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Green Markets and Private Provision of Public Goods

by

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Abstract

Many governments, nongovernmental organizations, and industries promote green markets as a decentralized mechanism to encourage private provision of environmental public goods. Green markets give consumers a new choice: instead of simply consuming a private good and making a donation to an environmental public good, consumers can purchase an impure public good that bundles characteristics of both activities. This paper develops a general model of private provision of a public good that includes the option to consume an impure public good. I use the model to investigate positive and normative consequences of introducing a green market. Despite the intent of green markets, I show that under quite reasonable assumptions, introducing a green market can have detrimental effects on both environmental quality and social welfare. I then derive conditions that are sufficient to rule out such unintended consequences. In general, the analysis applies to any market setting with opportunities to consume a private good, contribute to a pure public good, and obtain the characteristics of both activities jointly through an impure public good.

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1 Introduction

The economics literature on private provision of public goods has grown extensive over the last 25 years. The general assumption of theoretical research in this area is that individuals choose between consumption of a private good and contributions to a pure public good.[1] Models based on this assumption establish the important foundation for understanding private provision of public goods. Yet individuals increasingly have a third option: consumption of impure public goods that bundle private and public characteristics. This paper addresses fundamental questions about how the option to consume impure public goods affects private provision and social welfare.

Markets for "environmentally-friendly" goods and services exemplify the increased availability of impure public goods in the economy. The distinguishing feature of these markets—hereafter referred to as "green markets"—is joint production of a private good and an environmental public good. Consider two particular examples. First is the growing market for "green electricity," which is electricity generated with renewable sources of energy. Typically, consumers voluntarily purchase green electricity with a price premium that applies to all or part of their household's electricity consumption. In return, production of green electricity displaces pollution emissions from electricity generated with fossil fuels. Thus, consumers of green electricity purchase a joint product—electricity consumption and reduced emissions.[2] The second example is the market for price-premium, shade-grown coffee, which is coffee grown under the canopy of tropical forests rather than in open, deforested fields. Shade-grown coffee plantations provide important refuges for tropical biodiversity, including migratory birds. Thus, consumers of shade-grown coffee also purchase a joint product—coffee consumption and biodiversity conservation.[3]

Green markets are expanding in many sectors of the economy in response to a willingness to pay premiums for goods and services with relative environmental benefits. According to market research in the United States, green products account for approximately 9 percent of all new-product

[1] Standard treatments can be found in Bergstrom, Blume, and Varian (1986), Andreoni (1988), and Cornes and Sandler (1996).

[2] More than 80 public utilities in 28 states have developed optional green-electricity programs for their customers. Furthermore, green electricity is increasingly an option in states with competitive electricity markets. See Swezey and Bird (2000) for a status report of green electricity in the United States.

[3] Estimates of market revenues for shade-grown coffee are \$30 million per year in the United States, and projections of domestic-market potential exceed \$100 million (Commission for Environmental Cooperation, 1999). See Perfecto *et al.* (1996) and Tangley (1996) for more on the environmental benefits of shade-grown coffee.

introductions in the economy (Marketing Intelligence Service, 1999). Expansion of green markets worldwide has also prompted governments, nongovernmental organizations, and industries to establish green certification, or "eco-labeling," programs that cover thousands of products in more than 20 countries (U.S. EPA, 1993; OECD, 1997). These trends have generated enthusiasm to promote green markets as a decentralized mechanism to encourage private provision of environmental public goods.

Beyond green markets, it is common for goods and services to bundle private and public characteristics of various types. In many cases, bundling occurs because firms donate a percentage of their profits to a charitable cause. This practice ranges from goods such as cosmetics and ice cream to services such as credit cards and long-distance telecommunication. Furthermore, many charitable and nonprofit organizations, themselves, finance their activities in part through the sale of private goods, such as theater tickets or magazine subscriptions.[4] Finally, opportunities for "socially responsible investing," which combine a positive externality with investment return, are becoming increasingly popular.[5] In all these examples, the bundling of private and public characteristics implies that individuals face an impure public good.

In this paper, I develop a general model of private provision of a public good that includes the option for consumption of an impure public good. Building on the characteristics approach to consumer behavior (Lancaster, 1971; Gorman, 1980), I assume individuals derive utility from characteristics of goods rather than goods themselves. Individuals have the opportunity to consume a private good and make a contribution to a pure public good, with each activity generating its own characteristic. Additionally, the same private and public characteristics are available jointly through consumption of an impure public good.

The distinguishing feature of the model is the way that characteristics are available through more than one activity. As previously mentioned, the standard pure public good model has only a pure private good and a pure public good. In the original, impure public good model, Cornes and Sandler (1984, 1994) assume the private and public characteristics of the impure public good are

[4] A recent study of the United States nonprofit sector finds that commercial activities account for approximately 54 percent of all fund raising (Salamon, 1999). In the context of private provision of public goods, Posnett and Sandler (1986) consider this approach to fund raising, while Andreoni (1998) considers the role of "seed money."

[5] Investment portfolios based on some criteria of social responsibility have doubled in value form $1.185 trillion in 1997 to $2.16 trillion in 1999 (Social Investment Forum, 1999)

not available through any other means.[6] Vicary (1997, 2000) extends their basic setup to enable provision of the public characteristic through donations, but again, the private characteristic of the impure public good is otherwise unavailable. In contrast, the model developed here applies when individuals have the opportunity to consume a private good, contribute to a pure public good, and the characteristics of both activities are available jointly through consumption of an impure public good.

This generalization of the choice setting enables broad application of the model, with examples being those already discussed. In the context of green markets—the application I focus on throughout this paper—the model captures the fact that individuals typically have choices among a conventional (pure private) good, direct donations to an environmental (pure public) good, and a green (impure public) version of the good that jointly provides characteristics of the other two choices. For example, consumers of green electricity have options to purchase conventional electricity and donate directly to reduce emissions. Similarly, consumers of shade-grown coffee have options to purchase regular coffee and make donations to conserve tropical biodiversity.[7]

After establishing the basic model, I focus the analysis herein on three key questions. Will green markets actually lead to improvements in environmental quality? How will green markets affect social welfare? And how does the potential for induced changes in either environmental quality or social welfare depend on a green market's size?

Several results are quite striking. Despite the intent of green markets to improve environmental quality, I show that under reasonable assumptions, introducing a green market may in fact discourage private provision of an environmental public good. Furthermore, introducing a green market may diminish social welfare, even though it expands the consumption possibilities for the entire economy. If, however, a green market is sufficiently large, or private consumption is a gross complement for environmental quality, these counter intuitive results are no longer possible. While these results have implications for policies related to the promotion of green markets, the findings

[6]This setup provides the basis for other models that examine impure altruistic motives for private provision of public goods. These models interpret the private characteristic as a "warm glow" or "joy-of-giving" satisfaction that one experiences from making a donation. See Steinberg (1987), Andreoni (1989, 1990), and Ribar and Wilhelm (2002).

[7]Many nongovernmental organizations provide opportunities for donations to specific environmental causes. For example, the Clean Air Conservancy focuses on reducing air pollution emissions, and Rainforest Alliance focuses on conserving tropical biodiversity.

also apply generally to questions about the ways in which availability of impure public goods affects private provision and social welfare.

The remainder of this paper proceeds as follows. Section 2 introduces the model and analyzes individual behavior. Section 3 describes properties of the Nash equilibrium. For comparison purposes, Section 4 considers the economy prior to introduction of a green market. Sections 5 and 6 compare the economy with and without the green market to analyze green-market effects on environmental quality and social welfare. Section 7 considers the influence of a green market's size, in terms of the number of individuals in the economy. Section 8 summarizes and discusses the main conclusions. Finally, an appendix includes all proofs that are not immediate from the text.

2 The Model

Following the characteristics approach to consumer behavior (Lancaster, 1971; Gorman, 1980), individual preferences are specified over characteristics of goods rather than goods themselves. Assume for simplicity there are two characteristics, X and Y. Characteristic X has properties of a pure private good, while characteristic Y satisfies the non-rival and non-excludable properties of a pure public good. Interpret Y as environmental quality. Individual preferences are represented by a strictly increasing and strictly quasiconcave utility function

$$U_i = U_i(X_i, Y) \quad \text{for } i = 1, 2, ..., n,$$

where X_i is individual i's private consumption of X, and Y is aggregate provision of environmental quality. Specifically, $Y \equiv \sum_{i=1}^{n} Y_i$, where Y_i is individual i's private provision.[8]

Characteristics X and Y are available through three market activities: consumption of a conventional good c that generates characteristic X, direct donations y to improve environmental quality Y, and consumption of a green (or impure public) good g that generates X and Y jointly. In order to simplify notation, choose units of c, y, and g such that one dollar buys one unit of each. Furthermore, choose units of X and Y such that one unit of c generates one unit of X, and one

[8]In this paper, I focus on only private provision of Y; however, the model could readily include additional sources of environmental quality, such as naturally given levels and provision through public policy.

unit of y generates one unit of Y. Let $\alpha > 0$ and $\beta > 0$ characterize the green-market technology such that one unit of g generates α units of X and β units of Y. We then have a convenient interpretation of the relationships between market goods and characteristics: each dollar spent on c generates a dollar's worth of X; each dollar donated through y generates a dollar's worth of Y; and each dollar spent on g generates α dollars' worth of X and β dollars' worth of Y. Finally, assume the green-market technology is determined exogenously and is known by all individuals.[9]

Consider the maximization problem for each individual under the assumption of Nash behavior. Individuals choose non-negative quantities of market goods to solve

$$\begin{aligned} &\max_{c_i, y_i, g_i} \; U_i(X_i, Y) \\ \text{subject to } \; &X_i = c_i + \alpha g_i, \;\; Y = y_i + y_{-i} + \beta(g_i + g_{-i}), \\ \text{and } \; &c_i + y_i + g_i = w_i, \end{aligned} \tag{P1}$$

where i subscripts indicate individual i's consumption, $y_{-i} \equiv \sum_{j \neq i} y_j$, $g_{-i} \equiv \sum_{j \neq i} g_j$, and w_i is individual i's income. The first two constraints follow from the choice of units and the green-market technology. The second constraint also specifies the interrelationship of each individual's behavior. In particular, the Nash assumption implies that each individual i takes y_{-i} and g_{-i} as given. The third constraint is a standard budget constraint.

