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NEARLY OPTIMAL PRICING FOR MULTIPRODUCT FIRMS*

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1 Introduction

We study the pricing problem for a multiproduct firm facing consumers who may purchase more than one (and possibly all) of their products. Examples include cable television companies, professional sports teams, and online music stores. In these cases the firm has a rich set of alternative pricing schemes available. Such firms can simply sell their products at a uniform price. They can also set different prices for each of their products. There are also bundling possibilities. The firm could make the products only available as a complete bundle. Or the firm can bundle a subset of the products and offer the others as individual items. There are many options—for a firm with K products, there are $2^K - 1$ possible combinations of products that can be separately priced. Even if K is only 10, there are over a thousand prices that could potentially be set. This is a highly complex pricing problem for firms.

In reality firms almost never implement complex pricing structures. Indeed, the reverse seems more common: firms often employ remarkably few prices. Why is this? In this study we show that simple pricing strategies are often *nearly optimal*: in a broad class of models, it takes surprisingly few prices to obtain 99% of the profit from offering every possible bundle combination. But it is not just any small subset of prices. We introduce a new pricing strategy to the literature, called bundle-size pricing (BSP), which has the same number of prices as there are products. We find that BSP tends to be more profitable than offering the individual products priced separately, and that BSP very closely approximates the profits from mixed bundling.

BSP involves setting different prices for different sized bundles. For a firm with 3 goods, BSP sets one price for the purchase of any single good, a second price for the purchase of any 2 goods, and a third price for purchasing all 3. BSP has not been discussed in the prior literature on bundling, which has focused on a few other alternatives: mixed bundling (MB) in which the firm chooses prices for every combination of goods, component pricing (CP) in which the firm sets different prices for each of its products, and pure bundling (PB) in which consumers only option is to purchase all of the firm's products at a single price.

The prior research offers two results of relevance for a firm with K products. First, MB tends to be strictly more profitable than CP.[1] Second, it is possible that PB is more profitable than CP.[2] Hence, the implication for a firm with 10 products, say, would be: the best thing to do is set 1,023 prices under MB; and if that is not feasible (likely) then offering all products only as a single package may be more profitable than offering them individually (or perhaps not). Our

[1] See McAfee, McMillan and Whinston (1989).

[2] See Stigler (1963) and Adams and Yellen (1976).

findings offer a new suggestion: BSP will require only 10 prices and attains 99% of the profit from MB under most circumstances—even when demand is highly asymmetric across products.[3] This is a significant step forward in providing practical advice for multiproduct firms.

We show that BSP and MB both tend to drive consumers to purchase larger-sized bundles than they would under CP. This has the effect of reducing consumers' heterogeneity in valuations for the products, which was always the key insight of the bundling literature. Put differently, the demand for each of the firm's K products under BSP (where a product is defined by bundle-size) tends to be less heterogeneous than the demands for the K products under CP. With less heterogeneity, the firm can extract more surplus.[4] However, it may seem that CP would be more profitable when there is a high degree of demand asymmetry across products. In fact, BSP is also able to extract surplus from individuals with high demand for one product and not others—BSP does so by setting a high price for single-good bundles.

The heterogeneity-reduction effect of bundling also implies that different bundles of the same size do not need to be priced very differently if the bundles are large. Hence, prices for large-sized bundles under BSP tend to be very close to prices under MB. This is why BSP tends be a good approximation to MB. One interpretation of our findings, then, is that many of the prices a firm would set under MB are redundant.

Our analysis has two components. First, we perform a large number of numerical experiments covering a broad range of demand and cost scenarios. In each experiment we compute the optimal prices under CP, PB, BSP and MB, and the associated profits. Numerical analysis is necessary for this problem because the profit maximization problem is analytically intractable under all but the simplest assumptions about the distribution of consumers' tastes.[5] An obvious limitation to this approach is that we cannot be certain our results will transcend the particular parameter values we covered. For this reason, the second component of our analysis utilizes an estimated model. This allows us to demonstrate that our findings apply to an empirically relevant model.

The empirical analysis is based on a theater company that produces a season of 8 plays. It is an interesting setting in which to compare the profitability of different pricing schemes. On

[3] Since BSP nests PB, it is also the case that BSP is always at least as profitable as PB and is often significantly more profitable.

[4] In the extreme, if marginal costs are zero, as $K \to \infty$ MB pushes all consumers toward purchasing the full bundle (i.e. MB simply implements PB in this limit), and extracts the entire consumer surplus. See Bakos and Brynjolffson (1999).

[5] We are not the first to rely on numerical methods to analyze bundling problems. See also Schmalensee (1984) and Fang and Norman (2006).

the one hand, the plays differ in overall popularity, suggesting that component pricing may be important for profits. On the other hand, many customers attend multiple plays, suggesting that some form of bundling may also be profitable. With 8 goods, MB would require the firm to set 255 prices, which is clearly impractical. In considering simpler alternatives, how important is it for the firm to set high prices for high demand plays? What about offering discounts to consumers that attend multiple plays? Or some combination of these? And how do these alternatives compare to MB in terms of profits and consumer surplus?

A key feature of the theater data is that we observe the set of plays chosen by each customer. This allows us to identify the covariances in the joint distribution of consumers' tastes, which is a determinant of profitability under alternative bundling schemes. The estimated demand system reveals strong positive correlations in tastes for most pairs of plays, which tends to reduce the relative profitability of bundling-type strategies compared to CP. Nevertheless, we find that BSP is 4.0% more profitable than CP, and BSP attains 98.3% of the MB profits.

The remainder of the paper is organized as follows. In Section 2 we summarize the relevant prior literature. Section 3 describes the basic intuition underlying the various pricing strategies, and presents the results from an extensive set of numerical experiments. The empirical example is presented in Section 4. Section 5 concludes.

2 Prior Theoretical Literature

The bundling literature explores the idea that a multiproduct monopolist can increase profits by selling goods in bundles, even when there are no demand-side complementarities or supply-side economies of scope. If a firm sells two products, and consumers vary in their willingness-to-pay for each product, then Stigler (1963) shows by example that selling these two products as a bundle (PB) may yield higher profit than if sold separately (CP). Adams and Yellen (1976) introduce MB as an alternative to CP and PB, showing by example that MB can strictly dominate both CP and PB. They also explain why higher values of marginal cost tend to favor CP over PB: with bundling, individuals may consume products for which their willingness-to-pay is less than the marginal cost to the firm.[6]

Two subsequent papers show that bundling (PB or MB) dominates CP in a wide variety of circumstances. First, Schmalensee (1984) expands the analysis to demand systems where

[6]In the language of Adams and Yellen (1976), these are violations of the *exclusion* condition.

consumers' product valuations are drawn from a bivariate normal.[7] Due to the limited computer power at the time, Schmalensee does not compute optimal MB prices, instead focusing on CP and PB. His main finding is that PB can be more profitable than CP even when the correlation of consumers' valuations is non-negative.[8] Second, McAfee, McMillan and Whinston (1989) extend the prior results by showing that MB strictly dominates CP under rather general circumstances.[9]

All the above papers analyze two product monopoly problems. A few prior papers study bundling with more than two goods. Bakos and Brynjolfsson (1999) focus on the profitability of PB as the number of goods (K) goes to infinity. They show that if goods have zero marginal cost, then as K goes to infinity PB approximates perfect price discrimination.[10] This finding is particularly interesting in our context, since it provides an example of an incomplex alternative to MB that closely approximates the profitability of MB in a particular circumstance (i.e., large K).

Armstrong (1999) provides a more general but similar result to Bakos and Brynjolfsson (1999). He shows that a two-part tariff, in which consumers are charged a fixed fee and can then purchase any products at marginal cost, achieves approximately the same profit as perfect price discrimination if the number of products approaches infinity. In the special case of zero marginal cost the two-part tariff is equivalent to PB. The focus on settings with large numbers of products may be quite relevant for some firms, such as booksellers or supermarkets. But clearly these results are of questionable relevance to firms with 5 products, say.

Fang and Norman (2006) also examine the profitability of PB with more than two goods. In contrast to Armstrong (1999) and Bakos and Brynjolfsson (1999), they focus on finite K, and they seek to determine under what circumstances PB is an attractive pricing strategy. They confirm that increasing marginal cost tends to favor CP over PB, as Adams and Yellen (1976) had argued. They also show (by way of numerical experiments) that increasing the number of goods may favor PB over UP.

For a firm selling a finite number of goods, the prior literature can be easily summarized:

[7] A concern with this approach is that the bivariate normal implies negative valuations for some consumers which would impact the analysis in non-trivial ways, as noted by Salinger (1995). In all of the analysis in our study we allow for free disposal.

[8] The numerical examples in Stigler (1963) and Adams and Yellen (1976) somehow suggest the importance of negative correlation, as noted by Schmalensee (1984).

[9] McAfee, McMillan and Whinston (1989) also distinguish between firms that can monitor purchases or not. With monitoring, the firm can charge a price for the bundle of two that is higher than the sum of component prices. We limit our analysis to the no-monitoring case.

[10] Bakos and Brynjolfsson (1999) also show that, under certain conditions, increasing the number of goods under PB monotonically increases profit. Geng, Stinchcombe and Whinston (2005) extend the analysis of Bakos and Brynjolfsson to incorporate diminishing marginal utility.

MB is *always* more profitable than CP, and in *some* cases PB may also be more profitable than CP. We contend these results are of limited practical value—MB rapidly becomes impractical as the number of goods increases above a mere few, and even in the cases when PB is more profitable than CP it is conceivable there are other straightforward pricing schemes that will do even better.

Hence, we focus on the question: do there exist pricing schemes that involve few enough prices that firms would reasonably be able to implement them, and which tend to yield profits that are close to the MB level in a wide class of circumstances?

3 The Multiproduct Pricing Problem

In principle, multiproduct firms can employ a wide variety of pricing schemes. For a firm with K products, the optimal MB strategy requires setting $(2^K - 1)$ prices.[11] PB and UP require only one price to be set: the price for the bundle of all K products (in the PB case), or the per-product price (in the UP case). In between these extremes are CP, by which we mean setting K different prices for the K different products, and BSP, by which we mean setting K prices that depend on the number of products purchased. Note that MB nests all the simpler pricing strategies as special cases, so it will always be weakly more profitable than any of these alternatives. Similarly, CP nests UP as a special case, and BSP nests both UP and PB as special cases. CP and BSP are non-nested alternatives.

CP and BSP are of particular interest because the number of prices equals the number of products, which is a reasonable benchmark for practical pricing strategies. However, there are many other potential pricing strategies that also involve K prices, which are nested subsets of MB. The problem in these cases is that it is ex-ante unclear which subset of K prices to choose. There are also strategies (with more than K prices) that nest both CP and BSP.

In practice, multiproduct firms tend to use a broad range of different pricing/bundling strategies. Consider baseball teams, for example, which have 81 home games (products).[12] For the 2006 season the Los Angeles Dodgers offered several bundles of specific games, a discount for choosing any 27 games, and equal prices for all individual games. In contrast, for 2006 the San Francisco Giants did not offer any bundles or quantity discounts, but the Giants did vary prices by day of week and by opponent. Variation in pricing strategies is also evident in settings with

[11] We subtract 1 because the firm does not set the price for the outside good.

[12] Under MB, with 81 products a firm would choose 2.4×10^{24} prices.

fewer products. Consider the Steppenwolf Theater in Chicago that produces a 5-play season. In 2006-07 they offered a discount for the 5-play bundle at a variety of prices that vary by time-of-week, and equal prices for individual shows (also varying by time-of-week). In 2006-07 the San Francisco Opera had a 10-opera season and offered 37 bundles (combinations of specific operas and time-of-week), and equal prices for individual shows (also varying by time-of-week). These examples highlight the dramatic differences in pricing strategies implemented by different firms in similar settings. We have been unable to find an example of MB being used in practice for 3 or more products.

3.1 Examples with Two Goods and Two Consumer Types

In order to clarify the basic intuition that underlies the various pricing schemes, in this section we present select examples with two goods and two consumer types in which the optimal prices under the various schemes are simple to determine. The examples illustrate how each of CP, PB, and BSP may attain the highest profits in different settings. We adhere to the standard assumptions of the bundling literature: (i) consumers purchase one or zero units of each product; (ii) consumers' valuations for a bundle equal the sum of their valuations for the bundle's component products (i.e. products are neither complements or substitutes); and (iii) there is no resale.

The seminal papers on bundling pointed out that PB can be more profitable than CP because it may reduce heterogeneity in consumers' willingness-to-pay. Consider the following example. There are two consumers (A and B) and two products (1 and 2) with zero marginal costs. Each consumer has valuations v_1 and v_2 for the two products. Valuations are given by:

	v_1	v_2
A	4	1
B	1	4

In this case the optimal CP prices are 4 and 4, and the CP profit is 8. With PB both consumers value the bundle at 5, and so the PB price is 5, extracting the full surplus of 10. This is a textbook example of why bundling can increase profits even though there are no complementarities in demand or costs.

When is CP more profitable than PB? Intuition suggests that if the optimal CP price would be much higher for one product than the other, CP is likely to be better. But asymmetry of this sort isn't enough. Consider the example:

	v_1	v_2
A	4	1
B	4	1

Demand for good 1 is higher than for good 2, and CP accommodates this by charging a price that is 4 times higher for good 1. The CP profit is 10. But both consumers also equally value the bundle at 5, allowing PB to also obtain a profit of 10. CP does no better than PB, despite the substantial demand asymmetry. Note also that the above valuations provide an example of vertically differentiated products—both consumers agree that good 1 is preferred to good 2. It is interesting that PB may be equally profitable to CP in such a case.

For CP to significantly outperform PB, the valuations must exhibit a kind of within-product asymmetry. In particular, a large fraction of the extractable surplus must be concentrated on one product and one consumer type. For example:

	v_1	v_2
A	1	2
B	5	2

In this case CP charges 5 and 2, and extracts 9. PB sets a price of 7, sells only to type B and extracts 7. Intuitively, CP is able to charge a high price for product 1 in order to extract the large amount of surplus attributable to type B, but this does not rule out selling good 2 to both types. PB, on the other hand, by extracting as much surplus as possible from type B, ends up abandoning type A altogether.

So far we have shown how PB may be more profitable than CP, and how CP may be more profitable than PB. What about BSP? In all of the above examples BSP is equivalent to PB. This is trivially true in the first two examples, because PB extracts the full surplus.[13] In the last example, it is tempting to set BSP prices of 2 (for one good) and 7 (for two goods), but in that case type B would choose only good 1 rather than the bundle of two, so BSP can do no better than PB.[14]

[13]Note that PB can never yield strictly higher profit than BSP because PB is a constrained version of BSP.

[14]Put differently: in order to induce B to buy the bundle, the BSP price for the bundle of one has to be high—but doing this means that A doesn't buy anything. The best BSP can do is pool the two types together by charging the PB price.

Clearly BSP cannot do worse than PB. But under what circumstances will BSP be strictly more profitable than PB? Consider the following example:

	v_1	v_2
A	4	0
B	3	3

The optimal CP prices are 3 and 3 yielding a profit of 9. The optimal PB price is 4, which yields a profit of 8. BSP charges 4 for the purchase of any single good, and 6 for the purchase of both. Type A buys product 1, type B buys the bundle of both, and profits are 10. Intuitively, the reason BSP is more profitable is that the consumer with the highest valuation for a bundle of one is different from the consumer with the highest valuation for a bundle of two. BSP is able to extract more surplus by setting prices that separate the two consumers, whereas PB is forced to pool the two types by lowering the price of the bundle of two.

Loosely speaking, we expect BSP to outperform PB when (i) willingness to pay for the bundle of all K products is heterogeneous across consumers, and (ii) consumers (or consumer types) who have the highest willingness to pay for a bundle of size m are not necessarily the same as those with the highest willingness to pay for a bundle of size $n > m$. In Appendix A we provide a formal condition that is sufficient for BSP to be strictly more profitable than PB.

