A Simulation-Based Welfare Loss Calculation for Taxes on Labor Supply with Piecewise-Linear Budget Constraints *

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(Preliminary and Incomplete)

August, 2001

Abstract

Graduated income tax rates and income transfer programs create piecewise-linear budget constraints that consist of budget segments and kink points. With any change in these tax rules, each individual may switch between a kink point and a budget segment, between two budget segments, or between two kink points. For the welfare gain or loss from that tax change, we propose a simulation-based method that is easy to implement and to compute in a way that fully accounts for these possibilities, by introducing a stochastic specification into the model. Our method also provides information on changes in working hours.

*For their comments, we thank Peter Debaere, Gerald Oettinger, Dan Slesnick, Dale Stahl, Steve Trejo, and Rob Williams. We also would like to thank Jerry Hausman for time spent answering our questions.

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1 Introduction

Graduated income tax rates and income transfer programs create piecewise-linear budget constraints that are composed of a collection of budget segments and kink points. A considerable body of work estimates labor supply under such budget sets.[1] The key insight in this literature is that the consumer chooses the budget segment that provides maximum utility.

Economists also calculate welfare loss due to taxation of labor supply. Many estimate average and marginal welfare loss, and many evaluate the economic effects of proposed and real tax reforms. But one important aspect is often missing in these calculations of welfare gain or loss. With a change of tax, the individual may switch to another budget segment, switch to or from a kink point, or even switch to or from participating in the labor force. In general, a different budget segment produces a different net wage and a different virtual income. Even within the same budget segment, a change in tax rules will likely change both the net wage and the virtual income.

This paper proposes a relatively easy way to account fully for the variations that may arise from switching between different budget segments and kink points under piecewise-linear budget constraints, by introducing a stochastic specification into the model. When estimating labor supply, existing literature compares the money-metric utilities on each budget segment before and after a change in tax. The segment or kink point chosen will be the one that yields the highest utility level. As reviewed below, however, this logic has not been applied in using these estimates to calculate welfare cost when individuals switch from one kink point or segment to another when this behavior is estimated with error. Here, to calculate welfare cost for each person, we use a Monte Carlo method to account for the uncertainties that arise from estimating errors. For heterogeneous individuals, this method estimates the probability of switching from one segment or kink point to another. This method also identifies the change of working hours. Moreover, this method provides a natural way to aggregate welfare loss and the change in working hours for various types of

[1]See Hausman (1985) and Moffitt (1990) for surveys of this literature.

heterogeneous individuals.

The problem of the welfare loss from labor taxes under piecewise-linear budget constraints is essentially the same problem as calculating consumer surplus or willingness-to-pay in discrete choice models where choices are mutually exclusive. Similarly, in the labor supply model, one may choose only one budget segment or kink point. Small and Rosen (1981) were among the first to study systematically the effect of a price change on welfare for discrete choice models. However, their study did not account for the possibility of changing income. McFadden (1999) thoroughly discusses a willingness-to-pay problem in discrete choice models by explicitly comparing the choices that yield maximum utilities before and after changes in some specific attributes of arguments in the utility function. Possible changes in income, prices or attributes may change the choice that maximizes utility and hence affect the values of the compensating variation (CV) and equivalent variation (EV). While his study concerns fishing,[2] other examples concern housing [3] or wealth accumulation.[4]

Previous literature on calculating welfare loss of labor taxation with piecewise budget constraints is based on analytical solutions. Examples include Hausman (1983) and Blomquist (1983). In order to allow for this analytical solution in his study of the change from one piecewise-linear budget constraint to another, Hausman assumes that each person's new optimal choice is on a segment of the new budget constraint. Blomquist allows for

[2]In McFadden example, evaluating environmental damages at various fishing sites, the attributes include the quality and quantity of fish at each site. The CV or EV are those that equalize the maximum utilities before and after some change in fishing quality.

[3] In a study of housing and taxes, Berkovec and Fullerton (1992) use a simulation approach to calculate welfare loss. They employ eight mutually exclusive regimes, with discrete choices whether to hold owner housing, rental housing, and corporate equity. For each household, they compare the utility levels in each regime before and after a tax change. Within each regime, they consider what tax bracket the person would face. Since they study housing choice, however, they ignore the choice of working hours. The implicit assumption is that hours do not change in response to a change in tax rate.

[4]Hubbard, Skinner and Zeldes (1995) show that the often-assumed monotonic relationship between wealth and consumption may not be valid anymore due to piecewise-linear budget constraints. Also, the breakdown of this monotonic relationship may have important effects on wealth accumulation and life-cycle behavior.

kinks in the existing tax system, but calculates the welfare gains of moving to a proportional tax system (with no kinks). By using a simulation approach, we can allow for changes to or from a kink.

The framework we adopt here is pioneered by various studies of Hausman in the 1980's. Blundell and MaCurdy (1999) discuss several attractive features of this framework: It explicitly recognizes the institutional features of the tax system, and it readily incorporates the fixed cost of holding a job. However, some concerns on how to estimate labor supply in this framework have also emerged. The most notable concern is of Heckman (1983), that the budget set for each individual often cannot be accurately determined and that a special type of errors-in-variable bias results.[5] Yet recent paper by Gan and Stahl (2002) show that the Heckman concern can indeed be addressed in the Hausman framework by introducing measurement error in non-labor income. The measurement error in non-labor income creates the random budget set for econometricians. Such a the labor supply equation can be estimated in a framework of piecewise-linear budget constraints without suffering from the Heckman concern.