The structure of problem (P1) is such that individuals choose market goods to obtain desired levels of X and Y at the lowest cost. This implies that for some values of α and β, individuals will never choose particular market goods. For example, if $\alpha \geq 1$, individuals would never prefer consumption of c over g. This follows because generating α units of X costs α dollars through consumption of c, or one dollar through consumption of g. Therefore, if $\alpha \geq 1$, obtaining X through c is (weakly) more costly, and g has the added benefit of generating a positive amount of Y. In this case, it would never be optimal to consume c. Similar reasoning implies that individuals would never make a donation through y if $\beta \geq 1$, and individuals would never consume g if $\alpha+\beta < 1$.

[9] This setup is similar to a standard linear-characteristics model (see Deaton and Muellbauer, 1980), except that characteristic Y is a pure public good. Cornes and Sandler (1984, 1994) and Vicary (1997, 2000) use the same approach to model impure public goods. The main difference here is that both characteristics of the impure public good are available separately through a conventional private good and a pure public good. Another difference is that utility is specified over two characteristics rather than three. This simplification, which enables more direct comparison with the standard pure public good model, is possible because of the expanded choice set.

The following assumption is made to ensure the most interesting case, whereby c, y, and g are all viable in the market.

Assumption 1. *(i)* $\alpha < 1$, *(ii)* $\beta < 1$, *and* *(iii)* $\alpha + \beta > 1$.

Part (i) implies that c generates X at a lower cost than g; this maintains the possibility for consumption of c. Part (ii) implies that y generates Y at a lower cost than g; this maintains the possibility for donations y. Part (iii) implies that g generates X and Y jointly at a lower cost than c and y separately; this maintains the possibility for consumption of g. A further implication of part (iii) is uniqueness of each individual's solution to (P1). Since $\alpha + \beta \neq 1$, individuals will never be indifferent between spending a dollar on g or splitting it between c and y. Therefore, no two combinations of market activities can generate the same quantities of X and Y at the same cost; this ensures a unique solution to each individual's problem.

Let $\hat{c}_i$, $\hat{y}_i$, and $\hat{g}_i$ denote individual i's demand for market goods, each of which depends on the exogenous parameters $(w_i, \alpha, \beta, y_{-i}, g_{-i})$. We then have the following result about optimal consumption bundles.

Proposition 1. *The solution to problem (P1) will not have both* $\hat{c}_i > 0$ *and* $\hat{y}_i > 0$ *for any* i.

To prove this proposition, consider an individual who chooses both $\hat{c}_i > 0$ and $\hat{y}_i > 0$. By part (iii) of Assumption 1, g generates X and Y jointly at a lower cost than c and y separately. This implies that individual i could increase utility by increasing g_i and reducing c_i and y_i, thereby obtaining more X and Y. It follows that $\hat{c}_i > 0$ and $\hat{y}_i > 0$ cannot be part of an optimal solution to (P1). Thus, an individual would never consume the conventional good and make a direct donation, as she could always do better with some combination of the green good and the conventional good *or* a direct donation.

Using Proposition 1, an alternative and useful way to express the individual's problem is with implicit choices over characteristics. This transformation simplifies the 3-dimensional problem in goods space to a 2-dimensional problem in characteristics space. Since not both $\hat{c}_i > 0$ and $\hat{y}_i > 0$, we can write the budget constraint in (P1) as satisfying two separate inequality constraints: $c_i + g_i \leq w_i$ and $g_i + y_i \leq w_i$. One or both of these constraints will bind at an optimal solution:

the first constraint will bind if $\hat{y}_i = 0$, and the second constraint will bind if $\hat{c}_i = 0$. With these corresponding zero conditions and the identities that $X_i = c_i + \alpha g_i$ and $Y_i = y_i + \beta g_i$, it is possible to substitute c_i, y_i, and g_i out of the two inequality constraints. This yields the two budget constraints for the individual's problem with choices over X_i and private provision Y_i:

$$\max_{X_i, Y_i} \; U_i\left(X_i, Y_i + Y_{-i}\right)$$
$$\text{subject to} \;\; X_i + \varphi Y_i \leq w_i \;\; \text{and} \;\; \gamma X_i + Y_i \leq w_i, \tag{P2}$$

where $Y_{-i} \equiv y_{-i} + \beta g_{-i}$, $\varphi \equiv \frac{1-\alpha}{\beta}$, and $\gamma \equiv \frac{1-\beta}{\alpha}$. In the first constraint, $\varphi < 1$ represents the implicit price of Y in terms of X when the individual makes trade-offs between c_i and g_i. In the second constraint, $\gamma < 1$ represents the implicit price of X in terms of Y when the individual makes trade-offs between y_i and g_i.

It is also useful to express the individual's problem with choices over X_i and total provision of Y. Since Y_{-i} is exogenous in (P2), we can add φY_{-i} to both sides of the first constraint and Y_{-i} to both sides of the second. This yields the individual's "full-income" budget constraints.[10] Then, both (P1) and (P2) are equivalent to the following maximization problem:

$$\max_{X_i, Y} \; U_i\left(X_i, Y\right)$$
$$\text{subject to} \;\; X_i + \varphi Y \leq w_i + \varphi Y_{-i}, \;\; \gamma X_i + Y \leq w_i + Y_{-i}, \tag{P3}$$
$$\text{and} \;\; Y \geq Y_{-i},$$

where the final constraint requires individuals to choose a level of environmental quality no less than the level provided by others.

Figure 1 shows the feasible set in characteristics space for problem (P3). The frontier is piecewise linear with a kink at point E, which is the allocation that arises if the individual purchases g_i only. The segment EM with slope $-\frac{1}{\varphi} < -1$ corresponds to potential allocations when the individual makes trade-offs between c_i and g_i (i.e., the first full-income budget constraint). The segment BE with slope $-\gamma > -1$ corresponds to potential allocations when the individual makes trade-offs

[10] Full income refers to personal income plus the value of public-good "spillins" from provision by others. See Cornes and Sandler (1996) for more on full income, which is equivalent to Becker's (1974) concept of "social income."

between y_i and g_i (i.e., the second full-income budget constraint).[11]

We can now use maximization problem (P3) to solve for each individual's demand for total environmental quality and their own level of private provision. Let $q_i(w_i, Y_{-i})$ denote individual i's demand for Y ignoring the inequality constraint $Y \geq Y_{-i}$. Then, with the inequality constraint, the individual's demand for total environmental quality can be written as

$$\hat{Y} = \max\{Y_{-i}, q_i(w_i, Y_{-i})\}. \tag{1}$$

Subtracting Y_{-i} from both sides yields the individual's level of private provision:

$$\hat{Y}_i = \max\{0, q_i(w_i, Y_{-i}) - Y_{-i}\}. \tag{2}$$

This equation states that individuals may free ride with $\hat{Y}_i = 0$, or provide a positive amount of environmental quality with $\hat{Y}_i = q_i(w_i, Y_{-i}) - Y_{-i}$.

Note that solving for $\hat{Y}_i$ is sufficient to identify demand for X_i and all three market goods. Satisfying both budget constraints implies

$$\hat{X}_i = \begin{cases} w_i - \varphi \hat{Y}_i & \text{if } \hat{Y}_i \leq \beta w_i \\ \frac{1}{\gamma}(w_i - \hat{Y}_i) & \text{if } \hat{Y}_i \geq \beta w_i. \end{cases}$$

It follows that if $\hat{Y}_i \leq \beta w_i$, the individual does not make a direct donation, in which case $\hat{y}_i = 0$, $\hat{g}_i = \frac{1}{\beta}\hat{Y}_i$, and $\hat{c}_i = w_i - \hat{g}_i$. If, however, $\hat{Y}_i \geq \beta w_i$, the individual does not consume the conventional good, in which case $\hat{c}_i = 0$, $\hat{g}_i = \frac{1}{\alpha}\hat{X}_i$, and $\hat{y}_i = w_i - \hat{g}_i$.

3 Equilibrium

Each individual's best-response function in equation (2) fully specifies their equilibrium strategy. This strategy involves choosing a level of private provision $\hat{Y}_i$, where $0 \leq \hat{Y}_i \leq w_i$ and Y_{-i} is taken as given. A Nash equilibrium is defined as any vector of private provision levels $(Y_1^*, Y_2^*, ..., Y_n^*)$

[11] The dashed segment BM with slope -1 indicates the budget frontier when g_i in not available, and the individual makes trade-offs between c_i and y_i. This market scenario is discussed in Section 4.

that satisfies $Y_i^* = \hat{Y}_i$ for all i with $Y_{-i} = \sum_{j \neq i} Y_j^*$. Brouwer's fixed-point theorem guarantees existence of at least one such equilibrium. This section goes on to identify a sufficient condition for uniqueness of an equilibrium, and to solve for equilibrium levels of private provision for all individuals.

The following assumption is made henceforth to simplify notation and facilitate comparison with other models.

Assumption 2. *Preferences are identical for all* $i = 1, 2, ..., n$.

While this assumption is useful to highlight important features of the equilibrium, none of the results that follow rely on the condition of homogeneous preferences.[12]

In order to motivate the third and final assumption, it is convenient to define two additional functions. Let $h_c(w_i + \varphi Y_{-i})$ denote individual i's demand for total Y that arises from solving (P3) with only the *first* budget constraint. Let $h_y(w_i + Y_{-i})$ denote individual i's demand for total Y that arises from solving (P3) with only the *second* budget constraint. Assume both functions satisfy the following conditions.

Assumption 3. *(Normality)* $0 < h_c'(\cdot) \leq \eta < \frac{1}{\varphi}$ *and* $0 < h_y'(\cdot) \leq \mu < 1$.