Note also, in this example both consumers weakly prefer good 1 to good 2 (vertically differentiated products). This is analogous to a baseball team with one specific game that all consumers value more than any other game. Economists tend to cite the overwhelming popularity of certain baseball games, such as games between traditional rivals like the New York Yankees and the Boston Red Sox, as evidence of the need to change from UP to CP. It is interesting that in such a setting BSP may be more profitable than CP.

3.2 An Example with Two Goods and a Continuum of Consumer Types

The goal of any pricing strategy is to extract as much surplus as possible from consumers. If consumers are heterogeneous, then a strategy that is able to separate consumers according to willingness-to-pay will extract more surplus than a strategy which does not. CP separates consumers in a straightforward way: consumers with a high valuation for a given good can be separated from consumers with a low valuation. In the two type example, this can result in one type buying a single good and the other type buying both goods. The same pattern can arise under BSP, but the mechanism is quite different. For BSP to obtain separation of the two types, the type with the highest valuation for a single-good bundle must differ from the type with the highest valuation for the two-good bundle. If not, then BSP is equivalent to PB and there is pooling. This condition is inconsequential for whether there is separation under CP. Viewed this way, it is apparent that CP and BSP are very different screening devices.

With a continuum of consumer types the difference in screening becomes more complex. Consider the following example. Assume there are two goods both with zero marginal cost. Consumers' valuations for good 1 are uniformly distributed between zero and θ: $v_1 \sim U[0, \theta]$. And consumers' valuations for good 2 are uniformly distributed between zero and 1: $v_2 \sim U[0, 1]$. Also assume that v_1 and v_2 are uncorrelated. The virtue of this model is that we can derive analytic solutions for the optimal prices under both CP and BSP, as well as the associated profits (see Appendix B for details).[15] If $\theta = 1.7$ then the optimal CP prices are .85 and .5 for goods 1 and 2, respectively. Under BSP the optimal price for a single-good bundle is .9, and the price for the bundle of both goods is 1.1. In this example BSP is 5.6% more profitable than CP (even though the optimal CP prices vary by 70% across the two goods).

In Figure 1 we show how CP and BSP lead to different partitions of consumers (for $\theta = 1.7$). CP is the most straightforward: consumers to the right of .85 purchase good 1, and consumers above .5 purchase good 2 (with consumers in region A purchasing both). Under BSP consumers in the two regions labelled C purchase one product (good 1 for the lower right C, and good 2 for the upper left C). And under BSP consumers in regions A, D and E choose the bundle of both goods.

What is more interesting is which pricing scheme extracts more surplus from which consumers. In Figure 1, consumers located in region A purchase both goods under CP and BSP. Under CP, these consumers each pay 1.35 and under BSP they pay 1.1. Hence, the firm extracts more surplus from consumers in region A by using CP rather than BSP. CP also extracts more surplus from consumers in region B, since these consumers buy either good under CP and buy nothing under BSP. BSP, on the other hand, extracts greater surplus from consumers in regions C, D and E. Region E is particularly interesting because these consumers purchase nothing under CP, and under BSP purchase the bundle of both goods. Consumers in region D also increase the number of goods purchased (from a single good to two). In region C the number of goods consumed remains at one, but BSP extracts more surplus because the price for a single good (.9) is greater than both single prices under CP. To summarize these differences, in Figure 1 we

[15] A limitation of this model is that BSP is weakly more profitable than CP for all values of θ. Nevertheless, the model is helpful for demonstrating the differences between CP and BSP for a given value of θ.

shade the regions in which BSP extracts more surplus from consumers than CP.

There are four main points to take from Figure 1. First, BSP is more focused on getting consumers to purchase multiple goods than they would have under CP. Relative to CP, BSP raises the price for single-good buyers, and lowers the price for multi-good buyers. It is profitable to do this in this example, but there is downside: (i) by increasing the price for a single-good bundle, some consumers are excluded from purchasing anything who otherwise would have purchased something (region B); and (ii) consumers that would have purchased both goods under CP are given a discount under BSP with no change in their purchase choice (region A).

Second, from the figure it is apparent why negative correlation in consumers' valuations would increase the relative profitability of BSP. Note the downward trend of the shaded regions in which BSP extracts greater surplus than CP—negative correlation implies an increase in the fraction of consumers in these regions.[16] It is also apparent from the figure that BSP is capable of extracting more surplus from individuals in the tails with high valuations for one product and low valuations for the other (region C). Hence, it is wrong to presume that BSP is poor at extracting surplus from consumers with a high valuation for only one product.

The third point concerns the consequences of diminishing marginal utility. The model that underlies Figure 1 assumes the utility of the bundle equals the sum of the utilities of the two goods. For some products, however, it may be important to incorporate diminishing marginal utility into the analysis. In this case the utility of a bundle is lowered by some factor that increases with the size of the bundle. In the extreme, if diminishing marginal utility is so strong that individuals never consume more than one good, then CP is weakly more profitable than BSP (for any distribution of valuations).

Such reasoning suggests any degree of diminishing marginal utility will reduce the profitability of BSP relative to CP, since the value of bundles is lowered. But this is wrong. Diminishing marginal utility also reduces the profitability of CP, possibly by even more than it does for BSP. This is because CP also benefits from extracting surplus from individuals that purchase both goods (region A), and CP actually extracts more surplus from this set of consumers than BSP does. In other words, diminishing marginal utility reduces willingness-to-pay for the bundle of both goods, which also reduces the amount of surplus that CP can extract. In the counterfactuals based on our empirical analysis in Section 4, we indeed verify that incorporating

[16] Introducing correlation to the example will change the optimal BSP prices, changing the details of the figure. Note, however, the optimal CP prices do not depend on the correlation of consumer's valuations—each good is optimally priced independently of the other good, so correlation plays no role in the CP optimization problem. Hence, the figure would change in some ways, but it would be qualitatively similar and this point would still hold.

diminishing marginal utility can reduce the profitability of CP by even more than it does BSP. This is another appealing aspect of BSP from a firm's point of view.

Fourth, this example gives an indication of the complexity of the BSP pricing problem. In this simple case with two goods and independent, uniformly distributed taste distributions, the regions of integration for determining demand for different sized bundles are non-rectangular and non-contiguous (i.e. region C). Adding non-zero correlation or more goods will increase the complexity, and allowing for more realistic distributions of valuations (such as normal) precludes analytic solutions. This is why numerical methods are essential for solving the BSP optimization problem in more general settings.

3.3 Numerical Analysis with Continuous Types and More than Two Goods

Although the two-good examples described above are illustrative, our objective is to analyze multiproduct pricing strategies in a more general context, and in particular to allow for more than two products. As we have argued, when the number of products increases MB quickly becomes highly complex. So it is important to understand which subset of prices (if any) can capture a large fraction of the profit that MB would obtain. The above examples illustrate how any of CP, PB or BSP may be the most profitable in any given circumstance.

The results in this section are based on a broad range of computational experiments in which we solve for the optimal prices and profits for five different pricing strategies, as detailed in Table 1. We vary the number of goods in these experiments from two to five. The experiments cover 13 different parametric families for the distribution of consumers' valuations, all of which are continuous-type cases. The particular distributions were chosen because of their common use in demand analysis, and the boundaries of the parameter space are intended to encompass a broad range of plausible cases. Each experiment is performed under four different assumptions regarding costs: (i) all products have zero marginal cost; (ii) all products have positive and equal marginal cost; (iii) all products have positive but differing marginal cost (we set marginal costs equal to half of each product's mean valuation); and (iv) marginal costs are zero but there is a binding capacity constraint.[17]

In all experiments we assume a demand model in which consumer i's utility from purchasing

[17]For the experiments with capacity constraints we first find the optimal uniform price in the absence of any capacity constraint, and then set the capacity constraint equal to .9 times the demand for the most popular product under the optimal uniform price. This ensures that the capacity constraint will be binding for at least one product under UP regardless of the particular parameters of the taste distribution.

bundle j is equal to $V_i'D_j - p_j$, where V_i is a $K \times 1$ vector of valuations for the firm's K products, D_j is a $K \times 1$ vector of binary indicators for which of the K products are included in bundle j, and p_j is the price of bundle j.[18] Each consumer's problem is to choose the offered bundle that maximizes her utility. Consumers' product valuations are heterogeneous: V_i is drawn from a (multivariate) distribution F. Importantly, we allow for free disposal—if a consumer purchases a bundle that includes a product for which she has a negative valuation we assume zero utility from consuming that product.[19]

Table 2 describes the alternative assumptions on the distribution of consumers' valuations (F) that we include in our experiments. We include distributions with non-zero covariances in product valuations across products. This is important since correlation in tastes is a key determinant of the profitability of bundling, as the prior literature has noted. Exponential, logit, lognormal, and normal distributions are all commonly used in empirical studies of demand. The uniform distribution is often convenient in theoretical studies of demand and is also occasionally used in empirical work.

For each parametric distribution we perform experiments for many combinations of parameter values. To help others reproduce our findings, rather than randomly draw parameter values, we define a grid of uniformly spaced parameter combinations. The grid boundaries for each parametric family are shown in Table 2. In each case the boundaries were chosen so that the range of optimal prices is roughly similar across cases to help with comparability. It is also important that our experiments include cases with a high degree of demand asymmetry which can favor CP. Hence, we choose grid boundaries that allow optimal CP prices to vary by up to a factor of 10.[20]

It is conceivable a firm may consider bundling together products for which the optimal component prices vary by much more than a factor than 10. We have chosen instead to focus on settings where the component products are more similar. Baseball games are perhaps an ideal example, because it is conceivable that the most popular game would have have an optimal component price that is several times greater (though probably not more than 10 times greater) than the least popular game. See also our empirical example in the next section. Nevertheless, this focus is a caveat to our analysis.

[18]As in the two-type examples in the prior subsection, by assuming additive preferences we are ruling out consumption complementarities as a motivation for bundling.

[19]Schmalensee (1984) does not allow free disposal. Either assumption may be correct depending on the particular products.

[20]Specifically, the range of parameters for each distributional family is such that the optimal component prices (assuming zero marginal cost) vary from about 0.2 to 2.0.

The granularity of each grid of parameter values varies with the number of products so that we analyze approximately 200 parameter combinations for each class of distribution, regardless of the number of products. We consider 13 parametric families, 4 marginal cost assumptions, variation in the number of products from 2 to 5, and about 220 parameter combinations in each case—leading us to compute 5 sets of prices and profits in over 45,000 different examples. Numerical methods are used to find the optimal prices in each case. We calculate the demands for each bundle using a kernel-smoothed frequency simulator, as discussed in Hajivassiliou, McFadden, and Ruud (1996), using 10,000 simulated consumers and a logistic kernel with smoothing parameter 0.02.

Before summarizing the outcomes of these experiments, it is important to acknowledge the limitations inherent in this kind of computational analysis. Although we attempt to cover a large space of parameter values, the results clearly depend on the specific parameters we choose (i.e. the choice of grid). Further, there is no way for us to know whether we are under- or over-sampling the relevant (i.e., empirically plausible) combinations of parameters. So, for example, when we describe average outcomes, these should certainly *not* be interpreted as outcomes that would be expected in an empirical sense—they should be interpreted narrowly as the average of the experiments we performed.

3.4 Results from Numerical Analysis

Figure 2 provides a summary of the numerical experiments for three different assumptions about costs.[21] The figure shows box-plots depicting various percentiles of the distribution of profits under each pricing strategy relative to BSP. To construct a given box-plot we pool experiments across distributions of consumers' valuations and for $K = 2, ..., 5$.[22] Hence, while the figures reveal the range of outcomes, they hide the differences across distributions and across K.[23] In Figure 2 each box-plot indicates the 1st, 25th, 50th, 75th and 99th percentiles of the distribution of profit for a given pricing strategy relative to BSP.[24]

As expected, Figure 2 shows that MB is always more profitable than BSP (because MB nests BSP), and BSP is always more profitable than UP (because BSP nests UP). However, there are

[21]We exclude the experiments for positive and equal marginal costs because they add no further insight, but we do include these results in the tables we discuss below.

[22]Note that we pool across parameter combinations for a given parametric family as well as pooling across parametric families (in addition to pooling across K).

[23]Those differences are shown in Tables 3 to 6, discussed below, and in even more detail in Appendix C.

[24]We depict the 1st and 99th percentiles instead of the min and max of the distribution because occasionally optimization error leads to misleading values for these extremes.

two more substantive findings to be taken from Figure 2:

1. BSP tends to be more profitable than CP. Based on the 46,344 experiments we performed (across different cost assumptions and across different taste distributions), we find that BSP is more profitable than CP 91% of the time. Furthermore, BSP obtains 13% higher profit than CP, on average.

2. BSP tends to obtain profits that are within 1% of the profits from MB. Specifically, the profit from BSP is within 1% of MB in 60% of the 46,344 experiments we performed. And on average, we find that BSP yields 98% of the MB profits.

Figure 2 also shows that varying assumptions about costs has an impact on the relative profits of the different pricing strategies, but the effect is quite small. Under the assumption of zero marginal costs, BSP is more profitable than CP in 97% of the experiments, and BSP is within 1% of the MB profits in 75% of the experiments. In comparison, under the assumption of positive and unequal marginal costs, BSP is more profitable than CP in 87% of the experiments, and BSP is within 1% of the MB profits in 34% of the experiments (although even here BSP attains 97% of the MB profits, on average). This is to be expected since the prior literature has explained that increases in marginal costs make "exclusion" (i.e., preventing consumers from purchasing goods they value below marginal cost) relatively more important.

In the introduction we noted that prior research shows that PB can be more profitable than CP for finite K. We find that PB attains higher profit than CP in 61 percent of our numerical experiments. We also find that increasing the number of goods tends to favor PB over CP: for $K = 2, 3, 4, 5$, PB is more profitable in 53, 61, 64, and 66 percent of the experiments, respectively. Fang and Norman (2006) also find this pattern in their numerical experiments.

In Tables 3 through 6 we summarize the results at a less aggregate level, showing how the performance of each pricing strategy varies according to the parametric family for the joint distribution of consumers valuations (under each cost scenario). We report the 1st, 50th and 99th percentiles of the distribution of profits of each pricing strategy relative to the profit from BSP, for each parametric family of consumers'valuations. Hence, we pool together the results from experiments with differing numbers of products ($K = 2, ..., 5$).[25] In Appendix C we provide more detailed summary statistics. To conserve space, we omit five of the parametric families described in Table 2 from Tables 3 through 6, because they make no qualitative difference to

[25] To be more precise, for each combination of parameters of the taste distribution we calculate the ratio of profits under pricing strategy X to profits under BSP. The table reports various percentiles of this ratio across parameter combinations and across $K = 2, ..., 5$. There are around 900 experiments in each distribution.

any of the findings.[26] The detailed results for these distributions are, however, included in Appendix C.

The main point to take from Tables 3 through 6 is that the choice of parametric family may not be innocuous in terms of the profitability of different pricing strategies. For instance, for the logit distribution (which is one of the most commonly used in empirical research) BSP is always more profitable than CP regardless of the level of marginal costs.[27] The same is true for log-normal distributions.

The role of a given parametric family may also vary depending on which assumption about costs is applied. For example, when marginal costs are all zero, BSP is always more profitable than CP if valuations are exponentially distributed. However, unequal marginal costs or capacity constraints can change this.

Importantly, the normal distribution (including cases with independence, positive correlations, negative correlations and unequal variances) is sufficiently unrestrictive in the sense that either CP or BSP may be the most profitable under any assumption on costs. In the empirical example we analyze below, we assume normally distributed tastes.

3.5 Discussion of Numerical Analysis

It is interesting that BSP tends to perform so well relative to MB even in experiments where one good has much higher demand than the other goods. Perhaps the reason why BSP is such a close approximation to MB is that prices simply don't matter very much in our experiments. To examine this possibility, we computed MB profits in cases where the firm is mistaken about the distribution of consumers' valuations. Suppose the true distribution of consumers' valuations is joint normal with positive correlations, but the firm sets MB prices incorrectly assuming negative correlations. In unreported experiments we found this tends to yield around 15% lower profit than if the firm had correctly assumed positive correlations.[28] This provides some degree of assurance that profits are indeed sensitive to prices in our experiments.

