When Should Borrowers Refinance Their Mortgages?

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Abstract

We develop an analytically tractable option pricing model of a mortgage holders refinancing behavior. This reduces to the choice of an optimal interest rate differential. At this optimal differential, the value of the interest saved equals the sum of refinancing costs and the difference between the old 'in the money' refinancing option that is implicitly given up at the refinancing and the new 'out of the money' refinancing option that is implicitly acquired. For a reasonable range of parameter values, we calculate the differential to be bounded below by 100 basis points. Using a unique panel data set from a large financial institution, we find that during the 1990's over half of refinancers did so at differentials of 100 basis points or fewer. We further estimate that the fraction of early refinancers in the whole population is on the order of 30%. We conjecture that the pattern of early refinancing may arise because mortgage holders ignore the option value of waiting to refinance; rather, they choose to refinance at the point where the net present value of interest saved is equal to the refinancing cost. This behavior is consistent with the advice of most popular finance books and websites.

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1 Introduction

Households in the US hold over $10 trillion in real estate assets.[1] For two-thirds of households, real estate accounts for the majority of their assets. Almost all home buyers obtain mortgages and the total value of outstanding home mortgages approaches $5 trillion, greater than the value of US government debt.

Borrowers refinance their mortgages for several reasons. First, some households refinance to either pay down their mortgage or to increase their mortgage. The latter group are typically motivated by a desire to increase current consumption. Second, households refinance to take advantage of lower interest rates and thereby save on future interest payments.

In the current paper, we develop a simple, analytically tractable, dynamic optimization framework to model the borrower's refinancing decision. This reduces to the choice of an optimal interest rate differential. At this optimal differential, the value of the interest saved equals the sum of refinancing costs and the difference between the old 'in the money' refinancing option that is implicitly given up at the refinancing and the new 'out of the money' refinancing option that is implicitly acquired. Solving for these option values yields an equation for the optimal interest differential. The solution depends on the discount factor, refinancing costs, mortgage size, the standard deviation of the innovation in the mortgage interest rate, and an exogenous probability of moving.

We calculate the optimal differential for a range of parameter values and mortgage sizes. For a mortgage holder with zero probability of moving, the differential ranges from 100 basis points to 200 basis points, depending on the size of the mortgage. For mortgage holders with a significant probability of moving, the optimal differentials can be much larger.

We use a unique panel data set from a large financial institution to determine the range of differentials at which people actually refinance. For each mortgage, the data set has information on initial characteristics (terms and location), quarterly updated credit bureau information, and monthly updated information on transactions (e.g. whether the payment was made; if so, the amount). We initially confine ourselves to the subsample of people who choose to refinance within the same institution, so that we may see the initial and new interest rate; this yields 840 mortgages spanning the period December 1992 to December 2001. We find that over half of refinancers do so at differentials of 100 basis points or fewer, and over seventy-five percent at differentials of 125 basis points or fewer. Approximately five percent of refinancers do so at differentials strictly greater than 200 basis points, of whom some have differentials as high as 500 or 600 basis points.

These calculations may overstate the fraction of "early refinancers" in the entire population. Because interest rates are persistent, we are more likely to observe mortgage holders who have the opportunity to refinance at interest rates that are very close to their original mortgage interest rate than we are

[1] Flow of Funds Accounts of the United States, Board of Governors of the Federal Reserve System, September 15, 2000.

to observe mortgage holders who have the opportunity to refinance at larger — i.e., optimal — interest differentials. When we correct this bias, we find that early refinancers still constitute at least one quarter of the population of all refinancers.

These results are surprising, because all pre-existing research on refinancing has emphasized procrastination — i.e. late, slow, or non-existent refinancing — as the principal error that mortgage holders make. The mortgage industry has informally labeled such late refinancers "woodheads." We also find evidence for woodheads, but we focus our analysis on the new finding that suboptimally early refinancing is also a prominent feature of the data.

An analysis of top-selling personal finance books for the general public (e.g., *52 Weeks to Financial Fitness*) immediately reveals why so many households refinance too early. These books often advocate using a simple NPV rule for refinancing, which ignores the option value of waiting to refinance. Of 19 top-selling personal finance books, 15 offer such advice. We find similar suboptimal advice at personal finance sites on the internet.

With this NPV advice in mind, we calculate the differentials which would be predicted by the NPV rule, i.e. ignoring the option value of waiting to refinance. For the same set of parameters and mortgage sizes as for our optimal calculations above, we obtain differentials between 10 and 85 basis points, consistent with the actual refinancing behavior observed.

Section 2 presents the model and the results of our optimal interest differential calculations. Section 3 describes our data set and presents our estimates of the fraction of early refinancers. Section 4 presents possible explanations for our findings and discusses the related literature. Section 5 concludes.

2 Theory

We model the refinancing decision as a continuous-time dynamic optimization problem.[2]

Let M denote the amount of the mortgage and r represent the fixed interest rate on the original loan (suppressing all time subscripts throughout). To eliminate a state variable, we assume that until the mortgage is repaid or refinanced the household only pays interest on the mortgage: rM. With hazard rate λ the mortgage is repaid for exogenous reasons. The hazard rate λ is chosen to match the rate at which mortgage holders repay their mortgages for reasons unassociated with interest rate variability (i.e., 30-year horizon of the mortgage and associated repayment of principal, death of the homeowner, refinancing for liquidity reasons, and paying off the mortgage because of a move).

