Fair Treatment and Inflation Persistence*

by
John C. Driscoll
Brown University and NBER
Department of Economics, Brown University
Box B, Providence, RI 02912
Email:John_Driscoll@brown.edu
Homepage: http://www.econ.brown.edu/fac/John_Driscoll
and
Steinar Holden
University of Oslo and Norges Bank
Department of Economics, University of Oslo
Box 1095 Blindern, 0317 Oslo, Norway
email: steinar.holden@econ.uio.no
homepage: http://folk.uio.no/~sholden/

First draft: 28 March 2001
This version: 10 March 2002
Preliminary version
Comments are welcome

Abstract

Most wage-contracting models with rational expectations fail to replicate the persistence in inflation observed in the data. We develop a wage-contracting model in a setting following Bhaskar (1990) in which workers are concerned about being treated fairly: they care disproportionately more about being paid less than other, identical workers than they care about being paid more than them. This model generates a continuum of equilibria, where workers want to match the wage set by other workers. If workers' expectations are based on the past behavior of wage growth, these beliefs will be self-fulfilling and thus rational. Moreover, the multiplicity of equilibria is consistent with the idea that there may be a natural range of unemployment, rather than a single natural rate. We estimate the model on quarterly U.S. data over the period 1955-2000. We find evidence that the dynamics of the Phillips curve do change below unemployment rates of 4.7 and above rates of 6.5 percent.

Keywords: Inflation persistence, coordination problems, adaptive expectations.
JEL Classification numbers: E31, E3, E5.

* The paper has benefited from comments by V. Bhaskar, Jeff Fuhrer, Greg Mankiw, Ian McDonald, Ragnar Nymoen, Andrew Oswald, and participants at seminars at Boston University, the Boston Fed, the University of Oregon, the Kiel Institute, the University of Oslo, Rutgers University and the NBER Conference on Macroeconomics and Individual Decision Making. Steinar Holden is grateful to the NBER for its hospitality when most of this paper was written.

1 Introduction

In recent years the short run aggregate supply curve has been the subject of renewed interest. Much of the theoretical literature has converged on a Taylor (1980) and Calvo (1983) type relationship, where nominal wage or price stickiness is combined with the assumption of rational expectations; the result is sometimes referred to as the New Keynesian Phillips curve. However, as has been pointed out by Fuhrer and Moore (1995) and more recently by Taylor (1999) and Mankiw (2000), these models run into serious problems when confronted with data: the models predict stickiness in prices, but not in inflation, and are thus unable to explain the inertia of actual inflation. Furthermore, as shown by Ball (1994), the models predict that anticipated disinflation is expansionary, which seems in contrast to the experiences of many countries in the 1980s and 90s. Perhaps most intriguingly, Mankiw (2000) observed that the models predict that a contractionary monetary shock causing a delayed and gradual decline in inflation should cause unemployment to fall during the transition, in stark contrast to the received wisdom of the effect of monetary contractions.

In short, macroeconomists are faced with the puzzle that the standard formulation of the short run aggregate supply curve seems to be an empirical failure. The search for a model that is both theoretically and empirically satisfying has led to a number of different suggestions. Roberts (1998) and Ball (2000) have suggested different varieties of near-rational expectation formation, essentially involving a return to models with adaptive expectations. Fuhrer and Moore (1995) take a second approach by essentially building inflation persistence into the preferences of the workers. More recently, Mankiw and Reis (2001) have proposed a third type of explanation, based on the assumption that agents have incomplete knowledge of the economy, in the sense that information about macroeconomic conditions diffuses slowly through the economy. A fourth approach, suggested by Gali and Gertler (1999) and Sbordone (1999) relates inflation to a measure of marginal costs rather than to output or unemployment. However, all these suggestions have their weaknesses, and it seems fair to say that the profession is still looking for a satisfying alternative.

In this paper, we propose a fifth type of explanation for inflation persistence, based on coordination problems. Following Bhaskar (1990), we assume that workers are concerned about fair treatment, in the sense that they care disproportionately more about being paid less than other workers than they do about being paid more than other workers. When this assumption is incorporated in a standard wage bargaining model, the result is a continuum of rational expectations equilibria, in the form of a range for the wage growth for which each wage setter will aim for the same wage growth as set by the other wage setters. We suggest that the past behavior of wage growth may serve as a focal point for expectations, implying that adaptive expectations may in fact be rational. Thus, these expectations combine the merits of both types of expectations: by being adaptive they satisfy the empirical requirements alluded to above, but by being rational they are not subject to the standard critique of adaptive expectations that agents make systematic errors.

