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Non-Distortionary Properties of Environmental Taxes: Extension of Sandmo's Observation

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Abstract

This paper extends Sandmo's (1975) observation that when the revenue from environmental taxes, set at their corrective Pigovian levels, equals the government's revenue need, the environmental taxes are non-distortionary. In the extension, we find that a large component of environmental tax revenue is non-distortionary, whether or not the total environmental tax revenue meets the government's revenue need, and whether or not there are other (distortionary) taxes. We also find that this non-distortionary component has the same distributional effects as a lump-sum tax, suggesting a policy tradeoff between the social goals of efficiency and distribution.

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Introduction

This paper extends Sandmo's (1975) observation that when the revenue from environmental taxes, set at their corrective Pigovian levels, equals the government's revenue need, the environmental taxes are non-distortionary.[2] For the extension, we define the Pigovian revenue as the sum of the marginal environmental damages times the amount of the environmental harm, and the appropriated Pigovian revenue as the part of the Pigovian revenue used to finance a public good or offset other taxes. The extension says that the appropriated Pigovian revenue is non-distortionary, whether or not it equals the government's revenue need, and whether or not there are other (distortionary) taxes.

Because of tax interaction effects it may seem that such an extension could not go through (and we had doubts ourselves). Further, much of the recent literature has emphasized the distortionary nature of environmental taxes, see for example Parry (1997), Bovenberg and Goulder (forthcoming), Bovenberg and de Mooij (1997), Oates (1994), Fullerton (1997), Bovenberg and de Mooij (1994), and Goulder, Parry, Williams and Burtraw (1999),[3] and the conclusions just cited seem to be in conflict with a large component of environmental tax revenue being non-distortionary. But we find no inconsistency between our analytic results and the analytic results in the papers just cited. Some of the difference in perspective comes from different ways the analysis is set up, and some from the new results themselves. We decompose the environmental tax in a way that differs from its treatment in the recent

[2]He also observed that first-best efficiency is obtained. This additional observation does not go through in all the cases of the extension, however.

[3]Parry (1997, p. 10) wrote: "environmental taxes are likely to *increase* rather than *decrease* the costs associated with the tax system overall." In the forthcoming *Handbook of Public Economics* Bovenberg and Goulder (forthcoming, p. 1) summarized the current literature of second-best environmental taxes: "The recent work emphasizes two fundamental ideas. First, environmental taxes and other forms of regulations act as implicit taxes on factors of production because they raise the costs and prices of produced goods relative to the prices of factors, thereby lowering real factor returns. Second, these implicit taxes compound distortions posed by pre-existing factor taxes." Bovenberg and de Mooij (1997, p. 242) wrote that environmental taxes "tend to be more, rather than less, distortionary than other taxes." Oates (1994, p. 916): "where pollution taxes must exist alongside distorting levies, the various economic linkages between the demands for different goods and in their production will typically be the source of additional excess burden." Fullerton (1997, p. 245): "Even if the pollution tax helps solve an environmental problem, it likely worsens other tax distortions." Bovenberg and de Mooij (1994, p. 1085): "environmental taxes typically exacerbate, rather than alleviate, preexisting tax distortions." Goulder et al. (1999 p. 27 ms. version): "We find that pre-existing taxes significantly raise the costs of all environmental policies relative to their costs in a first-best world."

literature. Our conclusions on distortionary costs are based on total distortionary costs, including the own-price effects of environmental taxes. At least some of the conclusions cited above exclude own-price effects.[4] Still, it is a puzzle to see how the appropriated Pigovian revenue could be non-distortionary in the presence of other taxes and tax interaction effects, a puzzle we address in the latter part of the paper.

Pigovian revenues are large, probably in the $100s of billions annually.[5] The part of the Pigovian revenue appropriated for financing a public good or reducing other taxes arises from environmental (more precisely, externality) taxes or auctioned marketable permits. Not all of the Pigovian revenues are appropriable, but conceivably the appropriated Pigovian part of environmental taxes and auctioned allowances could become a large, perhaps the largest source of non-distortionary revenue in the tax system.

In related contributions we characterize conditions of symmetry between second-best environmental and other taxes (Theorem 1 and Corollary 1). This characterization simplifies and generalizes the analysis. Theorems 1 and 2 give first-order conditions FOC for second-best taxes. Theorem 3 establishes an equivalence relation between the appropriated Pigovian revenue and lump-sum taxes, and this leads directly to the extension. The equivalence rela-

[4] We thank Larry Goulder and Ian Parry for clarifying this source of divergence, personal communications, October, 2001. In Theorem 1 of our paper, the negative of own-price distortionary environmental cost is also the environmental benefit excluded in Goulder's (1995) "gross cost." This exclusion affects interpretations of distortionary cost, but not the first-order conditions for second-best equilibria.

[5] Consider just a few examples from the environmental damage assessment literature. The EPA's central estimate for the benefits of the Clean Air Act for the year 2000 is $71 billion (p. iii, 1999). Shrank and Lomax (2001) estimated $78 billion annual costs from wasted time and gasoline due to highway congestion (the estimate excludes costs of increased air pollution, costs of increased maintenence and capacity; the $78 billion estimate is for 68 urbanized areas which include about 75% of the urbanized areas in the US). Porter (1999) estimated an annual external cost from automobile air pollution of $27 billion, $60 billion from non-driver fatalities (p. 194), and cites the $90 billion annual cost of road administration, maintenance and capital outlay, much of which is externally borne cost, including the wear and tear from trucks from their heavy axle loadings (p. 161). Porter cites 9 other studies with estimates of external costs of driving in the range of $500 billion to $1 trillion annually (p. 194). Newman and Kenworthy (1999, p. 56) list 5 studies for the US and 3 for other countries (with some overlap with Porter's citations). These estimates range from about $400 to 800 billion in annual external costs for the US. Vickrey estimated the congestion cost of cars in Manhattan to be about $15 per car per trip into the island (personal communication). H. Uzawa (1974, p. 98) estimated the external cost of driving to be in the range of $3000 to 4000 in current dollar values per car per year, comparable to other estimates. Most of these estimates are based on average damages. Converting to marginal damages, appropriate for an estimate of the Pigovian revenue, would tend to increase estimates of Pigovian revenues. In the other direction, only a portion of Pigovian revenues are collectable and thereby appropriable.

tion also shows, perhaps counter-intuitively, that (adverse) distributional effects associated with lump-sum taxes carry over to the unregulated case, command-and-control, and grandfathered marketable allowances. Ways to reduce these effects, also perhaps counter-intuitively, are to increase the environmental tax level or reduce the number of marketable allowances.

Besides Sandmo, others have considered non-distortionary properties of environmental taxes. Kaplow (1996) showed that in the presence of distortionary taxes, taxes based on the marginal benefit principle can finance public goods without additional distortionary costs, and he applied this idea to environmental taxes, which produce the public good of environmental quality. Fullerton and Metcalf (2001) showed that environmental taxes and other forms of regulation can create rents that when taxed away lower the distortionary cost of the tax system, compared with forms of regulation that leave the rents with the producers. Compared with these papers on non-distortinary taxes, our contribution is to find the extension, the symmetry characterization, and the lump-sum equivalent distortionary and distributional effects of the appropriated Pigovian revenue.

Section I sets out the model. Section II provides intuition for the formal results to follow. Section III proves the results leading to the extension. Section IV explores how tax interaction effects fit with the non-distortionary property. Section V is on policy.

I. The Model

The model applies to environmental harms generated in production processes. In the model x_i, y_i, z, S and L_i are i's consumption of the private goods x and y, the public good z, the externality "smoke" S, and i's labor supply.

