Order Flows, Exchange Rates and Asset Prices

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Abstract

We develop an equilibrium model of international capital flows in which risk-averse home and foreign investors invest in both home and foreign equity markets with incomplete exchange rate risk trading. Exchange rates are determined by a price-elastic foreign exchange supply, so that private order flows move the exchange rate in line with recent evidence. The equilibrium prices for domestic and foreign equity and the exchange rate are obtained under asset market clearing. We derive the implications of incomplete exchange-rate risk trading for the correlation structure between asset returns and the exchange rate. The theoretical implications are confronted with the data.

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1 Introduction

The last 25 years have been characterized by a remarkable increase in international capital mobility. While gross cross-border transactions in bond and equity for the U.S. were equivalent at only 4 percent of GDP in 1975, this share increased to 100 percent in the early 1990s and has grown to 245 percent by 2000. Furthermore, a growing proportion of these capital flows consist of equity as opposed to bank loans or government bonds[1]. The increasing size and equity content of current capital flows suggests an integrated analysis of both domestic and foreign market returns, equity portfolio flows and exchange rates. To develop such a framework and confront it with the data is the principal contribution of this paper.

The increasing importance of international financial markets has not yet inspired a new financial market paradigm for exchange rate theory. This is even more surprising given the notoriously poor empirical performance of traditional models of exchange rate determination (Meese and Rogoff (1983a, 1983b)) and widespread pessimism about the explanatory scope of macro variables in general[2]. Empirical progress has however been made on the microstructure of exchange rate determination. Evans and Lyons (2001a,b), Lyons (2001), Rime (2001), Killeen *et al.* (2001), and Hau *et al.* (2001) show that net order flow from electronic brokerage systems have remarkably high correlation with contemporaneous exchange rate changes. These empirical results have been established both for inter-dealer order flow and more importantly for us, for customer-dealer order flow. Since net customer-dealer order flow in the foreign exchange market is at least partly determined by investors' desires for portfolio shifts, we recover an important linkage between exchange rate dynamics and investor behavior. Additional new (proprietary) data from global custodians which undertake a large proportion of global equity clearing have been analyzed by Froot *et al.* (2000). The data sources provide new insights into the empirical correlation structure between foreign returns, exchange rates and capital flows. Such work is so far entirely descriptive and does not propose any theoretical framework.

In this paper, we develop a model of international equity market interaction with market spill-overs solely based on financial structure, in particular forex market incompleteness. Exchange rates, portfolio equity flows and equity returns are jointly and endogenously determined. For simplicity we assume a world economy with, in each of two countries, a constant risk-free interest rate and an exogenous stochastic dividend process for the equity markets. Domestic and foreign investors are risk averse and maximize a simple myopic return trade-off between instantaneous expected return and its variance. They can invest in both the domestic and foreign equity and bond markets. Dividend payments and equity purchases are undertaken in local currency. The exchange rate is determined under market clearing in the forex market where private investor order flows from portfolio rebalancing and dividend repatriation meet a price-elastic forex supply of liquidity-providing financial institutions. Order flow drives the exchange rate in accordance with the empirical findings in the recent microstructure

[1] Based on data from the Bank for International Settlement (BIS), the London based research firm Cross Border Capital reports that during the period 1975-1984 bank loans accounted on average for 39.5% of total outflows from major industrialized countries (60.3% of inflows), while equities accounted for only 9.5% of outflows (6.1% of inflows). During the 1985-94 period these proportions are reversed. Bank loans accounted only for 8.3% of outflows (16.3% of inflows), while equities jumped to 35.9% of outflows (31.6% of inflows).

[2] Frankel and Rose (1995) summarize the situation by saying that "... no model based on such standard fundamentals like money supplies, real income, interest rates, inflation rates, and current account balances will ever succeed in explaining or predicting a high percentage of the variation in the exchange rate, at least at short- or medium-term frequencies." More recently Devereux and Engel (2002) argue that one cannot match some stylized facts regarding exchange rate volatility and disconnect without adding ingredients such as noise traders to the standard models.

literature.

Our most important structural assumption concerns incomplete forex risk trading. Foreign exchange derivative markets are among the most developed markets in the world. But how large a proportion of individual investors actually take advantage of this risk trading opportunity? Do U.S. investors holding European equity swap their currency risk with European investors holding U.S. equity with the symmetric exchange rate exposure? Direct evidence on representative portfolio holdings are difficult to obtain. Nevertheless, we can examine the forex hedging behavior of mutual funds and other institutional investors which manage a large proportion of U.S. foreign equity investments. Their lower transaction costs and higher financial sophistication make them better candidates for forex risk trading compared to individual investors. Do they swap forex risk with their foreign counterparts? Existing evidence strongly suggests that they do not. Levich *et al.* (1999) surveyed 298 U.S. institutional investors and found that more than 20 percent were not even permitted to hold derivative contracts in their investment portfolio. A further 25 percent of institutional investors were formally unconstrained, but did not trade in derivatives. The remaining 55 percent of institutional investors hedged only a minor proportion of their forex exposure. For the full sample, Levich et al. calculated that forex risk hedging concerned only 8 percent of the total foreign equity investment.[3] Portfolio managers cited monitoring problems, lack of knowledge and public and regulatory perceptions as most important reasons for the restricted forex derivative use. The development of the derivative market notwithstanding, only a minor proportion of the total macroeconomic forex return risk seems to be separately traded and eliminated. The typical foreign equity investor holds currency return and foreign equity return risk as a bundle.

1.1 Literature Review

How does our approach differ from the existing exchange rate literature? We highlight four aspects that distinguish our work from previous studies. The differences concern (1) the emphasis on equity flows relative to the new open macro literature, (2) the financial market incompleteness assumption relative to the real business cycle literature, (3) the endogeneity of the order flows relative to the forex microstructure literature and (4) the explicit modeling of the exchange rate relative to the finance literature.

First, macroeconomic theory has recently emphasized better microfoundations together with a more rigorous modelling of the dynamic current account. This approach is exemplified by Obstfeld and Rogoff (1995) and surveyed in Lane (2001). But international equity markets do not play an important role in this framework. While monopolistic profits occur in these models, they typically accrue entirely to domestic residents and therefore do not give rise to any equity flows. In the spirit of the traditional asset market approach to exchange rates (surveyed by Branson and Henderson (1985)), we resuscitate equity flows while introducing a more realistic equation for exchange rate determination than this literature.

Second, we emphasize financial market incompleteness. To the extent that real business cycle models do allow for international asset trade, they typically examine the resulting exchange rate dynamics in a complete market setting.[4] In this idealized setting all benefits from international

[3] We also consulted market experts in two large U.S. custodians. Independent sources at both State Street Bank and Citibank estimated the notional forex hedge at less than 10 percent. This confirms the survey evidence.

[4] Capital market incompleteness and the short sale constraint for foreign bonds set our model apart from the Lucas

exchange rate risk trading are realized. We argue that this assumption is at odds with current evidence on very low hedge ratios for foreign equity investment as discussed in the introduction. In our view the market succeeds in trading international equity fairly frictionlessly, but fails to realize the full benefit of trading the associated forex risk. We see this market incompleteness not related to the absence of the market (forex derivatives exist), but rather to transaction and agency costs of using them.

Third, our model is inspired by the new empirical literature on the microstructure of the forex market. Order flow is identified as an important determinant of exchange rate dynamics (Evans and Lyons (2001a,b); Rime (2001); Killeen *et al.* (2001); and Hau *et al.* (2001)). We interpret this literature as evidence for a price inelastic forex supply and explore its consequences for optimal international portfolio investment. The previous microstructure literature has always treated the forex order flows as exogenous model primitives and not itself as the object of equilibrium analysis as we do. Forex order flow moves exchange rates in our model in line with recent evidence, due to a portfolio balance effect. We do not incorporate any informational asymmetries in our set up, unlike Evans and Lyons (2001) or Evans (2002) for example.[5]

Fourth, our analysis relates to a recent literature on the international equity flows. Some of this work is entirely descriptive (Bekaert and Harvey, (2000); Bekaert *et al.* (2002) and Richards (2002)). Brennan and Cao (1997) and Griffin *et al.* (2002) also provide a theoretical analysis of foreign investment behavior. Paradoxically, both treated foreign investment like domestic investment by modelling only dollar returns. Instead of an exchange rate, home and foreign investors are separated by information asymmetries (Brennan and Cao) or by exogenous differences in return expectations (Griffin *et al.*). Unlike these models, our framework assumes that foreign and home investors are separated by an exchange rate and pursue investment objectives in the currency of their respective residence.

1.2 Model Implications

The model we develop has empirically testable implications regarding relative volatilities of equity prices and exchange rate, correlations between equity returns, exchange rates and portfolio flows. We highlight here the five main empirical predictions of our model:

1. Market incompleteness in combination with a price-elastic liquidity supply of currency balances generate exchange rates which are almost as volatile as equity returns.

2. Foreign market equity returns in local currency correlate negatively with currency returns of foreign investment.

3. Home equity market returns correlate positively with foreign market equity inflows.

4. Foreign equity market returns correlate negatively with foreign market equity inflows.

5. Foreign market equity inflows are associated with a foreign currency appreciation.

(1982) model and much of the stochastic dynamic general equilibrium literature. Our model is not however cast in a general equilibrium set up, since dividend processes and riskless rates are exogenous.

[5] The time horizon we are interested in (up to 1-4 quarters) also differentiates our study from this literature, which looks typically at very high frequency data.

The following section describes the model. In section 3, we solve the model for two special cases, namely the case of financial autarky and full integration in a complete market setting. These two polar cases provide two benchmarks for the general case of financial integration under market incompleteness explored in section 4. We describe the most important empirical implications and confront them with the data in section 5. Conclusions follow in section 6.

2 The Model

The world has two countries and a home and a foreign investor. Both investors are risk averse and can invest in risky home and foreign assets (equities) and in riskless domestic and foreign assets (bonds). Purchase of foreign assets by the home investor implies a forex order flow for foreign balances. Similarly all foreign dividend income of the home investors is repatriated and generates an opposite forex order flow. Foreign investors generate order flows in a symmetric way. Investors do not hold any monetary balances, and all their wealth is invested either in equity or riskless assets. The exchange rate is determined through a flow constraint, which balances the private order flow of the investors with a price-elastic supply of forex balances on the part of financial institutions. Forex order flow therefore generates an exchange rate response and allows the liquidity suppliers to make an intertemporal trading profit. For the sake of simplicity we do not provide any microfoundations for the liquidity supply.

The following four assumptions provide more detail on each element of the model structure. We start with the asset market structure:

Assumption 1: Asset Market Structure

A home (h) and a foreign (f) stock market provide exogenous stochastic dividend flows D_t^h and D_t^f in local currency. Home and foreign investors can invest in both stock markets. In addition, each investor can invest in a domestic bond providing a riskless constant return r or in a foreign bond providing a return r in the foreign currency. We impose a short sale constraint on the foreign riskless bond.

The domestic investor cannot acquire a short position in the foreign bond. Markets are therefore incomplete and risk trading opportunities are generally not fully exploited. In particular, foreign exchange exposure from foreign stock investment is not fully eliminated as it would be in a complete market setting. We believe that incomplete hedging of foreign investment is the more realistic description compared to a world of full international exchange-rate risk sharing[6].

Investors in our model are risk averse and their objective is to find an optimal trade-off between expected profit flow of their asset position and the instantaneous profit risk. Each investor measures profits in home currency.

Assumption 2: Investor Behavior

Home and foreign investors are risk averse and maximize (in local currency terms) a myopic mean-variance objective for the profit flow[7]. Domestic investors choose portfolio

[6] See our discussion in section 1.3.

[7] For the time horizons relevant for our exercise (1 day to 1-4 quarters), good prices can be considered to be sticky in local currency.

weights $K_t = (K_t^h, K_t^f)$ and foreign investors choose $K_t^* = (K_t^{f*}, K_t^{h*})$ so as to solve the optimization problem

$$
\begin{array}{lll}
\max_{K_t^h, K_t^f} & \mathcal{E}_t \int_{s=t}^{\infty} e^{-r(s-t)} & \left[d\Pi_s - \frac{1}{2}\rho d\Pi_s^2\right] ds \\
\max_{K_t^{f*}, K_t^{h*}} & \mathcal{E}_t \int_{s=t}^{\infty} e^{-r(s-t)} & \left[d\Pi_s^* - \frac{1}{2}\rho d\Pi_s^{*2}\right] ds
\end{array}
$$

where $\mathcal{E}_t$ denotes the rational expectation operator. Let $dR_t = (dR_t^h, dR_t^f)^T$ and $dR_t^* = (dR_t^{f*}, dR_t^{h*})^T$ denote the excess payoff (in local currency terms over the riskless asset) for domestic and foreign investors, respectively.

We define the excess stochastic profit flows for the domestic and foreign investor as

$$
\begin{aligned}
d\Pi_t &= K_t dR_t \\
d\Pi_t^* &= K_t^* dR_t^*,
\end{aligned}
$$

respectively. The investor risk aversion is given by ρ and the discount rate is given by r.

The myopic investor behavior simplifies the asset demand equations to linear functions in the fundamentals. Intertemporal hedging demand components are ignored under this utility specification. We highlight that both stock markets have to clear under the optimal asset demand. For simplicity we normalize the quantity of outstanding equity to one. This implies

$$
\begin{aligned}
K_t^h + K_t^{h*} &= 1 \\
K_t^f + K_t^{f*} &= 1
\end{aligned} \tag{1}
$$

as the two asset market clearing conditions.

