Asset Pricing with Liquidity Risk*

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Abstract

This paper studies equilibrium asset pricing with time-varying illiquidity. It is shown that a security's required return depends on its expected illiquidity and on the covariances of its own return and illiquidity with market return and market illiquidity. This gives rise to a liquidity-adjusted capital asset pricing model (CAPM) with four betas (covariances). Further, if a security's liquidity is persistent, a shock to its illiquidity results in low contemporaneous returns and high predicted future returns. The four-beta CAPM is tested cross-sectionally using the daily return and volume data on NYSE and AMEX stocks over the period 1963–1999. Robust evidence is presented in support of the model.

(Preliminary and Incomplete)

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1 Introduction

Empirically, various measures of liquidity vary over time both for individual stocks and for the market as a whole (Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2000), and Huberman and Halka (1999)). Hence, investors face uncertainty about liquidity, which raises the question: How does liquidity risk affect asset prices in equilibrium?

We answer this question by deriving explicitly a liquidity-adjusted capital asset pricing model (CAPM). The model shows that a security's required return depends on the sensitivity of its return and tradability to market downturns as well as to liquidity crises. Specifically, a security's required return depends on its expected illiquidity and on the covariances of its own return and illiquidity with market returns and market illiquidity. This gives rise to a liquidity-adjusted CAPM with four betas (covariances). Additionally, we show that returns co-move with liquidity and high illiquidity predicts high future returns, if liquidity is persistent. The model provides a theoretical foundation for a rich set of existing empirical findings (discussed below) and provides new testable implications. We test the model cross-sectionally using the liquidity measure suggested by Amihud (2002) and employing the daily return and volume data on NYSE and AMEX stocks over the period 1963–1999. We provide robust evidence that the model's implications are supported in data.

The existing theoretical literature on frictions and asset pricing has focused on various frictions with *deterministic* severity (for instance, Amihud and Mendelson (1986), Constantinides (1986), Vayanos (1998), Vayanos and Vila (1999), Gârleanu and Pedersen (2000)). This paper complements the literature by deriving pricing effects associated with *the risk of changes* in the liquidity of an individual security as well as in market liquidity. We discuss in turn the model's three main predictions and their empirical relevance.

First, the model shows that investors require a return premium for a security that is illiquid when the market as a whole is illiquid. The potential importance of this result follows from the empirically documented commonality in liquidity. In particular, Chordia, Roll, and Subrahmanyam (2000) find significant commonality in liquidity using daily data for NYSE stocks in 1992, Huberman and Halka (1999) find a systematic time-varying component of liquidity using daily NYSE data from 1996, and Hasbrouck and Seppi (2000) find weak commonality in liquidity for 30 Dow stocks over 15-minute intervals during 1994. The effect of commonality of liquidity on required returns has however not yet been tested. Empirically, we find support for this prediction in univariate tests, but the effect seems small when we control for the model's other risk factors.

Second, the model shows that investors are willing to pay a premium for a security that has a high return when the market is illiquid. Pastor and Stambaugh (2001) find empirical support for this effect using monthly data over 34 years with a measure of liquidity that they construct based on the return reversals induced by order flow. Consistently, we find empirical support for this prediction.

Third, the model implies that investors are willing to pay a premium for a security that is liquid when the market is down. This is another new testable prediction. It is supported empirically in most of our numerous specifications and robustness tests. Further, we demonstrate that the risk premium arising from this effect is economically significant.

To summarize, a security's required return depends on the sensitivity (covariance) of its return and illiquidity to changes in market return and market illiquidity. Over the period 1963-1999, the three covariances described above contribute on average a difference in risk premium between stocks with high expected illiquidity and low expected illiquidity of about 9.9% annually.[1]

Additionally, the model shows that, since liquidity is persistent,[2] *liquidity predicts future returns.* This is because a positive shock to illiquidity predicts high future illiquidity, which raises the required return. This theoretical prediction is consistent with the empirical findings of Amihud (2002) and Jones (2001).

Finally, the model shows that *liquidity co-moves with contemporaneous returns* if liquidity is persistent. A positive shock to illiquidity raises the required return, which depresses current prices and lowers contemporaneous returns. In support of this prediction, Amihud (2002) finds a negative relation between return and unexpected illiquidity for size portfolios, Chordia, Roll, and Subrahmanyam (2001), Jones (2001), and Pastor and Stambaugh (2001) find a negative relation between market return and illiquidity, and Amihud, Mendelson, and Wood (1990) find that stocks, whose liquidity worsened more during the 1987 crash, had more negative returns.

The paper is organized as follows. Section 2 describes the economy, Section 3 derives the liquidity-adjusted capital asset pricing model, Section 4

[1]As we show later, sorting stocks by expected illiquidity also produces a sorting on these three covariances.

[2]The persistence of liquidity is documented empirically by Amihud (2002), Chordia, Roll, and Subrahmanyam (2000, 2001), Hasbrouck and Seppi (2000), Huberman and Halka (1999), Jones (2001), and Pastor and Stambaugh (2001).

studies how liquidity predicts and co-moves with returns, Section 5 contains our empirical results, Section 6 concludes, and proofs are in the Appendix.

2 Assumptions

The model assumes a simple overlapping generations economy in which a new generation of agents is born at any time $t \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ (Samuelson (1958)). Generation t consists of N agents, indexed by n, who live for two periods, t and $t+1$. Agent n of generation t has an endowment at time t and no other sources of income, trades in periods t and $t+1$, and derives utility from consumption at time $t+1$. He has constant absolute risk aversion A^n so that his preferences are represented by the utility function $-E_t \exp(-A^n x_{t+1})$, where x_{t+1} is his consumption at time $t+1$.

There are I securities indexed by $i = 1, \ldots, I$ with a total of S^i shares of security i. At time t, security i pays a dividend of D_t^i, has an ex-dividend share price of P_t^i, and has an illiquidity cost of C_t^i, where D_t^i and C_t^i are random variables.[3] The illiquidity cost, C_t^i, is modeled simply as the per-share cost of selling security i. Hence, agents can buy at P_t^i but must sell at $P_t^i - C_t^i$. Short-selling is not allowed.

Uncertainty about the illiquidity cost is what generates the liquidity risk in this model. Specifically, we assume that D_t^i and C_t^i are autoregressive

[3]All random variables are defined on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, and all random variables indexed by t are measurable with respect to the filtration $\{\mathcal{F}_t\}$, representing the information commonly available to investors.

processes of order one, that is:

$$
\begin{aligned}
D_t &= \bar{D} + \gamma(D_{t-1} - \bar{D}) + \varepsilon_t \\
C_t &= \bar{C} + \gamma(C_{t-1} - \bar{C}) + \eta_t \,,
\end{aligned}
$$

where $\bar{D}$, $\bar{C} \in \mathbb{R}^I_+$ are positive real vectors, $\gamma \in [0, 1]$, and (ε_t, η_t) is an independent identically distributed normal process with mean $E(\varepsilon_t) = E(\eta_t) = 0$ and variance-covariance matrices $\text{var}(\varepsilon_t) = \Sigma^D$, $\text{var}(\eta_t) = \Sigma^C$, $E(\varepsilon_t \eta_t^\top) = \Sigma^{CD}$, and $\text{var}(\varepsilon_t - \eta_t) = \Gamma$ $(= \Sigma^D + \Sigma^C - \Sigma^{CD} - (\Sigma^{CD})^\top)$.

Finally, we assume that agents can borrow and lend at a risk-free real return of $r^f > 1$, which is exogenous. This can be interpreted as an inelastic world bond market, or a generally available production technology that turns a unit of consumption at time t into r^f units of consumption at time $t + 1$.

The assumptions with respect to agents, preferences, and dividends are strong. These assumptions are made for tractability, and, as we shall see, they imply natural closed-form results for prices and expected returns. The main result (Proposition 1) applies more generally, however. It applies with arbitrary utility functions as long as conditional expected net returns are normal,[4] it applies with arbitrary return distribution and quadratic utility, and it can be viewed as a result of near-rational behavior (for instance, by using a Taylor expansion of the utility function). See Huang and Litzenberger (1988), Markowitz (2000), and Cochrane (2001). Our assumptions allow us, additionally, to study return predictability caused by illiquidity (Proposition 2)

[4]The normal returns assumption is an assumption about endogenous variables, which is used in standard CAPM analysis (for instance, Huang and Litzenberger (1988)). This assumption is satisfied in the equilibrium of the model of this paper, and may also be satisfied in larger classes of models.

and the co-movements of returns and illiquidity (Proposition 3), producing insights that also seem robust to the specification.

The overlapping-generations model can capture investors' life-cycle motives for trade (as in Vayanos (1998), and Constantinides and Donaldson (2001)), or can be viewed as a way of capturing short investment horizons (as in De Long, Shleifer, Summers, and Waldmann (1990)) and the large turnover observed empirically in many markets.

3 Liquidity-Adjusted Capital Asset Pricing Model

This section shows that, under the stylized assumption of mean-variance investors, a liquidity-adjusted version of the Capital Asset Pricing Model (CAPM) applies which is characterized by four betas or covariances.

