Understanding Large Movements in Stock Market Activity

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Abstract

Financial data display rather striking scale-free characteristics that have been a topic of considerable interest. We propose a model of the microbehavior of financial markets that accounts for two empirically-observed facts, the power law distribution of returns with exponent 3, and the power law distribution of trading volume with exponent 1.5. We also show that the model is consistent with a number of additional empirically-found results, including equal-time codependences among return, volume, and number of trades.

Useful for quantifying earthquake risk is the empirical fact that the number of earthquakes larger than a given magnitude decreases as a power law of the magnitude [1–3]. Useful in quantifying economic risk is the empirical fact that in a given time interval Δt, the number of stock price fluctuations larger than a fixed magnitude decreases as a power law of the magnitude. The exponent value that characterizes this power law appears to be the same for quite different countries, different size of markets, and for different market trends (e.g., bull or bear markets). There is no satisfactory explanation of why there should be a power law decay, of why the exponent characterizing this decay should have an apparently "universal" value, or of why the exponent value should be approximately three [4,5]. Here

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we propose and develop a model that explains this empirical law, explains other power laws characterizing large movements in market activity, and explains a number of codependences characterizing the fluctuations of different quantities. The model is based on a plausible set of assumptions, and its quantitative predictions are in accord with known empirical facts.

We define p_t as the price of a given stock, and the stock price "return" r_t as the change in the logarithm of stock price, $r_t \equiv \ln p_t/p_{t-\Delta t}$. The probability that a return is in absolute value larger than x is found to be

$$P(|r_t| > x) \sim x^{-\zeta_r} \text{ with } \zeta_r \approx 3. \tag{1}$$

This "inverse cubic law" holds over as many as 80 standard deviations for some stock markets, for Δt ranging from one minute to one month, across different sizes of stocks, different time periods, and also for different stock market indices [6–9]. Mechanisms based on shocks of infinite variance cannot account for stock market fluctuations, as they imply $\zeta_r \leq 2$ [10,11]. Moreover, the most extreme events — including the 1929 and 1987 market crashes — conform to this law [12], demonstrating that crashes do not appear to be outliers of the distribution (1).

Power laws also describe the distributions of other fluctuating quantities. Emprical studies show that the distribution of trading volume V_t obeys [13,14]

$$P(V_t > x) \sim x^{-\zeta_{V_t}} \qquad \text{with} \qquad \zeta_{V_t} \approx 1.5, \tag{2}$$

while for the individual trade sizes [15] q_t

$$P(q_t > x) \sim x^{-\zeta_q} \qquad \text{with} \qquad \zeta_q \approx 1.5, \tag{3}$$

and for the number of trades N_t [16]

$$P(N_t > x) \sim x^{-\zeta_N} \qquad \text{with} \qquad \zeta_N \approx 3.4. \tag{4}$$

We will base our model on the distribution of the largest market participants, mutual funds [17]. Motivated by the fact that size distributions of several economic variables display power-law distributions (examples include incomes [18], cities [19] and firms [20]), we hypothesize that the tail of the distribution of mutual fund size S is also described by a power law

$$P(S > x) \sim x^{-\zeta_S}, \tag{5}$$

where S denotes the market value of the managed assets. To test this hypothesis, we use the Center for Research in Security Prices database to calculate ζ_S for the top 10% of the mutual funds for each year in the period 1961-1999, and find—using the Hill estimator—an average value $\zeta_S = 1.05 \pm 0.02$. Exponent values of approximately 1 have also been found for the cumulative distributions of city size and firm size, and the origins of this "Zipf" distribution are becoming better understood [21]. For the following derivation of the inverse cubic law, we use the value $\zeta_S = 1$.

A second important empirical fact motivating our model is that large traders have large price impacts [22,23]. A typical stock has a turnover (fraction of shares exchanged) of approximately 50% a year [24], which implies a daily turnover of approximately $50\%/250 = 0.2\%$—i.e., on average 0.2% of outstanding shares change hands each day. The 30th largest mutual fund owns about 0.1% of such a stock [25]. If its fund manager sells its holdings of this stock, the sale will represent half of the daily turnover, and so will impact both the price and the total volume [26,27].

The gist of our theory is that managers of large mutual funds receive "intuitions" about the future direction of the market and trade on them, while avoiding too much in annual transaction costs. The resulting outcomes are the power law distributions of returns and volume with exponents 3 and 1.5 respectively.

We present the theory in three steps.

(A) Square root price impact. The price impact Δp of a trade of size V has been established to be increasing and concave [28,29], and we hypothesize its functional form to be

$$r = \Delta p \sim V^{1/2}. \tag{6}$$

We investigate empirically the equivalent relation $E\left[r^2 \mid V\right] \sim V$ [30]. Figure 2 shows empirical values of the quantity $E\left[r^2 \mid V\right]$, which we find is affine in V for large V [31]. Relationship (6) implies [32]

$$\zeta_r = 2\zeta_V. \tag{7}$$

Hence the cubic law of return (1) can be explained by the half-cubic law of volumes (2) and the square root price impact (6). It remains to explain (2) and (6).