An additional market clearing condition applies to the foreign exchange market with an exchange rate E_t. We can measure the equity related capital outflows dQ_t of the home country (in foreign currency terms) as

$$
dQ_t = E_t K_t^{h*} D_t^h dt - K_t^f D_t^f dt + dK_t^f P_t^f - E_t dK_t^{h*} P_t^h. \tag{2}
$$

The first two terms capture the outflow if all dividends are repatriated. But investors can also increase their holdings of foreign equity assets. The net capital outflow due to changes in the foreign holdings, dK_t^f and dK_t^{h*} are captured by the third and fourth term. Let us for example denote the euro area as the foreign and the U.S. as the home country. Then dQ_t represents the net capital outflow induced by equity trade out of the U.S. into the euro area in euro terms. An increase in E_t (denominated in euro per dollar) corresponds to a dollar appreciation against the euro. Any capital outflow in our model is identical to a net demand in foreign currency as all investment is assumed to occur in local currency. We can therefore also identify dQ_t with the equity trade induced order flow for foreign currency in the foreign exchange (forex) market. Furthermore, the above investor capital outflow (or forex order flow) can be linearly approximated by

$$
dQ_t^D = (E_t - \overline{E})\overline{K}\overline{D}dt + (K_t^{h*} - K_t^f)\overline{D}dt + (D_t^h - D_t^f)\overline{K}dt + (dK_t^f - dK_t^{h*})\overline{P}. \tag{3}
$$

where the upper bar variables denote the unconditional means of the stochastic variables. We normalize $\overline{E}$ to 1. The linearization makes the analysis tractable by leading to linear asset demands.

The net forex order flow of investors is absorbed by liquidity-supplying banks which can buffer foreign exchange imbalances[8]. The following assumption characterizes the liquidity supply.

Assumption 3: Price-Elastic Excess Supply of Foreign Exchange

The foreign exchange market clears for a price-elastic excess supply curve with elasticity parameter κ. For an equilibrium exchange rate E_t, the excess supply of foreign exchange is given by

$$Q_t^S = -\kappa(E_t - \overline{E}),$$

where $\overline{E}$ denotes the steady state exchange rate level.

An increase in E_t (euro depreciation) decreases the excess supply of euro balances. This exchange rate-elastic excess supply may be generated by the intertemporal arbitrage of risk averse forex market makers, who sell dollars for euros when the exchange rate is high and buy dollars when the exchange rate is low. While it is possible to endogenize the elasticity parameter κ, we prefer the simpler parametric representation.

Market clearing in the forex market then requires $Q_t^S = Q_t^D$ and the foreign exchange rate is subject to the constraint[9]

$$-\kappa dE_t = (E_t - \overline{E})\overline{KD}dt + (K_t^{h*} - K_t^f)\overline{D}dt + (D_t^h - D_t^f)\overline{K}dt + (dK_t^f - dK_t^{h*})\overline{P}. \tag{4}$$

The exchange rate level is therefore tied to the relative dividend flows, $D_t^h - D_t^f$, the relative level of foreign asset holdings $K_t^{h*} - K_t^f$, and their relative changes $dK_t^{h*} - dK_t^f$. Foreign asset holdings follow from the optimal foreign asset demand and depend on the stochastic characteristics of the exchange rate.

It is straightforward to express the excess payoffs (over riskless asset) on a unit of home equity over the interval dt as dR_t^h. To characterize the foreign excess payoff dR_t^f in home currency we use a linear approximation around the steady state exchange rate $\overline{E}$ and the steady state price $\overline{P}$. Formally, excess payoffs are given as

$$\begin{aligned} dR_t^h &= dP_t^h - rP_t^h dt + D_t^h dt \\ dR_t^f &\approx -dE_t\overline{P} + dP_t^f - dE_t dP_t^f - r\left[P_t^f - \overline{P}(E_t - 1)\right]dt + \left[D_t^f - \overline{D}(E_t - 1)\right]dt \end{aligned}$$

for the home and foreign asset, respectively. Excess returns follow as $dR_t^h/\overline{P}$ and $dR_t^f/\overline{P}$, respectively. The exchange rate component of the foreign payoff is given by $-\overline{P}dE_t$ and the exchange rate return by $-dE_t$.

Finally, we have to specify the stochastic structure of the state variables spelled out in the following assumption:

Assumption 4: Stochastic Structure

[8] A generalization of the model consists in allowing for additional current account imbalances given by $CA_t dt = \gamma\left(\overline{E} - E_t\right)dt$. The current account for the U.S. is in deficit when the dollar is strong and vice versa (γ is the exchange rate elasticity of the current account). This generalization is straightforward.

[9] We show in section 4 that there is no trade in the foreign riskless bond in equilibrium and therefore the forex order flow results only from equity trade and dividend repatriation.

The home and foreign dividends follow independent Ornstein-Uhlenbeck processes with identical variance and mean reversion given by

$$\begin{aligned} dD_t^h &= \alpha_D(\overline{D} - D_t^h)dt + \sigma_D dw_t^h \\ dD_t^f &= \alpha_D(\overline{D} - D_t^f)dt + \sigma_D dw_t^f \end{aligned}$$

The innovations dw_t^h and dw_t^f are independent.

The mean reversion of all stochastic processes simplify the analysis considerably. We can now introduce variables F_t^h and F_t^f which denote the expected present value of the future discounted dividend flow,

$$\begin{aligned} F_t^h &= \mathcal{E}_t \int_{s=t}^{\infty} D_t^h e^{-r(s-t)} ds = f_0 + f_D D_t^h \\ F_t^f &= \mathcal{E}_t \int_{s=t}^{\infty} D_t^f e^{-r(s-t)} ds = f_0 + f_D D_t^f, \end{aligned}$$

with constant terms defined as $f_D = 1/(\alpha_D + r)$ and $f_0 = (r^{-1} - f_D)\overline{D}$. The risk aversion of the investors and the endogenous exchange rate variability imply that the asset price will generally differ from this fundamental value.

3 Two Special Cases

To sharpen our intuition of the mechanism at work, we first discuss two instructive special cases of the model. First we cover the extreme case in which no foreign asset holdings are allowed. We refer to this case as financial autarchy. Investors do not share internationally their domestic equity risk. The opposite extreme assumption is to allow for complete markets with full risk sharing of both the equity risk and the exchange rate risk. This latter possibility corresponds to the case of one integrated domestic market with two freely tradeable assets. The exchange rate is then a redundant price. As empirically most relevant we consider a third case in which equity is freely traded but the exchange rate risk is not. We spare the latter for section 4.

Solving the model always requires three steps. First we postulate a linear solution for the asset prices and the exchange rate. Second, we derive the optimal asset demand under the conjectured solution. Third, we impose the market clearing conditions, show that the resulting price functions are indeed of the conjectured form and finally solve for the coefficients. To provide for a more coherent exposition, we summarize our results in various propositions. All derivations are relegated to the appendix of the paper.

3.1 Equilibrium without Risk Sharing (Financial Autarchy)

In the absence of foreign asset holdings, all domestic assets are owned by domestic investors, hence by assumption

$$\begin{pmatrix} K_t^h & K_t^f \\ K_t^{f*} & K_t^{h*} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}.$$

This allows the following simple characterization of the equilibrium:

Proposition 1. (Equilibrium under Financial Autarchy)

Assume a two-country world in which home investors hold the domestic asset and foreign investors the foreign asset. The home and foreign stock market prices are given by

$$\begin{aligned} P_t^h &= p_0 + p_F F_t^h \\ P_t^f &= p_0 + p_F F_t^f \end{aligned}$$

with $p_0 = -\rho\sigma_R^2/r$ and $p_F = 1$. The (instantaneous) return volatility follows as $\sigma_R^2 = \sigma_D^2/(\alpha_D + r)^2$.

Proof: See Appendix A.

The asset prices are proportional to the fundamental values given by the expected discounted cash flows. The coefficient $p_0 < 0$ denotes the negative risk premium. It is proportional to the investor risk aversion ρ and the instantaneous variance of the excess return processes.

3.2 Equilibrium with Perfect Risk Sharing

A second special case allows for perfect risk sharing across the two financial markets. This is the relevant case if the market are complete and financial risk trading opportunities are exploited. We summarize the equilibrium characteristics as follows:

Proposition 2. (Equilibrium under Complete Markets)

The home and foreign stock market prices and the exchange rate are given by

$$\begin{aligned} P_t^h &= p_0 + p_F F_t^h \\ P_t^f &= p_0 + p_F F_t^f \\ E_t &= 1 \end{aligned}$$

where we define $p_0 = -\rho\sigma_R^2/2r$, and $p_F = 1$. The (instantaneous) return volatility follows as $\sigma_R^2 = \sigma_D^2/(\alpha_D + r)^2$. The domestic and foreign portfolio positions of the two investors are equal and constant with

$$\begin{pmatrix} K_t^h & K_t^f \\ K_t^{f*} & K_t^{h*} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}.$$

Proof: An identical riskless rate in the home and foreign country under complete markets implies a constant exchange rate, $E_t = 1$. The complete solution is derived in Appendix B.

First, we note that the exchange rate is constant. In a world of perfect risk sharing, the two country model is not different from one domestic economy with two asset markets. The asset prices are again proportional to their fundamental values, F_t^h and F_t^f, respectively. The risk sharing across the two investor groups implies that the asset price risk discount $p_0 < 0$ is only half as large as in the autarchy case for the same return volatility σ_R^2. This implies lower average asset returns under market integration. Evidence that financial integration indeed reduces market stock returns is provided by Bekaert and Harvey (2000), Henry (2000) and Stulz (1999) among others. These authors show reduced capital costs or excess returns on equity for emerging countries following their capital market liberalization.

We further highlight that complete forex risk trading implies no particular correlation structure between exchange rates and equity returns. The exchange rate is a redundant price and constant. This implication is of course at odds with the high exchange rate volatility observed in practise. But it provides a useful benchmark for the following section which explores the case of equity market integration under incomplete exchange rate risk trading.

4 Foreign Investment under Incomplete Markets

We now treat the case in which a foreign exchange market allows investment in the foreign equity but exchange rate risk trading is incomplete. If the exchange rate move stochastically, home investors with foreign equity holdings incur an additional exchange rate risk in addition to the risk of the stochastic dividend flow. Foreign investors hold the opposite risk due to ownership stakes in foreign equity. If this reciprocal exchange rate risk were tradeable, it could be perfectly eliminated as assumed in the perfect market case treated in section 3.2. But now we assume that such forex risk trading does not occur.

The non-tradeability of the forex risk not only excludes derivative contracts, but also requires that investors cannot short sell the foreign riskless asset. Short selling of foreign riskless assets effectively amounts to a separate trading of the exchange rate risk. Theoretically, investors should seek a short position in the foreign riskless asset equivalent to their foreign equity stake in order to sell the exchange rate risk to the foreign population, for which this risk represents a desirable hedge to their own foreign equity investment.

From this argument it is straightforward to show that the short-selling constraint is generally binding. If negative positions are desirable, but not available, then the best investors can do is to hold a zero position in foreign riskless assets. Proposition 3 formally expresses this insight:

Proposition 3 (Zero Foreign Riskless Asset Holdings):

Under incomplete markets, the inability to short sell foreign riskless assets is a binding constraint. In steady state investors do not hold foreign riskless assets.

Proof: See Appendix C.

Proposition 3 is useful because it considerably simplifies the equilibrium analysis. To calculate excess returns, we do not have to track investor positions in the foreign riskless assets, but only positions in home and foreign equity. This amounts to saying that the net forex order flow is correctly characterized by equation (3) since we can ignore the foreign bond positions. Even if a strictly positive foreign bond position is feasible, investors prefer a zero foreign bond holdings. Foreign investment then concerns only equity.

4.1 Exchange Rate Dynamics

Next we discuss the exchange rate dynamics under incomplete markets. Two principle equilibrium forces shape this dynamics. The first equilibrium tendency is governed by the elastic liquidity supply for forex order flow. Forex order flow dQ_t^D in equation (3) is accommodated by financial institutions which finance these home outflows according to an upward sloped supply curve. The elasticity of forex liquidity supply certainly influences the impact of net order flow on the exchange rate and indirectly the adjustment speed towards the steady state exchange rate, $\overline{E}$. We associate the supply induced mean reversion with a first characteristic root (labeled z). A second important parameter for the exchange rate dynamics is the mean reversion of the dividend processes. This mean reversion α_D is exogenous and any feedback effect from the exchange rate dynamics to the dividend process is ruled out by assumption.

An important simplifying feature of our model is its symmetry between the home and foreign country. Symmetry implies that the exchange rate can depend only on differences between home and foreign country variables, but not on country specific variables itself. Otherwise the symmetry would be broken. The symmetry requirement also implies that the exchange rate can only be a function of current and past relative dividend innovations, $dw_s = dw_s^h - dw_s^f$. These relative innovations are the only exogenous source of exchange rate dynamics.

Finally, we highlight the linearity of the model structure. The forex order flow constraint is linearized and the exogenous dividend dynamics is linear by assumption. Moreover, we have assumed a myopic mean-variance utility function which translates linear dividend, price and return processes into linear asset demands. It is therefore justified to restrict our attention to the class of linear exchange rate and price processes. The argument for two fundamental equilibrium forces explains why we focus on two state variables Δ_t and Λ_t, both of which depend for reasons of model symmetry on current and past relative dividend innovations dw_s only.

The following proposition 4 states the conjectured exchange rate process and derives its implications for the order flow constraint (4).