We are interested in how an asset's expected (gross) return,

$$r_t^i = \frac{D_t^i + P_t^i}{P_{t-1}^i},$$

depends on its relative illiquidity cost, defined as

$$c_t^i = \frac{C_t^i}{P_{t-1}^i},$$

on the market return,

$$r_t^M = \frac{\sum_i S^i (D_t^i + P_t^i)}{\sum_i S^i P_{t-1}^i},$$

and on the relative market illiquidity,

$$c_t^M = \frac{\sum_i S^i C_t^i}{\sum_i S^i P_{t-1}^i} \ .$$

To determine equilibrium returns, consider first an economy with the same agents in which asset i has a dividend of $D_t^i - C_t^i$ and no illiquidity cost. In this imagined economy, standard results imply that the CAPM holds (Markowitz (1952), Sharpe (1964), Lintner (1965), and Mossin (1966)). We claim that the equilibrium prices in the original economy with frictions are the same as those of the imagined economy. This follows from two facts: *(i)* the net return on a long position is the same in both economies; *(ii)* all investors in the imagined economy hold a long position in the market portfolio, and a (long or short) position in the risk-free asset. Hence, an investor's equilibrium return in the frictionless economy is feasible in the original economy, and is also optimal, given the more limited investment opportunities.

These arguments show that the CAPM in the imagined frictionless economy translates into a CAPM in net returns for the original economy with illiquidity costs. Rewriting the one-beta CAPM in net returns in terms of gross returns, we get a four-beta liquidity-adjusted CAPM which is the main testable[5] implication of this paper:

[5] Difficulties in testing this model arise from the fact that it makes predictions concerning conditional moments as is standard in asset pricing. See Hansen and Richard (1987), Cochrane (2001), and references therein. An unconditional version of (1) applies under stronger assumptions as discussed in Section 3.1.

Proposition 1 *In the unique linear equilibrium, the conditional expected return of security i is*

$$
\begin{aligned}
E_{t-1}(r_t^i - r^f) &= E_{t-1}(c_t^i) + \lambda_{t-1}\text{cov}_{t-1}(r_t^i, r_t^M) + \lambda_{t-1}\text{cov}_{t-1}(c_t^i, c_t^M) \\
&\quad - \lambda_{t-1}\text{cov}_{t-1}(r_t^i, c_t^M) - \lambda_{t-1}\text{cov}_{t-1}(c_t^i, r_t^M), \qquad (1)
\end{aligned}
$$

where λ_{t-1} is the "market price of risk,"

$$
\lambda_{t-1} = \frac{E_{t-1}(r_t^M - c_t^M) - r^f}{\text{var}_{t-1}(r_t^M - c_t^M)} = A\, P_{t-1}^M, \qquad (2)
$$

and $A = \left(\sum_n \frac{1}{A^n}\right)^{-1}$ is the aggregate risk aversion.

Equation (1) is simple and natural. It states that the required excess return is the expected relative illiquidity cost, $E_{t-1}(c_t^i)$, (as first found theoretically and empirically[6] by Amihud and Mendelson (1986)) plus four betas or covariances that depend on the asset's payoff and liquidity risks. As in the standard CAPM, the required return on an asset increases (linearly) with the covariance between the asset's return and the market return. This model yields three additional effects:

First, the return increases with the covariance between the asset's illiq-

[6] Empirically, Amihud and Mendelson (1986, 1989) find the required rate of return on NYSE stocks to increase with the relative bid-ask spread. This result is supported for amortized spreads for NYSE stocks by Chen and Kan (1996), and for Nasdaq stocks by Eleswarapu (1997), but is questioned for NYSE stocks by Eleswarapu and Reinganum (1993), and Chalmers and Kadlec (1998). Gârleanu and Pedersen (2000) find that adverse-selection costs are priced only to the extent that they render allocations inefficient. The ability of a market to allocate assets efficiently may be related to market depth, and, consistent with this view, the required rate of return has been found to decrease with measures of depth (Brennan and Subrahmanyam (1996) and Amihud (2002)). Easley, Hvidkjær, and O'Hara (2000) find returns to increase with a measure of the probability of informed trading.

uidity and the market illiquidity ($\text{cov}_{t-1}(c_t^i, c_t^M)$). This is because investors want to be compensated for holding a security that becomes illiquid when the market in general becomes illiquid. The potential empirical significance of this pricing implication follows from the presence of a time-varying common factor in liquidity, which is documented by Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2000), and Huberman and Halka (1999). These papers find that most stocks' illiquidities are positively related to market illiquidity, so the required return should be raised by the commonality-in-liquidity effect.

In this model, the risk premium associated with commonality in liquidity is caused by the wealth effects of illiquidity. Also, this risk premium would potentially apply in an economy in which investors can choose which securities to sell. In such a model, an investor who holds a security that becomes illiquid (that is, has a high cost c_t^i) can choose not to trade this security and instead trade other (similar) securities. It is more likely that an investor can trade other (similar) securities, at low cost, if the liquidity of this asset does not co-move with the market liquidity. Hence, investors would require a return premium for assets with positive covariance between individual and market illiquidity.

The second effect on expected returns is due to covariation between a security's return and the market liquidity. We see that $\text{cov}_{t-1}(r_t^i, c_t^M)$ affects required returns negatively because investors pay a premium for an asset with a high return in times of market illiquidity. Empirical support for this effect is provided by Pastor and Stambaugh (2001), who find that "the average return on stocks with high sensitivities to liquidity exceeds that for stocks with low sensitivities by 7.5% annually, adjusted for exposures to the market return as well as size, value, and momentum factors."

The third effect on required returns is due to covariation, $\text{cov}_{t-1}(c_t^i, r_t^M)$, between a security's illiquidity and the market return. This effect stems from investors' willingness to accept a lower expected return on a security that is liquid in a down market. When the market declines, the ability to sell easily is especially valuable. Hence, an investor is willing to accept a discounted return on stocks with low illiquidity costs in states of poor market return.

In this model, the conditional CAPM holds for net returns, that is, returns net of illiquidity costs. The analysis is, however, focused on gross returns. The focus on gross returns is motivated by several considerations. First, most empirical work uses some measure of gross returns. Second, illiquidity costs are hard to measure empirically, and are measured in various different ways. Third, the model shows interesting pricing implications of co-movements in individual and market gross return and liquidity. Empirical work has documented that some of these interactions are significant (Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2000), and Huberman and Halka (1999)) and priced (Amihud and Mendelson (1986), Amihud (2002), and Pastor and Stambaugh (2001)). Fourth, a pricing relation for gross returns and illiquidity, which is similar in spirit to (1), may hold in richer models in which net returns are not sufficient state variables. As argued above, some additional liquidity effects suggest risk premia of the same sign for the covariance terms in (1). These additional liquidity effects also suggest that the size of the risk premia need not be identical across the covariance terms. To accommodate the possibility of a richer liquidity framework, we consider the generalized relation:

$$E_{t-1}(r_t^i - r^f) = \alpha_{t-1} + \lambda_{t-1}^0 E_{t-1}(c_t^i) + \lambda_{t-1}^1 \text{cov}_{t-1}(r_t^i, r_t^M) \tag{3}$$
$$+\lambda_{t-1}^2 \text{cov}_{t-1}(c_t^i, c_t^M) - \lambda_{t-1}^3 \text{cov}_{t-1}(r_t^i, c_t^M) - \lambda_{t-1}^4 \text{cov}_{t-1}(c_t^i, r_t^M).$$

Here, λ^0 has been studied by Amihud and Mendelson (1986) and others (see Footnote 6), there is a large literature that studies λ^1 (see, for instance, Cochrane (2001)), and λ^3 is examined by Pastor and Stambaugh (2001). We are not aware of any empirical work that considers all these interactions. Within this framework, one can test the hypotheses that $\lambda^1 = \lambda^2 = \lambda^3 = \lambda^4 = \lambda$ and $\alpha = 0$.

3.1 An Unconditional Liquidity-Adjusted CAPM

To estimate the liquidity-adjusted CAPM, we derive an unconditional version. An unconditional result obtains, for instance, under the assumption of independence over time of dividends and illiquidity costs. Empirically, however, illiquidity and, to some extent, returns are persistent. Therefore, we rely instead on the following two considerations.

First, we make the simplifying assumption that the market prices of risk, α and $\lambda_0, \ldots, \lambda_4$, are constant over time. To justify this assumption, we note that the possible time-variation of these parameters is driven by constant absolute risk aversion in our model.[7] With constant relative risk aversion, however, the market price of risk is approximately constant.[8] See Friend and

[7]Constant market prices of risk means that the expected net return on the market is constant. The expected gross return, however, is time-varying because of the time-varying illiquidity.

[8]The (market wide) relative risk aversion could, of course, be time-varying because of

Blume (1975).

Second, we use the fact that for any random variables X and Y, it holds that

$$E(\mathrm{cov}_t(X,Y)) = \mathrm{cov}(X - E_t(X), Y) = \mathrm{cov}(X - E_t(X), Y - E_t(Y)). \quad (4)$$

Using the assumption of constant market prices of risk and the property (4) of covariances, we get the relation:

$$E(r_t^i - r_t^f) = \alpha + \lambda^0 E(c_t^i) + \lambda^1 \beta^{1i} + \lambda^2 \beta^{2i} - \lambda^3 \beta^{3i} - \lambda^4 \beta^{4i} \ , \quad (5)$$

where

$$\beta^{1i} = \mathrm{cov}(r_t^i, r_t^M - E_{t-1}(r_t^M)) \quad (6)$$

$$\beta^{2i} = \mathrm{cov}(c_t^i - E_{t-1}(c_t^i), c_t^M - E_{t-1}(c_t^M)) \quad (7)$$

$$\beta^{3i} = \mathrm{cov}(r_t^i, c_t^M - E_{t-1}(c_t^M)) \quad (8)$$

$$\beta^{4i} = \mathrm{cov}(c_t^i - E_{t-1}(c_t^i), r_t^M - E_{t-1}(r_t^M))\,. \quad (9)$$

4 Implications of Persistence of Liquidity

This section shows that persistence of liquidity implies that liquidity predicts future returns and co-moves with contemporaneous returns.