(B) Explaining the half-cubic law distribution of returns (2). Call V the size of the block a trader wants to trade, and V_t the aggregate volume which is comprised of individual blocks $V_t = \sum_{j=1}^{J} V_j$. Hence $\zeta_{V_t} = \zeta_V$. If each fund i of size S_i were to trade, at random, a volume V_i proportional to S_i (which we write as $V_i = a_i S_i$), then the distribution of individual volumes will follow $\zeta_V = \zeta_S = 1$. However, $\zeta_V > 1$ empirically, i.e. the distribution of volumes is less fat-tailed that the distribution of size. An intuitive reason for this fact is that large traders have large price impacts, and must moderate their trading to avoid paying too large costs of price impacts. The trader will also be careful to moderate his annual transaction costs, suggesting that an important quantity is the average proportional amount of funds of size S, defined to be the amount of transaction costs paid by the funds (as a fraction of the portfolio)

$$c\left(S\right) = \frac{\text{Annual amount lost by the fund in price impact}}{\text{Value } S \text{ of the assets under management of the fund}}. \tag{8}$$

For example, if funds of size S pay on average 1% in price impact a year, then $c\left(S\right) = 0.01$.

The preceding consideration motivates the following theorem.

Theorem. If the following conditions hold (i) Zipf's law for institutional investors $\zeta_S = 1$; (ii) Square root price impact (6); (iii) Funds trade in typical volumes $V \sim S^{\delta}$ with $\delta > 0$; (iv) Funds adjust trading frequency and/or volume so as to pay transactions costs $c\left(S\right) = C$ approximately independently of S, then returns and volumes are power law distributed with tail exponents

$$\zeta_r = 3, \zeta_V = 3/2. \tag{9}$$

The validity of conditions (i) and (ii) was shown above, while condition (iii) is given by virtually any model—e.g. the model discussed in the [33, sup]. Condition (iv) implies that large and small funds pay roughly similar annual price impact costs (say 1%). An alternative is that $c(S)$ increases with S. If, e.g. $c(S) \sim S$, then if fund A is ten times larger than fund B, and $c(S_B) = 1\%$, then A pays $c(S_A) = 10\%$/year in transaction costs. Such a fund A would soon be eliminated from the market. Thus it is plausible that $c(S) \approx C$, independent of S, will be assured by evolutionary forces. Indeed, a fund trading so much would likely be eliminated from the market, because most funds do not seem to possess superior information or insight about the markets [34].

To prove the theorem, we start by noting that with each block trade, the fund incurs a price impact proportional to $V \cdot \Delta p$. From condition (ii), this cost is $V^{3/2}$. If $F(S)$ is the annual frequency of trading, then the annual loss in transactions costs is $F(S) \cdot V^{3/2}$, i.e. a fraction

$$c(S) = F(S) \cdot (V(S))^{3/2} / S \tag{10}$$

of the value S of this portfolio. Condition (iii) implies that either $V(S)$ or $F(S)$ will adjust in order to satisfy

$$F(S) \sim S \cdot (V(S))^{-3/2} . \tag{11}$$

Condition (i) implies that the number of traders with size larger than S is $G(S) \sim S^{-1}$, so the density of traders of size S is $\rho(S) = -G'(S) \sim S^{-2}$. By condition (iii), volumes $V > x$ correspond to traders of size S such that $S^{\delta} > x$. There is a number $\rho(S) \sim S^{-2}$ of traders of size S and they trade with frequency given $F(S)$ in (11). Hence

$$P(V > x) \sim \int_{S^{\delta}>x} F(S)\,\rho(S)\,dS \sim \int_{S>x^{1/\delta}} S^{1-3\delta/2} S^{-2} dS \sim x^{-3/2}. \tag{12}$$

which leads to a power law distribution of volumes with exponent $\zeta_V = 3/2$. From (7), it follows that $\zeta_r = 3$.

The structure of the above result is quite general, and does not depend on many details of the trading strategy, such as specific values of δ. Rather, it relies on the empirically-grounded relation (6). We next propose an explanation for this relation.

(C) Price impact of trades. We consider the behavior of one stock, A, whose original price is 1 USD, so that the new price is 1 USD $+b\Delta p$, where Δp is the proportional price increase in the trade, and b is some real number [36]. The manager of a large mutual fund who wants to make a buy trade of size V offers a price increment Δp to realize his trades, and his broker contacts potential suppliers of shares. The number of liquidity providers available after he has waited a time T is of course a non-decreasing function of T, and we will take it to be proportional to T [37]. A liquidity provider i of size s_i is willing to supply q_i shares, for a price increase Δp, with the supply function $q_i \sim s_i \Delta p$.

After a time T, the active trader can buy a quantity of shares proportional to $\langle s \rangle \Delta p T$. The search process stops when the desired quantity is reached, i.e., when $\langle s \rangle \Delta p T = V$. Hence, the time needed to find the shares is

$$T = \frac{V}{\langle s \rangle \Delta p} \sim \frac{V}{\Delta p}. \tag{13}$$

There is a trade-off between cost of execution Δp, and the time to execution T if the trader wants to realize his trade in a short amount of time T, he will have to pay a large price impact $\Delta p \sim V/T$.

We consider the managers' trading problem. Let us assume that fund managers receive independent and identically distributed "intuitions" that tell them that a given stock is mispriced by an amount M, i.e., the difference between the fair value of the stock and the traded price is M. It does not matter whether these hunches reflect genuine insight, or the overconfidence of the traders in their judgement [35]. Consider, e.g., $M > 0$. During the time interval that the trader is trying to purchase the stock, the price of the stock continues to increase, in part because some of the liquidity providers will "front run" (purchase the stock themselves, in anticipation of the price impact of the large trader). So the price of the stock goes up at a rate μ [38]. The trader's goal is thus to maximize B, the perceived dollar benefit from trading [39],

$$B = V\left(M - \Delta p - \mu T\right). \tag{14}$$

The validity of (14) can be seen by observing that the trader expects the asset to have excess returns M, but after a delay of T, the price has increased by μT, so that the remaining mispricing is only $M - \mu T$. The total dollar profit per share B/V is the realized excess return $M - \mu T$ minus the price concession Δp, which gives (14).