(1)	$U^i(x_i, y_i, z, L_i, S)$	Utility for individual i, where U is homothetic, increasing in $x_i, y_i,$ and z and decreasing in L_i and S
(2)	$x = f(L_x, S)$	Production function for the private good x
(3)	$y = g(L_y)$	Production function for the private good y
(4)	$z = h(L_z)$	Production function for the public good z

where S is an externality harm in (1), and a factor of production in (2). Abatement opportunities are subsumed in the production function f. The model applies to externalities from congestion and depletion, as well as pollution.[6] We assume constant returns to scale in the production functions.[7]

There can be excise taxes on the two private goods x and y, and a labor tax. We normalize by letting labor be the numeraire good with the wage set at $w = 1$, by setting the excise tax on x equal to zero, and by passing back the excise tax on y to be a possibly separate tax on labor L_y in the y-industry. Write the (per unit) factor tax on L_x as v and the (per unit) factor tax on L_y as $\tilde{v}$. Labor used in producing the public good z is untaxed. The labor taxes v and $\tilde{v}$ are collected from producers, and the (per unit) environmental tax t is collected from the smoke emitting x-industry.[8]

Define i's marginal benefit from the public good by $\hat{p}_i = \frac{-U_z^i}{U_L^i}$ (where $U_z^i = \frac{\partial U^i}{\partial z}$ and $U_L^i = \frac{\partial U^i}{\partial L_i}$), and the sum of the marginal benefits of z as $\hat{p} = \sum_i \hat{p}_i$. Correspondingly, define the marginal damage of smoke to individual i as $\hat{t}_i = \frac{U_S^i}{U_L^i}$.[9] Define the (corrective) Pigovian level of the environmental tax as the sum of the marginal damages $\hat{t} = \sum \hat{t}_i$.

Define the Pigovian revenue as $\hat{t}S$ and divide it into two parts. One part, the *appropriated Pigovian revenue*, is appropriated by the government as revenue to finance the public good or reduce other taxes. The other part, the *Pigovian compensation*, is used to compensate the recipients of the environmental harm. Define the individual appropriated Pigovian revenue to be $\alpha_i S$, where α_i is a constant rate of appropriation chosen by the government, and define the aggregate appropriated Pigovian revenue to be αS, where $\sum \alpha_i = \alpha$. Define the individual compensation to each i to be $(\hat{t}_i - \alpha_i)S$, and the aggregate Pigovian compensation to be $(\hat{t} - \alpha)S$. By construction, the aggregate appropriated Pigovian revenue αS and the Pigovian compensation $(\hat{t} - \alpha)S$ add to the aggregate Pigovian revenue $\hat{t}S$.

[6]For example, in the problem of highway congestion where each commuter faces a congestion tax, each commuter has a production function that produces trips with the factors of her time, her car's gasoline, oil and capital depreciation, and congestion costs imposed on others, internalized through the congestion tax.

[7]While g and h are linear with CTS, we use the more general notation to allow for generalization to additional factors of production.

[8]We use capitals for private good prices and factors of production; and lower case for the private and public goods, wages and taxes.

[9]When individuals have their own individual abatement or defensive strategies, evaluating $\hat{t}_i$ at the level of the individual efficient harm avoidance prevents the "coming to the nuisance" problem identified by Coase and others; see Page (pp. 52-62, 1973).

Divide the environmental tax t into two parts, $t = \hat{t} + \tau$, where τ is the *surtax*. Thus the environmental tax t is greater or less than its corrective Pigovian level when τ is greater or less than 0. The surtax revenue τS is appropriated by the government to finance the public good or reduce other taxes. As we will soon see, these two decompositions reveal the underlying symmetry (and asymmetry) between environmental taxes and other taxes.

We consider two specifications of utility.[10] The first is that of a representative agent, where each i has the same utility function $U(x_i, y_i, z, L_i, S)$. The second specification is additively separable in labor, where i's utility is $U^i(x_i, y_i, z, S) - L_i$, a version of the form used in studying Groves taxes (see Green and Laffont, pp. 29-32, 1980). The net social benefit NSB is defined as the sum of the utilities of all the individuals.

The second-best problem is to find tax rates τ, v and $\tilde{v}$ and appropriation rate α to maximize the NSB, subject to the Walrasian equilibrium conditions and other possible constraints. Besides the market clearing equations, the Walrasian equilibrium conditions come from:

(5) $\max\limits_{x_i, y_i, L_i} \quad U^i(x_i, y_i, z, L_i, S) \qquad$ subject to $P_x x_i + P_y y_i = wL_i + (\hat{t}_i - \alpha_i)S - M_i$

(6) $\max\limits_{L_x, S} \quad P_x f(L_x, S) - (w + v)L_x - (\hat{t} + \tau)S$

(7) $\max\limits_{L_y} \quad P_y g(L_y) - (w + \tilde{v})L_y$

(8) $wL_z \;=\; \alpha S + \tau S + vL_x + \tilde{v}L_y + M$

where (5) is the consumers' maximization problem, $(\hat{t}_i - \alpha_i)$ is individual i's rate of compensation, and M_i is an ordinary (fixed constant) lump-sum tax on individual i. For the policy application we will set $M_i = 0$ (all i), but the M_i are useful for benchmark comparisons in the analysis. (6) is industry x's problem, and (7) is y's problem. The government's budget constraint (8) says that expenditures for labor to produce z equals the tax revenue sources of the appropriated Pigovian revenue, the surtax revenue, revenue from the tax on labor in the

[10] Espinosa and Smith (2002) found that computable general equilibrium estimates of second-best taxes can vary sensitively with separability specifications. The theorems and corollaries of the paper are sufficiently general to hold for both specifications of utility, one with no separability assumptions and the other with a strong assumption of separability.

x-industry and revenue from the tax on labor in the y-industry and the lump-sum revenue M where $\sum M_i = M$.

The case with constraints $\alpha_i = \tau = M_i = 0$ is an early case in the environmental economics literature. In this case, the environmental tax t equals its corrective Pigovian level $t = \hat{t}$ and the Pigovian revenue is returned on a marginal damage basis as compensation to the harm recipients. At the time, the focus of attention was on the corrective benefits of environmental taxes, not the distortionary costs of the tax system.

A program of complete marginal damage compensation ($\alpha_i = 0$ all i) is an ideal case, since it requires estimating all the individual $\hat{t}_i$, which can not be done without substantial error. In fact, some compensation is done, but there is relatively little regulatory emphasis placed on compensation, or on marginal damage compensation. In place of compensation, there is more regulatory emphasis on reducing the harm, and this is an alternative way of protecting the vulnerable. The case of complete marginal damage compensation, however, remains a useful analytic tool in much the same way that individual Lindahl prices remain a useful analytic tool.[11]

The case with α constrained by $\alpha_i = \hat{t}_i$ (all i), τ unconstrained, and $M_i = 0$ (all i) is a main case of the recent literature on environmental taxes and distortionary costs. In this case there are no compensation payments, all the environmental revenue $tS = (\hat{t} + \tau)S$ is appropriated by the government and the environmental tax $t = \hat{t} + \tau$ is unconstrained because τ is unconstrained.

For the Walrasian equilibrium to be an appropriate solution concept, we assume that the number of individuals n is large, x_i is a small fraction of x and consumer i negligibly affects the aggregate S through his purchases of x_i (this parallels the assumption that i's purchases have a negligible affect of prices P_x and P_y), y_i is a small fraction of y, and there are many small firms in each industry. The model generalizes models of Goulder, Parry, Williams

[11] When individual Lindahl taxes are set equal to the $\hat{p}_i$ and the α_i are set equal to 0, first-best efficiency is achieved with $\alpha = \tau = v = \tilde{v} = 0$. Lindahl taxes are the same concept as Pigovian taxes, except the former is applied to public goods, the latter to public bads. Pigovian and Lindahl taxes provide an extension of the competitive model to the cases of public goods and externalities, by privatizing public goods and bads with individual prices. From this benchmark the second-best problem arises when Lindahl taxes and ordinary lump-sum taxes are unavailable.

and Burtraw (1999), Bovenberg and de Mooij (1994), and Parry (1995), see Appendix.