Proposition 4 (Exchange Rate Dynamics):

Assume that (i) equity prices $P = (P_t^h, P_t^f)$ depend linearly on the exchange rate E_t and the dividend processes $D_t = (D_t^h, D_t^f)$ and (ii) the exchange rate has the following linear representation

$$E_t = 1 + e_\Delta \Delta_t + e_\Lambda \Lambda_t$$

with

$$\begin{aligned} \Delta_t &= D_t^h - D_t^f = \int_{-\infty}^t \exp[-\alpha_D(t-s)]\sigma_D dw_s \\ \Lambda_t &= \int_{-\infty}^t \exp[z(t-s)] dw_s \end{aligned}$$

where $z < 0$ and $dw_s = dw_s^h - dw_s^f$. This implies that the order flow constraint (4) is of the simple form

$$dE_t = k_1 \Delta_t dt + k_2 (E_t - 1) dt + k_3 dw_t,$$

where k_1, k_2 and k_3 represent undetermined coefficients.

Proof: The derivation is provided in Appendix C. We have to show that for a linear price and exchange rate equilibrium investor utility maximization implies optimal asset demands K_t^{h*}, K_t^f such that the expression $(K_t^{h*} - K_t^f)\overline{D}dt + (dK_t^f - dK_t^{h*})\overline{P}$ in equation (4) is linear in $E_t - 1$, Δ_t and dw_t.

Under linearity of the price and exchange rate processes, the order flow constraint simplifies to a differential equation in only two state variables Δ_t and $E_t - 1$. This allows us to characterize the exchange rate dynamics as a system of two first-order differential equations,

$$\begin{pmatrix} d\Delta_t \\ dE_t \end{pmatrix} = \begin{pmatrix} -\alpha_D & 0 \\ k_1 & k_2 \end{pmatrix} \begin{pmatrix} \Delta_t \\ E_t - 1 \end{pmatrix} dt + \begin{pmatrix} \sigma_D \\ k_3 \end{pmatrix} dw_t. \tag{5}$$

The associated characteristic polynomial follows as

$$\begin{vmatrix} -\alpha_D - \lambda & 0 \\ k_1 & k_2 - \lambda \end{vmatrix} = (-\alpha_D - \lambda)(k_2 - \lambda) = 0,$$

with characteristic roots $\lambda' = k_2$ and $\lambda'' = -\alpha_D$. A stable solution requires $k_2 < 0$. The exchange rate solution can then be written as a linear combination $e_\Delta \Delta_t + e_\Lambda \Lambda_t$ of the two eigenvectors

$$\Delta_t = \int_{-\infty}^{t} \exp[-\alpha_D(t-s)]\sigma_D dw_s \qquad \text{and} \qquad \Lambda_t = \int_{-\infty}^{t} \exp[k_2(t-s)]dw_s$$

as conjectured in proposition 4.

In order to find the solution parameters, we have to impose the market clearing conditions (1) and determine the steady state levels for the exchange rate, $\overline{E}$, the equity price, $\overline{P}$, and the foreign equity holding, $\overline{K}$. In order to obtain non-negative (steady state) prices ($\overline{P} > 0$) and positive (steady state) home and foreign holdings ($0 < \overline{K} < 1$), we have to restrict the parameter domain of our model. In particular we have to impose an upper bound $\overline{\rho}$ on the risk aversion and a lower bound $\underline{\kappa}$ on the elasticity of the forex liquidity supply.

Propositions 5 and 6 characterize the equilibrium properties:

Proposition 5: Existence and uniqueness of the Incomplete Market Equilibrium.

Home and foreign investors optimize their portfolio according to assumptions 1 to 4. For a sufficiently low risk aversion of the investors with $\rho < \overline{\rho}$ and a sufficiently price-elastic forex supply $\kappa > \underline{\kappa}$, there exists a unique stable linear equilibrium

$$\begin{aligned} P_t^h &= p_0 + p_F F_t^h + p_\Delta \Delta_t + p_\Lambda \Lambda_t \\ P_t^f &= p_0 + p_F F_t^f - p_\Delta \Delta_t - p_\Lambda \Lambda_t \\ E_t &= 1 + e_\Delta \Delta_t + e_\Lambda \Lambda_t \end{aligned}$$

where we define as F_t^h and F_t^f the expected present value of the future home and foreign dividend flows, respectively (as in section 3). The variable $\Delta_t = D_t^h - D_t^f$ represents the relative dividend flows for the two countries and Λ_t a weighted average of past relative dividend innovations decaying at an endogeneous rate $z < 0$ as defined in proposition 4.

Proof: For a full derivation see Appendix C.

Proposition 6: Characterization of the Incomplete Market Equilibrium.

For the incomplete market equilibrium, we can sign the price parameters as follows:

$$p_0 < 0,\ p_F = 1,\ p_\Delta > 0,\ e_\Delta < 0,\ p_\Delta \sigma_D + p_\Lambda > 0,\ e_\Delta \sigma_D + e_\Lambda < 0.$$

Portfolio holdings are given by

$$\begin{pmatrix} K_t^h & K_t^f \\ K_t^{f*} & K_t^{h*} \end{pmatrix} = \begin{pmatrix} 1-\overline{K} & \overline{K} \\ 1-\overline{K} & \overline{K} \end{pmatrix} + \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix} \frac{1}{2\rho} \left(m_\Delta \Delta_t + m_\Lambda \Lambda_t \right)$$

for the parameters $m_\Delta < 0$, and $m_\Lambda > 0$ defined in Appendix C.

Proof: For a derivation see Appendix C.

As in the previous complete market case, we find that investor risk aversion requires an equity risk premium in the form a price discount $p_0 < 0$. As before, a coefficient $p_F = 1$ implies that the equity price reflects the fundamental value of expected future dividends, F^h and F_t^f respectively. Moreover two new stochastic terms Δ_t and Λ_t influence asset prices and the exchange rate.

We conduct a more detailed dicussion of the empirical properties of this equilibrium in the next section.

5 Model Implications

We summarize the main empirical implications of our model. In particular, we focus on the relative volatility of exchange rate and equity returns, on the correlation between exchange rate returns, home and foreign equity returns, and home and foreign portfolio flows. Moreover, we highlight the dependence of the correlation structure on the fundamental parameters of the model. These consist of the risk aversion of investors given by ρ, the elasticity of the forex liquidity supply κ, the riskless rate r and the three parameters governing the dividend processes ($\overline{D}$, α_D, σ_D).

5.1 Relative Exchange Rate Volatility

In complete markets all exchange rate risk can be traded and state contingent contracts leave no particular role to the exchange rate itself. Hence, general equilibrium models with complete markets and flexible prices typically do not generate sufficient exchange rate volatility. If however forex risk is not separately traded, then investors choose their exchange rate exposure jointly with their foreign equity position. Differences in the market performance generate differences in dividend income between home and foreign investors. The resulting capital flows induces exchange rate movements under a price-elastic liquidity supply of currency balances and these movements affect in turn the investment positions of equity traders. We summarize this result as follows:

Implication 1: Relative Exchange Rate Volatility

Market incompleteness in combination with a price-elastic liquidity supply of currency balances can generate exchange rates which are almost as volatile as equity returns.

A more price inelastic supply (lower κ) will tend to generate a larger exchange rate impact for any given order flow and therefore allow for more exchange rate volatility. Figure 1 plots the volatility ratio of exchange rate returns and equity market returns,

$$\sqrt{\frac{\mathcal{E}_t(dE_t dE_t)}{\mathcal{E}_t(dP_t^h dP_t^h/\overline{P}^2)}}.$$

for a risk aversion parameter $0.1 < \rho < 12$ and a liquidity supply parameter $40 < \kappa < 900$. A highly price elastic forex liquidity supply (κ large) implies a low forex volatility. But as the supply elasticity decreases, substantial forex volatility can result as illustrated by the parametric plot. Incomplete forex risk trading and a less than fully price elastic forex liquidity supply can explain the high observed forex volatility documented in more detail in section 6.

5.2 Equity Returns and Exchange Rate Returns

Market incompleteness implies a negative correlation between foreign equity returns and exchange rate returns. A decreasing home equity price is associated with a home currency appreciation. For example a U.S. equity market price decrease ($dP_t^h < 0$, $dR_t^h < 0$) coincides ceteris paribus with a dollar appreciation ($dE_t > 0$).

Formally, we can derive the correlation of currency returns and home equity returns as

$$Corr\left[dE_t dR_t^h\right] = \frac{\mathcal{E}_t(dE_t dR_t^h)/\overline{P}dt}{\mathcal{E}_t(dR_t^h dR_t^h)/\overline{P}^2 dt} = -\frac{1}{\sqrt{2}\sqrt{1 - F(p_\Delta, p_\Lambda)}}$$

where we defined

$$F(p_\Delta, p_\Lambda) = \frac{2 f_D \sigma_D [p_\Delta \sigma_\Delta + p_\Lambda]}{[(f_D \sigma_D) + 2[p_\Delta \sigma_\Delta + p_\Lambda]]^2}.$$

For $p_\Delta \sigma_\Delta + p_\Lambda > 0$ the function $F(p_\Delta, p_\Lambda)$ is bounded by $0 < F(p_\Delta, p_\Lambda) \leq \frac{1}{4}$ and therefore

$$-\sqrt{\frac{2}{3}} < Corr\left[dE_t dR_t^h\right] \leq -\frac{1}{\sqrt{2}}.$$

This provides us with a surprisingly sharp prediction about the correlation structure of currency and equity returns. Figure 2 shows a parametric plot for the parameter range $0.1 < \rho < 12$ for the risk aversion and $40 < \kappa < 900$ for the currency supply elasticity. The correlation is almost constant over all parameters with $Corr\left[dE_t dR_t^h\right] \approx -0.71 \approx -1/\sqrt{2}$[10].

Given the symmetry of the model with respect to both countries, we also obtain

$$Corr\left[dE_t dR_t^h\right] = -Corr[dE_t dR_t^f]$$

or

$$Corr[dE_t(dR_t^h - dR_t^f)] < 0.$$

Therefore, exchange rate movements correlate negatively with differences in stock market returns. We summarize the result as follows:

> **Implication 2: Foreign Equity and Foreign Exchange Rate Return**
>
> Foreign market equity returns in local currency correlate negatively with currency returns of foreign investment. The same is true for foreign market equity excess returns.

[10] The exact magnitude of this correlation depends on the correlation of the two exogenous dividend flows. We assumed that this correlation is zero. If the dividends were positively correlated, we conjecture that $Corr\left[dE_t dR_t^h\right]$ would still be negative but its magnitude would be smaller.

5.3 Equity Returns and Portfolio Flows

Next, we turn to the model implications for portfolio flows. The mean reversion of the dividend processes implies that expected returns are high if dividend levels and stock prices are low. Repatriation of the dividend under differences in dividend income generate a capital inflow for the country with the low dividend flows, because home investors earn higher oversea dividend income than foreign investors. The inelastic liquidity supply of currency balances requires an off-setting capital outflow in the form of net purchases of foreign equity by home investors and net sales of home equity by foreign investors. Formally, we obtain

$$\mathcal{E}(dK_t^f dR_t^h/\overline{P}) = -\frac{1}{\overline{P}\rho}\left[(m_\Delta\sigma_D + m_\Lambda)\left(\frac{1}{2}f_D\sigma_D + (p_\Delta\sigma_D + p_\Lambda)\right)\right] > 0,$$

because $(m_\Delta\sigma_D + m_\Lambda) = \kappa\rho(e_\Delta\sigma_D + e_\Lambda)/\overline{P} < 0$ and $p_\Delta\sigma_D + p_\Lambda > 0$. Furthermore, we can take account of the model symmetry between the two countries as well as the constant net asset supply ($dK_t^f = -dK_t^{f*}$, $dK_t^h = -dK_t^{h*}$). This implies also

$$\mathcal{E}(dK_t^f dR_t^h)/\overline{P} = \mathcal{E}(dK_t^{h*} dR_t^f)/\overline{P} = -\mathcal{E}(dK_t^{f*} dR_t^h)/\overline{P} = -\mathcal{E}(dK_t^h dR_t^f)/\overline{P} > 0.$$

We plot the covariance of the foreign equity inflow and the home return in Figure 3. The covariance becomes larger as the risk aversion of the investor decreases. Under lower risk aversion, investors become more willing to give up diversification benefits in pursuit of higher expected returns. Portfolio movements increase in magnitude.

The above correlation concerns a cross-market effect from home returns to foreign equity inflows. We can also ask how home investors adjust their foreign market equity positions to foreign return expectations? Given that high foreign returns tend to coincide with negative currency return, we expect that home investors are net sellers of foreign equity. Formally,

$$\mathcal{E}(dK_t^f dR_t^f)/\overline{P} = \mathcal{E}(dK_t^{h*} dR_t^h)/\overline{P} = -\mathcal{E}(dK_t^{f*} dR_t^f)/\overline{P} = -\mathcal{E}(dK_t^h dR_t^h)/\overline{P} < 0.$$

The covariance of foreign market equity inflows and foreign market return expectations is quantitatively similar to the cross-market effect, but has the opposite sign.

We summarize the model implications as follows:

Implication 3: Home Equity Return and Foreign Market Portfolio Inflow

Home equity market returns correlate positively with foreign market equity inflows.

Implication 4: Foreign Equity Return and Foreign Market Portfolio Inflow

Foreign equity market returns correlate negatively with foreign market equity inflows.