This paper makes several contributions relative to existing welfare cost calculations. First, we calculate welfare cost using labor supply estimates that account for the Heckman concern. Second, earlier analytical approaches had to assume that each person's new indifference curve is tangent to a line segment on the new budget constraint, while our approach allows movement to or from a kink point. Third, earlier analytical approaches could not employ the entire estimated distributions of both error terms. For example, Hausman (1983) allows for measurement error (η) and heterogeneity in one of the preference parameters (β). To get a probability-weighted choice of hours, one needs to integrate over both distributions. To simplify, one might use just the mean of each distribution. Later we call

[5] MaCurdy, Green and Paarsch (1990) argue that the likelihood setup in Hausman's framework may create artificial constraints on the parameter values. In fact, the Hausman's random coefficient model does not subject to the MaCurdy's critique. The Triest (1990) dual random error model, suggested in Blundell and MaCurdy (1999) as a way to address the MaCurdy critique, is a special case of Hausman's random coefficient model. See Gan and Stahl (2002) for a detailed discussion on this point.

this the simple "Harberger" method (Harberger, 1964), because the person's choice is only one point. Instead, Hausman uses an approximation, evaluating the distribution at the means of intervals. Here, we employ the entire estimated distributions of both error terms. For *each* individual in the data set, our Monte Carlo simulation takes a thousand random drawings from the estimated distributions. For each drawing, it calculates the chosen segment or kink, and the resulting welfare cost. We then have a probability distribution of the welfare cost. Compared to the simple Harberger method, this procedure might be important, especially if the errors are large and the tax system is steeply graduated, because welfare cost increases with the square of the tax rate.[6] Fourth, not only do we recognize that a change in the tax system may change a person's optimal choice of segment or kink point, but also we consider the possibility that the EV or CV itself is a transfer that may again affect the person's choice. Finally, this simulation method is easy to implement and to calculate with no additional difficulty for a concave budget set.

In Section 2, we define and provide a framework to estimate the CV and EV under budget constraints that are piecewise linear. These budget constraints are discussed in Section 2.1, while the issues related to CV and EV under piecewise budget constraints are in Section 2.2. Then Section 3 provides a framework to estimate welfare loss using the simulation method. Section 4 offers empirical examples to compare the values of welfare loss derived from alternative methods. Section 5 concludes the paper.

2 A Basic Framework

In this study, we consider a static partial equilibrium labor supply model. The before-tax wage is constant, with no inter-temporal optimization of labor supply. All of the following variables are individual-specific, but we suppress the index for notational convenience.

[6] Suppose, for example, that the mean of the distribution places the person in the 20% tax bracket but that person actually has a 40% probability of being in the 30% bracket. The simple welfare cost is some constant times .2 squared (which is .04), while the true welfare cost involves the same constant times [(.6)(.2)(.2) + (.4)(.3)(.3)], which is .06. In this simple example, the welfare cost measure is raised by 50%.

We begin with a typical labor supply model of utility maximization with respect to choices about leisure and other consumption goods x. The hours of working are defined to be h, so $-h$ is leisure. The person's non-labor income is y, and the real wage is w. The indirect utility $v(w, y)$ is the maximum value of the direct utility $u(x, h)$ that can be obtained when facing the budget constraint:

$$\begin{aligned} & v(w, y) = \max_{x,h} u(x, h) \\ & \text{s.t.} \quad x - wh = y \end{aligned} \tag{1}$$

where the price of x is normalized to 1, and the cost of leisure is the wage rate w.

2.1 Budget segments and tax revenues

Graduated tax rates and income transfers imply different combinations of real wage rates and incomes in Equation (1). Let a tax bracket be represented by $\{t_j; Y_{j-1}, Y_j\}$, where t_j is the marginal tax rate for a person whose before-tax income lies within the interval $[Y_{j-1}, Y_j]$. Information about $\{t_j; Y_{j-1}, Y_j\}$ can often be found from tax tables. Note that the relevant budget set is based on after-tax income. Let the end points of the segment in a budget set that corresponds to bracket $\{Y_{j-1}, Y_j\}$ be $\{y^a_{j-1}, y^a_j\}$, where y^a refers to after-tax income. A complete characterization of budget segments requires information on working hours that correspond to the set $[y^a_{j-1}, y^a_j]$, and we denote these hours as as $[H_{j-1}, H_j]$. To calculate the location of each budget segment, we start with the first budget segment and proceed through all budget segments. Besides the before-tax wage rate w, another critical piece of information necessary is Y^n, the non-labor income this person may have. Let y^n be after-tax non-labor income, where the tax is calculated as if the person had no labor income. Then labor income pushes the person into successively higher tax brackets. We summarize information on budget segments in Table 1.

One interesting observation from Table 1 is that non-labor income affects the budget segments, since the end points of a budget segment are function of Y^n or y^n.

$$
\begin{aligned}
H_j &= (Y_j - Y^n)/w \\
y_j^a &= y^n + \textstyle\sum_{k=1}^{j}(1-t_k)(Y_k - Y_{k-1})
\end{aligned}
\tag{2}
$$

A change in non-labor income Y^n will lead to a change of the whole budget set. If Y^n is measured with error, the whole budget set will be measured with error. This point is used by Gan and Stahl (2002) as a way to resolve the critique that Heckman (1983) and MaCurdy et al. (1990) raise with respect to the Hausman labor supply estimates.