II. Intuition and Conjecture

In this section we rule out lump-sum taxes ($M_i = 0$) and constrain $\alpha \leq \hat{t}$ (no over-appropriation of the Pigovian revenue). The section sketches five intuitive ideas which we will refine and formalize in the following section. The first is a sketch (Figure 1) of the tasks in extending Sandmo's observation. In the figure Case (*ii*) is already done for us. Sandmo observed that when the Pigovian revenue equals the first-best revenue need, it can finance the revenue need by itself, replacing all other taxes and becoming a non-distortionary source of revenue. Unfortunately, as is well recognized, this case has little policy relevance because it is unlikely that the appropriated Pigovian revenue will exactly meet the revenue need. The most likely case is Case (*iii*) where the Pigovian revenue is less than the first-best revenue need, and the government uses some distortionary taxes, so that tax interaction effects arise. Proving the extension for this case will not be so easy, but it is the most important for policy application. For completeness, we consider Case (*i*), where the Pigovian revenue is higher than the first-best revenue need. The case also seems very unlikely, but straightforward. It seems that the government can appropriate what it needs of the Pigovian revenue to meet the first-best need, again setting other taxes to zero and obtaining non-distortionary first-best results.

The second idea is that there might be a symmetry between environmental and other taxes when the Pigovian revenue is not used to finance the public good or offset other taxes but instead to pay marginal damage compensation to recipients of the environmental harm. Figure 2(*i*) shows the demand for labor in the x-industry (the marginal revenue product of labor in the x-industry) paralleling the demand for smoke emissions in the same industry (the marginal revenue product of smoke). The after-tax price of labor ($w + v$) parallels the after-tax price of smoke ($t = \hat{t} + \tau$). The Walrasian market equilibration at the dot at a parallels the after-tax market equilibration at the dot at b.

But the parallel between the supply of labor and the sum of the marginal damages of smoke is more partial. Labor supply (to the x industry) is equilibrated to the after-tax price of labor (recall the the producer collects the tax in this model) at the dot at c. In contrast, there is no market equilibration between the sum of marginal damages at d and the smoke recipients' voluntary acceptance of smoke, because smoke is an externality (the absence of a market equilibration is symbolized by the absence of a dot, and this missing dot corresponds to $(\hat{t}_i - \alpha_i)S)$ dropping out of i's FOC in the consumers' maximization problem (5) because S is an externality.) But when $\alpha = 0$ the government pays compensation to the recipients, and the Pigovian compensation $\hat{t}S$ parallels the labor compensation wL_x. When $\alpha_i = 0$ (all i) each recipient gets marginal damage compensation, and with increasing marginal damages, each recipient of harm gets a surplus just as laborers get marginal cost compensation and a surplus with increasing marginal costs of labor.

The apparent symmetry when $\alpha = 0$ appears to break down when the government attempts to appropriate some or all of the Pigovian revenue. When the government attempts to appropriate some of the labor compensation by a per unit tax $\underline{v}$ collected from the laborers, this is equivalent to a higher tax v collected from the producers, as in Figure 2(ii). In result, there is less labor supplied with an increase in distortionary costs. In contrast, when the government appropriates some or all of the Pigovian revenue, by setting the per unit rate of appropriation $\alpha > 0$, the harm recipients involuntarily accept the smoke with less or no compensation, the producers' after-tax price of smoke $(\hat{t} + \tau)$ remains the same, and the amounts of smoke and distortionary cost apparently remain the same.

This second idea, of symmetry and asymmetry, leads to the third – that appropriation of the Pigovian revenue might be a non-distortionary source of revenue, even in the presence of the distortionary taxes τ and v (taxes τ and v are Figure 2(ii).

The third idea leads to the fourth – that as long as there is a revenue need, second-best analysis would imply increasing appropriation of the Pigovian revenue, until all of it is appropriated, Figure 3(i). If there is still a revenue need, it seems possible to augment the Pigovian revenue by decreasing the environmental tax, Figure 3(ii) Decreasing the environmental tax increases the amount of smoke, the increased smoke increases individual marginal damages and the sum of the marginal damages, and thus increases the product of the amount of smoke times the sum of the marginal damages, which is appropriated Pigovian revenue (since all the Pigovian revenue is being appropriated) and is (perhaps) a non-distortionary source of revenue. When and if τ decreases to zero at e, there appears to be no necessary reason for the process to stop. It might go on to f, where τ is negative, depending on the elasticity of smoke with respect to the environmental tax, tax interaction effects, and the subsidy cost of a negative surtax revenue.

And fifth, if the appropriated Pigovian revenue is non-distortionary in the same way that lump-sum taxes are non-distortionary, there may be policy tradeoffs between efficiency and equity goals. The next two sections refine and formalize these intuitions and conjectures.

III. Formal Results

For simplicity in the theorems and corollaries of this section, we will only consider internal Walrasian equilibria where $0 < \hat{t}, S, x, y, z < \infty$. Unless otherwise specified, environmental taxes are the only instruments of environmental regulation,[12] the theorems and corollaries below are for the model of (1) - (8), and utility is either in the form of a representative agent or additive separability of labor. Theorem 1 derives FOC for second-best τ, v and $\tilde{v}$, subject the the Walrasian equilibrium constraints, and the constraints of fixed α and M.

Theorem 1 (MB = MC). *For fixed α and M (and $\alpha_i = \alpha_j, M_i = M_j$, all i and j, with representative agent utility), the first-order conditions for second-best labor and environmental taxes are*

$$\frac{d(NSB)}{-U_L} \quad = \quad \overbrace{(\hat{p}h_L - 1)dL_z}^{\text{marginal benefit}} \quad + \quad \overbrace{\tau dS + vdL_x + \tilde{v}dL_y}^{-\text{ marginal distor'y costs}} \quad = \quad 0 \tag{9}$$

subject to the government budget constraint

$$\underbrace{wL_z}_{\text{expenditure}} \quad = \quad \underbrace{\alpha S + \tau S + vL_x + \tilde{v}L_y + M}_{\text{revenue sources}} \tag{8}$$

Proof. The main part of the proof, for (9), is in the Appendix. The second condition repeats the government's budget constraint. ■

[12] With modification, the theorems and corollaries carry over to situations where there are other regulatory controls. For example, auctioned marketable permits are similar to environmental taxes with $\alpha = 0$, "grandfathered" marketable permits are similar to environmental taxes with the environmental revenue returned to the producers, optimized command-and-control is similar to grandfathered non-marketable permits.

The restrictions of $\alpha_i = \alpha_j, M_i = M_j$ are used to preserve the world of equals, which is needed for the proof for the representative agent utilities. With additively separable utility, we don't need these restrictions for the proof. We interpret the theorem as follows. In (9), $\hat{p}h_L$, where $h_L = \frac{\partial h}{\partial L_z}$, is the value of the marginal product of a unit of labor in producing z, valued in units of labor, and 1 is the marginal cost of labor (in units of labor), so $(\hat{p}h_L - 1)$ is the net marginal benefit of the public good financed by taxes. The three terms $\tau dS, v dL_x$ and $\tilde{v} dL_y$ are differential increments of Harberger triangles of distortionary costs.[13] Thus Theorem 1 says that in the second-best equilibrium the marginal net benefits from an extra dollar of government revenue in producing more of the public good are just balanced by the increase in the distortionary costs of financing the extra dollar.

