Life Cycle Consumption and Portfolio Choice with Additive Habit Formation Preferences and Uninsurable Labor Income Risk.*

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Abstract

I consider a life cycle model of consumption, savings and portfolio allocation for a household with additive and endogenous habit formation preferences. When the exogenous state variable follows a finite–state Markov process, I characterize analytically the admissible habit–wealth region for every age and solve the problem numerically in this region. The boundary of the admissible region depends on the *worst possible path* of future labor income and on the habit persistence parameter and *does not* depend on the probability of the worst case path. The distance to the admissible boundary and the current habit jointly determine the optimal savings policy and portfolio allocation. This feature of the model may be helpful in addressing several standing issues in the life cycle portfolio choice literature. For example, it may shed light on the slow decumulation of savings by the old, if one considers a possibility of drastic reduction of disposable income due to sudden increase in nondiscretionary expenses (e.g. on health care). It may also help, with the introduction of severe (but unlikely) income shock due to unemployment or disability, to rationalize conservative equity portfolios and limited stock market participation observed in the data.

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1 Introduction

Additive habit formation preferences play an important role in the recent efforts to build empirically plausible model of asset prices.[1] Because the tractable general equilibrium models with additive habit formation are confined to the representative agent setting, the behavior of the individual agents implied by the additive habit formation preferences is not well understood. This paper fills the gap by considering a partial equilibrium problem of life cycle choice of consumption, savings and portfolio allocation for a household with additive and endogenous habit formation preferences.

[... to be added later ... On the rationale and the origin of the habit formation utility, empirical work supporting/rejecting nonseparabilities.]

The model in this paper brings together three key elements so far analyzed separately by a number of studies. First important feature of the model is that a household receives age-dependent labor income which is subject to uninsurable risk. This follows the tradition of the buffer stock theory of savings as, for example, in Deaton (1991). More recent research on the topic also includes the analysis of portfolio allocation in addition to the consumption and savings theme, as, for example, in Heaton and Lucas (1997), Viceira (2001), Cocco, Gomes, and Maenhout (1999), Gomes and Michaelidis (2001). The other two key features of the model are the finite horizon and the habit formation preferences with endogenous and additive habit. Although a number of theoretical analyses of portfolio choice with habit formation preferences is attempted so far, none of them, except Gomes and Michaelidis (2002), has considered habit formation in a life-cycle setting. For example, an infinite horizon complete-markets case is analyzed by Hindy, Huang, and Zhu (1997) who also study the effect of durability of consumption on portfolio allocation. Heaton and Lucas (1997) contains a treatment of the infinite horizon case with labor income uncertainty. Gomes and Michaelidis (2002) consider a life-cycle model with costly stock market participation and focus on the

[1] This is large and rapidly growing area of asset pricing finance. See, for example, Sandaresan (1989), Constantinides (1990), Detemple and Zapatero (1991), Heaton (1995), Chapman (1998), Campbell and Cochrane (1999).

case with the "catching-up-with-the-Joneses" ratio-habit preferences.[2]

Under the assumptions that current habit is a multiple of consumption in the previous period and that exogenous state variables follow a finite-state Markov process, I characterize analytically the admissible cash-habit region for every age and solve the problem numerically in this region. The boundary of the admissible region depends on the *worst possible* future path of labor income and on the habit persistence parameter. The feasibility constraints *do not* depend on the probability of the worst case scenario, they only depend on how low the labor income can fall. Thus, the worst case scenario of labor income plays a special role in this problem. This feature of the model may be very helpful in addressing several problems encountered so far by the life cycle portfolio choice models, among these problems are: explaining conservative equity holdings, limited stock market participation across all ages, and slow decumulation of savings.

The intuition for the feasibility constraints is as follows. When household has preferences with the endogenous and additive habit, it can not have too high consumption today, even if this consumption is within today's budget constraint. The reason is that consuming too much today may generate an unsustainable habit sequence along the low labor income path in the future. This constraint on consumption is tighter the lower the present value of earnings along the worst possible labor income path and the higher the habit persistence parameter. The boundary of the admissible cash–habit region corresponds to the *infimum* of the admissible cash for any given habit and the marginal utility at the boundary is infinite. The optimal savings and portfolio choice policies depend on how much cash is accumulated in excess of the infimum and on the current habit level. This introduces additional dynamics in wealth accumulation and portfolio allocation due to the time variation in risk aversion and the elasticity of intertemporal substitution.

The rest of the paper is organized as follows. In section 2 I set up the life cycle model with habit formation utility and derive the feasibility constraints. Calibrations are described in section 3. The results of the simulations are discussed in section 4 and section 5 con-

[2]Gomes and Michaelidis (2002) also report the results for some calibrations with the additive habit preferences but without a detailed analysis of the problem which is the main goal of the present paper.

cludes. Appenidx A contains a formal derivation of the feasibility constraints and appendix B describes the numerical method used to solve the model.

2 A life-cycle model

2.1 Household problem

There is one agent (household) in the model who lives a finite life with the deterministic horizon from t_0 to T and is characterized by the following habit formation utility function defined over the admissible consumption sequences $\{c_t\}_{t=t_0}^{T}$ s.t. $c_t > h_t \geq 0$: [3]

$$U = E_{t_0}\left\{\sum_{t=t_0}^{T}\beta^{t-t_0}\frac{(c_t-h_t)^{1-\gamma}}{1-\gamma}\right\}, \tag{1}$$

where $\beta < 1$ is the subjective discount factor, $\gamma > 0$ is the curvature parameter, and h_t denotes the time-t habit. Consumption paths such that $c_t \leq h_t$ for some t in some state with nonzero probability are assigned infinitely negative value of the expected utility. I assume that the habit is internal and depends only on one lag of consumption with the persistence parameter δ:

$$h_t = \delta c_{t-1}\ , \ \ h_{t_0} = h^0. \tag{2}$$

The household receives stochastic labor income y_t, which is a product of the deterministic age-dependent component $f(t)$ and the stochastic (transitory) shock $\tilde{\eta}_t$:

$$y_t = \tilde{\eta}_t f(t) \tag{3}$$

In the calibrations it will be assumed that during the retirement years y_t is nonstochastic.

There are two securities available to the household for saving: a risk-free bond and a risky stock. The bond pays a constant interest rate r_b and the stock represents a market index and pays stochastic return $r_{s,t}$. Neither security can be sold short.[4] Let the purchases

[3]There is an underlying probability space (Ω, F, P), where $F = \{F_\tau\}_{\tau=t_0}^{T}$ is a sequence (filtration) of the sigma algebras generated by the exogenous random variables (states) in the state vector z_t which is defined later. It is assumed that at any given time period τ, all control variables, i.e., consumption and asset holdings, are measurable functions with respect to F_τ. The outcomes of the exogenous variables Ω and the probability measure P are calibrated in section 3.

[4]For stocks this is not a binding constraint, for bonds, it depends on the calibration.

of bond and stock at time t be denoted b_t and s_t respectively. The total cash x_t available to the household at time t is defined as follows:

$$x_t = s_{t-1}(1 + r_{s,t}) + b_{t-1}(1 + r_b) + y_t \tag{4}$$

At every period $t = t_0, \ldots, T$ the household chooses consumption and purchases of bond and stock subject to the following constraints:

$$\begin{aligned} x_t &= c_t + b_t + s_t \\ b_t &\geq 0\ ,\ \ s_t \geq 0 \\ c_t &> h_t\ ,\ \ t = t_0, \ldots, T \end{aligned} \tag{5}$$

While the first constraint is a common budget constraint, the last system of inequality constraints on consumption is specific to the problem with habit formation preferences. The household must choose consumption so that it does not fall short of habit at any time regardless of the random realizations of the exogenous variables. In the next section, and formally in appendix A, I discuss how these constraints may be rewritten to make their economic intuition more transparent.

