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Owner-Occupied Housing in the Presence of Adjustment Costs: Implications for Asset Pricing and Nondurable Consumption

Marjorie Flavin
UCSD and NBER

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Department of Economics, 0508
University of California, San Diego
9500 Gilman Dr.
La Jolla, CA 92093-0508

e-mail: mflavin@ucsd.edu

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ABSTRACT

The paper generalizes the Grossman and Laroque (1990) model of optimal consumption and portfolio allocation in the context in which a durable good (or house) subject to adjustment costs is both an argument of the utility function and a component of wealth. Because the Grossman and Laroque version of the model considers a utility function in which the durable good is the sole argument, and thus abstracts completely from nondurable consumption, their analysis cannot address either a) the potential spillover effects of the adjustment costs of the durable good on the dynamics of nondurable consumption, or b) the implications for portfolio allocation of housing risk arising from variation in the relative price of housing. By incorporating a utility function that depends on nondurable consumption goods as well as the durable good, the model nests both the Grossman and Laroque model and the standard consumption-beta model.

Although the household incurs an adjustment cost when altering the holding of the durable good (or house), financial assets can be bought and sold costlessly. Consumption of the nondurable good can also be adjusted costlessly. Due to the adjustment costs associated with the durable good, the current house stock becomes a state variable that affects both the nondurable consumption choice and portfolio allocation. The analytical model shows that if the covariance matrix of asset returns is block diagonal in the sense that the return to housing is uncorrelated with the returns to financial assets, all households will hold a single optimal portfolio of risky assets, despite differences among households in terms of preferences or in terms of the state variables faced. While the state variables do not affect the composition of the optimal risky portfolio, they do affect the household's degree of risk aversion and therefore the allocation of the optimal portfolio between the optimal risky portfolio and the riskless asset. In the absence of nonnegativity constraints on the holdings of financial assets, asset pricing is consistent with the standard Capital Asset Pricing Model (CAPM).

Because nondurable consumption is costlessly adjustable, the usual first order condition equating the marginal utility of nondurable consumption with the marginal utility of wealth holds, the Euler equation holds, and asset pricing is also consistent with the consumption-beta model. Unlike the standard model in which utility is a function of a single, nondurable consumption good, the model does not imply an exact inverse relationship between risk aversion and the elasticity of intertemporal substitution. Like generalizations of the consumption-beta model which incorporate habit persistence, the housing model introduces an additional state variable to the household's optimization problem. Because the state variable moves slowly (when the state variable is interpreted as the habitual level of consumption, or habit) or is unchanged for substantial periods of time (when the state variable is interpreted as the house), both models can generate a low elasticity of intertemporal substitution of (nondurable) consumption without requiring a high degree of risk aversion.

The paper generalizes the Grossman and Laroque (1990) model of optimal consumption and portfolio allocation in the context in which a durable good (or house) subject to adjustment costs is both an argument of the utility function and a component of wealth. Because the Grossman and Laroque version of the model considers a utility function in which the durable good is the sole argument, and thus abstracts completely from nondurable consumption, their analysis cannot address either a) the potential spillover effects of the adjustment costs of the durable good on the dynamics of nondurable consumption, or b) the implications for portfolio allocation of housing risk arising from variation in the relative price of housing. By incorporating a utility function that includes nondurable consumption goods as well as the durable good as arguments, the model nests both the Grossman and Laroque model and the standard consumption-beta model.[1]

Like Grossman and Laroque, we assume that the household incurs an adjustment cost when altering the holding of the durable good (or house), although financial assets can be bought and sold costlessly. Consumption of the nondurable good can also be adjusted costlessly. When choosing a new house, the consumer takes into account the fact that housing will be fixed at the new level until the subsequent stopping time, when it is again worthwhile to incur the adjustment cost. Thus the home purchase decision is endogenous and fully rational, but, because of the adjustment cost, infrequent. Within a short interval during which stopping does not occur (i.e., the current house is not sold), the

[1] Beaulieu (1993) also develops a generalization of Grossman and Laroque (1990) in which the utility function depends on nondurable goods as well as a house. In Beaulieu's model, the relative price of the house in terms of the nondurable good is fixed. Due to the simplifying assumption that the relative price of the two goods is constant, housing is "risky" only because the household may be confronted with paying the adjustment cost; his approach does not allow for housing risk in the form of appreciation or depreciation of the value of the house relative to nondurable goods. Nevertheless, Beaulieu's analysis makes several of the points discussed below; in particular, he points out that adding the durable good (subject to costly adjustment) to the standard consumption-beta model drives a wedge between the elasticity of intertemporal substitution and the reciprocal of the coefficient of relative risk aversion. He also points out that while the Euler equation for nondurable consumption holds in the more general model, the fact that the marginal utility of nondurable consumption depends, at the household level, on the holding of the durable good, aggregation issues will preclude empirical applications of the model based on representative agent specifications.

household chooses its optimal portfolio and its level of consumption of nondurable goods conditional on its current real estate holdings. That is, due to the adjustment costs associated with the durable good, the current house stock becomes a state variable that affects both nondurable consumption choice and portfolio allocation.

The analytical model shows that if the covariance matrix of asset returns is block diagonal in the sense that the return to housing is uncorrelated with the returns to financial assets, all households will hold a single optimal portfolio of risky assets, despite differences among households in terms of preferences or in terms of the state variables faced. While the state variables do not affect the composition of the optimal risky portfolio, they do affect the household's degree of risk aversion and therefore the allocation of the optimal portfolio between the optimal risky portfolio and the riskless asset. Further, in the absence of nonnegativity constraints on the holdings of financial assets, asset pricing is consistent with the standard Capital Asset Pricing Model (CAPM).

Because nondurable consumption is costlessly adjustable, the usual first order condition equating the marginal utility of nondurable consumption with the marginal utility of wealth holds, the Euler equation holds, and asset pricing is consistent with the consumption-beta model. In essence, the implications of the traditional CAPM and the consumption-beta model exactly coincide under the assumptions of the model. Although household behavior if fully consistent with the consumption-beta model, empirical exploitation of the model's implications for risk premia by estimating the covariance of marginal utility with asset returns would require both a) the correct specification of household preferences, and b) observation of the value of the state variable (the house) at the household level. This discussion shows that it is not necessary to conclude that the consumption-beta model should be rejected on the basis of the extensive empirical evidence that the traditional CAPM outperforms the consumption-based CAPM in terms of predicting asset premia. Instead, one can interpret the poor performance of the consumption-beta model as an indication that we cannot infer the marginal utility of nondurable consumption with sufficient accuracy to exploit the implications of the model empirically.