It is well known in the literature that a person's optimal hours may be at a kink point instead of being on the interior of a segment, in the framework of piecewise-linear budget constraints. Define

$$
\begin{aligned}
S_j &\equiv \begin{cases} 1 & \text{if at segment j,} \\ 0 & \text{otherwise;} \end{cases} \\
K_j &\equiv \begin{cases} 1 & \text{if at kink j,} \\ 0 & \text{otherwise.} \end{cases}
\end{aligned}
\tag{3}
$$

The conditions determining the values of S_j and K_j require knowledge of the labor supply function. For example, consider a commonly estimated linear labor supply function

$$
h = \begin{cases} \alpha w_j + \beta y_j^v + s, & \text{if positive} \\ 0 & \text{otherwise} \end{cases}
\tag{4}
$$

where s includes $Z\gamma$ (the effect of) other socio-demographic variables Z and the statistical error. In this equation, y_j^v is virtual income, defined as the intercept of the line that extends this budget segment to the zero-hours axis. Given that labor supply function, the conditions for $S_j = 1$ or $K_j = 1$ are:

$$
\begin{aligned}
S_j = 1 \quad &\text{if} \quad H_{j-1} < \alpha w_j + \beta y_j^v + s < H_j \\
K_j = 1 \quad &\text{if} \quad \alpha w_{j+1} + \beta y_{j+1}^v + s \le H_j \le \alpha w_j + \beta y_j^v + s
\end{aligned}
\tag{5}
$$

If a budget set is globally convex, the highest indifference curve will either touch a single kink point or be tangent to a single segment. Only one of S_j or K_j will be 1. However, often a budget set is not convex due to the fixed cost of working or some income transfer program (such as AFDC). A possibility arises that more than one of the S_j and/or K_j will be 1. In this case, we must compare the utility levels for for all $S_j = 1$ and $K_j = 1$ and pick the segment or kink point that yields the highest utility level.

Another key variable in the calculation of welfare cost is the tax revenue from this person, which can be obtained based on the information in Table 1. Let the working hours be $h \in [H_{j-1}, H_j)$ as given in the table. Then the tax revenue R for this individual is given by

$$\begin{aligned} R &= R^n + \sum_{k=1}^{j-1} (H_k - H_{k-1}) w t_k + w(h - H_{j-1}) t_j \\ &= R_{j-1} + w(h - H_{j-1}) t_j \end{aligned} \tag{6}$$

where R^n is the tax revenue from non-labor income, and R_{j-1} is defined as the tax revenue if the working hours were $h = H_{j-1}$ (which may be obtained from the tax table and Table 1 when the wage rate w is given).

2.2 CV and EV under piecewise budget constraints

The welfare cost of the tax may be based on either the compensating variation (CV) or the equivalent variation (EV). In a simple proportional tax system, consider the case where a change in tax moves the pair of after-tax wage and virtual income from (w^0, y^0) to (w', y'). The CV and EV may be formally defined as:

$$\begin{aligned} u^0 = v(w^0, y^0) &= v(w', y' + CV) \\ v(w^0, y^0 - EV) &= v(w', y') = u' \end{aligned} \tag{7}$$

Calculating welfare cost in the framework of piecewise-linear budget constraints is similar to the problem of calculating willingness-to-pay in a discrete choice model. Let us consider a person's choice of hours before and after a change in tax under a piecewise-linear budget constraint. After a tax change, when a utility-maximizing individual chooses working hours that fall on a budget segment that provides the highest utility level, the chosen segment or kink point changes. We then compare the difference between the optimal utility levels and find a CV or EV value to equalize them. This basic idea is in McFadden (1999), but in our case the CV or EV is a transfer that may lead the person to choose yet another kink point or segment.

At any kink point where $K_j = 1$, we use the direct utility function $u(x, h)$, where $x = y_j^a$, and $h = H_j$. A person whose optimal hours are zero or negative does not participate in the labor force. The utility level of this person is $u(0, y_0^a)$, where $y_0^a = y_0^v = y^n$.

Let $'$ indicate a variable after a change in tax schedules, and suppose k^0 and k' are the total numbers of segments before and after the tax change. For a convex budget set, since only one of the S_js and K_js is 1, we can find the utility levels before and after a tax change as:

$$\begin{aligned} u^0 &= \textstyle\sum_{j=1}^{k^0} S_j^0 v(w_j^0, y_j^{v0}) + \sum_{j=0}^{k^0} K_j^0 u(y_j^{a0}, H_j^0) \\ u' &= \textstyle\sum_{j=1}^{k'} S_j' v(w_j', y_j^{v}{}') + \sum_{j=0}^{k'} K_j' u(y_j^{a}{}', H_j') \end{aligned} \tag{8}$$

Note, in general, that $S_j^0 \neq S_j'$ and $K_j^0 \neq K_j'$. Under the new tax regime, a person may switch to a different kink point or segment.

When the budget set is not convex, we must consider the possibility that more than one of the S_js and/or K_js is one. Define

$$\begin{aligned} v_j &\equiv v(w_j, y_j^v)S_j + (1 - S_j)m \\ u_j &\equiv u(y_j^a, H_j)K_j + (1 - K_j)m \end{aligned} \tag{9}$$

where m is a small number, $m < \min_j\{v(w_j, y_j^v), u(y_j^a, H_j)\}$. The utility levels before and after a change in tax can be written as

$$\begin{aligned} u^0 &= \max_j\{v_j^0, u_j^0; j = 1, \cdots, k^0\} \\ u' &= \max_j\{v_j', u_j'; j = 1, \cdots, k'\} \end{aligned} \tag{10}$$

where v_j and u_j are defined in (9).