The state vector for the household problem is defined as $z_t = [h_t, x_t, r_{s,t}, \tilde{\eta}_t, t]'$. The first two state variables are endogenous and the last three are exogenous. The household problem is to find the optimal sequences of consumption $\{c_t(z_t)\}$, bondholdings $\{b_t(z_t)\}$, and stockholdings $\{s_t(z_t)\}$ as functions of the state z_t for $t = t_0, \ldots, T$ subject to the constraints 5 to maximize the expected utility (1). For numerical solution it is more convenient to write the household problem as a sequence of stochastic Bellman equations. Let $V_t(z_t)$ denote the value function at time t and state z_t. Then, the household problem can be written as the following sequence of maximizations:

$$\begin{aligned} V_t(z_t) &= \max_{c_t, s_t, b_t} \frac{(c_t - h_t)^{1-\gamma}}{1-\gamma} + \beta E_t \left\{ V_{t+1}(z_{t+1}) \right\}, \\ &\quad \text{for } \ t = t_0, \ldots, T, \text{ s.t. (2), (4), and (5)} \\ V_{T+1} &\equiv 0 \end{aligned} \tag{6}$$

I assume no bequests, but the bequest function may be easily accommodated in (6) by appropriately defining V_{T+1}. Assuming that the exogenous variables in z_t follow a finite-state Markov process, I solve the problem (6) numerically and simulate the cross-sectional distributions of the variables of interest. Appendix B provides the details of the numerical solution.

2.2 Characterizing the feasible set of the endogenous variables

Constraints (5) restrict the feasible pairs of the endogenous variables (x_t, h_t) and contain important information about the economic structure of the problem. It is also important to analyze these constraints to develop the efficient numerical method. In appendix A I derive formally the restrictions on the admissible cash-habit pairs when the exogenous variables follow a finite-state Markov process. In this section I describe the intuition of the derivation and the economic interpretation of the constraints.

Constraints (5) imply that $x_t > h_t$ for all t, i.e., there must be sufficient total cash strictly exceeding the current habit. It should be noted that at any time t the constraints imply more than just $c_t > h_t$. It is not enough to restrict the household to consume today above today's habit. Consuming too much today may generate infeasible habits in the future. For example, consider a policy to consume all resources today, i.e., set $c_t = x_t > h_t$ $(t < T)$. If the following inequality holds:

$$\min_{\tilde{\eta}_{t+1}} y_{t+1} \leq h_{t+1} = \delta x_t, , \tag{7}$$

then in the worst case income shock tomorrow, the household does not have enough resources to consume above habit. When (7) is satisfied with equality, the marginal utility is infinite. Thus, increasing consumption too much today may be inadmissible in the worst case scenario tomorrow.

To characterize the admissible endogenous variables I start at time T and proceed backwards. At time T, there is no need to look forward anymore and the only constraint is to have cash above habit. The border of the admissible set for the endogenous variables is $x_T = h_T$. At time $T-1$, the household must have enough resources for the current habit, thus part of the border of admissible set is given by $x_{T-1} = h_{T-1}$. Another restriction emerges from the requirement that there should exists a savings and a portfolio policy at $T-1$ such that in the next period, under any realizations of the exogenous variables, they guarantee cash above habit. Appendix A shows that this requirement restricts the admissible pairs (x_{T-1}, h_{T-1}) to the following set:

$$\frac{\underline{y}_T}{1+r_b} + x_{T-1} - \left(1 + \frac{\delta}{1+r_b}\right) h_{T-1} > 0, \tag{8}$$

where $\underline{y}_T$ is the lowest possible labor income at T. In deriving the equation (8), I assume that the worst possible outcome for total cash is when the labor income has the lowest possible value and the stock return is negative. If habit and cash at time $T-1$ satisfy (8) with equality, the marginal utility is infinite. This is because even if at $T-1$ consumption is set equal to habit, leaving the *maximum possible* savings for tomorrow, there won't be enough cash in the worst case at time T to consume above habit no matter what portfolio is chosen at $T-1$. To see this, note that if $c_{T-1} = h_{T-1}$, then the household saves $x_{T-1} - h_{T-1}$. Assuming that in the worst state at T the stock return is negative, to *maximize* cash in the worst case the household should not buy any stock and invest only in bonds. Thus, the maximum cash in tomorrow's worst case is given by $\underline{x}_T = (1+r_b)(x_{T-1} - h_{T-1}) + \underline{y}_T$ and tomorrow's habit is $h_T = \delta h_{T-1}$. Note that we have $\underline{x}_T - h_T = 0$ if we assume (8) holds with equality. Thus, even if the household saves the maximum possible and in the safest possible portfolio, there will not be enough resources in the worst case outcome to exceed the habit at time T.

Constraints similar to (8) are derived in appendix A for a general time t. At every time t there are $T-t+1$ constraints as follows:

$$0 < \sum_{k=1}^{\tau} \frac{\underline{y}_{t+k}}{(1+r_b)^k} + x_t - h_t \left(\sum_{k=0}^{\tau} \left(\frac{\delta}{1+r_b} \right)^k \right) \ , \ \tau = 0, \ldots, T-t \tag{9}$$

where the convention is to set the first sum to zero for $\tau = 0$. For future reference, it is convenient to adopt the following notation:

$$Y_{t,t+\tau} = \sum_{k=1}^{\tau} \frac{\underline{y}_{t+k}}{(1+r_b)^k} \ , \ Y_{t,t} \equiv 0$$

$$\Delta_\tau = \sum_{k=0}^{\tau} \left(\frac{\delta}{1+r_b} \right)^k = \frac{1 - \left(\frac{\delta}{1+r_b} \right)^{\tau+1}}{1 - \frac{\delta}{1+r_b}}.$$

Then (9) can be compactly written as:

$$0 < Y_{t,t+\tau} + x_t - h_t \Delta_\tau \ , \ \ \tau = 0, \dots, T-t \tag{10}$$

The interpretation of (10) is very intuitive. Note that $Y_{t,t+\tau}$ is the time-t present value of the *minimum possible* labor income received between t and $t+\tau$. Tuse, the first two terms in the equation above correspond to the minimum possible present value of available resources for a given time horizon τ. On the other hand, $h_t\Delta_\tau$ is the *infimum* of the present value of the future habits up to time $t+\tau$ when the current habit is h_t. To see this, note that if the household consumes $h_{t+\tau} + \epsilon$ each period between t and $t+\tau$, where $\epsilon > 0$ is a very small number, the limit, as ϵ goes to zero, of the present value of habits generated by this consumption sequence is equal to $h_t\Delta_\tau$. The equation (10) is a present value constraint on cash and habits and it says that, if the household consumes close to the minimum possible and cautiously saves only in bonds, there must be enough assets today plus the future labor income in the worst case to cover the habit acquired up to today. Thus the consumption can not be decided based upon the budget constraint alone, the household must consider what habit the consumption will generate so that (10) is satisfied for all periods in the future, and in doing so the household will refer to the worst case scenario for labor income, regardless of its probability as long as the probability is positive.

2.3 When do the constraints on the endogenous states matter?

Two papers, Heaton and Lucas (1997) and Michaelidis and Gomes (2002), compute the solutions to the additive habit formation models with endogenous additive habits without refering to the constraints discussed above. Thus, it is instructive to examine under what conditions these constraints become important for the solution of the problem.