[2] We model only fixed rate mortgages, since the refinancing problem is not relevant for adjustable rate mortgages. See Campbell and Cocco (2001) for a recent paper on the choice between the two types of mortgages. See Dun and McConnell (1981a, 1981b), Timmis (1985), Dunn and Spatt (1986), Johnston and Van Drunen (1988), Chen and Ling (1989), Kau and Keenan (1995), and Stanton (1995) for other endogenous mortgage refinancing models. Our paper provides the first analytically tractable model of optimal mortgage refinancing.

Let r_M represent the market interest rate. Define $x = r - r_M$, and assume x is Brownian motion with drift,

$$dx = \alpha dt + \sigma dW, \tag{1}$$

where W is a standard Wiener process.

The cost of refinancing is given by $C(M) = F + fM$. The consumer is risk neutral and has discount factor ρ. Finally, we assume that the borrower has built up enough equity in the property so that defaulting is not a relevant choice.

Given these assumptions, the solution to the borrower's problem will be characterized by a stopping rule: refinance if and only if $x \geq x^*$. We now turn to the derivation of x^*.

We can decompose the value function of the borrower's mortgage obligation into two terms: the option value of being able to refinance and the NPV of the expected payments on the mortgage if the borrower were unable to refinance.

$$V(x, r, M) = W(x) - \frac{(r+\lambda)M}{\rho+\lambda}, \tag{2}$$

where $W(x)$ is the value of the option to refinance the mortgage. Intuitively, the second term is the expected value of flow payments, $(r+\lambda)M$, discounted by the effective discount rate, $\rho+\lambda$, where ρ captures the discounting due to impatience and λ captures the discounting due to the hazard rate that the mortgage will be repaid.

Using Bellman's principle of optimality and Ito's lemma, we obtain the following differential equation for W in the continuation region of the state space:

$$\begin{aligned}
\rho V &= -rM + \alpha V_x + \frac{1}{2}\sigma^2 V_{xx} - \lambda(M+V) && (3)\\
&= -rM + \alpha W' + \frac{1}{2}\sigma^2 W'' - \lambda(M+V) && (4)\\
&= -rM + \alpha W' + \frac{1}{2}\sigma^2 W'' - \lambda\left(M + W - \frac{(r+\lambda)M}{\rho+\lambda}\right) && (5)
\end{aligned}$$

Intuitively, ρV represents the required rate of return, αV_x represents the increase in value associated with drift in x, $\frac{1}{2}\sigma^2 V_{xx}$ represents the increase in value associated with variability in x, and $-\lambda(M+V)$ represents the decrease in value arising from a λ hazard of a Poisson event that causes the mortgage to be repaid (thereby losing any option value of refinancing).

Substituting for V yields

$$\rho\left(W - \frac{(r+\lambda)M}{\rho+\lambda}\right) =$$

$$-rM + \alpha W' + \frac{1}{2}\sigma^2 W'' - \lambda\left(M + W - \frac{(r+\lambda)M}{\rho+\lambda}\right).$$

Simplifying, yields

$$\rho W = \alpha W' + \frac{1}{2}\sigma^2 W'' - \lambda W. \tag{6}$$

Note that this last expression is a second-order differential equation in W. The original value function V has been eliminated from the analysis.

The associated value matching and smooth pasting conditions are:

$$\begin{aligned} V(x^*, r, M) &= V(0, r_M, M) - C(M) && (7) \\ V_x(x^*, r, M) &= \frac{M}{\rho + \lambda}. && (8) \end{aligned}$$

The value matching equation implies that the value of the program just before refinancing, $V(x^*, r, M)$, equals the value of the program just after refinancing, $V(0, r_M, M) - C(M)$. The smooth pasting condition implies that moving r_M below the threshold point $r - x^*$ does not change any option value terms — since the consumer is going to instantaneously refinance anyway — and only reduces the NPV of future interest payments.

Substituting equation 2 into equation 7 yields

$$W(x^*) - \frac{(r+\lambda)M}{\rho + \lambda} = W(0) - \frac{(r + \lambda - x^*)M}{\rho + \lambda} - C(M), \tag{9}$$

which implies that

$$W(x^*) = W(0) + \frac{x^* M}{\rho + \lambda} - C(M). \tag{10}$$

Substituting equation 2 into equation 8 yields

$$W'(x^*) = \frac{M}{\rho + \lambda}.$$

The only remaining piece of boundary information is the limit property

$$\lim_{x \to -\infty} V(x, r, M) = -\frac{(r+\lambda)M}{\rho + \lambda}.$$

As the market interest rate becomes arbitrarily large — and hence x goes to $-\infty$ — the option value of refinancing vanishes and the value of the program converges to the NPV of the interest payments. This in turn implies,

$$\lim_{x \to -\infty} W(x) = 0.$$

With these three pieces of boundary information, we are now ready to solve for the value functions V and W. Since a solution for W trivially implies a solution for V (recall equation 2), we focus our attention on W.

Using standard differential equation solution methods, one can show that the option value function $W(x)$ has a solution of the form $W(x) = Ke^{sx}$, with exponent

$$s = \frac{-\alpha + \sqrt{\alpha^2 + 2\sigma^2(\rho + \lambda)}}{\sigma^2}.$$

The remaining two parameters, K and x^*, solve the following system of equations derived from the value matching and smooth pasting conditions.

$$Ke^{sx^*} = K + \frac{x^* M}{\rho + \lambda} - C(M) \tag{11}$$

$$Kse^{sx^*} = \frac{M}{\rho + \lambda}. \tag{12}$$

We solve numerically these two equations for different values of λ,σ, F, f and ρ to obtain values of the optimal differential x^*. We choose λ so that the expected time until future full repayment, $\frac{1}{\lambda}$, is infinite (i.e. $\lambda = 0$), twenty years ($\lambda = 0.05$) and ten years ($\lambda = 0.1$). This range of values reflects the fact that most fixed-rate mortgages are either fifteen or, more commonly, thirty years, and that personal considerations may cause termination of the mortgage contract at an even earlier date. The gradual repayment of principal implies that the average duration of a 30-year mortgage is approximately 20 years. Hence, a realistic calibration for λ would lie between 0.05 and 0.20 for most homeowners. We explore slightly lower values to bias down the optimal interest rate differentials. Such a bias lowers the estimated proportion of early refinancers.

