# An Empirical Test of a Contingent Claims Lease Valuation Model

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**Abstract**

Though there are several theoretical lease pricing models, they have not been systematically estimated or tested using real data. The existing empirical literature on leases typically tries to explain *one* period's lease payment, ignoring how fast those payments are scheduled to grow, or whether the lease contains option features. This paper bridges the gap between the two literatures by providing the first empirical investigation of an internally consistent lease pricing model. We develop a no-arbitrage based valuation model that allows us to calculate each lease's NPV taking into account both the contractual payment amounts and any embedded options. Using a proprietary data set of several hundred leases from suburban malls in 11 states, we then compare the NPV of each lease with various characteristics of the leases and underlying properties, to see what characteristics are important, and how the underlying valuation model can be improved.

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# 1 Introduction

Leases are one of the most important financing sources for US corporations.[1] Leases are in many ways very similar to corporate bonds. Both are contracts in which one party promises to make set payments to another over some period of time. In both cases, the period of the payments may be long or short, the payments may be fixed or adjust over time according to some rule, and the contracts may or may not contain option-like features. In the case of corporate bonds, the most common options are the options to default, to call the bond (i.e. to repurchase it at some fixed price), and to convert it to a fixed share of the firm's equity. In the case of lease contracts, there is again a default option, there may be cancellation options (effectively making the lease callable), and there are often also various equity-like features in which future payments are tied to various economic variables such as sales and CPI growth.

Despite the huge importance of these markets to the US economy, and the recent explosion of research, both theoretical and empirical, into the valuation of corporate debt [see, for example, Black and Scholes (1973), Merton (1974), Black and Cox (1976), Geske (1977), Jarrow and Turnbull (1995), Longstaff and Schwartz (1995), Leland (1994), Leland and Toft (1996), Duffie and Singleton (1999), Anderson and Sundaresan (2000), Collin-Dufresne and Goldstein (2001), and Huang and Huang (2002), among many others], leases have remained relatively understudied. Though there are several existing theoretical models of lease pricing [see, for example, Miller and Upton (1976), Brennan and Kraus (1982), McConnell and Schallheim (1983), Schallheim and McConnell (1985), Grenadier (1995b)], they have not been systematically estimated or empirically tested. On the other hand, while there is an empirical literature that tries to determine the factors that affect lease rates [see, for example, Glascock, Jahanian, and Sirmans (1990), Benjamin, Boyle, and Sirmans (1990)], this usually proceeds by regressing the current period's lease payment on various explanatory right hand side variables. The problem is that there is no way to interpret the regression coefficients

[1] Although the exact value of the outstanding stock of commercial and manufacturing real estate leases in the U.S. is not known, the economic value of these positions can be approximated through the value of non-residential, non-agricultural structures in the United States. In the third quarter of 2001, the Department of Economic Analysis, Bureau of Economic Analysis, estimates that the value of these assets was $4.8 trillion. Forty three percent of the outstanding stock of non-residential structures was leveraged and many of these mortgages have been securitized. The direct form of securitization is through the commercial mortgage backed securities market, with issuance in 2001 of about $60 billion (*Inside ABS and MBS*, ) and the indirect form is through off-balance sheet financing such as synthetic leasing, with an estimated issuance of about $50 billion through the end of 2001. (*The Real Estate Journal*, 2/25/02). A second impediment, to accurate measurement of the lease positions of firms arises from the lack of standardized reporting mechanisms for lease positions on corporate balance sheets. For example, Cisco Systems is the lessee for a $1.1 billion synthetic lease on its corporate real estate. This position appears on its balance sheet as restricted cash. In 2000, the dollar amount of rental expenses was more than 41% of the dollar amount of interest expenses for all active companies reported in COMPUSTAT. These rentals, of course, are likely to include more than real estate lease expenditures.

obtained, since the value of a lease depends not just on its current payment amount, but also on how fast those payments will grow over time, what options there are to renew or cancel the lease, and on the lease's maturity.

This paper bridges the gap between these two literatures. We develop a no-arbitrage based valuation model that allows us to calculate a single summary measure of each lease's NPV, taking into account both the contractual payment amounts and any embedded options. We then compare the NPV of each lease with various characteristics of the leases and underlying properties, to see what we can learn about what characteristics are important, and in turn what this tells us about how the underlying valuation model can be improved. Focusing on the model's NPV estimates, rather than just a single payment, allows us to express all the leases on a consistent basis, regardless of their maturity, payment structure, and renewal/cancellation options.

In our empirical analysis, we use a proprietary data set of seven hundred and eight leases from properties located in 11 states. The leases are a subset of a portfolio of leases assembled by the lead underwriter for a \$559,155,971 commercial mortgage pool consisting of 132 fixed-rate, first lien mortgage loans. For each of the 47 properties in our sample we have detailed information about the contract structure of the leases including the base rent levels, the treatment of expense pass-throughs, the renewal options, the reset periods and level of rent changes, the percentage options, and the maturities on the contracts. We also have detailed information about the local submarket, tenant mix, mortgage contract, and a recent appraisal for each of the properties. There is considerable variability in the leases which provides a unique opportunity to analyze the cross-sectional variation in lease contract structures across locations and properties.

After reviewing the relevant literature in Section 2, we present the model in Section 3, discussing our assumptions about service flows, building value, the term structure of lease rates and how these concepts can be combined into a lease valuation model that is empirically tractable. Section 4 discusses our data set development, and Section 5 presents our estimation strategy, a variant of that used in Quigg (1993). The estimation results are reported in Section 6. We find that there are important differences across metropolitan areas in the term structure of spot lease rates and that our valuation strategy leads to interpretable results concerning the net present value effects of various lease contract elements. Section 7 concludes the paper.

## 2 Lease Literature

The existing literature on real estate leases has focused on three primary modelling strategies. One important set of papers has applied methods of option pricing theory to value leases and embedded options such renewal or cancelation options, performance options, sale-leaseback contracts, and upward only adjusting leases [See Miller and Upton (1976), McConnell and Schallheim (1983), Schallheim and McConnell (1985), Grenadier (1995b), Grenadier (1996), and Ambrose, Hendershott, and Klosek (2002))]. A key observation from this work is that lease contracts giving tenants control over the same space for the same period of time must be priced equivalently in a competitive market. In the option pricing context, the value of leasing an asset over a given holding period is economically equivalent to a portfolio consisting of buying the building and simultaneously writing an, equivalent holding period, European call option on the building with an exercise price of zero. Grenadier exploits this idea and develops models based on fundamental economic uncertainty and competitive interaction among real estate producers to endogenously solve for equilibrium processes for rent, new construction, and asset prices. Thus, he explicitly equates underlying fundamentals to the shape of the spot rent curve in a given real estate market.[2]

A second set of papers builds upon economic theories of agency. Most of this literature is based on issues related to moral hazard and appropriate transfer of incentives between landlords and tenants. None of the papers in this literature considers general principles of optimal security design in a real estate leasing context. Instead, these studies focus on one or two specific dimensions of lease contracts that are found in the commercial real estate market [See Benjamin, Boyle, and Sirmans (1990, 1992), Hendershott and Ward (2001), Lee (1995), Miceli and Sirmans (1995), Pashigan and Gould (1998), Wheaton (2000)]. The most commonly studied contract is the percentage rent lease. In these contracts, the tenant's payments include a base rent per square foot and a contingent performance rent set as a percentage of gross sales exceeding a threshold sales level. The only paper that considers cancellation and downsizing options in commercial leases is Mooradian and Yang (2000).