Any two consecutive constraints from (9) for τ and $\tau+1$ intersect at the point given by:

$$h^*_{t,\tau} = \frac{y_{t+\tau+1}}{\delta^{\tau+1}}$$

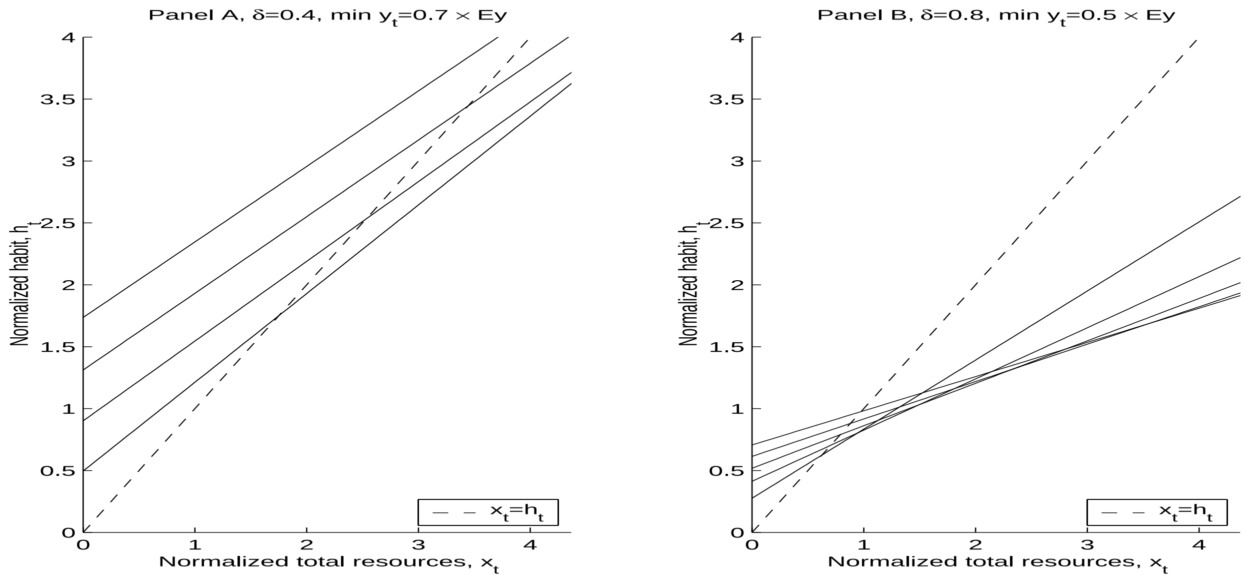

Figure 1: Constraints on endogenous state vaiables. The region below the low envelope of the lines defines admissible states.

$$x^*_{t,\tau} \;=\; \frac{y_{t+\tau+1}}{\delta^{\tau+1}}\Delta_\tau - Y_{t,t+\tau}$$

The first intersection for $\tau = 0$ occurs at the point $x^*_{t,0} = h^*_{t,0} = \frac{y_{t+1}}{\delta}$. And all the subsequent intersections lie to the northwest of this point. If the lower bound on labor income is high and the habit persistence is low, this intersection point will be far away from the actual habit level occuring in simulations. In this case, the only relevant constraint to keep track of is $x_t > h_t$. To see this, consider a numerical example shown on figure 1, where cash and habit are normalized by the expected labor income and the risk free interest rate is assumed to be $r_b = 1\%$. On panel A, the persistence parameter δ is 0.4 and the minimum labor income is 0.7 times its expected value. In this case, the first intersection of the constraints occurs at the point $(x^*_{t,0}, h^*_{t,0}) = (1.75, 1.75)$. These numbers are selected to be within the range of the calibrations reported by the two papers mentioned above. In the same calibrations, the simulated consumption is close to the average income, i.e. $c_t \approx 1$. Therefore, the steady habit level is on the order of 0.4 times the expected income. Thus, even the first intersection is far away from the range of realizations of the endogenous variables. For such parameter values, it is not necessary to pay attention to all constraints in (10). Since the endogenous variables in the computations are usually confined to a rectangle $[h_l, h_h] \times [x_l, x_h]$, if h_h is well below the first intersection, the only relevant constraint in this rectangle is $x_t > h_t$. As the lowest labor income realization falls or the persistence parameter increases, more constraints will be within a given rectangle.[5] Figure 1, panel B shows an example with the tighter constraints for $\delta = 0.8$ and the lowest possible income equal to 0.5 times the expected income. The lower envelope of all lines on figure 1 defines an infimum of cash necessary to sustain a given habit h_t. As discussed above, at this border, the expected marginal utility is infinite.

2.4 A convenient transformation

For computations, it is convenient to transform the cash variable x_t. I define $x'_t(x_t, h_t)$ as the distance, going horizontally, to the left from (x_t, h_t) to the lower envelope of the constraints. This distance corresponds to the difference between the actual x_t and the infimum of admissible resources that the household must have given the habit h_t:

$$x'_t = x_t - \max_{\tau} \{h_t \Delta_\tau - Y_{t,t+\tau}\} \ , \ \ x'_t > 0 \tag{11}$$

The feasibility constraint $x'_t > 0$ ensures that a pair (x'_t, h_t) corresponds to an admissible pair (x_t, h_t). The endogenous states used throughout the computations are (x'_t, h_t).

There are two advantages of using x'_t instead of the usual x_t, both are related to the computational properties of the problem. First, the transformation allows to use the standard fast interpolation routines for the rectangular regions rather than the slower routines for the irregularly shaped areas as those shown on figure 1. Second, the transformation helps to reduce the dimensionality of the problem. Usually the computational algorithm can reduce the number of grid points by using a nonuniform grid and by concentrating the grid points in the areas where the gradient of the value function is high. For this problem, the gradient of the value function is high everywhere in the vicinity of the lower envelope of the constraints

[5] At this point if one does not take into account these additional constraints, the numerical procedure fails. Consistent with that, Heaton and Lucas (1997) reported numerical difficulties when experimenting with the values of the habit persistence δ higher than 0.5 without taking into account the constraints other than $x_t > h_t$.

