

## Understanding the Evolution of Inequality During Transition: The Optimal Income Taxation Framework

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Abstract: What explains the spectacular increases in inequality of disposable income in transitional economies of Central and Eastern Europe? There are at least two possible explanations. First, the pre-tax distribution of income became more unequal because of the shift to a market economy. Second, the degree of progressivity of the income tax system declined. But each of these factors is in turn determined by other structural changes associated with transition—notably, the decrease in public provision of key public goods, the decrease in non income tax revenue sources such as profits from public production, and perhaps a decline in society's inequality aversion. This paper develops a framework in which these different forces on inequality can be assessed. Using a simple two-type and two-sector optimal income tax model with endogenous wages, we first of all show that a decrease in the provision of public goods could indeed lead to increasing “inherent” inequality, in other words inequality in market incomes. It then deploys the Mirrlees model of optimal non-linear taxation to assess the relative impacts of this increase in inherent inequality, the decreasing sources of non income tax revenue, and possible declines in inequality aversion, to get a numerical feel for their possible impacts on inequality

## 1. Introduction

Tax/transfer system reform was central to the transition process from the centrally planned economy to a market-type economy in Central and Eastern Europe (CEE). In the old fiscal system a large share of tax revenue came directly or indirectly from state-owned firms. The new fiscal system is in turn designed to be compatible with future EU membership of CEE countries. A personal income tax, a value added tax and entrepreneurial profits tax are all largely modelled on western counterparts. The introduction of the new fiscal system, in concert with other structural features of the transition, has had profound indirect and direct distributional effects. One common characteristic of the transition in Central and Eastern Europe has been an increase in income inequality. Both market and disposable income inequality has risen in these countries during the 1990s. Driving this increase in inequality have been a variety of factors. In the pre reform situation the requirement of government expenditure was largely met from non-tax revenue as the profits of public production, taxation of enterprise profits and commodity transactions. Privatisation of state owned firms surely has had significant consequences for income inequality. Other factors such as trade liberalization, changes to the level and composition of government spending including declines in the provision of public goods, and changes in the wage setting process, have all tended to raise inequality. At the same time, it can be argued that these societies have become less averse per se to inequality<sup>1</sup>.

This paper develops a framework in which these different forces on inequality can be assessed. We start by surveying the salient empirical facts on income inequality and redistribution based on the Luxembourg Income Study (LIS) database in Section 2. In Section 3 we indicate the potentially important channels for changes in market income inequality, or “inherent” inequality, using a simple two-type and two-sector optimal income tax model with endogenous wages. We argue that a reduction in public goods provision can indeed lead to an increase in inherent inequality in such models. Section 4 accepts an increase in inherent inequality but looks at optimal redistribution in the face of this increase, and also when sources of non income tax revenue disappear as the structure changes, and as aversion to inequality falls—all forces that, it can be argued, have been present in the transition process. Section 5 concludes the paper with a discussion of directions for further research.

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<sup>1</sup> According to Atkinson and Micklewright (1992) CEE countries were during the 1970s and 1980s very egalitarian in their disposable income distribution compared with comparable western market economies.

## 2. The Basic Facts

This section sets the stage by reviewing empirical findings on income inequality and the extent of redistribution in the transition countries. Data on income distribution shown in Table 1 and 2 are obtained from the LIS. The relatively high quality of this data source has been commented on elsewhere (see Atkinson-Brandolini, 2001) The income concepts employed are market income (MI), social retirement benefits (P) added to market income (MI+P) and disposable income (DI),<sup>2</sup> with household size being allowed for by deflating by the square root of the number of household members.

Table 1 provides estimates of the change in the disposable income distribution. In the period considered, the Gini-coefficient of disposable income rose markedly, as did the various decile ratios. Table 2 shows that the inequality of market incomes also rose markedly, a factor confirmed by Table 3, which shows significant increases in the decile ratios of the gross earnings of employees. However, interestingly, Table 2 shows that the extent of redistribution, as measured by the difference in the increase of market income inequality and the increase of disposable income inequality, actually increased. For example, between 1986 and 1995 in Poland the Gini coefficient for market income increased by over 20 percentage points. But the disposable income Gini only rose by around 10 points. Thus on one measure, the extent of redistribution increased by more than 10 percentage points.