Empirically, liquidity is *time-varying and persistent* (which means that $\gamma > 0$).[9] This model shows that persistent liquidity implies that returns are

habit formation, for instance. In order to keep the model relatively simple, we do not try to capture such effects.

[9]See Amihud (2002), Chordia, Roll, and Subrahmanyam (2000, 2001), Hasbrouck and

predictable. Intuitively, high illiquidity today predicts high expected illiquidity next period, implying a high required return.

Proposition 2 *Suppose that $\gamma > 0$, and that $q \in \mathbb{R}^I$ is a portfolio*[10] *with $\gamma D^q_{t-1} + (1-\gamma) E(D^q_t + P^q_t \mid D^q_{t-1} = \bar{D}^q, C^q_{t-1} = \bar{C}^q) > 0$. Then, the conditional expected return increases with illiquidity,*

$$\frac{\partial}{\partial C^q_{t-1}} E_{t-1}(r^q_t - r^f) > 0. \tag{10}$$

Proposition 2 relies on a mild technical condition, which is satisfied, for instance, for any portfolio with positive current and mean dividend and positive mean price. The proposition states that the conditional expected return depend positively on the current illiquidity cost, that is, the current liquidity predicts the return.

Jones (2001) finds empirically that the expected annual stock market return increases with the previous year's bid-ask spread and decreases with the previous year's turnover. Amihud (2002) finds that illiquidity predicts excess return both for the market and for size-based portfolios.

Predictability of liquidity further implies a negative conditional covariance between contemporaneous returns and illiquidity. Naturally, when illiquidity is high, the required return is high also, which depresses the current price, leading to a low return. This intuition applies as long as liquidity is persistent ($\gamma > 0$) and innovations in dividends and illiquidity are not too correlated ($q^\top \Sigma^{CD} q$ low for a portfolio q) as is formalized in the following

Seppi (2000), Huberman and Halka (1999), Jones (2001), and Pastor and Stambaugh (2001).

[10]For any $q \in \mathbb{R}^I$, we use the obvious notation $D^q_t = q^\top D_t$, $r^q_t = \frac{\sum_i q^i (D^i_t + P^i_t)}{\sum_i q^i P^i_{t-1}}$ and so on.

proposition.

Proposition 3 *Suppose $q \in \mathbb{R}^I$ is a portfolio such that $\gamma > r^f \frac{q^\top \Sigma^{CD} q}{q^\top \Sigma^C q}$. Then,* $\mathrm{cov}_{t-1}(c_t^q, r_t^q) < 0$.

Consistent with this result, Chordia, Roll, and Subrahmanyam (2001), Jones (2001), and Pastor and Stambaugh (2001) find a negative relation between the market return and measures of market illiquidity, Amihud (2002) finds a negative relation between the return on size portfolios and their corresponding unexpected illiquidity, and Amihud, Mendelson, and Wood (1990) argue that the 1987 crash was in part due to an increase in (perceived) market illiquidity.

5 Empirical Results

In this section, we estimate and test the liquidity-adjusted CAPM as specified in Equation (5). We do this in four steps:

(i) We estimate, in each month of our sample, a measure of illiquidity, c, for each individual security. (Section 5.1.)

(ii) We form a "market portfolio" and sets of 25 test portfolios sorted based on illiquidity and size, respectively. For each portfolio and each month, we compute its return and illiquidity. (Section 5.2.)

(iii) For the market portfolio as well as the test portfolios, we estimate the innovations in illiquidity, $c_t^i - E_{t-1}(c_t^i)$. (Section 5.3.)

(iv) Using these illiquidity innovations and the portfolios' returns, we estimate their liquidity betas. Finally, we consider the empirical fit of the

(unconditional) liquidity-adjusted CAPM by running cross-sectional regressions based on the empirical methodology of Fama and MacBeth (1973). To check the robustness of our results, we do the analysis with a number of different specifications. (Section 5.4.)

5.1 The Illiquidity Measure

Liquidity is (unfortunately) not an observable variable. There exist, however, many proxies for liquidity. Some proxies, such as the bid-ask spread, are based on market microstructure data that is available for less than a decade. We want a longer time series in order to study the effect on expected returns. Therefore, we follow Amihud (2002) in estimating illiquidity using daily CRSP data only. In particular, Amihud (2002) defines the illiquidity of stock i in month t as

$$ILLIQ_t^i = \frac{1}{D_t^i} \sum_{d=1}^{D_t^i} \frac{|R_{td}^i|}{V_{td}^i}, \tag{11}$$

where R_{td}^i and V_{td}^i are, respectively, the return and dollar volume on day d in month t, and D_t^i is the number of days in month t.

The intuition behind this illiquidity measure is as follows. A stock is illiquid — that is, has a high value of $ILLIQ_t^i$ — if the stock's price moves a lot in response to little volume. Amihud (2002) shows that this measure is positively related to measures of price impact and fixed trading costs and gives further justification.

Admittedly, this is a noisy measure of illiquidity, which makes it harder for us to find an empirical connection between returns and illiquidity. This problem is alleviated in part, however, by considering portfolios rather than individual stocks.

5.2 Portfolios

The data for our empirical analysis is obtained from CRSP (Center for Research in Security Prices, University of Chicago). Specifically, we employ daily return and volume data from July 1st, 1962 till December 31st, 1999 for all common shares (CRSP sharecodes 10 and 11) listed on NYSE and AMEX.[11]

We form a market portfolio for each month t during this sample period based on stocks with price, at beginning of month, between 5 and 1000, and with at least 15 days of return and volume data in that month.

We form 25 illiquidity portfolios for each year y during the period 1964 to 1999 by sorting stocks with price, at beginning of year, between 5 and 1000, and return and volume data in year $y-1$ for at least 150 days. We compute the annual illiquidity for each eligible stock as the average over the entire year $y-1$ of daily illiquidities (computed analogously to monthly illiquidity calculation in (11)). The eligible stocks are then sorted into 25 portfolios, $p \in \{1, 2, \ldots, 25\}$, based on their year $y-1$ illiquidities.

We also form 25 size portfolios, $p \in \{1, 2, \ldots, 25\}$, for each year y during the period 1964 to 1999 by ranking the eligible stocks (as above for illiquidity Portfolios) based on their market capitalization at the beginning of year y.

For each portfolio p (including the market portfolio), we compute its

[11]Since volume data in CRSP for Nasdaq stocks is available only from 1982 and includes inter-dealer trades, we employ only NYSE and AMEX stocks for sake of consistency in the illiquidity measure.

return in month t, as

$$r_t^p = \sum_{i \text{ in } p} w_t^{ip} r_t^i, \tag{12}$$

where the sum is taken over the stocks included in portfolio p in month t, and where w_t^{ip} are either equal weights or value-based weights, depending on the specification.[12]

Similarly, we compute the illiquidity of a portfolio, p, as

$$ILLIQ_t^p = \sum_{i \text{ in } p} w_t^{ip} ILLIQ_t^i, \tag{13}$$

where, as above, w_t^{ip} are either equal weights or value-based weights, depending on the specification.

The model's results are phrased in terms of value-weighted returns (and value-weighted illiquidity). Several studies, however, focus on equal-weighted return and illiquidity measures, for instance Amihud (2002) and Chordia, Roll, and Subrahmanyam (2000). Further, computing the market return and illiquidity as equal-weighted averages is a way of compensating for the over-representation in our sample of large liquid securities, as compared to the aggregate wealth portfolio in the economy. Hence, we estimate and test our model with equally weighted average for the market portfolio and both equal

[12]The returns, r_t^i, are adjusted for stock delisting to avoid survivorship bias, following Shumway (1997). In particular, the last return used is either the last return available on CRSP, or the delisting return, if available. While a last return for the stock of -100% is naturally included in the study, a return of -30% is assigned if the deletion reason is coded in CRSP as 500 (reason unavailable), 520 (went to OTC), 551–573 and 580 (various reasons), 574 (bankruptcy) and 584 (does not meet exchange financial guidelines). Shumway (1997) obtains that -30% is the average delisting return, examining the OTC returns of delisted stocks. Amihud (2002) employs an identical survivorship bias correction.

and value weighted averages for the test portfolios based on illiquidity and size sorts.

5.3 Innovations in Illiquidity

Illiquidity is persistent. The auto-correlation of the market illiquidity, for instance, is 0.942. Therefore, we focus on the innovations, $c_t^p - E_{t-1}(c_t^p)$, in illiquidity when computing liquidity betas (as explained in Section 3.1).

Another problem with *ILLIQ* is that it is measured in "percent per dollar," whereas the model in specified in terms of "dollar cost per dollar invested." To adjust for this, we scale the illiquidity measure by the market index, P_{t-1}^M, in the previous period. Specifically, P_{t-1}^M is the ratio of the capitalizations of the market portfolio at the end of month $t-1$ and of the market portfolio at the end of July 1962. Hence, to predict illiquidity, we run the following regression for each portfolio as well as for the market:

$$ILLIQ_t^p\, P_{t-1}^M = a_0 + a_1 ILLIQ_{t-1}^p\, P_{t-1}^M + a_2 ILLIQ_{t-2}^p\, P_{t-1}^M + u_t\ . \tag{14}$$

We use the same date for the market index (P_{t-1}^M) in all terms of the regression in order to make sure that we are measuring innovations only in illiquidity, not changes in P^M. The residual, u, of this regression is interpreted as the standardized liquidity innovation, $c_t^p - E_{t-1}(c_t^p)$, that is,

$$c_t^p - E_{t-1}(c_t^p) = u_t\ . \tag{15}$$

Figure 1 shows the innovations in (equal-weighted) market illiquidity, $c_t^M - E_{t-1}(c_t^M)$, scaled to have unit standard deviation. These innovations appear to be serially uncorrelated and are close to iid. The measured innovations in market illiquidity are high during periods that anecdotally were characterized by liquidity crisis, for instance, in 11/1973, 10/1987, the oil crisis and the stock market crash, respectively. Also, there is a string of relatively large shocks in 6–10/1998, the period in which Russia defaulted and Long-Term Capital Management suffered large losses. The correlation between this measure of innovations in market illiquidity and the measure of innovations in liquidity used by Pastor and Stambaugh (2001) is -0.33.[13] (The negative sign is due to the fact that Pastor and Stambaugh (2001) measure liquidity, whereas we follow Amihud (2002) in considering *il*liquidity.)