If we combine these two implications we obtain that net foreign stock ownership increase by the domestic residents should correlate with high return expectations in the home market relative to the foreign market:

$$\mathcal{E}(dK_t^f - dK_t^{h*})(dR_t^f - dR_t^h) < 0.$$

5.4 Exchange Rate Returns and Portfolio Flows

Finally, the model implies a restriction on the covariance between the foreign equity inflow and the exchange rate return as

$$\mathcal{E}(dE_t dK_t^f) = -\frac{\kappa}{\overline{P}}(e_\Delta \sigma_D + e_\Lambda)^2 < 0.$$

and similarly

$$\mathcal{E}(dE_t dK_t^f) = -\mathcal{E}(dE_t dK_t^{h*}) = -\mathcal{E}(dE_t dK_t^{f*}) = \mathcal{E}(dE_t dK_t^h) < 0$$

The covariance $\mathcal{E}(dE_t dK_t^f)$ is plotted in Figure 4 for the same parameter range as in the previous graphs. It is larger for both low risk aversion and a low forex liquidity supply elasticity[11]. The low risk aversion is required to allow for portfolio shifts under different return expectations. A low supply elasticity assures capital flows have a large exchange rate impact. If we combine the first two of the above equations we get:

$$\mathcal{E}(dK_t^f - dK_t^{h*}, dE_t) < 0$$

We summarize the last model implication as follows:

Implication 5: Forex Return and Foreign Market Portfolio Inflow

Currency returns on domestic investments correlate negatively with foreign market equity inflows, or, equivalently in our model, net foreign stock ownership increase by the domestic residents is associated with an appreciation of the foreign currency.

6 Evidence

[Preliminary and Incomplete]

Finally, we explore the empirical validity of the model predictions highlighted in section 5. Data are abundantly available for exchange rates and equity market prices. To measure return correlations accurately, we use daily exchange rate returns and equity market index returns for the period 1/1/1980 to 1/1/2002. We also use monthly data series for these same variables to check our results on a longer time period (1974:1 to 2001:12). The stock index and the exchange rate data are from Datastream-MSCI. Statistical information on equity flows is more difficult to obtain. Here we use the so-called TIC data produced by the U.S. Treasury department. Available on a monthly frequency since 1977, the TIC data record transactions in portfolio equities between U.S. residents and residents of foreign countries.[12] They allow us to compute net purchases of foreign equities by U.S. residents (dK_t^f) and net purchases of U.S. equities by foreigners (dK_t^{h*}).

Since cross-border equity flows have been growing sizably in the last decade[13], we use two normalized measures of net purchases. We normalize flows by market capitalization (this is the natural measure in our model) and alternatively by the average flows over the previous 12 months (as Brennan

[11] Remember that $e_\Delta \sigma_D + e_\Lambda$ is a function of κ and ρ.

[12] For a thorough presentation of these data see Warnock *et al.* (2001). In particular, the TIC data records transactions based on the residency of the seller and of the buyer. For example, a German equity sold in London by a US resident to a UK Bank will be recorded as a sale of a foreign security by a US resident to the UK. To the extent that the operation was actually performed in euro and not in Sterling or that it was realized on the behalf of a German equity trader who cares about profits in euro and not in Sterling, this may cause a problem for our inference.

[13] See Portes and Rey (1999) for a detailed study of the properties of these flows.

and Cao (1997)). The results appear not to depend on any particular normalization. The stock market capitalization data are from S&P Emerging Markets Database. A detailed data documentation is provided in the appendix.

The model applies to countries with developed equity markets. It therefore seems appropriate to focus on OECD countries and their dollar exchange rates. Ideally, sample countries should have freely floating dollar exchange rates and large equity market capitalizations relative to their GDP. Moreover, the ideal sample country should also have a high equity content in its international capital flows. Given these restrictions and data availability, our final sample consists of 14 developed countries. We present results for the full sample and also for the later periods (1990-2001 and 1995-2001) since cross border equity flows have increased sizably in the 1990s. We therefore expect a better model fit for recent years.

6.1 Relative Exchange Rate Volatility

First, we compute monthly realized volatility measures based on the sum of squared daily logarithmic returns.[14] The standard deviations of monthly realized volatility are averaged over all months. All equity return are denominated in local currency and exchange rate returns are calculated relative to the U.S. dollar. In Table 1, we report the ratio of standard deviation of exchange rate and equity returns (columns d and e). The volatility ratios vary between 0.16 (Canada) and 0.94 (Austria) for the period 1980-2001 with a mean of 0.66 and a median of 0.68. Exchange rate volatility therefore tends to be lower than equity index volatility, but is still of a comparable magnitude. Our theoretical framework can explain such high exchange rate volatility by assuming a relatively price inelastic forex liquidity supply. Comparing the entire period 1980-2001 (column d) to the subsample 1995-2001 (column e), we find that volatility ratios decline over time for most countries. The reduction in the ratio can mostly be attributed to a decrease in exchange rate volatility. We can speculate that the liquidity supply in the forex market (parameter κ in our model) might have become more price elastic over time. Forex market might have become more developed along with the development of equity markets.

Qualitatively similar results are obtained for the volatility ratio based on the conventional standard deviation of monthly returns (Table 2 columns d and e) or daily returns (Table 3 columns d and e).

6.2 Equity Returns and Exchange Rate Returns

Next, we compute realized monthly correlations between exchange rates and equity returns obtained from daily data (Table 1 columns a,b and c).[15] The model predicts a negative correlation between exchange rate returns and stock returns and a negative correlation between exchange rate returns and excess stock market returns. For example, an increase in the equity return of the UK stock market is associated with a depreciation of the Sterling vis-à-vis the dollar. The empirical results reported in Table 1 (column a) are very striking. Most of our countries display a statistically significant negative correlation between their dollar exchange rate and equity index returns. Again, we break the full sample period 1980-2001 (Table 1 column a) into subsamples 1990-2001 (column b) and 1995-2001

[14] Realized logarithmic standard deviations have been shown to be approximatively gaussian whereas the unconditional distributions of returns are highly right-skewed (see Diebold *et al.* (2001)).

[15] We also compute excess returns defined as the local currency equity return minus the U.S. equity return in dollars and their correlation with currency returns. The results (not reported) are very similar.

(column c). International equity trading has outgrown GDP for most OECD countries only in the 1990. Parallel to this equity market development, we find a remarkable increase in the magnitude of the negative correlations across the three subsamples for most countries. Our median estimates for the realized correlations is -0.15 for the period 1995-2001.

The only countries for which the correlation is consistently significantly positive on the three subsamples are Australia and Canada. We conjecture that this can be explained by the fact that the Australian and the Canadian economies are small open economies strongly affected by natural resource prices. Chen and Rogoff (2001) show indeed that for Canada and Australia world commodity price fluctuations explain well movements in the exchange rate even at the quarterly frequency. They underline the specificity of their results to this restricted class of countries and their inability of discovering similar empirical regularities for other developed countries.

Japan is the only other country of the sample for which the results are also mixed. The correlation of the Japanese exchange rate with stock index returns is positive for the first two subsamples and turns negative only for the most recent subsample. We also highlight that Japan has a lower equity content of capital flows compared to other developed countries (except for recent years). Again we complement these average realized correlation measures with conventional correlation measures at a monthly frequency (Table 2 columns a,b and c) and at the daily frequency (Table 3 columns a,b and c). The results are virtually the same as before.

The negative correlation between exchange rate and equity returns has been noted by other researchers. Using quarterly data, Brooks *et al.* (2001) compute the excess return of a European over an U.S. equity index and find a negative correlation with the euro-dollar exchange rate return. In other words, a dollar depreciation vis-à-vis the euro is associated with a positive excess return of the U.S. stock market vis-à-vis the European stock market. They discard their finding as 'counter-intuitive' (p. 17), since it contradicts the popular view that a strengthening U.S. equity market should be mirrored by a strengthening dollar. However, our model predicts the negative correlation as the equilibrium outcome. The correlation evidence deserved to be highlighted not only because of its strong statistical significance and increasing magnitude. It also stands out relative to the empirical failure of uncovered interest parity for the same set of countries and at the same horizons.

6.3 Equity Returns and Portfolio Flows

Equity flow data are unfortunately (publicly) available only at a monthly frequency. In the absence of daily data, we cannot calculate realized month correlations and therefore use conventional monthly correlation measures. This is bound to introduce larger measurement errors and lowers the quality of our inference. In Table 4 (columns a and b), we provide the correlations between U.S. equity returns and equity inflows into foreign countries $Corr(dK_t^f, dR_t^h)$.[16] The model predicts a positive sign. The empirical results are mostly in accordance with this prediction. For the first subsample 11 correlations are positive out of 14; for the second subsample, 7 are positive. In Table 5 we also look at a related measure, namely the correlations of net foreign stock ownership increase by US residents with excess stock returns in the foreign markets, formally $Corr(dK_t^f - dK^{h*}, dR_t^f - dR_t^h)$. Theory predicts a negative sign. Again the results are rather mixed. Altogether, 6 correlations out of 14 are negative

[16] Normalizing net purchases by market capitalization or using previous flows as in Brennan and Cao (1997) does not make much difference for our results.

for the first subsample and 9 for the second subsample.

The relative weakness of the flow results compared to the price results of the previous sections may be due to the relatively imprecise flow data we use. Indeed other studies, which use proprietary daily flow data find significant positive correlations between US returns and inflows into foreign markets. This result is one of the main empirical findings of Griffin et al. (2002) and Richards (2002) which both document significant positive correlations of the U.S. returns with inflows into 8 Asian equity markets for recent periods.

Table 4, columns d and e, show the correlations between portfolio flows into a market and equity returns in that same market $Corr(K_t^f, dR_t^f)$. We confirm once more the empirical results that Griffin et al. (2002) and Richards (2002) obtained for 8 Asian equity markets: flows into foreign markets and returns in those markets tend to be positively correlated (11 positive correlations in the first subsample and 11 in the more recent period). This time however, the results are at odds with our model. As explained in section 5, since high foreign returns tend to coincide with low exchange rate returns, in our model domestic investors tend to be net sellers of foreign equities when foreign returns are high. The positive correlation found in the data may come from a price pressure effects, which are absent from our set up.

6.4 Exchange Rate Returns and Portfolio Flows

In Table 5, columns c and d, we show the correlations between exchange rate returns and net foreign stock ownership increase by US residents, $Corr(dK_t^f - dK^{h*}, dE_t)$. Our theory predicts that they are negatively correlated. The empirical results are mostly in accordance with the theory: 9 countries out of 14 have correlations which are rightly signed for both parts of the subsample. A net equity inflow in the local equity market is therefore usually associated with an appreciation of the local currency. [TO BE FINISHED]

7 Conclusions

[PRELIMINARY]

The contribution of this paper is twofold. First, we develop a simple model of international equity trading in an environment where the currency risk is not hedged and equity flows generate exchange rate movements due to a price-elastic forex liquidity supply. In our framework, international spill-overs and multi-market interaction are solely based on financial structure, namely forex market incompleteness. Second, we document stylized facts regarding correlations and relative volatility of equity returns, exchange rates and portfolio investment flows and confront them with the implications of our model. We show that in the presence of risk averse investors a price-elastic forex supply generates high exchange rate volatility and, perhaps surprisingly, a negative correlation between equity and currency returns for a large range of parameter values. These two predictions are strongly borne out in the data both at the daily and monthly frequency. The model also provides clear predictions on correlation structures of international equity flows with exchange rate and equity market returns. High U.S. equity returns are associated with foreign market equity inflows and appreciating foreign currencies. We find weak support for the first prediction. It seems to be borne out strongly however in two other recent studies which use high frequency data on equity flows. We also find some support for the second prediction in our data. The statistical evidence for the flow correlations is in general not as strong as for the price correlations. Again, the low quality of the flow data for our purposes and its low measurement frequency might be at fault.

Our analysis can be extended in various directions. We conjecture that the results are robust to a positive correlation between the dividend flows, which was so far ignored. Internationally correlated equity market risk is *per se* devoid of risk trading benefits and therefore does alter the allocation problem for the remaining uncorrelated equity return risk. A more interesting extension would take account of differences in opinion concerning the international equity returns between the home and foreign investors. This would introduce an additional new trading motive and source of equity flows ignored in our model specification.

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Appendix A: Equilibrium under Financial Autarchy

Proposition 1:

We conjecture a linear price equilibrium of the form

$$\begin{aligned} P_t^h &= p_0 + p_F F_t^h \\ P_t^f &= p_0 + p_F F_t^f. \end{aligned}$$

Given $F_t^h = f_0 + f_D D_t^h$, with $f_D = 1/(\alpha_D + r)$ and $f_0 = (r^{-1} - f_D)\overline{D}$, the excess return in the home country follows as

$$dR_t^h = dP_t^h - rP_t^h dt + D_t^h dt = p_F f_D dD_t^h + D_t^h dt - r\left(p_0 + p_F f_0 + p_F f_D D_t^h\right) dt$$

Market clearing requires $\mathcal{E}_t \frac{(dR_t^h)}{\rho\sigma_R^2 dt} = 1$ and this implies for the price coefficients

$$\begin{aligned} p_0 &= -\frac{\rho\sigma_R^2}{r} \\ p_F &= 1. \end{aligned}$$

The instantaneous return volatility is given follows as

$$\sigma_R^2 = \mathcal{E}_t(f_D\sigma_D dw_t^h)^2 = \frac{\sigma_D^2}{(\alpha_D + r)^2}.$$

Appendix B: Equilibrium under Complete Market

Proposition 2:

We conjecture a linear price system of the form

$$\begin{aligned} P_t^h &= p_0 + p_F F_t^h \\ P_t^f &= p_0 + p_F F_t^f \\ E_t &= 1. \end{aligned}$$

The fundamental processes are given by: $F_t^h = f_0 + f_D D_t^h$, $F_t^f = f_0 + f_D D_t^f$. Prices P_t^h, P_t^f, E_t evolve according to

$$\begin{aligned} dP_t^h &= p_F dF_t^h = p_F f_D \alpha_D \overline{D} dt - p_F f_D \alpha_D D_t^h dt + p_F f_D \sigma_D dw_t^h \\ dP_t^f &= p_F dF_t^f = p_F f_D \alpha_D \overline{D} dt - p_F f_D \alpha_D D_t^f dt + p_F f_D \sigma_D dw_t^f \\ dE_t &= 0. \end{aligned}$$

Next we give a linear approximation of the excess returns for home investor in the home market, R_t^h, and foreign market, R_t^f , respectively. Similarly, we state the foreign investor's excess returns (in foreign currency terms) in the foreign market, R_t^{f*}, and home market, R_t^{h*}, respectively.

$$\begin{aligned} dR_t^h &= dP_t^h - rP_t^h dt + D_t^h dt \\ dR_t^f &\approx -dE_t\overline{P} + dP_t^f - dE_t dP_t^f - r[P_t^f - \overline{P}(E_t - 1)]dt + [D_t^f - \overline{D}(E_t - 1)]dt \\ dR_t^{f*} &= dP_t^f - rP_t^f dt + D_t^f dt \\ dR_t^{h*} &\approx dE_t\overline{P} + dP_t^h + dE_t dP_t^h - r\left[P_t^h + \overline{P}(E_t - 1)\right] dt + \left[D_t^h + \overline{D}(E_t - 1)\right] dt \end{aligned}$$

with dP_t^h, dP_t^f and dE_t given above.