To better understand why BSP tends to obtain higher profits than CP in our numerical experiments in Table 7 we compare prices and market shares under CP, BSP and MB. The

[26] We leave out: Lognormal(-), Lognormal(+), Normal(+/-), Normal(v-), and Normal(v+).

[27] Although the presence of capacity constraints can favor CP over BSP with logit demand.

[28] These experiments were performed for $K = 2, ..., 5$ and with both zero and positive marginal costs. We also considered a variety of other examples of mistaken beliefs. The results were qualitatively the same in all cases.

table documents how close the CP prices and BSP prices are to the MB prices, as well as the closeness of the market shares. For each possible bundle of a given size we compute absolute price differences (as a percentage of the MB price), and average these differences across experiments. For example, based on all of our experiments with $K = 3$, including all cost scenarios, CP prices for individual component sales (bundle size equals one) tend to differ from the MB prices by 29.5%. In contrast, BSP prices in the same experiments tend to differ from MB prices by 65.7%. Hence, Table 7 reveals that CP prices for small-sized bundles (for any given K) tend to be closer to the MB prices than BSP does. But for large-sized bundles, and especially for the bundle of all K products, the BSP prices are typically very close to the MB prices, unlike CP.

The fact that prices for large-sized bundles under BSP tend to be close to the MB prices for the same bundles stems from two sources. Consider an example in which there are 5 goods ($K = 5$), and consider the prices for the various bundles containing 4 of these 5 goods (there are 5 such bundles). Under BSP these bundles are equally priced, while under MB there may be 5 different prices for these bundles. The results in Table 7 indicate that: (i) the average price of these 5 bundles under MB is close to the uniform price under BSP; and (ii) there is not much variation in prices across these 5 bundles under MB. The second of these features is an interesting consequence of heterogeneity-reduction. That is to say, as bundle-size increases, the demand for alternative bundles of the same size becomes similar. Hence, different bundles of the same size do not need to be priced very differently if the bundles are large. This is why BSP prices tend to be an especially good approximation of MB prices for large-sized bundles.

Consider also the market shares shown in Table 7. Under BSP and MB the tendency is for the majority of consumers to purchase the bundle of all K products, while under CP there are relatively few sales of the full bundle. For example with $K = 3$, CP sells a single good to 38% of consumers, while BSP and MB sell a single good to only 12% and 14% respectively. Meanwhile BSP and MB sell the full bundle to 29% and 27% of consumers, respectively, and CP sells the full bundle to only 8% of consumers. Hence, pricing under CP tends to be a better approximation to MB for small-sized bundles than BSP, while BSP tends to be a better approximation to MB than CP for the large-sized bundles. But the large-size bundles matter more—MB tends to sell many more large-sized bundles than CP, and BSP does about as well as MB in this respect.

Table 8 shows the consequences for social surplus. BSP and MB tend to yield significantly higher total output and higher profits (as we have seen in the previous tables). The table also shows that BSP and MB tend to also reduce the dead weight loss by significant amounts, relative to CP. Interestingly, the table also indicates that BSP and MB tend to result in lower consumer surplus than CP. In our experiments, apparently BSP and MB are more like perfect price discrimination. This is because of the heterogeneity-reduction effect: there is less heterogeneity in consumers' valuations for bundles of multiple goods than there is for individual goods.

4 Estimation of Joint Distribution of Consumers' Valuations

An obvious limitation of the numerical experiments in Section 3 is that we cannot be certain our results will transcend the particular parameter values we covered. For this reason, the second component of our analysis utilizes an estimated model. This allows us to demonstrate that our findings apply to an empirically relevant model.

In this section we address the problem of how to estimate the joint distribution of consumers' valuations from available data. It is important that such an approach allow for non-zero covariances in tastes, and the ability for individuals to purchase multiple products. We estimate such a demand model using data from a theater company that offers an 8-play season. Based on these estimates, we compute the profitability of each pricing strategy.

Several features make this empirical example an appealing context to study multiproduct pricing. First, the plays differ in their overall popularity, making it plausible that CP would be a sensible pricing strategy. Second, many consumers attend more than one play, making it plausible that bundling strategies may also be profitable. Third, individuals do not consume multiple units of the same play. Fourth, the assumption of no demand or cost interdependencies is reasonable. Fifth, we are confident there is insignificant resale activity—this is not rock concerts or professional sports. For all of these reasons our empirical example is a remarkably clean setting that allows us to abstract from complicating factors, in the same manner as the theoretical models of bundling.

A by-product of the analysis is that we measure the impact of each pricing strategy on consumer welfare. This is interesting because bundling, like price discrimination more generally, has ambiguous affects on consumer welfare relative to uniform pricing.[29] To the best of our knowledge, there is one prior empirical analysis of bundling. Crawford (2006) tests the hypothesis that consumers' demand for a bundle of cable channels becomes less heterogeneous as more channels are added to the bundle, which he finds to be the case. Based on a calibrated demand model, Crawford argues that adding a top-15 cable channel to a bundle and re-optimizing prices

[29]See Leslie (2004) for a similar empirical analysis of the welfare effects of price discrimination, which also happens to be in the context of theater ticket pricing.

leads to 5.5% lower consumer surplus, and 6.0% higher profit.[30]

4.1 Data Summary

The data for our empirical analysis come from *TheatreWorks*, a theater company based in Palo Alto, California. We observe all ticket sales for *TheatreWorks*' 2003–2004 season, which consisted of 229 performances of 8 different plays or musicals. Table 9 provides summary information for each of the 8 plays. A total of 69,207 tickets were sold to the eight plays. A substantial proportion of the tickets go to subscribers: 55,697 (i.e. 80%) of tickets were purchased as part of a subscription package. A given individual may purchase tickets to multiple plays (whether as a subscriber or not), and there were 18,457 individuals that attended at least one play.

Consumers may purchase tickets to individual plays at a uniform price. *TheatreWorks* also offers three subscription packages: (i) the full 8-play season; (ii) any combination of 5 plays; or (iii) a pre-specified bundle of 3 plays.[31] These subscriptions were offered at discounted prices, in the sense that the per-play price was significantly lower for subscriptions than for ordinary box office sales for individual plays. Table 10 summarizes the purchase options and their average prices.[32] There were 5,139 buyers of the 8-play bundle, 2,794 buyers of a 5-play bundle, and 205 buyers of the 3-play bundle. The data are rich enough that we also observe non-subscribers that buy tickets to multiple plays. Among non-subscribers, we observe 8,131 buyers of a single play, 1,409 buyers of two plays, 555 buyers of three plays (different to the pre-set bundle of three) and 224 buyers of four plays.

One caveat regarding the data for non-subscribers, which represent 13,510 ticket sales, is that we observe the name and address for non-subsciption purchases only if the ticket is mailed to the buyer. This allows us to infer instances of non-subscribers who purchase tickets to multiple plays. But if tickets are anonymously purchased at the box-office we have no way of knowing if that individual also purchased tickets to other plays. This would lead us to under-count the number of non-subscribers that attend multiple events. However, the name-and-address data actually cover 45% of the tickets purchased by non-subscribers. We assume the sample of non-subscribers for whom we observe mailing information is a random subset of all non-subscibers.

[30] Two studies in the marketing literature use survey response data to estimate demand and compare profits from UP, PB and MB: Venkatesh and Mahajan (1993) and Jedidi, Jagpal and Manchanda (2003).

[31] The pre-specified bundle consisted of the only 3 plays that were performed at *TheatreWorks*' secondary venue, a smaller theater in Palo Alto, CA.

[32] In fact prices also vary by time-of-week (but not by play). We therefore report simple (unweighted) averages of these prices. Note also, prices do not vary by seat quality. This is because the venues are small enough that the variation in seat quality is fairly minor.

This allows us to extrapolate the same pattern of multi-play purchases to the full sample of non-subscribers.

The popularity of the flexible 5-play subscription is a particularly important feature of the data. Observing which five plays these subscribers selected allows us to identify the covariance of tastes across plays—e.g., if we observe that two plays tend to be included together disproportionately often in the five-play combination, we know that tastes for those two plays are positively correlated. Conversely, if another pair of plays is rarely included in the same bundle, we can infer that tastes for those two plays are negatively correlated. Table 11 summarizes the correlations implied by the *pick-five* purchases: it reports the difference between the empirical correlations of the choices and the correlations that would be expected if tastes were independent.[33] The patterns make intuitive sense. For example, tastes for *Bat Boy*, described in the brochure as a "wacky new musical," are positively correlated with tastes for *Memphis*, described as a "rafter-rattling musical comedy." Conversely, tastes for *Bat Boy* are negatively correlated with *All My Sons*, a classic Arthur Miller drama billed as an "intense, compelling tale of love, greed, and personal responsibility."

The set of products for each consumer to pick from includes each individual play, the preset bundle of three, the full bundle of eight, and 56 possible combinations of five plays. However, an individual consumer may create other bundles by adding individual plays. For example, a consumer can bundle any four plays, although there is no discount for doing so. Or a consumer can purchase a bundle of five, and add a sixth play at the regular single play price. Defining products to include every conceivable bundle implies 255 choices. In fact we observe zero sales of bundles of six or seven plays. We therefore ignore these combinations in the estimation. Hence, we model the demand for 219 different bundles, plus an outside alternative, giving a total of 220 possible choices. Lastly, we note that capacity constraints are infrequently binding—only 27 of the 229 performances were sold out—leading us to abstract from their impact in the estimation. In the subsequent counterfactual analyses we check whether capacity constraints are binding.

4.2 Empirical Model

The empirical specification is based on an underlying model of individual consumer utility maximization, and follows the approach in the theoretical literature on bundling. The firm offers $j = 1, ..., J-1$ bundles containing combinations of the $k = 1, ..., K$ products. There is also a J^{th}

[33] Since each consumer selected five plays of eight, the pairwise correlations will be nonzero even if tastes are independent. The expected correlation if plays are chosen independently is -1/7.

option for consumers which is the outside alternative. We assume the net utility to consumer i from option j is given by

$$u_{ij} = \begin{cases} V_i'D_j - \alpha p_j & : \quad j = \{1, ..., J\} \\ 0 & : \quad j = J \end{cases}$$

where V is a $K \times 1$ vector of valuations for the individual plays, D_j is a $K \times 1$ vector of indicators for whether each play is included in bundle j, p_j is the price of the bundle, and $\alpha > 0$ measures the sensitivity to price. As mentioned above, we estimate demand for 219 bundle options, plus an outside alternative with zero utility for all consumers. Hence, in our notation $J = 220$. As always in the bundling literature, we assume there are no demand-side complementarities from consuming particular plays together.

We allow for two classes of consumers: theater-lovers and regular consumers. In fact the data support this description, as we explain below in the subsection on identification. Formally, we assume that consumers' product valuations are distributed according to a K-dimensional bimodal normal distribution:

$$\begin{aligned} V_i &= \theta_i + \epsilon_i, \quad \text{where} \\ \theta_i &= \begin{cases} \bar{\theta} & : \quad \text{probability } \lambda \\ 0 & : \quad \text{probability } (1-\lambda); \quad \text{and} \end{cases} \\ \epsilon_i &\sim \mathrm{N}(\mu, \Sigma). \end{aligned}$$

In this notation, μ is a $K \times 1$ vector of means, Σ is a $K \times K$ variance-covariance matrix, and $\bar{\theta}$ is a scalar additive component (equal for all plays). There are two kinds of consumers. A fraction λ of consumers are theater-lovers who like attending performances regardless of the specific play, although they may also like some plays more than others. A fraction $(1 - \lambda)$ are regular consumers with no particular preference for seeing plays in general. In our approach, the only difference between theater-lovers and regular consumers is the scalar additive component to utility: $\bar{\theta}$. In the next subsection we discuss the identification of these features of the model.

The conditional means of V are not separately identified from the variances. Intuitively, increasing the variance in valuations for a particular play and increasing the mean of the valuations for that play both lead to higher demand for the play. To address this we impose the restriction that all mean terms equal zero: $\mu(k) = 0$, $\forall k$, leaving the variance-covariance matrix unconstrained.[34] In fact we also estimated the model based on the restriction that all variances

[34] We also normalize the variance of valuations for play (1) to equal 1: $\Sigma(1, 1) = 1$.

equal one and the mean terms are unconstrained. But we found that version to be too restrictive in the following sense: BSP is always more profitable than CP, even in counterfactuals where we dramatically increase the asymmetry across products by making the mean valuations for each play very different across plays. In contrast, by allowing free variances it is possible that either CP or BSP may be the most profitable, depending on the particular values of the variance terms. We viewed this as a desirable attribute for the model. This approach has the implication that a high quality play will have a higher variance in consumers' valuations—i.e. our model captures quality via the variance terms rather than the means, which is unconventional in the literature.

To incorporate free disposal, we assume $V(k) = \max\{V(k), 0\}$, $\forall k$. Including α, Σ, $\bar{\theta}$ and λ, we estimate a total of 38 parameters. Let Θ denote the set of parameters to be estimated. Let n_j denote the observed number of consumers choosing option j. To determine the number of consumers choosing the outside alternative, we assume a market size of 100,000.[35] It is impossible to know the correct market size, but this seems reasonable based on the population of Palo Alto and the surrounding suburbs.[36]

The model's parameters are estimated via simulated maximum likelihood. We simulate the probability of a consumer choosing each option. For a given set of parameters, Θ, we draw ns simulated consumers based on the above distribution of product valuations. For each simulated consumer we compute the optimal bundle choice, implying predicted market shares for all 220 choices: s_j, such that $\sum_{j=1}^{J} s_j = 1$. The log-likelihood function is given by:

$$l(\cdot, \Theta) = \sum_{j=1}^{J} n_j \log s_j(P, \Theta),$$

where P is the $(J-1)$-dimensional vector of observed prices. The vector of estimated parameters is the value of θ that maximizes the log-likelihood function.

As is well known, simulated maximum likelihood estimators are biased due to the convexity of the log operator, but are consistent as long as the number of simulation draws (ns) scales up in proportion to $\sqrt{N}$, the squareroot of the sample size. In practice, we try values of ns ranging from 50,000 to 500,000.

[35] We check robustness of our estimates to alternative values of market size.

[36] In 2000, the population of Palo Alto was 58,598. Two nearby suburbs are Mountain View (population 70,708) and Los Altos (population 27,693). The combined population is 156,999, and we consider two thirds of this to be a reasonable measure of the market of potential consumers.

4.3 Identification

What variation in the data serves to identify each parameter of the demand model? The variance terms, $\Sigma(k,k)$, are identified by the plays' relative overall market shares: relatively high share plays must have relatively higher variances. Note, however, that the observed ranking of market shares need not be a one-to-one mapping with the estimated play variances, because the covariance terms in Σ also have an impact on choice probabilities. For example, a given play can have a high market share either because the variance in valuations is high, or because it has a strong positive correlation with another high-variance play. We suspect the choices of single-play buyers are particularly informative about the variances, because covariances play less of a role for them than for multi-play buyers.

The covariance terms themselves are identified by the bundle combinations chosen by multi-play buyers, such as the *pick-five* subscribers. Pairs of plays that consumers choose to bundle relatively often will have more positive covariances. It is tempting to expect the estimated covariances to be similar to the empirical covariance matrix shown in Table 11. However, the estimated covariances are based on a model in which we control for mean play qualities, and prices, and we utilize the complete dataset.[37] Hence, we only expect some degree of similarity. Note also, while a large fraction of consumers choose to subscribe to the full season of all eight plays, this does not necessarily imply strong positive covariances, because other features of the model can explain this particular behavior, as we explain below.

To identify the degree of price sensitivity, α, price variation comes in the form of differences in per-play prices across bundles. In particular, one specific 3-play bundle is offered at a discount (\$36.20 per play) while all other 3-play combinations have no discount (\$40.80 per play). The taste distribution alone may explain why a specific 3-play bundle is more popular than other 3-play bundles. Hence, α is identified by the extent to which demand for the discounted 3-play bundle exceeds the demand implied by the taste distribution alone. We also assume there are no complementarities in demand between these particular plays, which seems reasonable in this context.