Also problematic for the consumption-beta model is the empirical evidence that the elasticity of intertemporal substitution is fairly low. According to the standard consumption-beta model, in which utility is a function only of nondurable goods, a single parameter governing the curvature of the utility function determines the household's behavior toward risk as well as its willingness to substitute consumption intertemporally. Given the tight link between risk aversion and intertemporal substitution, parameterizations of the utility function that are consistent with the empirical evidence of a low elasticity of intertemporal substitution imply an implausibly high degree of risk aversion. However, the tight link between intertemporal substitution and risk aversion is broken in the generalized version of the model in which utility depends on a durable good subject to adjustment costs in addition to nondurable consumption. Under the plausible assumption of imperfect intratemporal substitutability between the two goods, the model can generate a low elasticity of intertemporal substitution of nondurable consumption, even though risk aversion, as manifest in portfolio decisions, is moderate or low. Other modeling approaches that break the link between the elasticity of intertemporal substitution and behavior toward risk include a) replacing preferences based on expected utility with a more general class of recursive preferences, and b) using a preference specification which exhibits habit persistence. Along several dimensions, the housing model is more closely related to generalizations of the consumption-beta model that incorporate habit persistence than to the models based on recursive preferences.

Section 1: Analytical model

In an important paper, Grossman and Laroque (1990) analyze optimal consumption and portfolio allocation in a context in which utility is derived solely from an illiquid durable good. They show that even modest transactions costs associated with adjustment of the quantity of the durable good will prevent the household from continuously equating the marginal utility of consumption with the marginal utility of wealth and therefore cause the consumption based CAPM to fail. Consumption (that is, consumption of the flow of services from the durable good) and marginal utility are constant for significant periods of time, despite fluctuations in the marginal utility of wealth, because the transactions costs preclude continuous, or even frequent, adjustment of the stock of the durable good.

Flavin and Yamashita (1999) consider a generalization of the Grossman and Laroque model in which current utility is a function of both a durable good, that is, a house, H, and a nondurable good, C. The nondurable good, C, has the ideal attributes of being infinitely divisible and costlessly adjustable. As in Grossman and Laroque, once the household purchases a particular house, no adjustments to the size (or any other attribute such as location) can be made without selling the existing house and incurring an adjustment cost proportional to the value of the house, and purchasing a new house. The implications of the model for the household's portfolio allocation problem are analyzed in Flavin and Yamashita (forthcoming); this paper considers the implications for asset pricing and for the behavior of nondurable consumption.

The household maximizes expected lifetime utility:

(1) $$U = E_0 \int_0^{\infty} e^{-\delta t} u(H_t, C_t) dt$$

Much of Grossman and Laroque (1990) is devoted to analytical and numerical characterization of the optimal stopping times, $\tau_1, \tau_2, \tau_3, \ldots$, at which the household optimally incurs the adjustment cost and reoptimizes over H. In Grossman and Laroque, the stopping times are endogenous in the sense that the household adjusts its holding of the durable good when the stochastic evolution of wealth creates too great a disparity between the existing stock of the durable and the frictionless optimal stock. In addition to the endogenous stopping times modeled by Grossman and Laroque, our version of the model permits "exogenous stopping" in the sense that the adjustment of H is caused by some event which is exogenous with respect to the evolution of wealth. Examples of exogenous events which might induce stopping are: a) death, in which the house is sold and the proceeds transferred to the heirs, b) change in job location, c) retirement, d) change in marital status, and e) acquisition or emancipation of children.

Each house is a distinct good, differing from every other house (at a minimum) in terms of its exact location. For the purposes of the analytical model, we assume that the house is not subject to physical depreciation. Using the nondurable good as numeraire, define:

(2)

P_t = house price (per square foot) in the household's current market

P_t' =house price (per square foot) in the region to which the household relocates in the next move

As in Grossman and Laroque, we abstract from labor income or human wealth, and assume that wealth is held only in the form of financial assets and the durable good. The household can invest in a riskless asset and in any of n risky financial assets. Unlike the durable good, holdings of the financial assets can be adjusted with zero transaction cost.

Thus wealth is given by:

(2) $$W_t = P_t H_t + B_t + \underline{X}_t \underline{\ell}$$

where $\underline{X}_t$ = (1xn) vector of amounts (expressed in terms of the nondurable good) held of the risky assets and $\underline{\ell}$ = (nx1) vector of ones. B_t is the amount held in the form of the riskless asset.

All financial assets, including the riskless asset, may be held in positive or negative amounts.[2]

Assuming that dividends or interest payments are reinvested so that all returns are received in the form of appreciation of the value of the asset, let b_{it} = the value (per share) of the ith risky asset, and assume that asset prices follow an n-dimensional Brownian motion process:

$$db_{it} = b_{it}\left((\mu_i + r_f)dt + d\omega_{it}\right) \quad (3)$$

The vector $\underline{\omega}_{Ft} \equiv (\omega_{1t}, \omega_{2t}, \ldots, \omega_{nt})$ follows an n-dimensional Brownian motion with zero drift and with instantaneous covariance matrix Σ, the corresponding vector of expected excess returns on risky financial assets is $\underline{\mu} \equiv (\mu_1, \mu_2, \ldots, \mu_n)$, and r_f is the riskless rate. The ith element of $\underline{X}_t$ in equation (3) is given by $X_{it} \equiv N_{it} b_{it}$ where N_{it} is the number of shares held of asset i. Since asset prices, b_{it}, are taken as exogenous, the household determines X_{it} by its choice of N_{it}. To simplify the notation, the model is expressed using X_{it} rather than N_{it} as the choice variable representing the portfolio decision.

House prices also follow a Brownian motion:

$$\begin{aligned} dP_t &= P_t\left((\mu_H + r_f)dt + d\omega_{Ht}\right) \\ dP_t' &= P_t'\left((\mu_{H'} + r_f)dt + d\omega_{H't}\right) \end{aligned} \quad (4)$$

where ω_{Ht} and $\omega_{H't}$ are Brownian motions with zero drift, instantaneous variance σ_P^2 and $\sigma_{P'}^2$, respectively, and instantaneous covariance σ_H.

Stacking equations (4) and (5), define the ((n+2)x1) vector $d\underline{\omega}_t$:

[2] Flavin and Yamashita (1999) considers the model under the alternative assumption that the household must hold nonnegative amounts of all financial assets other than mortgages.