This assumption simply requires that both characteristics X and Y are normal: individuals want more of both when they have more full income. The parameters η and μ imply that the slope of each function is bounded away from $\frac{1}{\varphi}$ and 1, respectively.

Assumption 3 differs in one important way from the normality assumption in the standard model of private provision of a pure public good. The standard model makes no distinction between characteristics and goods, so the normality assumption applies to the private and public goods directly (e.g., Bergstrom, Blume, and Varian, 1986; Andreoni, 1988). In contrast, Assumption 3 applies to characteristics and not goods. As a result, Assumption 3 requires normality of g, while there is no such requirement for c or y. This distinction is important in the context of green markets

[12] The assumption of homogeneous preferences is standard in models of privately provided public goods. Nevertheless, all subsequent analysis could readily incorporate heterogeneous preferences. Following Andreoni (1988), utility functions could include a vector of parameters that distinguish different types of individuals. Such a parameterization, however, would complicate the analysis with no change in the main results.

because it admits the possibility that c is inferior. It is easy to envision scenarios in which more income induces less demand for the conventional version of a good.[13]

A technical implication of Assumption 3 is that best-response functions are guaranteed to have slopes bounded within the interval $(-1, 0]$. To see this, rewrite equation (1) using our newly defined functions:

$$\hat{Y} = \max\{Y_{-i}, \min\{h_c(\cdot), \max\{\beta w_i + Y_{-i}, h_y(\cdot)\}\}\}. \tag{3}$$

The different possibilities in this expression correspond to different allocations on the individual's budget frontier in Figure 1. Demand for total environmental quality must always satisfy $\hat{Y} \geq Y_{-i}$. Demand $\hat{Y}$ will equal $h_c(\cdot)$ on segment EM if and only if $Y_{-i} \leq h_c(\cdot) \leq \beta w_i + Y_{-i}$. Furthermore, demand $\hat{Y}$ will equal $h_y(\cdot)$ on segment BE if and only if $h_y(\cdot) \geq \beta w_i + Y_{-i}$, in which case $h_y(\cdot) < h_c(\cdot)$ by normality of Y.[14] Now subtracting Y_{-i} from both sides of (3), we can rewrite best-response functions in equation (2) as

$$\hat{Y}_i = \max\{0, \min\{h_c(\cdot) - Y_{-i}, \max\{\beta w_i, h_y(\cdot) - Y_{-i}\}\}\}. \tag{4}$$

With Assumption 3, the slope of this expression with respect to changes in Y_{-i} is clearly bounded within the interval $(-1, 0]$.

These bounds on best-response functions are sufficient for existence of a unique Nash equilibrium. Cornes, Hartley, and Sandler (1999) show this result for the standard pure and impure public good models. Their proof generalizes to the model developed here and is relied upon for the following proposition.

[13] For example, more income may induce less demand for conventional electricity (or conventional coffee) and more demand for green electricity (or shade-grown coffee).

[14] To see that $h_y(\cdot) \geq \beta w_i + Y_{-i}$ and normality of Y imply $h_y(\cdot) < h_c(\cdot)$, note that individual i has an endowment at $X_i = \alpha w_i$ and $Y = \beta w_i + Y_{-i}$. The value of this endowment, which is equivalent to full income, depends on the implicit prices of characteristics: $m_i(p_X, p_Y) = p_X \alpha w_i + p_Y(\beta w_i + Y_{-i})$. Demand for Y, ignoring the constraint $Y \geq Y_{-i}$, can be expressed as a function $\hat{Y}(p_X, p_Y, m_i(p_X, p_Y))$. Then, $h_y(\cdot) = \hat{Y}(\gamma, 1, m_i(\gamma, 1)) = \hat{Y}(1, \frac{1}{\gamma}, m_i(1, \frac{1}{\gamma}))$, where the second equality follows because demand is homogeneous of degree zero. It is also the case that $h_c(\cdot) = \hat{Y}(1, \varphi, m_i(1, \varphi))$. Writing the demand functions in this way, the only difference between $h_y(\cdot)$ and $h_c(\cdot)$ is a decrease in p_Y from $\frac{1}{\gamma} > 1$ to $\varphi < 1$. A standard result of demand theory with endowment income is that if a good is normal and net demand is non-negative, a decrease in the good's own price results in strictly greater demand for the good (see Varian, 1992, p. 145). It follows that normality of Y and $h_y(\cdot) \geq \beta w_i + Y_{-i}$ implies $h_y(\cdot) < h_c(\cdot)$.

Proposition 2. *There exists a unique Nash equilibrium.*

Let Y^* denote the equilibrium level of aggregate environmental quality. We can now solve for each individual's equilibrium level of private provision Y_i^*. The following proposition shows how Y_i^* for all i depends on w_i and Y^*.

Proposition 3. *If Y^* is the equilibrium level of environmental quality with the green market, then there exist three critical levels of income $\underline{w} < \tilde{w} < \overline{w}$ such that for all i*

$$Y_i^* = \begin{cases} 0 & \text{if } w_i \leq \underline{w} \\ \frac{1}{\varphi}(w_i - \underline{w}) & \text{if } w_i \in (\underline{w}, \tilde{w}) \\ \beta w_i & \text{if } w_i \in [\tilde{w}, \overline{w}] \\ w_i - \overline{w}(1-\beta) & \text{if } w_i > \overline{w}, \end{cases}$$

where

$$\underline{w} \equiv h_c^{-1}(Y^*) - \varphi Y^*, \quad \tilde{w} \equiv \frac{\underline{w}}{\alpha}, \quad \text{and} \quad \overline{w} \equiv \frac{h_y^{-1}(Y^*) - Y^*}{1-\beta}.$$

The different possibilities in Proposition 3 have an intuitive interpretation in terms of demand for market goods. Individuals with sufficiently low income ($w_i \leq \underline{w}$) free ride and purchase only the conventional good c. All individuals with greater income provide positive amounts of environmental quality, and their level of provision increases with income. Among these individuals, those with lower income ($w_i \in (\underline{w}, \tilde{w})$) continue purchasing c and provide environmental quality through purchases of the green good g. They spend all their income above $\underline{w}$ on private provision and face a price of φ for Y, which implies $Y_i^* = \frac{1}{\varphi}(w_i - \underline{w})$. As income increases, these individuals substitute away from c and toward more g. Eventually, individuals with higher income ($w_i \in [\tilde{w}, \overline{w}]$) begin to purchase g only, which implies $Y_i^* = \beta w_i$. Finally, individuals with the highest income ($w_i > \overline{w}$) continue purchasing g and provide further environmental quality through direct donations y. They spend all their income above $\overline{w}(1-\beta)$ on private provision and face a price of unity for Y, which implies $Y_i^* = w_i - \overline{w}(1-\beta)$.

The following definition partitions individuals into four convenient sets based on the three critical levels of income in Proposition 3.

Definition 1. *Individual i is in set F (free riders) if $w_i \leq \underline{w}$; set C (contributors) if $w_i \in (\underline{w}, \tilde{w})$; set G (all greens) if $w_i \in [\tilde{w}, \overline{w}]$; and set D (donors) if $w_i > \overline{w}$.*

Note that these sets correspond to different loci on the individual's budget frontier in Figure 1. In particular, sets F, C, G, and D correspond to point M, the interior of segment EM, point E, and the interior of segment BE, respectively.

Other implications of Proposition 3 relate to equilibrium consumption of X and individual levels of utility. All i in set F spend all their income on c, resulting in $X_i^* = w_i$. All i in set C spend $\underline{w}$ on X and face a price of unity, resulting in $X_i^* = \underline{w}$. All i in set G spend all their income on g, resulting in $X_i^* = \alpha w_i$. Finally, all i in set D spend $\overline{w}(1-\beta)$ on X and face a price of γ, resulting in $X_i^* = \alpha\overline{w}$.

Differences in individual utility arise only from differences in consumption of X. This follows because all i benefit from the total level of environmental quality Y^*. Utility rises with income in sets F and G because X_i^* rises with income. In contrast, there is a single level of utility for all i in C and a single, although higher, level of utility for all i in D. Thus, $U_F < U_C < U_G < U_D$, where U_k is the level of utility for any individual in set k.

Many of the equilibrium results of this model share features with the standard model of private provision of a pure public good. The next section demonstrates similarities and differences between the two models. In particular, the standard model is shown as a special case of the model developed here.

4 Model Without a Green Market

In order to analyze the effects of introducing a green market, we must consider the economy prior to availability of the green good g. The constraint $g_i = 0$ for all i captures this scenario. Rewriting the individual's problem in (P3) with this constraint yields the following:

$$\max_{X_i, Y} U(X_i, Y) \quad \text{subject to} \quad X_i + Y = w_i + Y_{-i} \text{ and } Y \geq Y_{-i}, \tag{P4}$$

where there is now a single budget constraint, and implicit prices of X and Y are both unity. Referring back to Figure 1, the frontier of the individual's budget set in this problem is the dashed segment BM, compared to BEM when g is available.

Maximization problem (P4) is identical to the standard model for private provision of a pure public good.[15] In this case, each individual's optimal choice of total environmental quality can be written as

$$\check{Y} = \max\left\{Y_{-i}, h\left(w_i + Y_{-i}\right)\right\}, \tag{5}$$

where $h\left(w_i + Y_{-i}\right)$, with no subscript, indicates individual i's demand for Y in (P4) ignoring the constraint $Y \geq Y_{-i}$. Then, without the green market, the individual's private provision is

$$\check{Y}_i = \max\left\{0, h\left(\cdot\right) - Y_{-i}\right\}. \tag{6}$$

Maintaining normality of X and Y implies $0 < h'\left(\cdot\right) \leq \varepsilon < 1$, which continues to guarantee existence of a unique Nash equilibrium through Proposition 2.[16]

It is possible to characterize properties of the Nash equilibrium without the green market in much the same way as with the green market. Let Y^+ denote the equilibrium level of environmental quality without the green market. Invert $h\left(\cdot\right)$ in equation (5) and add Y_i^+ to both sides. Solving for private provision yields $Y_i^+ = w_i - h^{-1}\left(Y^+\right) + Y^+$ for an individual with positive provision. Define a critical level of income $w^+ \equiv h^{-1}\left(Y^+\right) - Y^+$. Then for the economy without the green market, the following proposition parallels Proposition 3.

