Imperfect Knowledge, Inflation Expectations, and Monetary Policy

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Abstract

This paper investigates the role that imperfect knowledge about the structure of the economy plays in the formation of expectations, macroeconomic dynamics, and the efficient formulation of monetary policy. Economic agents rely on an adaptive learning technology to form expectations and continuously update their beliefs regarding the dynamic structure of the economy based on incoming data. The process of perpetual learning introduces an additional layer of dynamic interactions between monetary policy and economic outcomes. We find that policies that would be efficient under rational expectations can perform poorly when knowledge is imperfect. In particular, policies that fail to maintain tight control over inflation are prone to episodes in which the public's expectations of inflation become uncoupled from the policy objective and stagflation results, in a pattern similar to that experienced in the United States during the 1970s. More generally, we show that policy should respond more aggressively to inflation under imperfect knowledge than under perfect knowledge.

KEYWORDS: Inflation targeting, policy rules, rational expectations, learning, inflation persistence.

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1 Introduction

Rational expectations provides an elegant and powerful framework that has come to dominate thinking about the dynamic structure of the economy and econometric policy evaluation over the past 30 years. This success has spurred further examination into the strong information assumptions implicit in many applications. Thomas Sargent (1993) concludes that "rational expectations models impute much *more* knowledge to the agents within the model ... than is possessed by an econometrician, who faces estimation and inference problems that the agents in the model have somehow solved" (p. 3, emphasis in original).[1] Researchers have proposed refinements to rational expectations that respect the principle that agents use information efficiently in forming expectations, but nonetheless recognize the limits to and costs of information-processing and cognitive constraints that influence the expectations-formation process (Sargent 1999, Evans and Honkapohja 2001, Sims 2001). In this study, we allow for a form of imperfect knowledge in which economic agents rely on an adaptive learning technology to form expectations. This form of learning represents a relatively modest deviation from rational expectations that nests it as a limiting case. We show that the resulting process of perpetual learning introduces an additional layer of interaction between monetary policy and economic outcomes that has important implications for macroeconomic dynamics and the efficient formulation of monetary policy.

Our work builds on the extensive literature relating rational expectations with learning and the adaptive formation of expectations (Bray 1982, Bray and Savin 1984, Marcet and Sargent 1989, Woodford 1990, Bullard and Mitra 2001). A key finding in this literature is that under certain conditions an economy with learning converges to the rational expectations equilibrium (Townsend 1978, Bray 1982, 1983, Blume and Easley 1987). However,

[1] Missing from such models, as Benjamin Friedman (1979) points out, "is a clear outline of the way in which economic agents derive the knowledge which they then use to formulate expectations." To be sure, this does not reflect a criticism of the traditional use of the concept of "rationality" as reflecting the optimal use of information in the formation of expectations, taking into account an agent's objectives and resource constraints. The difficulty is that in Muth's (1961) original formulation, rational expectations are not optimizing in that sense. Thus, the issue is not that the "rational expectations" concept reflects too much rationality but rather that it imposes too little rationality in the expectations formation process. For example, as Sims (2001) has recently pointed out, optimal information processing subject to a finite cognitive capacity may result in fundamentally different processes for the formation of expectations than those implied by rational expectations. To acknowledge this terminological tension, Simon (1978) suggested that a less misleading term for Muth's concept would be "model consistent" expectations (p. 2).

until agents have accumulated sufficient knowledge about the economy, economic outcomes during the transition depend on the adaptive learning process (Lucas 1986). Moreover, in a changing economic environment, agents are constantly learning and their beliefs converge not to a fixed rational expectations equilibrium, but to an ergodic distribution around it (Sargent 1999, Evans and Honkapohja 2001). In this paper, we investigate the macroeconomic implications of such a process of perpetual learning.[2]

As a laboratory for our experiment, we employ a simple linear model of the U.S. economy with characteristics similar to more elaborate models frequently used to study optimal monetary policy. We assume that economic agents know the correct structure of the economy and form expectations accordingly. But, rather than endowing them with complete knowledge of the parameters of these functions—as would be required by imposing the rational expectations assumption—we posit that economic agents rely on finite memory least squares estimation to update these parameter estimates. This setting conveniently nests rational expectations as the limiting case corresponding to infinite memory least squares estimation and allows varying degrees of imperfection in expectations formation to be characterized by variation in a single model parameter.

We find that even marginal deviations from rational expectations in the direction of imperfect knowledge can have economically important effects on the stochastic behavior of our economy and policy evaluation. An interesting feature of the model is that the interaction of learning and control creates rich nonlinear dynamics that can potentially explain both the shifting parameter structure of linear reduced form characterizations of the economy and the appearance of shifting policy objectives or inflation targets. For example, sequences of policy errors or inflationary shocks, such as experienced during the 1970s, could give rise to stagflationary episodes that do not arise under rational expectations with perfect knowledge.

[2] Our work also draws on some other strands of the literature relating to learning, estimation, and policy design. One such strand has examined the formation of inflation expectations when the policymaker's objective may be unknown or uncertain, for example during a transition following a shift in policy regime (Taylor 1975, Bomfim et al, 1997, Erceg and Levin, 2001, Kozicki and Tinsley, 2001, Tetlow and von zur Muehlen, 2001). Another strand has considered how policymaker uncertainty about the structure of the economy influences policy choices and economic dynamics (Sargent, 1999, Balvers and Cosimano 1994, Wieland 1998, and others). Finally, our work relates to explorations of alternative approaches for modeling aggregate inflation expectations, such as Ball (2000), Mankiw and Reis (2001) and Carroll (2001).

Indeed, the critical role of the formation of inflation expectations for understanding the success and failures of monetary policy is a dimension of policy that has often been cited by policymakers over the past two decades but has received much less attention in formal econometric policy evaluations. An important example is the contrast between the stubborn persistence of inflation expectations during the 1970s when policy placed relatively greater attention on countercyclical concerns and the much improved stability in both inflation and inflation expectations following the renewed emphasis on price stability in 1979. In explaining the rationale for this shift in emphasis in 1979, Federal Reserve Chairman Volcker highlighted the importance of learning in shaping the inflation expectations formation process:[3]

> It is not necessary to recite all the details of the long series of events that have culminated in the serious inflationary environment that we are now experiencing. An entire generation of young adults has grown up since the mid-1960's knowing only inflation, indeed an inflation that has seemed to accelerate inexorably. In the circumstances, it is hardly surprising that many citizens have begun to wonder whether it is realistic to anticipate a return to general price stability, and have begun to change their behavior accordingly. Inflation feeds in part on itself, so part of the job of returning to a more stable and more productive economy must be to break the grip of inflationary expectations.
> (Statement before the J.E.C., October 17, 1979.)

This historical episode is a clear example of inflation expectations becoming uncoupled from the intended policy objective and illustrates the point that the design of monetary policy must account for the influence of policy on expectations.

We find that policies designed to be efficient under rational expectations can be quite inefficient when knowledge is imperfect; in particular, the efficient response to inflation is more aggressive than would be optimal with perfect knowledge. This deterioration in performance is particularly severe when policymakers put a high weight on stabilizing real economic activity relative to price stability. We show that economic performance can be improved significantly by placing greater emphasis on controlling inflation and inflation

[3] Indeed, we would argue that the shift in emphasis towards greater focus on inflation was itself influenced by the recognition of the importance of facilitating the formation of stable inflation expectations—which had been insufficiently appreciated earlier during the 1970s. See Orphanides (2001) for a more detailed description of the policy discussion at the time and the nature of the improvement in monetary policy since 1979. See also Christiano and Gust (2000) and Sargent (1999) for alternative explanations of the inflationary episode of the 1960s and 1970s.

expectations. We find that policies emphasizing tight inflation control can facilitate learning and provide better guidance for the formation of inflation expectations. Such policies mitigate the negative influence of imperfect knowledge on economic stabilization and yield superior macroeconomic performance. Thus, our findings provide analytical support for monetary policy frameworks that emphasize the primacy of price stability as an operational policy objective, for example the inflation targeting approach as discussed by Bernanke and Mishkin (1997) and as adopted by several central banks over the past decade or so.

2 The Model Economy

We consider a stylized model that gives rise to a nontrivial inflation-output variability tradeoff and in which a simple one-parameter policy rule represents optimal monetary policy under rational expectations.[4] In this section, we describe the model specification for inflation and output and the central bank's optimization problem; in the next two sections, we take up the formation of expectations by private agents.

Inflation is determined by a modified Lucas supply function that allows for some intrinsic inflation persistence,

$$\pi_{t+1} = \phi \pi^e_{t+1} + (1-\phi)\pi_t + \alpha y_{t+1} + e_{t+1}, \quad e \sim \text{iid}(0, \sigma_e^2), \tag{1}$$

where π denotes the inflation rate, π^e is the private agents' expected inflation rate based on time t information, y is the output gap, $\phi \in (0,1)$, $\alpha > 0$, and e is a serially uncorrelated innovation. As discussed by Clark et al (1999), Lengwiler and Orphanides (forthcoming), and others, this specification incorporates an important role for inflation expectations for determining inflation outcomes while also allowing for some inflation persistence that is necessary for the model to yield a nontrivial inflation-output gap variability tradeoff.[5]

The output gap (the percent deviation of real output from potential output) is determined by the real rate gap (the difference between the short-term real interest rate and the

[4]Since its introduction by Taylor (1979), the practice of analyzing monetary policy rules using such an inflation-output variability tradeoff has been adopted in a large number of academic and policy studies.

[5]We have also examined the "New-Keynesian" variant of the Phillips curve studied by Gali and Gertler (2000) and others, which also allows for some intrinsic inflation inertia. As we report in section 6, our main findings are not sensitive to this alternative.

equilibrium real interest rate),

$$y_{t+1} = -\xi(r_t - r^*) + u_{t+1}, \quad u \sim \text{iid}(0, \sigma_u^2). \tag{2}$$

where r is the short-term real interest rate, r^* is the equilibrium real rate, and u is a serially uncorrelated innovation. Note that a monetary policy action at period t affects output in the following period, reflecting the lag in the monetary transmission mechanism.