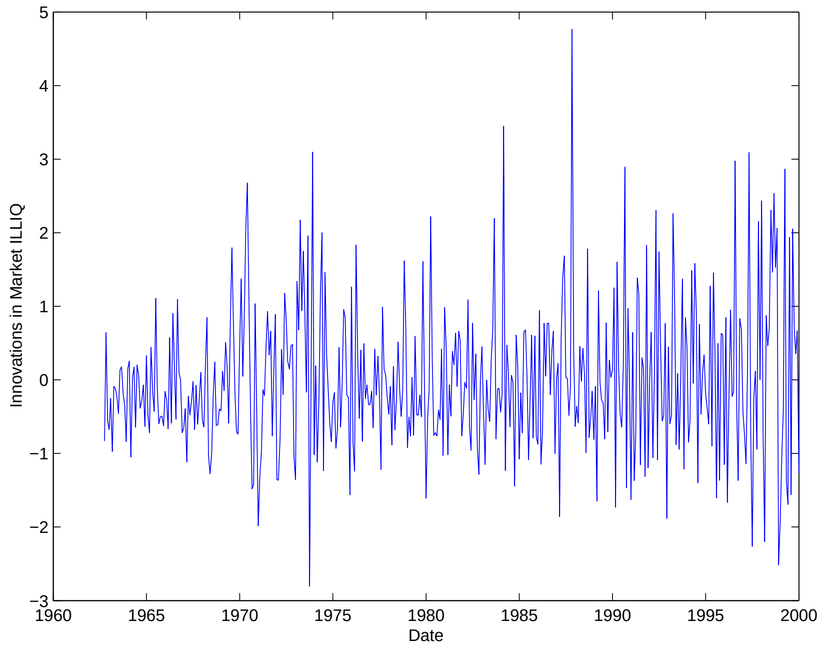

Figure 1: Standardized innovations in market illiquidity from 1962-1999.

[13]We thank Pastor and Stambaugh for providing their data on innovations in market liquidity.

5.4 How Liquidity Risk Affects Returns

In this section, we estimate the liquidity-adjusted CAPM (5) using the portfolios based on the sorting by illiquidity. The properties of these portfolios are reported in Table 1. The four betas or the covariances, β^{1i}, β^{2i}, β^{3i} and β^{4i}, for each portfolio are computed as per Equation (6) using the entire time-series, i.e., all monthly return and illiquidity observations for the portfolio and the market portfolio from the beginning of year 1964 till end of year 1999. Similarly, average illiquidity $E(c)$ for a portfolio is computed using the entire time-series of monthly illiquidity observations for the portfolio.

We see from Table 1 that the sort on past illiquidity successfully produces portfolios with monotonically increasing average illiquidity from portfolio 1 through portfolio 25. We further see that stocks that are illiquid — that is, have high values of $E(c)$ — also tend to have a small market capitalization and a low turnover.

Further, illiquid stocks tend to have high liquidity *risk*: they have large values of β^{2p} and large negative values of β^{3p} and β^{4p}. This is an interesting result on its own. It says that a stock, which is illiquid in absolute terms (c), also tends to have a lot of commonality in liquidity with the market ($\text{cov}(c^i, c^M)$), a lot of return sensitivity to market liquidity ($\text{cov}(r^i, c^M)$), and a lot of liquidity sensitivity to market returns ($\text{cov}(c^i, r^M)$).

This result is confirmed by considering the correlation among the betas, reported in Table 2. This correlation of betas is not just a property of the liquidity-sorted portfolios, it also exists on an individual stock level, as is seen in Table 3. While this correlation is theoretically intriguing, it makes it hard to empirically distinguish the separate effects of illiquidity and the individual liquidity betas, as discussed further below.[14]

Using the portfolios' betas and the illiquidity, we estimate our model (5), and subsets of its coefficients, by running cross-sectional regression using the method of Fama and MacBeth (1973). To be precise, in each month over the period 1964–1999, we run a cross-sectional regression of the excess returns on the 25 test portfolios with explanatory variables being the portfolio characteristics, and the estimated coefficients are averaged over all months. We consider different specifications by employing combinations of these characteristics.

The results are reported in Table 4. We see that the λ's, i.e., the respective market prices of liquidity risks have signs that are consistent with the model's prediction. In particular, a security's required return is increasing in its level of β^2 and decreasing in its level of β^3 and β^4. The market prices of liquidity risks are significant at conventional levels in univariate regressions. The statistical significance is reduced as more coefficients are estimated simultaneously, but some betas remain significant. For instance, the coefficient related to β^4 is significant whenever it is included. This lack of ability to identify all the coefficients jointly may be due, at least in part, to the co-linearity of the different kinds of illiquidity risk. Of course, we must also entertain the possibility that not all the risk factors are empirically relevant.

The effect of liquidity risk on required returns seems economically impor-

[14]We have not been able to construct portfolios which allow us to better identify the separate beta effects. For instance, we have considered portfolios based on predicted liquidity betas, similar to the approach taken by Pastor and Stambaugh (2001). These results are not reported as they produce similar results to those from the illiquidity and size portfolios. Using these portfolios does not, however, improve statistical power. We attribute this, in part, to the difficulty of predicting liquidity betas, and, in part, to the co-linearity of the liquidity betas.

tant. To get a perspective on the magnitude of the effect, we consider first the univariate coefficients from Table 4. The difference in annualized expected return between portfolio 1 and 25 that can be attributed to a difference in β^2, i.e., in the covariance, $\text{cov}_{t-1}(c_t^i, c_t^M)$, between the portfolio illiquidity and the market illiquidity, is

$$\lambda^2(\beta^{2,p_{25}} - \beta^{2,p_1}) \cdot 12 = 8.2\%.$$

Similarly, the annualized return difference stemming from the difference in β^3, i.e., in the covariance, $\text{cov}_{t-1}(r_t^i, c_t^M)$, between the portfolio return and the market illiquidity, is

$$\lambda^3(\beta^{3,p_{25}} - \beta^{3,p_1}) \cdot 12 = 6.2\%,$$

and the effect of β^4, i.e., in the covariance, $\text{cov}_{t-1}(r_t^i, c_t^M)$, between the security's return and the market illiquidity, is

$$\lambda^4(\beta^{4,p_{25}} - \beta^{4,p_1}) \cdot 12 = 8.8\%.$$

Of course, we cannot believe all the univariate effects at the same time. The multivariate regression in Table 4 shows that, controlling for β^3 and β^4, the effect of β^2 becomes insignificant. The effects of β^3 and β^4 remain, however, economically significant, with annualized effects of 2.7% and 7.2%, respectively, a total effect of 9.9%.

Further, as predicted by the model and consistent with the findings of Amihud (2002), absolute illiquidity $E(c)$ contributes a positive and significant risk premium in the univariate specification. Its effect, however, becomes insignificant when it is included in the specification along with β^4.

To test the robustness of our results, we estimate the model with a number of different specifications. We control for size and volatility (Table 5), we consider annual data (Table 6), we use monthly data excluding the month of January (Table 7), we consider the sub-period 1964–1981 (Table 8), and the sub-period 1982–1999 (Table 9), and finally value-weighted portfolios (Table 10).[15] We see that the signs of the beta coefficients are stable across all specifications of the model. We find some, but not all, beta coefficients that are significant at conventional levels in almost all specifications.

To further test the robustness of results, we re-estimate our model with size-based portfolios. Table 11 shows the properties of equal-weighted size-based portfolios confirming that small sized stocks are illiquid (in absolute terms as measured by $E(c)$) and also have high liquidity risk (as measured by the three betas β^{2p}, β^{3p} and β^{4p}). Table 12 shows the results of Fama-MacBeth regressions with equal-weighted portfolios and Table 13 shows the results with value-weighted portfolios. Again, the results are generally consistent with the model, although weaker. With the size portfolios, the strongest effect is that related to β^3.

[15]The subsets of the model that we estimate depend on the specification. This is because of the severity of the co-linearity problem depends on the specification. For example, in Table 4, β^1 and β^3 are not employed together since their correlation across 25 equally weighted illiquidity portfolios is as high as -0.98, and in addition, in Table 10, β^2 and β^4 are not employed together since their correlation across 25 value weighted illiquidity portfolios is greater than 0.95 (not reported).

6 Conclusion

This paper considers the effect of liquidity risk. The paper develops a simple pricing formula that shows that investors should worry about a security's performance and tradability both in market downturns and when liquidity "dries up." Said differently, the required return of a security i is increasing in the covariance, $\mathrm{cov}_{t-1}(c_t^i, c_t^M)$, between its illiquidity and the market illiquidity, and decreasing in the covariance, $\mathrm{cov}_{t-1}(r_t^i, c_t^M)$, between the security's return and the market illiquidity, and decreasing in the covariance, $\mathrm{cov}_{t-1}(c_t^i, r_t^M)$, between its illiquidity and market returns.