The optimal price impact Δp maximizes B with, according to (13), $T = aV/\Delta p$, i.e., Δp maximizes $V\left(M - \Delta p - \mu aV/\Delta p\right)$, which gives

$$\Delta p \sim V^{1/2}. \tag{15}$$

The time to execution is $T \sim V/\Delta p \sim V^{1/2}$, and the number of "chunks" in which the block is divided is $N \sim T \sim V^{1/2}$ [40,41]. This last relation gives

$$\zeta_N = 3 \tag{16}$$

which is close to the empirical value of 3.4 [42].

The model's predictions for power law exponents are robust to additional sources of noise. To see this, we observe that while the model provides a mechanism for a source of return R_{it} which has $\zeta_R = 3$, it is plausible that the observed return is $r_{it} = a_{it}R_{it} + b_{it}$, where a_{it} and b_{it} are independent stochastic disturbances. For example, b_{it} captures news that affect prices and not volume. Because of the general properties of power laws, we have $\zeta_r = \min\left(\zeta_R, \zeta_a, \zeta_b\right)$. So if our theory of R_{it} captures the first order effects (i.e. those with dominating power law, so that $\zeta_a, \zeta_b \geq \zeta_R$), its predictions for the power law exponents of the "noised up" empirical counterpart $\widetilde{r_{it}}$ will still be true, as we have $\zeta_r = \zeta_R = 3$.

The proposed model provides an explanation for the tail behavior of returns, volume and number of trades, taken individually. It also makes predictions for the *joint* behavior of these variables. We now use the *Trades and Quotes* data base [43] to test these predictions. We estimate empirically the conditional relationships between returns and several measures of trading activity. To generate their counterparts in the proposed model, we use Monte-Carlo simulations. In a given time interval Δt, there will be J "rounds" where a big trader creates one or more trades. Each round creates a volume V_j, a return $\pm V_j^{1/2}$ and a number of trades $V_j^{1/2}$. Then total volume, number of trades, and returns, will be the sum of these individual quantities [44] $V \equiv \sum_{j=1}^{J} V_j$, $N \equiv \sum_{j=1}^{J} V_j^{1/2}$ and $r \equiv \sum_{j=1}^{J} \varepsilon_j V_j^{1/2}$, with $\varepsilon_j = \pm 1$. As a measure of trade imbalance, we use N', the number of buyer-initiated trades minus the number of seller-initiated trades [45], and V', the number of shares exchanged that come from a buy order minus the number of shares exchanged that come from a sell order.

In Fig. 3(A) we compare the price impact function $E[r|V']$ produced by the model against the data, and the agreement is satisfying. We observe that aggregation over several trades flattens the shape of the price impact function.

We study a variant of Figure 3(A) in Figure 3(B), which plots $E[V'|r]$. Somewhat surprisingly, the shape is now roughly linear, a feature matched by the model. The cause is again the aggregation over several trades. Figure 3(C), $E[N|V']$, tests the model prediction that periods with large volume imbalance V' are periods where a large number N of trades are made. One sees that the data display relationships that are very similar to those predicted by the model. Figures 3(A), (B), and (C) support the view that large returns and large numbers of trades go together with large volume imbalances V'.

It is an important feature of the proposed model that large desires to trade create many trades. Indeed in the proposed model, we have

$$|N'| \sim N \tag{17}$$

in the extreme events they are dominated by one large trader who wants to trade a volume V_j, and creates a number $V_j^{1/2}$ of orders, so that N_j, N, $|N_j'|$ and $|N'|$ have the same order of magnitude, V_j. Relation $|N'| \sim N$ expresses that most trades have the same sign, i.e., move the price in the same direction, the desired direction of the large trader. This contrasts with a simple alternative model where each desire to trade would create only one trade, as in a competitive market. In this alternative model we would have

$$N' = \sum_{i=1}^{N} \varepsilon_i \tag{18}$$

where $\varepsilon_i = \pm 1$, leading to $|N'| \sim N^{1/2}$ in the tail events. We investigate relation (17) directly in Figure 3(D), which plots $E[N|N']$. We indeed find $E[N|N'] \sim |N'|$ while the alternative model (18) would counterfactually predict $E[N|N'] \sim N'^2$. Figure 3(E) probes our view that in periods of high volume imbalance, most trades change the price in the same direction, Eq. (18). Indeed, in this figure the data and the model exhibit a similar sharp transition of N'/N as V' changes sign.

Thus the proposed model agrees with the empirical facts presented in Figure 3. Existing theories need special assumptions that will make them posit relations between return, volume and number of trades that are close to the predictions of the proposed model. Further, existing theories do not account for the empirical values of the power law exponents. For instance, in the efficient market view [46,47], prices reflect news about fundamentals. Hence the efficient market theory can only assume that it is the nature of news to have an exponent equal to 3. Similarly, given that the power law exponent of mutual fund size is 1, simple models of trading would predict a volume proportional to fund size, hence an exponent of 1 for volume, in contrast to the value of approximately 1.5 found in the empirical data.