The central bank's objective is to design a policy rule that minimizes the loss, denoted by $\mathcal{L}$, equal to the weighted average of the asymptotic variances of the output gap and of deviations of inflation from the target rate,

$$\mathcal{L} = (1-\omega)Var(y) + \omega Var(\pi - \pi^*), \tag{3}$$

where $Var(z)$ denotes the unconditional variance of variable z, and $\omega \in (0, 1]$ is the relative weight on inflation stabilization.

The central bank sets its instrument, the short-term (ex ante) real interest rate r_t, after private agents set their expectations for inflation in period $t+1$, π^e_{t+1}, but before time $t+1$ innovations are observed. We assume that the central bank has perfect knowledge regarding the structural parameters of the model, α, ϕ, ξ, and r^*. With this assumption, we can reformulate the policy instrument in terms of the choice at time t of the intended level of output gap in period $t+1$, $x_t = -\xi(r_t - r^*)$.[6] Hence, the realization of the output gap in period $t+1$ equals the intended output gap plus the control error, u_{t+1},

$$y_{t+1} = x_t + u_{t+1}. \tag{4}$$

This completes the description of the structure of the model economy, with the exception of the expectations formation process that we examine in detail below.

3 The Perfect Knowledge Benchmark

We begin by considering the "textbook" case of rational expectations with perfect knowledge in which private agents know the structure of the economy and the central bank's

[6] Note that here we abstract from the important complications associated with the real-time measurement of the output gap and and the equilibrium real interest rate for formulating the policy rule. See Orphanides (1998) and Laubach and Williams (2001) for analyses of these issues.

policy. In this case, expectations are rational in that they are consistent with the true data generating process of the economy (the model). In the following section, we use the resulting equilibrium solution as a "perfect knowledge" benchmark against which we compare outcomes under imperfect knowledge, in which case agents do not know the structural parameters of the model, but instead must form expectations based on estimated forecasting models.

Under the assumption of perfect knowledge, the evolution of the economy and optimal monetary policy can all be expressed in terms of two variables, the current inflation rate and its target level. These variables determine the formation of expectations and the policy choice, which, together with serially uncorrelated shocks, determine output and inflation in period $t+1$. Specifically, we can write the monetary policy rule in terms of the inflation gap,

$$x_t = -\theta(\pi_t - \pi^*), \tag{5}$$

where $\theta > 0$ measures the responsiveness of the real rate gap to the inflation gap.

Given this monetary policy rule, inflation expectations are given by:

$$\pi^e_{t+1} = \frac{\alpha\theta}{1-\phi}\pi^* + \frac{1-\phi-\alpha\theta}{1-\phi}\pi_t. \tag{6}$$

Inflation expectations depend on the current level of inflation, the inflation target, and the parameter θ measuring the central bank's responsiveness to the inflation gap. Substituting this expression for expected inflation into equation (1) yields the rational expectations solution for inflation for a given monetary policy,

$$\pi_{t+1} = \frac{\alpha\theta}{1-\phi}\pi^* + (1 - \frac{\alpha\theta}{1-\phi})\pi_t + e_{t+1} + \alpha u_{t+1}. \tag{7}$$

One noteworthy feature of this solution is that the first-order autocorrelation of the inflation rate, given by $1 - \frac{\alpha\theta}{1-\phi}$, is decreasing in θ and is invariant to the value of π^*. Note that the rational expectations solution can also be written in terms of the "inflation expectation gap"—the difference between inflation expectations for period $t+1$ from the inflation target, $\pi^e_{t+1} - \pi^*_t$,

$$\pi^e_{t+1} - \pi^*_t = \frac{1-\phi-\alpha\theta}{1-\phi}(\pi_t - \pi^*). \tag{8}$$

Equations (5) and (6) close the perfect knowledge benchmark model.

3.1 Optimal Monetary Policy under Perfect Knowledge

For the economy with perfect knowledge, the optimal monetary policy, θ^P, can be obtained in closed form and is given by:[7]

$$\theta^P = \frac{\omega}{2(1-\omega)}\left(-\frac{\alpha}{1-\phi} + \sqrt{\left(\frac{\alpha}{1-\phi}\right)^2 + \frac{4(1-\omega)}{\omega}}\right) \quad \text{for} \quad 0 < \omega < 1. \tag{9}$$

In the limit, when ω equals unity (that is, when the policymaker is not at all concerned with output stability), the policymaker sets the real interest rate so that inflation is expected to return to its target in the next period. The optimal policy in the case $\omega = 1$ is given by: $\theta^P = \frac{1-\phi}{\alpha}$, and the irreducible variance of inflation, owing to unpredictable output and inflation innovations, equals $\sigma_e^2 + \alpha^2\sigma_u^2$. More generally, the optimal value of θ depends positively on the ratio $\frac{1-\phi}{\alpha}$, and the parameters α and ϕ enter only in terms of this ratio. In particular, the optimal policy response is larger the greater the degree of intrinsic inertia in inflation, measured by $1-\phi$.

The greater the central bank's weight on inflation stabilization, the greater is the responsiveness to the inflation gap, and the smaller the first-order autocorrelation in inflation. Differentiating equation (9) shows that the policy responsiveness to the inflation gap is increasing in ω, the weight the central bank places on inflation stabilization. As a result, the autocorrelation of inflation is decreasing in ω, with a limiting value approaching unity when ω approaches zero, and zero when ω equals one. That is, if the central bank cares only about output stabilization, the inflation rate becomes a random walk, while if the central bank cares only about inflation stabilization, the inflation rate displays no serial correlation. And, as noted, this model yields a nontrivial monotonic tradeoff between the variability of inflation and the output gap for all values of $\omega \in (0, 1]$. These results are illustrated in Figure 1. The top panel of the figure shows the variability tradeoff described by optimal policies for values of ω between zero and one. The lower panel plots the optimal values of θ against ω.

[7] See Clark, Goodhart, and Huang (1999) and Orphanides and Wieland (2000) for examples of the method of solving for the optimal policy. Note that owing to the linear-quadratic structure of the model, the distributions of the innovations do not influence the equilibrium determination of the expectations and policy functions.

4 Imperfect Knowledge

As the perfect knowledge solution shows, private inflation forecasts depend on knowledge of the structural model parameters and policymaker preferences. In addition, these parameters influence the expectations formation function nonlinearly. We now relax the assumption that private agents have perfect knowledge of all structural parameters and the policymaker's preferences. Instead, we posit that agents must somehow infer the information necessary for forming expectations by observing historical data, in essence acting like econometricians who know the correct specification of the economy but are uncertain about the parameters of the model.

In particular, we assume that private agents update the coefficients of their model for forecasting inflation using least squares learning with finite memory. We focus on least squares learning because of its desirable convergence properties, straightforward implementation, and close correspondence to what real-world forecasters actually do.[8] Estimation with finite memory reflects agents' concern for changes in the structural parameters of the economy. To focus our attention on the role of imperfections in the expectations formation process itself, however, we deliberately abstract from the introduction of the actual uncertainty in the structure of the economy which would justify such concerns in equilibrium.

We follow Sargent (1999) and Evans and Honkapohja (2001) by modeling finite memory or "perpetual learning" by assuming agents use a constant gain in their recursive least squares formula that places greater weight on more recent observations. This algorithm is equivalent to applying weighted least squares where the weights decline geometrically with the distance in time between the observation being weighted and the most recent observation. This approach is closely related to the use of fixed sample lengths or rolling-

[8]This method of adaptive learning is closely related to optimal filtering where the structural parameters are assumed to follow random walks. Of course, if private agents know the complete structure of the model—including the laws of motion for inflation, output, and the unobserved states and the distributions of the innovations to these processes—then with this knowledge they could compute efficient inflation forecasts that could outperform those based on recursive least squares. However, uncertainty regarding the precise structure of the time-variation in the model parameters is likely to reduce the real efficiency gains from a method optimized to a particular model specification relative to a simple method such as least-squares learning. Further, once we begin to ponder how economic agents could realistically model and account for such uncertainty precisely, we quickly recognize the significance of respecting (or the absurdity of ignoring) the cognitive and computational limits of economic agents.

window regressions to estimate a forecasting model (Friedman 1979). In terms of the mean "age" of the data used, a rolling-regression window of length l is equivalent to a constant gain κ of $2/l$. The advantage of the constant gain least squares algorithm over rolling regressions is that the evolution of the former system is fully described by a small set of variables, while the latter requires one to keep track of a large number of variables.

4.1 Least Squares Learning with Finite Memory

Under perfect knowledge, the predictable component of next period's inflation rate is a linear function of the inflation target and the current inflation rate, where the coefficients on the two variables are functions of the policy parameter θ and the other structural parameters of the model, as shown in equation (6). In addition, the optimal value of θ is itself a nonlinear function of the central bank's weight on inflation stabilization and the other model structural parameters. Given this simple structure, the least squares regression of inflation on a constant and lagged inflation,

$$\pi_i = c_{0,t} + c_{1,t}\pi_{i-1} + v_i, \tag{10}$$

yields consistent estimates of the coefficients describing the law of motion for inflation (Marcet and Sargent (1988) and Evans and Honkapohja (2001)). Agents then use these results to form their inflation expectations.[9]

To fix notation, let X_i and c_i be the 2×1 vectors, $X_i = (1, \pi_{i-1})'$ and $c_i = (c_{0,i}, c_{1,i})'$. Using data through period t, the least squares regression parameters for equation (10) can be written in recursive form:

$$c_t = c_{t-1} + \kappa_t R_t^{-1} X_t(\pi_t - X_t' c_{t-1}), \tag{11}$$

$$R_t = R_{t-1} + \kappa_t(X_t X_t' - R_{t-1}) \tag{12}$$

where κ_t is the gain. With least squares learning with infinite memory, $\kappa_t = 1/t$, so as t increases, κ_t converges to zero. As a result, as the data accumulate this mechanism

[9] Note that here we assume that agents employ a reduced form of the expectations formation function that is correctly specified under rational expectations. Instead, agents may be uncertain of the correct form and estimate a more general specification, for example, a linear regression with additional lags of inflation which nests (10). In section 6, we also discuss results from such an example.

converges to the correct expectations functions and the economy converges to the perfect knowledge benchmark solution. As noted above, to formalize perpetual learning—as would be required in the presence of structural change—we replace the decreasing gain in the infinite memory recursion with a small constant gain, $\kappa > 0$.[10]

With imperfect knowledge, expectations are based on the perceived law of motion of the inflation process, governed by the perpetual learning algorithm described above. The model under imperfect knowledge consists of the structural equation for inflation (1), the output gap equation (2), the monetary policy rule (5), and the one-step-ahead forecast for inflation, given by

$$\pi_{t+1}^{e} = c_{0,t} + c_{1,t}\pi_t, \tag{13}$$

where $c_{0,t}$ and $c_{1,t}$ are updated according to equations (11) and (12).