The model also shows why high illiquidity predicts high future returns, and why contemporaneous liquidity and returns co-move.

Hence, the model helps explain the existing empirical evidence related to liquidity risk. Further, its novel predictions are consistent with our empirical findings. In particular, we find, in a variety of specifications, that certain, though not all, liquidity risks are priced. To summarize, over the period 1963-1999, the three covariances described above contribute on average a difference in risk premium between stocks with high expected illiquidity and low expected illiquidity of about 9.9% annually.

While the model gives clear predictions that seem to have some bearing in the data, it is decidedly simplistic. The model and the empirical results are suggestive of further theoretical and empirical work. In particular, it would be of interest to answer questions such as: What explains the time-variation in liquidity? Why are stocks that are illiquid in absolute terms also more sensitive to liquidity risk, in the sense of high values of all three liquidity betas? How does dynamic trading affect the pricing of liquidity risk?

A Appendix

Proof of Proposition 1:

We first solve the investment problem of any investor n at time t. We assume, and later confirm, that the price at time $t+1$ is normally distributed conditional on the time t information. Hence, the investor's problem is to choose optimally the number of shares, $y^n = (y^{n,1}, \ldots, y^{n,I})$, to purchase according to

$$\max_{y^n \in \mathbb{R}^I_+} \left(E_t(W^n_{t+1}) - \frac{1}{2} A^n \operatorname{var}_t(W^n_{t+1}) \right),$$

where

$$W^n_{t+1} = (P_{t+1} + D_{t+1} - C_{t+1})^\top y^n + r^f(e^n_t - P_t^\top y^n),$$

and e^n_t is this agent's endowment. If we disregard the no-short-sale constraint, the solution is

$$y^n = \frac{1}{A^n} \left(\operatorname{var}_t(P_{t+1} + D_{t+1} - C_{t+1})\right)^{-1} \left(E_t(P_{t+1} + D_{t+1} - C_{t+1}) - r^f P_t\right).$$

We shortly verify that, in equilibrium, this solution does not entail short selling. In equilibrium, $\sum_n y^n = S$, so equilibrium is characterized by the condition that

$$P_t = E_t(P_{t+1} + D_{t+1} - C_{t+1}) - A \left(\operatorname{var}_t(P_{t+1} + D_{t+1} - C_{t+1}) S\right),$$

where $A = \left(\sum_n \frac{1}{A^n}\right)^{-1}$. The unique stationary linear equilibrium is

$$P_t = \frac{1}{r^f - 1}\left(\frac{r^f(1-\gamma)}{r^f-\gamma}(\bar{D}-\bar{C}) - A\left(\frac{r^f}{r^f-\gamma}\right)^2 \Gamma S\right) + \frac{\gamma}{r^f-\gamma}(D_t - C_t), \tag{A.1}$$

where $S = (S^1, \ldots, S^I)$ is the total supply of shares.

With this price, conditional expected net returns are normally distributed, and any investor n holds a fraction $A/A^n > 0$ of the market portfolio $S > 0$ so he is not short selling any securities. Therefore, our assumptions are satisfied in equilibrium.

Finally, since investors have mean-variance preferences, the conditional CAPM holds for net returns. See, for instance, Huang and Litzenberger (1988). Rewriting in terms of net returns yields the result stated in the proposition. □

Proof of Proposition 2:

The conditional expected return on a portfolio q is computed using (A.1):

$$\begin{aligned} E_{t-1}(r_t^q) &= E_{t-1}\left(\frac{B + r^f D_t^q - \gamma C_t^q}{B + \gamma D_{t-1}^q - \gamma C_{t-1}^q}\right) \\ &= \frac{B + r^f(1-\gamma)\bar{D}^q + r^f\gamma D_{t-1}^q - \gamma(1-\gamma)\bar{C}^q - \gamma^2 C_{t-1}^q}{B + \gamma D_{t-1}^q - \gamma C_{t-1}^q}, \end{aligned}$$

where,

$$B = \frac{r^f - \gamma}{r^f - 1}\, q^\top \left(\frac{r^f(1-\gamma)}{r^f-\gamma}(\bar{D}-\bar{C}) - A\left(\frac{r^f}{r^f-\gamma}\right)^2 \Gamma S\right).$$

The conditional expected return depends on C_{t-1}^q in the following way:

$$
\begin{aligned}
&\frac{\partial}{\partial C_{t-1}^q} E_{t-1}(r_t^q - r^f) \\
&= \frac{\gamma}{(r^f-\gamma)^2 P_{t-1}^2}\Big[-\gamma\left(B+\gamma D_{t-1}^q - \gamma C_{t-1}^q\right) \\
&\quad + \left(B + r^f(1-\gamma)\bar{D}^q + r^f\gamma D_{t-1}^q - \gamma(1-\gamma)\bar{C}^q - \gamma^2 C_{t-1}^q\right)\Big] \\
&= \frac{\gamma}{(r^f-\gamma)P_{t-1}^2}\Big[\gamma D_{t-1}^q + (1-\gamma)E(D_t^q + P_t^q \mid D_{t-1}^q = \bar{D}^q, C_{t-1}^q = \bar{C}^q)\Big].
\end{aligned}
$$

This partial derivative is greater than 0 under the conditions given in the proposition. □

Proof of Proposition 3:

The conditional covariance between illiquidity and return for a portfolio q is:

$$
\begin{aligned}
\mathrm{cov}_{t-1}(c_t^q, r_t^q) &= \frac{1}{(P_{t-1}^q)^2}\,\mathrm{cov}_{t-1}(C_t^q\,,\,P_t^q + D_t^q) \\
&= \frac{1}{(P_{t-1}^q)^2(r^f-\gamma)}\,\mathrm{cov}_{t-1}(C_t^q\,,\,r^f D_t^q - \gamma C_t^q) \\
&= \frac{1}{(P_{t-1}^q)^2(r^f-\gamma)}\,(r^f\,q^\top\Sigma^{CD}q - \gamma\,q^\top\Sigma^C q),
\end{aligned}
$$

which yields the proposition. □

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Table 1: **Properties of illiquidity portfolios.** This table reports the properties of the odd–numbered portfolios from the 25 equally-weighted illiquidity portfolios formed for each year y during the period 1964 to 1999. The portfolios are formed by sorting on their annual illiquidities the common shares listed on NYSE and AMEX with price, at beginning of year, between 5 and 1000, and return and volume data in year $y-1$ for at least 150 days. The annual illiquidity for each eligible stock is the average over the entire year $y-1$ of daily illiquidities (computed analogously to monthly illiquidity calculation in Equation (11)). The four betas or the covariances, β^{1p}, β^{2p}, β^{3p} and β^{4p}, are computed for each portfolio as per Equation (6) using the entire time-series, i.e., all monthly return and illiquidity observations for the portfolio and the market portfolio over the sample period. The monthly innovations in portfolio illiquidity and market illiquidity are computed using an AR(2) specification for the standardized illiquidity series as described in Section 5.3. The monthly innovations in market portfolio return are computed using an AR(2) specification for the raw (not standardized) market return series that also employs available market characteristics at the beginning of the month. The average illiquidity $E(c^p)$, the average return, $E(r^p)$, the turnover, and the market capitalization (size), are computed analogously for each portfolio as averages of the respective monthly characteristics over the entire sample period. Finally, the standard deviation of returns for a portfolio, $\sigma(r^p)$, is the average of the standard deviation of daily returns for the portfolio's constituent stocks computed each month. The characteristics have been scaled as indicated in the table for ease of reporting.

	β^{1p} (* 10^3)	β^{2p} (* 10^{15})	β^{3p} (* 10^9)	β^{4p} (* 10^9)	$E(r^p)$ (%)	$\sigma(r^p)$ (%)	$E(c^p)$ (* 10^6)	turnover	size (ml $)
1	1.99	0.02	−0.96	−0.01	12.30	1.61	0.01	2.37	972.05
3	2.28	0.02	−1.16	−0.03	11.77	1.72	0.04	2.55	179.10
5	2.54	0.04	−1.35	−0.08	13.84	1.85	0.08	2.60	96.29
7	2.69	0.04	−1.45	−0.10	14.53	1.99	0.13	2.68	59.07
9	2.78	0.14	−1.53	−0.23	16.25	2.04	0.23	2.54	38.44
11	2.88	0.22	−1.57	−0.40	16.00	2.13	0.36	2.58	26.32
13	2.89	1.36	−1.63	0.59	16.15	2.16	0.57	2.37	18.96
15	3.00	0.67	−1.65	−1.03	17.75	2.24	0.90	2.17	14.01
17	2.95	0.69	−1.73	−1.22	17.71	2.29	1.45	2.02	10.04
19	2.95	1.08	−1.79	−2.29	18.26	2.37	2.40	1.86	7.29
21	2.98	2.30	−1.78	−3.39	20.97	2.43	4.32	1.71	5.02
23	2.99	4.29	−1.79	−5.29	21.91	2.60	8.88	1.56	3.17
25	2.97	12.14	−1.73	−10.47	24.39	3.32	42.83	1.40	1.32

Table 2: **Beta correlations for illiquidity portfolios.** This table reports the correlations of the four betas or the covariances, β^{1p}, β^{2p}, β^{3p} and β^{4p}, for the 25 equally-weighted illiquidity portfolios. The portfolios are formed for each year y during the period 1964 to 1999 by sorting on their annual illiquidities the common shares listed on NYSE and AMEX with price, at beginning of year, between 5 and 1000, and return and volume data in year $y-1$ for at least 150 days. The annual illiquidity for each eligible stock is the average over the entire year $y-1$ of daily illiquidities (computed analogously to monthly illiquidity calculation in Equation (11)). The four betas are computed for each portfolio as per Equation (6) using the entire time-series, i.e., all monthly return and illiquidity observations for the portfolio and the market portfolio over the sample period. The monthly innovations in portfolio illiquidity and market illiquidity are computed using an AR(2) specification for the standardized illiquidity series as described in Section 5.3. The monthly innovations in market portfolio return are computed using an AR(2) specification for the raw (not standardized) market return series that also employs available market characteristics at the beginning of the month.