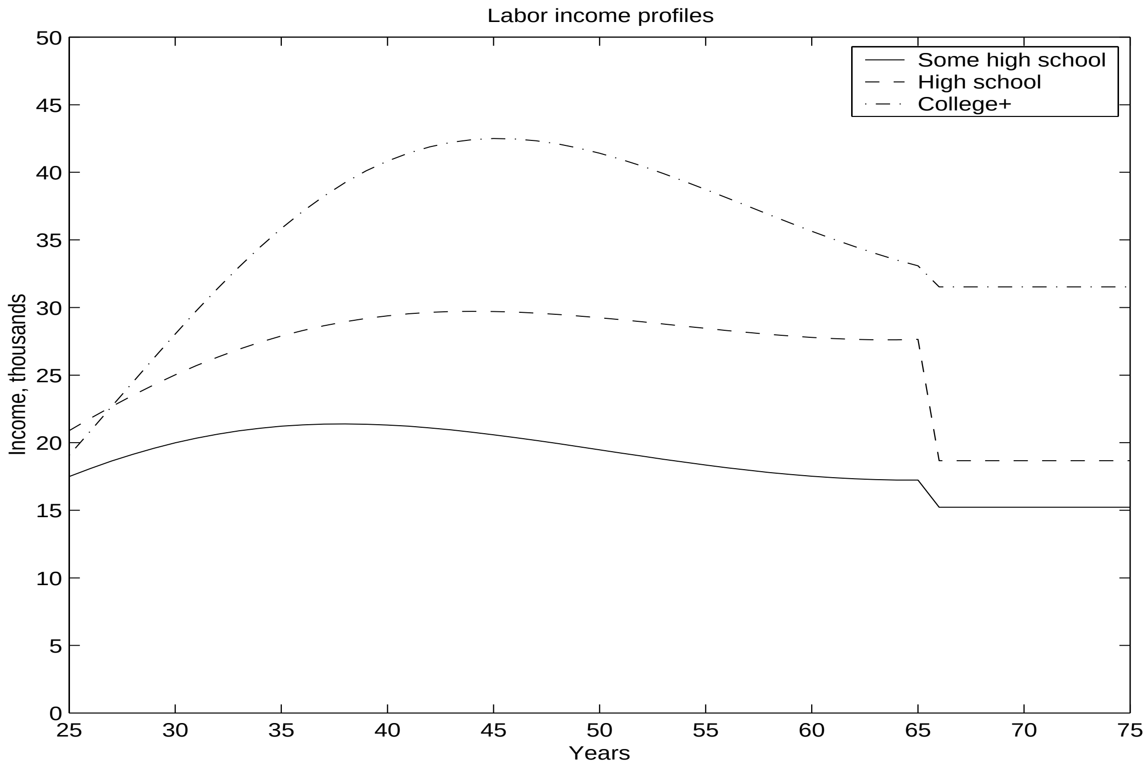

Figure 2: Age-income profiles from Cocco, Gomes and Maenhout (2000) for three educational cohorts.

in figure 1. Thus, if one were to use an untransformed x_t for the computations, the grid on x_t would have to be very dense throughout the chosen domain for x_t since the area with the high gradient of the value function depends jointly on cash and habit and stretches from low to high values of x_t. On the other hand, the transformation maps all the points that are close to the admissible border to the points with the low values of x'_t, allowing for the grid on x'_t to be more dense near zero and less dense away from zero.

The solutions to the stochastic Bellman equations (6) are computed on the grid in a rectangle $[0, h_h] \times [0, x'_h]$ using cubic approximations for the points that do not fall on the grid. Appendix B provides further details of the computational procedure.

3 Calibration of the model

The household is assumed to live from 25 till 75 years of age. The retirement begins at the age of 65 at which time the nonfinancial income y_t becomes certain. The age-income profile $f(t)$ is taken from Cocco, Gomes and Maenhout (2000). They estimate age-income profiles from the PSID data using cubic polynomials and report the estimated coefficients for three cohorts divided by education: high school drop-outs, high school graduates, and college graduates. Figure 3 shows the three profiles reconstructed using the tables from their paper. Unless otherwise noted, I use the age-income profile for high school graduates in the calibrations that follow. The utility function parameters are set as follows: $\beta = 0.95$ and $\gamma = 5$. Later I also consider $\gamma = 2$ in the comparative statics exercises. The habit persistence parameter δ is considered in the range of 0.5-0.9.[6]

From the earlier discussion it is clear that the distribution of income shock is crucial for the feasible set of the endogenous variables. Typically, the income shock $\tilde{\eta}_t$ is modeled as a finite-state Markov process with several symmetric outcomes around the mean of 1 to match the estimated transitory volatility of income.[7] For the problem with habit formation it is important to consider the skewness of the labor income shock distribution because the worst labor income realizations are important here. I assume that shock $\tilde{\eta}_t$ is i.i.d. and consider two different calibrations of the shock, symmetric and asymmetric. For the symmetric distribution, the transitory shock $\tilde{\eta}$ is assumed to have two values around the mean of 1 using the standard deviation of 27%.[8] Another calibration assumes the low outcome to be 0.3 and the high outcome to be 1.05. with the probabilities of 0.05 and 0.95 respectively. The current version of this calibration is only experimental and is not matching any empirical estimates. It is intended only as an illustration of the effect of the asymmetric

[6]Heaton and Lucas (1997) in the infinite horizon model and Gomes and Michaelidis (2001) in the finite horizon model use the values of the persistence below 0.5. They find that for these values of the persistence, the model with habit and the model without habit do not have significantly different implications for the portfolio choice, but that the wealth accumulation patterns are different across the two types of models.

[7]See, for example, Storesletten, Telmer and Yaron (1999), Cocco *et. al.* (2000), Heaton and Lucas (1997).

[8]This is the figure reported in Cocco, *et. al.* (2000). Heaton and Lucas (1996, 1997) and Storesletten, *et. al.* (1999) estimate somewhat higher volatility of the transitory shock.

shock. Next version of the paper will have an accurate calibration of the asymmetric income shock which would require more than one exogenous state for the shock variable.[9]

Stock returns are also calibrated to be i.i.d. Markov process with two symmetric outcomes around the mean. The mean is set at 6% and the standard deviation is 15%, thus the two outcomes for $r_{s,t}$ are -9% and 21%. Bond return is assumed to be constant and set at 1%.

For every calibration, a cross section of 10,000 households is simulated to compute the distributions for the variables of interest: wealth, consumption, and portfolio composition. The initial values for habit and financial wealth are set so that to be uniformly admissible in simulations under all reported parameter values. The initial habit for a given calibration is set to $12 \times \delta$, thus, the consumption in period $t_0 - 1$ is assumed to be 12 ($'000), uniformly in all calibrations. For such a habit to be admissible in all calibrations with the low labor income floor (asymmetric distribution case) and the high habit persistence δ, the initial wealth of all households is set to 15 ($'000). While this higher level of the initial wealth is only necessary for a few calibrations, it is maintained throughout the paper for the results to be comparable across different calibrations.[10]

4 Simulation results

4.1 Savings and portfolio policies

[... to be completed later ...]

To better understand the intuition for the results obtained in simulations, it helps to examine the computed savings and portfolio policies. Two main properties emerge throughout the life cycle:

[9] More realistic calibration exercise, which is obviously due here, requires a non-i.i.d. income process which increases the computational time by a multiple of the number of states which require separate value functions. As is, the i.i.d. case, the model takes about 7-10 hours to compute one calibration (depending on the workload) on a 4-processor SUN workstation. Given the tight deadlines for the conferences which this paper was prepared for, a more realistic calibration of the asymmetric shock case was postponed until the next draft.

[10] The alternative is to set the initial habit at a lower level so that the usual assumption of zero starting financial wealth is in the admissible range. However, this leads to unrealistically low consumption early in life for the simulations with high habit persistence.

1. The savings rate (out of the discretionary resources $x_t - h_t$) increases when the household is closer to the admissible region's boundary. As x'_t goes to zero, the savings rate approaches 1 asymptotically. Why?: The future expected marginal utility is higher near the admissible border and approaches infinity, thus it is optimal to save more and consume less today.

2. Portfolio allocation becomes more conservative closer to the admissible boundary, converging to 100% investment in the risk free asset. Why?: The risk aversion goes to infinity approaching the boundary. Stock is a risky asset and its presence in the portfolio decreases the *lower bound* on the future wealth. If the household invests too aggressively near the admissible boundary, there may be a chance of not having enough resources to consume above habit in the worst case scenario.

4.2 Symmetric income shock distribution

Figure 3 plots the means of the cross sectional distributions of financial wealth, consumption, and portfolio share invested in stocks. The results are reported for three values of the habit persistence parameter $\delta = 0.9$, 0.7, and 0.5. The effect of increasing δ is to increase wealth accumulation throughout the life cycle. This is consistent with the results reported in Gomes and Michaelidis (2001) for lower habit persistence. Other things equal, higher δ narrows the admissible region for the endogenous states, so that any endogenous state that is admissible under the two different values of δ becomes (weakly) closer to the boundary of the feasible region for the higher value of δ. Consistent with savings policies discussed above, this has an effect of increased savings out of the discretionary resources $x_t - h_t$ and faster wealth accumulation.