These facts set up our basic analytical questions. What explains the increase in market inequality? Given this increase, what explains the increased degree of redistribution especially if, as is often argued, the degree of inequality aversion also fell during the transition period? The next two sections take up these questions.

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<sup>2</sup> See Milanovic (2000) p. 375 note 7 and LIS (<http://lisweb.ceps.lu/techdoc.htm>)

Table 1  
Income (disposable) inequality measures

Country	Year	Gini-coefficient	Percentile Ratio(90/10)	Percentile Ratio(90/50)	Percentile Ratio(80/20)
Czech-Republic	1992	0.207	2.37	1.55	1.73
	1996	0.259	3.01	1.79	2.04
Hungary	1991	0.283	3.39	1.82	2.18
	1994	0.323	4.19	2.09	2.42
Poland	1986	0.271	3.51	1.77	2.32
	1992	0.274	3.42	1.84	2.17
	1995	0.318	4.04	1.89	2.37
Russia	1992	0.393	6.66	2.40	3.45
	1995	0.447	9.39	2.82	3.95
Slovak Republic	1992	0.189	2.25	1.49	1.68

Table 2. Gini (G) coefficients and redistribution in transition economies

Country	Year	G(MI)	G(MI+P)	G(DI)	RD=G(MI)-G(DI)
Czech-Republic	1992	43.7	30.0	21.7	-22.0
Hungary	1991	52.0	39.2	30.3	-21.7
Poland	1986	39.9	33.5	29.1	-10.8
	1992	45.9	36.3	33.8	-12.7
	1995	60.6	50.9	38.8	-21.7
Russia	1992	56	47.2	45.2	-10.8
	1995	62	50.0	48.8	-13.2
Slovak Republic	1992	43.0	32.0	20.9	-22.1

Source: Milanovic(2000) based on LIS data.

Table 3 Distribution of gross earnings of employees(P90/P10)

Country	1989	1990	1991	1992	1993	1994	1995	1996	1997
Czech-republic	2.43		2.60	2.75	3.20	3.14	3.70	2.86	2.98
Hungary		3.40		3.56	3.70	3.75			4.17
Poland	2.43		2.85	2.91	3.01	3.40	3.35	3.48	3.53
Russia	3.33	3.36	4.28	8.17	15.55	9.41	9.96	9.60	10.40
Ukraine				3.12	5.51		5.74	5.74	

Source: Flemming-Micklewright (2000), Appendix B

### 3. Public Provision and Market Inequality

Consider the following model, a modified version of the model in Naito (1999)<sup>3</sup>. There are two types of workers in the economy: Workers of type 1 are less skilled and earn income  $w_1$ . The more skilled workers, type 2, earn a wage  $w_2 (> w_1)$ . The number of workers of each type is 1. Workers supply labour, denoted by  $l$ , and consume two types of goods: a normal private good  $x$ , and a quasi-private good  $g$ . The latter good is provided by the state sector. Preferences are represented by a strictly monotone, strictly quasi-concave, and twice differentiable utility function by  $v(x, l, g)$ . Workers maximise  $v(x, l, g)$  with respect to his or her labour supply, subject to a given tax schedule,  $T(y)$ , and the budget constraint  $x = y - T(y)$ , where  $y = wl$  denotes workers gross income.

The good  $x$  is produced in the private sector according to an aggregate, constant returns to scale, production function  $H(l_1^x, l_2^x)$ , where  $l_1^x$  and  $l_2^x$  denote the labour inputs in the private sector. The good  $g$  in turn is produced according to the aggregate production function  $G(l_1^g, l_2^g)$ , where  $l_1^g$  and  $l_2^g$  are the labour inputs in the public sector. Note that the same technology is used to produce both goods. They have thus similar producer prices as well. For simplicity, the prices for both goods are normalised to unity. This specification captures two important features of the model. First, the wage rates are endogenous in a similar way as in Stern (1982) or Stiglitz (1982). In the following,

$\Omega = \frac{w_1}{w_2}$  depicts the relative wage of the low-skilled type. Assuming a competitive labour market,