In this paper, we treat the after-tax wage rates on all segments of the budget set as integral parts of the "price", since they are pre-determined by the tax table. An analogy is a pricing system that depends on quantity. A lump-sum transfer such as CV or EV increases the optimal quantity, which might yield a different price. Because each piece of the entire nonlinear budget constraint affects the final chosen budget segment or kink point, the new net wage is natural to treat as part of the CV or EV calculation. A lump-sum transfer of CV or EV will change a person's entire budget set. The new budget set is still piece-wise linear, in a way that corresponds to tax rules, but the extra transfer means that the person can buy more of both goods. For the end point of a budget segment j, H_j does not change, but y_j^a and virtual income y_j^v do change – by the amount of lump sum transfer. As a consequence, the optimal working hours change. Therefore, it is entirely possible that a person moves to a different segment or a kink point. Let $''$ represent variables after the person is given the EV.

$$\begin{aligned} v_j'' &= v(w_j^0, y_j^{v0} - EV)S_j'' + (1 - S_j'')m \\ u_j'' &= u(y_j^{a0} - EV, H_j^0)K_j'' + (1 - K_j'')m \end{aligned} \tag{11}$$

In (11), the values of S_j'' and K_j'' are functions of the unknown EV, and m is the same as in (9). A correct measure of EV must take this complication into account, as the solution to:

$$EV : u' = \max_j\{v_j'', u_j''; j = 0, \cdots, k^0\} \tag{12}$$

where u' is defined in Equation (10). Because S_j and K_j depend on the unknown EV, a solution to (12) must be obtained iteratively. A similar calculation can be undertaken for CV.

In order to compare these procedures to those suggested in Hausman (1983), we first rewrite Hausman's methods in our notation. In particular, consider the expenditure function, Equation (2.4) in Hausman (1983). The calculation of EV based on such an expenditure function depends on the condition that a person must fall on a particular segment. In our notation,

$$\begin{aligned} &\text{let} \quad u_{j^0}^0 \equiv \max_j\{v(w_j^0, y_j^{v0});\ j = 1, \cdots, k^0\}, \\ &\text{then} \quad u_{j^0}^0(w_{j^0}, y_{j^0}^v - EV) = \max_j\{v'(w_j', y_j^{v\prime});\ j = 1, \cdots, k'\} \end{aligned} \tag{13}$$

A comparison of (13) and (12) reveals two differences: First, (13) does not consider a kink point, and second, it does not consider the case that a transfer of EV may further change the chosen segment. Hausman (1983) mentions that calculation of (13) by integration over the error terms' distributions is numerically difficult when the budget set is concave. Therefore, he uses various simplifications. Using the EV in (12) and earlier equations, however, these simplifications are no longer necessary. Also, the simulation method based on (12) is not affected by whether the budget set is concave or convex.

3 Welfare Loss Based on Stochastic Simulations

In this section, we introduce a stochastic specification into the model of the previous section, and we provide a simulation-based method to calculate welfare loss.

3.1 Specifying the utility function

Estimates based on Equation (10) require complete knowledge of a person's direct and indirect utility functions. Two approaches have been proposed in the previous literature. In the first approach, one may start with an assumed utility specification and then solve for demand functions including leisure demand (labor supply). For example, Dickens and Lundberg (1993) uses a CES-type of utility function. After estimating the corresponding demand function, they can calculate welfare loss. In the second approach, introduced in Hausman (1981a), one starts with and estimates a specification of the demand function, such as a linear specification, and "recovers" the utility function for that demand function by using Roy's identity. That is, using

$$\frac{\partial v(w,y)/\partial w}{\partial v(w,y)/\partial y} = h, \tag{14}$$

one can solve a differential equation to get $v(w,y)$. Unfortunately, as pointed out in Slesnick (1998), closed-form solutions to Equation (14) can only be obtained for a limited class of demand functions.

Nevertheless, we adopt the second approach and use labor supply functions that yield closed-form solutions. In particular, we consider a linear labor supply function as in (4). Following Hausman (1981b), when $h > 0$, the corresponding indirect utility function is

$$v(y,w) = e^{\beta w}\left(y + \frac{\alpha}{\beta}w - \frac{\alpha}{\beta^2} + \frac{s}{\beta}\right) \tag{15}$$

When a person is at a kink point, the indifference curve is not tangent to the budget set, so the utility level can only be obtained from the direct utility function. At a kink point j, the direct utility function corresponding to the labor supply function in (4) is:

$$u(y_j^a, H_j) = \exp\left(\frac{\beta y_j^a + s - H_j}{H_j - \alpha/\beta}\right)\left(\frac{H_j - \alpha/\beta}{\beta}\right) \tag{16}$$

3.2 A stochastic specification and simulation procedures

So far, we have discussed how to obtain utility functions from empirically estimated labor supply functions, but these functions are estimated with stochastic error. Part of this error may represent the deviation between actual hours and desired hours (which econometricians do not observe). Amother part may be a deliberate effort by the econometrician to represent the heterogeneity of preferences or to represent specification errors. A typical example is in random coefficient models where parameters of the model are assumed to be randomly distributed, and where the task of the estimation is to obtain the parameters of that random distribution.

