of GDP (vertical axis), for model and data. The horizontal axis is model's GDP $\left(w_{i} L_{i}\right)$.


Figure 3: The figure shows two scatter plots, by $\operatorname{OECD}(19)$ country. The left (right) panel shows inward MP (imports) as share of GDP (vertical axis), for model and data. The horizontal axis is model's GDP $\left(w_{i} L_{i}\right)$.


[^0]:    *We benefited from comments by participants at the Annual Meeting of the Society for Economic Dynamics (2007), the Annual Conference on Macro and Development at U. Pittsburgh (2007), and seminars at New York University, NBER ITO Spring Meeting (2008), FRB New York, Pennsylvania State University, University of British Columbia, the Econometrics Society Meetings (2008) at Pittsburgh, and University of Texas-Austin. We have also benefited from comments and suggestions from Costas Arkolakis, Russell Cooper, Alexander MongeNaranjo, Joris Pinkse, and Jim Tybout. All errors are our own.
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[^1]:    ${ }^{1}$ One exception is Rodríguez-Clare (2007), who evaluates the contribution of trade and diffusion of ideas to the overall gains from openness using a Ricardian model that incorporates both of these channels (along the lines of Krugman, 1979, and Eaton and Kortum, 2006). Another exception is Garetto (2007), who develops a model with both trade and vertical MP.

[^2]:    ${ }^{2}$ This is similar to the way that Grossman and Rossi-Hansberg (2008) model offshoring of tasks.
    ${ }^{3}$ In contrast to some recent models (e.g., Helpman, Melitz and Yeaple, 2004), our model has no fixed costs of production, no fixed costs of exporting or MP, and no firm-level heterogeneity.
    ${ }^{4}$ We avoid refering to this type of MP as vertical MP because the main motivation for BMP in our model is to avoid trade costs rather than allocating the different elements of the production process across locations according to their comparative advantage.

[^3]:    ${ }^{5}$ Note that this implies that there will be a share $\lambda_{i}^{M} / \lambda_{i}$ for which $x_{i}^{M} \leq \lambda_{i}^{N}$. For these goods $z_{i}=1$ so there is no efficiency loss associated with MP.
    ${ }^{6}$ This result comes from having $\lambda$ ideas for each good (each associated with a cost parameter), all of which are independently drawn from an exponential distribution with parameter 1 . Then, the distribution of the best technology is exponential with parameter $\lambda$.

[^4]:    ${ }^{7}$ We use the following normalization: $\sum_{i=1}^{I} w_{i} L_{i}=1$.

[^5]:    ${ }^{8}$ There is not even exports from the country where the multinational technology originates, $y_{n i i}^{M}=0$ for $n \neq i$.
    ${ }^{9}$ Note that if $h<k$, there would be no MP. But since the good-specific efficiency losses will necessarily be high for some goods, then even when trade is more costly than MP, $k<h$, there is trade between countries.

[^6]:    ${ }^{10}$ The expressions for $s^{N}$ and $s^{M}$ can be derived formally by using the results above that $s_{n i}^{N}=\psi_{n i}^{N} / \psi_{n}$ and

[^7]:    ${ }^{13}$ The equilibrium analysis for the full model is analogous to the one carried out for the simple model and is therefore left for the Appendix.

[^8]:    ${ }^{14} \mathrm{An}$ alternative but equivalent way to define $D_{i i}^{T}$ is as $D_{i i}^{T}=X_{i i}^{T} / X_{i}^{T}$, where $X_{i i}^{T} \equiv X_{i}^{T}-M_{i}$ is spending on locally produced intermediates. It is easy to show that $X_{i i}^{T} / X_{i}^{T}=1-\sum_{n \neq i} D_{n i}$.
    ${ }^{15} \mathrm{An}$ alternative but equivalent way to define $D_{i i}$ is as $D_{i i}=X_{i i} / X_{i}$, where $X_{i i} \equiv X_{i}-M_{i}$ denotes the value added corresponding to final goods produced in country $n$. It is easy to show that $X_{i i} / X_{i}=1-\eta \sum_{n \neq i} D_{n i}$.