	β^{1p}	β^{2p}	β^{3p}	β^{4p}
β^{1p}	1.000	0.426	−0.979	−0.475
β^{2p}		1.000	−0.475	−0.879
β^{3p}			1.000	0.564
β^{4p}				1.000

Table 3: **Beta correlations for individual stocks.** This table reports the correlations of the four betas or the covariances, β^{1i}, β^{2i}, β^{3i} and β^{4i}, for the common shares listed on NYSE and AMEX during the period 1964–1999. The correlations are computed annually for all eligible stocks in a year and averaged over the sample period. A stock is included in year y if it has price, at beginning of year, between 5 and 1000, and return and volume data in year $y-1$ for at least 150 days. The four betas are computed for each stock as per Equation (6) using the entire time-series, i.e., all monthly return and illiquidity observations for the stock and the market portfolio over the sample period. The monthly innovations in market illiquidity are computed using an AR(2) specification for the standardized illiquidity series as described in Section 5.3. The innovations in stock illiquidity are computed using a similar AR(2) specification as for the market illiquidity but with AR(2) coefficients that are estimated for the market illiquidity. The monthly innovations in market portfolio return are computed using an AR(2) specification for the raw (not standardized) market return series that also employs available market characteristics at the beginning of the month.

	β^{1i}	β^{2i}	β^{3i}	β^{4i}
β^{1i}	1.000	0.093	−0.640	−0.114
β^{2i}		1.000	−0.145	−0.634
β^{3i}			1.000	0.134
β^{4i}				1.000

Table 4: **Return and liquidity risk.** This table reports the results of the Fama and Macbeth (1973) type regressions employed to test the liquidity-adjusted CAPM specified in Equation (5). It presents the means of the estimated coefficients from the monthly cross-sectional regressions over the period 1964–1999 of portfolio excess returns (%) for the 25 equally-weighted illiquidity portfolios with explanatory variables being the portfolio characteristics: the four betas or the co-variances, β^{1p}, β^{2p}, β^{3p} and β^{4p}, and the average portfolio illiquidity, $E(c^p)$. For ease of reporting the estimated coefficients, these characteristics are multiplied by 10^3, 10^{15}, 10^9, 10^9 and 10^6, respectively. The t-statistics reported in the parentheses are estimated using the standard Fama and Macbeth (1973) method. The reported R^2 for a specification is the average of the R^2's from the monthly cross-sectional regressions for that specification. The 25 illiquidity portfolios are formed for each year y during the period 1964 to 1999 by sorting on their annual illiquidities the common shares listed on NYSE and AMEX with price, at beginning of year, between 5 and 1000, and return and volume data in year $y-1$ for at least 150 days. The annual illiquidity for each eligible stock is the average over the entire year $y-1$ of daily illiquidities (computed analogously to monthly illiquidity calculation in Equation (11)). The four betas are computed for each portfolio as per Equation (6) using the entire time-series, i.e., all monthly return and illiquidity observations for the portfolio and the market portfolio over the sample period.

	constant	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$E(c^p)$	R^2
1	−0.661	0.495					0.270
	(−1.350)	(2.560)					
2	0.615		0.056				0.190
	(2.420)		(3.660)				
3	−0.324			−0.670			0.290
	(−0.870)			(−2.700)			
4	0.588				−0.070		0.230
	(2.320)				(−3.720)		
5	−0.015		0.041	−0.420			0.380
	(−0.050)		(3.680)	(−1.910)			
6	0.139			−0.310	−0.056		0.410
	(0.440)			(−1.400)	(−3.810)		
7	0.137		0.003	−0.310	−0.053		0.440
	(0.430)		(0.280)	(−1.400)	(−3.200)		
8	0.648					0.018	0.160
	(2.530)					(3.850)	
9	−0.045		0.018	−0.450		0.009	0.430
	(−0.140)		(1.580)	(−2.020)		(2.320)	
10	0.152			−0.300	−0.059	−0.001	0.460
	(0.490)			(−1.340)	(−2.600)	(−0.250)	
11	0.157		0.004	−0.290	−0.057	−0.002	0.490
	(0.510)		(0.450)	(−1.330)	(−2.540)	(−0.370)	

Table 5: **Return and liquidity risk: with control variables.** This table reports the results of the Fama and Macbeth (1973) type regressions employed to test the liquidity-adjusted CAPM specified in Equation (5) but augmented with control variables $ln(size)$ and $\sigma(r^p)$ for each portfolio. It presents the means of the estimated coefficients from the monthly cross-sectional regressions over the period 1964–1999 of portfolio excess returns (%) for the 25 equally-weighted illiquidity portfolios with explanatory variables being the portfolio characteristics: the four betas or the covariances, β^{1p}, β^{2p}, β^{3p} and β^{4p}, the average portfolio illiquidity, $E(c^p)$, the natural log of the portfolio's market capitalization at the beginning of the year, $ln(size)$, and the monthly average of the standard deviation of returns for the portfolio over the previous year, $\sigma(r^p)$. For ease of reporting the estimated coefficients, these characteristics are multiplied by 10^3, 10^{15}, 10^9, 10^9, 10^6, 10^{-6} (size) and 10^2, respectively. The t-statistics reported in the parentheses are estimated using the standard Fama and Macbeth (1973) method. The reported R^2 for a specification is the average of the R^2's from the monthly cross-sectional regressions for that specification. The 25 illiquidity portfolios are formed for each year y during the period 1964 to 1999 by sorting on their annual illiquidities the common shares listed on NYSE and AMEX with price, at beginning of year, between 5 and 1000, and return and volume data in year $y-1$ for at least 150 days. The annual illiquidity for each eligible stock is the average over the entire year $y-1$ of daily illiquidities (computed analogously to monthly illiquidity calculation in Equation (11)). The four betas are computed for each portfolio as per Equation (6) using the entire time-series, i.e., all monthly return and illiquidity observations for the portfolio and the market portfolio over the sample period.

	constant	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$E(c^p)$	$ln(size)$	$\sigma(r^p)$	R^2
1	0.560						−0.075	0.135	0.370
	(1.240)						(−1.820)	(0.960)	
2	2.626	−0.504					−0.200	−0.015	0.450
	(3.170)	(−2.130)					(−3.310)	(−0.110)	
3	0.807		0.014				−0.084	0.019	0.430
	(1.920)		(1.230)				(−2.020)	(0.130)	
4	2.440			0.720			−0.216	−0.018	0.430
	(3.170)			(2.040)			(−3.280)	(−0.130)	
5	1.240				−0.058		−0.096	−0.237	0.450
	(3.150)				(−3.030)		(−2.290)	(−1.510)	
6	2.262		0.004	0.640			−0.205	−0.037	0.470
	(2.870)		(0.350)	(1.870)			(−3.110)	(−0.250)	
7	1.451			0.150	−0.053		−0.119	−0.193	0.480
	(2.060)			(0.510)	(−2.680)		(−2.010)	(−1.220)	
8	1.542		−0.003	0.200	−0.054		−0.130	−0.203	0.520
	(2.060)		(−0.270)	(0.640)	(−2.670)		(−2.090)	(−1.230)	
9	1.287					0.009	−0.111	−0.145	0.440
	(3.000)					(1.770)	(−2.740)	(−0.780)	
10	2.026		0.001	0.380		0.007	−0.177	−0.141	0.510
	(2.680)		(0.100)	(1.110)		(1.200)	(−2.700)	(−0.750)	
11	1.459			0.130	−0.052	0.001	−0.120	−0.188	0.530
	(2.100)			(0.410)	(−2.220)	(0.170)	(−2.000)	(−1.040)	
12	1.496		−0.002	0.150	−0.053	0.002	−0.126	−0.205	0.560
	(2.030)		(−0.170)	(0.450)	(−2.320)	(0.250)	(−2.020)	(−1.110)	

Table 6: **Return and liquidity risk: annual data.** This table reports the results of the Fama and Macbeth (1973) type regressions employed to test the liquidity-adjusted CAPM specified in Equation (5). It presents the means of the estimated coefficients from the annual cross-sectional regressions over the period 1964–1999 of portfolio excess returns (%) for the 25 equally-weighted illiquidity portfolios with explanatory variables being the portfolio characteristics: the four betas or the covariances, β^{1p}, β^{2p}, β^{3p} and β^{4p}, and the average portfolio illiquidity, $E(c^p)$. For ease of reporting the estimated coefficients, these characteristics are multiplied by 10^3, 10^{15}, 10^9, 10^9 and 10^6, respectively. The t-statistics reported in the parentheses are estimated using the standard Fama and Macbeth (1973) method. The reported R^2 for a specification is the average of the R^2's from the annual cross-sectional regressions for that specification. The 25 illiquidity portfolios are formed for each year y during the period 1964 to 1999 by sorting on their annual illiquidities the common shares listed on NYSE and AMEX with price, at beginning of year, between 5 and 1000, and return and volume data in year $y-1$ for at least 150 days. The annual illiquidity for each eligible stock is the average over the entire year $y-1$ of daily illiquidities (computed analogously to monthly illiquidity calculation in Equation (11)). The four betas are computed for each portfolio as per Equation (6) using the entire time-series, i.e., all monthly return and illiquidity observations for the portfolio and the market portfolio over the sample period.