(5) $$d\underline{\omega}_t = \begin{bmatrix} d\omega_{1t} \\ \vdots \\ d\omega_{nt} \\ d\omega_{Ht} \\ d\omega_{H't} \end{bmatrix}$$

which has instantaneous ((n+2)x(n+2)) covariance matrix Ω:

(6) $$\Omega = \begin{bmatrix} \Sigma & 0 & 0 \\ 0 & \sigma_P^2 & \sigma_H \\ 0 & \sigma_H & \sigma_{P'}^2 \end{bmatrix}$$

Note that, in order to simplify the optimization problem, the covariance matrix Ω is assumed to be block diagonal. The block diagonality of Ω implies that housing prices both in the current market and in the next market are uncorrelated with the returns to financial assets. It is important to note that the block diagonality does not require an absence of correlation in regional house prices; the covariance matrix Ω allows for an arbitrary $\sigma_H \equiv \text{cov}(P_t, P'_t)$. Because the covariance matrix does not place any restrictions on the degree of correlation of regional housing prices, the model is sufficiently general to incorporate the role of housing investment in providing a hedge against the risk arising from variability in future housing costs. For given σ_P^2 and $\sigma_{P'}^2$, the extent to which homeownership provides a hedge against future housing costs will be increasing in σ_H.

Flavin and Yamashita (forthcoming) present empirical evidence that the block diagonality assumed in equation (7) is consistent with data on US house prices and asset returns. Table 1 reports an estimate of the covariance matrix using data from Case and Shiller (1989) based on repeat sales transactions prices for four cities – Atlanta, Chicago, Dallas, and San Francisco – and returns to four financial assets – T-bills, Treasury bonds, a stock index, and fixed-rate mortgages.[3] The correlation

Table 1: Expected Returns and Covariance Matrix – Case-Shiller Price Indices

	T-Bills	Bonds	Stocks	Atlanta	Chicago	Dallas	SF
Mean Return	-0.0038	0.0060	0.0824	0.05356	0.05363	0.07196	0.09787
Standard Deviation	0.0435	0.0840	0.2415	0.04200	0.06079	0.04872	0.06540
			Covariance Matrix				
T-Bills	0.001892						
Bonds	0.002505	0.007061					
Stocks	0.000201	0.004038	0.058329				
Atlanta	0.000525	0.002038	0.003202	0.001764			
Chicago	0.000277	0.002859	0.002211	0.001006	0.003696		
Dallas	-0.000127	-0.000769	-0.000825	0.000851	0.001327	0.002373	
San Francisco	-0.000580	0.000415	-0.000223	-0.000159	0.001931	0.000796	0.004277
			Correlation Matrix				
T- Bills	1.0000						
Bonds	0.68533	1.0000					
	(0.09103)						
Stocks	0.01912	0.19897	1.0000				
	(0.12498)	(0.12251)					
Atlanta	0.41871	0.38527	0.42041	1.0000			
	(0.24271)	(0.24663)	(0.23970)				
Chicago	0.15244	0.37332	0.20051	0.39412	1.0000		
	(0.26414)	(0.24794)	(0.26001)	(0.24563)			
Dallas	-0.08701	-0.12527	-0.09341	0.41585	0.44796	1.0000	
	(0.26625)	(0.26516)	(0.26632)	(0.24306)	(0.23895)		
San Francisco	-0.29702	0.05041	-0.01879	-0.05794	0.48567	0.24987	1.0000
	(0.25520)	(0.26692)	(0.26721)	(0.26681)	(0.23362)	(0.25878)	

Standard errors are in parentheses.

Source: Flavin and Yamashita (forthcoming).

[3] Case and Shiller (1989) used data from the Society of Real Estate Appraisers to construct a Weighted Repeated Sales Index for each of the four cities by extracting the date and sales price of houses that sold twice during the sample period (1970-1986). Using weighted least squares, the change in the log of the individual house price is regressed on a set of dummy variables to obtain an index for average house appreciation in each city.

between the housing returns and financial asset returns is not statistically significantly different from zero for any of the four cities.[4] Flavin and Yamashita (forthcoming) also use house price data from the Panel Study of Income Dynamics to check the assumption of zero correlation between returns to financial assets and housing returns. With the much larger sample size of the PSID, the estimates of the correlation between housing returns and financial asset returns are essentially zero, in terms of numerical magnitude as well as statistical significance.

Let $V(H,W,P,P')$ denote the supremum of household expected utility, conditional on the current values of the state variables (H,W,P,P'). At every moment, the household considers whether the disparity between the current size house and the frictionlessly optimal size house is sufficiently large to justify paying the transactions cost and reoptimizing over the house. House sales of this type are referred to as "endogenous" sales because they are triggered by the evolution of wealth, and therefore endogenous to the model. At time t=0, the Bellman equation is:

$$(7) \qquad V(H_0,W_0,P_0,P_0') = \sup_{\{C_t\},\{\underline{X}_t\},\tau} E\left[\int_0^{\tau} e^{-\delta t} u(H_0,C_t)dt + e^{-\delta\tau} V(H_\tau,W_\tau,P_\tau,P_\tau')\right]$$

where τ denotes the next stopping time. If it is optimal to stop immediately, i.e., $\tau=0$, the household sells the current house, pays the transactions cost, and buys a new house.[5] Thus the home purchase decision is endogenous and fully rational, but, because of the transactions cost, infrequent.

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[4] With the partial exception of San Francisco, the correlation between housing returns in two different cities is positive and approaching statistical significance at the 5% level. For all city pairs, the correlation of housing returns is statistically significantly different from unity, and for most pairs is about 0.4. Note, however, that the assumption of block diagonality does not require the absence of correlation of housing returns in different regions (i.e., does not require $\sigma_H = 0$).

[5] For the model studied by Grossman and Laroque, which considers only endogenous stopping, a conservative estimate of transactions costs equal to 5% of the value of the house implies that the average time between house purchases is 20 to 30 years.

Suppose that at time $t=0$, the household decides that it is not optimal to sell the house immediately (i.e., $\tau \neq 0$), so that the value function $V(H_0, W_0, P_0, P_0')$ strictly exceeds the maximum value attainable if the house were sold immediately. By continuity, there must be a time interval (0,s) sufficiently small that the possibility of stopping within that small interval can be ignored.[6] During such a time interval, wealth evolves according to:

(9) $$dW_t = \left[P_t H_0(\mu_H + r_f) + \underline{X}_t(\underline{\mu} + r_f) + r_f B_t - C_t\right]dt + \underline{X}_t d\underline{\omega}_{Ft} + P_t H_0 d\omega_{Ht}$$

or, rewriting in order to eliminate the term representing risk-free bonds,

(10) $$dW_t = \left[r_f W_t + P_t H_0 \mu_H + \underline{X}_t \underline{\mu} - C_t\right]dt + \underline{X}_t d\underline{\omega}_{Ft} + P_t H_0 d\omega_{Ht}$$

and the Bellman equation is:

(11) $$V(H_0, W_0, P_0, P_0') = \sup_{\{\underline{X}_t\},\{C_t\}} E\left[\int_0^s e^{-\delta t} u(H_0, C_t)dt + e^{-\delta s} V(H_0, W_s, P_s, P_s')\right]$$

subject to the budget constraint (10), and the process for house prices (5). Subtracting $V(H_0, W_0, P_0, P_0')$, dividing by s and taking the limit as $s \to 0$ gives:

(12) $$0 = \lim_{s\to 0} \sup_{\{\underline{X}_t\}\{C_t\}} E\left[\frac{1}{s}\int_0^s e^{-\delta t} u(H_0, C_t)dt + \frac{1}{s}\left(e^{-\delta s} V(H_0, W_s, P_s, P_s') - V(H_0, W_0, P_0, P_0')\right)\right].$$

Evaluating the integral and using Ito's lemma, equation (12) can be rewritten as:

(13) $$\begin{aligned} 0 = \sup_{\underline{X}_0, C_0} \Big\{ & u(H_0, C_0) - \delta V(H_0, W_0, P_0, P_0') + \frac{\partial V}{\partial W}(r_f W_0 + P_0 H_0 \mu_H + \underline{X}_0 \underline{\mu} - C_0) \\ & + \frac{\partial V}{\partial P} P_0 \mu_H + \frac{\partial V}{\partial P'} P_0' \mu_{H'} + \frac{1}{2}\frac{\partial^2 V}{\partial W^2}\left(\underline{X}_0 \Sigma \underline{X}_0^T + P_0^2 H_0^2 \sigma_P^2\right) + \frac{1}{2}\frac{\partial^2 V}{\partial P^2} P_0^2 \sigma_P^2 + \frac{1}{2}\frac{\partial^2 V}{\partial P'^2} P_0'^2 \sigma_{P'}^2 \\ & + \frac{\partial^2 V}{\partial W \partial P} P_0^2 H_0 \sigma_P^2 + \frac{\partial^2 V}{\partial W \partial P'} P_0 P_0' H_0 \sigma_H + \frac{\partial^2 V}{\partial P \partial P'} P_0 P_0' \sigma_H \Big\} \end{aligned}$$

Nondurable consumption satisfies the usual first order condition:

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[6] See Grossman and Laroque (1990), page 31.

(14) $$\frac{\partial u}{\partial C}=\frac{\partial V}{\partial W}$$

The vector of holdings of risky financial assets, $\underline{X}_0$, is chosen according to:

(15) $$0=\text{constant}+\frac{\partial V}{\partial W}\left(r_f W_0 + P_0 H_0 \mu_H - C_0\right)+\frac{1}{2}\frac{\partial^2 V}{\partial W^2}P_0^2 H_0^2 \sigma_P^2$$

$$+\sup_{\underline{X}_0}\left\{ \frac{\partial V}{\partial W}\underline{X}_0\underline{\mu}+\frac{1}{2}\frac{\partial^2 V}{\partial W^2}\underline{X}_0\Sigma\underline{X}_0^T \right\}$$

Thus from the first order condition for $\underline{X}_0$, the optimal holding of risky financial assets, stated as shares of wealth, is given by:

(16) $$\left(\frac{1}{W_0}\right)\underline{X}_0^T=\left[\frac{-\dfrac{\partial V}{\partial W}}{\dfrac{\partial^2 V}{\partial W^2}W_0}\right]\Sigma^{-1}\underline{\mu}$$

and the amount held of the riskless asset is:

(17) $$B_0 = W_0 - P_0 H_0 - \underline{X}_0 \underline{\ell}$$

In equation (16), the expression in square brackets is the reciprocal of the coefficient of relative risk aversion:

(18) $$RRA \equiv -\frac{\dfrac{\partial^2 V(W_t,H_t,P_t,P_t^{'})}{\partial W_t^2}}{\dfrac{\partial V(W_t,H_t,P_t,P_t^{'})}{\partial W_t}}W_t > 0$$

Note that, because the household's degree of risk version depends on the curvature of the value function, behavior toward risk will depend not only on the curvature of the one-period utility function, $u(H_t,C_t)$ but also on all of the state variables. The property that risk aversion varies with the state is also a feature of the version of the model considered by Grossman and Laroque (1990). In particular, they find that the household is less risk averse (in terms of the allocation of its portfolio between the risky and risk-free asset) shortly before purchasing a new house, and relatively more risk averse immediately after purchasing a new house.[7]

Section 2: Implications for asset pricing: CAPM vs. consumption-beta

From equation (16), all consumers hold risky assets in exactly the same proportion, despite differences among households in terms of preferences (i.e., in the specification of u(H,C)) or in terms of the state variables faced. The result that there is a single optimal portfolio of risky assets held by all consumers is consistent with the more restricted version of the model considered by Grossman and Laroque (1990). Like the corresponding result in Grossman and Laroque, the result does not require a specific assumption, such as constant relative risk aversion, on the instantaneous utility function. Further, the result does not require a specific assumption about the degree of substitutability between H and C; all that is required is a general instantaneous utility function $u(H_t, C_t)$. Note, however, that the derivation of equation (16) required the assumption that the covariance matrix is block diagonal as specified in equation (7); in the absence of this restriction the Grossman and Laroque result that all consumers hold risky assets in the same proportion would not survive in the more general model. Under a completely general covariance matrix (i.e., one that is not block diagonal), risky financial assets could be used to hedge either the risk associated with the current house, or to hedge the risk associated with the variability of future house prices. However, under the assumption of block diagonality, returns to financial assets are uncorrelated with both current house prices and with future house prices. In this case, even though the risk averse household will dislike the risk created by variability in P or P', the household is unable to hedge either of these types of risk with the portfolio of financial assets. Since, under block diagonality, there is no scope for using financial assets to hedge the risk from current or future house prices, the presence of the (risky) housing asset does not create

[7] See Grossman and Laroque (1990), pages 38-40.

any "distortion" of the optimal portfolio of risky financial assets as compared to the risky portfolio implied by the standard model which abstracts from housing altogether. While the composition of the optimal risky portfolio does not depend on the values of the state variables, the household's degree of risk aversion in general will depend on the values of the state variables. As in the standard model, the allocation of the overall portfolio between the optimal risky portfolio and the riskless asset will depend on the household's risk aversion.