Most of the agency models of lease contracting treat the landlord as the principal and the tenant as the agent. Benjamin, Boyle, and Sirmans (1990, 1992) argue that landlords should reward the positive externalities of anchor tenants by reducing their base rent. Furthermore, the landlord should purchase the long position in the percentage rent option by further

[2]In a recent extension of this work, Grenadier (2002) equates the equilibrium value of a lease to the equilibrium value of a portfolio contingent on the underlying building value. Building values are, in turn, determined by the optimal construction strategies of firms who face competition and uncertainty. In addition to the usual put/call parity conditions of option pricing solutions, Grenadier finds that strategic interactions in a competitive market structure significantly affect the value of the lease options. However, the underlying intuition from his prior work and its implications for empirical work remains essentially intact.

reducing the anchor's base rent. Miceli and Sirmans (1995) show that contracting structure depends on the relative risk aversion of the tenant and the landlord. They find that fixed contracts are the efficient contract if tenants are risk–neutral and landlords are risk–averse, whereas contracts with profit sharing mechanisms (percentage rents) are efficient if both are risk–averse. In the Lee (1995) model, the effects of both risk sharing and moral hazard lead to lease contracts with both base rent and a percentage of sales. Hendershott and Ward (2001) and Pashigan and Gould (1998) explore similar trade–offs between base rent and percentage rents and also show that higher base rents should correspond to lower percentage rents. Wheaton (2000) disagrees with these conclusions because they contradict recent empirical evidence that base rent and percentage rents are positively correlated in most U.S. retail leases. He proposes a model in which the landlord is the agent. The tenant contracts, using percentage rents, to align the tenant and landlord's incentives in future periods when the landlord expands, alters, or releases competing space.

A third modelling strategy considers efficient lease contracting mechanisms in models of optimal tenant allocation (Brueckner (1993) and Grenadier (1995a)). In these models again, the landlord is the principal and the tenant is the agent. The Brueckner (1993) model suggests that the optimal rent maximizing allocation of space across tenant types leads to discriminatory flat rents. His model generates percentage rents if each tenant must expend unobservable effort to expand sales. In this case, the landlord uses both price discrimination in flat rents and provides some incentive to encourage the spillover effects from tenant effort. The Brueckner (1993) model predicts that rents should be lower for tenants who produce positive externalities (the anchor tenants) but also suggests that rents will be lower when the sales of any one tenant is higher.[3]

There is relatively little empirical research on lease contracting largely because lease contracts are viewed as proprietary and strategic by most real estate companies. For this reason, lease contracting data are very difficult to obtain and there are no prior structural empirical models of lease valuation. All of the empirical studies are reduced form analyses of the relation between base rent at a given point in time and various lease contract and market features. Pashigan and Gould (1998) and Hendershott and Ward (2001) consider average *ex post* realizations of rent sorted by classes of tenant and lease contract so it is very difficult to isolate causal effects.[4]

[3]Although not a principal/agent model, Grenadier (1995a) also considers an optimal space allocation model in an option pricing framework. This model, allows the rent levels for different tenant types to vary stochastically over time, introduces interrelations among tenant demands, and accounts for landlord adjustment costs. Optimal contract mechanisms are not solved for and instead the focus is on the value of landlord flexibility in dynamically solving the optimal remixing exercise policy and in statically setting an initial optimal portfolio of tenant types.

[4]These studies apply aggregate data collected by the Urban Land Institute, *Dollars and Cents of Shopping*

All of the existing papers suffer from problems with the samples. Benjamin, Boyle, and Sirmans (1990, 1992) consider 103 leases, but they are all from one shopping center developer. They use percentage rent adjusted base rent as the dependent variable in their regressions. Wheaton (2000) considers 2,000 retail leases, again from a single mall operator on the East coast, and compares sample averages for base and percentage rents to national aggregate data provided by the Urban Land Institute. Mooradian and Yang (2000) report regression analyses of base rent on vacancy rate, leased square footage, and dummy variables for cancellation, downsizing, and sublet options for 311 commercial leases. Their sample of leases all have the same tenant. These empirical studies uniformly find strong relationships between *ex post* realizations of base rent and various contract structures. However, it is difficult to interpret the results given the problems with their limited samples of leases and their inability to control for the underlying structural characteristics of the lease.

A final limitation of the existing literature is the lack of information on the relation between lease contracting and access to debt financing. The credit analysis procedures of most commercial lending institutions focus on cash flow summaries of the aggregate lease portfolio of commercial properties. Although lenders collect contractual information for each lease, these contracts are rarely evaluated on a tenant–by–tenant basis. The software commonly used to generate the pro forma for the lease portfolio largely ignores the effects of uncertainty and option mechanisms in the leases.[5] For this reason the capital markets are typically not well informed about the lease contracting structure underlying commercial mortgages and little is known about how commercial mortgage performance is driven by the underlying characteristics of the leases.

To summarize, there are now a number of sophisticated theoretical models of the commercial lease contracting. To date, the underlying predictions of these models remain largely untested. There are two needed advances to do so. First, suitable data must be assembled. These data must include detailed information about the lease contracts over many tenants, cities, and debt financing structures. Second, testing these models requires a structural empirical specification. Only a structural specification can adequately account for the lease contracting characteristics in addition to the underlying economic fundamentals that tenants and landlords face when contracting. This data collection and modelling exercise are the

*Centers* summarizing the revenue performance of various classes of retail property controlling for tenant type. The problem with these data is that they are *ex post* realizations of specific contract structures and there is no way to control for specific contract structures. Thus for example, the analyst cannot distinguish a lucky realization on sales from higher percentage sharing rules or lower thresholds on the options.

[5]Argus is a popular lease evaluation software product that is widely used by commercial lenders. Rental cash flows are stabilized by ignoring the uncertainty arising from embedded options in the leases. Until very recently, Argus generated pro formas always assumed that leases reset "at market" on the lease maturity date further blunting the uncertainty of likely realized cash flows from a given lease portfolio.

subject of the next section.

# 3 The Model

We develop a simple contingent-claims model for valuing leases with a wide range of possible terms and embedded options, as a function of the instantaneous spot lease rate. This rate is taken as exogenously specified, in the spirit of Brennan and Kraus (1982), McConnell and Schallheim (1983) and Schallheim and McConnell (1985). We use no-arbitrage arguments to derive a partial differential equation for the lease value in terms of the underlying state variable.