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$(\ln r_{(i)} - \ln r_{(i+1)})_{1 \le i \le n} =^d \frac{1}{3} (u_i/i)_{1 \le i \le n}$ where the u_i are independent standard exponential variables. We calculate the p value that the $\ln r_{(m)} - \ln r_{(n)}$ has a value as large as the empirical one $p_{m,n} = P^{\text{theoretical}}(\ln r_{(m)} - \ln r_{(n)} \ge \text{empirical value of } \ln r_{(m)} - \ln r_{(n)})$. All the $p-$values are above the conventional significance level of 0.05. For instance, $p_{1,2} = 0.18$, $p_{1,10} = 0.24$, $p_{2,10} = 0.43$, $p_{1,100} = 0.35$, $p_{2,100} = 0.76$.

[13] Volume is measured in number of shares. Because the volumes V_{it} (resp. q_{it}) are normalized by their average value V_i (resp. q_i), defining volume as fraction of shares outstanding or dollar value makes no material difference in the results.

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[30] This relation is more robust to aggregation. Suppose that n i.i.d. blocks are traded, with volumes $V_1, ..., V_n$, with i.i.d. signs $\varepsilon_i = \pm 1$. Prices move because of these blocks, and because of some independent source of shocks u. Then $r = u + h \sum_{i=1}^{n} \varepsilon_i V_i^{1/2}$, and $E[r^2|V] = \sigma_u^2 + h^2 V$.

[31] For each bin V_i of V, we estimate $r_i^2 = E[r^2|V = V_i]$, and Δr_i^2 the half-width of its 95% confidence interval, using the results of B. Logan, C. Mallows, S. Rice and L. Shepp, *Annals Prob.* **1**, 788 (1973). Motivated by [30], we estimate a best fit $r_i^2 = g(V_i) = 0.90 + 0.31V, R^2 = 0.98$, by weighted least squares with weights $(\Delta r_i^2)^{-2}$. We find that for all values $V_i \geq 3$, we have $|r_i^2 - g(V_i)| \leq \Delta r_i^2$. Hence we cannot reject the linear form $E[r^2|V] = g(V)$ for V large enough, here $V \geq 3$.

[32] If $r = \Delta p = hV^{1/2}$, we get indeed $P(r > x) = P\left(hV^{1/2} > x\right) = P(V > x^2/h^2) \sim x^{-2\zeta_V}$.

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[36] Hence if $0 < b < 1$ the price impact has both a permanent component and a transitory

component that is for the trader a transaction cost.

[37] This could be because each phone call by the broker takes some minimum amount of time; or, because each trader is typically paying attention to a limited number of stocks, and look this stock A only once in a while only with a Poisson probability.

[38] If a liquidity provider i front-runs with probability π, and, when he does so, has a price impact $\langle I(s_i)\rangle$, we get $\mu = \pi \langle I(s_i)\rangle f$, where f is the number of providers contacted per unit of time. The specific origin of μ does not matter, just the fact that the price increases at a rate μ while the trade is not done.

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[43] Source `www.nyse.com`.

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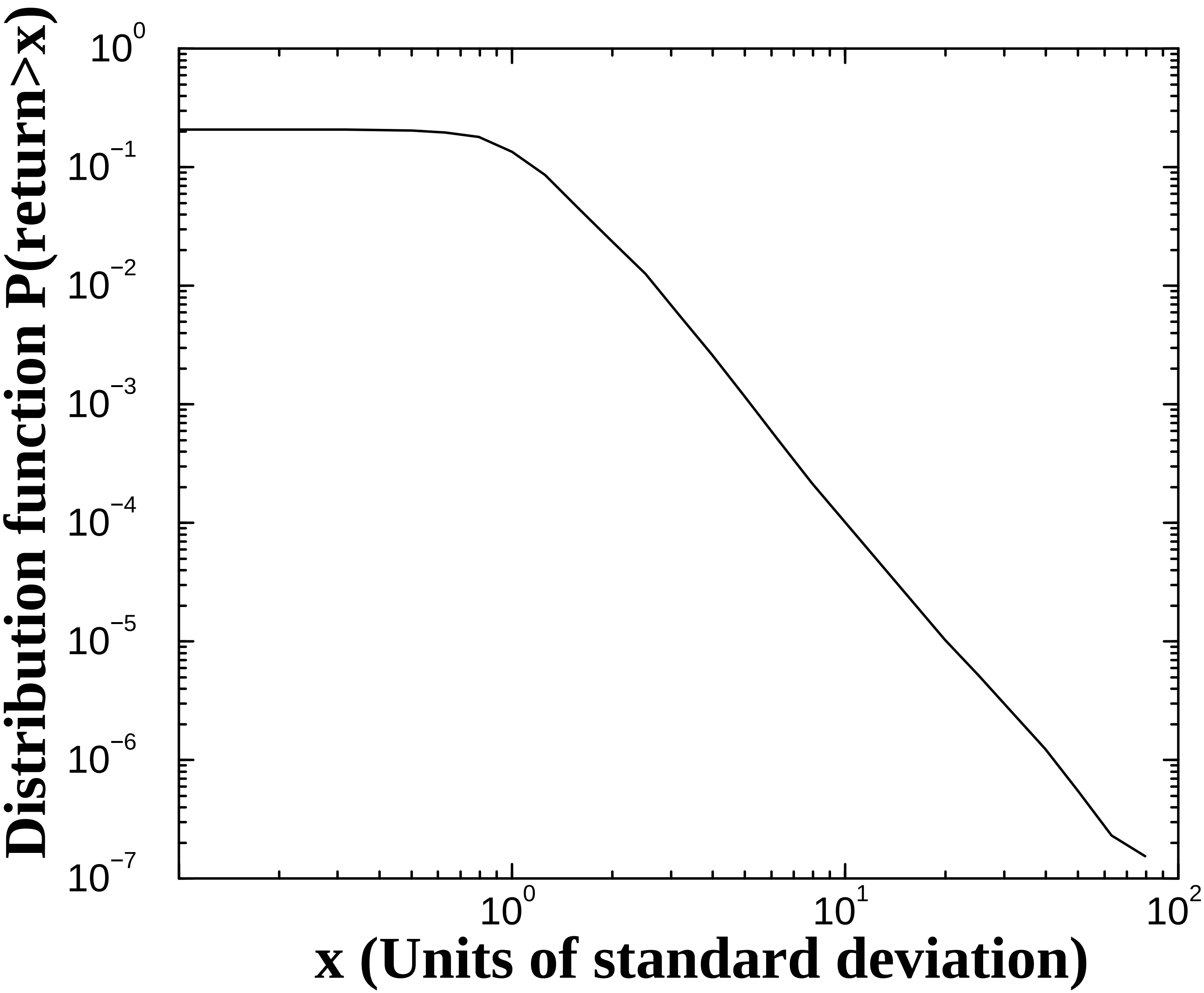
FIGURES