Proposition 4. *If Y^+ is the equilibrium level of environmental quality without a green market, then there exists a critical level of income w^+ such that for all i*

$$Y_i^+ = \begin{cases} 0 & \text{if } \; w_i \leq w^+ \\ w_i - w^+ & \text{if } \; w_i > w^+. \end{cases}$$

[15] Another way to see the pure public good model as a special case is to assume $\alpha + \beta \leq 1$. With this assumption, it is straightforward to show that problems (P3) and (P4) are equivalent.

[16] See Bergstrom, Blume, and Varian (1986) for an alternative proof that relies on normality to guarantee existence and uniqueness of a Nash equilibrium in the pure public good model.

This proposition states that if $w_i \leq w^+$, the individual provides no environmental quality and therefore free rides. If $w_i > w^+$, the individual spends all her income above w^+ on private provision, and since the price of Y is unity through donations y, it follows that $Y_i^+ = w_i - w^+$.

Without the green market, differences in utility still depend only on differences in consumption of X. Since the price of X is also unity through consumption of c, Proposition 4 implies that $X_i^+ = w_i$ for all i with $w_i \leq w^+$, and $X_i^+ = w^+$ for all i with $w_i > w^+$. Therefore, individual levels of utility rise with income for all those who free ride, while individual levels of utility are the same, although generally higher, for all those who make a donation.

In sum, Propositions 3 and 4 demonstrate the differences between private provision with and without a green market—or more generally, with and without a viable impure public good. With the impure public good, there are three critical levels of income that distinguish between four potential sets of individuals: those who free ride, those who contribute through the impure public good, those who spend all their income on the impure public good, and those who make a donation. Without the impure public good, however, there is only one critical level of income that distinguishes between two potential sets of individuals: those who free ride, and those who make a donation.

5 Environmental Quality

This section considers the effect of introducing a green market on equilibrium provision of environmental quality. In contrast to conventional wisdom—which suggests that green markets are always beneficial to environmental quality—the general result is that introducing a green market can either increase or decrease private provision of an environmental public good. Different possibilities are shown to depend on the distribution of income, and on whether environmental quality is a gross complement (or substitute) for private consumption.

To begin, consider two simple examples that demonstrate the possibility for both an increase and a decrease in environmental quality. Assume the economy consists of two individuals with identical incomes $w_i = w$ and utility functions $U_i = X_i^\rho + Y^\rho$. Let $w = 100$ and $\rho = .3$. Without the green market, it is straightforward to show that the equilibrium is symmetric with $Y_i^+ = \frac{w}{3}$ and $Y^+ = 2Y_i^+ \cong 66.7$. This level of environmental quality serves as a reference point for the following examples that include a green market.

Example 1: The technology parameters of the green market are $\alpha = \beta = .6$. Solving for a fixed point that satisfies best-response functions in (4) and letting $r \equiv \frac{\rho}{\rho-1}$ implies $Y_i^* = \frac{\varphi^{r-1}w}{2+\varphi^r}$ for $i = 1, 2$. We then have $Y^* = 2Y_i^* \cong 111.9$. This *increase* in environmental quality from Y^+ to Y^* is shown in Figure 2.

Example 2: This example differs only with respect to the technology parameters of the green market. Let $\alpha = .9$ and $\beta = .3$. Solving for a fixed point in this case implies $Y_i^* = \frac{w}{1+2\gamma^r}$ for $i = 1, 2$. We then have $Y^* = 2Y_i^* \cong 61$. This *decrease* in environmental quality from Y^+ to Y^* is shown in Figure 3.

Note that the green-market technology in Example 2, in comparison to Example 1, favors production of the private characteristic. This difference causes both individuals move to set C in Example 1, whereas both individuals move to set D in Example 2. It turns out that these examples can be generalized as follows.

Proposition 5. *After introducing a green market, it will always be the case that*
(a) $Y^+ < Y^*$ *if provision comes from set* C *only;*
(b) $Y^+ > Y^*$ *if provision comes from set* D *only and* Y *is a gross substitute for* X.

This proposition states that environmental quality will always increase if no individuals make a donation with the green market. Furthermore, environmental quality will always decrease if all individuals with positive provision make a donation with the green market, and environmental quality is a gross substitute for private consumption.

To gain an intuition for these results, it is useful to think of introducing a green market as having two effects on each individual. First is a "price effect" from a change in the relative prices of characteristics X and Y. Second is a "spillin effect" from a change in the level of environmental quality provided by others (Y_{-i}). Both effects contribute to changes in each individual's demand for Y. This, in turn, influences changes in the equilibrium level of environmental quality. Examples 1 and 2 illustrate these ideas.

In Example 1, both the price effect and the spillin effect stimulate demand for Y. Both individuals move to set C and therefore face a lower relative price of Y ($\varphi < 1$). On its own, this price effect stimulates demand for Y, which encourages private provision. Then, increased provision by one individual generates a positive spillin effect for the other individual through an increase in Y_{-i}. This spillin effect further stimulates demand for Y because an increase in Y_{-i} increases full income, and Y is normal. Figure 2 demonstrates both the positive price effect—with the steeper slope of the binding segment of the new budget constraint—and the positive spillin effect—with $Y^+_{-i} < Y^*_{-i}$. The overall result is an increase in the equilibrium level of environmental quality.

Example 2, in contrast, illustrates a case in which both the price effect and the spillin effect depress demand for Y. Individuals move to set D and therefore face a lower relative price of X ($\gamma < 1$). On its own, this price effect depresses demand for Y, which is a gross substitute for X because $\rho = .3$.[17] Consequent reductions in Y_{-i} then generate a negative spillin effect, as the reduction in full income further depresses demand for Y. Figure 3 demonstrates both the negative price effect—with the flatter slope of the binding segment of the new budget constraint—and the negative spillin effect—with $Y^+_{-i} > Y^*_{-i}$. In this case, the overall result is a decrease in the equilibrium level of environmental quality.

Considering both Examples 1 and 2, it is clear why introducing a green market will, in general, have an ambiguous effect on environmental quality. Availability of the green good changes the relative prices of characteristics. Price effects may then stimulate demand for environmental quality for some individuals, while depressing demand for others.[18] In such cases, the net effect on equilibrium environmental quality, after accounting for spillin effects, is generally ambiguous.

It is important to recognize, however, that a negative price effect on demand for environmental quality is only possible if Y is a gross substitute for X, as in the previous examples. If, on the other hand, Y is a gross complement for X, a more general result is possible.

Proposition 6. *Environmental quality will always increase after introducing a green market (i.e., $Y^+ < Y^*$) if environmental quality is a gross complement for private consumption.*

The intuition for this proposition is the following. Introducing a green market unambiguously stimulates demand for Y through the price effect. This induces some individuals to increase their

[17] Note that $U_i = X_i^\rho + Y^\rho$ and $\rho \in (0, 1)$ implies X and Y are gross substitutes.

[18] In general, these same possibilities arise for individuals moving to set G, as these allocations are simply corner solutions of sets C and D.

private provision. A consequence of increased provision by some individuals may be crowding out of provision by others, since best response functions are downward sloping. Any crowding out, however, must be less than one-to-one because the increase in spillins stimulates demand for Y (i.e., best response functions have slopes greater then -1). Therefore, the net effect on equilibrium environmental quality must be positive.

6 Social Welfare

We have seen the ways in which a green market can affect environmental quality. But how will introduction of a green market affect social welfare? Availability of a green good expands each individual's choice set over market goods. It also expands the consumption possibilities over characteristics for the entire economy. Nevertheless, a green market can still decrease social welfare. This section shows the different ways that social welfare can either increase or decrease after introducing a green market.[19]

Consider the most straightforward case: the green market increases both environmental quality and social welfare. Figure 2 provides an example. Environmental quality increases from Y^+ to Y^*, and utility increases from U_i^+ to U_i^* for both individuals. Returning to the notions of a price effect and a spillin effect from the previous section, it is clear in Figure 2 that both the lower price of Y and the increased spillins lead to positive income effects. This, in turn, leads to the increase in utility for both individuals, which implies a Pareto improvement with the green market.

Once again, the example in Figure 2 is representative of a more general result. The following proposition identifies a situation in which a green market leads to an unambiguous increase in social welfare.

Proposition 7. *Introducing a green market will always increase social welfare if provision comes from set C only.*

We saw in Proposition 5 that environmental quality will always increase if no individuals make a

[19]Note that with or without a green market, the equilibrium level of social welfare will fall short of the Pareto-efficient level. This is because, in both cases, individuals take no account of the external benefits of their own private provision of environmental quality. Therefore, conclusions about changes in social welfare are based on whether one inefficient equilibrium Pareto dominates another.

donation with the green market. Proposition 7 implies that social welfare will always increase as well. This result will be especially useful in the next section.

More generally, it is important to recognize that even when a green market increases environmental quality, social welfare may not increase. Introducing a green market may shift the burden of provision from one group of individuals to another, and despite a net increase in environmental quality, those individuals picking up the burden may become worse off. Figure 4 demonstrates this possibility, with individuals 1 and 2 shown in different panels. Income differs between the two individuals with $w_1 < w_2$.[20] Without the green market, w_1 is low enough so that individual 1 has zero provision and free rides on individual 2's provision. They enjoy utility levels U_1^+ and U_2^+. With the green market, individual 1 moves to set C and increases provision from zero to Y_1^*. Individual 2 moves to set D and decreases provision from $Y_2^+ = Y^+$ to Y_2^*. The net effect is an increase in environmental quality from Y^+ to Y^* and new levels of utility U_1^* and U_2^*. The important thing to note is that $U_1^+ > U_1^*$ and $U_2^+ < U_2^*$. Thus, introducing the green market makes individual 1 worse off and individual two better off. In this case, neither equilibrium Pareto dominates, despite the fact that the green market increases environmental quality.[21]

Now consider situations in which the green market decreases the level of environmental quality. Figure 3 demonstrates that it is still possible for social welfare to increase. Environmental quality decreases from Y^+ to Y^*, as private provision decreases from Y_i^+ to Y_i^* for both individuals. The decrease in spillins generates a negative income effect for both individuals. This, however, is more than offset by the positive income effect from the lower price of X that both individuals face. Thus, utility increases from U_i^+ to U_i^* for both individuals, despite the decrease in environmental quality.