Suppose the spot lease rate (equivalently, the instantaneous service flow) from a new building, $X_t$, follows a geometric Brownian motion process,

$$dX_t/X_t = \mu_x\, dt + \sigma_x\, dZ_t. \tag{1}$$

Write the value of an asset whose payoffs depend on $X_t$ (and possibly time) as $V(x,t)$, where $x$ is the current value of $X_t$. By Ito's Lemma, we can write

$$\frac{dV(x,t)}{V(x,t)} = m(x,t)\, dt + s(r,t)\, dZ, \tag{2}$$

where

$$m(x,t)\, V = V_t + \mu_x x V_x + \frac{1}{2}\sigma_x^2 x^2 V_{xx}, \tag{3}$$

$$s(x,t)\, V = \sigma_x x V_x. \tag{4}$$

This equation holds for any asset $V$. Since everything is driven by a single factor, the instantaneous returns on all assets depending on only $X_t$ and $t$ must be perfectly correlated. As a result, to prevent arbitrage the risk premium on any asset must be proportional to the standard deviation of its return.[6] Substituting for the asset's standard deviation from Equation (4), if the asset pays out dividends at rate $d$, we can thus write

$$m = r - \frac{d}{V} + q(x,t) x \sigma_x \frac{V_x}{V}, \tag{5}$$

where $q(x,t)$ is the price of risk. Substituting equation (5) into equation (3), assuming that

[6] Suppose this did not hold for two risky assets. We could then create a riskless portfolio of these two assets with a return strictly greater than $r$, leading to an arbitrage opportunity (see Ingersoll (1987)).

the price of risk is a constant,

$$q(x,t) \equiv \lambda/\sigma_x, \tag{6}$$

yields a partial differential equation that must be satisfied by any contingent claim,

$$\frac{1}{2}\sigma_x^2 x^2 V_{xx} + [\mu_x - \lambda]\, xV_x + V_t - rV + d = 0. \tag{7}$$

Note that in the case where $\lambda = \mu_x - r$, this equation reduces to the familiar Black and Scholes (1973) option pricing equation. Equation (7) can be used to price any contract whose payments depend on the instantaneous spot lease rate, including lease contracts with assorted embedded options, by suitable choice of boundary conditions. For example, consider the building itself, (assuming no depreciation). This is an infinitely lived asset which pays out $X_t$ at time $t$, so it must solve Equation (7), with $d$ replaced by $x$:

$$\frac{1}{2}\sigma_x^2 x^2 V_{xx} + [\mu_x - \lambda]\, xV_x + V_t - rV + x = 0. \tag{8}$$

Noting also that the value will not depend on $t$, it is simple to verify that the solution to this equation is

$$A(x) = \frac{x}{(r+\lambda) - \mu_x}. \tag{9}$$

This is just the standard perpetuity formula, where the expected return on the asset is $r+\lambda$, and the cash flows' (expected) growth rate is $\mu_x$.

## 3.1 Term Structure of Lease Rates

The partial differential equation above can be used to determine the term structure of fixed lease rates for a given instantaneous lease rate. For a given maturity, $T$, it must be the case that rolling over a sequence of instantaneous leases has the same present value as taking out a single lease with a constant periodic payment over the same time interval.

#### Rolling over short term leases

Consider first rolling over a sequence of instantaneous leases. The present value of the remaining payments is the value of an asset which pays out a dividend at rate $X_t$, and has value 0 at date $T$. It therefore solves the equation

$$\frac{1}{2}\sigma_x^2 x^2 V_{xx} + [\mu_x - \lambda]\, xV_x + V_t - rV + x = 0, \tag{10}$$

subject to the boundary condition that its value at maturity equals zero. It is simple to verify that the solution to this equation is

$$V(x) = \frac{x}{(r+\lambda) - \mu_x}\left[1 - e^{-\{(r+\lambda)-\mu_x\}T}\right], \tag{11}$$

where $T$ is the remaining time to maturity. This is just a version of the familiar annuity formula. Note that as $T$ grows large, this converges to the result in Equation (9).

**Fixed Lease Payments**

Now consider a long term lease with constant payment $d$ per period. The present value of the remaining payments is the value of an asset which pays out a dividend at (constant) rate $d$, and has value 0 at date $T$. It therefore solves the equation

$$\frac{1}{2}\sigma_x^2 x^2 V_{xx} + [\mu_x - \lambda]\, xV_x + V_t - rV + d = 0, \tag{12}$$

subject to the boundary condition that its value at maturity equals zero. It is simple to verify that the solution to this equation is

$$V(x) = \frac{d}{r}\left[1 - e^{-rT}\right], \tag{13}$$

where $T$ is the remaining time to maturity. Since the values of the two leases must be the same, we can immediately solve for the ratio of the long term to the instantaneous lease rate,

$$\frac{d}{x} = \left(\frac{r}{r+\lambda-\mu_x}\right)\left(\frac{1-e^{-\{(r+\lambda)-\mu_x\}T}}{1-e^{-rT}}\right) \tag{14}$$

# 4 Lease Data

Empirical analyses of real estate leases have been greatly hampered by the lack of comprehensive "lease-level" datasets that are needed to implement option valuation models such as that described above. Commercial mortgage underwriting tends to focus on "stabilized" lease cash flows using fixed assumptions about the exercise of the lease options and arbitrary probability "weights" on future uncertain cash flows. These underwriting evaluations tend to focus heavily on the anchor or credit tenants in the facility and on a common calendar date to compare the base rents between the subject property and a small set of "comparable" properties. Recent empirical papers on commercial real estate leases have implemented a similar strategy of focusing on a common period's base rent. Typically these papers estimate models in which the level of base rent is regressed on lease contract terms and other exogenous variables representing the level of economic activity in a region (See Glascock, Jahanian, and Sirmans (1990); Benjamin, Boyle, and Sirmans (1990); Benjamin, Boyle, and Sirmans (1992); and Mooradian and Yang (2000)). A serious limitation of this type of regression analysis is that a single period's payment does not summarize all of the necessary information about a lease contract. In particular, it does not control for the rate at which the lease payments will grow in the future, the effects of the compound options such as renewal and cancellation options, or uncertainty about the realized term of the lease. This means that we cannot really draw any conclusions from these regressions.

The primary advantage of our contingent claims valuation model is that, unlike studying a single period's lease payment, the model provides an appropriate metric, the net present value, that can be consistently compared across leases. Of course, since this measure takes into account contract features such as payment schedules and embedded options, a significant cost of our proposed strategy is the difficulty of assembling a suitable "lease-level" data set containing the information necessary to estimate the model. In particular, for each lease we need contractual information on the base rent, the number and exercise date of the renewal options, the date and level of each base rent reset period, and the maturities on the contracts. Assembling this information requires a detailed evaluation of highly idiosyncratic lease contracts for every tenant in each property.

Summary statistics for the leases in our data set are reported in Table 1. The mean contract maturity is 5.23 years, about 40% of the leases were written in 1995 and 1996, and the earliest leases were written in 1987. The average appraised value of the properties was about $5 million and the average monthly base rent in September 1997 was $11.02 per square foot. Approximately 27.5 percent of the leases had renewal options and the average number of renewals for those leases that had renewals was one. The maximum number of renewal options was six. The average leased square footage was 3,858 and the range of leased square footage varied from a maximum of 153,480 square feet and a minimum size of 120 square feet for professional office suites. Only .2% of our tenants had percentage rents and the breaks for these rents were the ratio of the initial stabilized rent to the percentage rate. Nearly all of the leases used expense passthroughs as the cost sharing mechanism so this lease feature is not explicitly considered. We had information on the level of tenant improvements for each lease, however, we do not know if it was the tenant or the landlord that paid for these up–front expenses or how they were amortized over time. The mean tenant improvement was $.22 a square foot, but most tenants did not receive tenant improvements.