FIG. 1. Cumulative distributions of the normalized 15-minute absolute returns of the 1000 largest companies in the *Trades and Quotes* database for the 2-year period 1994–1995. Regression fit yields $P(|r_t| > x) \sim x^{-\zeta_r}$ with $\zeta_r = 3.1 \pm 0.1$.

FIG. 2. Conditional expectation of the squared return r^2 given the volume V. The bands represent 95% confidence intervals, using the techniques of [31]. The theory predicts a relation $E[r^2|V] = aV + b$, the "square root" price impact of volume.

FIG. 3. Conditional expectations for (A) $E[r|V']$, (B) $E[V'|r]$, (C) $E[N|V']$, (D) $E[N|N']$, and (E) $E[N'/N|V']$. We form, for each interval $\Delta t = 15$ min the following quantities r returns, V_B (resp. V_S) number of shares exchanged in a buyer (resp. seller) initiated trade, N_B number of buyer (resp. seller) initiated trades, $V' \equiv V_B - V_S$, and $N' \equiv N_B - N_S$. The left panel shows the empirical values for the 116 most frequently traded stocks in the *Trades and Quotes* database for the 2-year period 1994–1995. Variables are normalized to unit variance after setting the mean to zero; for variables such as volume for which the variance is divergent, we have normalized by the first moment instead. The right panel shows the model's predictions.



Distribution function P(return>x)
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^0
10^1
10^2
x (Units of standard deviation)