As a result of higher savings under more persistent habit, the consumption is lower early in life for higher values of δ. The consumption profiles are uneventful, except for the last period where there is a jump. The jump occurs because of the discrete change in the marginal utility from period $T - 1$ to the last period T. The marginal utility of consumption at time

$T-1$ is given by

$$(c_{T-1}-h_{T-1})^{-\gamma}-\beta\delta E_{T-1}\left\{(c_T-\delta c_{T-1})^{-\gamma}\right\},$$

while in the last period T, the future habit does not matter since household dies and the marginal utility of consumption at time T is given by:

$$(c_T-\delta c_{T-1})^{-\gamma}.$$

Since the second term in the marginal utility for $T-1$ is negative, optimally equating the marginal utility at $T-1$ to the expected marginal utility at T, in general, requires an increase in time-T consumption. Simulations show that this consumption increase is lower for higher values of δ. There are two opposing forces affecting the size of the terminal consumption jump. First, higher persistence means that the second term in the marginal utility at $T-1$ is higher and therefore it would take a larger change in c_T to equate the marginal utilities across T and $T-1$. Second, with the higher δ the difference (c_T-h_T) is smaller so that the *derivative* of the marginal utility is higher and as a result it takes a smaller change in c_T to satisfy the optimality condition. Simulations show that the second effect dominates for higher values of δ. Thus, the size of the consumption jump is nonmonotone in δ since the jump vanishes as δ goes to zero.[11]

The bottom panel of Figure 3 shows the portfolio allocation in stocks throughout the life cycle.[12] As expected from the earlier overview of the portfolio policies, higher habit persistence reduces the stockholdings. The figure shows a common problem of the life cycle portfolio choice models: the predicted share invested in stocks is 100% throughout most of the working life of the household. This effect is a well known feature of the portfolio choice models with labor income and was first emphasized by Merton (1971) in a model where a

[11]This jump feature is rather unrealistic and may be avoided by defining the last period value function as a weighted average of the bequest function and the last period utility. In this case the marginal utility at T would be multiplied by some weight $\lambda\in(0,1)$ and there would be an additional term multiplied by $1-\lambda$ where c_T would negatively affect the amount of bequest, thus, the jump in consumption c_T would have to be smaller to equate the marginal utilities across $T-1$ and T.

[12]Technically, in the last period the portfolio share in stocks (α) is undefined since there are no savings. On the figure α at time $T=75$ is set to zero.

constant labor income substitutes for bonds in the financial portfolio.[13] The equity-only prediction is not supported by the empirical evidence however. For example, Carroll and Samwick (1997) and Ameriks and Zeldes (2000) summarize the data on portfolio allocation of households. The data suggest that even conditional on nonzero stockholdings, household portfolios are far from 100% equity. As to the life cycle shape of the portfolio share in equities, the evidence from the above studies is mixed and depends on how the estimation procedures control for age, time or cohort effects.[14] Figure 3 shows that simulated portfolios become interior only later in life. There are two effects present here. First, as the amount of human capital decreases towards the end of the life cycle, households begin to shift their portfolio allocation towards bonds which were earlier crowded out by the human capital. The second effect is specific to the habit formation utility. As households deplete assets but maintain the consumption level (hence the habit level), the endogenous state variable moves closer to the boundary of the admissible set and the risk aversion increases implying more conservative portfolios. For $\delta = 0.5$ and 0.7 households wait almost till retirement to start shifting their portfolios away from stocks. For $\delta = 0.9$, they start earlier, at the age of 55, but the model's implication for the portfolios of the young is still unrealistic.

Perhaps the most important lesson from the calibrations considered in this section is that the additive habit preferences *per se* do not resolve the main problem of portfolio allocation encountered by the life cycle models with labor income and the CRRA or the Epstein and Zin (1992) recursive preferences.[15] Even for the higher levels of habit persistence, the model is unable to generate conservative portfolios early in the life cycle. However, the calibrations attempted so far do not exploit one of the key properties of the habit formation preferences that the lower bound on labor income affects savings and portfolio choice. The symmetric distribution of the income shock is only designed to match the transitory volatility of income

[13] Also, see Jagannathan and Kocherlakota (1996) for further discussion of this argument and an excellent survey of the related literature.

[14] See Carroll and Samwick (1997) and Ameriks and Zeldes (2000) for details

[15] Gomes and Michaelidis (2001) consider the case of recursive preferences and find that it is only marginally improving the performance of the model relative to the case with the CRRA utility and high risk aversion coefficient.

and overstates the worst case scenario. In the next section I explore how the model reacts to the possibility of a low labor income outcome.

4.3 Asymmetric income shock distribution

A typical uncertainty in the individual labor income considered in the life cycle literature is that due to unemployment. The parameters for the labor income process are usually estimated from the panel data on earnings and the empirical procedures use the averages across individuals to obtain stable estimates.[16] Such averaging may underestimate both the lower bound on nonfinancial income received during the periods of unemployment and the duration of unemployment spell. Thus, prolonged unemployment and a possible termination of unemployment benefits escape the usual calibration procedure. Also there is a risk from permanent income shocks which reduce labor income without a possibility of reversal.

Labor income risk aside, there are other risks which may significantly affect household earnings. Even if labor income is certain, a household may encounter unexpected nondiscretionary expenses that substantially reduce its disposable income. An example of such expenses is the out-of-pocket expenses on health care which are especially likely to occur later in the life cycle.[17] While public and private health insurance often cover such expenses, not all households have access to insurance and often there are limits on insurance which may require a contribution of personal funds. Another example of a major income shock is that due to disability. For reasons of moral hazard, disability is only partially insurable and may entail a permanent and significant reduction of household income.

All risks mentioned above reduce nonfinancial income in the worst case scenario and therefore may have a significant impact on consumption and savings in the model with habit formation preferences. In this section I examine the calibration where the lower realization of labor income could be 0.3 times the expected value with 5% probability and the income

[16] Such calibrations of the labor income shock distribution parameters are abundant in the literature see, for example, Heaton and Lucas (1996), Storesletten, Telmer and Yaron (1998), Cocco, Gomes and Maenhout (1999) and many others. The most common data used for this purpose is PSID (Panel Study of Income Dynamics).

[17] Also examples in that category are the expenses in excess of the limits covered by the auto or home insurance.

shock is a two-state iid process with the second state income of 1.05 times the expected value. This calibration is not matching any empirical estimates and is intended only as an illustration of the effect of lowering the floor on the labor income realization relative to the symmetric case considered previously.

Similarly to the symmetric distribution case, I compute the calibrations for three values of habit persistence $\delta = 0.9$, 0.7, and 0.5. Once again, the three panels on figure 4 show the averages of the cross sectional distributions of financial wealth, consumption and portfolio share in stocks. The consumption profiles are qualitatively similar to the symmetric case analyzed previously and I only discuss wealth profiles and portfolio choice for this calibration.

The comparative statics of wealth accumulation with respect to habit persistence is the same as before, higher value of δ implies higher savings rates. Note that relative to the symmetric distribution case on figure 3, households accumulate more wealth for any given age for all values of δ. This is to be expected, since lowering the floor on the labor income realizations narrows the admissible region for the endogenous variables and any given admissible pair (x_t, h_t) becomes (weakly) closer to the admissible boundary. This also affects the portfolio composition in an interesting way.