$\Omega$  is a function of  $l_1/l_2$ ,  $\frac{w_1}{w_2} = \frac{H_1(l_1, l_2)}{H_2(l_1, l_2)}$ , where  $H_i$  ( $i=1,2$ ) denotes the partial derivatives of the

production function with respect to the labour inputs  $l_i^x$ . It captures the idea that the relative wage rate of type 1, determined at the market, is a decreasing function of  $l_1/l_2$ . It is also assumed that the

public sector minimizes costs with respect to the (shadow) wages  $r_1$  and  $r_2$  by the government. Thus the public sector minimizes production costs by equating the marginal rate of transformation

between unskilled and skilled workers to the ratio of equilibrium wage rates, i.e.  $\frac{r_1}{r_2} = \frac{G_1(l_1, l_2)}{G_2(l_1, l_2)}$ ,

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<sup>3</sup> See also Gaube (2000) and Pirttilä-Tuomala (2002)

where  $G_i$  ( $i=1,2$ ) in turn denotes the partial derivatives of the public sector production function with respects to labour inputs  $l_i^g$ .

Following the standard idea of Pareto-optimal taxation, the government maximises the utility of the less-skilled workers subject to the constraint that the skilled worker must stay at a given utility level. The government redistributes income by taxing income on a non-linear scale. It may also use a uniform public provision of  $g$  as a policy variable. We apply the information-based approach to tax policy by assuming that the government can observe the labour income  $y$ , but it does not observe the income earning abilities (the wage rates) of the workers. Therefore, the government must select the tax schedule subject to the self-selection constraint that the skilled worker has an incentive to work  $l_2 = y_2 / w_2$ , report income  $y_2$  and consume  $x_2$  instead of wishing to pretend to be the unskilled household, i.e. mimic, working  $y_1 / w_2 = w_1 l_1 / w_2 = \Omega l_1$ , reporting income  $y_1$ , and consuming  $x_1$ . The government chooses the optimal tax schedule (or labour – after-tax income) bundles to the two different worker types subject to the constraint that the skilled worker be at a given utility level, the self-selection constraint of the skilled worker, and the resource constraint of the economy ( $l_1 = l_1^x + l_1^g$ ,  $l_2 = l_2^x + l_2^g$ ). We concentrate here on the 'normal' case where the redistribution occurs from the skilled workers to the unskilled ones. Thus the self-selection constraint of the skilled workers is binding. The Lagrangean of the government optimisation problem can be expressed in terms of the controls  $x_i$ ,  $l_i$ ,  $l_i^x$ ,  $l_i^g$  ( $i = 1,2$ )

$$\begin{aligned}
L = & v(x_1, l_1, g) + \mathbf{d} [v(x_2, l_2, g) - \bar{v}^2] \\
& + \mathbf{m} [v(x_2, l_2, g) - v(x_1, \Omega l_1, g)] \\
& + \mathbf{r}_x [H(l_1^x, l_2^x) - x_1 - x_2] \\
& + \mathbf{r}_g [G(l_1^g, l_2^g) - 2g] + \mathbf{a}_1 [l_1 - l_1^x - l_1^g] + \mathbf{a}_2 [l_2 - l_2^x - l_2^g]
\end{aligned} \tag{1}$$

The first-order conditions are the following:

$$x_1 : v_x^1 - \mathbf{m} \hat{v}_x^2 - \mathbf{r}_x = 0, \tag{2}$$

$$l_1 : v_l^1 - \mathbf{m} \hat{v}_l^2 \left( \frac{\partial \Omega}{\partial l_1} l_1 + \Omega \right) + \mathbf{a}_1 = 0, \tag{3}$$

$$x_2 : (\mathbf{d} + \mathbf{m})v_x^2 - \mathbf{r}_x = 0, \quad (4)$$

$$l_2 : (\mathbf{d} + \mathbf{m})v_i^2 + \mathbf{a}_2 = 0, \quad (5)^4$$

$$l_i^x : \mathbf{r}_x H_i - \hat{\mathbf{m}}_i^2 \Omega_i l_i - \mathbf{a}_i = 0, i = 1, 2, \quad (6)$$

$$l_i^g : \mathbf{r}_g G_i - \mathbf{a}_i = 0, \quad (7)$$

where the hat terms refer to the so-called mimickers, i.e. type 2 workers when mimicking the choice of type 1 and  $\Omega_i = \frac{\partial \Omega}{\partial l_i^x}, i = 1, 2$ .<sup>5</sup>

Suppose that the government has chosen to produce a certain amount of consumption,  $g$ . Given this, suppose further that the government's income tax and public employment policy is optimal. We will now show that the marginal rate of transformation between these two types of

labour in public production is smaller than that one in the private sector, i.e.  $\frac{r_1}{r_2} < \frac{w_1}{w_2}$ .