When $\alpha = 0$ the FOC of Theorem 1 satisfy a permutation symmetry that will be useful later. In this symmetry the tax-quantity pairs can be permuted, without changing equations (9) & (8). For example, exchanging $\tilde{v}$ and τ, and exchanging L_y and S, leaves the equations of the FOC the same. In this symmetry, the surtax τ is symmetric with the taxes v and $\tilde{v}$ and the externality S is symmetric with the non-externalities L_x and L_y. When $\alpha \neq 0$ the symmetry still goes through in (9), but is broken in (8). Summarizing,

Corollary 1 (Symmetry). *In the second-best equilibrium*

(a) *if* $\alpha = 0$, *then* τ *is symmetric with other taxes* v *and* $\tilde{v}$

(b) *if* $\alpha \neq 0$, *then* τ *is asymmetric with other taxes* v *and* $\tilde{v}$

Corollary 1 formalizes the symmetry idea of Section II, and as we will see later it extends to tax interaction effects. Theorem 2 derives FOC for second-best α and M, subject to the Walrasian equilibrium constraints and upperbound constraints $\overline{\alpha}$ and $\overline{M}$ (either of which could be infinite). Write λ_1 and λ_2 for the shadow prices for $\overline{\alpha}$ and $\overline{M}$ respectively.

Theorem 2 (Appropriation of the Pigovian Revenue). *First-order conditions for second-best* α *and* M *are*

(1) *either* $(\alpha \leq \overline{\alpha}$ *and* $\lambda_1 = 0)$ *or* $(\alpha = \overline{\alpha}$ *and* $\lambda_1 > 0)$

(2) *either* $(M \leq \overline{M}$ *and* $\lambda_2 = 0)$ *or* $(M = \overline{M}$ *and* $\lambda_2 > 0)$

[13] More precisely the three terms are differential distortionary benefits or the negative of the marginal distortionary costs of taxes.

(3) $\lambda_1 = S\lambda_2$

Proof. These FOC come from Lagrangian conditions. See Appendix for details.

Conditions (1) and (2) are slack complementarity conditions for the upperbound constraints $\alpha \leq \overline{\alpha}$ and $M < \overline{M}$ respectively, and (3) draws a connection between the two. Either both constraints $\alpha \leq \overline{\alpha}$ and $M < \overline{M}$ are strictly binding simultaneously, or neither is strictly binding (for interior equilibria with $S > 0$). The case where constraints on lump-sum taxes are not strictly binding (the case when the shadow price of $\overline{M}$ is zero) is a defining property of first-best efficiency.

When lump-sum taxes M_i are not allowed and $\overline{M} = 0$, Theorem 2 says that as long as the constraint on α is not strictly binding, the constraint $M \leq \overline{M}$ will not be strictly binding either and there will still be first-best efficiency in the sense that the shadow price of $\overline{M}$ is zero. The case where the constraint on α is weakly binding corresponds to the case of Sandmo's observation (when $\overline{\alpha} = \hat{t}$), and the case where the constraint is non-binding corresponds to Case (i) of Section II.

We formalize the non-distortionary property of the appropriated Pigovian revenue as follows. So far we have been working with a general model where there can be both ordinary lump-sum taxes M and appropriated Pigovian revenue αS, either ordinary taxes or appropriated Pigovian revenues, or neither. But instead of working with a single general model, it is more convenient to compare two restricted versions of the general model.

Theorem 3 proves an equivalence relation between the appropriated Pigovian revenue and ordinary lump-sum taxes. The equivalence relation shows that the appropriated Pigovian revenue has the same non-distortionary properties as lump-sum taxes. Theorem 3 also shows that the distributional properties associated with ordinary lump-sum taxes carry over to the appropriated Pigovian revenue as well. A simple corollary establishes the extension of Sandmo's observation.

For Theorem 3 consider two restricted versions of model (1) - (8). In Model A no lump-sum taxes are allowed (by constraint $M_i = 0$ all i), but the Pigovian revenue can be appropriated. In Model B there is no appropriated Pigovian revenue (by constraint $\alpha_i = 0$, all i), but lump-sum taxes are allowed.

Now consider two sets of Walrasian equilibria. Set A contains interior Walrasian equilibria for Model A, and Set B contains interior Walrasian equilibria for Model B. Define the correspondence between Walrasian equilibria of Sets A and B as follows. Pick and fix τ, v and $\tilde{v}$, and an n-tuple of α_i (the choice is arbitrary as long as the choice leads to an internal equilibrium in Set A). From Set A find the Walrasian equilibrium by using the FOC from (5) - (7), shown as $A1, A2$, and $A3$ in the Appendix along with $L_z = \alpha S + \tau S + vL_x + \tilde{v}L_y$, the version of (8) without lump-sum taxes M. (If there are multiple equilibria, pick the one with maximum NSB or a tie of it.) For this equilibrium, write the Walrasian equilibrium values $(x_1', ..., x_n', y_1'..., y_n', z', L_1', ..., L_n', S')$.

Next, consider the Walrasian equilibria in Set B associated with the same τ, v and $\tilde{v}$, and individual lump-sum taxes M_i defined as follows. Set $M_i = \alpha_i S'$, where S' is the level of smoke in the Walrasian equilibrium picked in Set A. This means that M is $M = \alpha S'$. Model B has the same FOC $A1, A2$, and $A3$ as in Model A. Thus the first equilibrium's values $(x_1', ..., x_n', y_1'..., y_n', z', L_1', ..., L_n', S')$ are also equilibrium values for the second equilibrium, as long as each i's wealth is the same and the government revenue from M in (8) in Model B is the same as its revenue from $\alpha S'$ in Model A. These last two conditions are met by the construction of $M_i = \alpha_i S'$.

Going the other way, consider Walrasian equilibria in Set B. Pick and fix τ, v, and $\tilde{v}$ and an n-tuple of M_i (the choice is arbitrary as long as it leads to an internal Walrasian equilibrium in Set B). Write the equilibrium values $(x_1'', ..., x_n'', y_1''..., y_n'', z'', L_1'', ..., L_n'', S'')$. For the correspondence from Set B to A define $\alpha_i = \frac{M_i}{S''}$ (each i and for interior $S'' > 0$). Corresponding to these same τ, v, and $\tilde{v}$, with the same individual wealth and government revenue, we find a Walrasian equilibrium in Set A with identical equilibrium values as the values in the equilibrium in Set B. We conclude:

Theorem 3 (Equivalence). *When the government spends its budget, for every interior Walrasian equilibrium with appropriated Pigovian revenues $\alpha_i S$ but no lump-sum taxes ($M_i = 0$ for all i) there is another Walrasian equilibrium with no appropriated Pigovian revenues ($\alpha_i S = 0$ all i) and lump-sum taxes M_i with the same equilibrium values for*

$(x_1, ..., x_n, y_1 ..., y_n, z, L_1, ..., L_n, S)$, *and for every interior Walrasian equilibrium with lump-sum taxes but no appropriated Pigovian revenues there is another equilibrium with appropriated Pigovian revenues but no lump-sum taxes with the same equilibrium values.*

By the equivalence relation, with fixed values for τ, v and $\tilde{v}$, and corresponding $\alpha_i S$ and M_i, the corresponding Walrasian values for x_i, y_i, and L_i (all i) and for z and S are the same. This means that the Walrasian equilibrium with appropriated Pigovian revenue in the original model inherits the same distortionary and distributional effects as in the corresponding Walrasian equilibrium with the corresponding lump-sum taxes M_i. In the correspondence the appropriated Pigovian revenues $\alpha_i S$ are non-distortionary in the same sense that lump-sum taxes (of equal amount) are non-distortionary. And just as lump-sum taxes are non-distortionary whether or not there are other taxes, so too is the appropriated Pigovian revenue. In the Walrasian equilibria in Model A of Theorem 3 there are no restrictions on whether or not the Pigovian revenue equals the government's revenue need, whether or not there are constraints on $\tau, v, \tilde{v}$ and α, or whether or not there is a fixed constraint on government expenditures. Summarizing:

Corollary 2 (Extension of Sandmo's Observation. *The appropriated Pigovian revenue is non-distortionary, in the same way that lump-sum taxes are non-distortionary, whether or not it equals the government's revenue need, and whether or not there are other* (*distortionary*) *taxes.*[14]

IV. Tax Interaction Effects

Theorems 1 - 3 are general theorems, saying little directly about how tax interaction effects relate to the non-distortional property of the appropriated Pigovian revenue. This section resolves some of this remaining piece of the puzzle. We begin with the symmetry property.