Figure 1

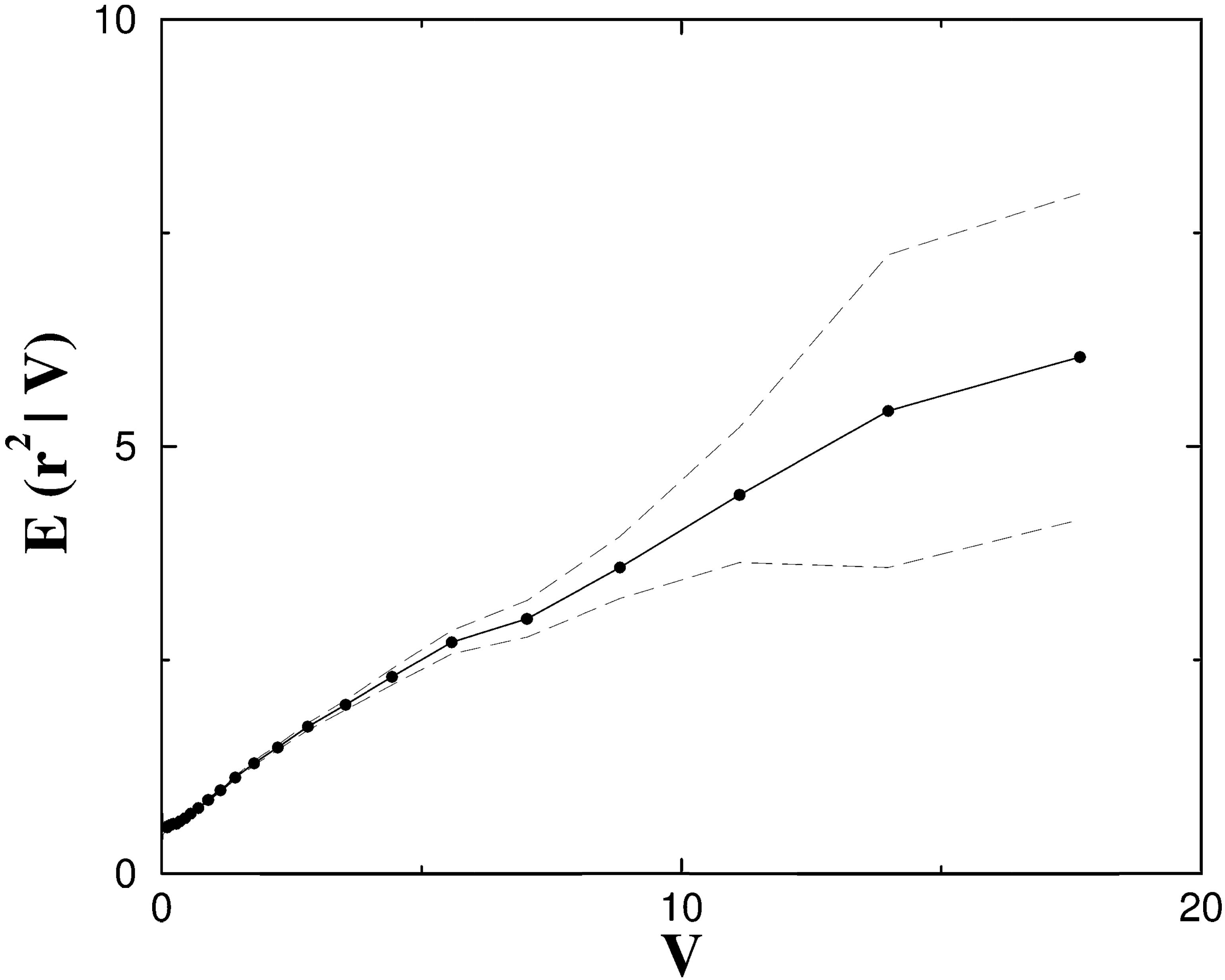


Figure 2

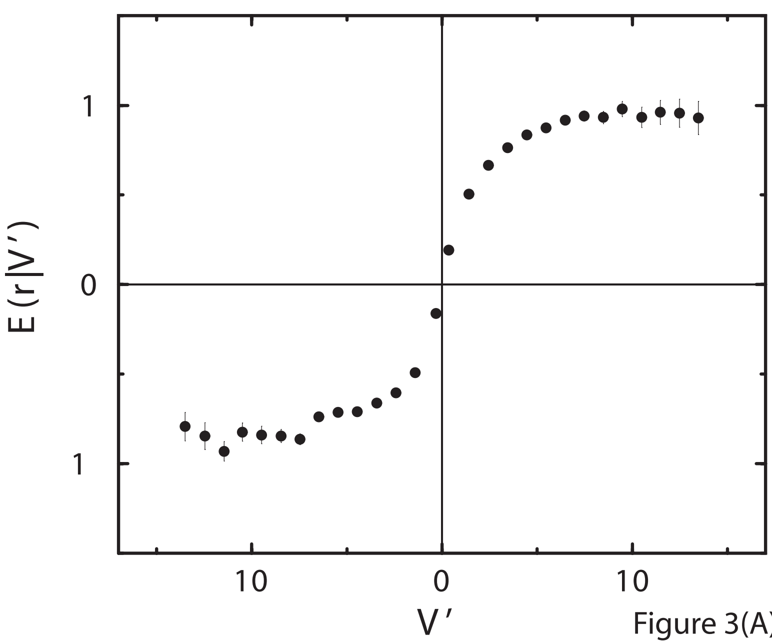


Figure 3(A)

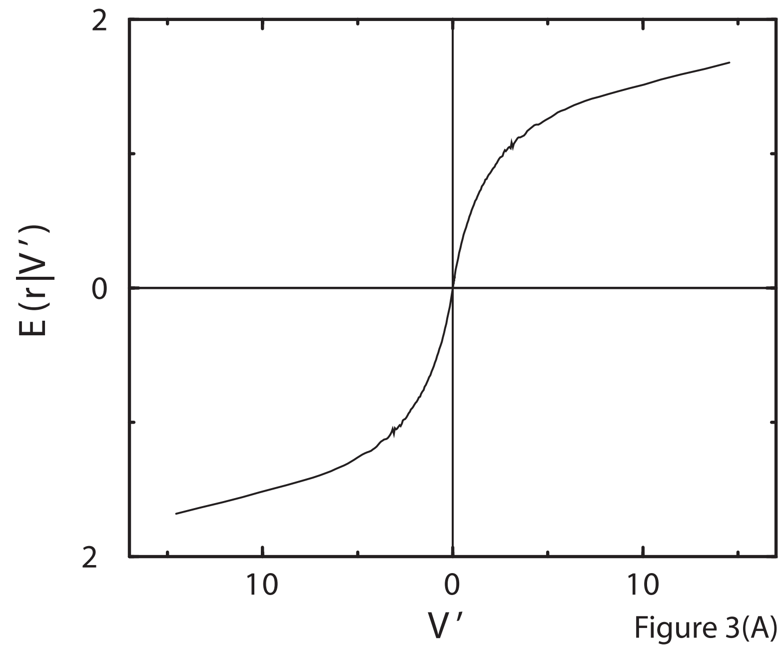
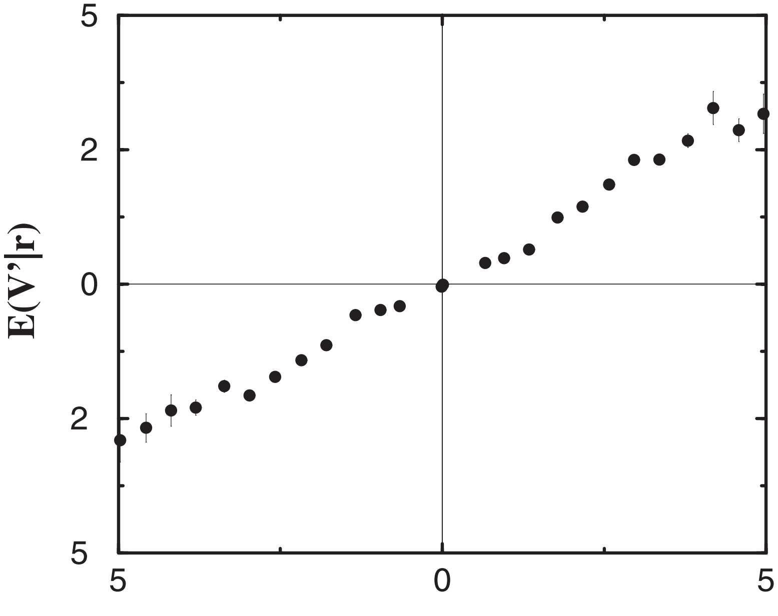