	constant	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$E(c^p)$	R^2
1	−14.272	8.847					0.360
	(−1.842)	(2.490)					
2	8.702		0.912				0.210
	(2.477)		(3.320)				
3	−8.163			−11.860			0.380
	(−1.545)			(−2.600)			
4	8.226				−1.167		0.270
	(2.396)				(−3.390)		
5	−3.566		0.604	−8.230			0.470
	(−0.792)		(3.640)	(−2.080)			
6	−1.006			−6.280	−0.860		0.490
	(−0.236)			(−1.690)	(−3.940)		
7	−0.981		−0.043	−6.270	−0.902		0.520
	(−0.228)		(−0.310)	(−1.680)	(−3.700)		
8	9.266					0.285	0.170
	(2.567)					(3.390)	
9	−4.019		0.261	−8.610		0.128	0.520
	(−0.877)		(1.440)	(−2.130)		(1.920)	
10	−0.340			−5.760	−1.064	−0.061	0.560
	(−0.076)			(−1.500)	(−3.090)	(−0.760)	
11	−0.315		0.022	−5.740	−1.054	−0.065	0.590
	(−0.071)		(0.140)	(−1.490)	(−3.170)	(−0.710)	

Table 7: **Return and liquidity risk: without January.** This table reports the results of the Fama and Macbeth (1973) type regressions employed to test the liquidity-adjusted CAPM specified in Equation (5). It presents the means of the estimated coefficients from the monthly cross-sectional regressions (excluding the month of January) over the period 1964–1999 of portfolio excess returns (%) for the 25 equally-weighted illiquidity portfolios with explanatory variables being the portfolio characteristics: the four betas or the covariances, β^{1p}, β^{2p}, β^{3p} and β^{4p}, and the average portfolio illiquidity, $E(c^p)$. For ease of reporting the estimated coefficients, these characteristics are multiplied by 10^3, 10^{15}, 10^9, 10^9 and 10^6, respectively. The t-statistics reported in the parentheses are estimated using the standard Fama and Macbeth (1973) method. The reported R^2 for a specification is the average of the R^2's from the monthly cross-sectional regressions for that specification. The 25 illiquidity portfolios are formed for each year y during the period 1964 to 1999 by sorting on their annual illiquidities the common shares listed on NYSE and AMEX with price, at beginning of year, between 5 and 1000, and return and volume data in year $y-1$ for at least 150 days. The annual illiquidity for each eligible stock is the average over the entire year $y-1$ of daily illiquidities (computed analogously to monthly illiquidity calculation in Equation (11)). The four betas are computed for each portfolio as per Equation (6) using the entire time-series, i.e., all monthly return and illiquidity observations for the portfolio and the market portfolio over the sample period.

	constant	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$E(c^p)$	R^2
1	0.131	0.106					0.260
	(0.280)	(0.590)					
2	0.391		0.020				0.180
	(1.520)		(1.430)				
3	0.200			−0.150			0.270
	(0.550)			(−0.640)			
4	0.383				−0.024		0.220
	(1.490)				(−1.380)		
5	0.344		0.019	−0.030			0.370
	(1.060)		(1.810)	(−0.150)			
6	0.405			0.010	−0.025		0.390
	(1.300)			(0.070)	(−1.790)		
7	0.402		0.004	0.010	−0.021		0.420
	(1.290)		(0.440)	(0.060)	(−1.280)		
8	0.401					0.007	0.150
	(1.560)					(1.620)	
9	0.328		0.007	−0.040		0.004	0.410
	(1.010)		(0.620)	(−0.220)		(1.210)	
10	0.005			−0.150	−0.049	−0.003	0.430
	(0.020)			(−0.650)	(−1.850)	(−0.480)	
11	0.389		0.003	0.000	−0.017	0.001	0.480
	(1.280)		(0.290)	(0.010)	(−0.780)	(0.250)	

Table 8: **Return and liquidity risk: 1964–1981.** This table reports the results of the Fama and Macbeth (1973) type regressions employed to test the liquidity-adjusted CAPM specified in Equation (5). It presents the means of the estimated coefficients from the monthly cross-sectional regressions over the period 1964–1981 of portfolio excess returns (%) for the 25 equally-weighted illiquidity portfolios with explanatory variables being the portfolio characteristics: the four betas or the covariances, β^{1p}, β^{2p}, β^{3p} and β^{4p}, and the average portfolio illiquidity, $E(c^p)$. For ease of reporting the estimated coefficients, these characteristics are multiplied by 10^3, 10^{15}, 10^9, 10^9 and 10^6, respectively. The t-statistics reported in the parentheses are estimated using the standard Fama and Macbeth (1973) method. The reported R^2 for a specification is the average of the R^2's from the monthly cross-sectional regressions for that specification. The 25 illiquidity portfolios are formed for each year y during the period 1964 to 1999 by sorting on their annual illiquidities the common shares listed on NYSE and AMEX with price, at beginning of year, between 5 and 1000, and return and volume data in year $y-1$ for at least 150 days. The annual illiquidity for each eligible stock is the average over the entire year $y-1$ of daily illiquidities (computed analogously to monthly illiquidity calculation in Equation (11)). The four betas are computed for each portfolio as per Equation (6) using the entire time-series, i.e., all monthly return and illiquidity observations for the portfolio and the market portfolio over the sample period.

	constant	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$E(c^p)$	R^2
1	−2.255	1.042					0.290
	(−3.160)	(3.400)					
2	0.478		0.092				0.200
	(1.210)		(3.740)				
3	−1.464			−1.350			0.320
	(−2.770)			(−3.430)			
4	0.436				−0.114		0.260
	(1.120)				(−3.740)		
5	−1.061		0.053	−1.030			0.410
	(−2.310)		(3.280)	(−3.010)			
6	−0.885			−0.900	−0.070		0.430
	(−2.030)			(−2.730)	(−3.240)		
7	−0.892		0.011	−0.900	−0.059		0.460
	(−2.040)		(0.850)	(−2.730)	(−2.490)		
8	0.537					0.028	0.170
	(1.340)					(3.750)	
9	−1.099		0.025	−1.060		0.011	0.450
	(−2.370)		(1.650)	(−3.080)		(1.960)	
10	−0.899			−0.910	−0.065	0.001	0.470
	(−2.080)			(−2.760)	(−2.090)	(0.200)	
11	−0.887		0.011	−0.900	−0.060	−0.001	0.500
	(−2.060)		(0.820)	(−2.750)	(−1.950)	(−0.070)	

Table 9: **Return and liquidity risk: 1982–1999.** This table reports the results of the Fama and Macbeth (1973) type regressions employed to test the liquidity-adjusted CAPM specified in Equation (5). It presents the means of the estimated coefficients from the monthly cross-sectional regressions over the period 1982–1999 of portfolio excess returns (%) for the 25 equally-weighted illiquidity portfolios with explanatory variables being the portfolio characteristics: the four betas or the covariances, β^{1p}, β^{2p}, β^{3p} and β^{4p}, and the average portfolio illiquidity, $E(c^p)$. For ease of reporting the estimated coefficients, these characteristics are multiplied by 10^3, 10^{15}, 10^9, 10^9 and 10^6, respectively. The t-statistics reported in the parentheses are estimated using the standard Fama and Macbeth (1973) method. The reported R^2 for a specification is the average of the R^2's from the monthly cross-sectional regressions for that specification. The 25 illiquidity portfolios are formed for each year y during the period 1964 to 1999 by sorting on their annual illiquidities the common shares listed on NYSE and AMEX with price, at beginning of year, between 5 and 1000, and return and volume data in year $y-1$ for at least 150 days. The annual illiquidity for each eligible stock is the average over the entire year $y-1$ of daily illiquidities (computed analogously to monthly illiquidity calculation in Equation (11)). The four betas are computed for each portfolio as per Equation (6) using the entire time-series, i.e., all monthly return and illiquidity observations for the portfolio and the market portfolio over the sample period.

	constant	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$E(c^p)$	R^2
1	0.934	−0.052					0.240
	(1.440)	(−0.230)					
2	0.615		0.056				0.190
	(2.420)		(3.660)				
3	−0.324			−0.670			0.290
	(−0.870)			(−2.700)			
4	0.588				−0.070		0.230
	(2.320)				(−3.720)		
5	−0.015		0.041	−0.420			0.380
	(−0.050)		(3.680)	(−1.910)			
6	0.139			−0.310	−0.056		0.410
	(0.440)			(−1.400)	(−3.810)		
7	0.137		0.003	−0.310	−0.053		0.440
	(0.430)		(0.280)	(−1.400)	(−3.200)		
8	0.648					0.018	0.160
	(2.530)					(3.850)	
9	−0.045		0.018	−0.450		0.009	0.430
	(−0.140)		(1.580)	(−2.020)		(2.320)	
10	0.152			−0.300	−0.059	−0.001	0.460
	(0.490)			(−1.340)	(−2.600)	(−0.250)	
11	0.157		0.004	−0.290	−0.057	−0.002	0.490
	(0.510)		(0.450)	(−1.330)	(−2.540)	(−0.370)	