From the equation (6) we see that only in the case that the second term is zero the production efficiency holds i.e.  $G_1/G_2 = H_1/H_2$ . But we also note that the term  $-\hat{\mathbf{m}}_i^2 l_i$  is positive. Thus the Diamond-Mirrlees efficiency theorem does not hold in this model. Given our assumptions about the public production function (6) implies the following results; to produce a given amount of consumption the government should employ more unskilled workers and less skilled workers than is necessary to minimize cost at the prevailing gross wage rates. This means that if the supply of low skilled workers becomes scarcer in the private sector, through hiring more of these workers into the public sector, this reduces the wage differentials of the workers. Thus, indirect redistribution through public sector employment will Pareto-improve welfare by mitigating the incentive problem of the non-linear income tax system. Or put it in terms of envelope arguments. If in the beginning the production efficiency holds, then the marginal change in hiring more low skilled workers to the public sector has no first order welfare costs. It affects only relative wages of the low skilled workers.

<sup>4</sup> There is no distortion at top in this model. This can be deduced by dividing (4) by (5). In other words the Stern-Stiglitz result- the marginal tax rate is negative for the skilled worker- does not hold in our model

<sup>5</sup> The function  $\Omega$  is homogeneous of degree zero in  $(l_1^x, l_2^x)$ . It implies  $\Omega_1 < 0$  and  $\Omega_2 > 0$ .

Given the optimal income tax and employment policy, we may also use the envelope argument to detect the change in the social welfare from an increase in the level of the publicly provided good as follows:

$$\frac{dL}{dg} = v_g^1 + (d + m)v_g^2 - \hat{m}_g^2 - \hat{m}_l^2 \frac{d\Omega}{dg} l_1 - 2r_g. \quad (8)$$

Our focus is, however, more in the production side of the economy, and therefore we concentrate on the case with the weakly separable (between consumption and labour (or leisure)) utility function. Rewriting (8) by substituting for  $r_x$  from (2) and (4) yields

$$\frac{dL}{dg} = -\hat{m}_l^2 \frac{d\Omega}{dg} l_1. \quad (9)$$

What is interesting in (9) is the link between the publicly provided private good and the wage structure of the economy (the term  $-\hat{m}_l^2 \frac{d\Omega}{dg} l_1$ ). If its provision leads to a relative increase in the wage rate for type 1 workers, then indirect redistribution through public provision will Pareto-improve welfare by mitigating the incentive problem of the non-linear income tax system.

These results bear resemblance to the interesting recent findings by Naito (1999) that if wage rates are endogenous, redistribution devices that otherwise would not be applied – in Naito's case public inputs and commodity taxation and in our case public sector employment combined with public provision of private goods – become welfare-improving. These theoretical results support the view that the privatisation and a decrease in public provision such as education, health care and social services may have been important factors in explaining increasing inherent inequality in transition economies during the 1990s.

#### 4. Optimal Non-linear Redistribution

An analytical framework for thinking through the relationship between inherent inequality and the extent of redistribution is put forward by James Mirrlees in his Nobel Prize winning paper (Mirrlees, 1971). It captures the central features in thinking about the evolution of redistribution

policy. Certain key elements of the Mirrlees model are useful for our purposes. First is the concept of inherent inequality reflecting among other things skilled /unskilled wage differentials, asset inequality and social norms. If there is no intervention by the government, the inherent inequality will be fully reflected in the disposable income. However, if the government wants to intervene – as seems to be the case in the transition countries – it will find the second component of the Mirrlees model, the egalitarian objectives of the government. And if the government tries to redistribute income from high-income people to low-income people, there will be incentive and disincentive effects. In other words the redistribution policy is the product of circumstances and objectives. Finally, the Mirrlees model has a revenue requirement from the tax/transfer system, to finance an exogenously given level of public goods. In this framework, we use numerical simulations to study questions such as how optimal redistribution might respond when inherent inequality increases, the government becomes less averse to inequality and the role of non-tax revenue decreases.<sup>6</sup>

It is useful to lay out the basic model, even though it is well known. There is a continuum of individuals, each having the same preference ordering, which is represented by an additive utility function  $u = U(x) - V(l)$  defined over consumption  $x$  and hours worked  $l$ , with  $U_x > 0$  and  $V_l < 0$  (subscripts indicating partial derivatives) and where  $V(\cdot)$  is convex. Workers differ only in the pre-tax wage  $w$  they can earn. There is a distribution of  $w$  on the interval  $(s, h)$  represented by the density function  $f(w)$ . Gross income  $y = wl$ .