A standard concern with demand estimation is the possibility that observed prices are correlated with unobserved demand shifters, which may bias parameter estimates. However, in the estimation we integrate over all unobserved demand components. There is no remaining error term that may be correlated with observed prices. Consider, for example, the discounted 3-play

[37] For example, the relatively large number of full season subscribers will encourage more positive covariances in the demand estimation.

bundle. We estimate the variances and covariances of the taste distribution—i.e., we control for the qualities of these plays, and we control for the tendency of consumers to want to bundle these particular plays together. The fact that this specific bundle is offered at a discount is exogenous variation for our purposes. Stated differently, we assume there are there are no bundle-specific error terms.

How do the data identify $\bar{\theta}$ and λ? This aspect of the model is important for explaining a key feature of the data. Suppose that $\bar{\theta} = 0$ (or equivalently, $\lambda = 0$). In this case, the probability of a consumer having high valuations for all eight plays is less than the probability of having high valuations for any five plays, say. Hence, the number of 8-play subscribers would be less than the number of 5-play subscribers. Similarly, the number of 5-play subscribers would be less than the number of 4-play subscribers, and so forth. But in Table 10 we see the reverse. This pattern can be partly explained by the lower per-play subscription prices for the 5-play and 8-play bundles. But this alone does not explain the large number of subscribers. This is why we distinguish theater-lovers in the demand model (i.e. the reason for including λ and $\hat{\theta}$).

Clearly, the relatively high fraction of 5-play and 8-play subscribers serves to identify $\bar{\theta}$ and λ. But how are these parameters separately identified? Since the number of single-play and 8-play buyers are both greater than the number of 2, 3, 4 or 5-play buyers, λ must not be too large or too small. If λ is near to one, nearly everyone is a theater-lover, and the model would predict a low level of single-play sales. If λ is near to zero, we have the same problem described in the previous paragraph. This logic implies that λ is identified by the ratio of subscribers (almost entirely 5-play and 8-play) to non-subscribers.

Applying similar logic, if $\bar{\theta}$ is very large then all theater-lovers will choose the 8-play bundle. If $\bar{\theta}$ is near zero then again we have the same problem described above. Hence, $\bar{\theta}$ should deliver an accurate prediction of the ratio of 5-play subscribers to 8-play subscribers. Hence, λ and $\bar{\theta}$ are identified by separate features of the data.

A final comment on the flexibility of the model. Notice from the last two columns of Table 9 that the rank-ordering of play popularity is different for non-subscribers (mainly single-play buyers) than it is for subscribers (which is driven by the tastes of *pick-five* buyers because full season buyers attend all plays). Our specification can explain this difference in the following way. The variance terms, $\Sigma(k, k)$, explain the relative popularity of plays among the single-ticket buyers. The covariance terms in Σ explain the popularity of certain pairings by the *pick-five* buyers, which also helps to explain differences in the popularity of plays that is contrary to the ranking by the single-ticket buyers. In other words, allowing for covariance in tastes gives us the flexibility to explain differences in play shares between single-play buyers and multi-play buyers.

4.4 Results

The parameter estimates for the structural demand model are presented in Table 12a (standard errors for variance-covariance matrix are reported in Table 12b). The variance coefficients from the distribution of ϵ vary from .85 to 1.19. The estimates for the covariances of ϵ vary from zero to .50. Many of the estimated covariances are near to zero: the absolute values of 11 out of the 28 covariance terms are less than .1. Collectively, these covariance terms seem to help substantially in fitting the data. A likelihood ratio test soundly rejects a model in which the covariances are constrained to be zero.[38]

It is important to note that Σ is the covariance matrix of ϵ. That is to say, Σ captures the correlation structure conditional on being a theater-lover, or conditional on not being a theater-lover. However, the correlation structure of the unconditional distribution of play valuations, V, also depends on the probability of being a theater-lover and the increment in utility for these consumers. Intuitively, taste correlations should be even more positive than for the unconditional distribution, because theater-lovers have a positive shift in the valuations of *all* plays.

When we compute pairwise correlation coefficients for the unconditional distribution of play valuations, we find that all correlations lie between .27 and .65 (the mean correlation coefficient is .44). This is important because positive correlation in the demand system tends to reduce the profitability of bundling-type strategies relative to component pricing. We return to this issue in the next subsection on counterfactual pricing experiments.

The estimate for consumers' sensitivity to price (α) is 7.23. To compute the implied price elasticity of aggregate demand we increase the price of all possible bundles of tickets by 1%, and measure the change in total total tickets sold to all plays. The resulting price elasticity of aggregate demand is 2.48. Interestingly, when we implement this calculation the demand for certain mid-sized bundles actually increases, despite increasing price for those bundles. Intuitively, a 1% increase in all prices causes some theater-lovers to substitute away from large to smaller-sized bundles.[39]

[38]The test statistic is 3,341, which easily exceeds the 5% critical value of 41.3 for the χ^2_{28} distribution. The p-value is effectively 0.

[39]For example, demand for one particular combination of 3 plays increases by 50% in response to a 1% increase in all prices.

The estimated probability of an individual being a theater-lover is .08. Table 10 shows that the total number of subscribers is 8,138. Hence, given a market size of 100,000 the estimated probability of being a theater-lover closely matches the fraction of potential consumers that actually subscribe to *TheatreWorks*. We estimate that theater-lovers' utility for any single play is higher than for regular consumers by an amount equal to 2.32 times the standard deviation of the conditional valuation of *A Little Night Music* (normalized to 1).

The large magnitude of the increment to utility for theater-lovers suggests that large-sized bundles would almost exclusively be purchased by theater-lovers. Indeed this is right. Based on our estimated demand model 88% of non-theater-lovers choose the outside option, 9.6% choose a single play, and 1.5% choose two plays. For theater-lovers, on the other hand, 99.4% attend at least one play, 52% subscribe to all eight plays, and 36% choose a bundle of five.

To evaluate the fit of the estimated model in Table 13 we present various measures of actual and predicted market shares. In the top portion of the table we show the play market shares for all consumers (ignoring the outside option). Even though the shares vary across plays from 11.1% to 15.3%, the actual and predicted shares are all within one percentage point of each other.

More importantly, our model fits the market shares for both single play buyers as well as *pick-five* subscribers, even though there are distinct differences in market shares between these two groups of buyers. For example, the most popular play among single-play buyers is play (4) with 27.4% share, more than twice that of the second most popular play among single-play buyers. For the *pick-five* buyers, the play shares are much more symmetric, with the most popular play being play (8) with a 14.6% share. In each case, the predicted shares are close to the actual shares. The estimated covariances play a critical role here. Note, for example, that the market share for play (8) among *pick-five* subscribers is high relative to its popularity among single-play purchasers. The model explains this fact partly by estimating relatively strong positive correlations between play (8) and all the other plays: the positive correlations mean that play (8) will tend to be purchased primarily in conjunction with other plays, lowering its market share as a single-play purchase.

Our estimates also fit the market shares by bundle size reasonably well. The actual fraction of all potential consumers that attend exactly one play is 8.1%, and we predict 9.1%. The actual fraction that choose to be *pick-five* subscribers is 2.3%, and we predict 3.2%. Lastly, the actual fraction that choose to subscribe to all eight plays is 5.1%, and we predict 4.4%.

4.5 Analysis of Alternative Pricing Strategies

In this subsection we compare the profitability of the various pricing schemes in the context of our estimated demand model. We also examine how particular changes in the model impact the relative profits of these different pricing structures.

Counterfactual Pricing Analysis

Using the estimated demand model, we compute profits and consumer surplus under each of UP, PB, CP, BSP and MB. We also compute the profit associated with the pricing scheme actually implemented by *TheatreWorks*, referred to as TW. Under TW the firm sets a uniform price for each play, a discount for one particular 3-play bundle, a discount for choosing any 5 plays, and a discount for the bundle of all 8 plays. In our baseline model we assume zero marginal costs and no capacity constraints, which seems reasonable given how few performances sold out. Below, we examine how capacity constraints impact the relative profits of the different pricing strategies.

Table 14 summarizes the results. The interpretation of the prices $(p_1, ..., p_8)$ varies across regimes, as explained in the note to the table. The revenue and consumer surplus (CS) results are normalized by the market size (i.e. figures are per consumer). Profits from the different pricing schemes vary from 28.73 under PB to 36.15 under MB (a difference of 26%). Hence, it is clear that the choice of price structure can be an important decision.

It is interesting that PB is the least profitable of the pricing strategies we examine. Bakos and Brynjolfsson (1999) and Fang and Norman (2006) show that PB becomes more profitable (relative to CP) as the number of goods increases (with zero marginal cost). But in this example with 8 goods, PB performs quite badly. Even UP is more profitable than PB in this setting (by 19%). This reinforces the point that PB is not necessarily a good option for firms.

Focusing on BSP in Table 14, we find that: (i) BSP attains 4.0% higher profit than CP, and (ii) BSP attains 98.3% of the profit from MB. These results are striking for a couple of reasons. MB requires the firm to set 255 distinct prices in this example, while BSP involves only 8 prices. In other words, about 3% of the number of prices are enough to yield more than 98% of the profit. But it's not just any 8 prices: CP also involves 8 prices, but attains only 94.5% of the MB profit. Also, as we have discussed, bundling-like strategies tend to be more profitable than CP in the presence of negatively correlated consumers' valuations. However, the estimated demand system includes a high degree of positive correlation in valuations, and BSP is still significantly more profitable than CP.

Under BSP the price per play varies between $24.52 (for one play) to $15.15 (for six plays). Interestingly, the price per play under BSP is non-monotonic in the number of plays. This is due to the distinction between regular consumers and theater-lovers. The firm wants to encourage non-theater-lovers to buy tickets to more than one play. But the firm does not want to offer progressively lower discounts as bundle-size increases up to eight plays, because theater-lovers dominate demand for large-sized bundles.

Note as well that under BSP the price for seeing just one play ($24.52) is higher than the highest price for any play under CP ($19.69). As explained in the previous section, BSP encourages consumers to purchase multiple plays by a combination of raising the price for one play and lowering prices for multiple-plays. By stimulating higher overall sales relative to CP, BSP not only increases profit, it also increases consumer surplus (hence, social surplus is also higher). This provides an example of a pricing strategy that increases *both* producer and consumer surplus.

TheatreWorks' actual pricing structure appears to perform quite well in this analysis. The profit under TW is 1.4% higher than under CP, despite the fact that TW involves half the number of prices as CP. The reason is that TW incorporates a degree of bundling into its structure, namely the *pick-five* and full season packages. Note that *TheatreWorks* is a non-profit organization, and arguably cares as much or more about consumer surplus than profit. According to our analysis, BSP is more attractive than TW in both dimensions. To put the differences in perspective, we estimate that switching from TW to BSP would increase profit by $87,000 and increase consumer surplus by $60,000. The difference between CP and BSP is $136,000 in profit and $206,000 in consumer surplus.

Model Perturbations

Given our estimates of demand, it is clear that BSP is the superior pricing strategy among the simple alternatives we consider. To evaluate the robustness of this conclusion, we ask how we would have to change the demand system to reverse the conclusion.

We have emphasized throughout this study that BSP is able to perform well even in the presence of highly asymmetric demand. As a measure of asymmetry, in the baseline model above, the highest price for a play under CP is 14.2% greater than the lowest price. But what if we amplified this difference? How much would we have to exacerbate the differences in plays' qualities to make CP more profitable than BSP? To examine this question, we took min-preserving spreads of the estimated variances (holding all other parameters fixed), and recomputed the optimal prices and profits under the various pricing strategies.[40] We find that BSP remains more profitable than CP even when the highest price for a play under CP (42.37) is 138% greater than the lowest price play (17.78). However, if the price difference increases to over 400% (prices of 89.74 and 17.79) then CP attains higher profit than BSP. Hence, increasing demand asymmetry favors CP, but it takes a remarkably high degree of asymmetry for CP to be more profitable than BSP.

Capacity constraints, which we have so far assumed away, could also favor CP over BSP. To assess the impact of capacity constraints, we assume capacity is equal for all plays, and is set to Δ% of the predicted sales for the most popular play under UP (where UP is computed under the assumption there are no capacity constraints). How small must Δ be (i.e., how tight does the capacity constraint have to be) for CP to be more profitable than BSP? We find that if $\Delta = 90$, BSP is still more profitable than CP, but if we lower it to 80 then CP becomes more profitable than BSP. Hence, it appears the greater is excess demand (i.e. the lower is capacity relative to demand) the more likely that CP is more profitable than BSP. Regardless, for *TheatreWorks* (as well as many other firms) the capacity for each good is to some extent endogenous. This makes it less likely that capacity constraints are a reason to prefer CP over BSP.[41]

While it is true that we can make CP more profitable than BSP by significantly amplifying demand asymmetries or by imposing relatively tight capacity constraints, note that our estimated demand model exhibits a high degree of positive correlation in tastes. This feature of demand, which arises from the presence of theater-lovers in the model, is clearly disadvantageous to bundling-type strategies like BSP. If instead there were no theater-lovers (i.e., if we set $\lambda = 0$), then the mean correlation coefficient in valuations for pairs of plays falls from .44 to .20, and BSP attains 11.5% higher profit than CP (compared to only 4.0% in the baseline model). Furthermore, reducing the correlation in tastes appears to offset other changes that tend to make BSP relatively less profitable. For example, with $\lambda = 0$, the capacity constraints described above would have to be set below 60% to make CP more profitable than BSP.

Finally, we argued in Section 3 that the inclusion of diminishing marginal utility may reduce the profit from CP by even more than it does for BSP. To verify this claim we generalize the

[40] That is, we hold the variance of the lowest-variance play at the estimated value, $min[\hat{\Sigma}(k,k)]$, and increase the remaining variance terms such that they differ from $min[\hat{\Sigma}(k,k)]$ by Δ times the corresponding differences in the actual estimates. At the same time, we inflate the covariances such that the correlations remain the same as in the actual estimates.

[41] In fact *TheatreWorks* offers more performances for some shows than for others, suggesting they choose capacities for each play (see Table 9).

utility function in the demand model in the following way:

$$u_{ij} = \begin{cases} V_i' D_j n_j^{\gamma} - \alpha p_j & : \quad j = \{1, ..., J\} \\ 0 & : \quad j = J \end{cases}$$

where n_j equals the number of goods in bundle j and γ is a parameter. We set $\gamma = -.2$ to capture diminishing marginal utility, and compute optimal prices holding all other parameters fixed at the estimated values under the baseline model.[42] Unsurprisingly the profits under all pricing schemes are lowered relative to the baseline. But now BSP attains 11.0% higher profit than CP, compared to 4.0% in the baseline model.[43] Hence, the inclusion of diminishing marginal utility can *increase* the profits of BSP relative to CP.

As we found in the numerical experiments discussed in Section 3, the above perturbations to the estimated demand model demonstrate that in some cases BSP is more profitable than CP, and in other cases it is the reverse. But what is most striking about the perturbations we analyze here is that it requires rather dramatic changes to the estimated model for CP to outperform BSP. That is to say, the relative profitability of BSP seems very robust in our analysis. This reinforces the findings of the numerical experiments: BSP appears to be an effective and practical pricing strategy for multiproduct firms to maximize profits in a broad class of settings.

5 Conclusion

We have examined the profitability of several incomplex pricing strategies for multiproduct firms, relative to the impractical ideal of mixed bundling. Rather than focus on a simplified and unrealistic model of demand, we have relied on computational methods to explore these issues in a wide variety of demand and cost scenarios. The analysis yields two main findings. First, bundle-size pricing tends to attain nearly the same level of profits as mixed bundling in a broad range of demand and cost scenarios. Hence, mixed bundling involves considerable redundancy—it includes many prices that are of negligible importance to profitability. Second, bundle-size pricing tends to be more profitable than component pricing, even in circumstances with a high degree of demand asymmetry across products.