Table 10: **Return and liquidity risk: value weighted.** This table reports the results of the Fama and Macbeth (1973) type regressions employed to test the liquidity-adjusted CAPM specified in Equation (5). It presents the means of the estimated coefficients from the monthly cross-sectional regressions over the period 1964–1999 of portfolio excess returns (%) for the 25 value-weighted illiquidity portfolios with explanatory variables being the portfolio characteristics: the four betas or the covariances, β^{1p}, β^{2p}, β^{3p} and β^{4p}, and the average portfolio illiquidity, $E(c^p)$. For ease of reporting the estimated coefficients, these characteristics are multiplied by 10^3, 10^{15}, 10^9, 10^9 and 10^6, respectively. The t-statistics reported in the parentheses are estimated using the standard Fama and Macbeth (1973) method. The reported R^2 for a specification is the average of the R^2's from the monthly cross-sectional regressions for that specification. The 25 illiquidity portfolios are formed for each year y during the period 1964 to 1999 by sorting on their annual illiquidities the common shares listed on NYSE and AMEX with price, at beginning of year, between 5 and 1000, and return and volume data in year $y - 1$ for at least 150 days. The annual illiquidity for each eligible stock is the average over the entire year $y - 1$ of daily illiquidities (computed analogously to monthly illiquidity calculation in Equation 11). The four betas are computed for each portfolio as per Equation (6) using the entire time-series, i.e., all monthly return and illiquidity observations for the portfolio and the (equally-weighted) market portfolio over the sample period.

	constant	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$E(c^p)$	R^2
1	−0.764	0.606					0.260
	(−1.750)	(3.230)					
2	0.678		0.073				0.180
	(2.930)		(3.270)				
3	−0.368			−0.800			0.260
	(−1.110)			(−3.300)			
4	0.641				−0.101		0.210
	(2.790)				(−3.480)		
5	−0.166		0.032	−0.630			0.350
	(−0.550)		(1.930)	(−2.820)			
6	−0.048			−0.520	−0.054		0.350
	(−0.160)			(−2.370)	(−2.340)		
7	0.707					0.015	0.150
	(3.040)					(3.000)	
8	−0.250			−0.700		0.006	0.340
	(−0.810)			(−3.090)		(1.510)	

Table 11: **Properties of size portfolios.** This table reports the properties of the odd–numbered portfolios from the 25 equally-weighted size portfolios formed for each year y during the period 1964 to 1999. The portfolios are formed by sorting on beginning of year market capitalizations the common shares listed on NYSE and AMEX with price, at beginning of year, between 5 and 1000, and return and volume data in year $y-1$ for at least 150 days. The four betas or the covariances, β^{1p}, β^{2p}, β^{3p} and β^{4p}, are computed for each portfolio as per Equation (6) using the entire time-series, i.e., all monthly return and illiquidity observations for the portfolio and the market portfolio over the sample period. The monthly innovations in portfolio illiquidity and market illiquidity are computed using an AR(2) specification for the standardized illiquidity series as described in Section 5.3. The monthly innovations in market portfolio return are computed using an AR(2) specification for the raw (not standardized) market return series that also employs available market characteristics at the beginning of the month. The average illiquidity $E(c^p)$, the average return, $E(r^p)$, the turnover, and the market capitalization (size), are computed analogously for each portfolio as averages of the respective monthly characteristics over the entire sample period. Finally, the standard deviation of returns for a portfolio, $\sigma(r^p)$, is the average of the standard deviation of daily returns for the portfolio's constituent stocks computed each month. The characteristics have been scaled as indicated for ease of reporting.

	β^{1p} (* 10^3)	β^{2p} (* 10^{15})	β^{3p} (* 10^9)	β^{4p} (* 10^9)	$E(r^p)$ (%)	$\sigma(r^p)$ (%)	$E(c^p)$ (* 10^6)	turnover	size (ml $)
1	3.13	10.10	−1.77	−8.13	23.27	2.93	6.35	2.10	0.78
3	3.11	4.76	−1.81	−7.78	19.58	2.75	2.57	2.12	2.10
5	3.17	9.94	−1.82	−2.23	19.47	2.62	1.53	2.26	3.63
7	3.08	1.38	−1.80	−1.81	20.29	2.46	0.98	2.16	5.64
9	3.07	0.98	−1.78	−1.48	16.49	2.40	0.65	2.43	8.34
11	3.01	0.47	−1.74	−0.94	16.21	2.28	0.43	2.37	11.92
13	2.98	0.38	−1.69	−0.58	16.16	2.17	0.31	2.44	17.13
15	2.79	0.27	−1.54	−0.35	14.76	2.06	0.21	2.33	24.97
17	2.59	0.17	−1.42	−0.29	15.49	1.94	0.15	2.27	37.20
19	2.57	0.07	−1.42	−0.16	14.63	1.85	0.09	2.27	57.78
21	2.39	0.04	−1.26	−0.08	14.05	1.76	0.05	2.17	97.16
23	2.13	0.01	−1.09	−0.04	12.88	1.64	0.03	2.00	186.96
25	1.75	0.00	−0.83	−0.01	12.97	1.50	0.01	1.52	971.87

Table 12: **Return and liquidity risk.** This table reports the results of the Fama and Macbeth (1973) type regressions employed to test the liquidity-adjusted CAPM specified in Equation (5). It presents the means of the estimated coefficients from the monthly cross-sectional regressions over the period 1964–1999 of portfolio excess returns (%) for the 25 equally-weighted size portfolios with explanatory variables being the portfolio characteristics: the four betas or the covariances, β^{1p}, β^{2p}, β^{3p} and β^{4p}, and the average portfolio illiquidity, $E(c^p)$. For ease of reporting the estimated coefficients, these characteristics are multiplied by 10^3, 10^{15}, 10^9, 10^9 and 10^6, respectively. The t-statistics reported in the parentheses are estimated using the standard Fama and Macbeth (1973) method. The reported R^2 for a specification is the average of the R^2's from the monthly cross-sectional regressions for that specification. The 25 size portfolios are formed for each year y during the period 1964 to 1999 by sorting on beginning of year market capitalizations the common shares listed on NYSE and AMEX with price, at beginning of year, between 5 and 1000, and return and volume data in year $y-1$ for at least 150 days. The four betas are computed for each portfolio as per Equation (6) using the entire time-series, i.e., all monthly return and illiquidity observations for the portfolio and the market portfolio over the sample period.

	constant	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$E(c^p)$	R^2
1	−0.127	0.301					0.330
	(−0.340)	(1.900)					
2	0.644		0.036				0.190
	(2.590)		(2.120)				
3	0.051			−0.420			0.330
	(0.170)			(−1.890)			
4	0.625				−0.048		0.220
	(2.530)				(−2.190)		
5	0.169		0.019	−0.330			0.410
	(0.610)		(1.720)	(−1.580)			
6	0.220			−0.280	−0.028		0.430
	(0.810)			(−1.380)	(−1.820)		
7	0.226		0.007	−0.280	−0.022		0.470
	(0.840)		(0.770)	(−1.360)	(−1.560)		
8	0.641					0.019	0.180
	(2.560)					(2.570)	
9	0.181		−0.011	−0.310		0.017	0.460
	(0.660)		(−1.050)	(−1.530)		(3.210)	
10	0.171			−0.320	0.013	0.017	0.480
	(0.630)			(−1.550)	(0.720)	(3.010)	
11	0.154		−0.010	−0.340	0.012	0.020	0.510
	(0.570)		(−1.000)	(−1.630)	(0.660)	(3.120)	

Table 13: **Return and liquidity risk: value weighted.** This table reports the results of the Fama and Macbeth (1973) type regressions employed to test the liquidity-adjusted CAPM specified in Equation (5). It presents the means of the estimated coefficients from the monthly cross-sectional regressions over the period 1964–1999 of portfolio excess returns (%) for the 25 value-weighted size portfolios with explanatory variables being the portfolio characteristics: the four betas or the covariances, β^{1p}, β^{2p}, β^{3p} and β^{4p}, and the average portfolio illiquidity, $E(c^p)$. For ease of reporting the estimated coefficients, these characteristics are multiplied by 10^3, 10^{15}, 10^9, 10^9 and 10^6, respectively. The t-statistics reported in the parentheses are estimated using the standard Fama and Macbeth (1973) method. The reported R^2 for a specification is the average of the R^2's from the monthly cross-sectional regressions for that specification. The 25 size portfolios are formed for each year y during the period 1964 to 1999 by sorting on beginning of year market capitalizations the common shares listed on NYSE and AMEX with price, at beginning of year, between 5 and 1000, and return and volume data in year $y-1$ for at least 150 days. The four betas are computed for each portfolio as per Equation (6) using the entire time-series, i.e., all monthly return and illiquidity observations for the portfolio and the (equally-weighted) market portfolio over the sample period.

	constant	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$E(c^p)$	R^2
1	−0.279	0.378					0.320
	(−0.790)	(2.480)					
2	0.670		0.066				0.200
	(2.720)		(2.840)				
3	−0.074			−0.540			0.320
	(−0.250)			(−2.500)			
4	0.644				−0.085		0.240
	(2.660)				(−2.690)		
5	0.092		0.034	−0.400			0.420
	(0.350)		(2.120)	(−1.990)			
6	0.113			−0.380	−0.040		0.420
	(0.440)			(−1.890)	(−1.760)		
7	0.062		0.048	−0.430	0.021		0.470
	(0.240)		(1.970)	(−2.080)	(0.600)		
8	0.672					0.030	0.200
	(2.730)					(2.840)	
9	0.092		0.033	−0.400		0.001	0.460
	(0.350)		(0.410)	(−2.000)		(0.010)	
10	0.050			−0.440	0.026	0.023	0.480
	(0.200)			(−2.120)	(0.670)	(1.960)	
11	0.055		0.024	−0.430	0.025	0.012	0.510
	(0.220)		(0.300)	(−2.120)	(0.630)	(0.300)	