The symmetry characterization of Corollary 1 extends to the tax interaction effects.

[14]Because the equivalence relation and this corollry 2 are proved for sets of Walrasian equilibria, which include the subsets of second-best Walrasian equilibria, the theorem and corollary can be proved without Theorems 1 and 2; but without these two theorems there would be less intuition of the equivalence relation and extension.

To see this use the implicit function theorem to write Walrasian equilibrium values of S, L_x, and L_y as functions of $\tau, v, \tilde{v}$, and α. Then write the marginal distortionary costs as

$$\tau dS = \tau \frac{\partial S}{\partial \tau} d\tau + \tau \frac{\partial S}{\partial v} dv + \tau \frac{\partial S}{\partial \tilde{v}} d\tilde{v} + \tau \frac{\partial S}{\partial \alpha} d\alpha$$

$$v dL_x = v \frac{\partial L_x}{\partial \tau} d\tau + v \frac{\partial L_x}{\partial v} dv + v \frac{\partial L_x}{\partial \tilde{v}} d\tilde{v} + v \frac{\partial L_x}{\partial \alpha} d\alpha$$

$$\tilde{v} dL_x = \tilde{v} \frac{\partial L_y}{\partial \tau} d\tau + \tilde{v} \frac{\partial L_y}{\partial v} dv + \tilde{v} \frac{\partial L_y}{\partial \tilde{v}} d\tilde{v} + \tilde{v} \frac{\partial L_y}{\partial \alpha} d\alpha$$

where $\tau \frac{\partial S}{\partial \tau} d\tau, v \frac{\partial L_x}{\partial v} dv$, and $\tilde{v} \frac{\partial L_y}{\partial \tilde{v}} d\tilde{v}$ are "own price" marginal tax interaction effects and the other terms on the right are "cross-price" marginal tax interaction effects. Substituting these tax interaction effects into (9) and (8), we find the same permutation symmetry applies to the tax interaction effects when $\alpha = 0$.

The permutation symmetry means that second-best taxes τ, v, and $\tilde{v}$ and factors S, L_x, and L_y can exchange roles when $\alpha = 0$, even though S is an externality and L_x and L_y are non-externalities. The factor taxes v and $\tilde{v}$ are ordinary Ramsey taxes in the second-best analysis, and when $\alpha = 0$, τ behaves like an ordinary Ramsey tax too. Ordinary second-best taxes are typically positive, but when there are complementarities one or more can be negative. By the symmetry property, when $\alpha = 0$ second-best τ are also typically positive but can be negative because of complementarities. In the symmetry case, tax interaction effects, themselves symmetric and reciprocal, offset each other, not completely because of complementarities, but in part. When $\alpha > 0, \tau$ is still a distortionary tax but it no longer behaves symmetrically with the other taxes.

Corollary 1 identifies the asymmetry as arising in the role of α in (8). Because S is a component of the appropriated Pigovian revenue αS, the own-price effect of τ on S plays an asymmetric role in second-best analysis when $\alpha > 0$. The aysmmetric case $\alpha > 0$ is also the case where there are lump-sum equivalent effects, because $\alpha S = M > 0$ in the equivalence correspondence, with $\alpha > 0$ and interior $S > 0$.

The constraint of revenue neutrality reveals more of how tax interaction effects work. We say there is *revenue neutrality* if the government's expenditure is constrained to $wL_z = \overline{G}$ for fixed $\overline{G}$.

Corollary 3 (Revenue Neutrality). *When there is revenue neutrality with expenditures* $wL_z = \overline{G}$*, the first-order conditions for second-best taxes are*

$$\text{(10)} \qquad \frac{d(NSB)}{-U_L} \;=\; \overbrace{\tau dS \;+\; v dL_x \;+\; \tilde{v} dL_y}^{-\text{ marginal distortionary costs}} \;=\; 0$$

subject to the government budget constraints

$$\text{(11)} \qquad \underbrace{d(\alpha S) + d(\tau S) + d(v L_x) + d(\tilde{v} L_y)}_{\text{marginal revenue}} = 0$$

Proof. With $w = 1$ and $L_z = \overline{G}$, we have $dL_z = d\overline{G} = 0$ and (9) & (8) reduce to (10) & (11). ■

The corollary says that in a second-best equilibrium with revenue neutrality, whether or not $\alpha = 0$, the marginal distortionary costs sum to zero, and with them so do the marginal tax interaction effects sum to zero. Consider a second-best equilibrium with revenue neutrality where second-best $\alpha = \overline{\alpha} = \hat{t} > 0$ and $\tau < 0$ (the usual case in the recent literature). Then the own-price tax interaction effect of τ, for an increase ($d\tau > 0$), is $\tau \frac{\partial S}{\partial \tau} d\tau$ and normally positive, because $\tau < 0, d\tau > 0$, and $\frac{\partial S}{\partial \tau}$ is normally negative. In this case the sum of the other tax interaction effects must be negative to satisfy (10). Thus an increase in τ exacerbates the distortionary of other taxes, in the second-best equilibrium of this case.

But at the same time the extension says that the appropriated Pigovian revenue is non-distortionary. The seeming conflict is resolved when we recall that in equilibrium the sum of the marginal distortionary costs, *including* the own-price distortionary cost of τ, is zero. In the second-best equilibrium a small change in τ, accompanied by small offsetting changes in v and $\tilde{v}$ to maintain the fixed revenue requirement, has no net effect on either the total distortionary costs or on the total revenue.[15]

[15]The case without revenue neutrality is a little more complicated.

Theorem 3 reveals more about how the tax interaction effects work. Even though there is an equivalence between Walrasian equilibria, the second-best analysis works differently in Models A and B. In Model B when the constraint $M \leq \overline{M}$ is strictly binding, dM must be non-positive. But in Model A when the constraint $\alpha \leq \hat{t}$ is strictly binding, there are more opportunities for $d(\alpha S)$. In particular it is possible to decrease τ and through the own-price tax interaction effect increase S and in this way the non-distortionary appropriated revenue, with $d(\alpha S) > 0$, consistent with the intuition of Figures 2 and 3.

So far the cross-price effects of τ on labor have received more attention than the own-price effects of τ on the appropriated Pigovian revenue. Usually both effects happen simultaneously, and the cross-price effect of τ on labor tends to decrease existing labor tax distortionary costs. Which is more decisive in explaining the finding that second-best τ is often negative?[16]

Two ways of decoupling the effects suggest that the own-price effect of τ in augmenting the appropriated Pigovian revenue may be more decisive. First, constrain $\alpha = 0$. Then there is no appropriated Pigovian revenue αS and no own-price effect of τ in increasing it ($\alpha S = (0)S = 0$). By Corollary 1, τ is symmetric with other taxes and typically positive. Thus, without the own-price effect of τ on the Pigovian revenue, second-best τ is typically postive.

Second, we decouple the two effects by allowing the Pigovian revenue to be appropriated but constraining $v = \tilde{v} = 0$, in which case there are no cross-price tax interaction effects of τ on labor. In this case we find second-best $\tau < 0$ as long as the elasticity of the environmental revenue tS is positive with respect to τ,[17] a condition easy to meet. Thus, without the cross-price of τ on labor, we often find second-best $\tau < 0$.

The lump-sum equivalent effect of the appropriated Pigovian revenue is an odd lump-sum effect. We are used to lump-sum sources of revenue that are fixed. The idea is that with fixed and unavoidable taxes like M_i no one can manipulate them on the margin. In the case of the appropriated Pigovian revenue αS, polluters can alter S and thus the revenue

[16]See Bovenberg and de Mooij's (1994) and Parry's (1995).
[17]See Corollary 3, Page and Zhange (2000).

source αS. Indeed the purpose of the Pigovian tax is to induce the polluters to alter their emission levels.