To illustrate the empirical relevance of our findings we estimate the demand facing a theater company that produces a season of 8 plays, and compute the profitability of each pricing scheme in this case. We find that bundle-size pricing is 4% more profitable than component pricing,

[42] Implicitly, $\gamma = 0$ in the baseline model.

[43] In this case, BSP attains 98.5% of the MB profits, compared to 98.3% under the baseline.

and bundle-size pricing attains more than 98% of the mixed bundling profits. Arguably, a limitation of our empirical analysis is that it concerns a fairly narrow setting. However, we see the simplicity of our example as a virtue: "bigger" examples invariably involve additional complexities (such as active resale markets, a much larger number of products, etc.) that make a clean empirical analysis infeasible.

Our results represent a significant push towards understanding the merits of feasible pricing schemes for multiproduct firms. What insight does the prior literature on bundling have for a firm with 5 products, say? A narrow reading of the literature would imply the firm should implement mixed bundling with 31 prices, which is unlikely to be practical for most firms. A broader interpretation of the literature would suggest the firm should consider some form of bundling—which is a powerful insight—but it is unclear exactly what form that should be. This paper suggests specific advice to such a firm; bundle-size pricing (5 prices) tends to attain about 99% of the mixed bundling profit, and is almost certainly more profitable than either component pricing or pure bundling.

An important theme of our findings is that bundling-like pricing schemes are often more profitable than component pricing. This is interesting because economists are prone to criticize firms for the lack of component pricing (e.g. movie cinemas). In fact, the appeal of bundling over component pricing is reflected in the pricing of some notable multiproduct firms. Major league baseball teams, for example, tend to employ bundling strategies (such as discounts for purchasing any 9 games) more often than they employ component pricing strategies (such as charging prices that vary by opponent or by day of the week).[44] Also, online music sellers almost never charge different prices for different music tracks, even though demand is dramatically stronger for some songs than others. But music is sold via subscriptions (a strategy akin to pure bundling) by at least two of the major online music stores.

More generally, our results suggest an additional explanation for the observed simplicity of multiproduct firms' pricing strategies. Other authors have proposed various theories to explain why, in practice, complex pricing schemes are costly to implement.[45] Our study is the first (to our knowledge) to quantify the benefits of complexity. Our findings suggest these benefits are generally small, so that the costs of complexity need not be large to make simplicity the best policy.

[44]We examined the pricing for all 30 major league teams during the 2006 season. 16 teams employed some form of bundling (not including season-ticket subscriptions), whereas only 7 charged prices that varied by opponent or by day of the week.

[45]See, for example, Blinder, Canetti, Lebow and Rudd (1998) and Kahneman, Knetsch and Thaler (1986).

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Appendix A

We noted in the text that BSP may be more profitable than PB if (i) willingness to pay for the bundle of all K products is heterogeneous across consumers, and (ii) consumers (or consumer types) who have the highest willingness to pay for a bundle of size m are not necessarily the same as those with the highest willingness to pay for a bundle of size $n > m$.

We can establish a more formal condition by using an approach similar to McAfee *et al.* They note that MB nests CP as a special case, and derive a condition on the joint distribution of tastes such that a local deviation from the CP prices yields an increase in profits. In our case, we know that BSP nests PB as a special case, and we can ask when a local deviation from the PB price will be profitable.

Since BSP allows consumers to pick their own bundles, any purchased bundle of m products will consist of the m products for which the consumer's valuations were highest. Let r_{im} denote consumer i's m^{th}-highest valuation (i.e., the m^{th} order statistic). Then consumer i's willingness to pay for a bundle of size m is just $y_{im} = \sum_{k=1}^{m} r_{ik}$; i.e., the sum of the first m order statistics. Using this notation, we can write a sufficient condition for BSP to yield higher expected profits than PB in terms of the joint distribution of $y_{i,K-1}$ and r_{iK}:

Proposition: Suppose a firm sells K products for which marginal costs are identical and equal to c, and let g denote the joint distribution of $y_{i,K-1}$ (a consumer's willingness to pay for a bundle of any $K-1$ products) and r_{iK} (the willingness to pay for the least preferred product). If p^* is the optimal PB price, then BSP is more profitable than PB if there exists a Δ such that

$$\textit{(i)} \qquad 0 < \Delta < c$$

$$\textit{(ii)} \qquad \int_0^{\Delta} \int_{p^*-r}^{\infty} g(y,r)\,dy\,dr > 0$$

To prove this, consider starting with BSP prices equal to the optimal PB price, $p_1 = p_2 = \ldots = p_K = p^*$, and then reducing the price of bundles with $K-1$ or fewer products to $\tilde{p}_{K-1} = p^* - \Delta$. The expected profits under these prices are

$$\begin{aligned} \tilde{\pi}(\Delta) \;=\; & (\,p^* - Kc\,) \int_{\Delta}^{\infty} \int_{p^*-r}^{\infty} g(y,r)\,dy\,dr \;\; + \\ & (\,p^* - \Delta - (K-1)c\,) \int_0^{\Delta} \int_{p^*-\Delta}^{\infty} g(y,r)\,dy\,dr \end{aligned}$$

The difference from the PB profits is then

$$\tilde{\pi}(\Delta) - \tilde{\pi}(0) \quad = \quad (c - \Delta) \int_0^{\Delta} \int_{p^*-r}^{\infty} g(y, r) dy dr + (\, p^* - \Delta - (K-1)c \,) \int_0^{\Delta} \int_{p^*-\Delta}^{p^*-r} g(y, r) dy dr$$

Conditions (i) and (ii) of the proposition simply guarantee that this difference is positive.

Note that when marginal cost is zero, condition (i) will not be met. However, this does not mean BSP cannot be more profitable than PB when marginal cost is zero: the proposition establishes a sufficient but not necessary condition for BSP profits to be higher than PB profits. So even if this kind of local change is not profitable, there may still be other (nonlocal) changes that are. However, the proposition does suggest that positive marginal costs make it more likely that BSP beats PB.

Appendix B

In this appendix we simply report the optimal prices and profits for the two-good model described in section 3.2. Consumers' valuations for the two goods are independent uniform random variables on $[0,1]$ and $[0,\theta]$, respectively. (Assume $\theta \geq 1$.) Marginal cost is 0.

Scheme	**Optimal prices**	**Optimal profits**
CP	$p_1^* = \frac{1}{2}$, $p_2^* = \frac{\theta}{2}$	$\pi^* = \frac{(1+\theta)}{4}$
PB	$p^* = \begin{cases} \sqrt{\frac{2\theta}{3}} & \text{if } \theta \leq 3/2 \\ \frac{1}{4} + \frac{\theta}{2} & \text{if } \theta > 3/2 \end{cases}$	$\pi^* = \begin{cases} \left(\frac{2}{3\theta}\right)^{3/2} & \text{if } \theta \leq 3/2 \\ \frac{1}{8\theta}\left(\theta + \frac{1}{2}\right)^2 & \text{if } \theta > 3/2 \end{cases}$
BSP	If $\theta \leq 1.756739614$: $p_1^* = \frac{(1+\theta)}{3}$ $p_2^* = \frac{1}{3}\left(2 + 2\theta - \sqrt{2\theta^2 - 2\theta + 2}\right)$ Otherwise: $p_1^* = p_2^* = \frac{1}{4} + \frac{\theta}{2}$	If $\theta \leq 1.756739614$: $\pi^* = \frac{\left(2\theta^2 - 2\theta + 2\right)^{3/2} - 3\theta^3 + 9\theta^2 + 9\theta - 3}{27\theta}$ Otherwise: $\pi^* = \frac{1}{8\theta}\left(\theta + \frac{1}{2}\right)^2$
MB	If $\theta \leq 2$: $p_1^* = \frac{2}{3}$, $p_2^* = \frac{2\theta}{3}$ $p_{12}^* = \frac{2}{3} + \frac{2\theta}{3} - \frac{\sqrt{2\theta}}{3}$ If $\theta > 2$: $p_1^* = \frac{2}{3}$, $p_2^* = \frac{\theta}{2} + \frac{1}{3}$ $p_{12}^* = \frac{\theta}{2} + \frac{1}{3}$	If $\theta \leq 2$: $\pi^* = \frac{2}{9}\left(1 + \theta + \frac{1}{3}\sqrt{2\theta}\right)$ If $\theta > 2$: $\pi^* = \frac{27\theta^2 + 36\theta - 4}{108\theta}$

Appendix C

Available on request from the authors (or via our home pages on the web). It's very long.

Table 1. Alternative pricing strategies

Initials	Name	Num. prices	Description
UP	Uniform pricing	1	Each product sold separately at a uniform price
PB	Pure bundling	1	Only option for consumers is the full bundle
CP	Component pricing	K	Each product sold separately at different prices
BSP	Bundle-size pricing	K	Prices depend only on number of purchased products
MB	Mixed bundling	$2^K - 1$	Separate prices for every possible combination of products

Table 2. Alternative taste distributions

Name	Description
Exponential	v_{ik}'s are independent exponential random variables with means between 0.2 and 2.0
Logit	v_{ik}'s are independent extreme value random variables with means between 0 and 2.5, and scale parameter = 0.25
Lognormal	v_{ik}'s are independent lognormal random variables; $log(v_{ik})$ has variance 0.25 and mean between -1.5 and 1
Lognormal(-)	$log(v_i)$ is a multivariate normal random vector with means ranging between -1.5 and 1, and negative pairwise correlations between products*
Lognormal(+)	$log(v_i)$ is a multivariate normal random vector with means ranging between -1.5 and 1, and positive pairwise correlations between products
Normal	v_{ik}'s are independent normal random variables with variances equal to 0.25, and means between -1 and 2.5
Normal(-)	v_i is a multivariate normal random vector with means ranging between -1 and 2.5, and negative pairwise correlations between products*
Normal(+)	v_i is a multivariate normal random vector with means ranging between -1 and 2.5, and positive pairwise correlations between products
Normal(+/-)	v_i is a multivariate normal random vector with means ranging between -1 and 2.5, and pairwise correlations are a mix of positive and negative values**
Normal(v)	v_{ik}'s are independent normal random variables with means equal to zero, and variances between 0.25 and 1.75
Normal(v-)	v_i is a multivariate normal random vector with means equal to zero, variances between 0.25 and 1.75, and negative pairwise correlations between products*
Normal(v+)	v_i is a multivariate normal random vector with means equal to zero, variances between 0.25 and 1.75, and positive pairwise correlations between products
Uniform	v_{ik}'s are independent uniform random variables on $[0, a_k]$, with a_k between 0.4 and 4

* For distributions with negatively correlated tastes, we set the pairwise correlation coefficients all equal to $\underline{r}/2$, where $\underline{r}$ is the smallest (i.e., most negative) value such that the covariance matrix remains positive definite. For K=(2, 3, 4, 5) the correlation coefficients are (-0.5, -0.25, -0.1667, -0.125) respectively.

** For $K > 2$, we assume tastes for one pair of products have a correlation of -.25 and for another pair of products 0.25.

Table 3. Percentiles of profits (as a fraction of BSP profits) with zero marginal costs

		Pricing Scheme			
Taste Distn.	percentile	UP	CP	PB	MB
	.01	0.632	0.724	0.995	1.000
Exponential	.50	0.752	0.812	1.000	1.001
	.99	0.878	0.977	1.000	1.041
	.01	0.532	0.838	0.956	1.000
Logit	.50	0.779	0.899	0.989	1.003
	.99	0.956	0.984	1.000	1.019
	.01	0.403	0.734	0.999	1.000
Lognormal	.50	0.698	0.809	1.000	1.000
	.99	0.861	0.945	1.000	1.002
	.01	0.623	0.806	0.972	1.000
Normal	.50	0.831	0.900	0.995	1.001
	.99	0.985	1.023	1.000	1.043
	.01	0.690	0.930	0.901	1.000
Normal(+)	.50	0.900	0.962	0.978	1.000
	.99	0.992	1.024	1.000	1.081
	.01	0.454	0.544	0.937	1.000
Normal(-)	.50	0.671	0.743	1.000	1.000
	.99	0.991	1.025	1.000	1.090
	.01	0.837	0.872	0.951	1.000
Normal(v)	.50	0.895	0.936	0.973	1.022
	.99	0.961	1.039	0.997	1.097
	.01	0.419	0.800	0.982	1.000
Uniform	.50	0.777	0.884	0.998	1.022
	.99	0.919	1.003	1.000	1.072

The table reports percentiles of the ratio of profits to BSP profits, calculated across roughly 900 experiments represented in each cell. For a detailed summary of the price strategies and the taste distributions see Tables 1 and 2, respectively.

Table 4. Percentiles of profits (as a fraction of BSP profits) with equal positive marginal costs

		Pricing Scheme			
Taste Distn.	percentile	UP	CP	PB	MB
Exponential	.01	0.720	0.758	0.788	1.000
	.50	0.812	0.859	0.975	1.004
	.99	0.946	1.001	0.994	1.048
Logit	.01	0.552	0.840	0.434	1.000
	.50	0.801	0.906	0.958	1.003
	.99	0.991	0.998	0.995	1.019
Lognormal	.01	0.563	0.728	0.841	1.000
	.50	0.743	0.828	0.995	1.000
	.99	0.935	0.971	1.000	1.004
Normal	.01	0.632	0.809	0.344	1.000
	.50	0.851	0.907	0.929	1.001
	.99	1.000	1.001	1.000	1.011
Normal(+)	.01	0.700	0.932	0.417	1.000
	.50	0.915	0.966	0.898	1.000
	.99	0.999	1.000	0.999	1.009
Normal(-)	.01	0.461	0.539	0.323	1.000
	.50	0.700	0.768	0.969	1.000
	.99	1.000	1.020	1.000	1.021
Normal(v)	.01	0.864	0.892	0.676	1.000
	.50	0.912	0.956	0.804	1.022
	.99	0.970	1.056	0.902	1.101
Uniform	.01	0.592	0.820	0.746	1.000
	.50	0.823	0.924	0.969	1.031
	.99	0.960	1.032	0.993	1.086

The table reports percentiles of the ratio of profits to BSP profits, calculated across the roughly 900 experiments represented in each cell. For a detailed summary of the price strategies and the taste distributions see Tables 1 and 2, respectively. Marginal cost is set to 0.2 for all products.

Table 5. Percentiles of profits (as a fraction of BSP profits) with unequal marginal costs

		Pricing Scheme			
Taste Distn.	percentile	UP	CP	PB	MB
	.01	0.657	0.824	0.812	1.000
Exponential	.50	0.839	0.940	0.910	1.054
	.99	0.939	1.058	0.988	1.172
	.01	0.373	0.834	0.829	1.000
Logit	.50	0.713	0.903	0.950	1.014
	.99	0.949	0.992	0.989	1.070
	.01	0.438	0.774	0.957	1.000
Lognormal	.50	0.720	0.888	0.989	1.034
	.99	0.911	0.993	0.997	1.081
	.01	0.466	0.799	0.798	1.000
Normal	.50	0.792	0.910	0.951	1.009
	.99	0.981	1.097	1.000	1.143
	.01	0.000	0.931	0.809	1.000
Normal(+)	.50	0.841	0.967	0.932	1.003
	.99	0.988	1.035	1.000	1.160
	.01	0.303	0.523	0.838	1.000
Normal(-)	.50	0.632	0.768	0.987	1.007
	.99	1.000	1.122	1.000	1.146
	.01	0.834	0.894	0.760	1.000
Normal(v)	.50	0.908	0.967	0.823	1.032
	.99	0.969	1.096	0.933	1.149
	.01	0.437	0.864	0.795	1.000
Uniform	.50	0.813	0.999	0.893	1.101
	.99	0.947	1.091	0.992	1.230

The table reports percentiles of the ratio of profits to BSP profits, calculated across the roughly 900 experiments represented in each cell. For a detailed summary of the price strategies and the taste distributions see Tables 1 and 2, respectively. Marginal cost equals 0.5 times consumers' mean product valuation.