When the stochastic errors enter into an objective function linearly, they tend to cancel out. In that case, a non-stochastic calculation might be sufficient. In our case, however, the welfare loss is a non-linear function of the stochastic errors. Comparing a stochastically specified model and a non-stochastic one, the welfare loss calculation may be significantly different. We show this difference below.

Researchers may obtain information from a stochastic model that would be difficult or impossible to obtain from a non-stochastic model. For example, if one is interested in the probability of switching segments, or of switching from participating in the labor force to non-participation, one can acquire this information rather easily in a stochastically-specified model. That information may be very hard to obtain from a non-stochastic model.

In this section, we consider a stochastic specification based on empirically estimated labor supply equations. The stochastic errors in different specifications of labor supply equations will have different forms. In Hausman (1981b), for example, the labor supply equation is:

$$h = \alpha w_j + (\overline{\beta} + \eta) y_j^v + z\gamma + \zeta \tag{17}$$

where $\overline{\beta}$ is the mean value of β, the coefficient on virtual income y_j^v. Equation (17) has two errors: η represents heterogeneity of preferences, and ζ is the error in measuring working hours. Another example of labor supply is Triest (1990), where the labor supply equation is:

$$h = \alpha w_j + \beta y_j^v + z\gamma + \eta + \zeta \tag{18}$$

In this equation, η is an optimization error. It is not observed by the econometrician but only observed by the individual to determine her segment or kink point. Again, ζ serves as measurement error for working hours. At a kink point in this model, we only have error η, but both η and ζ are present when a person is on a line segment.

For both (17) and (18), the indirect and direct utility functions are given in (15) and (16), respectively. Often, when labor supply equations are estimated, the density forms of η and ζ are assumed, and the parameters of the density functions are estimated. Our simulation procedure is based on random draws of η and ζ from the estimated densities. We now describe the basic procedure of the simulation method.

We start with the choice of labor supply equation (17) or (18), and then for each worker we take $I = 1,000$ draws of the error term $\epsilon \equiv (\eta, \zeta)$. The draws may come from a "known" parametric distribution specified and estimated for the labor supply function. Alternatively, it may come from the empirical distribution of the residuals of the labor supply function.

For the ith random draw, $\epsilon_i = (\eta_i, \zeta_i)$, we find the values of $S_{ij}^0, S_{ij}', K_{ij}^0$ and K_{ij}' from (5). Then from (9) and (10), we find the optimal segment or kink point in each of the two tax regimes, given ϵ_i. This procedure applies whether the budget set is concave or convex.

Let j_i^0 and j_i' be the optimal choice of segment or kink in the two tax regimes, given the *ith* draw of ϵ, and let u_i^0 be the optimal utility in the old tax regime given ϵ_i. We can obtain the EV_i, given ϵ_i, and u_i', using (11) and (12).

After a transfer of EV_i, note that the chosen segment or kink point may change again, and we use $''$ to indicate variables after this EV transfer. Solving (12) requires numerical iteration.

In addition, for any individual worker, we know the j_i^0 and j_i' for the ith draw, so it is easy to obtain the tax revenues in the two tax regimes $R_{j_i'}$ and $R_{j_i^0}$. The deadweight loss (DWL) for this person, just for the ith drawing from the whole distribution of ϵ is:[7]

$$DWL(\epsilon_i) \equiv -\left[EV_i - (R_{j_i'} - R_{j_i^0})\right] \tag{19}$$

where $R_{j_i^0}$ and $R_{j_i'}$ also change as ϵ_i changes.

Naturally, the mean of all these $DWL(\epsilon_i)$ can be made arbitrarily close to the expectation of DWL by increasing the number of draws I:

$$E(DWL) = \int DWL(\epsilon_i) dF(\epsilon_i) \approx \frac{1}{I} \sum_i^I DWL(\epsilon_i) \tag{20}$$

One may also calculate the probability of moving from segment j^0 to segment j'.

$$\text{Prob(segment } j^0 \Rightarrow \text{segment } j') = \frac{1}{I} \sum_{i=1}^I S_{j_i^0} \times S_{j_i'},$$

or the probability of moving from segment j^0 to kink j':

$$\text{Prob(segment } j^0 \Rightarrow \text{kink } j') = \frac{1}{I} \sum_{i=1}^I S_{j_i^0} \times K_{j_i'}$$

In addition, we can calculate the change of working hours. For each random draw ϵ_i, the working hours can be calculated using (4). If j_0' and j_i' are segments, then

$$\begin{aligned} h_i' &= \hat{\alpha} w_{j_i'} + \hat{\beta} y^v_{j_i'} + z\hat{\gamma} + \eta_i + \zeta_i \\ h_i^0 &= \hat{\alpha} w_{j_i^0} + \hat{\beta} y^v_{j_i^0} + z\hat{\gamma} + \eta_i + \zeta_i \end{aligned}$$

The difference between the two numbers of hours is the change in labor supply. The average from all random draws provides a number that converges to the true value of the change of working hours:

$$\Delta h = \frac{1}{I} \sum_{i=1}^I \left(h_i' - h_i^0 \right)$$

[7]See Mohring (1971) and Auerbach (1985). Since $EV < 0$ for a gain, we take the negative of $(EV - \Delta Rev)$ in order to show a positive number for a welfare gain from removing the tax (loss from having the tax).