Suppose that the aim of policy can be expressed as maximizing the following social welfare criterion

$$S = \int_s^h W(u(w)) f(w) dw, \quad (10)$$

where  $W(\cdot)$  is an increasing and concave function of utility. The government cannot observe individuals' productivities and thus is restricted to setting taxes and transfers as a function only of earnings,  $T[y(w)]$ . The government maximizes  $S$  subject to the revenue constraint

$$\int_s^h T(y(w)) f(w) dw = R \quad (11)$$

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<sup>6</sup> These questions were examined by Newbery (1997) in the framework of optimal linear taxation.

where in the Mirrlees tradition  $R$  is interpreted as the required revenue for essential public goods. The more non-tax revenue a government receives from external sources (as in the old fiscal system from state owned firms), the lower is  $R$ . In addition to the revenue constraint, the government faces incentive compatibility constraints. These in turn state that each  $w$ -individual maximizes utility by choice of hour. Totally differentiating utility with respect to  $w$ , and making use of workers utility maximization condition, we obtain the incentive compatibility constraints,

$$\frac{du}{dw} = -\frac{V_l}{w}. \quad (12)$$

Since  $T = wl - x$ , we can think of government as choosing schedules  $l(w)$  and  $x(w)$ . In fact it is easier to think of it choosing a pair of functions,  $u(w)$  and  $l(w)$ , which maximize welfare index (10) subject to the incentive compatibility condition (12) and the revenue requirement (11). Omitting details (for an exposition see Tuomala, 1990), the first order conditions of this problem imply a pattern of marginal rates<sup>8</sup>,  $t(z) = T'(z)$ , satisfying

$$\frac{t}{1-t} = (E^{-1} + 1)U_x \mathbf{m}(w) / \mathbf{l}w f(w) \quad (13)$$

where  $\lambda$  is the multiplier on the revenue constraint and

$$\mathbf{m}(w) = \int_s^w ((W'U_x - \mathbf{l})(1/U_x))f(p)dp. \quad (14)$$

is the multiplier on the incentive compatibility constraint. This latter satisfies the transversality conditions

$$\mathbf{m}(s) = \mathbf{m}(h) = 0. \quad (15).$$

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<sup>7</sup> The 1.order condition of individual's optimisation problem is only a necessary condition for the individual's choice to be optimal, but we assume here that it is sufficient as well. Assumptions that assure sufficiency are provided by Mirrlees (1976). Note also that while we here presume an internal solution for  $l$ , (12) remains valid even if individuals were bunched at  $l=0$  since, for them,  $du/dw=0$ .

<sup>8</sup> There are other works that have looked at alternative derivations and formulae for non-linear taxation, see Revesz (1989), Roberts (2000) and Saez (2001)

Finally, as in Atkinson-Stiglitz (1980)  $E = V'/V''$ . It is the elasticity of labour supply with respect to net wage, holding marginal utility of income constant, i.e.  $E$  is “compensated” wage elasticity in a rather unusual sense.

Unfortunately, however, as is well recognized in the non-linear taxation literature, closed form analytical results are few and far between.<sup>9 10</sup> It should be clear from (13) that the variation of the optimal marginal tax rate with the level of income is a complex matter, and that comparative statics of inequality and averages as parameters vary will not be available in closed form. This is a general feature on the optimal nonlinear income taxation literature (see Tuomala, 1990) where, following the lead of Mirrlees (1971) numerical calculations have proved useful in generating useful results<sup>11</sup>. We follow this route here. With these techniques, we can compute post tax income at each level of  $w$ , and thus calculate inequality of pre and post tax income as well as total income, for different values of key parameters. Our focus is on identifying the combined effects of greater inherent inequality (the standard deviation of  $w$ ), smaller inequality aversion and larger tax revenue requirement. We turn now to this task.