Table 6. Percentiles of profits (as a fraction of BSP profits) with capacity constraints

		Pricing Scheme			
Taste Distn.	percentile	UP	CP	PB	MB
	.01	0.455	0.818	0.652	1.000
Exponential	.50	0.859	0.947	0.857	1.069
	.99	1.227	1.246	1.000	1.303
	.01	0.436	0.858	0.743	1.000
Logit	.50	0.770	0.922	0.986	1.004
	.99	1.000	1.025	1.000	1.071
	.01	0.351	0.746	0.868	1.000
Lognormal	.50	0.700	0.831	1.000	1.000
	.99	0.934	0.953	1.000	1.061
	.01	0.583	0.821	0.666	1.000
Normal	.50	0.833	0.915	0.987	1.000
	.99	1.004	1.043	1.000	1.085
	.01	0.597	0.907	0.687	1.000
Normal(+)	.50	0.876	0.964	0.970	1.000
	.99	1.005	1.038	1.000	1.056
	.01	0.482	0.667	0.704	1.000
Normal(-)	.50	0.774	0.852	0.990	1.010
	.99	1.001	1.054	1.000	1.128
	.01	0.847	0.919	0.499	1.000
Normal(v)	.50	0.912	0.976	0.709	1.027
	.99	1.125	1.125	0.949	1.138
	.01	0.336	0.858	0.729	1.000
Uniform	.50	0.844	0.977	0.914	1.064
	.99	1.143	1.198	1.000	1.285

The table reports percentiles of the ratio of profits to BSP profits, calculated across the roughly 900 experiments represented in each cell. Marginal costs are zero, but there is a binding capacity constraint which varies by experiment. It is set by first computing the optimal uniform price under no capacity constraints, finding the quantity demanded for the most popular product, and setting the constraint equal to 0.9 times that quantity.

Table 7. Price differences and market shares, by bundle size

K=3

	Average price differences		Market shares		
Bundle size	$\lvert p_{CP} - p_{MB}\rvert$	$\lvert p_{BSP} - p_{MB}\rvert$	CP	BSP	MB
1	0.295	0.657	0.384	0.125	0.140
2	0.146	0.183	0.227	0.170	0.197
3	0.174	0.037	0.082	0.287	0.274

K=4

	Average price differences		Market shares		
Bundle size	$\lvert p_{CP} - p_{MB}\rvert$	$\lvert p_{BSP} - p_{MB}\rvert$	CP	BSP	MB
1	0.359	0.809	0.327	0.089	0.097
2	0.196	0.306	0.250	0.134	0.157
3	0.167	0.123	0.140	0.147	0.169
4	0.202	0.037	0.051	0.257	0.238

K=5

	Average price differences		Market shares		
Bundle size	$\lvert p_{CP} - p_{MB}\rvert$	$\lvert p_{BSP} - p_{MB}\rvert$	CP	BSP	MB
1	0.409	0.929	0.275	0.062	0.067
2	0.238	0.403	0.245	0.106	0.119
3	0.185	0.197	0.170	0.119	0.144
4	0.183	0.094	0.092	0.139	0.154
5	0.218	0.035	0.035	0.232	0.212

Price differences are calculated as a percent of the MB price, and then averaged across prices within bundle size and across experiments. Market shares are averages across experiments for bundles of a given size. For example, on average across experiments with $K = 3$, MB pricing leads 14.0% of consumers to purchase a single product.

Table 8. Average welfare effects

		CP	BSP	MB
$K = 3$	Total output	1.086	1.329	1.359
	Consumer surplus	0.590	0.526	0.523
	Producer surplus	0.929	1.072	1.089
	Total surplus	1.520	1.597	1.611
	Dead weight loss	0.367	0.290	0.276
$K = 4$	Total output	1.452	1.827	1.874
	Consumer surplus	0.796	0.692	0.670
	Producer surplus	1.261	1.489	1.516
	Total surplus	2.057	2.180	2.186
	Dead weight loss	0.493	0.370	0.364
$K = 5$	Total output	1.815	2.347	2.410
	Consumer surplus	1.000	0.850	0.808
	Producer surplus	1.590	1.913	1.951
	Total surplus	2.589	2.763	2.760
	Dead weight loss	0.619	0.445	0.448

Total output is calculated as the number of units sold of all K products combined. The cells report averages taken across experiments.

Table 9. Summary of ticket sales

Play	Type	Number of Performances	Average Attendance	Ticket sales (subscription)	Ticket sales (non-subscription)
A Little Night Music	Musical	30	294.87	7018	1828
All My Sons	Drama	33	233.85	6826	891
Bat Boy	Musical	30	263.93	6782	1136
Memphis	Musical	30	352.40	6999	3573
My Antonia	Drama	26	312.38	7002	1120
Nickel and Dimed	Drama	26	343.62	6800	2134
Proof	Drama	25	319.88	6885	1112
The Fourth Wall	Comedy	29	313.83	7385	1716
Total		229	302.21	55,697	13,510

Three plays (*Bat Boy*, *All My Sons*, and *The Fourth Wall*) were performed at the Lucie Stern Theater in Palo Alto (capacity=428). The remaining five were performed at the Mountain View Center for the Performing Arts (capacity=589).

Table 10. Sales by purchase option

Purchase option	Price per play ($)	Number of consumers
Non-subscription:		
1 play	40.80	8,131
2 plays	40.80	1,409
3 plays	40.80	555
4 plays	40.80	224
Subscription:		
3-play bundle	36.20	205
5-play pick	37.00	2,794
8-play bundle	34.55	5,139

The 3-play subscription bundle was for the specific three plays performed at the (smaller) Lucie Stern Theater in Palo Alto, which is why the per-play price is lower than the 5-play bundle. Consumers purchasing the 5-play subscription could combine any 5 plays of their choice.

Table 11. Correlation of tastes for *pick-five* subscribers

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) *A Little Night Music*	.000							
(2) *All My Sons*	-.026	.000						
(3) *Bat Boy*	.067	-.233	.000					
(4) *Memphis*	.072	-.081	.257	.000				
(5) *My Antonia*	.177	.067	-.086	-.037	.000			
(6) *Nickel and Dimed*	-.160	-.009	-.013	-.039	.001	.000		
(7) *Proof*	-.066	.210	-.030	-.057	-.094	.008	.000	
(8) *The Fourth Wall*	-.038	.034	.003	-.117	-.007	.196	.008	.000

This is the difference between the observed correlation matrix and the correlation matrix that would be expected if plays were chosen independently (i.e., no correlation in tastes). It is constructed for the 8,005 purchasers of the flexible 5-play subscription.

Table 12a. Estimated coefficients

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
				Covariances (Σ)				
(1)	1.0000							
(2)	**0.2442**	**0.8775**						
(3)	**0.0665**	**0.1721**	**0.8531**					
(4)	-0.0021	0.0072	**0.0734**	**1.1874**				
(5)	**0.3278**	**0.1753**	**0.1079**	**0.1479**	**0.8493**			
(6)	0.0018	**0.0865**	**0.0159**	**0.1301**	**0.0503**	**0.9746**		
(7)	**0.1690**	**0.2376**	**0.0723**	-0.0021	**0.2572**	-0.0033	**0.8526**	
(8)	**0.5048**	**0.4033**	**0.4006**	**0.4353**	**0.4035**	**0.4959**	**0.3903**	**1.0125**

	Estimate	Std. error
Price sensitivity (α)	**7.2256**	(0.0151)
Probability of theater-lover (λ)	**0.0827**	(0.0015)
Increment for theater-lovers ($\bar{\theta}$)	**2.3227**	(0.0059)

Standard errors for Σ are in Table 9a. Estimates that are significant at the 5% level are highlighted in bold.

Table 12b. Standard errors for estimated covariances (Σ) in Table 12a

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(2)	0.0314	0.0063						
(3)	0.0153	0.0343	0.0060					
(4)	0.0075	0.0225	0.0193	0.0087				
(5)	0.0075	0.0112	0.0099	0.0181	0.0064			
(6)	0.0150	0.0086	0.0079	0.0132	0.0179	0.0073		
(7)	0.0161	0.0128	0.0149	0.0154	0.0102	0.0202	0.0065	
(8)	0.0079	0.0131	0.0111	0.0140	0.0089	0.0116	0.0221	0.0050

Table 13. Actual and predicted market shares of each play

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Shares among all consumers (%)							
Actual	12.8	11.1	11.4	15.3	11.7	12.9	11.6	13.2
Predicted	12.7	12.1	12.0	14.4	11.8	12.8	11.9	12.4
	Shares among single-play consumers (%)							
Actual	13.1	10.9	8.9	27.4	9.4	13.0	8.2	9.1
Predicted	14.5	10.2	9.3	26.3	9.0	14.7	8.9	7.1
	Shares among *pick-five* subscribers (%)							
Actual	13.4	10.6	10.3	13.3	13.3	11.9	12.5	14.6
Predicted	12.5	12.4	12.6	12.0	12.6	12.7	13.1	12.2

Table 14. Counterfactual pricing

	UP	PB	TW	CP	BSP	MB
p_1	18.59		21.28	18.20	24.52	26.51
p_2				17.24	19.26	20.84
p_3			20.55	18.86	17.45	17.97
p_4				19.69	16.13	16.53
p_5			14.77	17.78	15.33	15.93
p_6				18.14	15.15	15.57
p_7				18.12	16.45	16.34
p_8		8.28	16.94	19.33	16.13	16.09
Revenue	34.14	28.73	34.67	34.18	35.54	36.15
CS	31.34	40.22	33.09	31.63	33.69	32.98

For UP, p_1 is the optimal uniform price for a single play. For PB, p_8 is the optimal per-play price for the bundle of all eight plays. TW is the pricing scheme currently employed by the theater company: p_1 is the single-play price, p_3 is the per-play price for a specific bundle of three plays, p_5 is the per-play price for any combination of five plays, and p_8 is the per-play price if you buy all eight. For CP, p_1-p_8 are the prices for the eight individual plays, and for BSP, p_1-p_8 are the per-play prices for any bundle containing the corresponding number of plays. For MB, p_1-p_8 are mean per-play prices for bundles of a given size (e.g. p_1 is the mean single-play price, p_2 is the mean price for all 2-play bundles, and so forth). The revenue and consumer surplus numbers are normalized by the market size—i.e., we report revenue per consumer.

Figure 1: Separation of consumers under CP and BSP

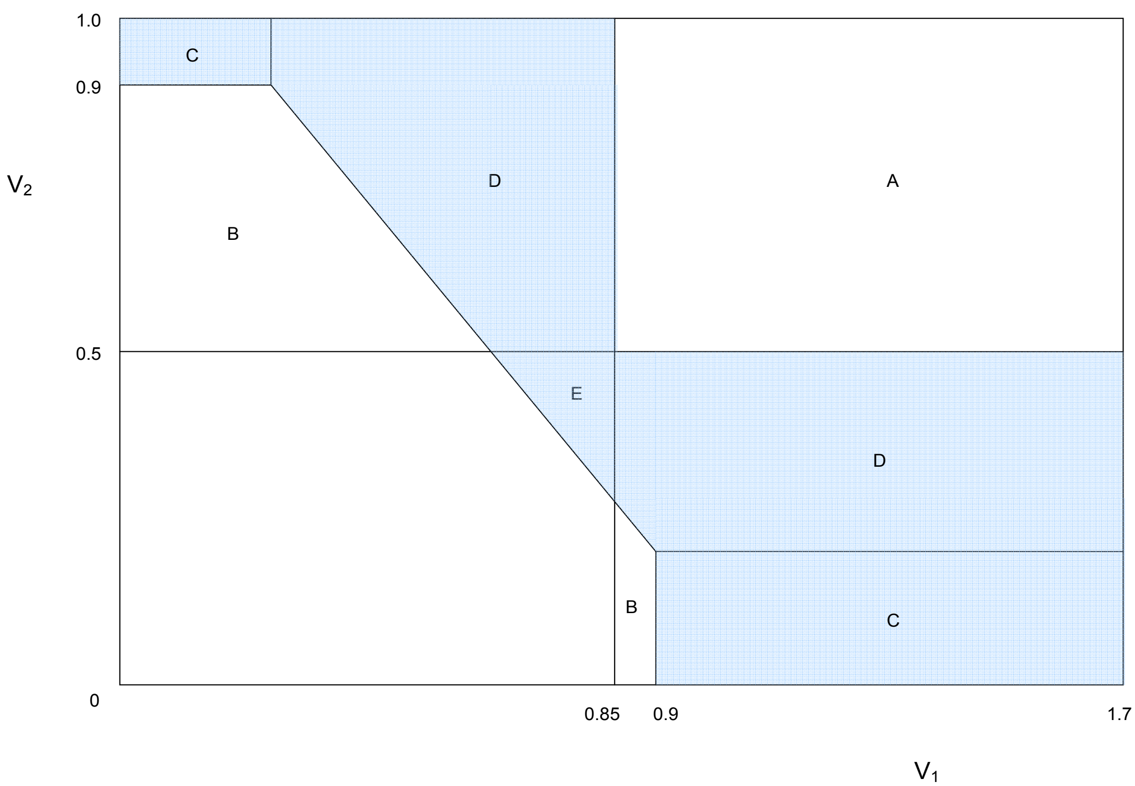
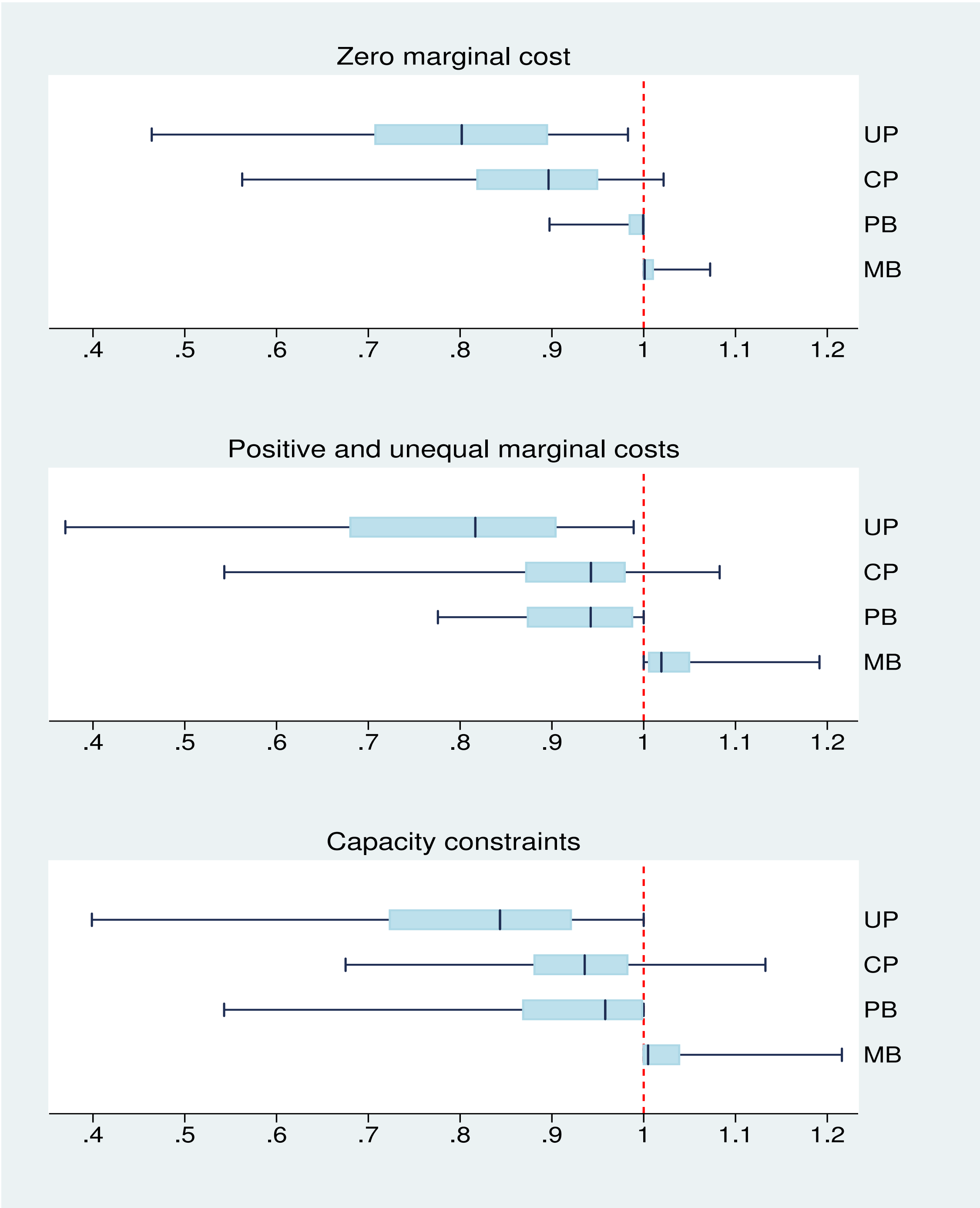


Figure 2: Distributions of profits for each pricing strategy, relative to BSP, under different assumptions on marginal costs



Each box-plot depicts the 1st, 25th, 50th, 75th and 99th percentile of the distribution of profit relative to the profit from MB.