The portfolio share invested in stocks looks hump-shaped and for the values of $\delta \geq 0.7$ the average portfolios are in the interior throughout the life cycle. Early in life, households have lower wealth and are more risk averse. They invest more conservatively in stocks at this time because if the labor income suddenly falls, stocks may also loose value at the same time, leaving a household with little resources. As households build up wealth, their portfolios become more heavily invested in stocks, reaching the peak approximately at 35 years of age. For lower values of δ, the peak is 100% equities, but for higher persistence of 0.9, the peak is at about 75%. After the age of 35, for $\delta = 0.5$ and 0.7 the portfolios change little until about 60 years of age. For $\delta = 0.9$, the portfolio share slowly declines and then stabilizes at about 50% in stocks before the retirement and then gradually declines during the retirement years. There are two opposite effects that are important for portfolio allocation in the middle of life cycle in this calibration. One effect is that as consumption increases so does the habit and portfolio becomes more conservative. A counterforce to that is the increasing income profile which allows to accumulate larger wealth and hence invest more in stocks. The second effect dominates for lower values of δ and the first effect is stronger for higher values of δ.

Jagannathan and Kocherlakota (1996) discuss a popular portfolio advise given by the financial advisors to reduce the exposure to stocks with age. They conclude that if the labor income is not tradable and is not highly correlated with stock returns, such popular advise may have some merits. This is because if the income from human capital resembles more that from a bond, then, as people age, they have to compensate for the decrease of human capital by buying more bonds in the financial portfolio. Note that under the high habit persistence, the portfolio share in stocks also declines with age, as the popular advise suggests. Besides the human capital argument mentioned above, the model with habit formation offers an additional argument for why such pattern is optimal. As households age, their consumption and habit grow and, at the same time, wealth is declining towards the end of the life cycle. Both of these effects work to increase the risk aversion and imply more conservative portfolios later in life.

The results of the exercise with the asymmetric income shock show the potential of the model to account for a number of interesting phenomena and warrant further investigation. The key advantage of this model over the one with the traditional CRRA preferences is in using the worst case scenario in the future as a reference for the optimal savings and portfolio policies. In some sense, the worst case scenario for labor income calibrated in this section (0.3 times the expected income) was not as bad as it can be. For example, Carroll (1992) estimates that with a very small probability labor income could be zero at an annual frequency. Even if the possibility of no income has a very small probability, as long as it is nonzero, a household with habit formation preferences will use this scenario as a reference point. To explore the properties of the model in this direction in the next version I will attempt calibrations with:

1. Prolonged unemployement risk

2. Permanent income shocks

3. Disability risk

4. Health expenses risk

4.4 Costly equity holdings

[... calibrations for this section are not complete yet and will be added in the next version ...]

One of the implications of the model is that all households invest in stocks, which is at odds with the limited stock market participation. A body of empirical research, beginning with Mankiw and Zeldes (1991), shows that not all households invest in equities. While frictionless portfolio choice models are usually unable to generate limited participation, several recent papers have emphasized that costly stockholdings may be responsible for the empirical observations.[18] So far the model abstracts from the higher costs of holding equities relative to the more liquid risk-free asset. Since portfolio allocation in this model depends on wealth, costly stock holding not only will generate limited participation, it will also affect portfolio allocation throughout the life cycle. Households who do not participate in the stock market would accumulate wealth slower, reaching the participation threshold wealth later in the life cycle. Simulations discussed earlier show that portfolios become more conservative towards the end of the life cycle. Thus, costly participation has a potential not only to explain zero equity holdings in households portfolios but also to contribute towards explaining conservative portfolios conditional on participation.

To account for the costly participation, the budget constraint in the model is changed so that in each period when the household invests in stocks it pays a fixed cost ϕ_t:

$$c_t = x_t - b_t - s_t - \phi_t I_{\{s_t>0\}} \ ,$$

where $I_{\{.\}}$ denotes the indicator function which is equal to zero if the condition is false and is equal to one otherwise.[19] This cost function is different from the one-time participation cost

[18]For example Vissing-Jorgensen (2000), Gomes and Michaelidis (2001).

[19]The participation cost does not affect the admissible region constraints since they are computed using

assumed, for example, in a life cycle model in Gomes and Michaelidis (2001).[20] The motivation for the periodic costs comes from the empirical observations summarized in Carroll and Samwick (1997), Ameriks and Zeldes (2000) and Vissing-Jorgensen (2000). These studies show that households enter and exit stock market, and particularly that there is a pattern of exits towards retirement. The usual implication of the one-time participation cost is that once the cost is paid, a household always has some stocks until the end of life cycle. When the cost is paid each period, the hump-shaped wealth pattern will generate limited stock market participation not only early in life as in the one-time cost case, but also towards the end of life cycle.

The cost assumed here is fixed in a sense that it does not depend on portfolio size or turnover. I consider the cost which is a constant fraction of the expected labor income $\phi_t = \phi f(t)$, so that the cost varies throughout life cycle. The idea of this assumption is to capture the opportunity cost of time which will be larger for a household with higher earnings.[21]

[... The simulations with costly stock market participation are work in progress and the results will be prepared for the next draft. ...]

4.5 Some comparative statics

In this section I consider perturbations of the model to understand the sensitivity of the results to the assumed parameters. In the previous sections I already discussed the comparative statics with respect to the habit persistence δ. To see how sensitive the results are to

the portfolio strategies that maximize the worst outcome for disposable wealth and stocks are not held in such portfolio strategies. Also, since the computation of the optimal portfolio allocation is done by the grid search and is not using any gradient methods, nondifferentiability of the budget constraint is not a problem for the numerical procedure.

[20] In addition to the motivation for this cost outlined in the main text there is another one from the computational prospective. When the cost is periodic, there is no need to track separate value functions for participants and nonparticipants, the value function is independent of whether a households was stockholder in the previous period or not and only depends on the state vector z_t.

[21] This opportunity cost may be associated with the additional time spend filing taxes, communicating with investment agent, spending time in front of the computer screen to check how the stock investment is performing, etc. In addition to that there may be components of the cost that are truly fixed and do not depend on one's earnings, such as fixed mutual fund fees, account set up costs or fixed broker charges.

the curvature coefficient, I consider a lower value of $\gamma = 2$. I only consider the asymmetric income shock case since under the symmetric distribution assumption, the portfolio choice is all equities for most of the life cycle even for $\gamma = 5$ (figure 3). The results of the calibration for $\gamma = 2$ are shown on figure 5 for three values of habit persistence $\delta = 0.9$, 0.7, and 0.5. Consumption profiles change little from the earlier results, only the terminal consumption jump becomes higher since for lower value of γ it takes a larger move in consumption to lower the marginal utility at the terminal date. Wealth accumulation profiles are a little lower than in the case of $\gamma = 5$, and the habit persistence δ still has a first order effect on wealth accumulation. Portfolio choice profiles shift upward so that households invest entirely or almost entirely in equities until the age of 60 (for $\delta = 0.9$) or throughout the life for lower values of δ. There is a jump in portfolio composition at retirement for $\delta = 0.9$. This is due to the fact that labor income uncertainty is eliminated at this time and the lowest possible realization of labor income in retirement years is higher than before the retirement. Whether this sudden change in portfolio composition shows up in simulations depends on whether the endogenous variables realizations are such that the transformation from x_t to x'_t is affected by the increased floor on labor income. If the habit is sufficiently low relative to wealth, then $x'_t = x_t - h_t$, and the transformation is not affected by the current floor on labor income. In this case the jump at retirement will not be observed.[22] If the habit is high relative to wealth, then the more general equation (11) is used for the transformation and the labor income floor enters the computation of x'_t. At retirement, x'_t increases by a discrete amount which decreases the risk aversion and results in a jump in α.