We identified two types of lease renewal/cancellations in leases. We had leases in which the renewals were exercisable at "market rents". Fifteen percent of the leases had this type of renewal. The second type of renewal option, present in about eleven percent of the sample, was exercisable at an *ex ante* fixed rent. This form of renewal option was usually, though not exclusively, associated with the anchor tenants.

The usual tenant mix in the suburban malls is one large high volume merchandiser, or a movie theater, as the anchor tenant, and small tenants that often include a video store, doctors' offices, nail and beauty salons, book stores, and restaurants. The suburban office were all low-rise buildings and the tenants appear to be independent professionals, accounting and law firms, travel agencies, insurance companies, restaurants, and some retail. The light industrial properties were one story tilt-up construction and the tenants are independent professionals and light manufacturing firms such as software companies, tee shirt printers, custom bicycle producers, and back–office financial services uses. The average age of the properties was 16.18 years and the average occupancy rate was 96.6%, ranging from fully occupied buildings to a low of 82% occupancy. The physical condition of the properties was excellent and 18.9% of the older properties had been recently renovated.

The lease documents indicated whether the tenant was an anchor, or not, and about 6% of the tenants were anchors. The tenant data also included a market for "credit" tenants. We separately verified this credit evaluation by checking for a credit rating for the listed tenant. The credit rating scheme we developed is intended to approximate the classifications used by the Urban Land Institute in their publication *Dollars and Cents of Shopping Centers* but with added information about credit worthiness. We classify a tenant as a *National Credit Tenant* if we could find an above investment grade bond rating for the firm. We classified tenants as *Regional Tenants* if the tenant was part of a regional chain of say grocery stores or restaurants. We classified tenants as *Local Tenants* otherwise. About 6% of our sample was classified as *National Credit Tenants* and 87.7% were small local tenants.

A unique feature of our data is that we also have information on the mortgage loan for which the lease portfolio of the property is the collateral. We can thus associate each lease with a mortgage loan contract which will enable us to analyze the relationship between debt financing and lease structures. The mortgages in the pool were originated during the first two quarters of 1997. The mortgage were usually bullet loans where the amortization maturity differed from the due date for the return of principal. The amortization maturities averaged 294 months and the balloon maturities averaged 124 months. The average loan amounts were $3.3 million and the mortgages ranged from $440,000 to $15.4 million. We have a property appraisal for each property and from that we obtain the market capitalization rate used by the appraiser for the local market. The average cap rate was 9.8%. The debt service coverage ratio (*debt service/net operating income*) was also provided. The average debt coverage ratio in the sample was 1.576 and the minimum was a surprisingly low 1.21. The loan–to–value ratios average 62.6% and the maximum was 78.7%. These underwriting levels were about standard in the market in 1997, where the average debt service coverage ratios for commercial mortgage backed security products was 1.32 and loan–to–value ratios average 69.7% nationally.

We assume that the term structure of spot leases is common to each of the fourteen metropolitan areas in which the properties are located: Atlanta, Baltimore, Denver, Detroit, Fort Worth, Las Vegas, Los Angeles, Madison, Orange, Philadelphia, Phoenix, Seattle, San Bernardino, and San Jose. We use a metropolitan area specific rental price index from 1987 through 1997, the National Real Estate Index obtained from Ernst and Young, to compute the standard deviation of the percentage rent changes. Those computed values are reported in the sixth column of Table 2.

# 5 Calculating Lease NPV

We can value leases with payments that vary over time, or that contain embedded options, by solving Equation (7) subject to appropriate boundary conditions. In general this needs to be done numerically. We implement the model using a binomial discrete-time approximation to the continuous-time model [see Cox, Ross, and Rubinstein (1979)].

As in the discussion of the term structure of lease rates in Section 3.1, renters should be indifferent between taking out a long term lease, with prespecified payment amounts and embedded options (and subject to the optimal exercise policy for those options), and rolling over a series of short term leases subject to the same termination policy. Given the model's parameters, $\mu$, $\lambda$, $r$, and $\sigma$, we can calculate an "NPV" for each lease, calculated from the perspective of the tenant, and defined as

$$\begin{aligned}\text{NPV} &= \text{PV(payments on rolled over short term lease)} \\ &\quad - \text{PV(payments on long term lease)}. \end{aligned} \tag{15}$$

If the model captures every important feature of the lease contract, this NPV ought to be zero. To the extent that factors not captured by the model are important in determining lease contract terms, they will show up as an NPV different from zero. The important point is that, unlike a single period's contractual payment, this NPV measure can be compared across different contracts, allowing us to explore the determinants of lease contract terms without worrying about whether all we are seeing is something related to the pattern of contractual cash flows, rather than a true economic relation between lease value and our explanatory variables.

## 5.1 Estimating the Model's Parameters

To use the model to calculate lease NPVs, we need values for the parameters $\mu$, $\lambda$, $r$, and $\sigma$. $r$ is the riskless interest rate, for which we use the 10 year Treasury rate. $\sigma$, the volatility of lease rates, is estimated separately for each city and each property type using the metropolitan rent series described above. The remaining parameters, $\mu$ and $\lambda$ only affect lease value via the difference, $\mu - \lambda$, so we do not need to estimate $\mu$ and $\lambda$ separately. Instead, for each city and each property type we find the value of $\mu - \lambda$ that minimizes the sum of squared NPVs for all corresponding leases. The idea is to give the model the best possible chance to fit observed lease contracts, then to see what explains the residuals.

In addition to the model's parameters, the NPV of a particular lease depends on the current value of the underlying state variable, $x$. The true instantaneous spot lease rate is not really observable, but we do have a rent series for each location, and assume that the spot lease rate is some constant multiple of the number in the rent series.[7] Since we do not know the exact contract whose terms are reported in the lease data files, we do not know this multiple, but instead this multiple becomes yet another parameter that gets estimated in order to set the sum of squared NPVs as close to zero as possible.

Finally, some tenants are so-called "anchor tenants". These are usually large tenants whose presence in turn attracts both other tenants and customers for those tenants. Since they increase profit for other tenants, they also increase the amount of rent they are willing to pay, and in general are able to negotiate more favorable terms for themselves as a consequence. We assume this benefit is proportional to the current spot lease rate, and value anchor tenants' leases by taking the same state variable as for a non-anchor tenant, then scaling it by a parameter that reflects this additional benefit. This parameter is also estimated.

We perform the estimation separately on each of twelve metropolitan areas. Our estimation strategy is similar to that of Quigg (1993), who also used a valuation model and observed market outcomes to back out the implied parameters of the model. Our observables are the base rent levels, the timing and base rent levels on the renewal options, and the maturities on the leases.