We emphasize that in the limit of perfect knowledge (that is, as $\kappa \to 0$), the expectations function above converges to rational expectations and the stochastic coefficients for the intercept and slope collapse to:

$$c_0^P = \frac{\alpha\theta\pi^*}{1-\phi},$$

$$c_1^P = \frac{1-\phi-\alpha\theta}{1-\phi}.$$

Thus, this modeling approach accommodates the Lucas critique in the sense that expectations formation is endogenous and adjusts to changes in policy or structure (as reflected here by changes in the parameters θ, π^*, α, and ϕ). In essence, our model is one of "noisy rational expectations." As we show below, although expectations are imperfectly rational in that agents need to estimate the reduced form equations they employ to form expectations, they are nearly rational in that the forecasts are close to being efficient.

5 Perpetual Learning in Action

We use model simulations to illustrate how learning affects the dynamics of inflation expectations, inflation, and output in the model economy. First, we examine the behavior of

[10]In terms of forecasting performance, the "optimal" choice of κ depends on the relative variances of the transitory and permanent shocks, similar to the relationship between the Kalman gain and the signal-to-noise ratio in the case of the Kalman filter. Here, we do not explicitly attempt to calibrate κ in this way, but instead examine the effects for a range of values of κ.

the estimated coefficients of the inflation forecast equation and evaluate the performance of inflation forecasts. We then consider the dynamic response of the economy to shocks similar to those experienced during the 1970s in the United States. Specifically, we compare the outcomes under perfect knowledge and imperfect knowledge with least squares learning that correspond to three alternative monetary policy rules to illustrate the additional layer of dynamic interactions introduced by the imperfections in the formation of inflation expectations.

In calibrating the model for the simulations, each period corresponds to about half a year. We consider values of κ of .025, .05, and .1, which roughly correspond to using 40, 20, or 10 years of data, respectively, in the context of rolling regressions. We consider two values for ϕ, the parameter that measures the influence of inflation expectations on inflation. As a baseline case, we set ϕ to 0.75, which implies a significant role for intrinsic inflation inertia, consistent with the contracting models of Buiter and Jewitt (1981) and Fuhrer and Moore (1995) and estimates by Brayton et al (1997).[11] In the alternative specification, we allow for a greater role for expectations and correspondingly down-weight inflation inertia by setting $\phi = .9$, consistent with estimates by Gali and Gertler (2000) and others. To ease comparisons between the two values of ϕ, we set α so that the optimal policy under perfect knowledge is identical in the two cases. Specifically, for $\phi = .75$, we set $\alpha = .25$, and for $\phi = .9$, we set $\alpha = .1$. In all cases, we assume $\sigma_e = \sigma_u = 1$.

The three alternative policies we consider correspond to the values of θ, $\{0.1, 0.6, 1.0\}$. These values represent the optimal policies under perfect knowledge for policymakers with preferences with a relative weight on inflation, ω, 0.01, 0.5, and 1, respectively. Hence, $\theta = 0.1$ corresponds to an "inflation dove" policymaker who is primarily concerned about output stabilization, $\theta = 0.6$ corresponds to a policymaker with "balanced preferences" who weighs inflation and output stabilization equally, and $\theta = 1$ corresponds to an "inflation hawk" policymaker who cares exclusively about inflation.

[11] Other estimates suggest an even smaller role for expectations relative to intrinsic inertia; see Fuhrer (1997), Roberts (2001), and Rudd and Whelan (2001).

5.1 The Performance of Least-Squares Inflation Forecasts

Even absent shocks to the structure of the economy, the process of least squares learning generates time variation in the formation of inflation expectations and thereby in the processes of inflation and output. The magnitude of this time variation is increasing in κ—which is equivalent to using shorter samples (and thus less information from the historical data) in rolling regressions. Table 1 reports summary statistics of the estimates of agents' inflation forecasting model based on stochastic simulations of the baseline model economy with $\phi = .75$. As seen in the table, the unconditional standard deviations of the estimates increase with κ. This dependence of the variation in the estimates on the rate of learning is portrayed in Figure 2, which shows the steady-state distributions of the estimates of c_0 and c_1 underlying Table 1. For comparison, the vertical lines in each panel indicate the values of c_0 and c_1 in the corresponding perfect knowledge benchmark.

Table 1: Least Squares Learning

	κ			
	0 (PK)	.025	.05	.10
$\theta = 0.1$				
Mean c_0	.00	.02	.01	-.01
SD c_0	–	.37	.68	1.40
Mean c_1	.90	.86	.83	.79
SD c_1	–	.11	.17	.25
Median c_1	.90	.89	.88	.87
$\theta = 0.6$				
Mean c_0	.00	.01	.01	.00
SD c_0	–	.25	.38	.59
Mean c_1	.40	.37	.35	.31
SD c_1	–	.20	.27	.37
Median c_1	.40	.39	.38	.36
$\theta = 1.0$				
Mean c_0	.00	.01	.01	.01
SD c_0	–	.24	.35	.52
Mean c_1	.00	-.02	-.03	-.06
SD c_1	–	.21	.29	.39
Median c_1	.00	-.02	-.03	-.06

The median values of the coefficient estimates are nearly identical to the values implied by the perfect knowledge benchmark; however, the mean estimates of c_1 are biased downward slightly. There is nearly no contemporaneous correlation between estimates of c_0 and c_1. Each of these estimates, however, is highly serially correlated, with first-order autocorrelations just below unity. This serial correlation falls only slightly as κ increases.

Note that a more aggressive policy response to inflation reduces the variation in the estimated intercept, c_0, but increases the magnitude of fluctuations in the coefficient on the lagged inflation rate, c_1. In the case of $\theta = 1$, the distribution of estimates of c_1 is nearly symmetrical around zero. For $\theta = 0.1$ and 0.6, the distribution of estimates of c_1 is skewed to the left, reflecting the accumulation of mass around unity, but the absence of much mass above 1.1.

Table 2: Forecasting Performance: Mean-squared Error

		$\phi = .75, \alpha = .25$			$\phi = .9, \alpha = .1$		
Forecast method	κ :	.025	.05	.10	.025	.05	.10
Perfect knowledge		1.03	1.03	1.03	1.01	1.01	1.01
$\theta = 0.1$							
LS (finite memory)		1.04	1.05	1.08	1.03	1.18	2.12
LS (infinite memory)		1.05	1.06	1.12	1.05	1.70	6.21
Long-lag Phillips curve		1.05	1.06	1.08	1.07	1.08	1.13
$\theta = 0.6$							
LS (finite memory)		1.04	1.04	1.05	1.01	1.01	1.04
LS (infinite memory)		1.06	1.09	1.14	1.10	1.19	1.43
Long-lag Phillips curve		1.05	1.06	1.10	1.06	1.12	1.29
$\theta = 1.0$							
LS (finite memory)		1.04	1.04	1.05	1.01	1.01	1.02
LS (infinite memory)		1.06	1.10	1.18	1.11	1.27	1.85
Long-lag Phillips curve		1.05	1.07	1.10	1.07	1.14	1.34

Finite-memory least squares forecasts perform very well in this model economy. As shown in Table 2, the mean-squared error of agents' one-step-ahead inflation forecasts is only slightly above the theoretical minimum given in the first line of the table (labeled "Perfect knowledge").[12] Only when both inflation displays very little intrinsic inertia and

[12]This is consistent with earlier findings regarding least squares estimation. Anderson and Taylor (1976),

the policymaker places very little weight on inflation stabilization does the performance of finite-memory least squares forecasts break down. Not surprisingly, given that we do not include any shocks to the structure of the economy, agents' forecasting performance deteriorates somewhat as κ increases. Nonetheless, finite-memory least squares estimates perform better than those with infinite memory (based on the full sample), and the difference in performance is more pronounced the greater the role of inflation expectations in determining inflation. In an economy where inflation is in part determined by the forecasts of other agents who use finite-memory least squares, it is better to follow suit rather than to use estimates that would have better forecast properties under perfect knowledge (Evans and Ramey 2001).

With imperfect knowledge, the private agents ability to forecast inflation depends on the monetary policy in place, with forecast errors on average smaller when policy responds more aggressively to inflation. This effect is more pronounced the greater the role of inflation expectations in determining inflation. The marginal benefit to tighter inflation control on agents' forecasting ability is greatest when the policymaker places relatively little weight on inflation stabilization. In this case, inflation is highly serially correlated, and the estimates of c_1 are frequently in the vicinity of unity. Evidently, the ability to forecast inflation deteriorates when inflation is nearly a random walk. As seen by comparing the cases of θ of 0.6 and 1.0, the marginal benefit of tight inflation control disappears once the first-order autocorrelation of inflation is well below one.