We assume  $w$  to be distributed lognormally with parameters  $m$  and  $\sigma$  (see Aitchison and Brown, 1957). This assumption is common in the literature, following Mirrlees (1971). For numerical simulations we choose  $\sigma = 0.39, 0.5, 0.7$  and  $1$  and  $m = -1$  ( and  $-0.8$ )<sup>12</sup>. The calculations were carried out for both the CES and CD utility functions

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<sup>9</sup> Equations (13) - (15) lead to the few qualitative conclusions available in this framework (see Tuomala, 1990). It can be shown that the marginal tax rate on income is nonnegative. This is more striking than it at first looks. It may very well be optimal to have the average tax rate less than zero, but it is never optimal to subsidize earnings at margin. An intuition is that it is cheaper to get people to given indifference curve by reducing average rate rather than by exacerbating deadweight loss through distorting their labour supply decisions. It can also be shown that the marginal tax rate is less than one. We also have the famous "end point" results. If wage distribution is bounded above, then the marginal tax rates at the top is zero. If it is optimal for least able individual to work then the marginal tax rate on least able is zero. An intuition behind these endpoint results is that only reason to have a marginal tax rate differing from zero is to raise an average tax rate above that point and lower it below i.e. equity considerations. But at the top is no one to take from and at the bottom there is no one to give to. So at the end points only efficiency considerations matter. Numerical solutions (Tuomala, 1990) have shown, however, that these results have very little practical relevance.

<sup>10</sup> There is a special case in which a closed-form solution can be obtained. This possibility was first shown by Lollivier and Rochet (1983) when preferences are quasi-linear in leisure and the social welfare function is utilitarian. Properties of the solution for this case were studied by Weymark (1986) for the discrete case and Ebert (1992) for the continuous case. Boddway et al (2000) provide a full characterization of the solution when preferences are quasi-linear in leisure. Diamond (1998) in turn shows that when preferences are quasi-linear in consumption, an explicit expression for the marginal rate can be derived in terms of the distribution of ability, the elasticity of the labour supply and the form of the social welfare function. He finds the marginal tax rate schedule to be u-shaped in his example. Dahan and Strawczynski (2000) clarifies this result showing that a rising marginal rates at high incomes depend on the joint assumptions of an unbounded distribution and quasi-linear preferences.

<sup>11</sup> Tuomala (1990) gives details of the computational procedure.

<sup>12</sup> As in Kanbur-Tuomala (1994) we also try to calibrate the lognormal distribution so that the income distribution inferred from the ability distribution matches the actual one. Of course it would be important to solve marginal tax rate

$$u = -\frac{1}{x} - \frac{1}{(1-l)} \quad (16)$$

$$u = \ln x + \ln(1-l) \quad (17)$$

where the elasticity of substitution between consumption and leisure, denoted by  $\epsilon$ , is 0.5 for CES and 1 for CD. The social welfare function of the government is specified<sup>13</sup> as  $W(u) = -\frac{1}{b}e^{-bu}$  so that  $\beta$  measures the degree of inequality aversion in the social welfare function of the government (in the case of  $\beta = 0$ , we define  $W = u$ ).  $R$  is specified as a fraction of national income, and is assumed to vary between -0.1 and 0.1.

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formula using the empirical earnings distribution. This is not possible to make directly because the earnings distribution is affected by the tax schedule itself. Saez (2001) makes an important innovation in this question. He calibrates the ability distribution so that given the utility function chosen and the actual tax schedule the resulting pre tax distribution replicates the empirical earnings distribution.

<sup>13</sup>For further discussion on the transformation of each individual's utility see Tuomala (1990)

**Table 4 (“The old fiscal sytem”)**

$\varepsilon=0.5$	$\beta = 1$	$\sigma=.39$	$m=-1$								
F(w)	R=-.1				R=0.0						
	x	z	MTR%	u	x	z	MTR%	u			
0.10	0.17	0.09	62	-7.63	0.16	0.10	65	-8.10			
0.50	0.20	0.18	56	-6.85	0.19	0.19	59	-7.24			
0.90	0.27	0.32	45	-5.78	0.26	0.33	47	-6.06			
0.99	0.38	0.49	28	-4.78	0.36	0.50	29	-4.99			
RD	0.55				0.51						
Decile ratio (P90/P10)	1.59	3.5			1.63	3.3					