[7] This assumption is not unreasonable given the homogeneity of the spot lease rate process which means that the values of many typical contracts will be some multiple of the current spot lease rate. This includes fixed streams of cash flows, where each cash flow is equal to some multiple of future spot lease rates is always proportional to the current spot lease rate. It also includes put and call options on the spot lease rate, as long as the exercise price is a multiple of the current spot lease rate.

## 6 Results

We use nonlinear least squares to jointly fit the parameters of our structural lease valuation model for each property class and metropolitan area. The fitted parameters include a parameter estimate for the property–class and metropolitan specific multiple of the unobservable instantaneous spot lease rate (reported in columns two and three of Table 2), the risk adjusted growth rate in rents, $\mu - \lambda$, (reported in columns four and five of Table 2), and a scaling parameter representing additional benefits ascribable to anchor tenants, due, say to the positive traffic externalities they generate (reported in columns seven and eight of Table 2). As previously discussed, the parameter for the spot rent volatility, $\sigma$, is calculated outside the model using rental data from each metropolitan area.

As shown, in columns two and three of Table 2, the fitted parameters for the metropolitan and property class–level multiple for the unobservable spot lease rent are positive and quite precisely estimated for most of the cities, except for the smaller sample size cities such as Baltimore, Detroit, Madison, Fort Worth, and San Jose. In contrast, the parameter estimates for the risk adjusted growth rates are highly variable and are quite imprecisely estimated for all of the subsamples as shown in columns four and five of Table 2. The final parameter, representing the anchor tenant's benefit multiple, appears in columns seven and eight when anchor tenants were present in the metropolitan–level property classes. These parameters are again quite precisely estimated for all but the smallest samples, and suggest that anchor tenants do indeed command significant discounts for positive externalities. Overall, our results indicate that tenants and landlords in these leasing submarkets behave in a manner that is consistent with optimal investment behavior (accepting zero valued NPV projects) and that lease values for the same space and time period within each submarket do appear to be priced equivalently as predicted in the option pricing literature.

To further consider the implications of our fitted lease valuation models by submarket, we follow the intuition of Grenadier (1995b) and plot the implied term structure of spot lease rates by property type and metropolitan area using the parameter estimates and equation 14. Figure 1 shows the implied spot lease rate for light industrial properties in four metropolitan areas: Baltimore, MD; Los Angeles and Orange, CA; and Seattle, WA. Figure 2 shows the implied term structure of spot lease rates for suburban office properties in three metropolitan areas: Phoenix, AZ and Los Angeles and Orange, CA. Finally, Figure 3 shows the results for retail properties in six metropolitan areas: Philadelphia, PA; Los Angeles and Orange, CA; Atlanta, GA; Phoenix, AZ; and Denver, CO. These curves represent the implied structure of forward rents as of September, 1997 in each case.

These plots highlight the variability in the shapes of the term structures across product types and metropolitan real estate markets. Los Angeles has a steeply downward sloping spot rent term structure in all three property types, whereas Orange has an upward sloping light industrial term structure and a flat term structure for retail and suburban office. The severe California recession in the late nineties appears to have been a more significant problem for the Los Angeles market than for the Orange market with an economic based focused on the port, its trucking hubs, and U.S. trade with Asia. Phoenix also has a strongly downward sloped term structure for both suburban and retail leases perhaps reflecting the overbuilding in the late nineties. In contrast, the Denver term structure is mildly upward sloping perhaps reflecting the perceived strength of the technology sector that was located there. These plots are again suggestive that our model is able to distinguish across metropolitan real estate markets and that differences in economic fundamentals can be inferred from the parameters of the lease contracting structures.

We further evaluate the reasonableness of our model by regressing the implied NPV values for each lease contract on a variety of lease contract and market indicators. Since in equilibrium, we would expect NPVs of zero for each lease contract, the estimated NPVs can be interpreted as residuals from the equilibrium NPV. The fixed effects regression reported in Table 3 should thus be interpreted as an exercise in residual diagnostics for our lease valuation model. If the model perfectly explained equilibrium lease contracting we would not expect to find statistically significant regressors. We estimate a fixed effects model where the fixed effects controls are for properties since many of the leases are physically located in the same properties.[8] Overall, as expected, the specification does not explain much of the variance in NPV residuals. Our diagnostic regression does, however, indicate several dimensions over which our valuation model is systematically missing important structure in the leases. In particular, it is not adequately controlling for the size effects of rented square footage on observed leased valuation. It appears that larger leases increase the benefits to tenants although at a decreasing rate, longer leases lead to higher residual NPV, and the number of renewal options significantly decreases residual NPV. In keeping with the literature on percentage leases, we also find that omitting controls for percentage rents in our lease valuation model significantly affects the NPV residuals. The negative and statistically significant parameter estimate for percentage rents, implies that tenants exposed to percentage rents also pay higher base rent as noted by Wheaton and in direct contradiction to many previous empirical and theoretical papers.

We compare our NPV residual diagnostics regression to a regression of *ex post* realized rent (the rent on September 9, 1997) on the same set of lease contract and tenant charac-

[8]We do not report the fixed effects parameters, although a $\chi^2$ test of joint insignificance was rejected at better than the 1% level

teristics. This reduced form specification is very common in the empirical lease literature, although in our case it is again a fixed effects model by property.[9] As shown in columns three and four of Table 3, a large percentage of the variance in realized rent is "explained" by contract elements and tenant characteristics. Surprisingly, the realized rent does not adequately account for the effects of market rate renewal options or percentage rents, and ascribes a statistically significant effect of the occupancy rate, the property type, and tenant improvements on realized rent levels. In contrast, these effects are not statistically significant (at the .05 level or better) factors of the true economic value of leases when the effects of the drift and volatility of spot lease rate, the embedded options, and the rent reset structure of leases are jointly accounted for in a full structural model of lease valuation.

Our final empirical evaluation of our lease valuation results, considers the relation between the economic value of leases, as measured both by contemporaneous fitted NPV and by contemporaneous realized rent rent, on mortgage underwriting criteria. In Table 4 we report the results of regressions of the originated mortgage loan–to–value ratio and debt service coverage ratio on lease value, tenant and property characteristics, and lease characteristics in a fixed effects model by loan.[10] As shown, the lease contracting structure and lease value (measure both by the contemporaneous estimated NPV and the realized rent) appear to have little effect on one of the major underwriting criteria in the commercial lending market. The only statistically significant effects on the level of the loan–to–value ratio are the property capitalization rate, recent renovations, and whether the property is retail. The debt service coverage ratio regression is report in column three and four of Table 4. This underwriting criterion is again unrelated to the economic value of the lease, as measured by contemporaneous NPV. The contemporaneous realization of rent, however, has a statistically significant and negative effect on the debt service coverage ratio, despite the fact that this measure provides little information about future rents. Surprisingly, creditworthy tenants do not affect the level of the ratio, however, being a local tenant, such as a beauty parlor, leads to statistically significant reductions in these ratios. It is also remarkable that space that requires higher tenant improvements leads to statistically significant reductions in the ratios and higher occupancy rates lead to statistically significant increases to it. These somewhat startling results suggest that there is very imperfect relationship between the true economic value of the lease portfolios of commercial properties and commercial mortgage underwriting criteria. Since the market values of commercial mortgages and commercial mortgage backed securities are wholly dependent on the performance of the underlying lease

[9] Again, we do not report the fixed effects parameters, although a $\chi^2$ test of joint insignificance was rejected at better than the 1% level

[10] Again, we do not report the fixed effects parameters, although a $\chi^2$ test of joint insignificance was rejected at better than the 1% level

portfolio, these results suggest that current underwriting practices may lead to systematic errors in evaluating the true economic risk of these positions.