We also bias down the optimal interest rate differential by ignoring the fact that the interest payments are tax deductible, but most refinancing costs (except the points paid) are not tax deductible. To take these tax issues into account all one needs to do is to scale up the transaction costs — $C(M)$ — by factor

$$\frac{\theta(1-\tau) + (1-\theta)}{(1-\tau)},$$

where τ is the mortgage holder's marginal tax rate and θ is the fraction of the refinancing costs that are tax deductible. We set $\theta = 1$, thereby biasing down our estimated interest rate differentials.

The standard deviation of interest rate innovations, σ, is chosen to correspond to the observed standard deviation of the first difference of the thirty-year mortgage rate, as compiled by Fannie Mae; as arguably interest rate variation has declined in recent years, we choose two values: .0121, corresponding to the sample 1971:04-2001:05, and .0069, corresponding to the sample 1990:01-2001:05. We choose values of three, five and seven percent for the discount factor ρ.

For convenience, we initially set $F = 0$, so that f can now be interpreted as the ratio of transactions cost to mortgage size. We allow f to be two or five percent. We note that this is below the three to six percent range provided by the Federal Reserve Board and Office of Thrift Supervision (1996) in a pamphlet advising consumers on refinancing, and is consistent with other estimates in the literature.[3]

Table 1 summarizes the results. The optimal differentials x^* lie between 60 and 220 basis points. Two-thirds of the values lie between 100 and 220, and the

[3]Bennett, Peach and Peristiani (1998) assert that the fixed cost alone is between 1.5 to 2.5% of the household's initial mortgage balance.

median differential is 120 basis points. As one might expect, having a shorter expected time until full repayment (i.e. bigger λ) increases the optimal differential. Increasing the volatility of changes in the mortgage rate also increases the differential, as does raising the refinancing cost and raising the discount factor.

In practice, refinancing transactions costs are likely to have both fixed and proportional components. The presence of fixed costs implies that the optimal refinancing differential will decrease with mortgage size, since the ratio of fixed costs to mortgage size will decline with mortgage size. In Table 2, we derive optimal refinancing differentials for transactions costs to be $\$2,000 + .01M$, and for parameter values of $\rho = .05$, $\lambda = .05$ and $\sigma = .0121$. The mortgage costs are picked to be consistent with the range reported in Federal Reserve Board and Office of Thrift Supervision (1996).[4] In computing these costs, note that we are also making the conservative assumptions that prepayment costs and the opportunity cost of the borrower's time are both zero.

The optimal differentials in Table 2 range from 101 basis points for a $300,000 mortgage to 183 basis points for a $50,000 mortgage, with a median value of 116 basis points. In practice, since the majority of mortgages in our data set lie between $100,000 and $200,000, the optimal refinancing differentials will lie between 110 and 138 basis points.

In the following section, we look at a data set of borrower behavior to see whether they follow these optimal rules.

3 Empirical Analysis

3.1 Data

We use a unique panel data set from a large financial institution. The entire data set includes a wide variety of mortgages (including Jumbo, Conventional, Low-to-Medium-Income (LMI)), for a total of over 212,000 mortgages, observed from the period December 1992 until December 2001.[5] We can observe three kinds of information:

1. Data on initial characteristics of the mortgage, including the mortgage amount, the initial terms (APR), the term structure (duration and fixed vs. variable) and property location (at the zip code level).

2. Credit bureau information, notably quarterly FICO score updates.

3. Transaction information, including the loan-to-value ratio and the actual record of monthly payments.

[4] They report a 3-6% range, based on the following fixed and proportional costs: application fees of $75-300, appraisal fees of $150-400, homeowner hazard insurance of $300-600, lenders' attorneys' review fees of $75-200, title search and title insurance of $400-600, home inspection fees of $175-350, loan origination fees of 1.0% and mortgage insurance of .5-1.0%. Their estimate, unlike our own calibration, may also be inclusive of points.

[5] The mortgages themselves may have originated as long ago as 1980, within or outside the financial institution.

We use only a subset of this information. We restrict our attention to conventional mortgages with a thirty-year fixed rate. This restriction limits the value of the mortgages to be between $50,000 and $300,000. This restriction is near-universal in the existing mortgage literature, in large part because the private sector treats the underwriting and portfolio management of the other types of mortgages quite differently. It is also plausible that the behavior of the borrowers of other types of mortgages will be different (e.g. FHA borrowers may act differently because of federal guarantees). We also exclude accounts which exhibit default or bankruptcy *before* they refinance or prepay.

Of the remaining mortgages, some of the records can not be used for our study because the mortgages are either prepaid, fully paid off, refinanced at another financial institution, or sold off on the secondary market for balance-sheet management purposes. Hence in order to characterize the refinancing behavior of borrowers, we need to further restrict the data set. We do so in two ways.

First, we restrict the data to those borrowers who refinanced within the same financial institution.[6] This leaves 1824 mortgages. We further restrict this group to be those who remain within the same zip code.[7] Finally, to exclude other motives which might involve either increasing or decreasing housing equity substantially, we confine ourselves to those who refinance for an amount within ten percent of the remaining mortgage balance. We also exclude people who refinance at negative differentials, since their motive for refinancing might be different.[8] This leaves 840 mortgages. We observe monthly information on the remaining 840 mortgages from December 1992 to December 2001.