Finally, there is the most counter intuitive possibility: introducing the green market decreases social welfare. Figure 5 provides an example.[22] As in Figure 3, environmental quality decreases, and the reduction in spillins generates a negative income effect for both individuals. In this case, however, the positive income effect from the lower price of X is not large enough to be offsetting. Therefore, utility declines from U_i^+ to U_i^* for both individuals, and the equilibrium without the

[20] Parameter values for this simulation are $w_1 = 100$, $w_2 = 250$, $\alpha = .7$, $\beta = .4$, and $\rho = .4$. Environmental quality increases from $Y^+ = 125$ to $Y^* \cong 130.7$.

[21] It can be shown that this scenario is possible regardless of whether Y is a gross substitute or complement for X.

[22] Parameter values for this simulation are $w_i = 100$, $\alpha = .95$, $\beta = .15$, and $\rho = .88$. Environmental quality decreases from $Y^+ \cong 66.7$ to $Y^* \cong 36.2$.

green market Pareto dominates the equilibrium with it.[23] This occurs despite the facts that with the green market, individuals have a broader choice set over market goods, and the green-market technology expands the consumption possibilities over characteristics for the entire economy. As the contrast between indifference curves in Figures 3 and 5 suggests, the greater the marginal rate of substitution between X and Y, when provision comes from set D, the greater the possibility for a decrease in social welfare. In such cases, the substitution effect is large relative to the income effect arising from the green market's expansion of the feasible set.

7 Green Markets in a Large Economy

Prior research shows that group size influences equilibrium results for private provision of a pure public good (Chamberlin, 1974; McGuire, 1974; Andreoni, 1988; Cornes and Sandler, 1996), an impure public good (Cornes and Sandler, 1984), and direct donations when an impure public good is available (Vicary, 1997, 2000). These findings raise questions about how the size of a green market will influence its effects on environmental quality and social welfare. This section analyzes the way that green-market effects change when the number of individuals in the market increases. The approach extends Andreoni's (1988) technique for the pure public good model.

To begin, use Proposition 3 to identify two conditions that must hold in equilibrium. First, the equilibrium level of environmental quality with a green market must satisfy

$$Y^* = \sum_{w_i \in (\underline{w}, \tilde{w})} \tfrac{1}{\varphi}\left(w_i - \underline{w}\right) + \sum_{w_i \in [\tilde{w}, \overline{w}]} \beta w_i + \sum_{w_i > \overline{w}} w_i - \overline{w}\left(1 - \beta\right),$$

where the summands represent provision from individuals in sets C, G, and D, respectively. Second, the critical level of income $\underline{w}$ must satisfy $\underline{w} = \kappa\left(Y^*\right)$, where $\kappa\left(Y^*\right) \equiv h_c^{-1}\left(Y^*\right) - \varphi Y^*$.[24] Taking the inverse of this expression yields $Y^* = \kappa^{-1}\left(\underline{w}\right)$. Note that, by Assumption 3, the slope of this inverse function is positive and bounded from above such that $0 < \partial \kappa^{-1}\left(w\right) / \partial w \leq \frac{\eta}{1-\varphi\eta}$.

[23] Note that this outcome is not possible if there is an increase in environmental quality. In order for a green market to unambiguously decrease social welfare, it must be the case that $Y_{-i}^{\cdot}$ decreases for all individuals. This implies $\sum_{i=1}^{n} Y_{-i}^{+} > \sum_{i=1}^{n} Y_{-i}^{*}$, which is equivalent to $(n-1)\,Y^{+} > (n-1)\,Y^{*}$ and $Y^{+} > Y^{*}$. Therefore, all individuals cannot be worse off with a green market if it increases environmental quality.

[24] A similar condition can be written for other critical levels of income ($\tilde{w}$ and $\overline{w}$); however, only one such condition is necessary here. Without loss of generality, I use the condition involving $\underline{w}$.

Now consider an economy with an arbitrary number of n individuals and a corresponding vector of incomes $(w_1, w_2, ..., w_n)$. Given the equilibrium level of environmental quality, denoted Y_n^*, we can write a function for average, private provision over all i:

$$\begin{aligned} A_n &= \frac{1}{n}\left[\sum_{w_i \in (\underline{w}_n, \tilde{w}_n)} \tfrac{1}{\varphi}(w_i - \underline{w}_n) + \sum_{w_i \in [\tilde{w}_n, \overline{w}_n]} \beta w_i + \sum_{w_i > \overline{w}_n} w_i - \overline{w}_n (1-\beta)\right] \\ &= \frac{\kappa^{-1}(\underline{w}_n)}{n}, \end{aligned}$$

where $\underline{w}_n$, $\tilde{w}_n$, and $\overline{w}_n$ are determined by their definitions in Proposition 3 with Y_n^*. Assume the distribution of income is characterized by a continuous probability density function $f(w)$ with support $0 \leq w \leq w_{max}$. We can then increase the number of individuals in the market by adding to the vector of incomes with random draws from $f(w)$. Then as n grows large and $n \to \infty$, average provision converges to

$$\lim_{n\to\infty} A_n = \int_{\underline{w}_\infty}^{\tilde{w}_\infty} \tfrac{1}{\varphi}(w - \underline{w}_\infty) f(w)\, dw + \int_{\tilde{w}_\infty}^{\overline{w}_\infty} \beta w f(w)\, dw + \int_{\overline{w}_\infty}^{w_{max}} w - \overline{w}_\infty (1-\beta) f(w)\, dw \tag{7a}$$

$$= 0, \tag{7b}$$

where $\underline{w}_n \to \underline{w}_\infty$, $\tilde{w}_n \to \tilde{w}_\infty$, and $\overline{w}_n \to \overline{w}_\infty$. The first equality follows by the law of large numbers. The second equality follows because $\underline{w}_n$ is bounded from above by w_{max}, which implies $\kappa^{-1}(\underline{w}_\infty)$ is finite.[25] The following lemma identifies a necessary condition for equating (7a) and (7b).

Lemma 1. $\underline{w}_\infty = w_{max}$.

This lemma implies that $\underline{w}_n \to w_{max}$ as $n \to \infty$. In words, the critical level of income that distinguishes between individuals who free ride and individuals who have positive provision converges to the maximum level of income in the economy.

[25] The easiest way to see that $\underline{w}_n \leq w_{max}$ for any size of the economy n is to recognize that normality of Y implies the wealthiest individual will always have positive provision. Then, there must exist some w_i such that $\underline{w}_n < w_i \leq w_{max}$ for any size of the economy n, which implies $\underline{w}_n \leq w_{max}$.

The fact that $\underline{w}_n$ converges to w_{max} as the economy grows large has several important implications, which are summarized as follows.

Proposition 8. *If an economy has n individuals, a green market, and incomes distributed according to a continuous probability density function $f(w)$ with $0 \leq w \leq w_{max}$, then the following statements describe the economy as n increases to infinity:*

(a) Only the wealthiest individuals have positive provision;

(b) The proportion of individuals with positive provision decreases to zero;

(c) Total provision increases to a finite level $\kappa^{-1}(w_{max})$;

(d) Average provision decreases to zero;

(e) Direct donations y decrease to zero;

(f) Only those who consume the green good g and the conventional good c have positive provision;

(g) Environmental quality is strictly greater than it would be without the green market;

(h) Social welfare is strictly greater than it would be without the green market.

To prove this proposition, we need only review some of the results shown previously. All individuals with $w_i \leq \underline{w}_n$ are free riders (by Proposition 3). Therefore, as $\underline{w}_n \to w_{max}$, only the wealthiest individuals provide environmental quality, and these individuals comprise a diminishing proportion, $1 - F(\underline{w}_n)$, of the population. By construction, $Y_n^* = \kappa^{-1}(\underline{w}_n)$, which is finite as $\underline{w}_n \to w_{max}$. Then by (7b), average provision decreases to zero. Since $\underline{w}_n \to w_{max}$, it follows that $\tilde{w} = \frac{\underline{w}_n}{\alpha} \to \frac{w_{max}}{\alpha} > w_{max}$, which implies that sets G and D are empty (by Definition 1). That is, no individual makes a donation y or consumes g only. But since provision must be positive for at least the wealthiest individual, positive provision must come from set C, which includes individuals who consume g and c. Then since provision comes from set C only, both environmental quality and social welfare must be strictly greater than they would be without the green market (by Propositions 5 and 7).

While parts (a)-(d) of Proposition 8 mirror results of the pure public good model, parts (e)-(h) are novel. A key finding is that, in larger markets, availability of a green good will tend to crowd out direct donations to improve environmental quality.[26] This may explain, in part, why

[26] Vicary (1997) finds a similar result, but further assumptions about demand for the impure alternative are necessary in his model, as individuals have no opportunity to obtain the private characteristic of the impure public good through other means.

many nonprofit organizations are increasingly turning toward commercial activities for fund raising. Rather than make direct donations, individuals purchase impure public goods. If this is the case, then the only consequence of introducing a green market is a decrease in the price of providing environmental quality. It follows that in a sufficiently large economy, introducing a green market unambiguously increases the level of environmental quality and social welfare.

8 Conclusions

This paper analyzes a new choice setting for private provision of a public good in order to capture the reality that impure public goods are increasingly prevalent in the economy. The model applies to any market setting where individuals have the opportunity to consume a private good, contribute to a pure public good, and the characteristics of both activities are available jointly through an impure public good. Many results of this setup differ from existing models of privately provided public goods.