But perhaps the more fundamental idea of a lump-sum revenue source is that it need not be fixed and unalterable but rather that each person either can not alter it or has no incentive to do so. In the case of the appropriated Pigovian revenue as a tax revenue source, each polluter j cares about the total amount of the environmental tax $t = \hat{t} + \tau$, and optimizes against tS_j (where S_j is j's emission). So each j can alter the appropriated Pigovian revenue αS, by altering his emission level S_j but has no incentive to do so, because he is already optimizing against tS_i and he doesn't care about how the Pigovian revenue is divided up (he doesn't get any part of it). Each i cares about his compensation $(\hat{t}_i - \alpha_i)S$, but can't affect it because α_i is chosen by the government, S is an externality, and $\hat{t}_i$ functions as a parameter fixed by the government.

Possibly this incentive structure has a parallel with a Groves mechanism like a pivot tax. In using a pivot tax to decide whether or not to provide non-divisible public good, each person reports a willingness to pay. Each person can alter the outcome by altering his reported willingness to pay but has no incentive to do so because of the incentives built into the pivot tax. Each person j would like to alter the pivot tax, but can't because j's tax is a function of everyone else's report except j's. If such a parallel exists, it would not be too surprising. The Pigovian tax, studied in this paper, is designed to internalize the externality costs of the polluters' actions. The pivot tax is designed to internalize the externality costs of the reports in making a decision that may harm or benefit others. In this interpretation the pivot tax is just another Pigovian tax, one directed toward information externalities.

V. Policy

The extension of Sandmo's observation and finding of a lump-sum equivalence effect is both good and bad news. Finding a large source of non-distortionary revenue is welcome news, but the likely companion of this news is adverse distributional effects associated with lump-sum sources of revenue. In the case of appropriated Pigovian revenue, the likely adverse distributional effects are associated with those most vulnerable to the environmental harms.

The most vulnerable have the highest marginal damages, but when the Pigovian revenue is fully appropriated ($\alpha_i = \hat{t}_i$ all i), they get no compensation.

Adverse distributional effects are especially large in the case of no regulation, where S is high compared with regulation and there is no compensation, and are likely to exist for command-and-control regulation and grandfathered marketable allowances, where there is typicaly little or no compensation either.[18]

When τ decreases, increasing S, and augmenting the appropriated Pigovian revenue, the most vulnerable are likely to bear the most severe harms of the increased S. Limiting the adverse distributional harms can be done by increasing compensation (by reducing α) or by reducing the harm S, both of which reduce the appropriated Pigovian revenue αS. But such a reduction decreases the source of non-distortionary revenue.

Thus there appear to be conflicting social goals: (a) to reduce the environmental harms S (a primary social goal of environmental regulation and one related to protecting the most vulnerable) and (b) to reduce the distortionary costs of the tax system. There has been much debate over whether policies can be designed to work toward both goals simultaneously (a "double dividend") or whether working toward one goal entails the sacrifice of the other (no double dividend).[19] The obvious first question is what is the starting point and what are the ending points over which the comparisons going to be made?

For the starting point we choose the case of no regulation. We model this baseline case "no regulation" in terms of constraints on τ and α to make later comparisons possible. We do this by observing that the case of no regulation corresponds to the constraints $\alpha_i = \hat{t}_i$ (all i) and $\tau = -\hat{t}$. With these constraints the environmental tax facing the polluters is $t = \hat{t} + \tau = \hat{t} - \hat{t} = 0$, so the polluters are unregulated. The appropriated Pigovian revenue is $\hat{t}S$ but the surtax revenue is $-\hat{t}S$ so the whole environmental revenue to the government is zero. With $\alpha_i = \hat{t}_i$ (all i) no one gets any compensation. But even though the

[18] The cases of no regulation, command-and-control, and marketable allowances can be modeled by setting appropriating the Pigovian revenue and then returning it lump-sum to the polluters. In these cases the non-distortionary benefits are lost, but the distributional effects remain.

[19] There are several and definitions in this debate, and we will add but a few words. The definition here is one of the earliest and one of the simplest versions.

non-distortionary appropriated Pigovian revenue is totally used up by financing the surtax subsidy, the distributional effects of the appropriated Pigovian revenue remain, because the equivalence theorem still goes through.

One ending point is the second-best equilibrium the main case of the recent literature ("recent literature"), where α is constrained to $\alpha \leq \hat{t}$ and τ is unconstrained. The other ending point is the symmetry case ("no lump sum"), where α is constrained to $\alpha = 0$ and τ unconstrained; this is the case where there is full marginal damage compensation, no lump-sum effects, but with the ordinary opportunity to raise surtax revenue through tax spreading. Our comparisons are for second-best equilibria, with the constraints of $M_i = 0$ (all i) and revenue neutrality.

The second obvious question is how are we going to measure the total changes in distortionary cost over the changes from the baseline of no regulation? Consider the first equality of (10) of Corollary 3 where the marginal NSB (normalized) equals the sum of the negative of the marginal distortionary costs. This means that integrated over some change in the constraints, when NSB goes up the total distortionary costs go down.

We will only consider cases that pass a cost-benefit test. We will say that if the NSB for the second-best equilibrium with constraints $\alpha_i = \tau = 0$ (all i) is higher than the NSB with no regulation, the test is passed. In the equilibrium of this "cost-benefit test case" (with constraints $\alpha_i = \tau = 0$, all i), the marginal benefits of regulation are equated with the marginal costs ($\tau = 0$) and there is no environmental tax revenue ($\alpha = 0$ and $\tau = 0$). Finally, we will use a basic principle of constrained maximization: when a constraint is relaxed, the maximand never gets smaller and sometimes gets larger.

Compare the move from "no regulation" to "recent literature" in two steps (see Figure 4). First move from "no regulation" to the "cost-benefit test case." With the cost-benefit test passed, NSB increases and distortionary costs decrease. Next move from the "cost-benefit test case" to "recent literature." Since the constraints are relaxed in this second more, NSB doesn't decline in this move. So over the two steps NSB increases and total distortionary costs decline. Estimates of second-best τ for the "recent literature" case are typically in the range of $-\hat{t} < \tau \leq 0$, with sometimes $\tau > 0$. With the environmental tax typically higher in "recent literature" than in "no regulation," we conclude that S typically declines over the whole move. With both distortionary costs and S reduced (typically in this case), we move toward both goals simultaneously, a double dividend.

Compare the move from "no regulation" to "no lump-sum" in two steps. First move from "no regulation" to the "cost-benefit test case." With the cost-benefit test passed, NSB increases and distortionary costs decrease. In the next step move from "cost-benefit" to "no-lump sum." With the constraint on τ relaxed, we conclude that NSB can't decrease, so NSB is higher in the complete move from "no regulation" to "no lump-sum." With the symmetry, second-best τ, is typically positive (depending on complementarities). We conclude that typically in moving from "no regulation" to "no lump-sum" distortinary costs and S both decline, another double dividend.

But the two social goals are not always in harmony. Consider a move from "no lump-sum" to "recent literature." Typically in the move distortionary cost decreases but S increases, a policy tradeoff with no double dividend. Going the other way from "recent literature" to "no lump-sum" we get the tradeoff the other way.

The point is that double dividends are common in moving from no regulation to some regulation, but in moving toward stricter regulation, tradeoffs are likely. Moreover, with potentially large amounts of non-distortionary revenue there is a lot of room in which to make compromises and tradeoffs. The well-known compromise struck in the Title IV SO_2 program is just one example. Environmentalists were willing to give up on compensation almost entirely (agreeing to $\alpha = 1$) in exchange for a reduction of of SO_2 emissions from about 20 million tons annually to about 10 million; polluters were willing to abate substantially in exchange for getting most of the appropriated Pigovian revenue, in the form of grandfathered marketable allowances; and the government was willing to transfer all but a trickle of the appropriated Pigovian revenue to the polluters (up to 3.5% of the allowances are auctioned) in exchange for a dramatically successful program.[20]

[20] Bovenberg and Goulder (2001) recommend a version of this compromise for controlling the greenhouse gas CO_2 but with about 90% of the appropriated Pigovian revenue retained as general revenue, and probably with taxes rather than allowances as the instrument.