These mortgages have a mean value of $166,874; the 25th and 75th percentiles are at $100,109 and $217,147 respectively. The average APR before refinancing is 8.5%, with 25th and 75th percentiles at 6.75% and 12.50%. After refinancing, the same values for the APR are 5.875%, 7.365% and 9.75%. Loan-to-value (LTV) ratios and credit scores are virtually unchanged before and after refinancing.

Figure 1 graphs the percentage of refinancers by interest rate differential. The profile peaks at 100 basis points and declines rapidly thereafter. Figure 2 graphs the cumulative percentage of refinancers by differential. Fifty-eight percent of borrowers refinance at differentials of 100 basis points or fewer, and 74 percent refinance at differentials of 125 basis points or fewer. About ten percent refinance at differentials of 200 basis points or higher.

Recall that in section 2, we found optimal refinancing rates to generally lie between 100 and 200 basis points, depending on the choice of parameters. For

[6] There is a possible sample selection problem here. If the financial institution had either aggressively marketed refinancing opportunities to its customers or if it had targeted certain regions or demographic groups for refinancing, then the sample could be biased. We know from internal discussions that the institution did not undertake such policies. In addition, some forms of marketing to demographic groups are illegal.

[7] It is possible that some of these refinancers are in fact moving to different houses for the same mortgage amount within the same zip code. We think this is an extremely small percentage.

[8] Approximately 100 people fall into this category.

any reasonable distribution of parameter values across the population of borrowers, then, there are two kinds of anomalous behavior in the data. First, large numbers borrowers refinance at rates significantly lower than the minimum value of the optimal differentials. Second, a smaller number of borrowers refinances at differentials which are too high.

Conditional on refinancing, we can estimate the fraction of people who refinance too early in two ways. First, we can apply a simple fixed cutoff; e.g. a cutoff of 100 basis points would suggest that 58 percent of refinancers do so too early. This method would be valid if refinancing costs were entirely or largely proportional. Since such costs have a large fixed component, a more accurate method is to compute an optimal differential for each borrower based on their mortgage size. Doing so, and also conservatively defining early refinancers as those who do so at or below 75% of their optimal differentials, and late refinancers as those who do so at or above 125% of their optimal differentials, yields an estimate for the fraction of early refinancers of 33.7%.

The analysis above suggests that, conditioning on refinancing, a large fraction of people do so too early. These calculations may overstate the fraction of "early refinancers" in the entire population. Because interest rates are persistent, we are more likely to observe mortgage holders who have the opportunity to refinance at interest rates that are very close to their original mortgage interest rate than we are to observe mortgage holders who have the opportunity to refinance at larger — i.e., optimal — interest differentials.

We address this bias by estimating parameters from the following model. We make the following assumptions:

1. λ is large for all households,

2. a fraction μ of households is myopic, in the sense that they use the NPV rule when determining the optimal time to refinance; this rule ignores the option value of waiting to refinance,

3. a fraction of p of households are procrastinators, in the sense that they always postpone refinancing (even if they think that they should do it),

4. myopia and procrastination are independent attributes (i.e., if you are a procrastinator then you have a μ chance of being myopic; if you are myopic then you have a p chance of being a procrastinator).[9]

We estimate μ as follows. Let H_α represent the set of households who enter the dataset and stay at their same home (with or without refinancing) at least until the spot market interest rate is αx_i^* percentage points below the interest rate of their original mortgage. Here x_i^* is the optimal refinancing interest rate differential for household i (the differential depends on the size of the mortgage held by household i). We set $\alpha = 1.25$˙ For each household, i, in H_α define a

[9]It is probably more realistic to assume that myopic agents are more likely to be procrastinators than non-myopic agents. If this were the case, we would estimate an even higher fraction of myopic agents.

time interval $[t^i_\alpha, T^i_\alpha]$, where t^i_α is the date at which that household first receives a mortgage and T^i_α is the date at which that household *first* hits the interest differential αx^*_i. We only include households that are originally customers of the financial institution and stay with that institution until at least time T^i_α.

Let E_α represent the set of early refinancers in H_α. We will define an early refinancer as any household that refinances at an interest differential less than $(0.75)\, x^*_i$ (during $[t^i_\alpha, T^i_\alpha]$). Let O_α represent the set of optimal refinancers in H_α. We define an optimal refinancer as any household that refinances at an interest differential between $(0.75)\, x^*_i$ and αx^*_i(during $[t^i_\alpha, T^i_\alpha]$).

We exclude all households who refinance at levels of mortgage debt that significantly differ from their original mortgage debt (more than a 10% deviation). We also exclude all households who hold mortgages that are not 30-year mortgages.

We estimate μ with

$$\widehat{\mu} = \frac{E_\alpha}{E_\alpha + O_\alpha}.$$

Note that $\widehat{\mu}$ is a consistent estimator of μ.

$$\begin{aligned}
\text{plim}_{H_\alpha \to \infty}\, \widehat{\mu} &= \text{plim}_{H_\alpha \to \infty} \frac{\frac{E_\alpha}{H_\alpha}}{\frac{E_\alpha + O_\alpha}{H_\alpha}} \\
&= \frac{\mu(1-p)}{\mu(1-p) + (1-\mu)(1-p)} \\
&= \mu
\end{aligned}$$

Imposing these restrictions yields 2095 mortgages in the set H_α, of which 223 are in E_α and 525 are in O_α, yielding a value for $\widehat{\mu}$ of 29.8 percent.

Figure 3 plots the number of refinancers by the difference between the differential at which they refinanced and their optimal differential. The large number of values below zero indicates the fraction of early refinancers.