Appendix C

Tables C1 through C4 report additional statistics corresponding to Tables 3 through 6. Each table represents numerical experiments for a given assumption about costs. Each cell in the table reports the 1st, 50th, and 99th percentile of profits (as a ratio of BSP profits) for the corresponding taste distribution and number of products (K).

Table C1. Profits relative to BSP, zero marginal costs

Taste Distribution	K	UP	CP	PB	MB
		Pricing Schemes			
Exponential	2	0.809	0.875	0.994	1.000
		0.866	0.887	1.000	1.000
		0.887	0.986	1.000	1.024
	3	0.700	0.802	0.997	1.000
		0.782	0.831	1.000	1.002
		0.816	0.954	1.000	1.041
	4	0.648	0.762	0.996	1.000
		0.735	0.789	1.000	1.002
		0.770	0.907	1.000	1.044
	5	0.601	0.723	0.998	1.000
		0.698	0.754	1.000	1.002
		0.734	0.870	1.000	1.046
Logit	2	0.685	0.924	0.952	1.000
		0.878	0.940	0.986	1.000
		0.972	0.988	1.000	1.017
	3	0.576	0.884	0.957	1.000
		0.793	0.908	0.987	1.003
		0.931	0.968	1.000	1.019
	4	0.547	0.854	0.967	1.000
		0.758	0.885	0.989	1.004
		0.906	0.949	1.000	1.019
	5	0.513	0.831	0.972	1.000
		0.730	0.870	0.991	1.004
		0.888	0.936	0.999	1.020
Lognormal	2	0.727	0.859	1.000	1.000
		0.810	0.876	1.000	1.000
		0.875	0.957	1.000	1.000
	3	0.591	0.799	1.000	1.000
		0.721	0.825	1.000	1.000
		0.800	0.911	1.000	1.001
	4	0.399	0.759	1.000	1.000
		0.668	0.789	1.000	1.000
		0.759	0.876	1.000	1.002
	5	0.350	0.729	0.999	1.000
		0.633	0.763	1.000	1.000
		0.733	0.849	1.000	1.004

Table C1, continued. Profits relative to BSP, zero marginal costs

Taste Distribution	K	Pricing Schemes			
		UP	CP	PB	MB
Lognormal(+)	2	0.771	0.934	0.999	1.000
		0.852	0.941	1.000	1.000
		0.935	0.979	1.000	1.000
	3	0.650	0.911	0.999	1.000
		0.806	0.920	1.000	1.000
		0.911	0.957	1.000	1.000
	4	0.436	0.899	0.999	1.000
		0.772	0.908	1.000	1.000
		0.900	0.941	1.000	1.001
	5	0.386	0.890	0.998	1.000
		0.751	0.900	1.000	1.000
		0.891	0.931	1.000	1.001
Lognormal(-)	2	0.514	0.549	0.985	1.000
		0.555	0.611	1.000	1.000
		0.845	0.924	1.000	1.035
	3	0.469	0.547	1.000	1.000
		0.543	0.635	1.000	1.000
		0.707	0.857	1.000	1.031
	4	0.381	0.542	1.000	1.000
		0.530	0.639	1.000	1.000
		0.671	0.810	1.000	1.028
	5	0.324	0.540	0.999	1.000
		0.521	0.633	1.000	1.000
		0.646	0.784	1.000	1.027
Normal	2	0.739	0.899	0.969	1.000
		0.920	0.960	0.998	1.000
		0.989	1.055	1.000	1.055
	3	0.667	0.852	0.973	1.000
		0.851	0.907	0.994	1.000
		0.976	1.000	1.000	1.041
	4	0.623	0.820	0.976	1.000
		0.805	0.879	0.995	1.001
		0.960	0.986	1.000	1.042
	5	0.580	0.795	0.976	1.000
		0.776	0.857	0.995	1.001
		0.945	0.981	1.000	1.021

Table C1, continued. Profits relative to BSP, zero marginal costs

Taste Distribution	K	Pricing Schemes			
		UP	CP	PB	MB
Normal(+)	2	0.794	0.955	0.936	1.000
		0.957	0.986	0.987	1.000
		0.996	1.004	1.000	1.012
	3	0.724	0.940	0.918	1.000
		0.914	0.966	0.975	1.000
		0.986	1.015	1.000	1.142
	4	0.680	0.932	0.901	1.000
		0.887	0.957	0.976	1.001
		0.977	1.034	1.000	1.065
	5	0.649	0.928	0.890	1.000
		0.869	0.949	0.976	1.001
		0.964	1.041	1.000	1.089
Normal(-)	2	0.521	0.543	0.858	1.000
		0.675	0.706	1.000	1.000
		0.992	1.061	1.000	1.075
	3	0.459	0.542	0.995	1.000
		0.695	0.765	1.000	1.000
		0.972	1.056	1.000	1.117
	4	0.457	0.544	0.996	1.000
		0.674	0.756	1.000	1.000
		0.947	1.001	1.000	1.045
	5	0.441	0.549	0.983	1.000
		0.655	0.740	1.000	1.000
		0.926	0.974	1.000	1.024
Normal(v)	2	0.916	0.954	0.964	1.000
		0.955	0.969	0.983	1.013
		0.961	1.056	0.998	1.095
	3	0.884	0.928	0.956	1.000
		0.920	0.944	0.971	1.017
		0.931	1.025	0.997	1.098
	4	0.853	0.896	0.957	1.001
		0.890	0.919	0.969	1.024
		0.897	0.981	0.994	1.089
	5	0.831	0.871	0.950	1.004
		0.864	0.898	0.969	1.030
		0.873	0.960	0.993	1.091

Table C1, continued. Profits relative to BSP, zero marginal costs

Taste Distribution	K	Pricing Schemes			
		UP	CP	PB	MB
Normal(v+)	2	0.906	0.982	0.937	1.000
		0.981	0.991	0.946	1.010
		0.983	1.046	0.988	1.067
	3	0.899	0.969	0.910	1.001
		0.960	0.981	0.921	1.014
		0.969	1.027	0.968	1.061
	4	0.872	0.964	0.899	1.002
		0.942	0.978	0.920	1.017
		0.964	1.018	0.952	1.058
	5	0.870	0.956	0.877	1.004
		0.933	0.973	0.904	1.022
		0.956	1.011	0.945	1.057
Normal(v-)	2	0.920	0.936	1.000	1.000
		0.945	0.959	1.000	1.000
		0.983	1.090	1.000	1.121
	3	0.855	0.867	0.991	1.000
		0.885	0.910	1.000	1.018
		0.904	0.990	1.000	1.092
	4	0.780	0.793	0.986	1.000
		0.796	0.842	1.000	1.014
		0.870	0.934	1.000	1.075
	5	0.741	0.758	0.985	1.000
		0.758	0.807	1.000	1.013
		0.842	0.892	1.000	1.077
Normal(+/-)	2	0.739	0.899	0.969	1.000
		0.920	0.960	0.998	1.000
		0.989	1.055	1.000	1.055
	3	0.647	0.817	0.953	1.000
		0.808	0.859	0.997	1.001
		0.973	1.003	1.000	1.039
	4	0.594	0.801	0.963	1.000
		0.767	0.831	0.994	1.001
		0.952	0.980	1.000	1.040
	5	0.564	0.786	0.962	1.000
		0.748	0.826	0.992	1.002
		0.928	0.973	1.000	1.024

Table C1, continued. Profits relative to BSP, zero marginal costs

Taste Distribution	K	Pricing Schemes			
		UP	CP	PB	MB
Uniform	2	0.501	0.916	0.989	1.000
		0.889	0.953	0.994	1.029
		0.919	1.004	1.000	1.050
	3	0.419	0.861	0.982	1.000
		0.806	0.901	0.999	1.022
		0.869	0.993	1.000	1.067
	4	0.433	0.824	0.980	1.001
		0.762	0.863	0.998	1.022
		0.826	0.971	1.000	1.074
	5	0.383	0.795	0.986	1.002
		0.730	0.835	0.998	1.022
		0.799	0.952	1.000	1.083

Table C2. Profits relative to BSP, positive and equal marginal costs

Taste Distribution	K	Pricing Schemes			
		UP	CP	PB	MB
Exponential	2	0.858	0.891	0.859	1.000
		0.891	0.922	0.985	1.002
		0.960	1.005	0.995	1.026
	3	0.777	0.832	0.832	1.000
		0.829	0.875	0.976	1.003
		0.918	1.006	0.993	1.043
	4	0.740	0.792	0.809	1.001
		0.789	0.837	0.970	1.005
		0.868	0.995	0.990	1.048
	5	0.708	0.753	0.773	1.001
		0.757	0.804	0.968	1.006
		0.828	0.984	0.989	1.054
Logit	2	0.693	0.924	0.447	1.000
		0.899	0.945	0.955	1.000
		0.995	1.000	0.995	1.017
	3	0.595	0.882	0.445	1.000
		0.812	0.917	0.958	1.003
		0.987	0.996	0.994	1.021
	4	0.568	0.855	0.459	1.000
		0.773	0.891	0.959	1.004
		0.977	0.992	0.993	1.021
	5	0.540	0.834	0.434	1.000
		0.746	0.872	0.961	1.004
		0.971	0.989	0.995	1.018
Lognormal	2	0.758	0.858	0.892	1.000
		0.853	0.905	0.997	1.000
		0.950	0.978	1.000	1.002
	3	0.645	0.795	0.864	1.000
		0.765	0.845	0.995	1.000
		0.897	0.955	1.000	1.003
	4	0.578	0.755	0.851	1.000
		0.718	0.802	0.994	1.000
		0.846	0.930	1.000	1.003
	5	0.539	0.724	0.833	1.000
		0.684	0.774	0.994	1.000
		0.815	0.909	1.000	1.010

Table C2, continued. Profits relative to BSP, positive and equal marginal costs

Taste Distribution	K	Pricing Schemes			
		UP	CP	PB	MB
Lognormal(+)	2	0.808	0.934	0.932	1.000
		0.907	0.958	0.996	1.000
		0.971	0.995	1.000	1.002
	3	0.715	0.912	0.907	1.000
		0.851	0.933	0.992	1.000
		0.942	0.986	1.000	1.002
	4	0.661	0.899	0.890	1.000
		0.826	0.920	0.990	1.000
		0.924	0.979	1.000	1.003
	5	0.625	0.890	0.873	1.000
		0.806	0.911	0.989	1.000
		0.905	0.974	1.000	1.004
Lognormal(-)	2	0.514	0.534	0.835	1.000
		0.568	0.625	0.999	1.000
		0.932	0.960	1.000	1.031
	3	0.485	0.527	0.878	1.000
		0.575	0.650	0.998	1.000
		0.872	0.928	1.000	1.025
	4	0.462	0.518	0.887	1.000
		0.560	0.647	0.998	1.000
		0.803	0.888	1.000	1.018
	5	0.447	0.518	0.879	1.000
		0.548	0.638	0.997	1.000
		0.755	0.854	1.000	1.018
Normal	2	0.766	0.900	0.636	1.000
		0.946	0.978	0.932	1.000
		1.000	1.000	1.000	1.008
	3	0.682	0.853	0.412	1.000
		0.868	0.918	0.921	1.001
		0.999	1.001	1.000	1.009
	4	0.640	0.822	0.329	1.000
		0.819	0.890	0.928	1.001
		0.996	1.000	0.999	1.011
	5	0.625	0.796	0.263	1.000
		0.789	0.863	0.931	1.001
		0.997	1.000	0.999	1.012

Table C2, continued. Profits relative to BSP, positive and equal marginal costs

Taste Distribution	K	Pricing Schemes			
		UP	CP	PB	MB
Normal(+)	2	0.808	0.956	0.608	1.000
		0.967	0.992	0.908	1.000
		1.000	1.002	0.999	1.005
	3	0.745	0.941	0.481	1.000
		0.927	0.971	0.895	1.000
		0.999	1.000	0.999	1.007
	4	0.705	0.935	0.423	1.000
		0.897	0.960	0.896	1.000
		0.998	1.000	0.998	1.008
	5	0.687	0.928	0.364	1.000
		0.880	0.953	0.892	1.000
		0.995	1.000	0.997	1.010
Normal(-)	2	0.522	0.539	0.600	1.000
		0.729	0.757	0.974	1.000
		1.000	1.029	1.000	1.029
	3	0.472	0.543	0.380	1.000
		0.758	0.791	0.972	1.000
		1.000	1.014	1.000	1.020
	4	0.460	0.543	0.323	1.000
		0.706	0.768	0.968	1.000
		0.998	1.000	1.000	1.018
	5	0.453	0.542	0.247	1.000
		0.683	0.755	0.968	1.000
		0.994	1.000	0.999	1.021
Normal(v)	2	0.929	0.967	0.794	1.000
		0.964	0.976	0.885	1.007
		0.972	1.065	0.910	1.091
	3	0.903	0.941	0.718	1.000
		0.929	0.957	0.828	1.015
		0.944	1.056	0.861	1.109
	4	0.875	0.910	0.676	1.002
		0.907	0.943	0.794	1.027
		0.921	1.028	0.825	1.103
	5	0.854	0.890	0.645	1.004
		0.885	0.922	0.775	1.031
		0.904	1.006	0.802	1.101

Table C2, continued. Profits relative to BSP, positive and equal marginal costs

Taste Distribution	K	Pricing Schemes			
		UP	CP	PB	MB
Normal(v+)	2	0.898	0.984	0.819	1.000
		0.978	0.989	0.874	1.005
		0.986	1.033	0.931	1.045
	3	0.884	0.970	0.780	1.000
		0.954	0.985	0.844	1.013
		0.975	1.035	0.875	1.063
	4	0.874	0.964	0.756	1.003
		0.944	0.988	0.817	1.025
		0.966	1.024	0.845	1.056
	5	0.856	0.956	0.750	1.006
		0.936	0.980	0.802	1.028
		0.957	1.019	0.825	1.061
Normal(v-)	2	0.950	0.950	0.808	1.000
		0.960	0.997	0.897	1.010
		1.000	1.119	0.925	1.129
	3	0.903	0.905	0.720	1.000
		0.921	0.946	0.843	1.023
		0.936	1.057	0.874	1.117
	4	0.854	0.855	0.648	1.002
		0.879	0.913	0.806	1.032
		0.903	1.030	0.853	1.126
	5	0.819	0.827	0.610	1.004
		0.841	0.879	0.793	1.029
		0.877	0.993	0.843	1.121
Normal(+/-)	2	0.766	0.900	0.636	1.000
		0.946	0.978	0.932	1.000
		1.000	1.000	1.000	1.008
	3	0.656	0.818	0.408	1.000
		0.824	0.862	0.911	1.001
		1.000	1.007	0.999	1.014
	4	0.614	0.799	0.320	1.000
		0.779	0.832	0.920	1.001
		0.997	1.000	0.998	1.016
	5	0.590	0.787	0.240	1.000
		0.761	0.825	0.916	1.002
		0.997	0.999	0.998	1.020