Finally, even though only one lag of inflation appears in the equations for inflation and inflation expectations, it is possible to improve on infinite-memory least squares forecasts by including additional lags of inflation in the estimated forecasting equation. This result is similar to that found in empirical studies of inflation, where relatively long lags of inflation help predict inflation (Staiger, Stock, and Watson 1997, Stock and Watson 1999, Brayton, Roberts, and Williams 1999). Evidently, in an economy where agents use adaptive learning, multi-period lags of inflation are a reasonable proxy for inflation expectations. This result may also help explain the finding that survey-based inflation expectations do not appear to

for example, emphasize that least squares forecasts can be accurate even when consistent estimates of individual parameter estimates are much harder to obtain.

be "rational" using standard tests (Roberts 1997, 1998). With adaptive learning, inflation forecast errors are correlated with data in the agents' information set; the standard test for forecast efficiency applies only to stable economic environments in which agents' estimates of the forecast model have converged to the true values.

5.2 Least Squares Learning and Inflation Persistence

The time variation in inflation expectations resulting from perpetual learning induces greater serial correlation in inflation. As shown in Table 3, the first-order unconditional autocorrelation of inflation increases with κ. The first column shows the autocorrelations for inflation under perfect knowledge ($\kappa = 0$); note that these figures are identical across the two specifications of ϕ and α. In the case of the "inflation dove" policymaker ($\theta = 0.1$), the existence of learning raises the first-order autocorrelation from 0.9 to very nearly unity. For the policymaker with moderate preferences ($\theta = 0.6$), increasing κ from 0 to 0.1 causes the autocorrelation of inflation to rise from 0.4 to 0.66 when $\phi = .75$, or to 0.93 when $\phi = .9$.

Table 3: Inflation Persistence: First-order Autocorrelation

		$\phi = .75, \alpha = .25$			$\phi = .9, \alpha = .1$		
θ	κ: 0	.025	.05	.10	.025	.05	.10
0.1	.90	.97	.98	.99	1.00	1.00	1.00
0.6	.40	.48	.55	.66	.62	.79	.93
1.0	.00	.03	.06	.12	.09	.18	.28

Thus, in a model with a relatively small amount of intrinsic inflation persistence, the autocorrelation of inflation can be very high, even with a monetary policy that places significant weight on inflation stabilization. Even for the "inflation hawk" policymaker whose policy under perfect knowledge results in no serial persistence in inflation, the perpetual learning generates a significant amount of positive serial correlation in inflation. As we discuss below, the rise in inflation persistence associated with perpetual learning in turn affects the optimal design of monetary policy.

5.3 The Economy Following Inflationary Shocks

Next, we consider the dynamic response of the model to a sequence of unanticipated shocks, similar in spirit to those that arose in the 1970s. The responses of inflation expectations and inflation do not depend on the "source" of the shocks, that is, on whether we assume the shocks are due to policy errors or to other disturbances. Note that under least squares learning, the model responses depend nonlinearly on the initial values of the states c and R. In the following, we report the average response from 1000 simulations, each of which starts from initial conditions drawn from the relevant steady-state distribution. The shock is 2 percentage points in period one and it declines in magnitude from periods two through eight. In period nine and beyond there is no shock. For these experiments we assume the baseline values for ϕ and α, and set $\kappa = 0.05$.

With perfect knowledge, the series of inflationary shocks causes a temporary rise in inflation and a decline in the output gap, as shown by the dashed lines in Figure 3. The speed at which inflation is brought back to target depends on the monetary policy response, with the more aggressive policy yielding a relatively sharp but short decline in output and a rapid return of inflation to target. With the inflation hawk or moderate policymaker, the peak increase in inflation is no more than 2-1/2 percentage points and inflation returns to its target within 10 periods. With the inflation dove policymaker, the modest policy response avoids the sharp decline in output, but inflation is allowed to rise to a level about 4-1/2 percentage points above target, and the return to target is more gradual, with inflation still remaining one percentage point above target after 20 periods.

Imperfect knowledge with learning amplifies and prolongs the response of inflation and output to the shocks, especially when the central bank places significant weight on output stabilization. The solid lines in the figure show the responses of inflation and output under imperfect knowledge for the three policy rules. The inflation hawk's aggressive response to inflation effectively keeps inflation from drifting away from target and the responses of inflation and output differ only modestly from those under perfect knowledge. In the case of balanced preferences, the magnitude of the peak responses of inflation and the output gap is a bit larger than under perfect knowledge, but the persistence of these gaps is markedly higher. The outcomes under the inflation dove, however, are dramatically different. The inflation dove attempts to finesse a gradual reduction in inflation without incurring a large decline in output but the timid response to rising inflation causes the perceived process for inflation to become uncoupled from the policymaker's objectives. Stagflation results, with the inflation rate stuck over 8 percentage points above target while output remains well below potential.

The striking differences in the responses to the shocks under imperfect knowledge are a product of he interaction between learning, the policy rule, and inflation expectations. The solid lines in Figure 4 show the responses of the public's estimates of the intercept and the slope parameter of the inflation forecasting equation under imperfect knowledge. Under the inflation hawk policymaker, inflation expectations are well anchored to the policy objective. The serially correlated inflationary shocks cause some increase in both estimates, but the implied increase in the inflation target peaks at only 0.3 percentage point (not shown in the figure). Even for the moderate policymaker who accommodates some of the inflationary shock for a time, the perceived inflation target rises by just one-half percentage point. In contrast, under the inflation dove policymaker, the estimated persistence of inflation, already very high owing to the policymaker's desire to minimize output fluctuations while responding to inflation shocks, rises steadily, approaching unity. With inflation temporarily perceived to be a near-random walk with positive drift, agents expect inflation to continue to rise. The policymaker's attempts to constrain inflation are too weak to counteract this adverse expectations process, and the public's perception of the inflation target rises by 5 percentage points. Despite the best of intents, the gradual disinflation prescription that would be optimal with perfect knowledge yields stagflation—the simultaneous occurrence of persistently high inflation and low output.

Interestingly, the inflation dove simulation appears to capture some key characteristics of the United States economy at the end of the 1970s, and it accords well with Chairman Volcker's assessment of the economic situation at the time:

> Moreover, inflationary expectations are now deeply embedded in public attitudes, as reflected in the practices and policies of individuals and economic institutions. After years of false starts in the effort against inflation, there is widespread skepticism about the prospects for success. Overcoming this legacy

> of doubt is a critical challenge that must be met in shaping–and in carrying out–all our policies.
>
> Changing both expectations and actual price performance will be difficult. But it is essential if our economic future is to be secure.
> (Statement before the Committee on the Budget, March 27, 1981)

In contrast to this dismal experience, the model simulations suggest that the rise in inflation—and the corresponding costs of disinflation—would have been much smaller if policy had responded more aggressively to the inflationary developments of the 1970s. Although this was apparently not recognized at the time, Chairman Volcker's analysis suggests that the stagflationary experience of the 1970s played a role in the subsequent recognition of the value of continued vigilance against inflation in anchoring inflation expectations.

6 Imperfect Knowledge and Monetary Policy

6.1 Naive Application of the Rational Expectations Policy

We now turn to the design of efficient monetary policy under imperfect knowledge. We start by considering the experiment in which the policymaker sets policy under the assumption that private agents have perfect knowledge when, in fact, they have only imperfect knowledge and base their expectations on the perpetual learning mechanism described above. That is, policy follows (5) with the response parameter, θ, computed using (9).

Figure 5 compares the variability pseudo-frontier corresponding to this equilibrium to the frontier from the perfect knowledge benchmark. The top panel shows the outcomes in terms of inflation and output gap variability with the baseline parameterization, $\phi = 0.75$. The bottom panel shows the results of the same experiment with the more forward-looking specification for inflation, $\phi = 0.9$. In both cases, the imperfect knowledge equilibrium shown is computed with $\kappa = 0.05$.

With imperfect knowledge, the perpetual learning mechanism introduces random errors in expectations formation, that is, deviations of expectations from the values that would correspond to the same realization of inflation and the same policy rule. These errors are costly for stabilization and are responsible for the deterioration of performance shown in Figure 5.

This deterioration in performance is especially pronounced for the policymaker who places relatively low weight on inflation stabilization. As seen in the simulations of the inflationary shocks reported above, for such policies the time variation in the estimated autocorrelation of inflation in the vicinity of unity associated with learning can be especially costly. Furthermore, the deterioration in performance relative to the case of perfect knowledge benchmark is larger the greater the role of expectations in determining inflation. With the higher value for ϕ, if a policymaker's preference for inflation stabilization is too low, the resulting outcomes under imperfect knowledge are strictly dominated by the outcomes corresponding to the naive policy equilibrium for higher values of ω.

6.2 Efficient Simple Rule

Next we examine imperfect knowledge equilibria when the policymaker is aware of the imperfection in expectations formation and adjusts policy accordingly. To allow for a straightforward comparison with the perfect knowledge benchmark, we concentrate on the efficient choice of the responsiveness of policy to inflation, θ^S, in the simple linear rule:

$$x_t = -\theta^S(\pi_t - \pi^*),$$

which has the same form as the optimal rule under the perfect knowledge benchmark.[13]

The efficient policy response with imperfect knowledge is to be more vigilant against inflation deviations from the policymaker's target relative to the optimal response under perfect knowledge. Figure 6 shows the efficient choices for θ under imperfect knowledge for the two model parameterizations; the optimal policy under perfect knowledge—which is the same for the two parameterizations considered—is shown again for comparison. The increase in the efficient value of θ is especially pronounced when the policymaker places relatively little weight on inflation stabilization, that is, when inflation would exhibit high serial correlation under perfect knowledge. Under imperfect knowledge, it is efficient for a policymaker to bias the response to inflation upward relative to that implied by perfect knowledge. This effect is especially pronounced with the more forward-looking inflation

[13]We note that this is only the restricted optimal rule within the family of rules that are optimal under rational expectations. With imperfect knowledge, the fully optimal policy would be a nonlinear function of all the states of the system, including the elements of c and R.

process. Indeed, in the parameterization with $\phi = .9$, it is never efficient to set θ below 0.6, the value that one would choose under balanced preferences ($\omega = 0.5$) under perfect knowledge.