In general equilibrium, the fact that all consumers hold risky assets in the same proportion implies that risk premia are determined by the standard CAPM. To see this, note that in equation (16), the expression in square brackets is a positive scalar that, for each household j, depends on preferences and on the household's vector of state variables. For household j, denote this scalar as s(j); denote the sum of s(j) across households as $S \equiv \sum_j s(j)$. Denote the total market value of risky asset i as M_i , and define the (nx1) vector $\underline{M} \equiv (M_1, M_2, ..., M_n)$. Market clearing requires:

$$\underline{M} = S\,\Sigma^{-1}\underline{\mu} \tag{19}$$

which implies

$$\underline{\mu} = \frac{1}{S}\Sigma\underline{M} \qquad S = \frac{\underline{M}^T\Sigma\underline{M}}{\underline{M}^T\underline{\mu}} \tag{20}$$

Eliminating S, (20) implies:

$$\underline{\mu} = \left[\frac{\underline{M}^T\underline{\mu}}{\underline{M}^T\Sigma\underline{M}}\right]\Sigma\underline{M} \tag{21}$$

Expressed in more familiar notation, equation (21) can be restated as:

$$E(r_i) - r_f = \frac{\operatorname{cov}(r_i - r_f, r_m - r_f)}{\operatorname{var}(r_m - r_f)}\left[E(r_m) - r_f\right] \tag{22}$$

since the matrix products in equation (21) have the following interpretations:

$\underline{M}^T\underline{\mu}$= expected excess return on the market portfolio $= \mu_m - r_f$

(23) $\underline{M}^T\Sigma\underline{M}$ =variance of return on the market portfolio $= \text{var}(r_m - r_f)$

$\Sigma\underline{M}$ = vector whose ith element represents the covariance of the return to risky asset i with the return to the market portfolio, i.e., ith element $= \text{cov}(r_i - r_f, r_m - r_f)$

Asset prices are also consistent with the consumption-beta model; the implications of the traditional CAPM and consumption-beta model exactly coincide in this setting. Because nondurable consumption is costlessly adjustable, households continuously equate the marginal utility of nondurable consumption with the marginal utility of wealth, and satisfy an Euler equation for each of the financial assets. Denoting the marginal utility of nondurable consumption of household j in period t as:

(24) $$\lambda_{jt} = \frac{\partial u(H_t, C_t)}{\partial C_t}$$

the set of Euler equations for the time interval (t, t+s) imply:

(25) $$\begin{aligned} E(r_{it+s}) - r_f &= \frac{-\text{cov}(r_{it+s} - r_f, \lambda_{jt+s})}{E(\lambda_{jt+s})} \\ E(r_{mt+s}) - r_f &= \frac{-\text{cov}(r_{mt+s} - r_f, \lambda_{jt+s})}{E(\lambda_{jt+s})} \end{aligned}$$

Even if households are identical in the sense that they have the same preferences (i.e., the same utility function $u(H_t, C_t)$), differences across households in the values of the state variables (including H_t and W_t) will create cross-sectional dispersion in the marginal utility of nondurable consumption, λ_{jt}. Nevertheless, since all households are satisfying the Euler equations for nondurable consumption, equation (25) will hold for all households. Rewriting equation (25) to express the risk premium on an individual risky asset in terms of the risk premium on the market portfolio gives:

$$(26) \qquad E(r_{it+s}) - r_f = \frac{\text{cov}(r_{it+s} - r_f, \lambda_{jt+s})}{\text{cov}(r_{mt+s} - r_f, \lambda_{jt+s})}[E(r_{mt+s}) - r_f]$$

Comparing equations (22) and (26), the model implies that

$$(27) \qquad \beta_i \equiv \frac{\text{cov}(r_{it+s} - r_f, \lambda_{jt+s})}{\text{cov}(r_{mt+s} - r_f, \lambda_{jt+s})} = \frac{\text{cov}(r_{it+s} - r_f, r_{mt+s} - r_f)}{\text{var}(r_{mt+s} - r_f)}$$

Thus the basic implication of the model is that risk premia on individual assets will be proportional to the risk premium on the market portfolio, and that an asset's beta can be expressed either in terms of the covariance of the asset's return with the marginal utility of consumption or in terms of the covariance of the asset's return with the market portfolio; in theory, equation (27) provides two alternative ways of obtaining empirical estimates of a unique vector of betas. In practice, of course, either approach to estimating the betas is compromised by serious measurement issues. In terms of the traditional CAPM approach, we do not observe the return on the complete market portfolio and consequently rely on a proxy (such as the return to a broad stock index). In terms of the consumption-beta approach, we do not directly observe the marginal utility of nondurable consumption at the household level, λ_{jt}. To estimate the risk premia using the consumption-beta approach in (27), we would need a) to make an assumption about the functional form of the utility function $u(H_t, C_t)$ and b) to have data on the state variable H_t as well as data on nondurable consumption at the household level. Thus it is not necessary to conclude that the consumption-beta model should be rejected on the basis of the extensive empirical evidence that the traditional CAPM outperforms the consumption-based CAPM in terms of predicting asset premia. In this setting, households behave in exactly the manner prescribed by the consumption-beta model. Instead, one can interpret the poor empirical performance of the consumption-beta model as an indication that, in practice, we cannot infer the marginal utility of nondurable consumption with sufficient accuracy to exploit the empirical implications of the model.

None of the preceding analytical results depend on any specific assumptions on the functional form of the utility function. In order to study the relationship between risk aversion and intertemporal substitution, we now assume that the instantaneous utility function is of the CES form:

$$(28) \quad u(H_t, C_t) = \frac{\left[\gamma H_t^{\alpha} + (1-\gamma) C_t^{\alpha}\right]^{\frac{1-\rho}{\alpha}}}{1-\rho} \qquad \alpha \leq 1, \quad 0 \leq \gamma \leq 1, \qquad 1 \neq \rho > 0$$

The parameter ρ determines the degree of curvature of the utility function with respect to the composite good. In the special case in which nondurable consumption is the sole argument of the utility function, the household's relative risk aversion would be completely determined by the curvature parameter (i.e., if $\gamma = 0$ then RRA=ρ). However, in the model considered here, the coefficient of relative risk aversion does not, in general, coincide with the parameter governing the curvature of the one-period utility function. For this reason, the parameter ρ will be referred to as "the curvature parameter" rather than "the risk aversion parameter". The parameter α governs the degree of intratemporal substitutability between housing and nondurable consumption goods. If α=1 the two goods are perfect substitutes. The limiting case of $\alpha \rightarrow -\infty$ implies Leontief preferences, i.e., no substitutability between goods:

$$(29) \quad u(H_t, C_t) = \frac{\left[\min(\gamma H_t, (1-\gamma) C_t\right]^{1-\rho}}{1-\rho}$$