Returns in foreign currency terms for the foreign investor are identical to those of the home investor because of a constant exchange rate, hence $dR_t^{h*} = dR_t^h$, and $dR_t^{f*} = dR_t^f$. Finally, we consider the correlation structure of returns. Let Ω denote the covariance of the returns (dR_t^h, dR_t^f) (in home currency terms) for the home investor and Ω^* the corresponding covariance of the return (dR_t^{f*}, dR_t^{h*}) (in foreign currency) for the foreign investor. Symmetry of the two country model implies

$$\Omega = \Omega^* = \begin{pmatrix} \Omega_{11} & \Omega_{21} \\ \Omega_{21} & \Omega_{22} \end{pmatrix},$$

where Ω_{11}denotes the variance of domestic investment, Ω_{22} the variance of foreign investment, and Ω_{21} the covariance risk. Generally, we have

$$\Omega^{-1} = \frac{1}{\det \Omega} \begin{pmatrix} \Omega_{22} & -\Omega_{21} \\ -\Omega_{21} & \Omega_{11} \end{pmatrix} = \begin{pmatrix} \Omega_{11}^{-1} & \Omega_{21}^{-1} \\ \Omega_{21}^{-1} & \Omega_{22}^{-1} \end{pmatrix}$$

with $\det \Omega = \Omega_{11}\Omega_{22} - \Omega_{21}\Omega_{21}$.

For the special case of complete markets with a constant exchange rate, we have $\mathcal{E}_t \left(dE_t dP_t^h \right) = 0$, $\Omega_{21} = 0$, and $\Omega_{11} = \Omega_{22} = \sigma_R^2$. Therefore,

$$\Omega^{-1} = \frac{1}{\sigma_R^4} \begin{pmatrix} \Omega_{11} & 0 \\ 0 & \Omega_{22} \end{pmatrix} = \frac{1}{\sigma_R^2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{\sigma_R^2} \mathbf{1}_{2\times 2}.$$

The first-order condition for the asset demands is given by

$$\begin{pmatrix} K_t^h & K_t^f \\ K_t^{f*} & K_t^{h*} \end{pmatrix} = \frac{1}{\rho dt} \mathcal{E}_t \begin{pmatrix} dR_t^h & dR_t^f \\ dR_t^{f*} & dR_t^{h*} \end{pmatrix} \Omega^{-1}$$

Market clearing in the two stock markets ($K_t^h + K_t^{h*} = 1$, $K_t^f + K_t^{f*} = 1$) implies the price coefficients

$$\begin{aligned} p_0 &= -\frac{\rho \sigma_R^2}{2r} \\ p_F &= 1. \end{aligned}$$

For the portfolio positions we get

$$\begin{pmatrix} K_t^h & K_t^f \\ K_t^{f*} & K_t^{h*} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}.$$

Appendix C: Equilibrium under Incomplete Markets

Proposition 3 (Short Selling Constraint is always Binding):

Define $\mathbf{x}_t = (x^h, x^f, x^b)^T$ as the $(1x3)$ vector of holdings in home equity, foreign equity, and foreign bonds, respectively. Denote by $\mathbf{dR}_t = (dR^h, dR^f, dR^b)$ the corresponding $(1x3)$ excess returns vector on investing $\overline{P}$ with $\mathcal{E}(dR_t^h)$, $\mathcal{E}(dR_t^f)$ given in appendix B and $\mathcal{E}(dR_t^b) \approx -\overline{P} dE_t$.

We call $\mathbf{\Omega} = \mathcal{E}(\mathbf{dR}_t^T \mathbf{dR}_t)$ the $(3x3)$ covariance matrix of the excess returns. We show that the constraint $x^b \geq 0$ is binding. The second-order condition for an interior maximum is that $\mathbf{\Omega}$ be positive semi-definite. To show that the constraint is binding, we just have to prove that the unconstrained maximization produces an interior solution with $x^b < 0$. Let

$$\mathbf{\Omega} = \mathcal{E}(\mathbf{dR}_t^T \mathbf{dR}_t) = \begin{pmatrix} \sigma_{pp} & -\overline{P}\sigma_{pe} + \sigma_{fh} & -\overline{P}\sigma_{pe} \\ -\overline{P}\sigma_{pe} + \sigma_{fh} & \overline{P}^2\sigma_{ee} + 2\overline{P}\sigma_{pe} + \sigma_{pp} & \overline{P}^2\sigma_{ee} + \overline{P}\sigma_{pe} \\ -\overline{P}\sigma_{pe} & \overline{P}^2\sigma_{ee} + \overline{P}\sigma_{pe} & \overline{P}^2\sigma_{ee} \end{pmatrix} dt,$$

where we define $\mathcal{E}(dR_t^h dR_t^h) = \mathcal{E}(dR_t^f dR_t^f) = \sigma_{pp}dt$, $\mathcal{E}(dE_t dR_t^h) = -\mathcal{E}(dR_t^f dE_t) = \sigma_{pe}dt$, $\mathcal{E}(dR_t^f dR_t^h) = \sigma_{fh}dt$ and $\mathcal{E}(dE_t dE_t) = \sigma_{ee}dt$. Inverting the symmetric covariance matrix allows us to compute the optimal unconstrained portfolio holdings $\mathbf{x}_t = (x^h, x^f, x^b)$. In particular we have:

$$\begin{aligned}
\det \mathbf{\Omega}\rho x^f &= [-\overline{P}^2\sigma_{ee}\sigma_{fh} - \overline{P}^2\sigma_{pe}^2]\mathcal{E}(dR_t^h)dt \\
&\quad +[\overline{P}^2\sigma_{ee}\sigma_{pp} - \overline{P}^2\sigma_{pe}^2]\mathcal{E}(dR_t^f)dt \\
&\quad +[-\overline{P}^2\sigma_{pp}\sigma_{ee} - \overline{P}\sigma_{pe}\sigma_{pp} + \overline{P}^2\sigma_{pe}^2 - \overline{P}\sigma_{fh}\sigma_{pe}]\mathcal{E}(dR_t^b)dt \\
\det \mathbf{\Omega}\rho x^b &= [\overline{P}^2\sigma_{fh}\sigma_{ee} + \overline{P}\sigma_{fh}\sigma_{pe} + \overline{P}^2\sigma_{pe}^2 + \overline{P}\sigma_{pe}\sigma_{pp}]\mathcal{E}(dR_t^h)dt \\
&\quad +[-\overline{P}^2\sigma_{pp}\sigma_{ee} - \overline{P}\sigma_{pe}\sigma_{pp} + \overline{P}^2\sigma_{pe}^2 - \overline{P}\sigma_{fh}\sigma_{pe}]\mathcal{E}(dR_t^f)dt \\
&\quad +[\overline{P}^2\sigma_{pp}^2\sigma_{ee} + 2\overline{P}\sigma_{pe}\sigma_{pp} + \sigma_{pp}^2 - \overline{P}^2\sigma_{pe}^2 + 2\overline{P}\sigma_{pe}\sigma_{fh} - \sigma_{fh}^2]\mathcal{E}(dR_t^b)d
\end{aligned}$$

In steady state excess returns are given by $\mathcal{E}(d\overline{R}^h) = (\overline{D} - r\overline{P})dt$; $\mathcal{E}(d\overline{R}^f) = (\overline{D} - r\overline{P})dt + \overline{P}\sigma_{pe}dt$; $\mathcal{E}(d\overline{R}^b) = 0$ and for symmetric steady state holdings $\mathbf{x}_t = (1 - \overline{K}, \overline{K}, \overline{x}^b)$ we obtain

$$\begin{aligned}
\det \mathbf{\Omega}\rho &= [-\overline{P}^2\sigma_{ee}\sigma_{fh} - 2\overline{P}^2\sigma_{pe}^2 + \overline{P}^2\sigma_{ee}\sigma_{pp}](\overline{D} - r\overline{P})dt + [\overline{P}^2\sigma_{ee}\sigma_{pp} - \overline{P}^2\sigma_{pe}^2]\overline{P}\sigma_{pe}dt \\
\det \mathbf{\Omega}\rho\overline{x}^b &= [\overline{P}^2\sigma_{fh}\sigma_{ee} + 2\overline{P}^2\sigma_{pe}^2 - \overline{P}^2\sigma_{pp}\sigma_{ee}](\overline{D} - r\overline{P})dt \\
&\quad +[-\overline{P}^2\sigma_{pp}\sigma_{ee} - \overline{P}\sigma_{pe}\sigma_{pp} + \overline{P}^2\sigma_{pe}^2 - \overline{P}\sigma_{fh}\sigma_{pe}]\overline{P}\sigma_{pe}dt.
\end{aligned}$$

and taking the sum implies ($\sigma_{pp} + \sigma_{fh} > 0$)

$$\det \mathbf{\Omega}\rho(\overline{K} + \overline{x}^b) = -(\sigma_{pp} + \sigma_{fh})\overline{P}^2\sigma_{pe}^2 dt < 0.$$

Since $\det \mathbf{\Omega}\rho\overline{K} > 0$, it follows that $\overline{x}^b < 0$. Hence, the constraint $\overline{x}^b \geq 0$ is in fact binding and investors hold zero foreign bond in the steady state.

Proposition 4 (exchange rate dynamics):

The first-order condition for the investor asset demands (for risk aversion ρ) is given by

$$\begin{pmatrix} K_t^h & K_t^f \\ K_t^{f*} & K_t^{h*} \end{pmatrix} = \frac{1}{\rho dt}\mathcal{E}_t \begin{pmatrix} dR_t^h & dR_t^f \\ dR_t^{f*} & dR_t^{h*} \end{pmatrix} \Omega^{-1}$$

The excess returns are of the form (with symmetric ones for the foreign investor):

$$\begin{aligned}
dR_t^h &= \alpha_0^h dt + \alpha_D^h D_t^h dt + \alpha_\Delta^h \Delta_t dt + \alpha_\Lambda^h \Lambda_t dt + p_F f_D \sigma_D dw_t^h + p_\Delta \sigma_\Delta dw_t + p_\Lambda \sigma_\Lambda dw_t \\
dR_t^f &= \alpha_0^f dt + \alpha_D^f D_t^f dt + \alpha_\Delta^f \Delta_t dt + \alpha_\Lambda^f \Lambda_t dt \\
&\quad -\overline{P}e_\Delta \sigma_D dw_t - \overline{P}e_\Lambda \sigma_\Lambda dw_t + p_F f_D \sigma_D dw_t^f - p_\Delta \sigma_\Delta dw_t - p_\Lambda \sigma_\Lambda dw_t
\end{aligned}$$

where $\alpha_0^j, \alpha_D^j, \alpha_\Delta^j, \alpha_\Lambda^j$ are sets of coefficients ($j = h, f$). Substitution then implies

$$K_t^{h*} - K_t^f = \frac{1}{\rho}\left[m_\Delta \Delta_t + m_\Lambda \Lambda_t\right]$$

where we define coefficients

$$\begin{aligned} m_\Delta &= 2p_\Delta(\alpha_D + r)(\Omega_{12}^{-1} - \Omega_{22}^{-1}) - 2[(\alpha_D + r)\overline{P} - \overline{D}]e_\Delta\Omega_{22}^{-1} \\ m_\Lambda &= 2p_\Lambda(-z + r)(\Omega_{12}^{-1} - \Omega_{22}^{-1}) - 2[\overline{P}(-z + r) - \overline{D}]e_\Lambda\Omega_{22}^{-1}. \end{aligned}$$

Moreover

$$dK_t^{h*} - dK_t^f = \frac{1}{\rho}\left[-\alpha_D m_\Delta \Delta_t dt + z m_\Lambda \Lambda_t dt\right] + \frac{1}{\rho}\left[m_\Delta\sigma_\Delta + m_\Lambda\sigma_\Lambda\right] dw_t.$$

Finally, we substitute

$$\Lambda_t = \frac{1}{e_\Lambda}(E_t - \overline{E}) - \frac{e_\Delta}{e_\Lambda}\Delta_t$$

and find that the term $(K_t^{h*} - K_t^f)\overline{D}dt + (dK_t^f - dK_t^{h*})\overline{P}$ is linear in $E_t - \overline{E}$, Δ_t and dw_t. Substitution into the forex order flow constraint (4) implies a representation

$$dE_t = k_1\Delta_t + k_2(E_t - \overline{E}) + k_3 dw_t.$$

Proposition 5 (Existence and Uniqueness of the Incomplete Market Equilibrium):

The two market clearing conditions $K_t^h + K_t^{h*} = 1$ and $K_t^{f*} + K_t^f = 1$ imply each 4 symmetric parameter contraints (for D_t^h, D_t^f, Λ_t,constant) given as

$$\begin{aligned} p_0 &= \frac{-\rho\det\Omega - \mathcal{E}_t(dE_t dP_t^f)(-\Omega_{12} + \Omega_{11})}{r(\Omega_{11} - 2\Omega_{12} + \Omega_{22})} && (6) \\ p_F &= 1 && (7) \\ p_\Delta &= -e_\Delta\frac{[(\alpha_D + r)\overline{P} - \overline{D}](\Omega_{21} + \Omega_{11})}{(\alpha_D + r)(\Omega_{11} + 2\Omega_{21} + \Omega_{22})} && (8) \\ p_\Lambda &= -e_\Lambda\frac{[(-z + r)\overline{P} - \overline{D}](\Omega_{21} + \Omega_{11})}{(-z + r)(\Omega_{11} + 2\Omega_{21} + \Omega_{22})} && (9) \end{aligned}$$

The forex order flow constraint (4) implies an additional 3 constraints (for $\Delta_t, \Lambda_t, dw_t$) given by

$$\begin{aligned} e_\Delta\left(\overline{KD} - \kappa\alpha_D\right) + m_\Delta\frac{1}{\rho}\left(\overline{D} + \alpha_D\overline{P}\right) &= -\overline{K} && (10) \\ e_\Lambda\left(\overline{KD} + \kappa z\right) + m_\Lambda\frac{1}{\rho}\left(\overline{D} - z\overline{P}\right) &= 0 && (11) \\ e_\Delta\kappa\sigma_\Delta + e_\Lambda\kappa - m_\Delta\frac{1}{\rho}\overline{P}\sigma_\Delta - m_\Lambda\frac{1}{\rho}\overline{P} &= 0 && (12) \end{aligned}$$

These 7 equations determine the 7 price parameters $p_0, p_F, p_\Delta, p_\Lambda, e_\Delta, e_\Lambda, z$.