Acknowledging imperfect knowledge can significantly improve stabilization performance relative to outcomes obtained when the policymaker naively adopts policies that are efficient under perfect knowledge. Figure 7 compares the loss to the policymaker with perfect and imperfect knowledge for different preferences ω. The top panel shows the outcomes for the baseline parameterization, $\phi = .75, \alpha = .25$; the bottom panel reports the outcomes for the alternative parameterization of inflation, $\phi = .9, \alpha = .1$. The payoff to reoptimizing θ is largest for policymakers who place a large weight on output stabilization, with the gain huge in the case of $\phi = .9$. In contrast, the benefits from reoptimization are trivial for policymakers who are primarily concerned with inflation stabilization regardless of ϕ.

The key finding that the public's imperfect knowledge on the part of the public raises the efficient policy response to inflation is not unique to the model considered here and carries over to models with alternative specifications. In particular, we find the same result when the equation for inflation is replaced with the "New Keynesian" variant studied by Gali and Gertler (2000) and others. Moreover, we find that qualitatively similar results obtain if agents include additional lags of inflation in their forecasting models.

6.3 Dissecting the Benefits of Vigilance

In order to gain insight into the interaction of imperfections in the formation of expectations and efficient policy, we consider a simple example where the parameters of the inflation forecast model vary according to an exogenous stochastic process.

From equation (6) recall that expectation formation is driven by the stochastic coefficient expectations function:

$$\pi^e_{t+1} = c_{0,t} + c_{1,t}\pi_t. \tag{14}$$

For the present purposes, let $c_{0,t}$ and $c_{1,t}$ vary relative to their perfect knowledge benchmark values; i.e., $c_{0,t} = c_0^P + v_{0,t}$ and $c_{1,t} = c_1^P + v_{1,t}$, where $v_{0,t}$ and $v_{1,t}$ are independent zero mean normal distributions with variances σ_0^2 and σ_1^2.

Substituting expectations into the Phillips curve and rearranging terms, results in the following reduced form characterization of the dynamics of inflation in terms of the control variable x:

$$\pi_{t+1} = (1 + \phi v_{1,t})\pi_t + \frac{\alpha}{1-\phi} x_t + \alpha u_{t+1} + e_{t+1} + \phi v_{0,t}. \tag{15}$$

In this case, the optimal policy with stochastic coefficients has the same linear structure as the optimal policy with fixed coefficients and perfect knowledge, and the optimal policy response is monotonically increasing in the variance σ_1^2.[14]

Although informative, the simple case examined above ignores the important effect of the serial correlation in v_0 and v_1 that obtains under imperfect knowledge. The efficient choice of θ cannot be written in closed form in the case of serially correlated processes for v_0 and v_1, but a set of stochastic simulations is informative. Consider the efficient choice of θ for our benchmark economy with balanced preferences, $\omega = 0.5$. Under perfect knowledge, the optimal choice of θ is approximately 0.6. Instead, simulations assuming an exogenous autoregressive process for either c_0 or c_1 with a variance and autocorrelation matching our economy with imperfect knowledge suggest an efficient choice of θ approximately equal to 0.7—regardless of whether the variation is due to c_0 or to c_1. For comparison, with the endogenous variation in the parameters in the economy with learning the efficient choice of θ is 0.75.

As noted earlier, for a fixed policy choice of policy responsiveness in the policy rule, θ, the uncertainty in the process of expectations formation with imperfect knowledge raises the persistence of the inflation process relative to the perfect knowledge case. This can be seen by comparing the solid and dashed lines in the two panels of Figure 8 which plot the persistence of inflation when policy follows the RE-optimal rule and agents have perfect

[14] See Turnovsky (1977) and Craine (1979) for early applications of the well-known optimal control results for this case. For our model, specifically, the optimal response can be written as:

$$\theta = \frac{\alpha(1-\phi)s}{(1-\phi)(1-\omega) + \alpha^2 s},$$

where s is the positive root of the quadratic equation:

$$0 = \omega(1-\omega)(1-\phi)^2 + (\omega\alpha^2 + (1-\omega)(1-\phi)^2\phi^2\sigma_1^2)s + (\phi^2\sigma_1^2 - 1)\alpha^2 s^2.$$

While the optimal policy response to inflation deviations from target, θ, is independent of σ_0^2, the variance of the $v_{0,t}$ differentiation reveals that it is increasing in σ_1^2, the variance of $v_{1,t}$. As $\sigma_1^2 \to 0$, of course, this solution collapses to the optimal policy with perfect knowledge.

and imperfect knowledge, respectively. This increase in inflation persistence complicates stabilization efforts as it raises, on average, the output costs associated with restoring price stability when inflation deviates from its target.

The key benefit of adopting greater vigilance against inflation deviations from the policymaker's target in the presence of imperfect knowledge comes from reducing this excess serial persistence of inflation. More aggressive policies reduce the persistence of inflation, thus facilitating its control. The resulting efficient choice of reduction in inflation persistence is reflected by the dash-dot lines in Figure 8.

7 Conclusion

In this paper, we examine the effects of a relatively modest deviation from rational expectations resulting from perpetual learning on the part of economic agents with imperfect knowledge. The presence of imperfections in the formation of expectations makes the monetary policy problem considerably more difficult than would appear under rational expectations. Using a simple linear model, we show that although inflation expectations are nearly efficient, imperfect knowledge raises the persistence of inflation and distorts the policymaker's tradeoff between inflation and output stabilization. As a result, policies that appear efficient under rational expectations can result in economic outcomes significantly worse than would be expected by analysis based on the assumption of perfect knowledge. The costs of failing to account for the presence of imperfect knowledge are particularly pronounced for policymakers who place relatively greater value on stabilizing output: A strategy emphasizing tight inflation control can yield superior economic performance, in terms of both inflation and output stability, than policies that appear efficient under rational expectations. More generally, policies emphasizing tight inflation control reduce the persistence of inflation and the incidence of large deviations of expectations from the policy objective, thereby mitigating the influence of imperfect knowledge on the economy. In addition, tighter control of inflation makes the economy less prone to costly stagflationary episodes. These results highlight the value of continued vigilance against inflation in anchoring inflation expectations and fostering macroeconomic stability.

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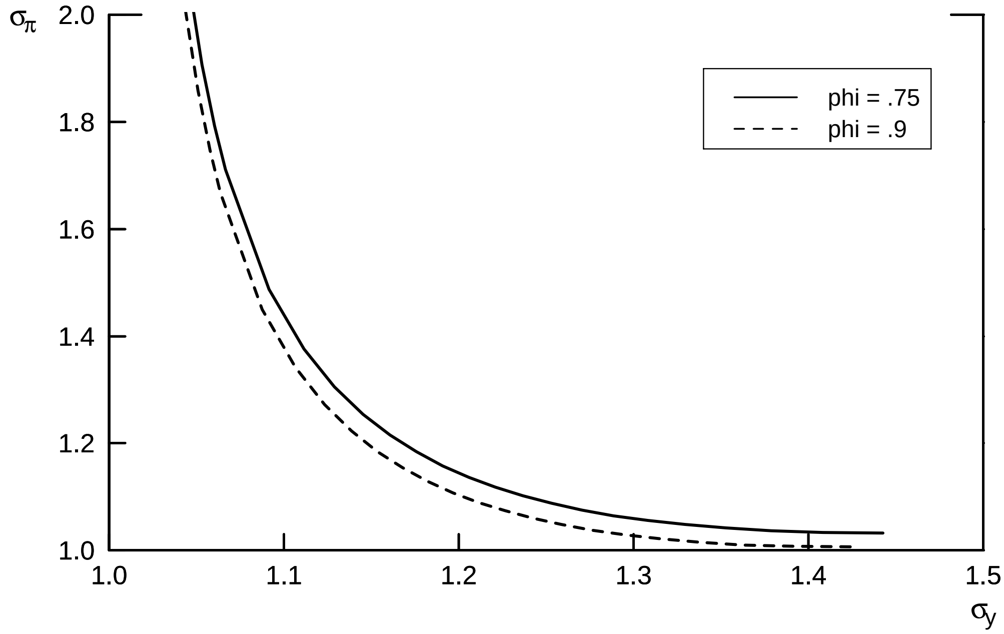
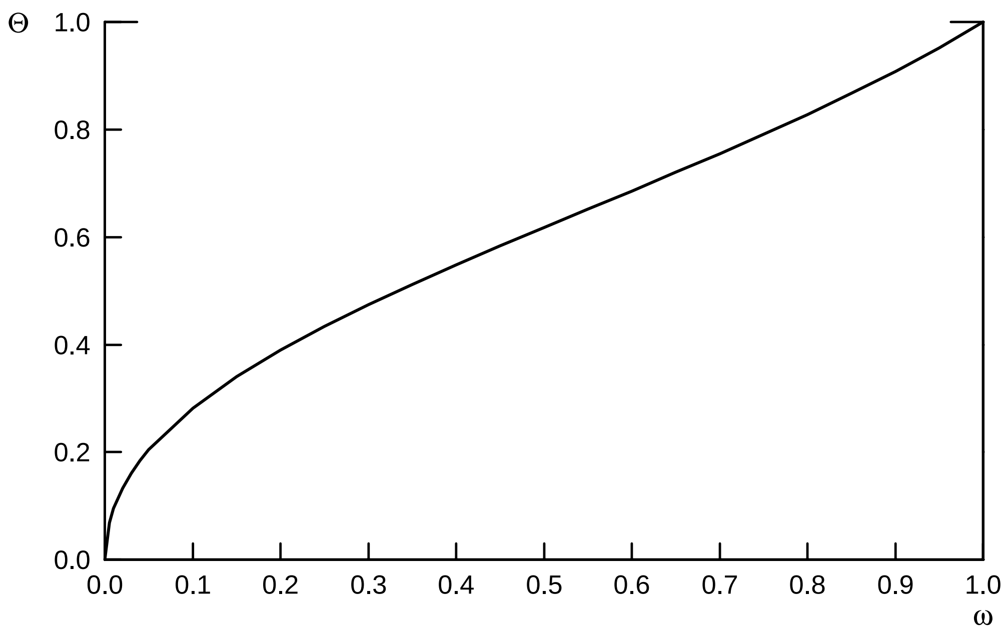
Figure 1