For the CES utility function given in equation (28), the result that the value function $V(H_t, W_t, P_t, P_t')$ is homogeneous of degree (1-ρ) in H and W can be established by an argument parallel to that in Theorem 2.1 of Grossman and Laroque (1990). Further, if t is a date at which the household moves, the value function at t depends on H_t only to the extent that H_t is a component of wealth; H_t does not appear in the value function as a separate state variable. Thus, if t is a stopping time, the value function can be written in the form:

$$(30) \qquad V(H_t, W_t, P_t, P_t') = k(P_t, P_t')W_t^{1-\rho}$$

where $k(P_t, P_t')$ is a function of house prices which does not depend on W_t, which implies

$$(31) \qquad RRA \equiv -\frac{\dfrac{\partial^2 V(W_t, H_t, P_t, P_t')}{\partial W_t^2}}{\dfrac{\partial V(W_t, H_t, P_t, P_t')}{\partial W_t}} W_t = \rho$$

To summarize, it is the curvature of the value function that reflects preferences toward risk and determines the composition of the optimal portfolio. If we assume that the instantaneous utility function has curvature with respect to the composite good as defined by the parameter ρ in equation (29), and t is a stopping time (i.e., t is one of the infrequent moments when the household is reoptimizing over H), then the coefficient of relative risk aversion coincides exactly with ρ in these circumstances. If t is not a stopping time, risk aversion will depend on the values of the state variables as well as parameters such as ρ. The state-dependence of preferences toward risk and portfolio composition is examined through numerical simulation by Grossman and Laroque (1990) in the context of their simplified version of the model.

Section III: The elasticity of intertemporal substitution of nondurable consumption

In the standard version of the consumption-beta model, it is assumed that 1) the lifetime utility function is determined within an expected utility framework, 2) the one-period utility function is time-separable, and 3) the utility function depends solely on a single, costlessly adjustable nondurable good. Under these assumptions, the curvature of the utility function immediately determines both risk aversion and the elasticity of intertemporal substitution. Further, it is an implication of the standard version of the model that the elasticity of intertemporal substitution is the reciprocal of the coefficient of relative risk aversion. In response to the large body of empirical work that demonstrated consistent rejection of this implication of the standard model, various authors have considered more general versions of the model by 1) relaxing the assumption of expected utility or by 2) relaxing the assumption of time-separable preferences. In both of these more general specifications, the model no longer has the implication that the elasticity of intertemporal substitution is equal to the reciprocal of the coefficient of relative risk aversion. In our model, we maintain assumptions 1) and 2) by using a standard time-separable expected utility framework, and consider the implications for the elasticity of intertemporal substitution after relaxing assumption 3) by making the utility function depend on the durable good subject to adjustment costs as well as nondurable consumption.[8]

Consider a time t=0 such that, due to the transactions costs, it is not optimal for the household to sell the house immediately and reoptimize over H. Having decided, for the moment, to maintain the level of housing services at H_0, the elasticity of intertemporal substitution (EIS) of nondurable consumption is:

[8] The point that an adjustment cost associated with durable goods will in general effect the dynamics of nondurable consumption was made in Bernanke (1985). In the context of the Permanent Income model based on quadratic preferences, Bernanke allows utility to depend on durable goods as well as nondurable goods in a potentially nonseparable way. For tractability, he models the adjustment costs associated with durable goods as a quadratic function of the change in the stock of durables; given the quadratic specification of preferences and adjustment costs, he is able to derive and estimate closed form solutions for the behavior of durable and nondurable consumption goods. Quadratic adjustment costs will induce adjustment dynamics very different from the specification of adjustment costs used by Grossman and Laroque, in which the adjustment cost is proportional to the entire stock of the durable — under the quadratic specification the adjustment will take the form of a series of small adjustments over a number of periods, while under proportional adjustment costs, the household will maintain a given stock of the durable over a long period and ultimately make a single, large adjustment. When the durable good is interpreted as a house, as in the current paper, modeling the adjustment cost as proportional to the stock seems more plausible than the quadratic function of the change in the stock. However, in Bernanke's paper, "durable goods" refers to durable goods as defined in the NIPA classification; that is, vehicles, furniture, clothing, etc. Since "durable goods" in his model refers to a collection of smaller individual goods, as opposed to a single indivisible good, the specification of adjustment costs as quadratic in the change in the total stock of durable goods is more plausible. While Bernanke's model allows for nonseparability between durable goods (as defined by the NIPA) and nondurable goods and services, empirical estimation of the model indicates that the restriction implied by separability cannot be rejected.

(32) $$\text{EIS} \equiv \frac{-\dfrac{\partial u(H_0,C_t)}{\partial C_t}}{C_t \dfrac{\partial^2 u(H_0,C_t)}{\partial C_t^2}} = \frac{-1}{(\alpha-1)\left[1-\dfrac{(1-\gamma)C_t^\alpha}{\gamma H_0^\alpha+(1-\gamma)C_t^\alpha}\right]-\rho\left[\dfrac{(1-\gamma)C_t^\alpha}{\gamma H_0^\alpha+(1-\gamma)C_t^\alpha}\right]}$$

Using the notation $\varpi_t \equiv \dfrac{(1-\gamma)C_t^\alpha}{\gamma H_0^\alpha+(1-\gamma)C_t^\alpha}$, note that

(33) $$0 \le \varpi_t = \frac{(1-\gamma)C_t^\alpha}{\gamma H_0^\alpha+(1-\gamma)C_t^\alpha} \le 1$$

so that the EIS takes the form:

(34) $$\text{EIS} = \frac{1}{(1-\varpi_t)(1-\alpha)+\varpi_t\rho}$$

In the special case in which the instantaneous utility function depends on nondurable consumption alone ($\tilde{u}(C_t) = \dfrac{C_t^{1-\rho}}{1-\rho}$), the EIS is ρ^{-1}, that is, we get the implication that the elasticity of intertemporal substitution is simply the inverse of the coefficient of relative risk aversion. In the more general case in which housing appears as an argument of the utility function, the elasticity of intertemporal substitution will equal the inverse of the curvature parameter ρ only if α, which reflects the intratemporal substitutability of the two goods, happens to obey the restriction $\alpha=1-\rho$.