4 Explanations for Early Refinancing

4.1 Popular Advice

Borrowers have many sources for potential advice on mortgage refinancing, including mortgage brokers, financial advisers, books and websites. We examine recommendations from the last two sources.[10]

We sampled 19 representative personal finance books. We chose books that were on top-ten sales lists at Amazon and Barnes & Noble web sites. We also chose books that were on the personal finance shelves at Wordsworth and Barnes & Noble stores. Details about our selection process and the result of our analysis appear in Appendix A.

[10] Although of course many of these books and sites are themselves written by people in the first two categories; hence the advice is likely to be similar.

Of the 19 books that we identified, 15 provided a break-even calculation of some sort. Six provided this calculation as their only guideline, and made no comment about waiting for extra profit (or in one case, even discouraged waiting). Nine other books discussed the break even calculation in addition to other rules of thumb.

For websites, we entered the words "mortgage refinancing advice" into a popular internet search website, Google (`http:\\www.google.com`), and examined the top ten sites which offered information on refinancing. Two of these sites provided a fixed interest-rate differential of two percent. The other eight sites offered a refinancing calculator based on a break-even criterion.[11]

4.2 The NPV Rule

Many books and websites advocate using a break-even calculation, in which the cost of refinancing is compared with the net present value of interest payments saved. This calculation effectively ignores the option value of waiting to refinance and therefore leads to suboptimally early refinancing.

It is straightforward to solve for the differentials implied by the NPV rule. Given refinancing costs $C(M)$, exponential discount rate ρ, and exogenous probability of full repayment λ, the criterion becomes

$$C(M) = \int_0^\infty e^{-(\rho+\lambda)t} x^\mu M dt, \tag{13}$$

where x^μ denotes the new myopic criterion.

This implies the following simple formula for x^μ:

$$x^\mu = (\rho + \lambda)\frac{C(M)}{M}. \tag{14}$$

Note that the myopic differential x^μ does not depend on the variance of the innovation in the mortgage rate.

We can readily compare the naive interest differential to the optimal interest differential. Using our previous characterization of the optimal threshold we can show that

$$x^* - x^\mu = \frac{1}{s}\left(1 - \frac{1}{e^{sx^*}}\right) > 0,$$

since $s > 0$ and $x^* > 0$. It may be more intuitive to recall equation 10, which relates the interest savings at the optimal threshold to the costs of refinancing, including transactions costs, $C(M)$, and the loss of an in the money refinancing option in exchange for an out of the money refinancing option,

$$\frac{x^* M}{\rho + \lambda} + W(0) - W(x^*) - C(M) = 0$$

[11]A somewhat broader search on the terms "mortgage refinancing rule of thumb" found that of the top 100 websites, 80 offered just a fixed interest-rate-differential while 20 also offered a mortgage calculator based on a break-even calculation. Of those 80, 30 suggested a differential of 2%, 25 of between 1 and 2%, and 15 of 1% or less (one as low as 1/8th percent).

The analogous equation for x^μ is

$$\frac{x^\mu M}{\rho+\lambda} - C(M) = 0.$$

Since $W(0) - W(x^*) < 0$, it follows immediately that $x^* > x^\mu$.

Tables 3 and 4 replicate Tables 1 and 2 for the same sets of parameters and mortgage sizes. Note that the "myopic differentials" range between 6 basis points and 85 basis points in table 3, and between 17 and 50 basis points in table 4. This covers the entire range of differentials at which we observe early refinancings in the data above. Hence this particular kind of myopia is a leading candidate for explaining the tendency of some people to refinance too early.

4.3 Comparison with Previous Results (Incomplete)

Our research is the first to show that many households are too *quick* to refinance. Previous studies have emphasized other aspects of the refinancing decision.

Several papers have looked at refinancing for consumption smoothing purposes, including Hurst (1999) and Hurst and Stafford (2002). The latter shows that households which are liquidity constrained use 60% of cashed-out equity for current consumption purposes. Stanton (1995) uses the motivation of consumption smoothing to explain some apparent empirical anomalies. He notes that some fixed rate mortgages are prepaid even when the current mortgage rate are above the household's coupon rate.

LaCour-Little (1999) distinguishes among various sources of prepayment: borrower mobility, liquidity demand, and interest-rate variation, using a loan level data set that isolates "pure" refinancing behavior as opposed to the "general" prepayment behavior. After excluding non-interest-rate driven prepayments, the study concludes that borrower and loan characteristics are significant factors driving prepayment behavior. This is especially true if the option is at-the-money as opposed to in- or out-of-the-money.

Leroy (1996) develops a model which interprets points as a device serving to separate borrower type with high and low prepayment probability.

Stanton (1995) develops a model of mortgage prepayment where mortgage holders face heterogeneous transaction costs. The model indicates that mortgage holders act as though the transaction costs far exceed the explicit costs incurred in refinancing. In addition, Stanton finds that mortgage holders typically delay refinancing for more than a year beyond the optimal refinancing date.

Bennett, Peach, and Peristiani (2000), simulate the threshold at which individuals will refinance a mortgage loan conditional not only on the market conditions but also on individual borrower characteristics. For example, they predict that a person with good credit history and 70% loan-to-value ratio could refinance at an interest rate differential of 70 basis points to 140 basis points. Giliberto and Thibodeau (1989) look at survey data of 4,000 households from 1981 to 1986 and show that from the number of households that refinanced with this group of households, 40% refinanced at 300 basis points or above, 76% refinanced at 200 basis points or above, and 84% refinanced at 100 basis points or above. We conjecture that our results differ sharply from theirs, since interest rates were falling rapidly in the early 1980's, implying that mortgage holders had little time to refinance too early and instead typically found themselves refinancing too late.