# 7 Conclusions

This paper has developed a flexible contingent claims lease valuation model, and estimates the model using a new dataset giving detailed contract information on 708 retail leases from suburban malls in 11 different states. Unlike prior empirical studies, which regress a single period's lease payment on various explanatory right hand side variables, we analyze the behavior of the NPVs of different leases, estimated using the model. This has the advantage of allowing us to handle, in a consistent framework, leases which differ in their initial payments, how fast the payments grow over time, their maturity, and what options there are to renew or cancel the lease. We find that the NPVs are systematically related to several lease and property characteristics, which suggests directions in which the model could be improved in future. We also find that there appears to be a very imperfect relationship between the economic value of lease portfolios and the underwriting criteria used in commercial lending.

Table 1: Summary Statistics for Properties, Leases, and Mortgages

| | Variable Name | Mean | Standard Deviation | Minimum | Maximum |
|---|---|---|---|---|---|
| Tenant and Property Characteristics | Age of Property | 16.1800 | 8.298 | 1.000 | 41.000 |
| | Anchor Lease | 0.059 | 0.236 | 0.000 | 1.000 |
| | Appraised Property Value (000) | $5,056.574 | $3,657.392 | $850.000 | $21,400.000 |
| | National Credit Tenant | 0.064 | 0.244 | 0.000 | 1.000 |
| | Regional Tenant | 0.059 | 0.236 | 0.000 | 1.000 |
| | Local Tenant | 0.877 | 0.328 | 0.000 | 1.000 |
| | Leased Square Footage (000) | 3.858 | 10.636 | .120 | 153.480 |
| | Occupancy Rate | 0.966 | 0.043 | 0.820 | 1.00 |
| | Recently Renovated | 0.189 | 0.392 | 0.000 | 1.000 |
| | Suburban Office | 0.184 | 0.387 | 0.000 | 1.000 |
| | Retail Mall | 0.451 | 0.500 | 0.000 | 1.000 |
| | Light Industrial | 0.366 | 0.481 | 0.000 | 1.000 |
| Lease Characteristics | Maturity | 5.230 | 5.670 | 1.000 | 50.000 |
| | Number of Lease Renewal Options | 0.171 | 0.601 | 0.000 | 6.000 |
| | Lease Renewal at Market Rent | 0.147 | 0.354 | 0.000 | 1.000 |
| | Lease Renewal at Fixed Rent | 0.107 | 0.310 | 0.000 | 1.000 |
| | Percentage Rent Rate | 0.002 | 0.010 | 0 | 0.080 |
| | Tenant Improvements psf | 0.222 | 0.151 | 0 | 0.50 |
| | Realized Monthly Rent (9/97) | $11.020 | $5.129 | $2.97 | $30.00 |
| | Ten Year Treasury at Origination | 0.061 | 0.001 | 0.055 | 0.081 |
| Mortgage Characteristics | Capitalization Rate | 0.098 | 0.012 | 0.076 | 0.127 |
| | Debt Service Coverage Ratio | 1.576 | .310 | 1.200 | 2.910 |
| | Loan-to-Value Ratio | .626 | .107 | .300 | 0.787 |
| | Loan Principal (000) | $3,314.717 | $2,766.173 | $440.000 | $15,400.000 |
| | Balloon Payment Period | 124.51 | 20.59 | 84 | 192 |
| | Amortization Period | 294.81 | 37.132 | 180 | 360 |
| | Origination Rate | .0922 | 0.009 | 0.0825 | 0.113 |

Table 2: Coefficient Estimates for the Lease Valuation Model

| Lease Contract Type | City | N | Spot Lease Rate Multiple | Std. Error | Risk Adj. Growth Rate $\mu - \lambda$ | Std. Error | Spot Rent Volatility $\sigma$ | Anchor Tenant Multiple | Std. Error |
|---|---|---|---|---|---|---|---|---|---|
| Light Industrial | Baltimore | 20 | 0.778 | (0.568) | -0.008 | (0.176) | 0.043 | 0.657 | (0.121) |
| | Detroit | 23 | 2.018 | (1.671) | -0.432 | (0.610) | 0.031 | 1.675 | (0.637) |
| | Las Vegas | 18 | 1.481 | (0.331) | -0.019 | (0.105) | 0.028 | 1.014 | (0.629) |
| | Los Angeles | 89 | 1.132 | (0.689) | 0.023 | (0.288) | 0.069 | | |
| | Madison | 17 | 1.758 | (1.552) | 0.157 | (0.198) | 0.052 | 0.027 | (0.117) |
| | Orange | 64 | 1.620 | (0.384) | -0.106 | (0.125) | 0.064 | 0.891 | (0.341) |
| | Seattle | 22 | 0.934 | (0.337) | 0.022 | (0.094) | 0.032 | 1.012 | (0.034) |
| Suburban Office | Los Angeles | 38 | 0.504 | (0.119) | 0.098 | (0.028) | 0.056 | | |
| | Orange | 28 | 0.623 | (0.126) | -0.002 | (0.057) | 0.068 | | |
| | Phoenix | 64 | 0.821 | (0.167) | 0.060 | (0.071) | 0.065 | 1.004 | (0.501) |
| Retail Malls | Atlanta | 24 | 1.078 | (0.461) | -0.027 | (0.131) | 0.026 | | |
| | Fort Worth | 37 | 0.774 | (1.885) | 0.009 | (0.070) | 0.042 | 0.379 | (1.372) |
| | Denver | 26 | 1.521 | (0.627) | -0.061 | (0.099) | 0.041 | 0.627 | (0.201) |
| | Detroit | 25 | 1.204 | (0.592) | -0.005 | (0.130) | 0.034 | 0.938 | (0.341) |
| | Las Vegas | 31 | 0.831 | (0.217) | -0.006 | (0.007) | 0.040 | | |
| | Los Angeles | 11 | 1.406 | (0.705) | 0.004 | (0.112) | 0.054 | 0.487 | (0.369) |
| | Orange | 34 | 1.102 | (0.417) | -0.004 | (0.005) | 0.038 | 0.363 | (0.399) |
| | Philadelphia | 27 | 0.864 | (0.286) | 0.018 | (0.069) | 0.059 | 0.798 | (0.283) |
| | Phoenix | 74 | 1.171 | (0.408) | -0.008 | (0.007) | 0.034 | 0.974 | (0.447) |
| | San Bernardino | 31 | 1.259 | (0.393) | -0.002 | (0.005) | 0.039 | 0.675 | (0.344) |
| | San Jose | 9 | 1.311 | (1.657) | -0.002 | (0.336) | 0.055 | 0.821 | (0.769) |