Efficient Policy Frontier with Perfect Knowledge



Optimal Policy Response to Inflation



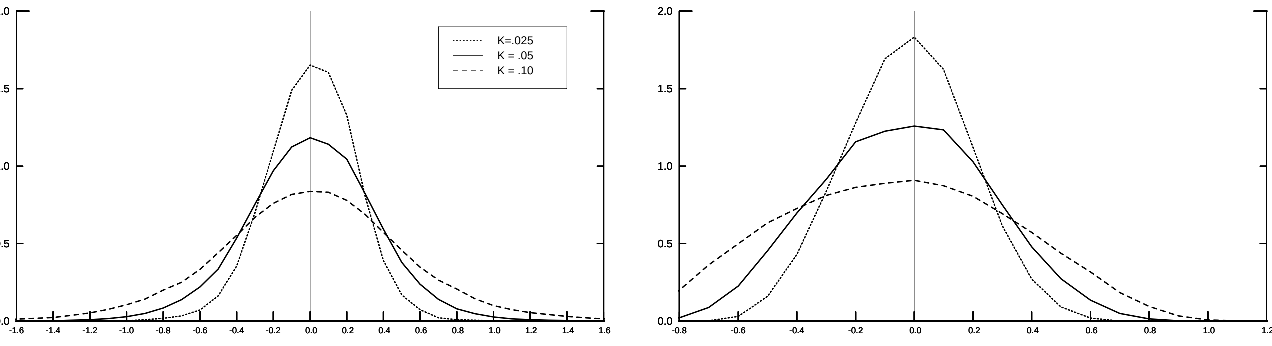
Notes: The top panel shows the efficient policy frontier corresponding to optimal policies for different values of the relative preference for inflation stabilization ω, for the two specified parameterizations of α and ϕ. The bottom panel shows the optimal response to inflation corresponding to the alternative weights ω, which are identical for the two parameterizations.

Figure 2

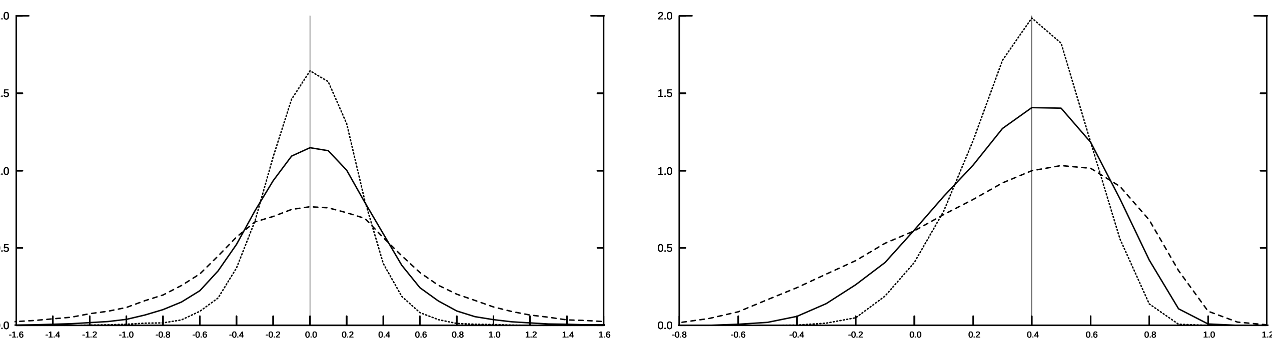
Estimated Expectations Function Parameters
($\phi = .75, \alpha = .25$)

Intercept | Slope

Inflation Hawk: $\theta = 1$



Balanced Preferences: $\theta = .6$



Inflation Dove: $\theta = .1$

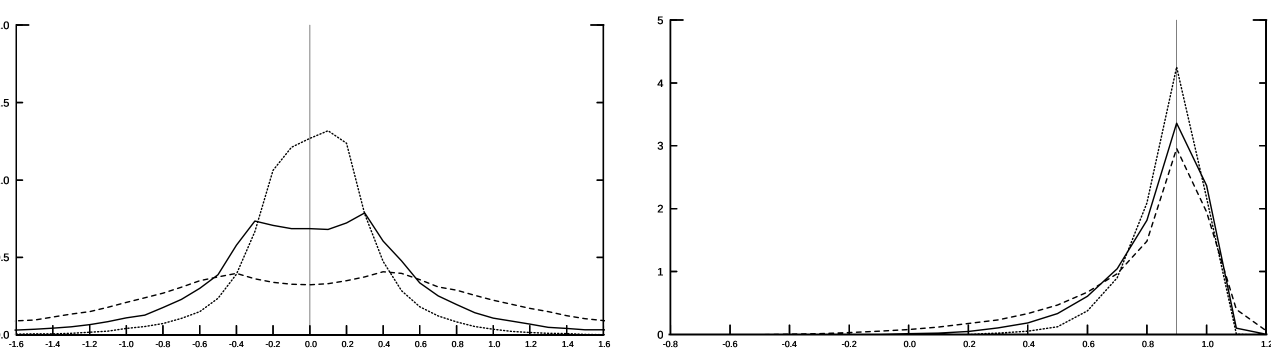
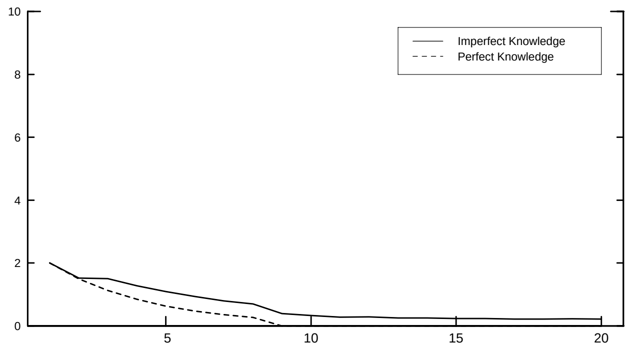