5 Conclusion

Choosing when to refinance a mortgage is one of the most important and most frequent option-pricing problems faced by most people. We develop an option-pricing model of refinancing in which the borrower's goal is to save on interest payments. We show that, for a reasonable range of parameter values, the model predicts optimal interest-rate differentials that are bounded below by 100 basis points.

Using a unique data set from a large financial institution, we compute the distribution of differentials at which people actually do refinance. We find that, conditional on refinancing, over half refinance at differentials below 100 basis points. Because interest rates are persistent, we are more likely to observe mortgage holders who have the opportunity to refinance at interest rates that are very close to their original mortgage interest rate, than we are to observe mortgage holders who have the opportunity to refinance at larger — i.e., optimal — interest differentials. When we correct this bias, we find that early refinancers still constitute at least one quarter of the population of all refinancers.

We examine the popular financial advice literature, and find that it advocates a combination of rules of thumb based on fixed interest-rate differentials and a "break-even" rule which compares the net present value of interest saved to the refinancing cost. Since this rule ignores option value considerations, it recommends suboptimally low refinancing differentials. We show that the rule predicts differentials between 6 and 85 basis points, consistent with many of the refinancing differentials that we observe in our sample.

6 References

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Table 1
Optimal Refinancing Differentials With Purely Proportional Transactions Costs

λ	σ	f	ρ	x^*
0	0.0069	0.02	0.03	0.0060
0	0.0069	0.02	0.05	0.0070
0	0.0069	0.02	0.07	0.0077
0	0.0069	0.05	0.03	0.0097
0	0.0069	0.05	0.05	0.0114
0	0.0069	0.05	0.07	0.0127
0	0.012	0.02	0.03	0.0079
0	0.012	0.02	0.05	0.0091
0	0.012	0.02	0.07	0.0100
0	0.012	0.05	0.03	0.0127
0	0.012	0.05	0.05	0.0147
0	0.012	0.05	0.07	0.0163
0.05	0.0069	0.02	0.03	0.0080
0.05	0.0069	0.02	0.05	0.0086
0.05	0.0069	0.02	0.07	0.0091
0.05	0.0069	0.05	0.03	0.0133
0.05	0.0069	0.05	0.05	0.0144
0.05	0.0069	0.05	0.07	0.0154
0.05	0.012	0.02	0.03	0.0104
0.05	0.012	0.02	0.05	0.0111
0.05	0.012	0.02	0.07	0.0117
0.05	0.012	0.05	0.03	0.0170
0.05	0.012	0.05	0.05	0.0182
0.05	0.012	0.05	0.07	0.0194
0.1	0.0069	0.02	0.03	0.0094
0.1	0.0069	0.02	0.05	0.0098
0.1	0.0069	0.02	0.07	0.0103
0.1	0.0069	0.05	0.03	0.0159
0.1	0.0069	0.05	0.05	0.0168
0.1	0.0069	0.05	0.07	0.0177
0.1	0.012	0.02	0.03	0.0120
0.1	0.012	0.02	0.05	0.0126
0.1	0.012	0.02	0.07	0.0131
0.1	0.012	0.05	0.03	0.0200
0.1	0.012	0.05	0.05	0.0210
0.1	0.012	0.05	0.07	0.0220

Note:λ denotes probability of full repayment, σ the standard deviation of the innovation in the mortgage rate, f the proportional transactions cost, ρ the discount rate and x^* the optimal refinancing differential.

Table 2
Optimal Refinancing Differentials With Fixed and Proportional Transactions Costs, By Mortgage Size

M	x^*
50,000	.0183
75,000	.0154
100,000	.0138
125,000	.0128
150,000	.0120
175,000	.0115
200,000	.0111
225,000	.0108
250,000	.0105
275,000	.0103
300,000	.0101

Note: Differentials calculated for $F = 2,000$, $f = .01$, $\lambda = .05$, $\rho = .05$, $\sigma = .012$. M denotes mortgage size in dollars and x^* denotes the optimal refinancing differential.

Table 3
Myopic Refinancing Differentials With Purely Proportional Transactions Costs

λ	σ	f	ρ	x^{μ}
0	0.0069	0.02	0.03	0.0006
0	0.0069	0.02	0.05	0.0010
0	0.0069	0.02	0.07	0.0014
0	0.0069	0.05	0.03	0.0015
0	0.0069	0.05	0.05	0.0025
0	0.0069	0.05	0.07	0.0035
0	0.012	0.02	0.03	0.0006
0	0.012	0.02	0.05	0.0010
0	0.012	0.02	0.07	0.0014
0	0.012	0.05	0.03	0.0015
0	0.012	0.05	0.05	0.0025
0	0.012	0.05	0.07	0.0035
0.05	0.0069	0.02	0.03	0.0016
0.05	0.0069	0.02	0.05	0.0020
0.05	0.0069	0.02	0.07	0.0024
0.05	0.0069	0.05	0.03	0.0040
0.05	0.0069	0.05	0.05	0.0050
0.05	0.0069	0.05	0.07	0.0060
0.05	0.012	0.02	0.03	0.0016
0.05	0.012	0.02	0.05	0.0020
0.05	0.012	0.02	0.07	0.0024
0.05	0.012	0.05	0.03	0.0040
0.05	0.012	0.05	0.05	0.0050
0.05	0.012	0.05	0.07	0.0060
0.1	0.0069	0.02	0.03	0.0026
0.1	0.0069	0.02	0.05	0.0030
0.1	0.0069	0.02	0.07	0.0034
0.1	0.0069	0.05	0.03	0.0065
0.1	0.0069	0.05	0.05	0.0075
0.1	0.0069	0.05	0.07	0.0085
0.1	0.012	0.02	0.03	0.0026
0.1	0.012	0.02	0.05	0.0030
0.1	0.012	0.02	0.07	0.0034
0.1	0.012	0.05	0.03	0.0065
0.1	0.012	0.05	0.05	0.0075
0.1	0.012	0.05	0.07	0.0085

Note:λ denotes probability of full repayment, σ the standard deviation of the innovation in the mortgage rate, f the proportional transactions cost, ρ the discount rate and x^{μ} the myopic refinancing differential.