Table 3: Determinants of Lease Residual NPV versus Realized Rental Rate (9/97): Property Fixed Effects Regression Model

| | NPV Residuals (psf) | | Realized Rent (9/97)(psf) | |
|---|---|---|---|---|
| | Coefficient | Std. Error | Coefficient | Std. Error |
| Itercept | -2951.579** | (1290.896) | 67.248*** | (10.204) |
| Age of Property | -110.087 | (141.851) | -1.365 | (1.121) |
| Anchor Lease | -33.386 | (60.083) | -.771 | (0.475) |
| National Credit Tenant | -33.247 | (63.319) | 0.238 | (0.500) |
| Local Tenant | -35.984 | (47.781) | 0.038 | (0.332) |
| Leased Square Footage | 17.357*** | (3.539) | -0.197*** | (0.027) |
| Sq. Leased Square Footage | -0.086*** | (0.025) | -0.001*** | (0.00002) |
| Lease Maturity | -13.733*** | (3.918) | 0.141*** | (0.031) |
| Number of Lease Renewal Options | 102.193** | (43.738) | -1.695*** | (0.345) |
| Lease Renewal at Market Rent | -79.964 | (47.781) | 0.324 | (0.331) |
| Lease Renewal at Fixed Rent | -120.592 | (76.89) | 1.951*** | (0.607) |
| Percentage Lease Rate | -3133.891** | (1466.384) | 19.048 | (11.591) |
| Property Occupancy Rate | 1158.785 | (2193.067) | -37.119** | (17.335) |
| Property Recently Renovated | -3362.936 | (3750.345) | 33.811 | (19.645) |
| Property Suburban Office | 3795.451 | (3635.792) | -23.958 | (28.739) |
| Property Retail Mall | 152.834 | (538.609) | 10.363** | (4.258) |
| Tenant Improvements | 5618.173 | (5814.837) | -92.236** | (45.964) |
| Ten Year Rate at Lease Origination | -362.053 | (7522.712) | 13.236 | (59.465) |
| Adjusted $R^2$ | .1265 | | .835 | |
| $F$ Statistic | 2.90*** | | 67.34*** | |
| $N$ | 709 | | 709 | |

*** .01 level of statistical significance

** .05 level of statistical significance

Table 4: The Determinants of the Mortgage Origination Loan-to-Value Ratio and Debt Service Coverage Ratio: Loan Fixed Effects Regression Model

| | Loan-to-Value Ratio | | Debt Service Coverage Ratio | |
|---|---|---|---|---|
| | Coefficient | Std. Error | Coefficient | Std. Error |
| Itercept | 0.603*** | ( 0.072) | 1.423*** | (0.282) |
| Age of Property | -0.00006 | (0.0004) | 0.002 | (0.002) |
| Anchor Lease | -0.008 | ( 0.010) | 0.018 | (0.039) |
| National Credit Tenant | -0.015 | (0.011) | 0.008 | (0.043) |
| Local Tenant | 0.007 | (0.008) | -0.075*** | (0.031) |
| Capitalization Rate | -0.512*** | (0.062) | 0.802*** | (0.243) |
| Leased Square Footage | 0.0002 | (0.0006) | -0.003 | (0.002) |
| Sq. Leased Square Footage | -0.000003 | (0.000004) | 0.00001 | (0.00001) |
| Realized Rent (9/97) | 0.0005 | (.0007) | -0.009*** | 0.003 |
| 9/97 Residual NPV | -0.000001 | (0.000006) | 0.00002 | (0.00002) |
| Remaining Lease Maturity | -0.0006 | (0.0007) | 0.003 | (0.003) |
| Number of Lease Renewal Options | 0.014 | (0.007) | -0.028 | (0.029) |
| Lease Renewal at Market Rent | 0.004 | (0.006) | -0.031 | (0.025) |
| Lease Renewal at Fixed Rent | -0.020 | (0.012) | 0.021 | (0.046) |
| Property Occupancy Rate | -0.056 | (0.078) | 0.654*** | (0.308) |
| Property Recently Renovated | -0.063*** | (0.009) | 0.199*** | (0.038) |
| Property Suburban Office | 0.030 | (0.019) | -0.174** | (0.074) |
| Property Retail Mall | 0.113*** | (0.012) | -0.236*** | (0.047) |
| Tenant Improvements (psf) | 0.013 | (0.029) | -0.450*** | (0.113) |
| Percentage Rent | 0.968 | (0.209) | -0.839 | (0.823) |
| Adjusted $R^2$ | .789 | | .604 | |
| $F$ Statistic | 66.45*** | | 27.66*** | |
| $N$ | 543 | | 543 | |