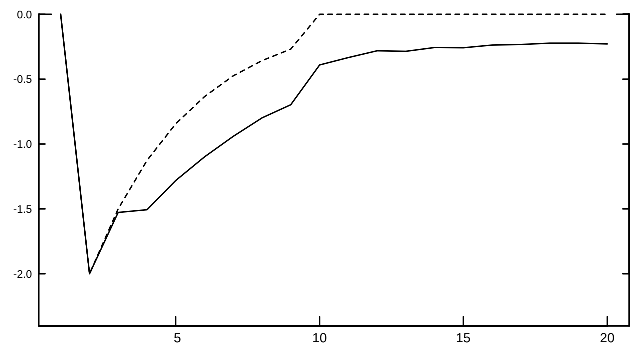
Figure 3

Evolution of Economy Following Inflation Shocks
$(\phi = .75, \alpha = .25)$

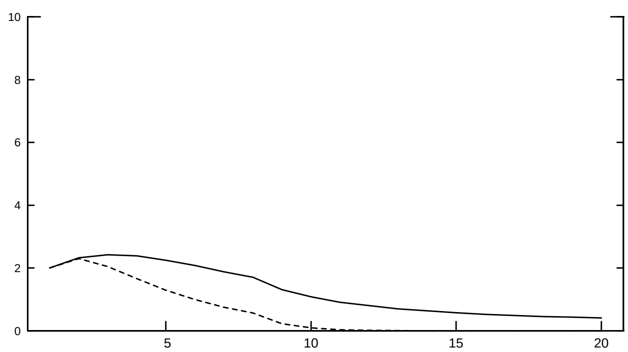
Inflation | Output

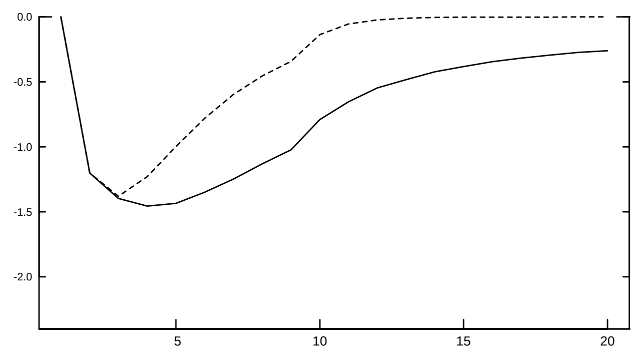
Inflation Hawk: $\theta = 1$



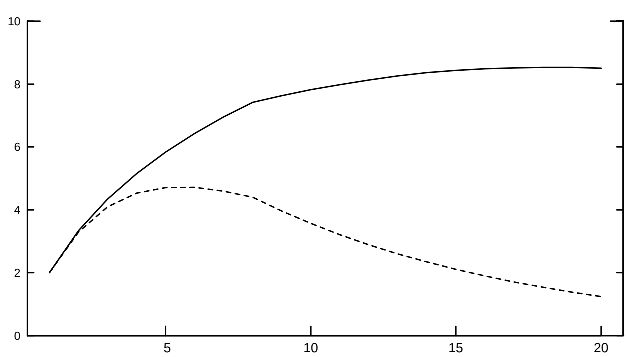


Balanced Preferences: $\theta = .6$





Inflation Dove: $\theta = .1$



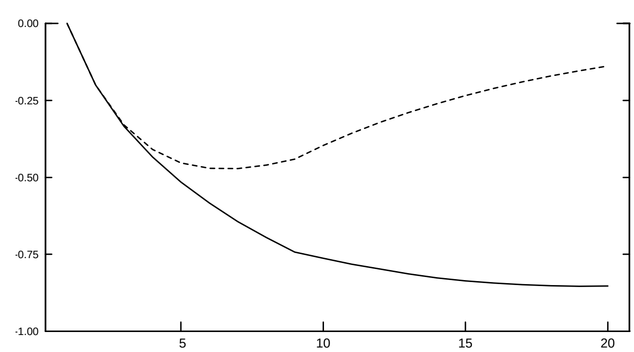
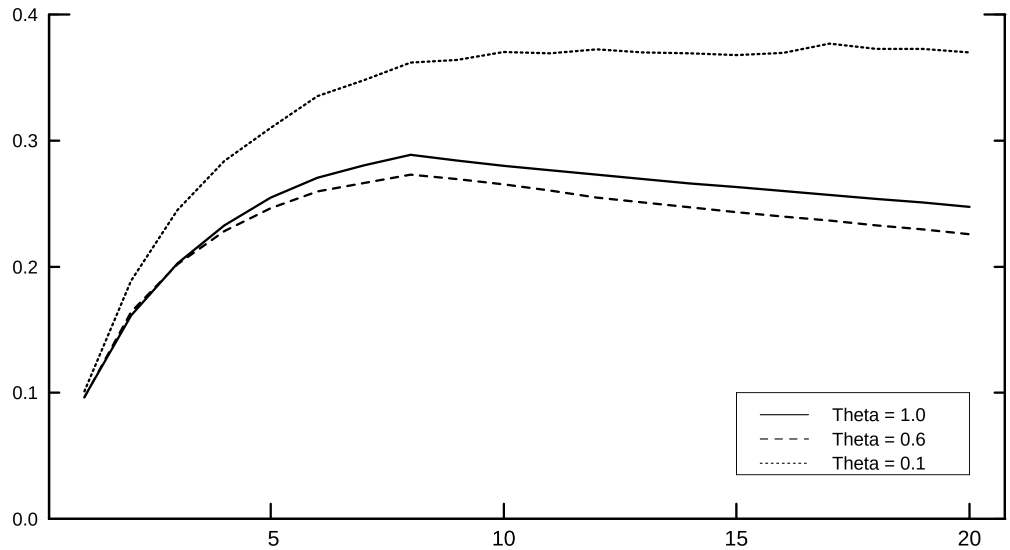


Figure 4

Estimated Intercept Following Inflation Shocks
$(\phi = .75, \alpha = .25)$



Estimated Slope Following Inflation Shocks

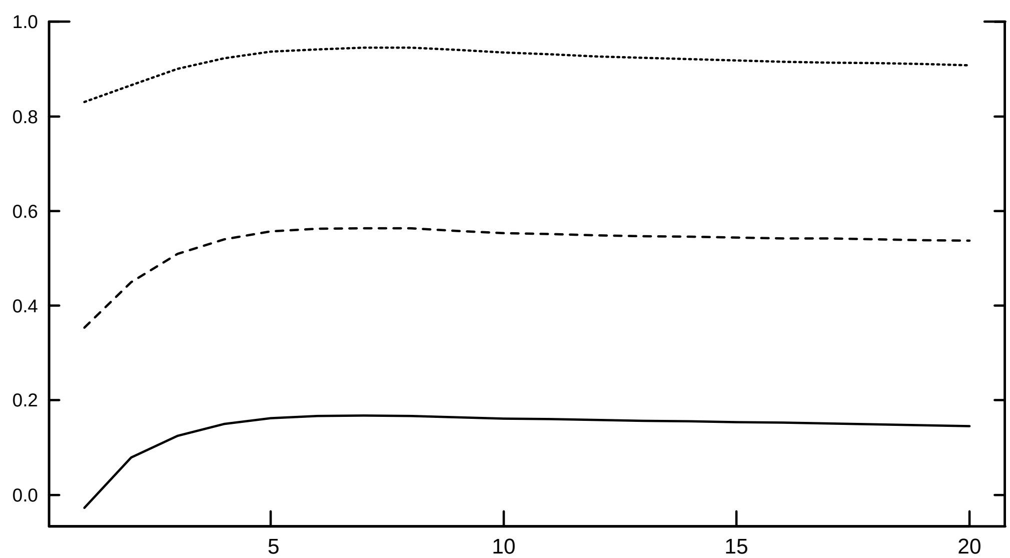
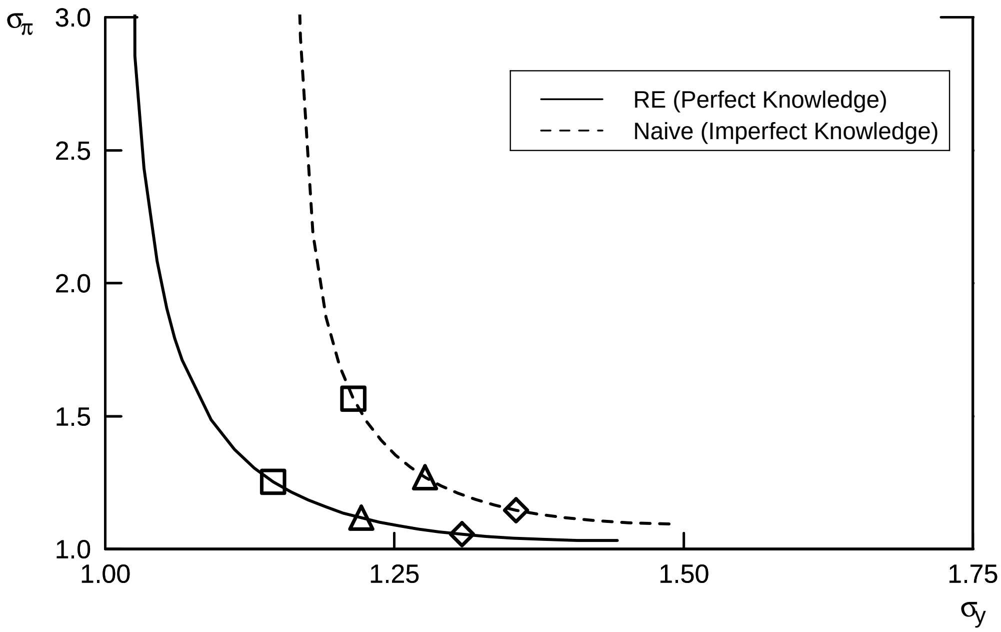
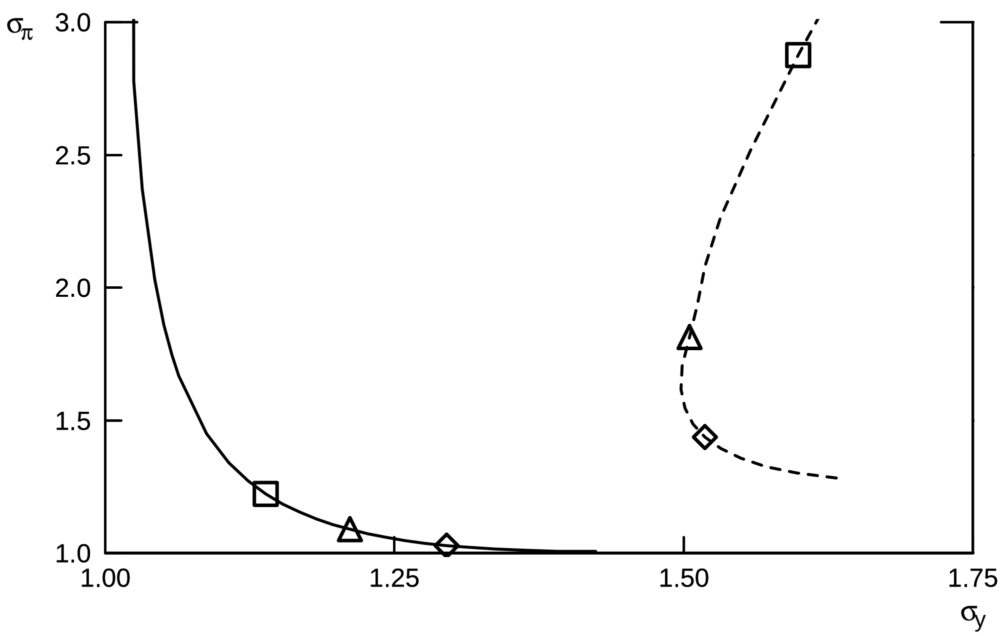


Figure 5

Outcomes with RE-policy, $(\phi = .75, \alpha = .25)$



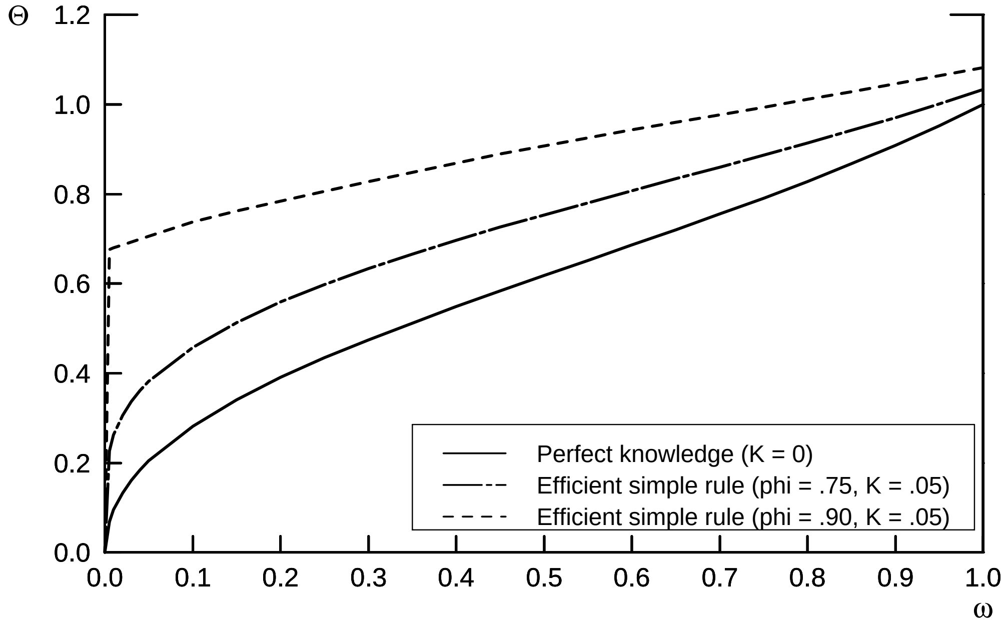
Outcomes with RE-policy $(\phi = .9, \alpha = .1)$



Notes: Each panel shows the efficient frontier with perfect knowledge and corresponding outcomes when the RE-optimal policies are adopted while, in fact, knowledge is imperfect. The square, triangle, and diamond correspond to preference weights $\omega = \{0.25, 0.5, 0.75\}$, respectively.