Combining wage setting with the price setting behavior of firms, the range of possible rates of wage growth transforms into a range of equilibrium rates of output. Intuitively, if wage setters expect other wage setters to set a low nominal wage growth, each wage setter will follow the lead by the others and aggregate wage growth will be low. For a given level of nominal aggregate demand (determined by monetary policy), real aggregate demand and thus output will be high. On the other hand, if wage setters expect other wage setters to set a high money wage growth, they will also set a high wage growth. For given nominal aggregate demand, real aggregate demand and output will be low.

However, outside the range of equilibria, the labor market is sufficiently tight or slack that it dominates workers' concern for fair treatment. If the labor market is too tight, workers will aim at higher wages than others; if labor market is too slack, workers must accept lower wages than others, and in both cases the continuum of equilibria collapses to a single point. With forward-looking agents, the model then resembles Taylor (1980)'s canonical formulation.

The paper also contributes to the considerable literature on macromodels with multiple equilibria (see Cooper (1999) for a survey). In particular, the paper draws heavily on Bhaskar (1990), who also derives a range of output equilibria based on similar assumptions on preferences (but within a different wage setting framework).[1] Bhaskar mentions that the continuum of equilibria may induce inertia in nominal wage growth, but does not pursue this idea.

We confront the model with US quarterly data for unemployment and CPI inflation for the period 1955 –2000. The results are generally favorable. First, as in previous studies, and consistent with our theory, we find that inflation is highly persistent. Secondly, as emphasized by, for example, Staiger, Stock and Watson (1997), we find that the relationship between inflation and unemployment is much noisier than

[1] McDonald (1995) surveys theories and evidence on models with a range of equilibria.

standard theory would suggest. We also find some evidence that there are bounds for unemployment. Again, this is in line with our prediction that there is a range of equilibria, not a unique natural rate. However, the prediction that inflation will react strongly to output outside the range receives more mixed results: we find a strong increase in inflation for unemployment rates below the range, as suggested by the model, but we do not find the corresponding strong decrease for high unemployment.

The paper is organized as follows: section 2 presents the model and describes the resulting dynamics of inflation; section 3 discusses the empirical implications and specification for the estimates; section 4 discusses the data used and empirical results; and section 5 concludes.

2 The model

We consider an economy consisting of K symmetric firms, each producing a different good. In each firm there are L/K insiders, who bargain jointly with the firm over their wage. After the wage determination, each firm sets the price of its product, facing a downward sloping demand curve. All agents are fully aware over how the economy works, so they can predict what other agents will do at the same and later stages of the model.

Each firm j has a constant returns to scale production function $Y_{jt} = N_{jt}$, where Y_{jt} is output, N_{jt} is employment, and the t subscript indicates the time period. The real profits of the firm are

(1) $\Pi_{jt} = (P_{jt}Y_{jt} - X_{jt}N_{jt})/P_t,$

where P_{jt} is the price of output, X_{jt} is the nominal wage in firm j, and

(2) $$P_t = \left(\frac{1}{K}\sum_j P_{jt}^{1-\eta}\right)^{\frac{1}{1-\eta}} \qquad \eta > 1,$$

is the aggregate price level. The demand function facing each firm has a constant elasticity

(3) $$Y_{jt} = (P_{jt}/P_t)^{-\eta}\, Y_t/K,$$

where Y_t is aggregate output.[2]

We now turn to the payoff function of the workers. Following Bhaskar (1990) we assume that workers are concerned with fair treatment, and resent being treated worse than identical workers elsewhere. Furthermore, their dissatisfaction from being paid less than identical workers in other firms is greater than the benefit from being paid more. Formally, the utility function of the workers is non-differentiable at the wage level of other workers, so that the left-hand derivative is greater than the right-hand derivative.