Appendix

Specialization to Earlier Models

To specialize our model (1) - (8) to the analytic model in Goulder et al. (1999), set

$$U^i(x_i, y_i, z, L_i, S) = u(x_i, y_i, z, \overline{L} - L_i) - \phi(S)$$

(representative agent); unit isoquant $x = 1 = f(L_x, S)$; where $L_x = 1 + c(a), S = e_0 - a$, a is the abatement parameter, $c(a)$ is the convex cost of abatement in terms of labor, the numeraire, and f has constant returns to scale; $g(L_y) = L_y$; $h(L_z)$ is a step function with $h(L_z)$ a high number for $L_z \leq \overline{G}$ and $\overline{G}$ is the revenue constraint, and $h(L_z) = 0$ for $L_z > \overline{G}$; $v = \tilde{v}$ (no excise taxes); and $\alpha = \hat{t}$ (government appropriates all the Pigovian revenue, and the environmental revenue becomes tS). The models of Bovenberg and Mooij (1994) and Parry (1995) can be specialized from the model (1) - (8) in a similar way.

Strategy to Solve for Walrasian Equilibria

Step (i). Fix α and the admissible candidate taxes and prices $\tau, v, \tilde{v}, P_x, P_y, \hat{t}$ and $\hat{p}$, with $w = 1$. Maximize the producers' profits to find their candidate supplies x and y, emissions of S, and derived demands for L_x, L_y and L_z. Maximize each i's utility subject to his budget constraint to find i's demands x_i and y_i and supply L_i. Calculate the aggregate candidate $\sum L_i$ (the candidate labor supply), $L_x + L_y + L_z$ (labor demand), $\sum x_i$ (demand for good x), x (supply of good x), $\sum y_i$ (demand for good y), and y (supply of good y). *Step* (ii). Adjust the candidate prices P_x, P_y and the candidate $\hat{p}$ and $\hat{t}$ so that the markets clear, the candidate $\hat{t} = \sum \frac{U_S^i}{U_L^i}$, and the candidate $\hat{p} = \sum \frac{-U_z^i}{U_L^i}$. This is the Walrasian equilibrium for the fixed α and fixed taxes τ, v and $\tilde{v}$.

Strategy to Solve for Second-Best Walrasian Equilibria

Step (iii). Maximize NSB over the admissible taxes τ, v, and $\tilde{v}$ for fixed α. The result in Theorem 1 is the second-best efficient Walrasian equilibrium for fixed α and M.

Step (iv). Theorem 2 optimizes over α and M as well.

Proof of Theorem 1

Step (i). Fix $v, \tilde{v}, \tau$ and α. Fix the candidate $P_x, P_y, \hat{t}$, and set $w = 1$. From (5) - (7) and using $w = 1$, the FOC conditions for the producers and consumers are

$$(A1) \qquad P_x f_L(L_x, S) - 1 = v, \quad P_x f_S(L_x, S) - \hat{t} = \tau \quad \text{and} \quad P_y g_L(L_y) - 1 = \tilde{v}.$$

$$(A2) \qquad -U_x/U_L = P_x \quad \text{and} \quad -U_y/U_L = P_y$$

Step (ii). Adjust the candidate P_x, P_y and $\hat{t}$ to solve the market-clearing equations

$$(A3) \qquad L_x + L_y + L_z = \sum L_i, \quad x = \sum x_i \quad \text{and} \quad y = \sum y_i$$

and equalize the candidate $\hat{t}$ to $\sum \frac{-U_S^i}{U_L^i}$.

Step (iii). For utilities in the representative agent form, the net social benefits is $NSB = \sum_i U(x_i, y_i, z, L_i, S)$. Then

$$\begin{aligned}
d(NSB) &= \sum(U_x dx_i + U_y dy_i + U_z dz + U_L dL_i + U_S dS) \\
\tfrac{d(NSB)}{-U_L} &= \sum(P_x dx_i + P_y dy_i - \tfrac{U_z}{U_L} dz - dL_i - \tfrac{U_S}{U_L} dS) \quad \text{(by (A2))} \\
&= P_x \sum dx_i + P_y \sum dy_i - dz \sum \tfrac{U_z}{U_L} - \sum dL_i - dS \sum \tfrac{U_S}{U_L} \\
&= P_x dx + P_y dy + \hat{p} dz - \sum dL_i - \hat{t} dS \\
&= P_x(f_L dL_x + f_S dS) + P_y g_L dL_y + \hat{p} h_L dL_z - (dL_x + dL_y + dL_z) - \hat{t} dS \\
&\qquad \text{(by (2), (3), (4) and (A3))} \\
&= (P_x f_L - 1) dL_x + (P_x f_S - \hat{t}) dS + (P_y g_L - 1) dL_y + (\hat{p} h_L - 1) dL_z
\end{aligned}$$

$$(A4) \qquad \frac{d(NSB)}{-U_L} = v dL_x + \tau dS + \tilde{v} dL_y + (\hat{p} h_L - 1) dL_z \quad \text{(by (A1))}$$

which is what we need to show for the main part of Theorem 1, for a representative agent.

For the case of additive separability, *Steps* (i) and (ii) are the same, with $NSB = \sum_i (U^i(x_i, y_i, z, S) - L_i)$. So $d(NSB) = \sum(U_x^i dx_i + U_y^i dy_i + U_z^i dz + U_S^i dS - dL)$ and the rest of the proof follows as before. ■

Proof of Theorem 2

Write i's utility as $U^i(x_i, y_i, z, L_i, S)$, for either its representative agent or additively separable form. We maximize $NSB = \sum U^i$ subject to the upperbound constraints on α and M, the constraints (A1), (A2), (A3), the government's budget constraint (5), and the sum of the individuals' budget constraints.

Form the Lagrangian:

$$\begin{aligned}
\mathcal{L} \;=\; & \sum U^i(x_i, y_i, z, L_i, S) - \lambda_1(\alpha - \overline{\alpha}) - \lambda_2(M - \overline{M}) - \lambda_3(P_x f_L - 1 - v) - \lambda_4(P_x f_S - \hat{t} - \tau) \\
- \; & \lambda_5(P_y g_L - 1 - \tilde{v}) - \lambda_6(U_x + U_L P_x) - \lambda_7(U_y + U_L P_y) - \lambda_8(L_x + L_y + L_z - \sum L_i) \\
- \; & \lambda_9(f(L_x, S) - \sum x_i) - \lambda_{10}(g(L_y) - \sum y_i) - \lambda_{11}(z - h(L_z)) \\
- \; & \lambda_{12}(L_z - \alpha S - \tau S - v L_x - \tilde{v} L_y - M) - \lambda_{13}(P_x x + P_y y - w \sum L_i - \hat{t} S + \alpha S + M)
\end{aligned}$$

To show (1), note that by the envelope theorem $V(\overline{\alpha}) = \frac{\partial \mathcal{L}}{\partial \overline{\alpha}} = \lambda_1$ is the value function for the upperbound constraint $\overline{\alpha}$, and $\lambda_1 \geq 0$ by the direction of the inequality constraint $\alpha \leq \overline{\alpha}$. So for a second-best equilibrium λ_1 must either be zero or the constraint $\alpha \leq \overline{\alpha}$ must be strictly binding. A corresponding argument leads to (2).