I apply the model in particular to green markets, which offer impure public goods that jointly provide a private good and an environmental public good. Green markets fit the model because in addition to the green good, consumers typically have opportunities to consume a conventional version of the good, and make direct donations to the associated environmental cause. Despite widespread enthusiasm for green markets to improve environmental quality and increase social welfare, questions remain about the positive and normative consequences of introducing a green market. I return to the three questions posed at the outset of this paper in order to highlight important conclusions.

Will green markets actually lead to improvements in environmental quality? In general, green markets will change the level of environmental quality. The surprising result, however, is that green markets will not necessarily improve environmental quality. Introducing a green market changes implicit prices of both private consumption and environmental quality. These changes in prices may encourage some individuals to provide more of the environmental public good, while encouraging others to provide less. If environmental quality is a gross substitute for private consumption, introducing a green market may either increase or decrease environmental quality. If, on the other hand, environmental quality is a gross complement for private consumption, introducing a green market will always increase environmental quality.

How will green markets affect social welfare? The potential green-market effects on social welfare are also surprising—green markets can either increase or decrease social welfare. The most intuitive possibility is for a green market to increase both environmental quality and social welfare. It is the case, however, that introducing a green market may shift some of the burden of provision from one set of individuals to another. Therefore, even with an increase in environmental quality, some individuals may be worse off with a green market. Alternatively, if a green market decreases environmental quality, it is still possible for social welfare to increase. This may occur if the green market induces substitution away from environmental quality and toward private consumption. Finally, a decrease in environmental quality may decrease social welfare. This result is possible despite the facts that the green market expands the choice set over market goods for every individual, along with the consumption possibilities over characteristics for the entire economy.

How does a green market's size influence its effects on environmental quality and social welfare? Several of these results are related to established theory on private provision of a pure public good. When the number of potential participants in a green market increases, the proportion of individuals with positive provision decreases, only the wealthiest provide, average provision decreases, and aggregate provision increases to a finite level. Several other results on the influence of a green market's size are new. When the number of potential participants in a green market increases, direct donations decrease, and the proportion of aggregate provision through the green market increases. Thus, green markets tend to crowd out direct donations to improve environmental quality. Furthermore, in economies that are sufficiently large, the effects on environmental quality and social welfare are no longer ambiguous—introducing a green market increases both environmental quality and social welfare.

In conclusion, we should be cautious about relying on impure public goods as a mechanism to encourage private provision of public goods. In the context of green markets, this paper demonstrates how unintended consequences may arise. In particular, markets designed to improve environmental quality and increase social welfare may, in fact, be detrimental to both. Furthermore, caution should apply even under the most favorable circumstances. Free riding plays a central role in green markets, just as it does in most market settings involving private provision of public goods. Therefore, green markets alone will never induce efficient levels of environmental quality. This, of course, leaves a continued role for government intervention. Future research should examine the relationship between green markets and more centralized forms of environmental policy.

Appendix

Proof of Proposition 3

In equilibrium, equation (3) must hold with $Y^* = \hat{Y}$ for all i. If $Y^* = h_c(w_i + \varphi Y_{-i})$, invert $h_c(\cdot)$, add φY_i^* to both sides, and rearrange to get $Y_i^* = \frac{1}{\varphi}(w_i - \underline{w})$. If $Y^* = h_y(w_i + Y_{-i})$, invert $h_y(\cdot)$, add Y_{-i} to both sides, and rearrange to get $Y_i^* = w_i - \overline{w}(1-\beta)$. Substituting these expressions into equation (4) yields equilibrium private provision for all i:

$$Y_i^* = \max\left\{0, \min\left\{\frac{1}{\varphi}(w_i - \underline{w}), \max\{\beta w_i, w_i - \overline{w}(1-\beta)\}\right\}\right\}. \tag{A1}$$

With (A1), we can verify that $\underline{w} < \tilde{w} < \overline{w}$. By definition, $\underline{w} < \tilde{w}$ since $\tilde{w} = \frac{\underline{w}}{\alpha}$ and $\alpha < 1$. To show that $\tilde{w} < \overline{w}$, recall from equations (3) and (4) that if $h_y(w_i + Y_{-i}) - Y_{-i} \geq \beta w_i$ for any level of w_i, then $h_y(w_i + Y_{-i}) - Y_{-i} < h_c(w_i + \varphi Y_{-i}) - Y_{-i}$ by normality of Y. This implies in (A1) that if $w_i - \overline{w}(1-\beta) \geq \beta w_i$ for any level of w_i, then $w_i - \overline{w}(1-\beta) < \frac{1}{\varphi}(w_i - \underline{w})$. Simplifying the first inequality yields $w_i \geq \overline{w}$. Combining the first and second inequality and rearranging terms yields $w_i > \tilde{w}$. Then, to satisfy $w_i \geq \overline{w}$ and $w_i > \tilde{w}$ for any level of w_i, it must be true that $\tilde{w} < \overline{w}$.

With these critical levels of income, the different possibilities in Proposition 3 follow directly from (A1). In particular, $Y_i^* = 0$ if $w_i \leq \underline{w}$, $Y_i^* = \frac{1}{\varphi}(w_i - \underline{w})$ if $w_i \in (\underline{w}, \tilde{w})$, $Y_i^* = \beta w_i$ if $w_i \in [\tilde{w}, \overline{w}]$, and $Y_i^* = w_i - \overline{w}(1-\beta)$ if $w_i > \overline{w}$. Q.E.D.

Proof of Proposition 5

The following lemma is stated and proved first.

Lemma A1: *If* $h(w_i + Y'_{-i}) \geq Y'_{-i}$ *and* $h(w_i + Y'_{-i}) \geq h_c(w_i + \varphi Y''_{-i})$, *then* $Y'_{-i} > Y''_{-i}$.

Proof: Starting with endowment income $X_i = w_i$ and $Y = Y'_{-i}$, the condition $h(w_i + Y'_{-i}) \geq Y'_{-i}$ implies that net demand for Y is non-negative with relative prices $p_X = p_Y = 1$. Then, changing relative prices so that $p_X = 1$ and $p_Y = \varphi$, demand for Y is given by $h_c(w_i + \varphi Y'_{-i})$. Normality of Y implies $h(w_i + Y'_{-i}) < h_c(w_i + \varphi Y'_{-i})$.[27] Then if $h(w_i + Y'_{-i}) \geq h_c(w_i + \varphi Y''_{-i})$, it must be

[27] See Varian (1992, p. 145) for further explanation of this point, which is a standard result of demand theory for a price change with endowment income.

true that $Y'_{-i} > Y''_{-i}$. Q.E.D.

Part (i): Assume to the contrary that $Y^+ \geq Y^*$. Then for all i with $w_i > w^+$, which includes at least the wealthiest individual (by normality of Y), the following inequality must hold:

$$Y^+ = h\left(w_i + Y^+_{-i}\right) \geq \max\left\{Y^*_{-i}, h_c\left(w_i + \varphi Y^*_{-i}\right)\right\} = Y^*.$$

Then by Lemma A1, $Y^+_{-i} > Y^*_{-i}$ for all i with $w_i > w^+$.

Now let the notation $\hat{Y}(Y_{-i})$ and $\check{Y}(Y_{-i})$ serve as a shorthand for best response functions in (4) and (6), respectively. Then, it must hold for all i with $w_i > w^+$ that

$$Y_i^+ = \check{Y}_i\left(Y^+_{-i}\right) \leq \check{Y}_i\left(Y^*_{-i}\right) \leq \hat{Y}_i\left(Y^*_{-i}\right) = Y_i^*.$$

The first inequality follows because best response functions have slopes ≤ 0, and we have shown that $Y^+_{-i} > Y^*_{-i}$. The second inequality follows because a decrease in the price of Y (from 1 to $\varphi < 1$) must weakly increase demand for Y, holding Y_{-i} constant. If, however, $Y_i^+ \leq Y_i^*$ for all i with $w_i > w^+$, then it is not possible for $Y^+_{-i} > Y^*_{-i}$ for all i with $w_i > w^+$, since no individuals reduce their provision with the green market. Therefore, the assumption that $Y^+ \geq Y^*$ leads to a contradiction. It must then be true that $Y^+ < Y^*$, which proves part (i).

Part (ii): Assume to the contrary that $Y^+ \leq Y^*$. Consider all i with $w_i > \overline{w}$, which includes at least the wealthiest individual (by normality of Y and the assumption that provision comes from set D only). Then for all such individuals, the following inequality must hold:

$$Y^+ = \max\left\{Y^+_{-i}, h\left(w_i + Y^+_{-i}\right)\right\} \leq h_y\left(w_i + Y^*_{-i}\right) = Y^*.$$

This inequality implies that $Y^+_{-i} < Y^*_{-i}$ for all i with $w_i > \overline{w}$. This follows because the assumption that Y is a gross substitute for X implies $h\left(w_i + Y'_{-i}\right) > h_y\left(w_i + Y''_{-i}\right)$ for any $Y'_{-i} \geq Y''_{-i}$.

It must also hold for all i with $w_i > \overline{w}$ that

$$Y_i^+ = \check{Y}_i\left(Y^+_{-i}\right) \geq \check{Y}_i\left(Y^*_{-i}\right) \geq \hat{Y}_i\left(Y^*_{-i}\right) = Y_i^*.$$

The first inequality follows because best response functions have slopes ≤ 0, and we have shown that $Y_{-i}^{+} < Y_{-i}^{*}$. The second inequality follows because the assumption that Y is a gross substitute for X implies that a decrease in the the price of X (from 1 to $\gamma < 1$) must weakly decrease demand for Y, holding Y_{-i} constant. The condition that $Y_i^{+} \geq Y_i^{*}$ for all i with $w_i > \overline{w}$ implies that all individuals with positive provision after introducing the green market had weakly greater provision before. Hence, it is not possible for $Y_{-i}^{+} < Y_{-i}^{*}$ for all i with $w_i > \overline{w}$, since by assumption, they are the only ones with positive provision with the green market. Therefore, the assumption that $Y^{+} \leq Y^{*}$ leads to a contradiction. It must then be true that $Y^{+} > Y^{*}$, which proves part (ii). Q.E.D.