There is considerable empirical support for an assumption of this kind. First, several experimental studies report asymmetric effects of pay differences on levels of satisfaction. Austin, McGinn and Susmilch (1980) employ a design in which subjects are randomly divided into three groups. In one group, the subjects read a story in which they are rewarded less pay than another identical worker; in the second, they receive equal pay; in the third, they receive more pay. The subjects are then asked to rate their

[2] Equation (3) can be derived by assuming Dixit-Stiglitz preferences; see Blanchard and Kiyotaki (1985) for an early implementation in a macroeconomic model of price setting.

satisfaction and fairness. The difference in satisfaction between the group paid more and the group paid equally is much smaller than the difference in satisfaction between the group paid equally and the group paid less. Ordonez, Connolly and Coughlan (2000) have subjects read a story in which a focal MBA graduate and one or two other comparison MBA graduates receive job offers. The number of comparison graduates and the salaries all three receive are varied across the subjects. The reported decrease in satisfaction when one of the comparison graduates has a higher offer is more than four times higher than the reported increase in satisfaction when one of the comparison graduates has a lower offer.

Second, several studies report asymmetric aversion to inequity. Loewenstein, Thompson and Bazerman (1989) report that subjects show strong aversion against disadvantageous inequality; while many subjects also exhibit aversion to advantageous inequality, this effect seems to be significantly weaker than the aversion to disadvantageous inequality. Goeree and Holt (2000) document the existence of asymmetric inequality aversion in experiments of alternating offers bargaining. Fehr and Schmidt (1999) develop a theory of inequity aversion and show that it is able to explain a number of seemingly puzzling findings in different economic situations.

Third, our assumption is also in accord with experiments on loss aversion, by Kahneman and Tversky (1979) and others. These indicate that outcomes are not perceived neutrally; rather, the value function appears to be steeper for losses than for gains.[3]

Finally, although for our results below we only require that workers have asymmetries in preferences and not outcomes (e.g. effort), it is worth noting that Akerlof (1984) reports that studies on the relationship between pay and effort generally find stronger evidence for the withdrawal of services by workers who think they are underpaid, than the positive effect on the effort of "overpaid" workers.

Formally, we assume that the payoff function of a representative worker is

$$(4) \qquad V_{jt} = V\left(\frac{X_{jt}}{P_t}, \frac{X_{jt}}{X_{Jt}}, \frac{X_{jt}}{X_{Gt}}\right) \equiv \frac{X_{jt}}{P_t}\left(\frac{X_{jt}}{X_{Jt}}\right)^{\alpha + D_{jt}\Phi}\left(\frac{X_{jt}}{X_{Gt}}\right)^{\lambda}$$

$$0 < \phi, \lambda < 1, \; 0 < \alpha + \phi < 1,$$

where X_{Jt} is the average wage of workers in the same group, X_{Gt} is the average wage of workers in the other group, and D_{jt} is a dummy variable being one if $X_{jt} < X_{Jt}$ and zero otherwise. The payoff is continuous in real and relative wages, and strictly increasing in the real wage. One would expect workers to prefer being paid more than others (α positive); however, for the sake of generality we also allow for the possibility that workers dislike inequality even if they gain themselves (α negative). In any case (4) implies workers always prefer higher wages for given wages of others. The key assumption is that the payoff is assumed to be non-differentiable at the point where wages are equal to the wages of other workers in the same group, $X_{jt} = X_{Jt}$, so that a loss

[3] Our approach is related to Lye, McDonald and Sibly (2001), who also use a model with loss aversion to derive Phillips-curve like equations. However, Lye, McDonald and Sibly relate the loss aversion to workers' own past wages rather than the wages of other workers. Furthermore, Lye, McDonald and Sibly do not focus on inflation persistence.

in payoff of a reduction in the relative wage is strictly greater than the gain in payoff of an increase in the relative wage. The non-differentiability only applies to workers in the same group. One possible justification for this difference is that the workers in different groups are different, so that the notion of equal wages for identical workers does not apply to workers in other groups. Allowing the comparison to workers in other groups to be non-differentiable would strengthen our results.

With the exception of the non-differentiability assumption, the results are robust to plausible variations in preferences. Working hours are treated as fixed, and are not included. Employment is not included in (4), which could be justified by the assumption that insiders are always employed, as variation in employment is undertaken by the firm adjusting the hiring of new workers. However, the qualitative results would hold also if workers were concerned about employment, or if working hours were allowed to vary. Moreover, the qualitative results would not be affected if the payoff were an arbitrary strictly increasing function of the real wage, rather than a linear function.