For (3), note that in the second-best equilibrium,

$$\frac{\partial \mathcal{L}}{\partial \alpha} = -\lambda_1 + \lambda_{12} S - \lambda_{13} S = -\lambda_1 + S(\lambda_{12} - \lambda_{13}) = 0 \quad \text{and}$$

$$\frac{\partial \mathcal{L}}{\partial M} = -\lambda_2 + \lambda_{12} - \lambda_{13} = -\lambda_2 + (\lambda_{12} - \lambda_{13}) = 0; \quad \text{so directly}$$

$$\lambda_1 = S\lambda_2 \quad \blacksquare$$

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Figure 1. Three Cases to Consider for the Extension

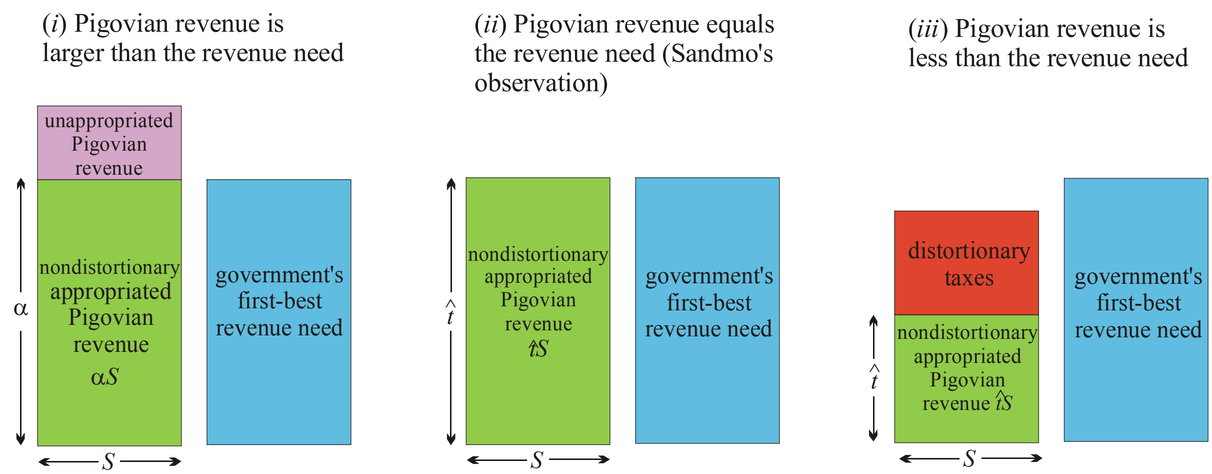
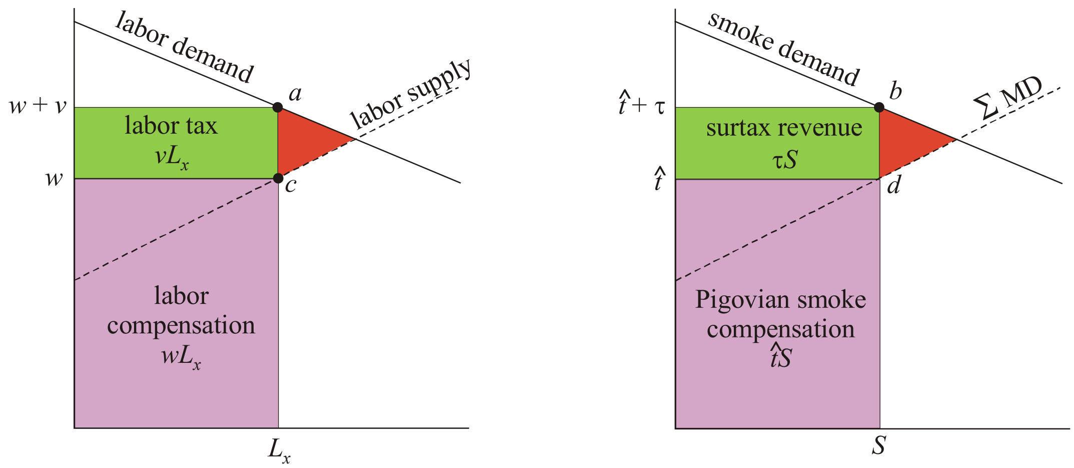


Figure 2. Symmetry and Asymmetry between Environmental and Other Taxes

(*i*) Symmetry when ($\alpha = 0$)



(*ii*) Asymmetry when ($\alpha > 0$)

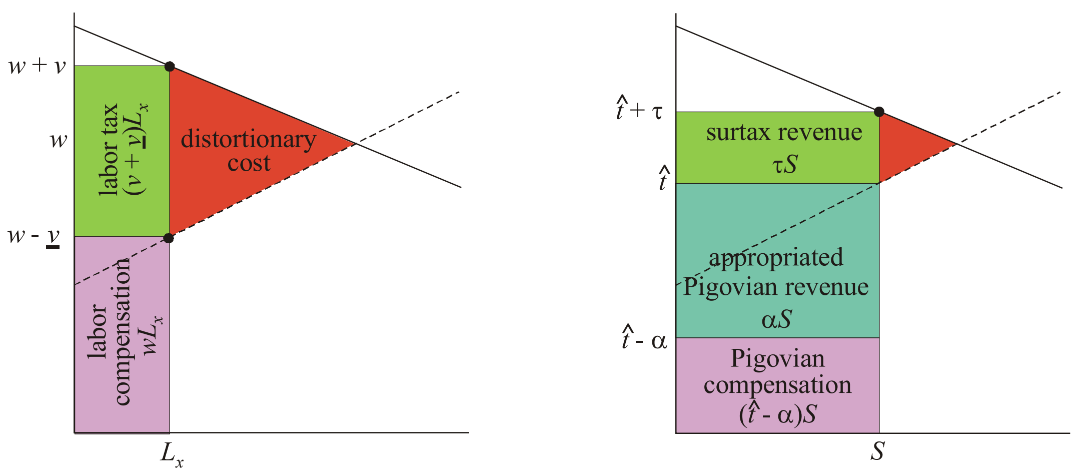


Figure 3. Increasing the Pigovian Revenue by Decreasing τ

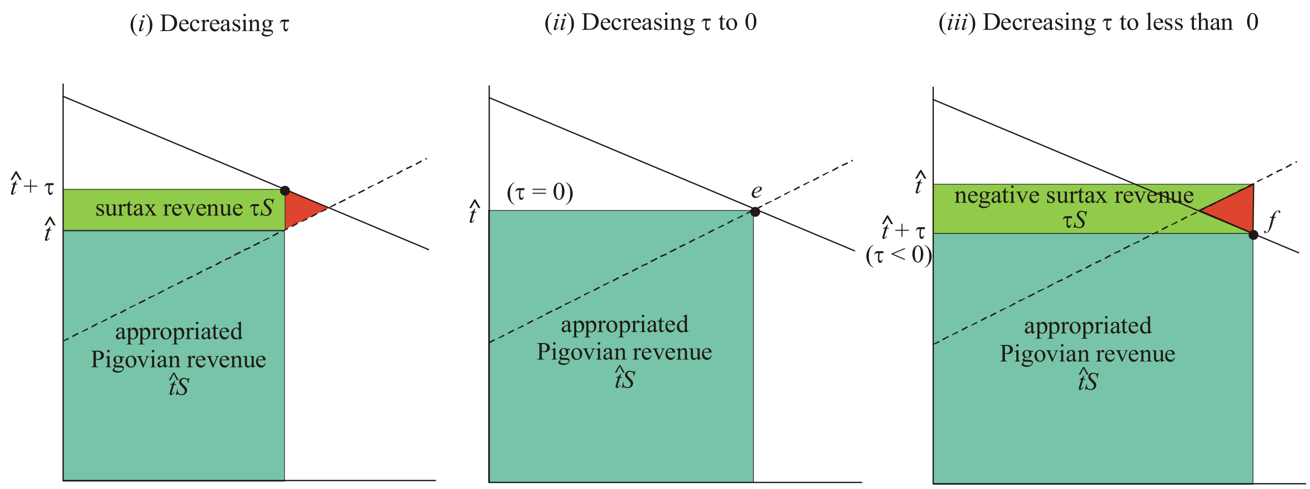


Figure 4. Two Goals: Reduce Distortionary Costs & Reduce Environmental Harm
(Constraints in Parentheses)