Proof of Proposition 6

This proof extends the proof of part (i) in Proposition 5. Assume to the contrary that $Y^{+} \geq Y^{*}$. Then for all i with $w_i > w^{+}$, which includes at least the wealthiest individual (by normality of Y), the following inequality must hold:

$$Y^{+} = h\left(w_i + Y_{-i}^{+}\right) \geq \max\left\{Y_{-i}^{*}, \min\left\{h_c\left(w_i + \varphi Y_{-i}^{*}\right), \max\left\{\beta w_i + Y_{-i}^{*}, h_y\left(w_i + Y_{-i}^{*}\right)\right\}\right\}\right\} = Y^{*}.$$

Then by Lemma A1, it must be the case that $Y_{-i}^{+} > Y_{-i}^{*}$ for all i with $w_i > w^{+}$.

It must also hold for all i with $w_i > w^{+}$ that

$$Y_i^{+} = \check{Y}_i\left(Y_{-i}^{+}\right) \leq \check{Y}_i\left(Y_{-i}^{*}\right) \leq \hat{Y}_i\left(Y_{-i}^{*}\right) = Y_i^{*}.$$

The first inequality follows because best response functions have slopes ≤ 0, and we have shown that $Y_{-i}^{+} > Y_{-i}^{*}$. The second inequality follows because Y is a gross complement for X, which implies that a decrease in the price of Y from (1 to $\varphi < 1$) or a decrease in the price of X from (1 to γ) must weakly increase demand for Y, holding Y_{-i} constant. The remainder of the proof is identical to that of part (i) of Proposition 5. Q.E.D.

Proof of Proposition 7

It is sufficient to show that $U\left(X_i^+, Y^+\right) < U\left(X_i^*, Y^*\right)$ for all i. We know from part (i) of Proposition 5 that $Y^+ < Y^*$. Therefore, we need only show that utility increases even if $X_i^+ > X_i^*$. Assume that $X_j^+ > X_j^*$ for some individual j. It must then be true that $w^+ > \underline{w}$. Since $Y_{-j}^+ = \sum_{w_i > w^+}^{i \neq j} w_i - w^+$ and $Y_{-j}^* = \sum_{w_i > \underline{w}}^{i \neq j} \frac{1}{\varphi}\left(w_i - \underline{w}\right)$, it also follows that $Y_{-j}^+ \leq Y_{-j}^*$. Without the green market, optimization implies $X_j^+ + Y^* - Y_{-j}^* = w_j$. With the green market, optimization implies $X_j^* + \varphi(Y^* - Y_{-j}^*) = w_j$. Then since $Y_{-j}^+ \leq Y_{-j}^*$, we have that $X_j^+ + \varphi(Y^+ - Y_{-j}^*) < w_i$. Therefore, (X_j^*, Y^*) is strictly and directly revealed preferred to (X_j^+, Y^+) for any j with $X_j^+ > X_j^*$. It must then be true that $U\left(X_i^+, Y^+\right) < U\left(X_i^*, Y^*\right)$ for all i. Q.E.D.

Proof of Lemma 1

Suppose $\underline{w}_\infty \neq w_{max}$. If $\underline{w}_\infty > w_{max}$, then $\underline{w}_n$ converges to a value greater than the maximum level of income. It follows that $Y^* = 0$ for some n because $Y_i^* = 0$ for all i if $\underline{w}_n > w_{max}$. This, however, contradicts normality of Y, which requires positive provision from at least the wealthiest individual. Now suppose $\underline{w}_\infty < w_{max}$. Then there exists a number θ such that $\underline{w}_\infty < \theta < w_{max}$. As $n \to \infty$, we will observe $w_i > \theta$ infinitely often. It follows that

$$
\begin{aligned}
0 &< \int_{\min\{\theta, \tilde{w}_\infty\}}^{\tilde{w}_\infty} \tfrac{1}{\varphi}\left(w - \underline{w}_\infty\right) f(w)\, dw + \int_{\max\{\tilde{w}_\infty, \min\{\theta, \overline{w}_\infty\}\}}^{\overline{w}_\infty} \beta w f(w)\, dw \\
&\quad + \int_{\max\{\theta, \overline{w}_\infty\}}^{w_{max}} w - \overline{w}_\infty (1 - \beta) f(w)\, dw \\
&\leq \int_{\underline{w}_\infty}^{\tilde{w}_\infty} \tfrac{1}{\varphi}\left(w - \underline{w}_\infty\right) f(w)\, dw + \int_{\tilde{w}_\infty}^{\overline{w}_\infty} \beta w f(w)\, dw + \int_{\overline{w}_\infty}^{w_{max}} w - \overline{w}_\infty (1 - \beta) f(w)\, dw \\
&= \lim_{n \to \infty} A_n,
\end{aligned}
$$

which contradicts (7b). Therefore, since both $\underline{w}_\infty > w_{max}$ and $\underline{w}_\infty < w_{max}$ lead to contradictions, it must be true that $\underline{w}_\infty = w_{max}$. Q.E.D.

References

Andreoni, James. "Privately Provided Public Goods in a Large Economy: The Limits of Altruism." *Journal of Public Economics* 35 (February 1988): 57-73.

Andreoni, James. "Giving with Impure Altruism: Applications to Charity and Ricardian Equivalence." *Journal of Political Economy* 97 (December 1989): 1447-58.

Andreoni, James. "Impure Altruism and Donations to Public Goods: A Theory of Warm-Glow Giving." *Economic Journal* 100 (June 1990): 464-77.

Andreoni, James. "Toward a Theory of Charitable Fund-Raising." *Journal of Political Economy* 106 (December 1998): 1186-1213.

Becker, Gary S. "A Theory of Social Interactions." *Journal of Political Economy* 82 (November - December 1974): 1063-93.

Bergstrom, Theodore C.; Blume, Lawrence E.; and Varian, Hal R. "On the Private Provision of Public Goods." *Journal of Public Economics* 29 (February 1986): 25-49.

Chamberlin, John. "Provision of Collective Goods as a Function of Group Size." *American Political Science Review* 68 (June 1974): 707-16.

Commission for Environmental Cooperation. "Measuring Consumer Interest in Mexican Shade-Grown Coffee: An Assessment of the Canadian, Mexican and US Markets." Montreal, Canada, 1999.

Cornes, Richard; Hartley, Roger; and Sandler, Todd. "Equilibrium Existence and Uniqueness in Public Good Models: An Elementary Proof Via Contradiction." *Journal of Public Economic Theory* 1 (October 1999): 499-509.

Cornes, Richard, and Sandler, Todd. "Easy Riders, Joint Production, and Public Goods." *Economic Journal* 94 (September 1984): 580-98.

Cornes, Richard, and Sandler, Todd. "The Comparative Static Properties of the Impure Public Good Model." *Journal of Public Economics* 54 (July 1994): 403-21.

Cornes, Richard, and Sandler, Todd. *The Theory of Externalities, Public Goods and Club Goods, Second Edition.* Cambridge, U. K.: Cambridge University Press, 1996.

Deaton, Angus, and Muellbauer, John. *Economics and Consumer Behavior.* Cambridge, U.K.: Cambridge University Press, 1980.

Gorman, William M. "A Possible Procedure for Analyzing Quality Differentials in the Egg Market." *Review of Economic Studies* 47 (October 1980): 843-56.

Lancaster, Kelvin. *Consumer Demand: A New Approach.* New York, NY: Columbia University Press, 1971.

Marketing Intelligence Service. "ProductScan Online." Naples, New York: Marketing Intelligence Service Ltd., 1999.

McGuire, Martin C. "Group Size, Group Homogeneity, and the Aggregate Provision of a Pure Public Good Under Cournot Behavior." *Public Choice* 18 (Summer 1974) 107-26.

Organisation for Economic Co-operation and Development. "Eco-labelling: Actual Effects of Selected Programms." Paris, France, 1997.

Perfecto, Ivette; Rice, Robert A.; Greenberg, Russell; and Van der Voort, Martha E. "Shade Coffee: A Disappearing Refuge for Biodiversity." *Bioscience* 46 (September 1996): 598-608.

Posnett, John, and Sandler, Todd. "Joint Supply and the Finance of Charitable Activity." *Public Finance Quarterly* 14 (April 1986): 209-22.

Ribar, David C., and Wilhelm, Mark O. "Altruistic and Joy-of-Giving Motivations in Charitable Behavior." *Journal of Political Economy* 110 (April 2002): 425-57.

Salamon, Lester M. *America's Nonprofit Sector: A Primer, Second Edition.* New York, NY: Foundation Center, 1999.

Social Investment Forum. "1999 Report on Responsible Investing Trends in the United States." Washington, D.C., 1999.

Steinberg, Richard S. "Voluntary Donations and Public Expenditures in a Federalist System." *American Economic Review* 77 (March 1987): 24-36.

Swezey, Blair, and Bird, Lori. "Green Power Marketing in the United States: A Status Report, Fifth Edition." NREL/TP-620-28738. Golden, CO: National Renewable Energy Laboratory, 2000.

Tangley, Laura. "The Case of Missing Migrants." *Science* 274 (November 1996): 1299-1300.

U.S. Environmental Protection Agency. "Status Report on the Use of Environmental Labels Worldwide." Washington, D.C., 1993.

Varian, Hal R. *Microeconomic Analysis, Third Edition.* New York, NY: W. W. Norton & Company, Inc., 1992.

Vicary, Simon. "Joint Production and the Private Provision of Public Goods." *Journal of Public Economics* 63 (February 1997): 429-45.

Vicary, Simon. "Donations to a Public Good in A Large Economy." *European Economic Review* 44 (March 2000): 609-18.