Figure 3(A)



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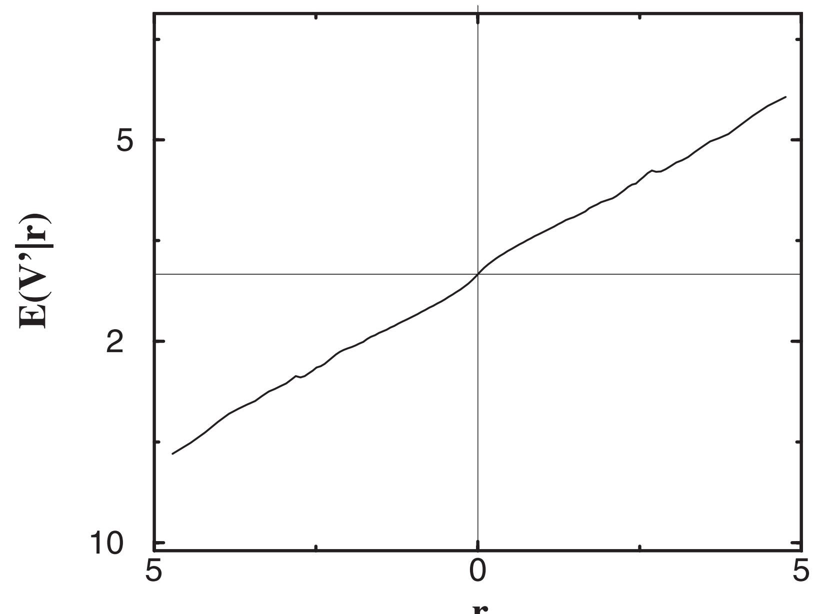


Figure 3(B)

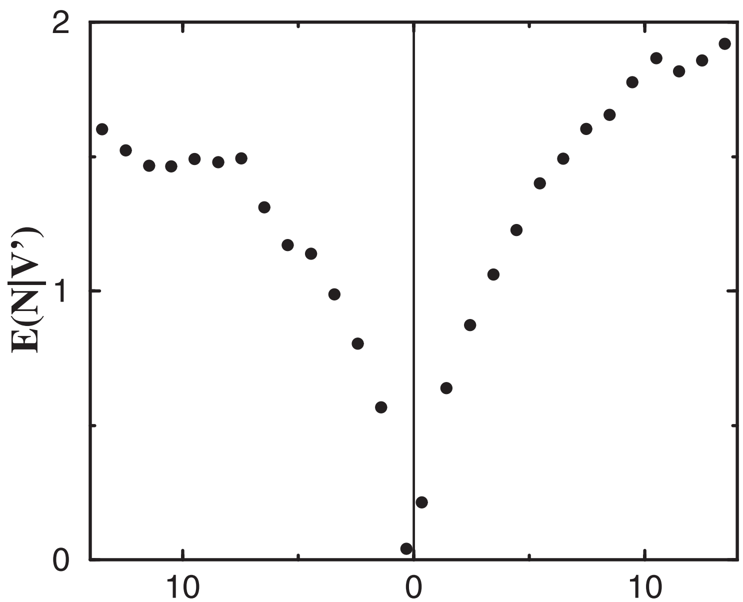


Figure 3(C)

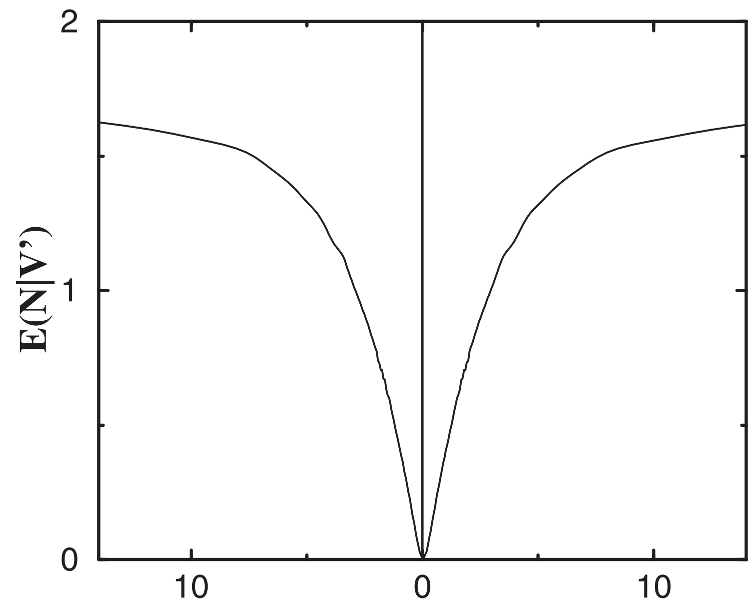


Figure 3(C)