In fact, the key features of the model are also robust to much more profound variations of the model assumptions. As noted by Bhaskar (1990), the particular assumptions concerning wage setting can be relaxed: qualitatively the same features can be derived in models with individual wage bargaining or with efficiency wages considerations.

Returning to the model, it is well known that maximization of profits with a constant elasticity of demand implies that the price is set as a mark-up over the marginal cost (which is here simply the wage), that is, the first order condition of the profit maximization problem implies

(5) $P_{jt} = \mu X_{jt}$, where $\mu = \eta/(\eta-1) > 1$.

As profits are concave in P_{jt}, the first-order condition is sufficient to ensure a unique maximum, and the optimal price is independent of the price set by other firms. The indirect payoff function of the firm, as functions of the real wage and aggregate output, can be found by use of (1), (3), (5) and the production function $Y_{jt} = N_{jt}$.

(6) $\Pi_{jt} = \Pi(X_{jt}/P_t, Y_t) = (\mu-1)(X_{jt}/P_t)^{1-\eta}\mu^{-\eta}Y_t/K$

Wage setting

Wage setting takes place simultaneously in all firms. Each firm is small so the wage setters in a single firm is assumed to take the values of the aggregate variables X_t, P_t and Y_t as exogenous in the negotiations. However, the parties will take into consideration that the employment level of the firm depends on the wage level as implied by (5) and (3).

We assume that the outcome of the wage negotiations is given by the Nash bargaining solution.

(7) $$X_{jt} = \arg\max \Omega_{jt}, \quad where\ \Omega_{jt} = \left[\Pi\left(\frac{X_{jt}}{P_t}, Y_t\right) - \Pi_0\right]\cdot\left[V\left(\frac{X_{jt}}{P_t}, \frac{X_{jt}}{X_{Jt}}, \frac{X_{jt}}{X_{Gt}}\right) - V_0\right]$$

and subject to $\Pi \geq \Pi_0$ and $V \geq V_0$, and labor demand as implied by (5) and (3). As argued by Binmore, Rubinstein and Wolinsky (1986), the appropriate interpretation of the threat points of the parties depends on the force that ensures that the parties reach an agreement.

We assume that if no agreement is reached (which will not happen in equilibrium), there is a risk that negotiations break down. Let $V_{0t} = V_0(Y_t)$ be the expected payoff of the workers in this event; higher aggregate output is associated with higher aggregate employment, and thus makes it easier for the workers to find a new job, increasing the expected payoff for job losers. The expected payoff of the firm in the case of a breakdown of the negotiation is for simplicity set to zero. Inserting $V_0 = V_{0t}$ and $\Pi_0 = 0$ in the Nash maximand, and taking into consideration the non-differentiability of the payoff function of the union, the first order conditions for the Nash bargain require that the left- and right-hand derivatives of the Nash maximand satisfy the following inequalities[4]

(8) $$\frac{d\Omega^-{}_{jt}}{dX_{jt}} = \frac{d\Pi_{jt}}{dX_{jt}}\left(V_{jt} - V_{0t}\right) + \Pi_{jt}\frac{dV^-}{dX_{jt}} \geq 0 \quad .$$

and

(9) $$\frac{d\Omega^+{}_{jt}}{dX_{jt}} = \frac{d\Pi_{jt}}{dX_{jt}}\left(V_{jt} - V_{0t}\right) + \Pi_{jt}\frac{dV^+}{dX_{jt}} \leq 0 \quad .$$

In addition to (8) and (9), we know that either $X_{jt} = X_{Jt}$, or (8) or (9) hold with equality.

Let $X^-_t = X^-(X_{Jt}, X_{Gt}, Y_t, P_t)$ and $X^+_t = X^+(X_{Jt}, X_{Gt}, Y_t, P_t)$ denote the wage levels X_{jt} for which (8) and (9) respectively hold with equality. As shown in the appendix, we know that $X^-(X_{Jt}, X_{Gt}, Y_t, P_t) > X^+(X_{Jt}, X_{Gt}, Y_t, P_t)$. Furthermore, in the appendix we also show the following result

[4] To ensure existence of an interior solution, we must have that $\eta > 2 + \alpha + \phi + \lambda$.