Moreover, for steady state levels $\overline{P} > 0, \overline{D} > 0, \overline{\Lambda} = 0$ and $0 < \overline{K} < 1$ we have:

$$\begin{aligned} \overline{P} &= p_0 + \frac{\overline{D}}{r} + p_\Lambda\overline{\Lambda} = p_0 + \frac{\overline{D}}{r} \\ \overline{K} &= \frac{\rho\left[\Omega_{11} - \Omega_{21}\right] - \mathcal{E}_t(dE_t dP_t^f)}{\rho\left(\Omega_{11} - 2\Omega_{21} + \Omega_{22}\right)}. \end{aligned}$$

The respective covariances are given by

$$\begin{aligned} \Omega_{11}/dt &= (f_D\sigma_D)^2 + 2[p_\Delta\sigma_\Delta + p_\Lambda]^2 + 2f_D\sigma_D[p_\Delta\sigma_\Delta + p_\Lambda] \\ \Omega_{12}/dt &= -2(p_\Delta\sigma_\Delta + p_\Lambda)^2 - \left[2(p_\Delta\sigma_\Delta + p_\Lambda) + f_D\sigma_D\right]\overline{P}\left(e_\Delta\sigma_D + e_\Lambda\right) - 2(p_\Delta\sigma_\Delta + p_\Lambda)f_D\sigma_D \\ \Omega_{22}/dt &= (f_D\sigma_D)^2 + 2[\overline{P}\left(e_\Delta\sigma_D + e_\Lambda\right) + p_\Delta\sigma_\Delta + p_\Lambda]^2 + 2f_D\sigma_D[\overline{P}\left(e_\Delta\sigma_D + e_\Lambda\right) + p_\Delta\sigma_\Delta + p_\Lambda] \end{aligned}$$

and furthermore

$$\overline{\Omega}/dt = 2\left(f_D\sigma_D\right)^2 + 2[\overline{P}\left(e_\Delta\sigma_D + e_\Lambda\right)]^2. \tag{13}$$

where we defined $\overline{\Omega}(z) = \Omega_{11} + 2\Omega_{21} + \Omega_{22} > 0$ as the instantaneous variance of the total market portfolio of all domestic and foreign equity.

Combining equations (11) and (9) and the definition of $\overline{\Omega}(z)$ we obtain an expression which characterizes the root z of the system as

$$\frac{\rho}{2}\left(\overline{KD} + \kappa z\right)\overline{\Omega} = f(z). \tag{14}$$

for a function $f(z) = [(-z+r)\overline{P} - \overline{D}]\left(\overline{D} - z\overline{P}\right)$.

The function $f(z)$ represents a convex parabola and has two intersects with the x-axes at $z_1 = -\overline{D}/\overline{P} + r \leq 0$ and $z_2 = \overline{D}/\overline{P} \geq 0$. Since $\frac{\rho}{2}\left(\overline{KD} + \kappa z\right)\overline{\Omega}$ is upward sloping (given $d\overline{\Omega}/dz > 0$), and positive for $z = 0$, it intersects the parabola at least once (for some negative z) and at most twice. The first intersection z is negative and the second one, if it exists, is positive. We discard the positive root anyway because it is unstable.

Assume the forex supply is sufficiently price elastic with $\kappa > \overline{\kappa} = \overline{KDP}/(\overline{D} - r\overline{P}) = \overline{KDP}/(-rp_0)$. Then $\frac{\rho}{2}\left(\overline{KD} + \kappa z\right)\overline{\Omega}(z)$ intersects the x-axis to the right of $z_1 = -\overline{D}/\overline{P} + r$ and the root z is confined to the interval $z \in [-\overline{D}/\overline{P} + r, -\overline{KD}/\kappa]$. This implies $(-z+r)\overline{P} - \overline{D} < 0$. Moreover, we require that $-\alpha_D < -\overline{D}/\overline{P} + r$ or $(\alpha_D + r)\overline{P} - \overline{D} > 0$. The latter condition can be rewritten as $\alpha_D\overline{P} > rp_0$, where p_0 represents the risk discount on the asset price. We can make p_0 sufficiently small by setting a low upper threshold value for the investor risk aversion, hence require $\overline{\rho}_1 > \rho$.

With these two conditions on κ and ρ we can now sign the parameters and show that:

$$e_\Delta < 0; \; p_\Delta > 0$$

Uniqueness of the Equilibrium:

We first note that there is a unique stable negative root $z < 0$. Moreover, equation (14) can be rewritten as

$$\overline{\Omega} = 2\left(f_D\sigma_D\right)^2 + 2[\overline{P}\left(e_\Delta\sigma_D + e_\Lambda\right)]^2 = \frac{[(-z+r)\overline{P} - \overline{D}]\left(\overline{D} - z\overline{P}\right)}{\frac{\rho}{2}\left(\overline{KD} + \kappa z\right)} > 0$$

A necessary condition for the existence of a real solution for $\overline{e} = e_\Delta\sigma_D + e_\Lambda$ is

$$V(\rho, \kappa) = \frac{[(-z+r)\overline{P} - \overline{D}]\left(\overline{D} - z\overline{P}\right)}{\rho\left(\overline{KD} + \kappa z\right)} - \left(f_D\sigma_D\right)^2 \geq 0.$$

This condition is satisfied only if $\rho\left(f_D\sigma_D\right)^2$ is sufficiently small or risk aversion is below a certain threshold $\rho < \overline{\rho}_2$. We now take $\rho < \min(\overline{\rho}_1, \overline{\rho}_2) = \overline{\rho}$. Given $\overline{e} \equiv e_\Delta\sigma_D + e_\Lambda < 0$ (shown in proposition 6), we can then rewrite equation (14) in linear form as

$$e_\Delta\sigma_D + e_\Lambda = -\frac{1}{\overline{P}}\sqrt{V(\rho, \kappa)} \tag{15}$$

We define a vector $\mathbf{e} = (e_\Delta, e_\Lambda, m_\Delta, m_\Lambda)$ and matrices

$$\mathbf{A} = \begin{pmatrix} \sigma_D & 1 & 0 & 0 \\ \left(\overline{KD} - \kappa\alpha_D\right) & 0 & \frac{1}{\rho}\left(\overline{D} + \alpha_D\overline{P}\right) & 0 \\ 0 & \left(\overline{KD} + \kappa z\right) & 0 & \frac{1}{\rho}\left(\overline{D} - z\overline{P}\right) \\ \kappa\sigma_\Delta & \kappa & -\frac{1}{\rho}\overline{P}\sigma_\Delta & -\frac{1}{\rho}\overline{P} \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} -\frac{1}{\overline{P}}\sqrt{V(\rho, \kappa)} \\ -\overline{K} \\ 0 \\ 0 \end{pmatrix}$$

so that the linear system $\mathbf{Ae} = \mathbf{b}$ summarizes the 4 equations (10), (11), (12) and (15). For $\det(\mathbf{A}) \neq 0$ there exists therefore a unique solution for $\mathbf{e}$. Furthermore, using the solution to $\mathbf{e} = \mathbf{A}^{-1}\mathbf{b}$, we can show that $d\overline{\Omega}/dz > 0$.

Next we show that this implies also a unique solution for the price coefficients $\mathbf{p} = (p_\Delta, p_\Lambda)$. Note that

$$(\Omega_{11} + \Omega_{12})/dt = (f_D \sigma_D)^2 - [2(p_\Delta \sigma_\Delta + p_\Lambda \sigma_\Lambda) + f_D \sigma_D] \, \overline{P} \, (e_\Delta \sigma_D + e_\Lambda)$$

is linear in $\mathbf{p}$ for a fixed vector $\mathbf{e}$. The equations (8) and (9) are therefore of the form $\mathbf{Cp} = \mathbf{d}$, where we define

$$\mathbf{C} = \begin{pmatrix} 2c_\Delta \overline{P}\overline{e}\sigma_\Delta - 1 & 2c_\Delta \overline{P}\overline{e} \\ 2c_\Lambda \overline{P}\overline{e}\sigma_\Delta & 2c_\Lambda \overline{P}\overline{e} - 1 \end{pmatrix}, \qquad \mathbf{d} = \begin{pmatrix} c_\Delta \left[(f_D\sigma_D)^2 - f_D\sigma_D \overline{P}\overline{e} \right] \\ c_\Lambda \left[(f_D\sigma_D)^2 - f_D\sigma_D \overline{P}\overline{e} \right] \end{pmatrix}$$

with $\overline{e} \equiv e_\Delta \sigma_D + e_\Lambda$, $\overline{\Omega} = \Omega_{11} + 2\Omega_{21} + \Omega_{22}$ and additional constants

$$c_\Delta = \frac{e_\Delta[(\alpha_D + r)\overline{P} - \overline{D}]}{(\alpha_D + r)\overline{\Omega}}, \qquad c_\Lambda = \frac{e_\Lambda[(-z + r)\overline{P} - \overline{D}]}{(-z + r)\overline{\Omega}}.$$

For $\det(\mathbf{C}) \neq 0$ we can invert $\mathbf{C}$ and obtain a unique solution for $\mathbf{p}$. Finally, the coefficient p_0 is uniquely determined by equation (6).

Hence the uniqueness of the equilibrium.

Proposition 6:

We have aleady shown that

$$p_F = 1; e_\Delta < 0; p_\Delta > 0$$

and we have trivially that:

$$p_0 < 0$$

Proof that $e_\Delta \sigma_D + e_\Lambda < 0$:

The symmetry of the model implies $\mathcal{E}_t(dE_t dR_t^h) = -\mathcal{E}_t(dE_t dR_t^f)$. Furthermore,

$$\mathcal{E}_t(dE_t dR_t^h)/dt = (e_\Delta \sigma_D + e_\Lambda) \left[f_D \sigma_D + 2 \left(p_\Delta \sigma_D + p_\Lambda \right) \right] < 0$$

amounts to showing that $\overline{e} \equiv e_\Delta \sigma_D + e_\Lambda < 0$ as long as $f_D \sigma_D + 2 \left(p_\Delta \sigma_D + p_\Lambda \right) > 0$. To simplify notation we define

$$k_1 = \frac{(\overline{KD} - \alpha_D \kappa)\overline{P}}{(\overline{D} + \alpha_D \overline{P})}, \qquad k_2 = \frac{(\overline{KD} + z\kappa)\overline{P}}{(\overline{D} - z\overline{P})}.$$

Clearly, $k_1 < 0$ and $k_2 < 0$ under the parameter constraints of proposition 5. Moreover, $k_1 - k_2 < 0$, because (for $\alpha_D > -z$) we find

$$(\overline{D} - z\overline{P})(\overline{KD} - \alpha_D \kappa) - (\overline{D} + \alpha_D \overline{P})(\overline{KD} + z\kappa) = -(\alpha_D + z) \left[\overline{D}\kappa + \overline{PKD} \right] < 0.$$

Substituting equations (10) and (11) into (12) implies

$$e_\Delta \sigma_\Delta \left[\kappa + k_1 \right] + e_\Lambda \left[\kappa + k_2 \right] = \frac{-\overline{KP}\sigma_\Delta}{(\overline{D} + \alpha_D \overline{P})} < 0.$$

Subtracting the term $e_\Delta \sigma_\Delta (k_1 - k_2) > 0$ (because $e_\Delta < 0$) from the left hand side implies

$$e_\Delta \sigma_\Delta \left[\kappa + k_2 \right] + e_\Lambda \left[\kappa + k_2 \right] < 0$$

and also

$$e_{\Delta}\sigma_{\Delta} + e_{\Lambda} < 0$$

since $\kappa + k_2 > 0$ is trivially fulfilled (for $\kappa > 0, \overline{K} > 0, \overline{D} > 0, \overline{P} > 0$).