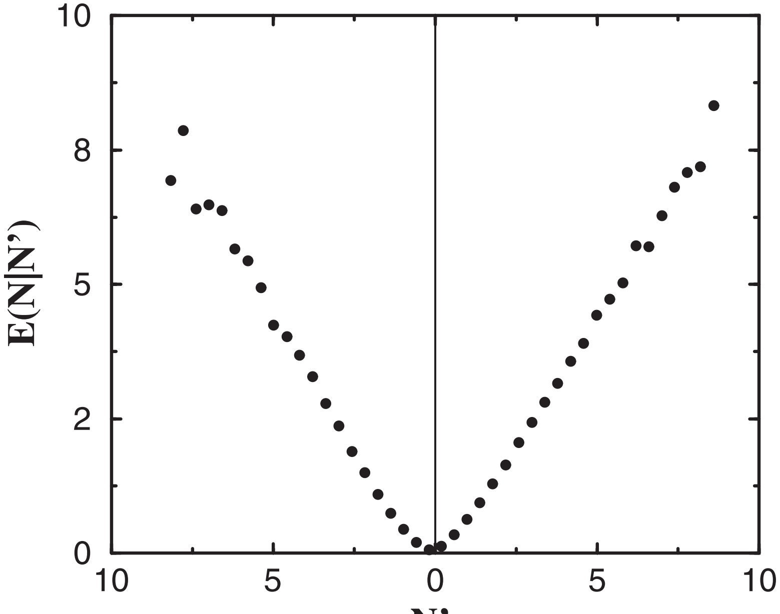


Figure 3(D)

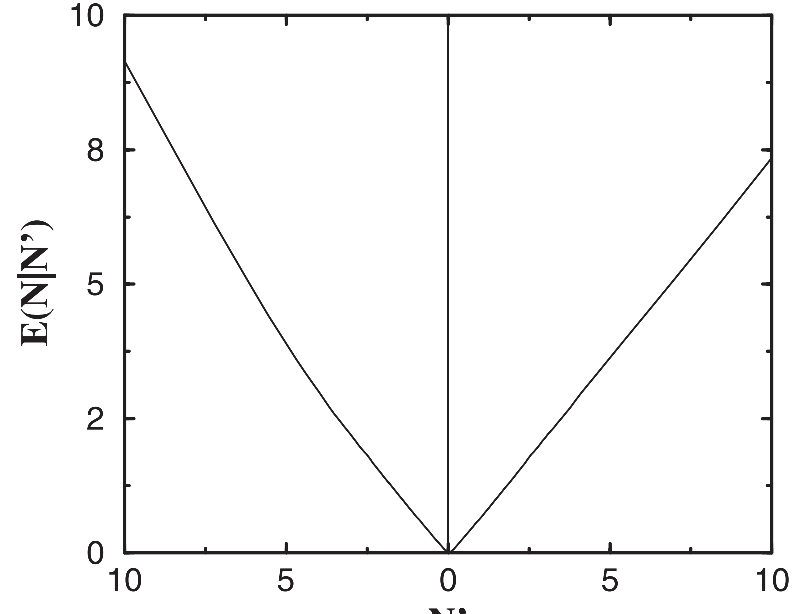
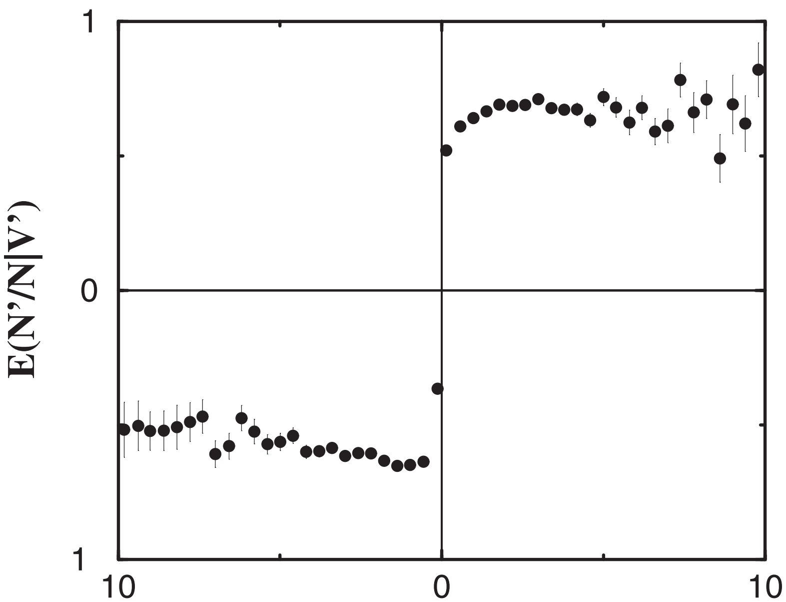


Figure 3(D)



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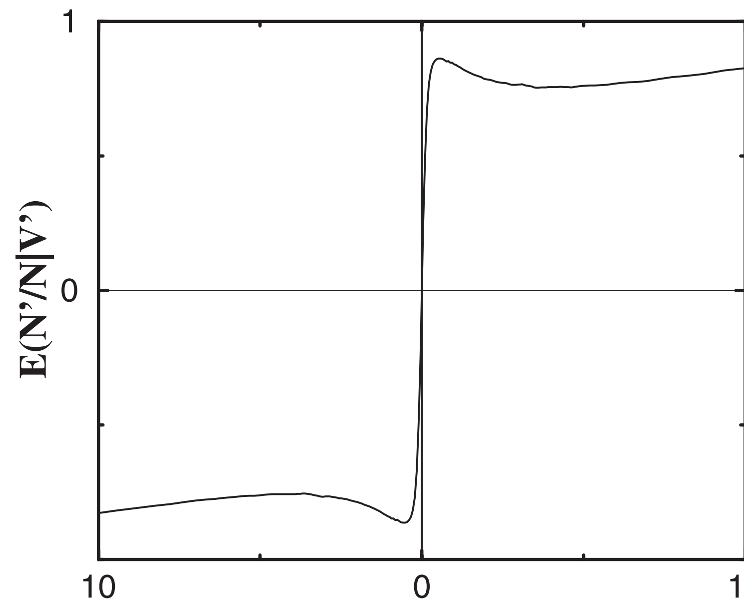

Figure 3(E)