Table C2, continued. Profits relative to BSP, positive and equal marginal costs

Taste Distribution	K	Pricing Schemes			
		UP	CP	PB	MB
Uniform	2	0.724	0.925	0.746	1.000
		0.911	0.982	0.970	1.023
		0.971	1.027	0.994	1.072
	3	0.648	0.875	0.782	1.000
		0.847	0.933	0.970	1.029
		0.948	1.043	0.989	1.099
	4	0.589	0.838	0.790	1.001
		0.805	0.897	0.965	1.032
		0.906	1.009	0.982	1.083
	5	0.540	0.814	0.772	1.002
		0.775	0.872	0.968	1.033
		0.883	1.005	0.981	1.086

Table C3. Profits relative to BSP, positive and unequal marginal costs

Taste Distribution	K	Pricing Schemes			
		UP	CP	PB	MB
Exponential	2	0.835	0.936	0.908	1.000
		0.915	0.974	0.934	1.031
		0.941	1.038	0.992	1.079
	3	0.717	0.888	0.863	1.001
		0.859	0.950	0.916	1.050
		0.910	1.050	0.981	1.127
	4	0.646	0.855	0.830	1.002
		0.828	0.923	0.890	1.058
		0.879	1.060	0.969	1.161
	5	0.606	0.819	0.807	1.003
		0.801	0.902	0.875	1.075
		0.864	1.068	0.964	1.192
Logit	2	0.606	0.923	0.829	1.000
		0.829	0.947	0.952	1.005
		0.960	1.003	0.993	1.047
	3	0.477	0.881	0.836	1.000
		0.733	0.915	0.948	1.013
		0.918	0.989	0.984	1.058
	4	0.415	0.850	0.832	1.000
		0.686	0.886	0.948	1.016
		0.888	0.969	0.983	1.074
	5	0.342	0.828	0.832	1.001
		0.650	0.863	0.953	1.018
		0.859	0.957	0.984	1.078
Lognormal	2	0.737	0.911	0.964	1.000
		0.813	0.945	0.989	1.005
		0.911	0.996	0.997	1.031
	3	0.583	0.836	0.957	1.000
		0.740	0.904	0.989	1.032
		0.860	0.982	0.997	1.053
	4	0.490	0.789	0.957	1.001
		0.683	0.865	0.989	1.037
		0.814	0.961	0.996	1.073
	5	0.435	0.758	0.950	1.003
		0.644	0.835	0.989	1.045
		0.777	0.950	0.996	1.085

Table C3, continued. Profits relative to BSP, positive and unequal marginal costs

Taste Distribution	K	Pricing Schemes UP	CP	PB	MB
Lognormal(+)	2	0.760	0.958	0.971	1.000
		0.851	0.976	0.992	1.000
		0.960	0.995	1.000	1.015
	3	0.620	0.931	0.973	1.001
		0.792	0.960	0.992	1.015
		0.936	0.984	0.999	1.022
	4	0.534	0.920	0.972	1.001
		0.739	0.950	0.991	1.020
		0.922	0.981	0.997	1.028
	5	0.497	0.911	0.967	1.003
		0.721	0.945	0.990	1.023
		0.912	0.975	0.998	1.031
Lognormal(-)	2	0.511	0.540	0.949	1.000
		0.583	0.674	0.989	1.012
		0.944	1.055	1.000	1.185
	3	0.442	0.528	0.979	1.000
		0.566	0.758	0.993	1.048
		0.814	0.997	0.998	1.189
	4	0.401	0.523	0.981	1.000
		0.536	0.726	0.995	1.049
		0.767	0.972	0.999	1.211
	5	0.333	0.514	0.981	1.000
		0.515	0.699	0.995	1.049
		0.724	0.944	0.999	1.230
Normal	2	0.686	0.901	0.863	1.000
		0.909	0.969	0.966	1.002
		0.987	1.135	1.000	1.198
	3	0.571	0.852	0.838	1.000
		0.820	0.922	0.949	1.008
		0.968	1.093	1.000	1.129
	4	0.509	0.818	0.825	1.000
		0.760	0.889	0.946	1.011
		0.953	1.030	1.000	1.076
	5	0.438	0.788	0.001	1.000
		0.718	0.859	0.946	1.013
		0.950	1.049	1.000	1.107

Table C3, continued. Profits relative to BSP, positive and unequal marginal costs

Taste Distribution	K	UP	CP	PB	MB
		Pricing Schemes			
Normal(+)	2	0.726	0.958	0.878	1.000
		0.944	0.988	0.955	1.000
		0.995	1.021	1.000	1.158
	3	0.623	0.941	0.828	1.000
		0.866	0.971	0.932	1.003
		0.982	1.023	1.000	1.160
	4	0.001	0.934	0.809	1.000
		0.827	0.958	0.926	1.004
		0.976	1.044	1.000	1.088
	5	0.000	0.927	0.778	1.000
		0.765	0.952	0.924	1.006
		0.964	1.022	0.999	1.090
Normal(-)	2	0.423	0.534	0.805	1.000
		0.691	0.772	0.992	1.004
		1.000	1.163	1.000	1.163
	3	0.351	0.526	0.865	1.000
		0.688	0.825	0.988	1.005
		0.963	1.082	1.000	1.127
	4	0.000	0.523	0.825	1.000
		0.620	0.774	0.986	1.007
		0.934	1.038	1.000	1.098
	5	0.303	0.514	0.858	1.000
		0.601	0.747	0.980	1.009
		0.918	1.021	1.000	1.095
Normal(v)	2	0.897	0.968	0.876	1.000
		0.961	0.979	0.884	1.011
		0.969	1.107	0.951	1.135
	3	0.875	0.942	0.810	1.000
		0.924	0.966	0.830	1.024
		0.942	1.096	0.905	1.154
	4	0.842	0.911	0.772	1.002
		0.902	0.952	0.802	1.042
		0.916	1.060	0.875	1.151
	5	0.821	0.892	0.758	1.004
		0.877	0.933	0.790	1.046
		0.892	1.034	0.859	1.148

Table C3, continued. Profits relative to BSP, positive and unequal marginal costs

Taste Distribution	K	Pricing Schemes			
		UP	CP	PB	MB
Normal(v+)	2	0.857	0.985	0.855	1.000
		0.978	0.992	0.873	1.006
		0.986	1.060	0.965	1.072
	3	0.847	0.973	0.822	1.001
		0.950	0.993	0.852	1.020
		0.973	1.062	0.920	1.088
	4	0.841	0.964	0.793	1.004
		0.939	0.999	0.827	1.039
		0.965	1.056	0.891	1.092
	5	0.820	0.956	0.784	1.006
		0.928	0.994	0.815	1.045
		0.956	1.050	0.875	1.097
Normal(v-)	2	0.936	0.953	0.879	1.000
		0.956	1.000	0.901	1.015
		1.000	1.178	0.956	1.202
	3	0.879	0.914	0.821	1.001
		0.916	0.951	0.849	1.032
		0.926	1.097	0.895	1.180
	4	0.842	0.863	0.796	1.002
		0.865	0.919	0.819	1.045
		0.896	1.052	0.864	1.184
	5	0.798	0.830	0.776	1.003
		0.826	0.889	0.810	1.045
		0.869	1.014	0.847	1.184
Normal(+/-)	2	0.686	0.901	0.863	1.000
		0.909	0.969	0.966	1.002
		0.987	1.135	1.000	1.198
	3	0.543	0.814	0.807	1.000
		0.780	0.870	0.943	1.005
		0.971	1.026	1.000	1.060
	4	0.000	0.798	0.768	1.000
		0.720	0.827	0.934	1.009
		0.949	1.010	1.000	1.064
	5	0.000	0.784	0.002	1.000
		0.674	0.812	0.930	1.013
		0.941	1.000	1.000	1.095

Table C3, continued. Profits relative to BSP, positive and unequal marginal costs

Taste Distribution	K	Pricing Schemes			
		UP	CP	PB	MB
Uniform	2	0.682	0.945	0.835	1.000
		0.896	1.034	0.953	1.073
		0.947	1.067	0.993	1.122
	3	0.524	0.904	0.805	1.000
		0.837	1.005	0.892	1.098
		0.919	1.084	0.983	1.176
	4	0.438	0.872	0.795	1.001
		0.797	0.995	0.892	1.116
		0.890	1.091	0.978	1.204
	5	0.418	0.851	0.783	1.003
		0.774	0.967	0.883	1.120
		0.872	1.101	0.981	1.250

Table C4. Profits relative to BSP, zero marginal costs with capacity constraints

Taste Distribution	K	Pricing Schemes UP	CP	PB	MB
Exponential	2	0.750	0.909	0.660	1.000
		0.894	0.989	0.899	1.030
		1.267	1.277	1.000	1.298
	3	0.377	0.863	0.681	1.000
		0.855	0.943	0.855	1.073
		1.128	1.159	1.000	1.281
	4	0.422	0.828	0.616	1.000
		0.850	0.929	0.842	1.072
		1.078	1.154	1.000	1.305
	5	0.455	0.802	0.650	1.000
		0.842	0.931	0.841	1.089
		1.098	1.197	1.000	1.303
Logit	2	0.479	0.910	0.773	1.000
		0.854	0.957	0.985	1.000
		1.000	1.039	1.000	1.045
	3	0.436	0.892	0.719	1.000
		0.779	0.925	0.985	1.005
		0.995	1.016	1.000	1.074
	4	0.408	0.868	0.795	1.000
		0.745	0.910	0.985	1.004
		0.980	0.999	1.000	1.057
	5	0.457	0.848	0.856	1.000
		0.716	0.896	0.989	1.006
		0.979	0.989	1.000	1.083
Lognormal	2	0.667	0.843	0.848	1.000
		0.795	0.894	1.000	1.000
		0.953	0.986	1.000	1.016
	3	0.353	0.805	0.869	1.000
		0.714	0.841	1.000	1.000
		0.871	0.920	1.000	1.042
	4	0.334	0.773	0.884	1.000
		0.667	0.809	1.000	1.000
		0.858	0.941	1.000	1.065
	5	0.330	0.743	0.868	1.000
		0.634	0.789	1.000	1.000
		0.828	0.919	1.000	1.070

Table C4, continued. Profits relative to BSP, zero marginal costs with capacity constraints

Taste Distribution	K	UP	CP	PB	MB
Lognormal(+)	2	0.698	0.846	0.877	1.000
		0.839	0.940	1.000	1.000
		0.975	0.975	1.000	1.005
	3	0.344	0.880	0.887	1.000
		0.777	0.920	1.000	1.000
		0.938	0.950	1.000	1.030
	4	0.344	0.878	0.885	1.000
		0.753	0.910	1.000	1.000
		0.934	0.977	1.000	1.068
	5	0.353	0.843	0.850	1.000
		0.730	0.901	1.000	1.000
		0.919	0.946	1.000	1.041
Lognormal(-)	2	0.593	0.731	0.895	1.000
		0.720	0.794	1.000	1.000
		0.926	0.992	1.000	1.073
	3	0.380	0.675	0.837	1.000
		0.639	0.767	0.978	1.001
		0.816	0.977	1.000	1.132
	4	0.321	0.643	0.864	1.000
		0.602	0.742	0.968	1.001
		0.801	0.913	1.000	1.075
	5	0.313	0.628	0.860	1.000
		0.579	0.724	0.970	1.000
		0.782	0.859	1.000	1.053
Normal	2	0.673	0.888	0.741	1.000
		0.924	0.960	0.992	1.000
		1.040	1.062	1.000	1.082
	3	0.609	0.868	0.631	1.000
		0.850	0.924	0.986	1.003
		0.998	1.012	1.000	1.054
	4	0.589	0.835	0.666	1.000
		0.804	0.903	0.986	1.001
		0.996	1.026	1.000	1.143
	5	0.535	0.813	0.643	1.000
		0.773	0.883	0.984	1.000
		0.994	1.001	1.000	1.085

Pricing Schemes: UP, CP, PB, MB.

Table C4, continued. Profits relative to BSP, zero marginal costs with capacity constraints

Taste Distribution	K	Pricing Schemes UP	CP	PB	MB
Normal(+)	2	0.701	0.900	0.757	1.000
		0.944	0.981	0.978	1.000
		1.008	1.044	1.000	1.056
	3	0.650	0.918	0.739	1.000
		0.893	0.968	0.969	1.000
		0.998	1.018	1.000	1.060
	4	0.597	0.916	0.687	1.000
		0.856	0.960	0.968	1.000
		1.005	1.020	1.000	1.032
	5	0.590	0.900	0.679	1.000
		0.833	0.953	0.968	1.000
		0.996	1.010	1.000	1.044
Normal(-)	2	0.565	0.725	0.854	1.000
		0.884	0.930	0.996	1.000
		1.003	1.068	1.000	1.099
	3	0.504	0.682	0.728	1.000
		0.799	0.878	0.987	1.030
		1.000	1.044	1.000	1.120
	4	0.482	0.671	0.712	1.000
		0.749	0.846	0.990	1.005
		0.998	1.012	1.000	1.129
	5	0.464	0.657	0.670	1.000
		0.719	0.827	0.988	1.008
		0.988	1.001	1.000	1.146
Normal(v)	2	0.827	0.962	0.716	1.000
		0.934	0.996	0.835	1.006
		1.166	1.166	1.000	1.166
	3	0.868	0.953	0.591	1.000
		0.928	0.985	0.706	1.034
		0.992	1.050	0.900	1.118
	4	0.858	0.919	0.523	1.000
		0.910	0.962	0.667	1.032
		0.938	1.029	0.890	1.135
	5	0.843	0.918	0.484	1.000
		0.896	0.951	0.622	1.031
		0.935	1.002	0.921	1.130

Table C4, continued. Profits relative to BSP, zero marginal costs with capacity constraints

Taste Distribution	K	UP	CP	PB	MB
		Pricing Schemes			
Normal(v+)	2	0.805	0.973	0.710	1.000
		0.962	0.992	0.838	1.000
		0.992	1.046	0.987	1.067
	3	0.868	0.964	0.602	1.000
		0.948	1.002	0.890	1.033
		1.001	1.041	0.972	1.070
	4	0.850	0.955	0.671	1.000
		0.938	0.999	0.874	1.035
		0.995	1.042	0.952	1.084
	5	0.855	0.961	0.616	1.000
		0.934	0.993	0.856	1.036
		0.998	1.029	0.945	1.068
Normal(v-)	2	0.837	0.959	0.815	1.000
		0.970	1.014	0.918	1.021
		1.063	1.154	1.000	1.177
	3	0.886	0.934	0.572	1.000
		0.932	0.980	0.720	1.054
		0.986	1.116	0.960	1.191
	4	0.860	0.882	0.547	1.000
		0.898	0.953	0.715	1.021
		0.958	1.111	0.939	1.266
	5	0.831	0.886	0.468	1.000
		0.874	0.929	0.591	1.026
		0.933	1.077	0.822	1.251
Normal(+/-)	2	0.673	0.888	0.741	1.000
		0.924	0.960	0.992	1.000
		1.040	1.062	1.000	1.082
	3	0.614	0.850	0.714	1.000
		0.837	0.904	0.978	1.010
		1.003	1.025	1.000	1.091
	4	0.568	0.829	0.691	1.000
		0.779	0.877	0.980	1.004
		0.987	1.009	1.000	1.087
	5	0.534	0.803	0.665	1.000
		0.753	0.867	0.979	1.004
		0.986	0.997	1.000	1.108

Table C4, continued. Profits relative to BSP, zero marginal costs with capacity constraints

Taste Distribution	K	UP	CP	PB	MB
		Pricing Schemes			
Uniform	2	0.279	0.928	0.726	1.000
		0.880	0.979	0.885	1.027
		1.076	1.197	1.000	1.248
	3	0.391	0.915	0.726	1.000
		0.848	0.985	0.910	1.067
		1.158	1.197	1.000	1.245
	4	0.413	0.881	0.758	1.009
		0.843	0.979	0.910	1.080
		1.108	1.192	1.000	1.304
	5	0.336	0.855	0.775	1.009
		0.821	0.962	0.920	1.092
		1.136	1.198	1.000	1.285