Consider a household with preferences characterized by modest curvature of the utility function with respect to the composite good; for example, assume $\rho=2$. Because the EIS is the reciprocal of a weighted average of ρ and $(1-\alpha)$, a low value of ρ does not necessarily imply a high elasticity of intertemporal substitution. As an extreme example, consider the EIS as $\alpha \to -\infty$ (that is, the limiting cases in which the intratemporal substitutability of the two goods approaches zero). In this case, the EIS of nondurable consumption approaches zero, regardless of the value of ρ. In the opposing extreme case of perfect intratemporal substitutability between the two goods ($\alpha = 1$), the EIS will generally exceed the inverse of the curvature parameter. Thus, depending on the parameter governing intratemporal substitution, the two good model implies the following relationship between the elasticity of intertemporal substitution and the curvature of the utility function:

$$(35)\qquad \begin{aligned} &\text{EIS} \to 0 && \text{for} \quad \alpha \to -\infty \\ &\text{EIS} < \rho^{-1} && \text{for} \quad \alpha < 1-\rho \\ &\text{EIS} = \rho^{-1} && \text{for} \quad \alpha = 1-\rho \\ &\text{EIS} \geq \rho^{-1} && \text{for} \quad \alpha = 1 \end{aligned}$$

Thus even if the curvature of the utility function with respect to the composite good is modest (i.e., ρ is small), the plausible assumption of imperfect intratemporal substitutability between the two goods can easily generate a low elasticity of intertemporal substitution of nondurable consumption.

Section IV: Comparison with the recursive utility framework of Epstein and Zin

In an important series of papers, Epstein and Zin (1989, 1991) show that the tight link between risk aversion and intertemporal substitution which characterizes the standard model can be broken by replacing the time-additive expected utility preference model with a generalized model of preferences based on Kreps and Porteus (1978). In this approach, preferences toward risk are embodied in the function μ ($\mu_{t+1} = \mu\left(U_{t+1} \middle| I_{t+1}\right)$) which relates the conditional distribution of next period's value function (U_{t+1}) to its certainty equivalent, μ_{t+1}. Lifetime utility, U_t, is then defined with an aggregator function, θ:

$$(36)\qquad U_t = \theta\left(C_t, \mu_{t+1}\right)$$

as a function of current consumption and the certainty equivalent of lifetime utility in t+1. Preferences regarding intertemporal substitution are embodied in the aggregator function and may be varied independently of preferences toward risk. For example, the functional form for the aggregator function suggested in Epstein and Zin (1991) is:

$$(37) \qquad U_t = \left[(1-\beta)C_t^{\lambda} + \beta\mu_{t+1}^{\lambda}\right]^{\frac{1}{\lambda}} \qquad 0 \neq \lambda < 1$$

The specification in (37) implies that, when future consumption is deterministic, the elasticity of intertemporal substitution is constant and equal to:[9]

$$(38) \qquad \text{EIS} = \frac{1}{1-\lambda}$$

After estimating Euler equations generated by their model under a general parameterization of risk aversion, Epstein and Zin conclude that "Risk preferences do not differ statistically from the logarithmic specification."[10] Under the log specification for risk aversion, their recursive utility model implies[11]

$$(39) \qquad E_t \Delta \ln C_{t+1} = \frac{1}{1-\lambda}\left[\ln\beta + E_t \ln(1+R_{t+1})\right]$$

The expected utility model can be obtained as a special case of Epstein and Zin's generalized preference model if (for the case of logarithmic risk preferences), $\lambda = 0$. However, their empirical work suggests that $\lambda < 0$, so that the expected utility model is rejected. Depending on the value of λ the recursive utility framework implies that consumers favor early resolution of uncertainty ($\lambda > 0$), favor late resolution of uncertainty ($\lambda < 0$) or are indifferent to the timing of the resolution of uncertainty ($\lambda = 0$). Thus the estimates of the EIS reported by Epstein and Zin (1991) are interpreted as evidence that the expected utility framework can be rejected as too restrictive, and, further, that consumers prefer late resolution of uncertainty.

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[9] See Epstein and Zin (1991), page 266.
[10] ibid, page 282.
[11] ibid, page 269.

In essence, Epstein and Zin maintain the assumption of a single, nondurable good, and dispense with expected utility, then interpret the small empirical estimates of the elasticity of intertemporal substitution as evidence that preferences are inconsistent with the expected utility framework. In my approach, the expected utility framework is maintained, and the estimates of a low elasticity of intertemporal substitution are interpreted as a consequence of imperfect substitutability between housing and the nondurable consumption good. To take a particularly simple case, consider the special case in which the curvature of the utility function with respect to the composite good is given by the log specification. In this case, the utility function (28) becomes:

$$u(H_t, C_t) = \frac{1}{\alpha}\ln\left[\gamma H_t^{\alpha} + (1-\gamma)C_t^{\alpha}\right] \tag{40}$$

and the elasticity of intertemporal substitution is:

$$EIS = \frac{1}{1-\alpha(1-\varpi_t)} \tag{41}$$

In this approach, one can maintain both the expected utility framework and the log specification of the utility function and interpret small empirical estimates of the EIS as an indication that housing and nondurable consumption goods are imperfect substitutes (i.e., $\alpha < 0$).

Section V: Comparison with models of habit persistence

Models of habit persistence provide another approach for breaking the tight relationship between the elasticity of intertemporal substitution and risk aversion. In particular, papers by Abel (1990), Campbell and Cochrane (1998, 1999), Constantinides (1990), Ferson and Constantinides (1991), Heaton (1995) and Sundarson (1989) examine the macroeconomic and asset pricing implications of a variety of models incorporating preferences which exhibit habit persistence. Of the many models of habit persistence contained in the literature, the model posed by Constantinides (1990) provides a convenient comparison to the housing model, as Constantinides considers the effects of habit persistence in an infinite horizon, continuous time model that, like the housing model, incorporates a portfolio decision and abstracts from labor income. That is, Constantinides considers the lifetime utility function:

(42) $$U = E_0 \int_0^{\infty} e^{-\delta t} u(c_t, h_t) dt$$

where h_t is the state variable representing the "habit".

In particular, Constantinides parameterizes the instantaneous utility function as:

(43) $$u(c_t, h_t) = \frac{(c_t - h_t)^{1-\rho}}{1-\rho}$$

and models "habit" as an exponentially weighted distributed lag of past consumption:

(44) $$h_t = e^{-at} h_0 + b \int_0^t e^{a(s-t)} c_s ds$$

In this specification, the consumption habit, h_t, can be interpreted as the subsistence level of consumption in the sense that marginal utility becomes infinite at $c_t = h_t$. For the parameter values $h_0 = b = 0$, the model specializes to the standard time-separable case.