Table 4
Myopic Refinancing Differentials With Fixed and Proportional Transactions Costs, By Mortgage Size

M	x^{μ}
50,000	.0050
75,000	.0037
100,000	.0030
125,000	.0026
150,000	.0023
175,000	.0021
200,000	.0020
225,000	.0019
250,000	.0018
275,000	.0017
300,000	.0017

Note: Differentials calculated for $F = 2,000$, $f = .01$, $\lambda = .05$, $\rho = .05$, $\sigma = .012$. M denotes mortgage size in dollars and x^{μ} denotes the myopic refinancing differential.

Appendix A: Personal Finance Literature

To find books that featured advice on mortgage refinancing, we executed a methodical search at online and conventional booksellers. Our online searches were conducted at the two largest online booksellers, Barnes&Noble.com (bn.com) and Amazon.com. In an effort to keep our online searches unbiased, we evaluated any viable book that appeared as a top ten bestseller at either site under keyword searches for "mortgages", "mortgage refinancing", or "refinancing". Only books that were out of print or obviously off the topic, like books on trading mortgage-backed securities, were ignored. At conventional bookstores Barnes & Noble and Wordsworth, we evaluated books shelved in the personal finance or real estate sections that had index entries for "refinancing" or "mortgage refinancing". All told, our search resulted in nineteen books containing advice on mortgage refinancing.

A table summarizing the books' advice is provided at the end of this appendix. Following is an explanation of the various categories in the table, with sample quotations provided for each type of advice. Of the nineteen books we found, fifteen provided some form of a break-even or NPV calculation. Such a calculation determines the number of months after which the savings from refinancing balance the initial costs. Six books advise readers to refinance whenever their planned stay exceeds this break-even point. Following this advice to the letter would, of course, result in no profit from refinancing. In the table that follows, these six books are coded as "strongly" advocating the break-even calculation. A prototypical example of such advice is provided in *Keys to Mortgage Financing and Refinancing*: "Compare the savings to the costs to find the amount of time before you break even . . . if the borrower plans to stay in the home at least [this long], it pays to refinance the loan."

Those books that provided a break-even calculation in addition to other rules of thumb are coded as "weakly" advocating the break-even rule. Nine books fell into this category. *The Wall Street Journal Guide to Understanding Personal Finance* recommends the break-even calculation but also states, "The rule of thumb is that it pays to refinance if you can get an interest rate at least two percentage points lower than you're currently paying." More qualitative advice is provided in *Talking Money*: if you plan to "stay in the house long enough not only to [break even] but to save additional money, then it would [be] a smart move."

Finally, four books specifically debunked the fixed rule of a two percent interest rate gap. *Mortgages for Dummies* stated simply, "Don't let the two-percent rule intimidate you . . . If you don't plan to sell your house in the next few years . . . interest rate spreads smaller than two percent are perfectly fine to justify refinancing."