*** .01 level of statistical significance

** .05 level of statistical significance

Figure 1: Term Structure of Lease Rates: Industrial

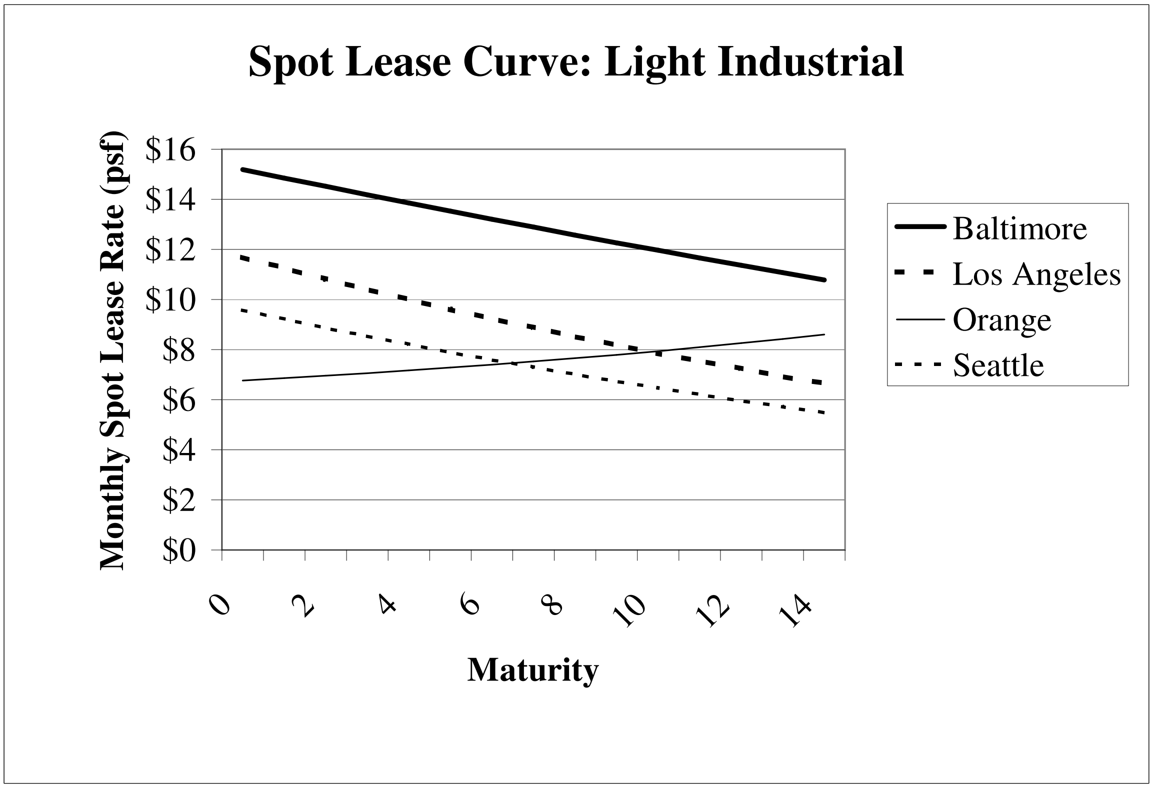


Figure 2: Term Structure of Lease Rates: Office

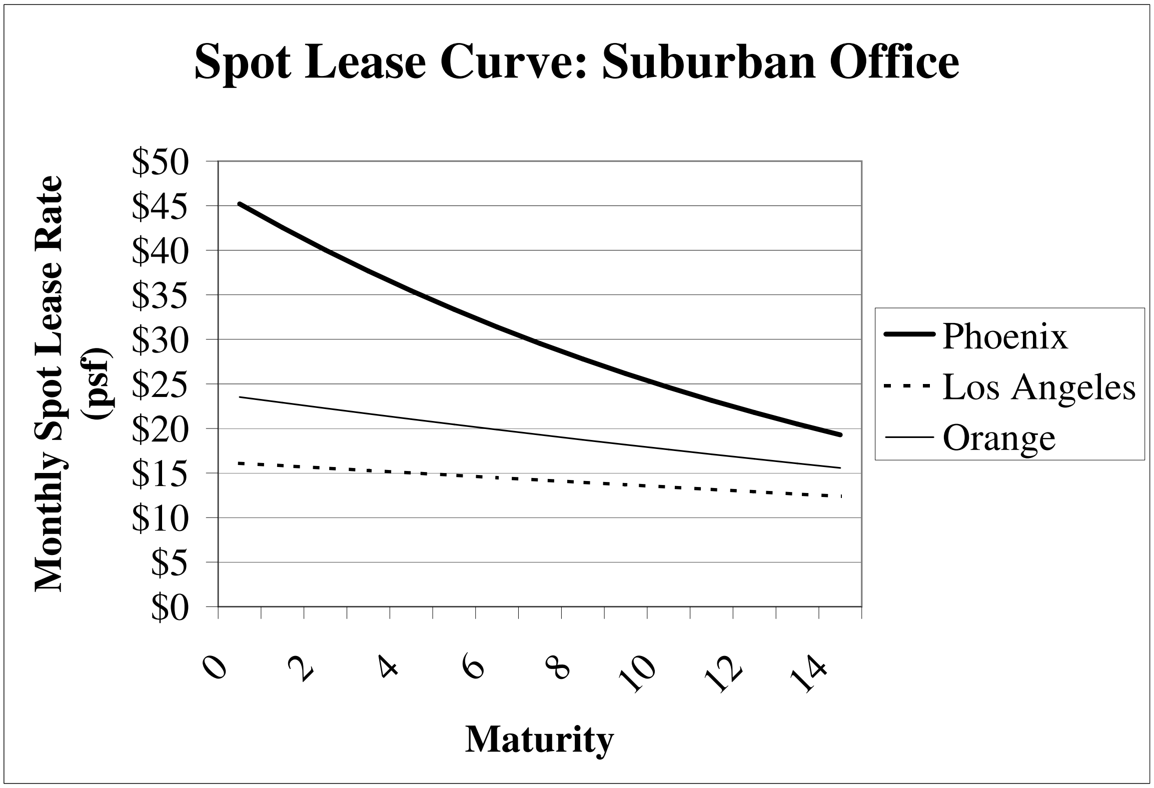
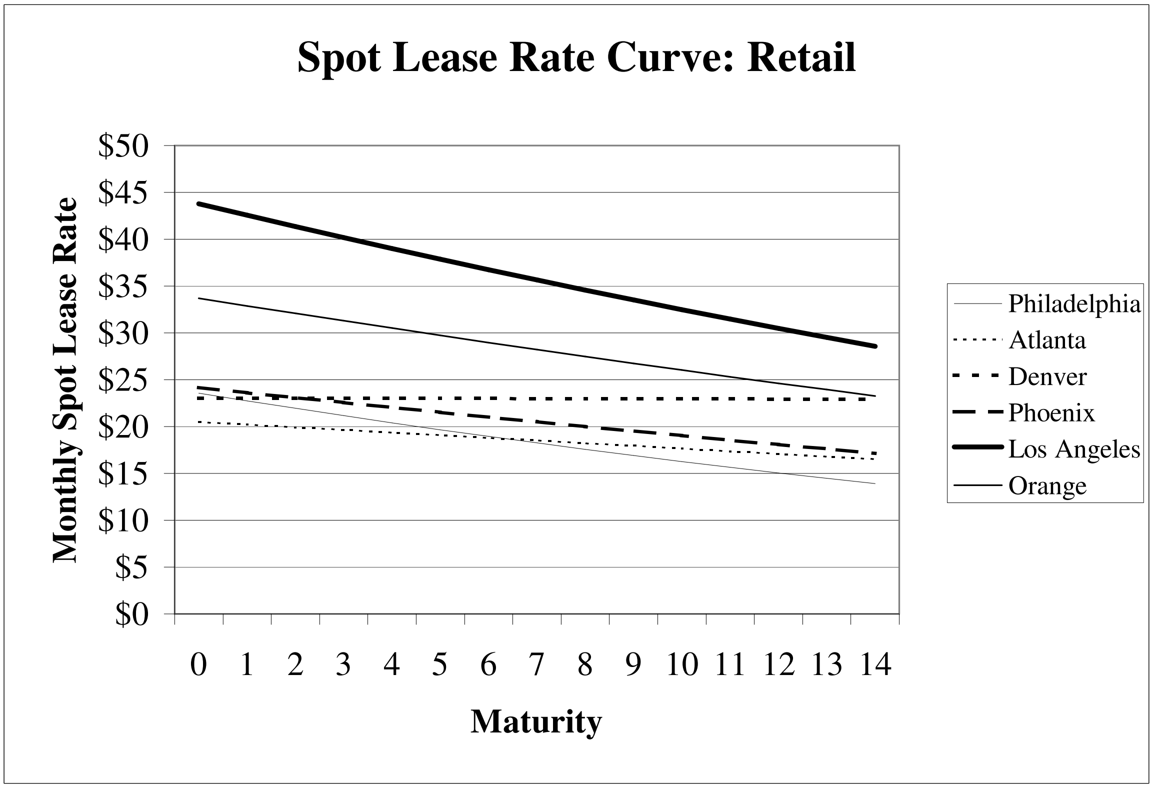


Figure 3: Term Structure of Lease Rates: Retail



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