In the general case, the value function depends on the state variable representing habit, h_t, as well as wealth, W_t:

(45) $$V(W_t, h_t) = \max_{\alpha(s), c(s)} E_t \int_t^{\infty} e^{-\delta s} \frac{(c_s - h_s)^{1-\rho}}{1-\rho} ds$$

where $\alpha(s)$ denotes the fraction of the portfolio invested in the risky asset. Defining relative risk aversion as in equation (18) as the curvature of the value function, Constantinides shows that risk aversion is not constant across time, as in the time-separable case, but instead varies with the ratio of the two state variables, $\frac{h_t}{W_t}$. That is, the degree of relative risk aversion at time t is given by:

$$\text{(46)}\qquad RRA = -\frac{\dfrac{\partial^2 V(W_t,h_t)}{\partial W_t^2}}{\dfrac{\partial V(W_t,h_t)}{\partial W_t}} W_t = \frac{\rho}{1-\left[\dfrac{h_t}{W_t(r+a-b)}\right]}$$

where a and b are the parameters which govern the strength of habit persistence and r denotes the risk-free rate of return.[12] Thus in contrast to the time-separable case, in which relative risk aversion is constant and completely determined by the curvature of the utility function, ρ, in the presence of habit persistence the household's degree of relative risk aversion depends on the ratio of habit to wealth. For a given value of ρ, relative risk aversion is an increasing function of the ratio of habit (or subsistence) to wealth.

Like the degree of relative risk aversion, Constantinides shows that the elasticity of intertemporal substitution will be time-varying and a function of the state variable, h_t, in addition to the curvature parameter, ρ. In the habit persistence model, the elasticity of intertemporal substitution is:

$$\text{(47)}\qquad EIS \equiv \frac{\dfrac{-\partial u(c_t,h_t)}{\partial c_t}}{c_t\dfrac{\partial^2 u(c_t,h_t)}{\partial c_t}} = \frac{1-\dfrac{h_t}{c_t}}{\rho}$$

[12] The model imposes the restriction that $0<b<r+a$ so that the expression in square brackets is nonnegative.

Because h_t is the subsistence level of consumption, the specification of the utility function in equation (43) implies that $\frac{h_t}{c_t} < 1$ and therefore that habit persistence reduces the elasticity of intertemporal substitution.

To underscore the common elements of the housing model and the habit persistence approach, it is useful to consider a slightly more general CES utility function that nests the utility functions used in both models (i.e., equations (28) and (43)). That is, consider the utility function

$$(48) \qquad u(x_t, c_t) = \frac{\left[\theta_1 x_t^\alpha + \theta_2 c_t^\alpha\right]^{\frac{1-\rho}{\alpha}}}{1-\rho} \qquad \alpha \leq 1, \quad 1 \neq \rho > 0$$

where x_t is an unspecified state variable and c_t is nondurable consumption. When the state variable is identified as the house ($x_t = H_t$), the assumption that utility is increasing in both goods implies the restriction $0 < \theta_i$. If, instead, the state variable is interpreted as the habitual level of consumption ($x_t = h_t$), the plausible parameter restriction would be $\theta_1 < 0$ and $\theta_2 > 0$, since the partial derivative of utility with respect to habit is negative. For the utility function in (48), the elasticity of intertemporal substitution is:

$$(49) \qquad EIS \equiv \frac{-\dfrac{\partial u(x_t, c_t)}{\partial c_t}}{c_t \dfrac{\partial^2 u(x_t, c_t)}{\partial c_t^2}} = \frac{-1}{(\alpha - 1)\left[1 - \dfrac{\theta_2 c_t^\alpha}{\theta_1 x_t^\alpha + \theta_2 c_t^\alpha}\right] - \rho\left[\dfrac{\theta_2 c_t^\alpha}{\theta_1 x_t^\alpha + \theta_2 c_t^\alpha}\right]}$$

Constantinides assumes that utility depends on the difference between current consumption and habit, and therefore sets $\theta_1 = -1$, $\theta_2 = 1$, and $\alpha = 1$ to get equation (47). For a given value of ρ, the implied EIS is a decreasing function of the ratio of habit to current consumption. However, as Constantinides points out, under this specification, the degree of habit persistence required to explain the equity premium puzzle is extremely high; that is, to fit the data, the habitual, or subsistence level, of consumption, h_t, is, on average, equal to 80% of the level of consumption, c_t. When the state variable is interpreted as the house, the parameter assignment $\alpha = 1$ implies, implausibly, that housing and nondurable goods are perfect substitutes. Under the interpretation of equation (48) provided by the housing model, a high degree of curvature of the utility function with respect to nondurable consumption is the result of imperfect substitutability between the two goods (i.e., $\alpha < 0$). In light of the parallel implications of the two models in terms of household behavior toward risk, and in terms of the dynamics of nondurable consumption, the housing model might be thought of as a "structural"[13] model of behavior that looks like habit persistence at the aggregate level.

In summary, it is evident that the restriction that households' relative risk aversion and elasticity of intertemporal substitution are simply and exactly reciprocals of one another is not a robust implication of the basic consumption-beta framework, but instead requires three assumptions: expected utility preferences, time-separability of preferences, and an instantaneous utility function which depends only on a single, costlessly adjustable, consumption good. Relaxing any one of these three assumptions – by introducing recursive preference, habit persistence, or a durable good subject to adjustment costs – relaxes the tight link between these two crucial aspects of household behavior. While any of the three generalized models can be used to reconcile empirical evidence that households, while not highly risk averse, are nevertheless strongly averse to intertemporal substitution of consumption, the models generate different implications on other aspects of household behavior. For example, the habit persistence model and the housing model both generate consumption smoothness by introducing a state variable. In contrast to the model based on recursive preferences, the habit persistence and housing models both imply that the household's current choices (with respect to

[13] Pun intended.

nondurable consumption and portfolio composition) will depend not only on current wealth, but also on the path of wealth. That is, in a comparison of two households that are identical in terms of their preferences and current wealth but differ in terms of the historical path of wealth, the two households may differ in terms of their optimal level of nondurable consumption and their optimal portfolio composition because the households may face different values of the state variables (habit or current housing stock). In contrast, the generalized model based on recursive preferences implies that optimal consumption and portfolio composition will depend on current wealth, but not on the path of wealth.

The habit persistence model and the housing model have a long list of common features: both retain the expected utility framework, both explain the smoothness of consumption of nondurable goods by introducing an additional state variable, and both imply that a household with stable preferences will nevertheless display variation over time in the degree of relative risk aversion and the elasticity of intertemporal substitution. However, the two approaches have important differences. To generate smooth consumption and a low elasticity of intertemporal substitution, the habit persistence model locates the "rigidity", or the cause of sluggish adjustment, in household preferences. Since the crucial state variable, "habit", is not directly observable, the parameters of habit formation (equation (44)) are inferred by choosing the degree of habit persistence that is consistent with observable behavior, such as the elasticity of intertemporal substitution. In contrast, the housing model identifies costs of adjustment of the house as the source of the "rigidity", and relies on an observable state variable, the house. The implications of the model for the elasticity of intertemporal substitution depend on the degree of intratemporal substitution between housing and nondurable consumption, but, unlike the degree of habit persistence, this preference parameter can be estimated directly.

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