RD = the extent of redistribution

**Table 5 (“The old fiscal system”)**

$\varepsilon=1$	$\beta = 1$	$\sigma=.39$	$m=-1$								
F(w)	R=-.1				R=0.0						
	x	z	MTR%	u	x	z	MTR%	u			
0.10	0.11	0.08	30	-2.68	0.10	0.07	33	-2.57			
0.50	0.17	0.15	28	-2.39	0.15	0.15	30	-2.30			
0.90	0.27	0.28	24	-2.00	0.25	0.28	25	-1.94			
0.99	0.41	0.44	20	-1.64	0.38	0.45	18	-1.60			
RD	0.31				0.37						
Decile ratio (P90/P10)	2.48	3.58			2.50	3.97					

**Table 6a (“The new fiscal system”)**


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$\varepsilon=.$ 5	$\sigma=.$ 5	$m=-$ 1						
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F(w)	R=0.0				R=0.0				m=-.8
	x	z	MTR%	u	x	Z	MTR%	u	
0.10	0.15	0.10	51	-8.67	.18	.11	50	-7.44	
0.50	0.20	0.20	50	-7.13	.24	.24	48	-6.17	
0.90	0.29	0.37	45	-5.58	.35	.43	43	-4.94	
0.99	0.49	0.59	38	-4.33	.51	.68	35	-3.85	
RD	0.48				.50				
Decile ratio (P90/P10)	1.93	3.7			1.90	3.83			

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**Table 6b (“The new fiscal system”)**


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$\varepsilon =$ 1	$\beta =$ 0	$\sigma=.$ 5	$m=-$ 1					
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F(w)	R=0.0				
	x	z	MTR%	u	
0.10	0.09	0.06	30	-2.79	
0.50	0.15	0.15	29	-2.59	
0.90	0.24	0.32	26	-2.05	
0.99	0.47	0.57	22	-1.41	
RD	0.45				
Decile ratio (P90/P10)	2.9	5.3			

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**Table 7 (“The new fiscal system”)**

$\varepsilon=0.5 \quad \beta=0 \quad \sigma=.7 \quad m=-1$								
F(w)	R=0.0				R=.1			
	x	z	MTR%	u	x	z	MTR%	u
0.10	0.16	0.06	56	-7.89	0.14	0.06	60	-8.86
0.50	0.20	0.17	60	-6.89	0.19	0.18	63	-7.22
0.90	0.31	0.45	57	-4.95	0.30	0.47	60	-5.41
0.99	0.54	0.91	45	-3.79	0.55	0.96	37	-3.85
RD	0.74				0.73			
Decile ratio (P90/P10)	1.94	7.56			2.14	7.91		

**Table 8 (“The new fiscal system”)**

$\varepsilon=.5. \quad \beta=0 \quad \sigma=1. \quad m=-1$								
F(w)	R=0.0				R=.1			
	x	z	MTR%	u	x	z	MTR%	u
0.10	0.17	0.02	55	-7.69	0.16	0.02	59	-7.55
0.50	0.21	0.14	68	-6.35	0.20	0.15	71	-6.67
0.90	0.35	0.55	71	-4.65	0.33	0.61	72	-4.88
0.99	0.70	1.61	58	-3.21	0.67	1.65	59	-3.31
RD	0.92				0.93			
Decile ratio (P90/P10)	2.06	27.5			2.06	30.1		

Tables 4-8 give net income ( $x$ ), gross income ( $z$ ), optimal marginal tax rates (MTR) and utility levels ( $u$ ) at various percentiles of the ability distribution.<sup>14</sup> Tables 4 and 5 reflect "the old fiscal system" ( $\beta = 1$ ,  $\epsilon = 0.5$  or  $1$ ,  $\sigma = 0.39$ ,  $R = -0.1, 0.0$ ) and tables 6a b, 7 and 8 in turn "the new one" ( $\beta = 0$ ,  $\epsilon = 0.5$  or  $1$ ,  $\sigma = 0.5, 0.7$  and  $1$ ,  $R = 0.0, 0.1$ ). Tables 4-8 also provide the decile ratio (P90/P10) for net income and gross income and the extent of redistribution measured as the proportional reduction between the decile ratio for market income,  $z$ , and the decile ratio for disposable income,  $x$ ,