All the estimated factors are calculated conditional on the wage rate w, virtual income y^v, and other socio-demographic variables z. We can easily integrate over these factors to get the population average. In practice, we just repeat the previous process for each successive individual in the sample, and take the average of all individuals (applying sample weights, if available).

4 Examples

Example 1: The welfare loss of taxation for a married woman. As in Hausman (1981b), we consider a married woman whose husband is earning a fixed \$10,000. Her wage rate is \$4.15 an hour. She works full time (1,925 hours per year) and files a joint return. The tax regime she faces is shown in Table 2, the federal tax brackets of 1975 (the sample year for Hausman, 1981b). The "new tax" regime is no tax at all. We choose this example for several reasons: First, this example is considered in Hausman (1981b), where he estimates labor supply using the data from Panel Study of Income Dynamics (PSID) and applies the estimates to calculate welfare loss. Second, the standard deduction for a married couple filing a joint return in 1975 creates a concave budget set.[8] Third, we consider only one person in this example to illustrate that she could be at different segments or kink points in the stochastic specification. It is also easy to compare the results with a traditional welfare cost calculation such as the Harberger triangle.

The estimated hours equation is given by

$$h = \alpha w_j + \beta(y_j^v - FC) + z\gamma + \zeta \tag{21}$$

where FC is the fixed cost of working, \$1,260 per year. Hausman estimates that $\alpha = 0.4608$, with a standard error .106, and β is a random coefficient representing variations in taste, with a truncated normal distribution (i.e., $\beta = \beta_k$ where $\beta_k \sim N(2.0216,\ 0.5262^2)$ and

[8] In 1975, for income below \$11,875, the standard deduction for a married couple filing a joint return was \$1,900. But when total income is between \$11,875 and \$16,250, the standard deduction is \$1,900 plus 16% of the income that exceeds \$11,875.

$\beta_k < 0$). The mean of the truncated normal, $E(\beta) = -.123$. Also, $\zeta \sim N(0,\ 0.2801^2)$. Then $z\gamma = 0.2595$, obtained from the equation

$$z\gamma = h - \hat{\alpha} w_j + E(\beta)(y_j^v - \hat{FC}) \tag{22}$$

where $h = 1,925$, and j is the chosen segment. At the means of the parameters and the error distribution, the marginal tax rate for this woman is 28%.

Table 3 shows that the DWL estimate using our stochastic specification is very different from the one using a simple Harberger triangle approximation.[9] For example, the welfare loss based on the Harberger triangle is about 29.8% of tax revenue for this woman, but the mean ratio of stochastic (DWL/tax revenue) is 72.4%. Although both of these numbers are very large, the estimate by Hausman is in between, at 58.1% of tax revenue. These numbers are large because of the large compensated elasticity from the Hausman estimates.

In the stochastic specification, this person has probabilities of being on different segments or kink points as shown in Table 4. The probability that this working woman chooses the segment with the 28% tax rate (segment 5) is 47.2%. The sum of the probabilities of choosing kink points is 13.1%. Generally speaking, segments or kink points closer to segment 5 have higher probabilities, with two exceptions. First, kink points 1 and 2 have zero probabililities, since the budget set is concave around these two kink points. Second, the probability of being at kink point 0 (not working) is positive (.32) because of the fixed cost of working in this model. Not working yields higher utility for those random draws where the optimal working hours are relatively small. In the new tax regime with no tax at all, the person has 96% probability of working, and 4% chance of not working.

Example 2: The Tax Reform Act of 1986 for married women.

The parameter estimates used in this example are from Triest (1990), and they are applied to a data set of married women extracted from PSID (1983).[10] Table 5 lists the

[9] For this purpose, we use Equation (4) in Browning (1987) for DWL as a function of the compensated labor supply elasticity, the fixed gross wage rate, and the marginal tax rate.

[10] We extract data from PSID following the procedure in Triest (1990). There appears some difference between our data and the Triest data. In our data set, there are 1,136 observations while the Triest data

tax rates and income brackets for both tax regimes. We assume all individuals take the standard deduction and file jointly.

The basic labor supply function in Triest (1990) appears above as Equation (18). In our data set, we observe each person's working hours and wages before the Tax Reform Act of 1986. The non-labor income is calculated from the husband's income and other family income. We can therefore derive the budget constraint for each person, and determine the chosen segment or kink point. If the chosen segment is j, with observed net wage w_j and virtual income y_j^v, then we can use Triest's parameter estimates $\hat{\alpha} = .235$ and $\hat{\beta} = -.022$ to calculate:

$$z\gamma = h - \hat{\alpha} w_j - \hat{\beta} y_j^v$$

The random errors η_i and ζ_i are distributed as $\eta \sim N(0, 0.67^2)$ and $\zeta \sim N(0, 0.77^2)$. We take 1,000 random draws of joint distribution of (η, ζ) for each individual. Averaging over the 1,136 individuals, Table 6 shows the equivalent variation, the change in working hours, the change in tax revenue, and the welfare gain from this tax reform. Interestingly, the working hours are lower in the new tax regime, indicating that the income effect dominates. The tax revenue is lower. Since EV on average is negative, the utility level in the new tax regime is higher. The welfare gain from this tax reform is 5.42% of the tax revenues in the old regime, which is significantly lower than that from the first example, for two reasons. First, the new tax regime in the first example is no tax at all. In the current example, the tax rates are lower but still significant. Second, different labor supply estimates are used. Hausman's estimates are significantly larger in magnitude for both parameters α and β. If the Triest estimates were used in the first example, the compensated elasticity would be only .430 instead of 1.084.