It is also possible to prove that:

$$p_{\Delta}\sigma_{D} + p_{\Lambda} > 0$$

Table 1: Realized Monthly Correlations and Volatility Ratios for Daily Returns

Reported are average realized monthly correlations of daily (log) equity index returns (in local currency) with (log) dollar exchange rate returns, $-Corr(dE_t dR_t^f)$, and realized monthly volatility ratios (in standard deviations), $\sqrt{Var(dE_t)}/\sqrt{Var(dR_t^f)}$, for various time periods. Note that the exchange rate E is expressed in foreign currency per dollar ($dE > 0$ corresponds to a dollar appreciation), and the index f represents one of the 14 foreign countries. The model predicts $-Corr(dE_t dR_t^f) < 0$. We denote *** significance at the 1% level, ** at the 5% level and * at the 10% level for the first 3 columns.

Country	(a)	(b)	(c)	(d)	(e)
	1/1/80-1/1/02	1/1/90-1/1/02	1/1/95-1/1/02	1/1/80-1/1/02	1/1/95-1/1/02
	$-Corr(dE_t dR_t^f)$	$-Corr(dE_t dR_t^f)$	$-Corr(dE_t dR_t^f)$	$\sqrt{\frac{Var(dE_t)}{Var(dR^f)}}$	$\sqrt{\frac{Var(dE_t)}{Var(dR_t^f)}}$
Australia	0.0966***	0.0564***	0.0694***	0.5715	0.7278
Austria	-0.0697**	-0.1220***	-0.1781***	0.9371	0.6962
Canada	0.0701***	0.0617***	0.1008***	0.1626	0.1922
Denmark	-0.1069***	-0.1615***	-0.2268***	0.7552	0.6316
France	-0.1349***	-0.2424***	-0.3022***	0.6811	0.5654
Germany	-0.089***	–0.1867***	-0.2748***	0.7154	0.5849
Italy	-0.0645***	-0.0782***	-0.1316***	0.5834	0.5014
Japan	0.0460***	0.0161	-0.0159	0.7194	0.6668
Netherlands	-0.2462***	-0.3383***	-0.3476***	0.7331	0.6241
Norway	-0.0799***	-0.1392***	-0.1342***	0.5337	0.5781
Spain	-0.1223***	-0.1875***	-0.2261***	0.6724	0.5470
Sweden	-0.0705***	-0.0599***	-0.0876***	0.5419	0.5016
Switzerland	-0.1612***	-0.2748***	-0.3300***	0.9177	0.7372
U.K.	-0.0487***	-0.1734***	-0.2150***	0.6874	0.4960

Table 2: Monthly Correlations and Volatility Ratios

Reported are correlations of monthly (log) stock returns (in local currency) with monthly (log) exchange rate returns, $-Corr(dE_t dR_t^f)$,and volatility ratios as the standard deviations of monthly returns, $\sqrt{Var(dE_t)}/\sqrt{Var(dR_t^f)}$, for various time periods. Note that the exchange rate E is expressed in foreign currency per dollar ($dE > 0$ corresponds to a dollar appreciation), and the index f represents one of the 14 foreign countries. The model predicts $-Corr(dE_t dR_t^f) < 0$. We denote *** significance at the 1% level, ** at the 5% level and * at the 10% level for the first column.

Country	(a)	(b)	(c)	(d)	(e)
	1/74-12/01	1/90-12/01	1/95-12/01	1/74-12/01	1/95-12/01
	$-Corr(dE_t dR_t^f)$	$-Corr(dE_t dR_t^f)$	$-Corr(dE_t dR_t^f)$	$\sqrt{\frac{Var(dE_t)}{Var(dR_t^f)}}$	$\sqrt{\frac{Var(dE_t)}{Var(dR_t^f)}}$
Australia	0.2872***	0.2774	0.34	0.4702	0.7691
Austria	-0.1280**	-0.2586	-0.2765	0.5734	0.4927
Canada	0.3568***	0.3844	0.4641	0.2569	0.2536
Denmark	-0.2407***	-0.3404	-0.4115	0.6248	0.4955
France	-0.0875	-0.3589	-0.3720	0.5107	0.4888
Germany	-0.1422**	–0.2965	-0.3856	0.5931	0.4382
Italy	-0.0928	-0.1652	-0.2439	0.4415	0.3619
Japan	0.0776	0.0300	-0.1192	0.6589	0.7703
Netherlands	-0.2789***	-0.3690	-0.3634	0.6449	0.5257
Norway	-0.0858	-0.1411	-0.0015	0.3809	0.4147
Spain	-0.0828	-0.2053	-0.2718	0.5034	0.3972
Sweden	-0.1536***	-0.1994	0.007	0.4546	0.3428
Switzerland	-0.2183***	-0.3334	-0.3262	0.7277	0.5785
U.K.	-0.0437	-0.2344	-0.2805	0.5104	0.5545

Table 3: Daily Correlations and Volatility Ratios

Reported are correlations of daily (log) stock returns (in local currency) with monthly (log) exchange rate returns, $-Corr(dE_t dR_t^f)$,and volatility ratios as the standard deviations of monthly returns, $\sqrt{Var(dE_t)}/\sqrt{Var(dR_t^f)}$, for various time periods. Note that the exchange rate E is expressed in foreign currency per dollar ($dE > 0$ corresponds to a dollar appreciation), and the index f represents one of the 14 foreign countries. The model predicts $-Corr(dE_t dR_t^f) < 0$. We denote *** significance at the 1% level, ** at the 5% level and * at the 10% level for the first column.

Country	(a)	(b)	(c)	(d)	(e)
	1/1/80-1/1/02	1/1/90-1/1/02	1/1/95-1/1/02	1/1/80-1/1/02	1/1/95-1/1/02
	$-Corr(dE_t dR_t^f)$	$-Corr(dE_t dR_t^f)$	$-Corr(dE_t dR_t^f)$	$\sqrt{\frac{Var(dE_t)}{Var(dR_t^f)}}$	$\sqrt{\frac{Var(dE_t)}{Var(dR_t^f)}}$
Australia	0.0874***	0.0677	0.0825	0.5978	0.7230
Austria	-0.0685***	-0.0993	-0.1854	0.7381	0.5697
Canada	0.1011***	0.0877	0.1167	0.3098	0.2652
Denmark	-0.0819***	-0.1319	-0.1935	0.6786	0.5185
France	-0.14***	-0.2337	-0.3054	0.5915	0.4579
Germany	-0.1197***	–0.1635	-0.2676	0.5788	0.4252
Italy	-0.0649***	-0.0828	-0.1897	0.4810	0.3998
Japan	0.0630***	0.0602	0.0330	0.5927	0.5954
Netherlands	-0.2211***	-0.3103	-0.3352	0.6175	0.4596
Norway	-0.0921***	-0.1437	-0.0880	0.4333	0.4699
Spain	-0.1165***	-0.1813	-0.2317	0.5648	0.4186
Sweden	-0.0838***	-0.0601	-0.0380	0.4666	0.3474
Switzerland	-0.1979***	-0.2815	-0.3443	0.7666	0.5685
U.K.	-0.0359***	-0.1674	-0.2223	0.6577	0.4398

Table 4: Monthly Correlations of Foreign Inflows and Equity Returns

Reported are correlations of net U.S. purchases of foreign equities with U.S. stock returns, $Corr(dK_t^f dR_t^h)$, and with foreign stock returns (in local currency), $Corr(dK_t^f dR_t^f)$ for various time periods. Net purchases are normalized by market capitalisation. The theory predicts $Corr(dK_t^f dR_t^h) > 0$ and $Corr(dK_t^f dR_t^f) < 0$.

Country	(a)	(b)	(c)	(d)	(e)
	1/78-12/01	1/90-12/01	1/95-12/01	1/78-12/01	1/90-12/01
	$Corr(dK_t^f dR_t^h)$	$Corr(dK_t^f dR_t^h)$	$Corr(dK_t^f dR_t^h)$	$Corr(dK_t^f dR_t^f)$	$Corr(dK_t^f dR_t^f)$
Australia	0.0458	0.0579	0.0686	-0.0534	-0.1224
Austria	-0.0327	-0.0220	-0.0955	-0.1168	-0.1576
Canada	0.0242	-0.0350	-0.0639	0.1681	0.1286
Denmark	0.0420	-0.0135	0.0412	-0.0388	-0.0979
France	0.1860	0.1513	0.1609	0.1201	0.0465
Germany	0.0337	-0.0532	-0.0415	0.2541	0.1963
Italy	0.0108	-0.0087	0.1071	0.1691	0.1876
Japan	0.2320	0.2338	0.31	0.3241	0.4110
Netherlands	-0.0051	-0.1819	-0.1245	0.1993	0.0972
Norway	0.0217	0.0211	0.0714	0.0384	0.0391
Spain	0.0852	-0.0347	-0.0233	0.1090	0.0559
Sweden	0.1233	0.1134	0.2145	0.1432	0.1451
Switzerland	-0.0476	0.0463	0.0156	0.0822	0.2163
U.K.	0.0418	0.0413	0.0541	0.0561	0.1190

Table 5: Monthly Correlations of Portfolio Shifts and Relative Equity Returns

Reported are correlations of net foreign stock ownership increase by U.S. residents, $dNF_t = dK_t^f - dK_t^{h*}$ (defined as net U.S. purchases of foreign equities minus net foreign purchases of U.S. equities), with relative equity return on the foreign index, $d\Delta R_t = dR_t^f - dR_t^h$, and the exchange rate return, dE_t. The theory predicts $Corr(dNF_t, d\Delta R_t) < 0$ and $-Corr(dNF_t, dE_t) > 0$.

Country	(a)	(b)	(c)	(d)
	1/78-12/01	1/95-12/01	1/78-12/01	1/95-12/01
	$Corr(dNF_t,\, d\Delta R_t)$	$Corr(dNF_t,\, d\Delta R_t)$	$-Corr(dNF_t,\, dE_t)$	$-Corr(dNF_t,\, dE_t)$
Australia	-0.0118	-0.1494	0.0124	-0.0010
Austria	-0.1401	-0.0513	-0.1124	0.2740
Canada	0.0468	0.0621	0.0669	0.0942
Denmark	-0.0446	-0.0040	-0.0881	-0.0295
France	-0.0014	-0.1983	0.1139	0.1814
Germany	0.0563	-0.0790	-0.0882	0.1210
Italy	0.0267	-0.0749	-0.0209	0.1936
Japan	0.0887	0.3261	0.0539	-0.0620
Netherlands	0.1421	0.2031	-0.0585	-0.0279
Norway	0.0511	0.0923	0.0453	-0.0125
Spain	-0.0685	-0.0028	0.0553	0.1939
Sweden	0.0581	-0.0054	0.0369	0.3620
Switzerland	0.0618	0.0036	0.1374	0.3052
U.K.	-0.0055	-0.0097	0.0565	0.0716

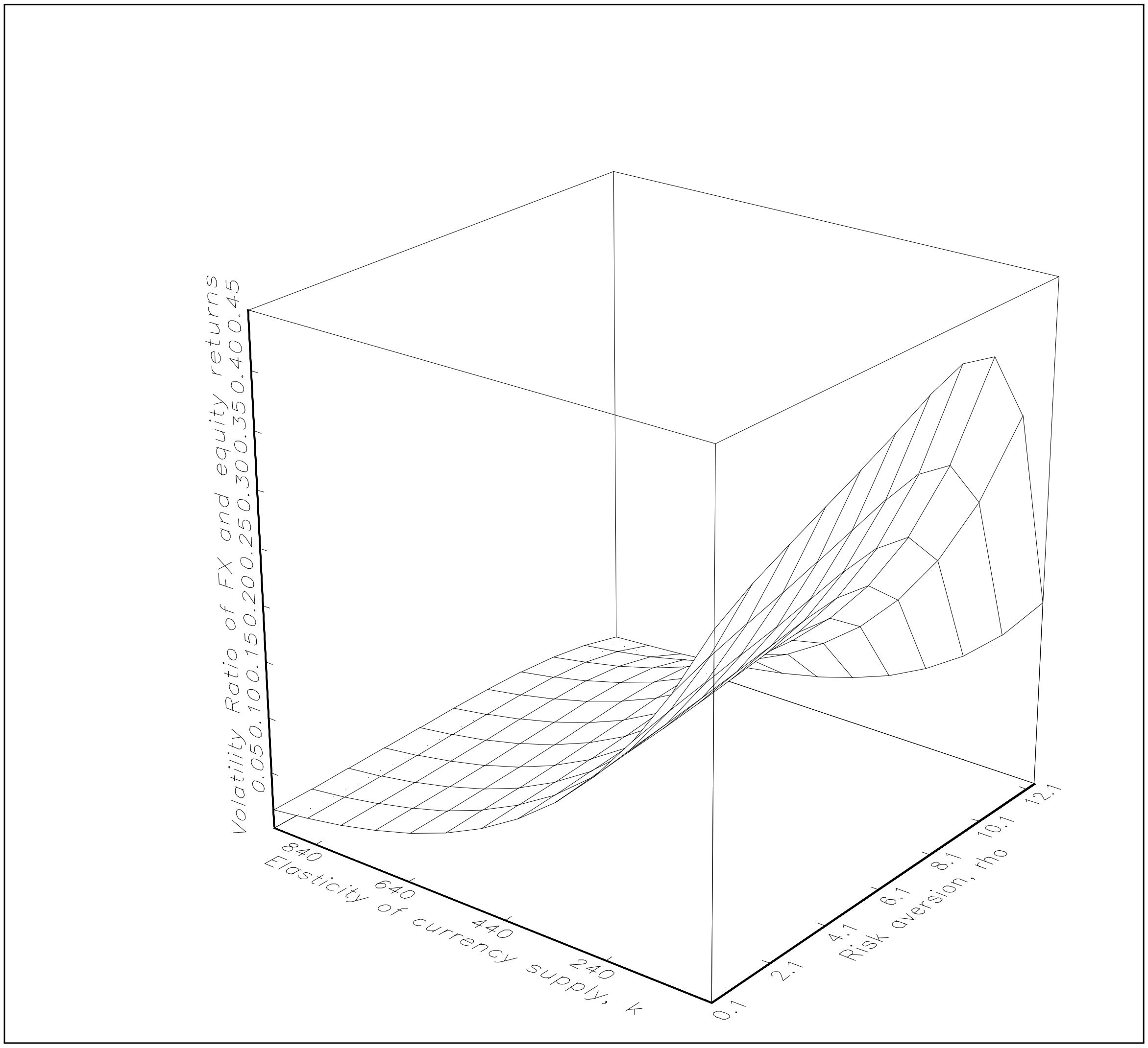

Figure 1: The volatility ratio of forex returns and equity returns is plotted for investor risk aversion parameters $0.1 < \rho < 12$ and an elasticity of forex liquidity supply $40 < \kappa < 900$. The dividend process parameters are $\overline{D} = 1$, $\alpha_D = 0.25$ and $\sigma_D = 0.1$.