Figure 1
Percent Refinancing By Interest Differential

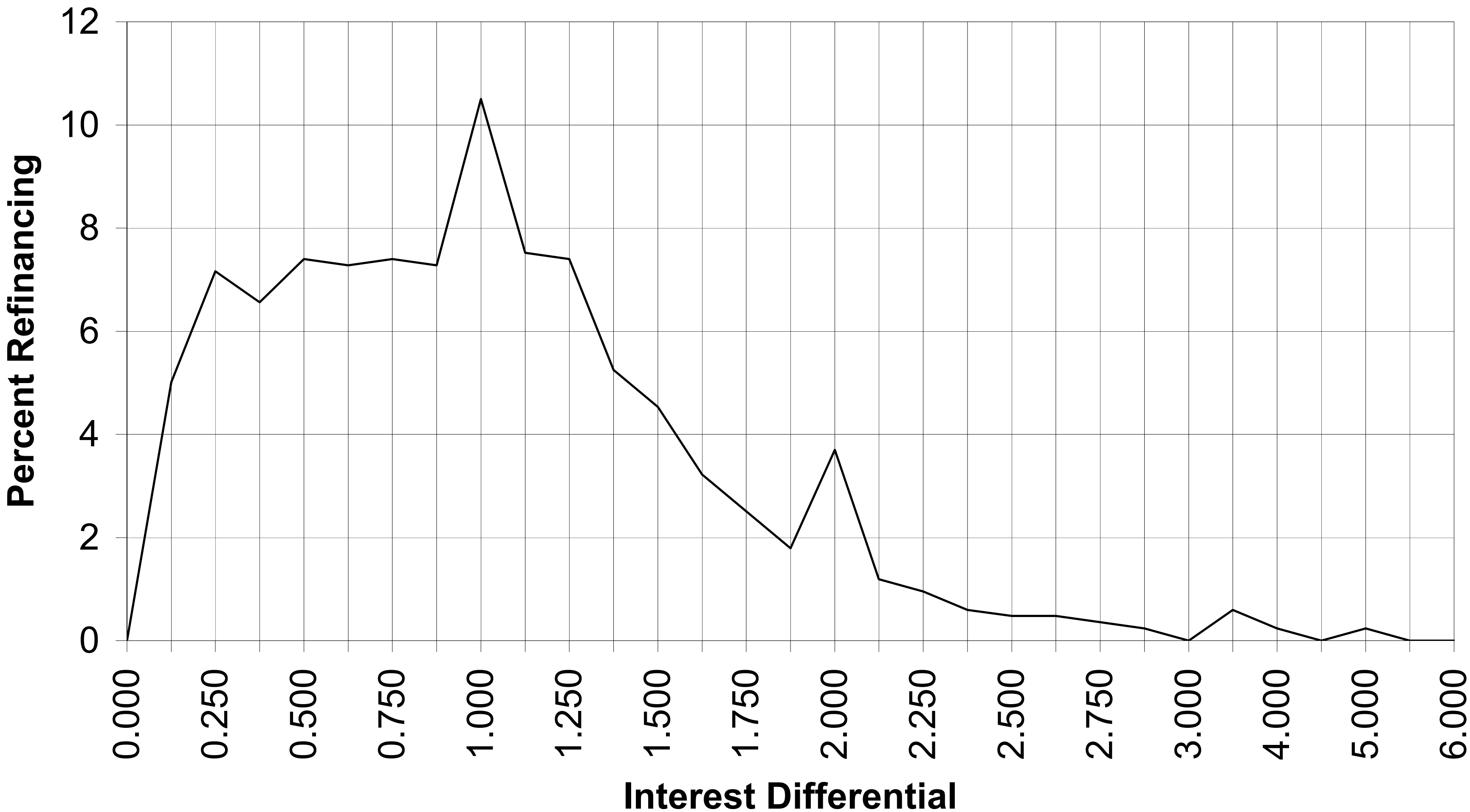

Figure 2
Cumulative Percent Refinancing By Interest Differential

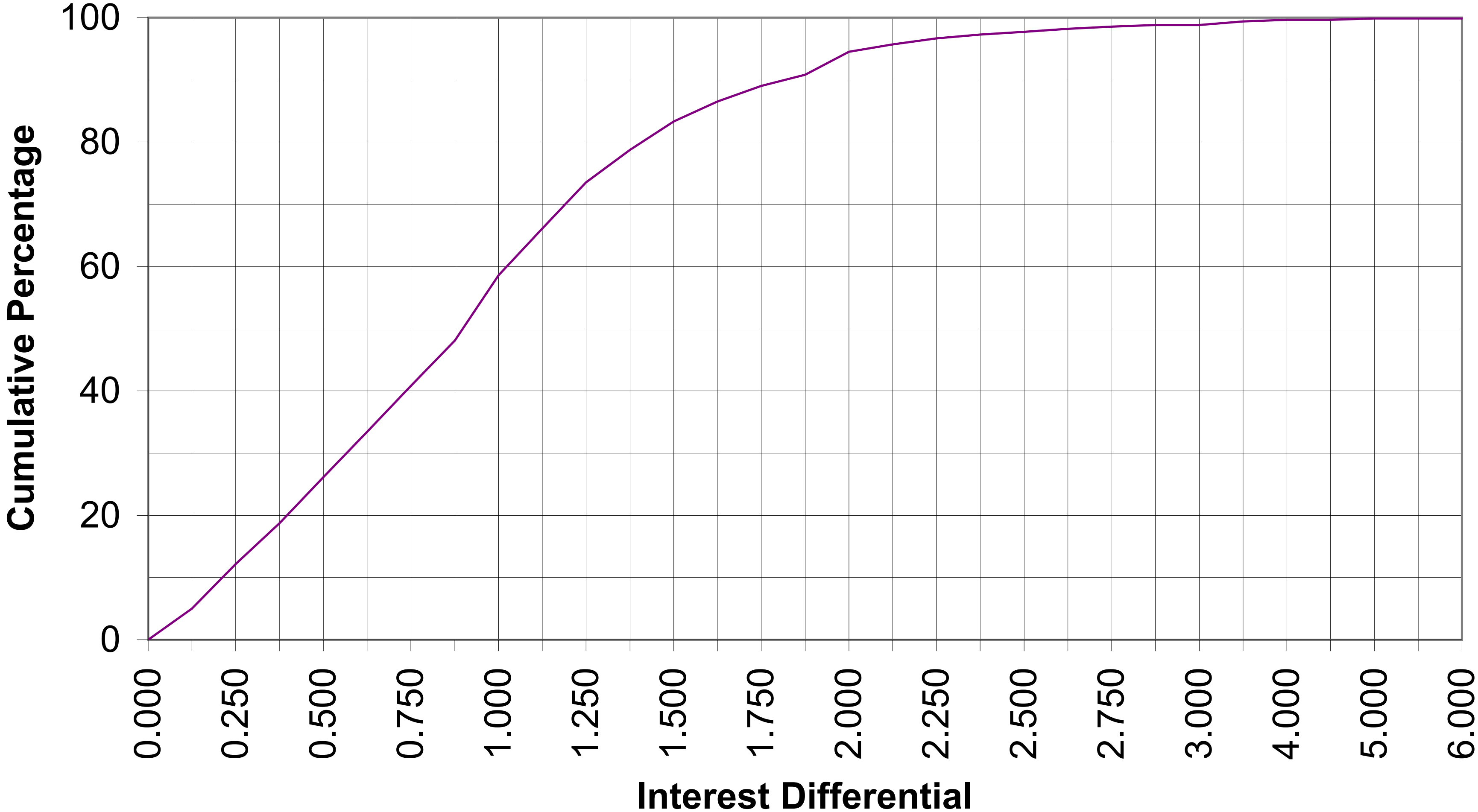

Figure 3
Number of Refinancers By Difference Between Actual Differential and Optimal Differential