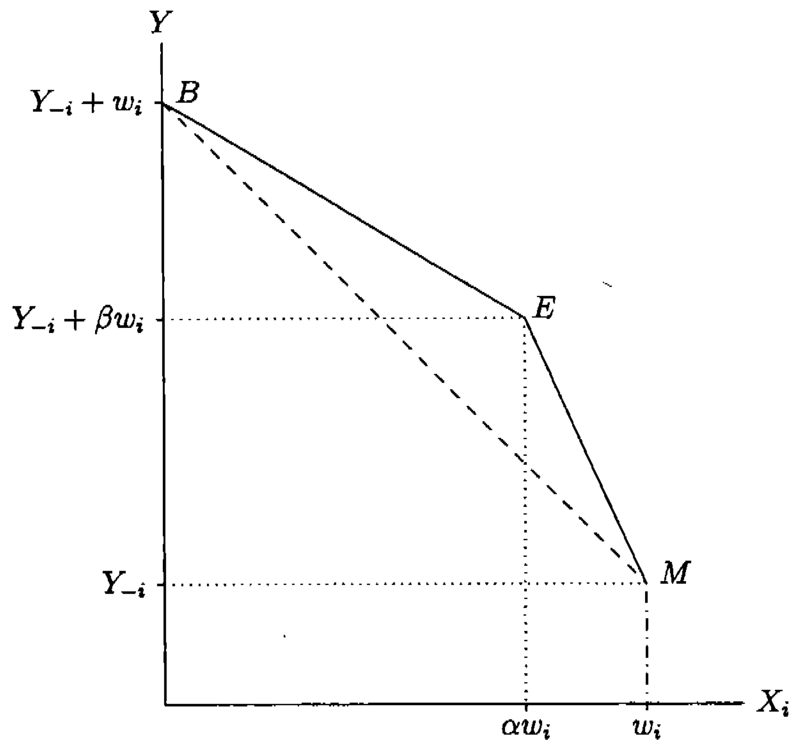

Figure 1: Budget set in characteristics space

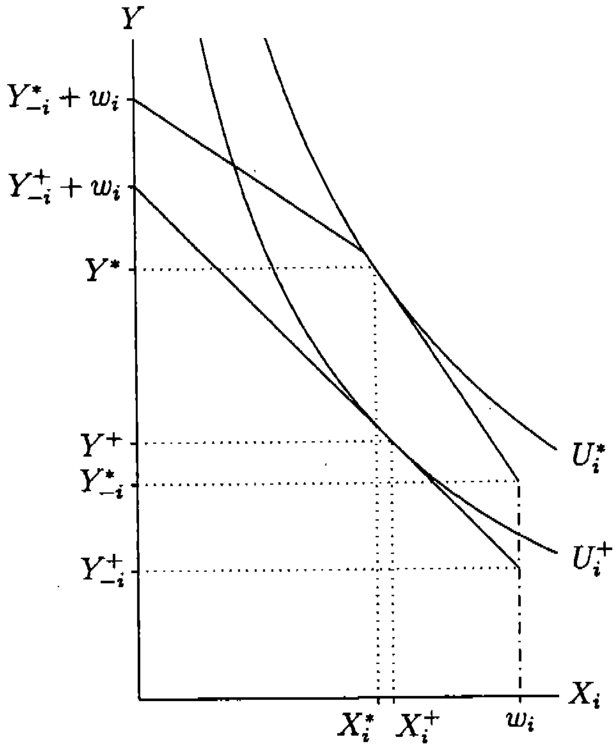

Figure 2: Green market increases both environmental quality and social welfare

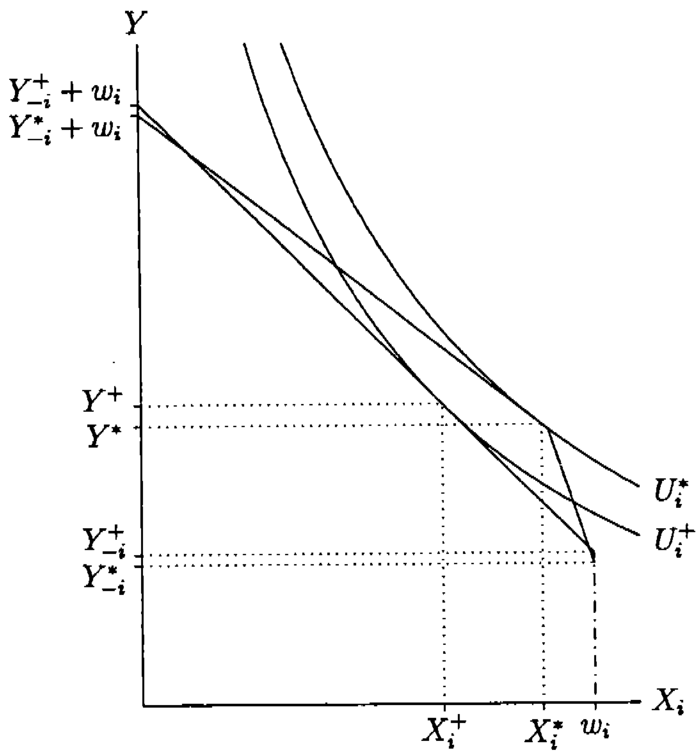

Figure 3: Green market decreases environmental quality and increases social welfare

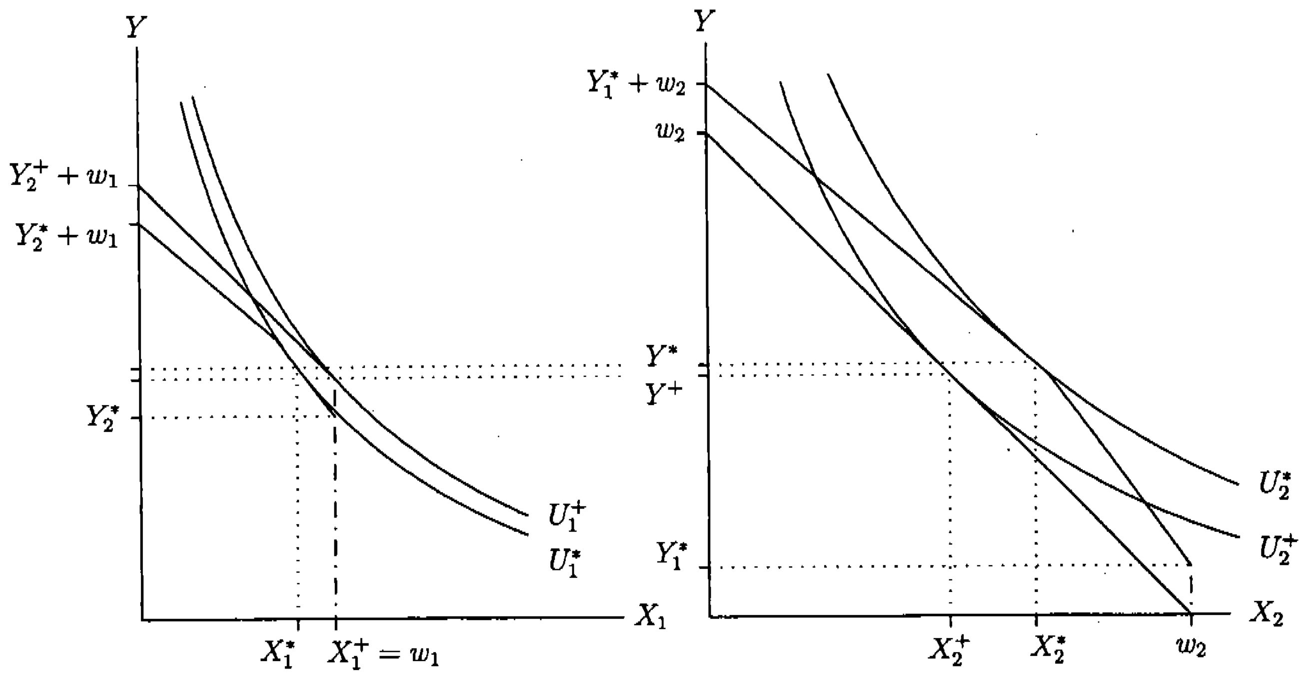

Figure 4: Green market increases environmental quality, decreases utility for individual 1, and increases utility for individual 2

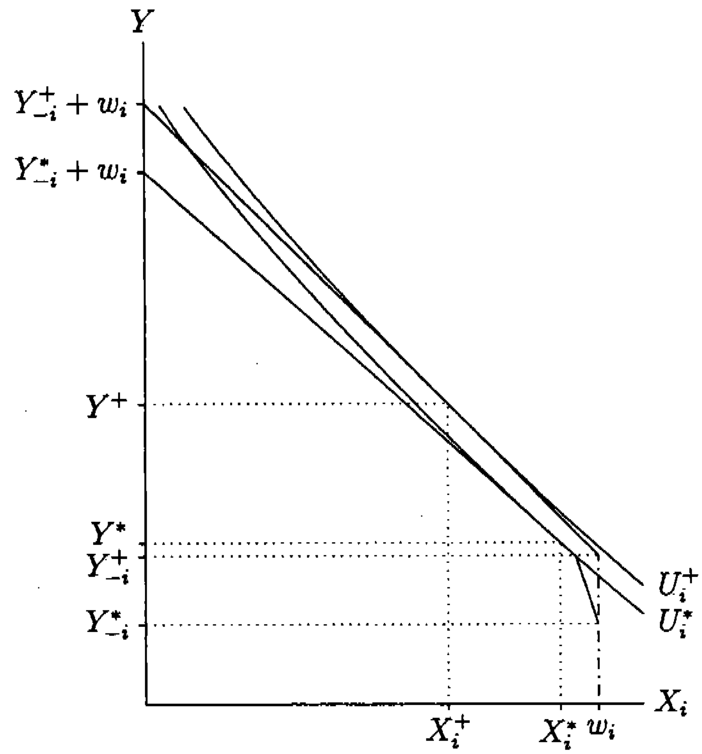

Figure 5: Green market decreases both environmental quality and social welfare