Figure 6

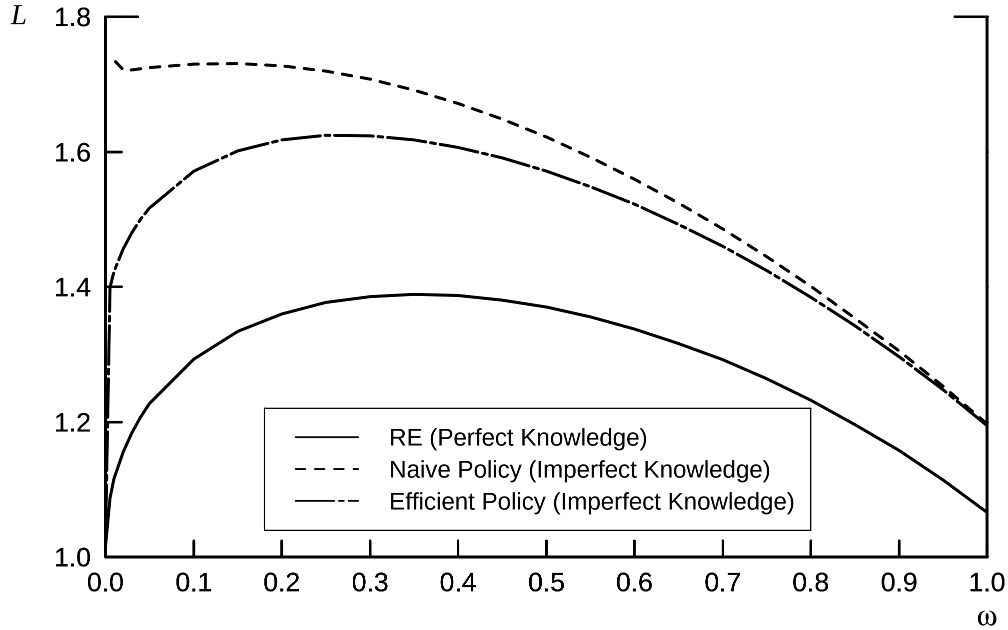
Efficient Policy Response to Inflation



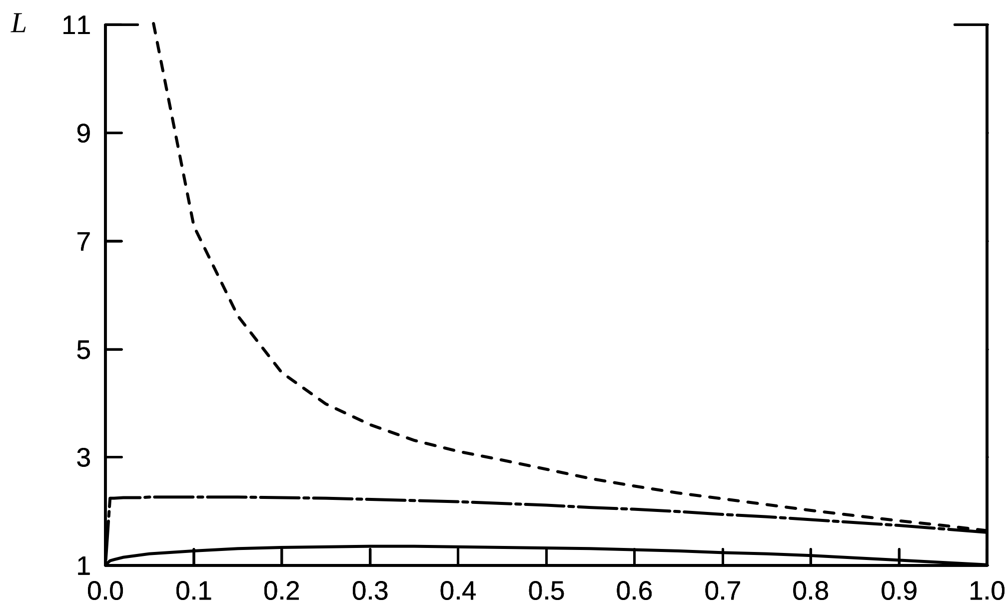
Notes: The solid line shows the optimal value of θ under perfect knowledge for alternative values of the relative preference for inflation stabilization ω. The dashed and dashed-dotted lines show the efficient one-parameter policy under imperfect knowledge for the two parameterizations of the model.

Figure 7

Policymaker Loss ($\phi = .75, \alpha = .25$)



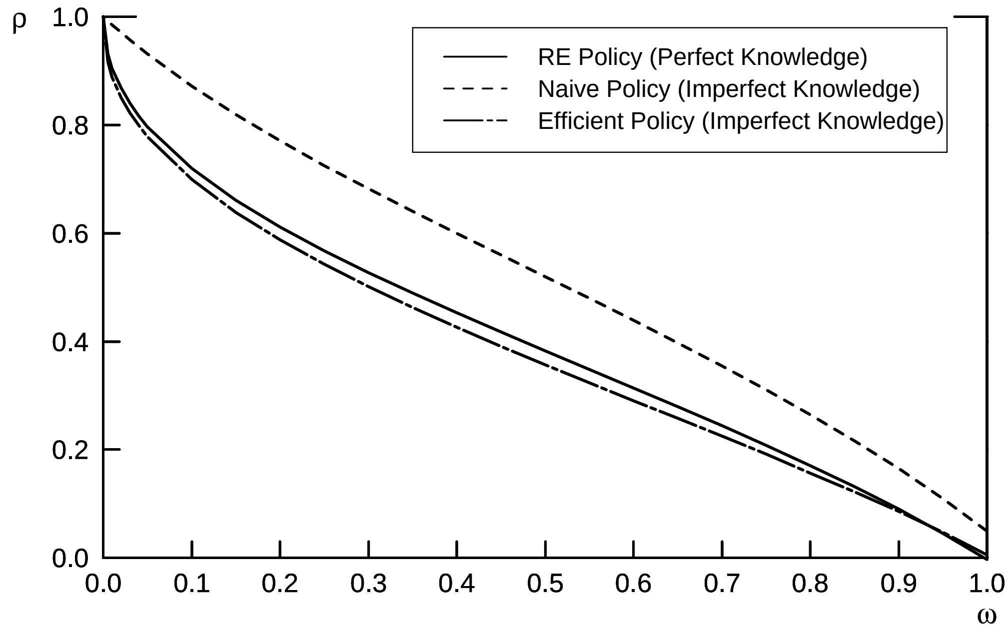
Policymaker Loss ($\phi = .9, \alpha = .1$)



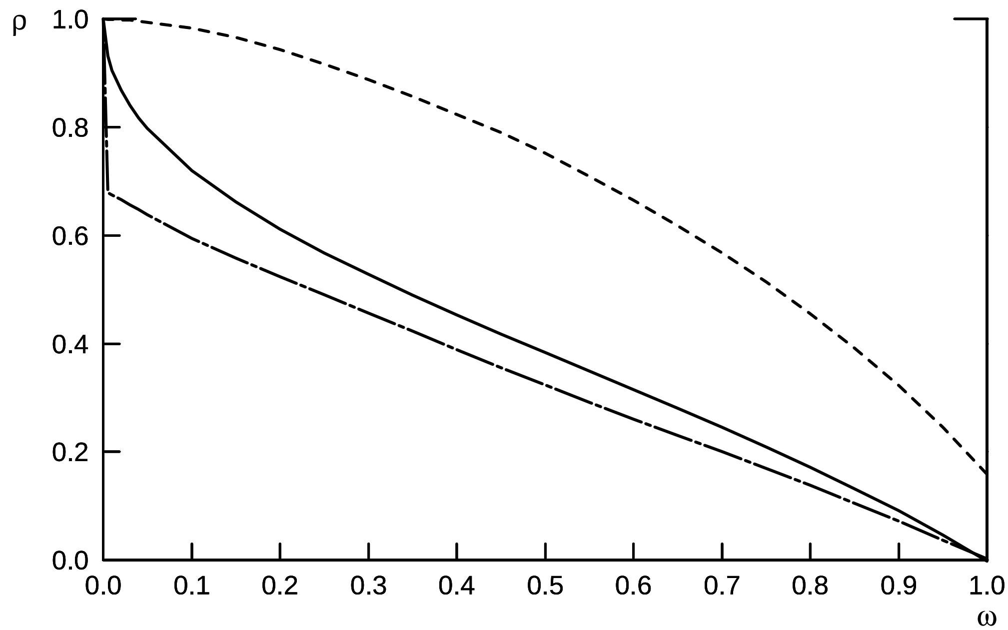
Notes: The two panels show the loss corresponding to alternative values of the relative preference for inflation stabilization ω for different assumptions regarding knowledge and different model parameterizations. The solid line shows the case of perfect knowledge. The dashed line shows the outcomes assuming the policymaker chooses θ assuming perfect knowledge when knowledge is in fact imperfect. The dashed-dotted line shows the outcomes for the efficient one-parameter policy under imperfect knowledge.

Figure 8

Inflation Persistence ($\phi = .75, \alpha = .25$)



Inflation Persistence ($\phi = .9, \alpha = .1$)



Notes: The figure shows the population first-order autocorrelation of inflation corresponding to policies based on alternative inflation stabilization weights ω. For each value of ω, the solid line shows the inflation persistence in the benchmark case of rational expectations with perfect knowledge. The dashed line shows the corresponding persistence when policy follows the RE-optimal solution but knowledge is imperfect. The dash-dot line shows the persistence associated with the efficient one-parameter rule with imperfect knowledge.