Finally, note the large standard error on the EV and the welfare effect. In this example, the "expected" welfare gain is \$157 per person, but the actual welfare gain has 15.7% chance of being negative.

has only 978 observations. The summary statistics between our data set and the Triest data are very close. One possible explanation is the new version of PSID has less missing values.

Example 3: The tax change of 2001 for married women.

In this example, we apply the parameter estimates of Gan and Stahl (2002) to the Economic Growth and Tax Reconcilation of 2001. Their model assumes measurement error in non-labor income Y^n,

$$Y^n = Y^{n*} - \eta \tag{23}$$

where Y^n is observed non-labor income, Y^{n*} is the true non-labor income (known to the individual herself but not to the econometrician). The measurement error in non-labor income produces a random budget set: the end points of each segments are random variable. Such a model is not subjected to Heckman critique. In fact, it is precisely in-line with the insights in Heckman (1983). Gan and Stahl show that such a model yields very different parameter estimates, and performs better statistically. The labor supply equation is given by

$$h = \alpha w_j + \beta y_j^v + z\gamma + \zeta \tag{24}$$

We use the same data set as in Gan and Stahl (2002): married women in the Current Population Survey (CPS) in March 2001 between the age 25 and 55. The data set has 16,829 observations. The parameter estimates and summary statistics are listed in Table 7. One interesting aspect of the parameter estimates is the large standard deviation of measurement error in nonlabor income. The estimates are used to evaluate the effect of the Economic Growth and Tax Relief Reconcilation Act of 2001, or simply the Bush Tax Cut. Here, we only consider the changes in marginal tax rates. In this case, the changes in marginal tax rates are phased in, so we use the tax rates after 2006 when all changes are fully implemented. Table 8 compares the regimes before and after the Bush Tax Cut. Again, we assume all of these married women take the standard deduction and file jointly.

We take 50 random draws of the error in equation (24), $\zeta \sim N(0, 0, 65^2)$, and for each ζ, we apply 50 randomly drawn $\eta \sim N(0, 1.33^2)$. Note a different η will yield a different budget constraint.

Averaging 16,829 individuals, Table 9 shows the equivalent variation, the change in working hours, the change in tax revenue, and the welfare gain from this tax reform. The net result is almost no change in working hours. The tax revenue is lowered by $663, and the welfare gain from this tax reform is 2.5% of revenue in the old tax regime.

5 Conclusion

The calculation of welfare loss suggested in this paper depends on estimates of labor supply. An ongoing debate concerns how to estimate the labor supply function under piecewise-linear budget constraints, but recent estimates are able to address the Heckman concern within Hausman's framework. The first contribution of this paper, relative to existing literature, is to calculate the welfare cost of labor taxes using labor supply estimates that address this concern. Second, we allow each individual to move from any kink or linear segment of the original budget constraint to any kink or linear segment of the new budget constraint. Third, we use a Monte Carlo simulation method in order to employ the entire estimated distribution of each error term. Thus, we need not assume that the person chooses one particular point, which would ignore the fact that labor supply is estimated with error. Fourth, we account for the fact that the equivalent variation is a transfer that itself would change the choices of this individual. And finally, the method we propose is relatively easy to implement and to calculate.

Using this new method, we calculate the welfare effect of three illustrative labor tax changes. First, we employ the example of Hausman (1981b) with one married woman who works full time. We show that the welfare effect of eliminating the tax system in this example using the stochastic evaluation is significantly larger than when using a simple Harberger triangle approximation. Second, we employ Triest's (1990) estimates to consider 1,136 married women in the data PSID (1984). In this case, we show that the mean welfare gain from the Tax Reform Act of 1986 is 3.27% of the original tax revenue. In the third case, we apply the estimates from Gan and Stahl (2002) to a recent data set from the CPS (March, 2001) to investigate the welfare gains of the Bush Tax Cut of 2001. We find almost no change in working hours, and the mean welfare gain from this tax cut is 2.5% of the old revenue.

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Table 1: Summary of budget segments

	Budget Segment 1	Budget Segment $j > 1$
function for after-tax income y^a	$y^a = y^n + w(1-t_1)h$	$y^a = y^a_{j-1} + w(1-t_j)(h - H_{j-1})$
kink points for income y^a	$y^a_0 = y^n$	$y^a_j = y^a_{j-1} + w(1-t_j)(H_j - H_{j-1})$ $= y^n + \sum_{k=1}^{j}(1-t_k)(Y_k - Y_{k-1})$
kink points for working hours h	$H_0 = 0$ $H_1 = (Y_1 - Y^n)/w$	$H_j = (Y_j - Y^n)/w$
virtual income y^v	$y^v_1 = y^n$	$y^v_j = y^a_{j-1} - w(1-t_j)H_{j-1}$ $= Y^n(2 - t_1 - t_j) - Y_j(1-t_j)$ $+ \sum_{k=1}^{j}(1-t_k)(Y_k - Y_{k-1})$

We define t_1 as the first tax rate applied to labor income of this person (after taxation of non-labor income). Using the person's non-labor income, t_j and Y_j are also individual-specific, but can be found from the tax table.