It is clear from this calibration that the curvature parameter γ has a significant effect on the predicted portfolio dynamics over the life cycle. Thus, it will be necessary to investigate the sensitivity of all interesting results to the value of γ. The amount of human capital also affects portfolio choice and therfore I will investigate what differences does the model imply for different educational cohorts. These calibrations are postponed until it will become clear which cases are interesting enough to explore their comparative static.

[22]Also, if the wealth is sufficiently high there won't be any significant change in the risk aversion and no sudden change in portfolio composition.

5 Summary and conclusions

This paper solves a life cycle model of consumption and portfolio choice with the additive habit formation utility. Under the assumptions that habit is a multiple of consumption in the previous period and that exogenous variables follow a finite-state Markov process I derive feasibility constraints for the endogenous variables of the model – habit and cash. I show that the feasibility constraints depend on the worst possible scenario for future labor income and on the habit persistence parameter. These constraints are derived analytically and the model is solved for a broader range of parameterizations than was previously considered in the literature. A distinct feature of this model is that the optimal savings and portfolio choice policies depend on the distance to the border of the feasible set and therefore depend on the worst case labor income. As a consequence, the results are sensitive to the distributional assumptions about the labor income process. In the next version of the paper this aspect of the model will be investigated in a greater detail to include calibrations of the "disaster" events that significantly reduce household income. Introducing costly stock market participation should also improve the model by making its predictions more realistic. Not only it will help generate zero stockholdings, but also it will make the portfolios of stockholders more conservative since costly participation slows down wealth accumulation.

The model presented in this paper shows the potential of the additive habit formation preferences to shed light on certain aspects of the household portfolio choice. It appears that with realistically calibrated income uncertainty and sufficiently high habit persistence coefficient δ, the model can generate portfolios much more conservative than those typically predicted by the models with time-separable CRRA preferences and high values of risk aversion (Cocco *et. al.* (1999)). Another distinct and interesting feature of this model is that it can predict *lower* allocation in stocks for *younger* households than for middle aged households. The persistence parameter δ required to make reasonable predictions about the portfolio allocation is on the order of 0.8 and higher.[23] General equilibrium habit formation models, for example, Constantinides (1990) and Campbell and Cochrane (1999), require

[23]The required δ may be lower when the labor income process is calibrated to include very bad outcomes.

similar or even higher habit persistence for the resolution of the asset pricing puzzles. In these equilibrium models the role of high habit persistence is to increase the risk aversion of a representative agent and hence the variability of the stochastic discount factor without increasing the variability of the aggregate consumption. In the partial equilibrium model of portfolio choice, higher habit persistence tightens the feasibility constraints, increasing the risk aversion for any given wealth-habit pair. Habit formation also gives rise to the time variation of the risk aversion. In a general equilibrium model this feature generates time varying moments of asset returns. In a life-cycle model, in addition to human capital, the time varying risk aversion contributes to changes in portfolio allocation. A portfolio choice model with habit formation preferences implies that wealth accumulation and portfolio choice strongly depend on how close a household is to the admissible cash level for a given habit level. In a context of a general equilibrium model this implication can be related to wealth inequality and to asset markets participation. It would be interesting to explore this link in a general equilibrium setting with heterogeneous agents.

References

Ameriks, John, and Stephen Zeldes, 2000, How do household portfolio shares very with age?, *working paper, Columbia University.*

Campbell, John Y., and John H. Cochrane, 1999, By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205–51.

Carroll, Christopher D., 1992, The buffer-stock theory of savings: Some macroeconomic evidence, *Brooking Papers on Economic Activity* 2, 61–156.

Chapman, David, 1998, Habit formation and aggregate consumption, *Econometrica* 66, 1223–30.

Cocco, João F., Francisco J. Gomes, and Pascal J. Maenhout, 1999, Consumption and portfolio choice over the life-cycle, *manuscript, Harvard University.*

Constantinides, George M., 1990, Habit formation: A resolution of the equity premium puzzle, *Journal of Political Economy* 98, 519–543.

Deaton, Angus, 1991, Saving and liquidity constraints, *Econometrica* 59, 1221–1248.

Detemple, Jerome B., and Fernando Zapatero, 1991, Asset prices in and exchange economy with habit formation, *Econometrica* 59, 1633–57.

Epstein, Larry G., and Stanley E. Zin, 1989, Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.

Gomes, Francisco, and Alexander Michaelidis, 2001, Life-cycle asset allocation: A model with borrowing constraints, uninsurable labor income risk and stock market participation costs, *working paper, London School of Economics.*

———, 2002, Portfolio choice with habit formation: A life-cycle model with uninsurable labor income risk, *working paper, London School of Economics.*

Haliassos, Michael, and Carol C. Bertaut, 1995, Why do so few hold stocks?, *The Economic Journal* pp. 1110–1129.

Heaton, John, 1995, An empirical inverstigation of asset pricing with temporally dependent preference specifications, *Econometrica* 63, 681–717.

———, and Deborah J. Lucas, 1997, Market frictions, savings behavior, and portfolio choice, *Macroeconomic Dynamics* 1, 76–101.

Hindy, Ayman, Chi-Fu Huang, and Steven H. Zhu, 1997, Optimal consumption and portfolio rules with durability and habit formation, *Journal of Economic Dynamics and Control* 21, 525–550.

Jagannathan, Ravi, and Narayana R. Kocherlakota, 1996, Why should older people invest less in stocks than younger people, *Minneapolis Federal Reserve Quarterly Review* 20, 11–23.

Mankiw, Gregory N., and Stephen P. Zeldes, 1991, The consumption of stockholders and nonstockholders, *Journal of Financial Economics* 29, 97–112.

Merton, Robert C., 1971, Optimum consumption and portfolio choice rules in a continuous-time model, *Journal of Economic Theory* 3, 373–413.

Poterba, James M., and Andrew A. Samwick, 1997, Household portfolio allocation over the life cycle, *the NBER working paper No. 6185*.

Sandaresan, Suresh M., 1989, 1989, *The Review of Financial Studies* 2, 73–89.

Storesletten, Kjetil, Chris Telmer, and Amir Yaron, 1998, Persistent idiosyncratic shocks and incomplete markets, *manuscript, Carnegie Mellon University*.

Viceira, Luis M., 2001, Optimal portfolio choice for long horizon investors with nontradable labor income, *Journal of Finance* 55, 1163–98.

Vissing-Jørgensen, Annette, 1999, Towards an explanation of household portfolio choice heterogeneity: Nonfinancial income and participation cost structures, *manuscript, University of Chicago*.

Appendix A: Feasibility constraints for endogenous states

[... to be completed ...]

Appendix B: Numerical method

[... to be completed ...]

Figure 3: Symmetric distribution of income shock, various values of δ, $\gamma = 5$. Financial wealth is the total holdings of bond and stock. Consumption and financial wealth are in thousands of dollars (inflation adjusted with 1992 as a base year).