[^9]:    ${ }^{16}$ This measure includes both local sales in $n$ and exports to any other country, including the home country $i$.

[^10]:    ${ }^{17}$ The logic is that for goods that country $n$ actually imports from $i$, we must have $p_{n}(u) / p_{i}(u)=1 / k_{n i}$, whereas for goods that are not imported we must have $p_{n}(u) / p_{i}(u) \leq 1 / k_{n i}$. This implies that if $i$ exports something to $n$ then $1 / k_{n i}=\max p_{n}(u) / p_{i}(u)$.
    ${ }^{18}$ Eaton and Kortum (2002) calculate $\log p_{i} /\left(p_{n} k_{n i}\right)$ as $D_{n i}=\max _{2 u} r_{n i}(u)-(1 / 50) \sum_{u=1}^{50} r_{n i}(u)$. Using this measure as a proxy for trade costs yields very similar results to the ones using $k_{n i}$. An alternative measure for trade costs that we use is the residual of regressing (log of) $k_{n i}$ on (log of) bilateral distance, source, and host country dummies. Again, results are very similar to the ones using directly $k_{n i}$.
    ${ }^{19}$ In future work, we plan to estimate $\lambda_{i} / L$ using GDP data ( $w_{i} L_{i}$ in the model).

[^11]:    ${ }^{20}$ Note that we have as many moments as number of parameters to estimate. Thus, using the identity matrix as optimal weighting matrix does not affect estimates.
    ${ }^{21}$ In this preliminary estimation, we report parameters' estimates using only one $\varepsilon$ matrix
    ${ }^{22}$ In principle, the model is able to match the moments perfectly, but the computation is time intensive and for this version of the paper we did not let the algorithm continue until the match was perfect.
    ${ }^{23}$ With no diffusion, Eaton and Kortum (2002) are able to recover $\theta$ from a OLS gravity equation without an intercept. Rodríguez-Clare (2007) shows that such intercept arises from the inclusion of diffusion on top of trade as a way to share ideas. With a very different methodology, Rodríguez-Clare estimates $\theta=0.22$ (very close to our estimate).

[^12]:    ${ }^{24}$ The role of $a$ as a key parameter leading intra-firm trade also can be seen when we double the intra-firm trade share observed in the data, from 0.07 to 0.15 , but keep $\rho=4$ (not shown in the text). In that case, $a$ increases from 0.34 to 0.54 .

[^13]:    ${ }^{25}$ This is true as long as a the ratio of total stock of ideas $\lambda_{i}$ to size $L_{i}$ is constant across countries. In this case, there is no trade in goods produced with global ideas.

[^14]:    ${ }^{26}$ In a model without intra-firm trade and "bridge" MP, this parameter is exactly the (inverse of) the elasticity of normalized trade shares to trade costs.

[^15]:    ${ }^{27}$ The correlation between data and model GDP is 0.99 .

[^16]:    ${ }^{28}$ This can be obtained by noting that the structure of wages will be the same as $L_{i}$ grows at rate $g_{L}$ for all $i$, but $p_{i}$ will fall at rate $\theta(1+\eta) g_{L}$.

[^17]:    ${ }^{29}$ This is just a bit lower than Eaton and Kortum's actual estimated gains of $3.5 \%$. Differences result because of alternative measures of trade costs and the general equilibrium structure of the model.

[^18]:    ${ }^{30}$ What is the relationship between $c_{n} Z_{n}$ and $X_{n}^{T} ? c_{n} Z_{n}$ is the total cost of the input bundle produced in $n$, which is used to produce intermediate goods in country $n$, and by country $n$ multinationals abroad. $X_{n}^{T}$ is total spending on intermediate goods in $n$, which does not include the cost of labor used to produce the input bundle.