Table 2: Tax tables for example 1

Income	Rates
0 – \$1,900	.0
\$1,900 – \$2,900	.14
\$2,900 – \$5,900	.16
\$5,900 – \$9,900	.19
\$9,900 – \$11,800	.22
\$11,800 – \$13,900	.185
\$13,900 – \$16,250	.21
\$16,250 – \$17,900	.25
\$17,900 – \$21,900	.28
\$21,900 – \$25,900	.32
\$25,900 – \$29,900	.36
\$29,900 – \$33,900	.39
\$33,900 – \$37,900	.42
\$37,900 – \$41,900	.45
\$41,900 – \$45,900	.48
\$45,900 +	.50

Table 3: Welfare loss from the 1975 tax system

before-tax wage = 4.15; before-tax non-labor income = $10,000

Deterministic evaluation	
working hours	1,925
marginal tax rate	.28
compensated elasticity	1.084
tax revenue	$1,815
Harberger DWL	$339
DWL as % of tax revenue	29.8%
Means using stochastic evaluation[a]	
old working hours	2,019 (753)
new working hours	2,386 (943)
tax revenue	$1856 ($187)
EV[b]	-$3,257 ($895)
DWL[c]	$1,401 ($716)
DWL as % of tax revenue	72.4% (32.8%)

a. Standard errors are in parenthesis.

b. $EV < 0$ means a gain from removal of the tax.

c. $DWL \equiv -(EV - \Delta Rev)$.

Table 4: The probability that this working woman is on each initial budget segment or kink

	segments		kink points
number	marginal tax rate	probabilities	probabilities
0			.032
1	.22	.020	.0
2	.185	.042	.0
3	.21	.111	.021
4	.25	.128	.039
5	.28	.472[a]	.039
6	.32	.086	.0

a. Using only the mean of the distribution, this woman would be on segment 5 in the old tax regime.

Table 5: Tax tables for example 2

Old Tax Regime		New Tax Regime	
Income	Rates	Income	Rates
0 – $3,400	0	0 – $3,000	.11
$3,400 – $5,500	.11	$3,000 – $28,000	.15
$5,500 – $7,600	.13	$28,000 – $45,000	.28
$7,600 – $11,900	.15	$45,000 – $90,000	.35
$11,900– $16,000	.17	$90,000 –	.385
$16,000 – $20,200	.19		
$20,200 – $24,600	.23		
$24,600 – $29,900	.26		
$29,900 – $35,200	.30		
$35,200 – $45,800	.35		
$45,800 – $60,000	.40		
$60,000 – $85,600	.44		
$85,600 – $109,000	.48		
$109,000 +	.50		

Table 6: The effect of the Tax Reform Act of 1986

	old tax regime	new tax regime	difference
tax revenue	\$5,791	\$5,374	-\$417
	(\$4,359)[a]	(\$3,612)	(\$764)
working hours	1,117	1,091	-27
	(748)	(745)	(265)
EV[b]			-\$574
			(\$1,217)
welfare effect[c]			\$157
			(\$1,047)
welfare effect as a % of old tax revenue			5.42%
			(17.75%)

a. Standard errors are in parenthesis.

b. $EV < 0$ means a gain.

c. The welfare gain is $-(EV - \Delta Rev)$.

Table 7: Estimation Results and Summary Statistics from the CPS (March, 2001)[a]

	Parameter estimates	Summary Statistics
working hours(per year)		1,353
		(954)
constant	-.218	
	(.027)	
wage (in $)	.00012	16.16
	(0.000034)	(25.5)
non-labor income (in $1000)	-0.00176	58.59
	(.00013)	(59.3)
# kids in ages 0-5	-.214	.369
	(.008)	(.675)
# kids in ages 6-18	-.088	.880
	(.0046)	(1.06)
age-40	-.012	.36
	(.0036)	(2.65)
unemployment rate (%)	-.011	4.01
	(.0037)	(1.61)
education (in years)	.034	10.14
	(.0020)	(3.14)
# of observations		16,829
% of labor participations		75.2%
std dev of measurement error	1.33	
in non-labor income	(.12)	
std dev of optimization error	.65	
	(.0034)	
Elasticities (evaluated at means)		
uncompensated	.012	
compensated	.012	

a. Married women between age 25 and 55.

Table 8: Tax tables for example 3

Old Tax Regime		New Tax Regime	
Income	Rates	Income	Rates
0 – $7,600	0	0 – $7,600	0
$7,601 – $51,450	.15	$7,601 – $13,600	.10
$51,451 – $113,550	.28	$13,601 – $51,450	.15
$113,551 – $169,050	.31	$51,451 – $113,550	.25
$169,051– $295,950	.36	$113,551 –$169,050	.28
$295,951 +	.396	$169,051 –$295,950	.33
		$295,951 +	.35

Table 9: The effect of the Reducing Marginal Tax Rates in 2001 Tax Cut

	old tax regime	new tax regime	difference
tax revenue	$10,364	$9,700	-663$
	($16,886)[a]	($15,300)	($1,595)
working hours	1,171	1,174	3
	(516)	(509)	(29)
EV[b]			-$1,305
			($2,159)
welfare effect[c]			$1,082
			($2,198)
welfare effect as a % of old tax revenue			2.5%
			(12.8%)

a. Standard errors are in parenthesis.

b. $EV < 0$ means a gain.

c. The welfare gain is $-(EV - \Delta Rev)$.