Consider first the progressivity of the tax structure as a function of market inequality and revenue requirement. Tables 4-8 show that optimal tax/transfer systems become more progressive when inequality increases,  $\sigma = 0.5$ ,  $\sigma = 0.7$  and  $\sigma = 1.0$ , and when  $R$  becomes more negative (i.e. more non tax revenue). To understand this, we can combine the results of two earlier studies. Kanbur-Tuomala (1994) show that with greater market income "inherent" inequality optimal marginal tax rates increase with income over the majority of the population. On the other hand we know from Immonen-Kanbur-Keen-Tuomala (1998) that as the revenue requirement becomes negative so that for example non-tax revenue is available the minimum income requirement for the poor can be met without clawing back revenue with a high marginal tax rate. Thus we have low marginal tax rates on the poor. In other words, optimal progressivity, taking into account incentive effects, increases with higher inherent inequality and with non-tax revenue. Thus, while the increasing inherent inequality would have induced a partially correcting "optimal" increase in progressivity of the tax/transfer system, the decrease in non-tax revenue (and hence increase in the revenue requirement from the tax system) that was also seen in the transition would have been a force for decreasing progressivity.

In tables 4-8 we see what happens when the government becomes less averse to inequality, inherent inequality increases and the revenue requirement also increases. The extent of optimal redistribution, denoted by  $RD$ , increases as a consequence of increasing the wage dispersion. This is just what we can see in those transition countries having at least two observations (see table 2). On the other hand the pattern of marginal tax rates is quite different. With parameters ( $\epsilon = 0.5, \sigma = 0.7, \beta = 0, R = 0.0$ .) the marginal tax rate increases with income up to the 74 %; with  $\epsilon = 1.0$ , on the other hand, it increases with income up to 69 %. An interesting question is when might an increase in inherent inequality, an increase in the tax revenue requirement, and a decrease in inequality aversion, be roughly offsetting? We see in tables 5 and 6b ( $\epsilon = 1$ ) that in terms of marginal tax rate structure the effect of increasing the wage

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<sup>14</sup> With the utility function we use, there is "bunching"—all those below a critical value of  $w$  choose not to work. Their pre tax income is thus zero and their post tax income is whatever the optimal tax and transfer regime gives them.

dispersion from  $\sigma = 0.39$  to  $\sigma = 0.5$  is the same as moving from  $\beta = 1$  to  $\beta = 0$ . If the extent of redistribution is the criterion then the cases ( $\epsilon = 0.5, \beta = 1, \sigma = 0.39, R = 0.0$ ) (table 6a) and ( $\epsilon = 0.5, \beta = 0, \sigma = 0.5, m = -0.8, R = 0.0$ ) (table 6a) are roughly speaking the same. Thus given this criterion the effect of increasing the inherent inequality from  $\sigma = 0.39$  to  $\sigma = 0.5$  (and  $m = -1$  to  $m = -0.8$ ), is the same as moving from  $\beta = 1$  to  $\beta = 0$ . In other words if market income inequality has risen as much as represented by this move, then it would need a considerable decrease in concern for equity to offset the case for the same extent of redistribution.

## 5. Conclusions

In this paper we argue that an analysis of the evolution of pre and post tax income inequality in the transition economies of Central and Eastern Europe can be structured, and the different forces in play understood, through the framework of optimal income taxation. Using the simple two-type and two-sector optimal income tax model we first of all show that a decrease in public provision of public goods may have been an important factor in explaining increasing pre-tax (“inherent”) inequality in transition economies during the 1990s. We also ask, in the framework of non-linear optimal tax theory, how redistribution might respond when inherent inequality increases, the government becomes less averse to inequality and the role of non-tax revenue decreases, all of which happened during transition. We use numerical simulations to study these questions. We discuss when these forces are offsetting and when they reinforce each other as governments choose tax/transfer schedules optimally in response to them, in trying to understand the stylised facts of pre and post tax income inequality during transition. While the increase in inherent inequality induces a response of greater progressivity, this is counteracted by the tendency of the other two forces to decrease progressivity. Overall “optimal” progressivity thus increases, but not sufficiently to overcome the increase in inherent inequality, which leads to an increasing post tax inequality. And these are precisely the stylised facts of inequality and progressivity during transition that we set out to investigate.

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