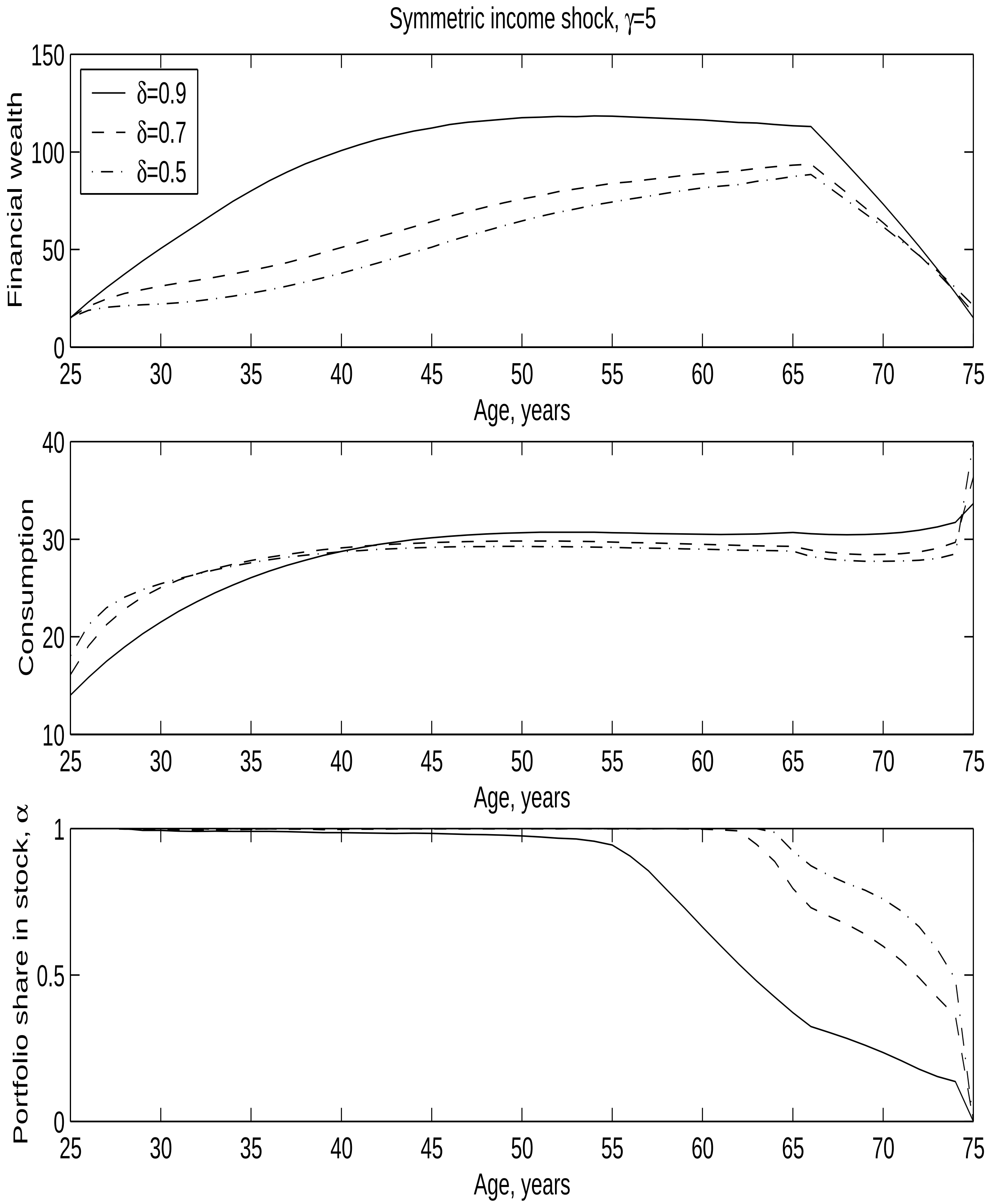

Figure 4: Asymmetric distribution of income shock, various values of δ, $\gamma = 5$. Financial wealth is the total holdings of bond and stock. Consumption and financial wealth are in thousands of dollars (inflation adjusted with 1992 as a base year).

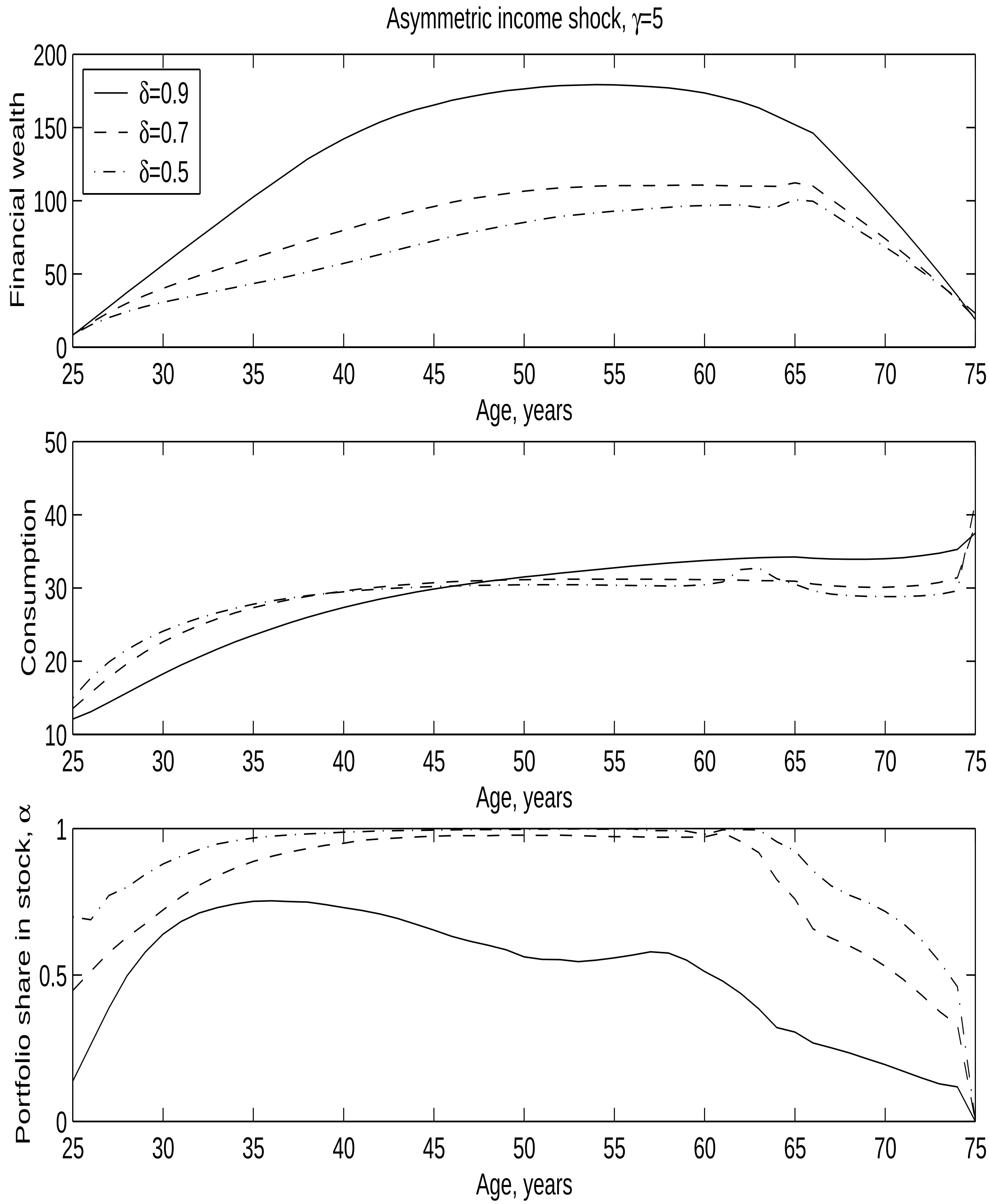

Figure 5: Asymmetric distribution of income shock, various values of δ, $\gamma = 2$. Financial wealth is the total holdings of bond and stock. The bottom panel shows α, the fraction of financial wealth invested in stock. Consumption and financial wealth are in thousands of dollars (inflation adjusted with 1992 as a base year).