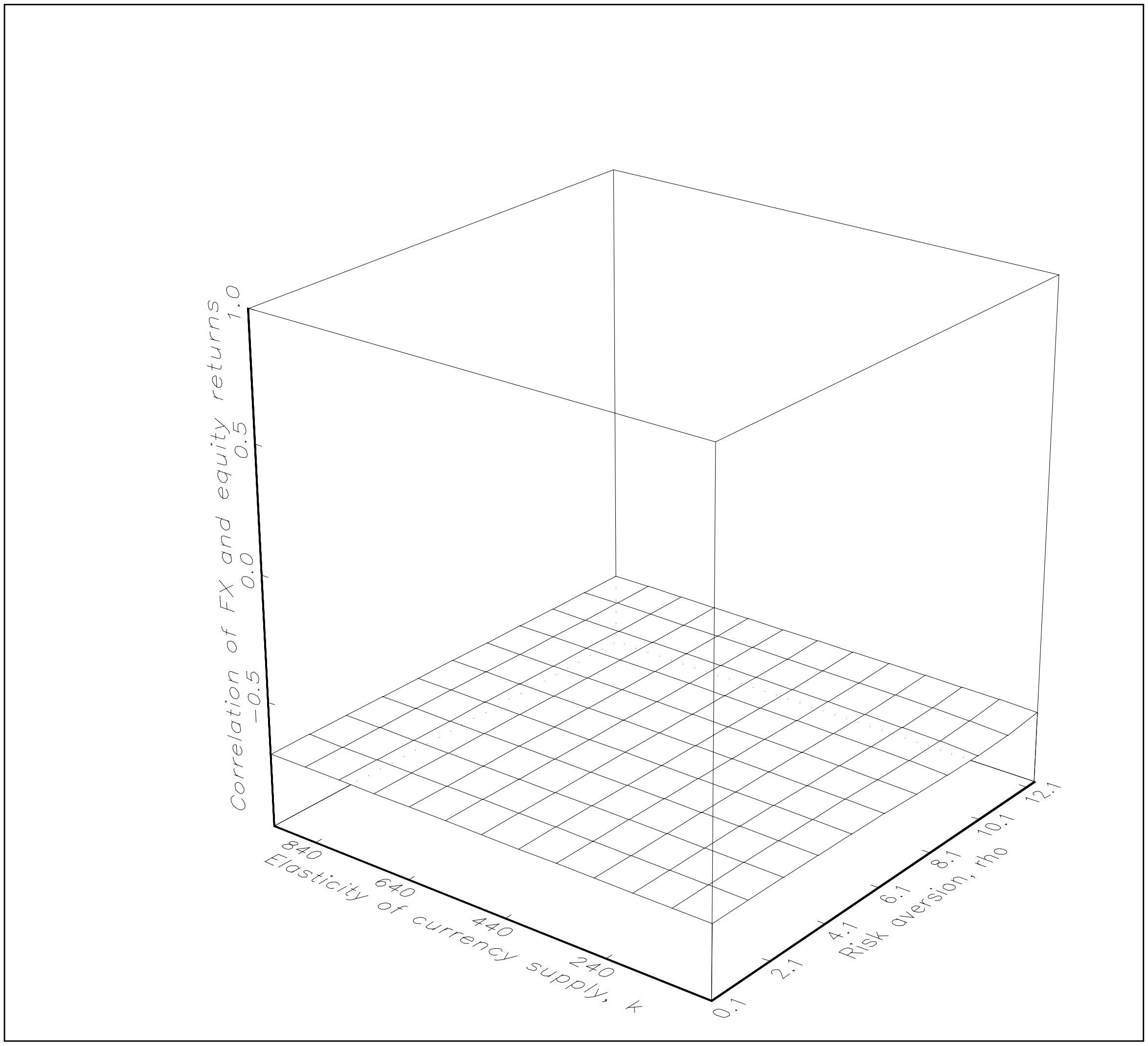

Figure 2: The correlation $Corr[dE_t dR_t^h]$ between forex returns and equity returns is plotted for investor risk aversion parameters $0.1 < \rho < 12$ and an elasticity of forex liquidity supply $40 < \kappa < 900$. The dividend process parameters are $\overline{D} = 1$, $\alpha_D = 0.25$ and $\sigma_D = 0.1$.

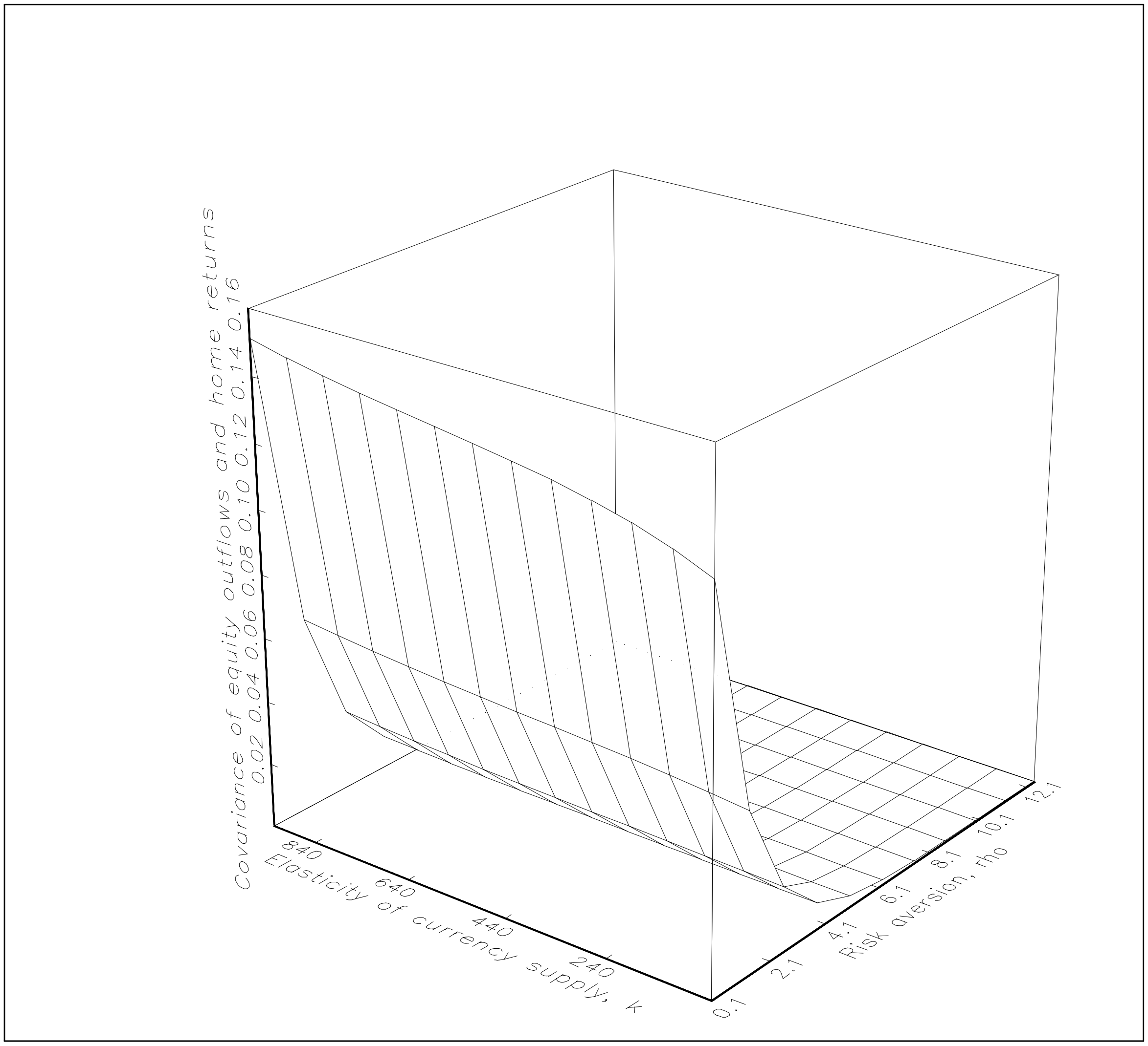

Figure 3: The covariance between equity market inflow dK_t^f and the home market return $dP_t^h/\overline{P}$ is plotted for risk aversion parameters $0.1 < \rho < 12$ and an elasticity of forex liquidity supply $40 < \kappa < 900$. The dividend process parameters are $\overline{D} = 1$, $\alpha_D = 0.25$ and $\sigma_D = 0.1$.

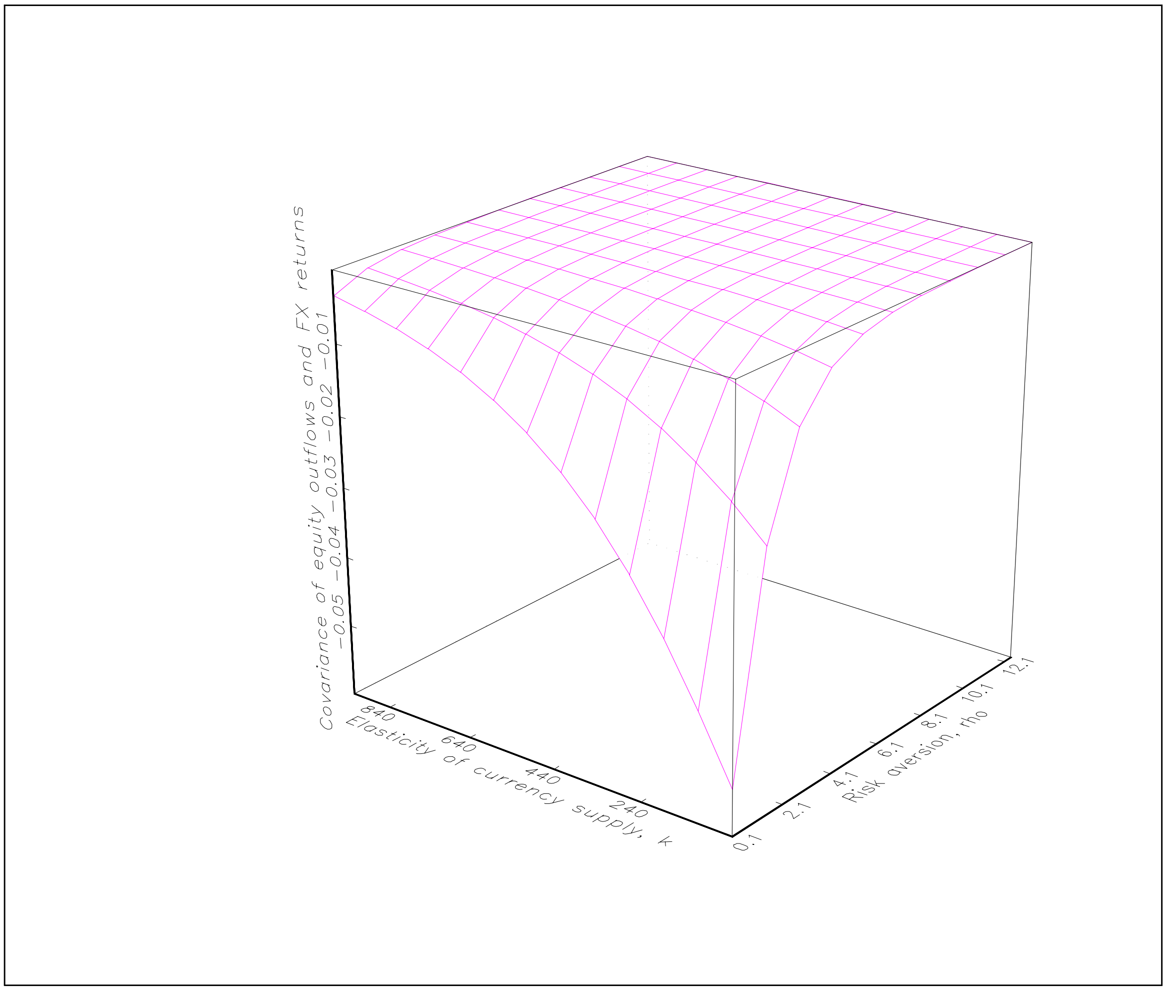

Figure 4: The covariance between foreign inflows dK_t^f and dE_t is plotted for risk aversion parameters $0.1 < \rho < 12$ and an elasticity of forex liquidity supply $40 < \kappa < 900$. The dividend process parameters are $\overline{D} = 1$, $\alpha_D = 0.25$ and $\sigma_D = 0.1$.