Result 1: There exists a unique outcome X_{jt} to the wage bargaining in firm j, given by

(i)	If	$X_{Jt} > X^{-}_{t}$;	$X_{jt} = X^{-}_{t}$
(ii)	If	$X_{Jt} \in [X^{+}_{t}, X^{-}_{t}]$	$X_{jt} = X_{Jt}$
(iii)	If	$X_{Jt} < X^{+}_{t}$;	$X_{jt} = X^{+}_{t}$

The intuition is in fact fairly simply. If the average wage in the group is within the range $[X^{+}_{t}, X^{-}_{t}]$, the wage setting in firm j will match this wage. However, if the average wage in the group is higher than the upper boundary X^{-}_{t}, workers in firm j are not able to match this wage, but they are able to obtain a "high" wage X^{-}_{t} because of the high marginal utility of wages when they have lower wages than the rest of the group. If the average wage in the group is below the lower boundary X^{+}_{t}, workers in firm j will obtain more than the average wage in the group, but they nevertheless only obtain a "low" wage X^{+}_{t} because of the low marginal utility of wages when they have higher wages than average.

In a symmetric equilibrium within each group, all wage setters in the same group set the same wage, thus we can focus on case (ii) of Proposition 1 where $X_{jt} = X_{Jt}$.[5] We set $X_{jt} = X_{Jt}$ in (8) and (9), and take into consideration that firms set prices as a markup over wages as given from (5) by imposing $P_t = \mu(X_{Jt})^{1/2}(X_{Gt})^{1/2}$. Letting lower case letters denote logs, (8) and (9) can be rewritten as (the threat point $V_0(Y_t)$ is approximated by a log linear function, c.f. the appendix)

[5] As discussed by Bhaskar (1990), we cannot be sure that a symmetric equilibrium will be realized. We discuss this below.

(10) $$x_{Jt} \leq x_{Gt} - \theta^- + \frac{\gamma_0}{2} y_t$$

$\theta^+ > \theta^- > 0; \gamma_0 > 0$

(11) $$x_{Jt} \geq x_{Gt} - \theta^+ + \frac{\gamma_0}{2} y_t$$

(10) and (11) can be used to derive bounds for output that must be satisfied in an overall symmetric equilibrium, where the same wage is set for both groups. Setting $x_{Jt} = x_{Gt}$ and imposing equality in (10) and (11), we can solve for $y_t = 2\theta^-/\gamma_0 \equiv y^L$ and $y_t = 2\theta^+/\gamma_0 \equiv y^H > y^L$. It is then immediate that (10) and (11) are satisfied for any $x_{Jt} = x_{Gt}$ if and only if $y_t \in [y^L, y^H]$. It turns out that any output level in this range is consistent with an overall symmetric equilibrium in the model.

The intuition for the range of equilibrium output levels is based on the feature that the range of equilibrium wage levels in Result 1 is transformed into a range for output. As long as output is within the range $[y^L, y^H]$, workers in any individual firm are in a position to obtain the same wage as workers in other firms; no more and no less. Output above y^H is not consistent with equilibrium, because then all workers would be in a stronger position in the wage setting and they would all obtain higher wages than the others, which is clearly impossible (formally, $y_t > y^H$ would imply that $x_{Jt} > x_{Gt}$ from (11), thus violating the symmetric equilibrium condition $x_{Jt} = x_{Gt}$). Analogously, output below y^L would imply that all workers would get lower wages than the others, which is also impossible.

Note that the existence of a range depends on the non-differentiability parameterized by ϕ. If ϕ=0, so that the left- and right-hand derivatives (8) and (9) are equal, so the range for wages in Proposition 1 would collapse to a single point. Likewise, ϕ=0 would imply that $\theta^- = \theta^+$, implying that $y^L = y^H$ so that output is uniquely determined.

To complete the model, we also need to specify the demand side of the economy. For simplicity, we assume that aggregate demand (equal to aggregate output in equilibrium) is given by the quantity equation

(12) $y_t = m_t - p_t.$

where the nominal money stock m_t is set by the central bank, prior to the wage and price setting.[6]

For completeness, we include the price level in logs as

(13) $p_t = \ln \eta + ½ (x_{Jt} + x_{Gt}).$

An (overall) symmetric equilibrium, where the same wage is set for both group, is a quadruple (x_{Jt}, x_{Gt}, y_t, p_t) that satisfies: $x_{Jt} = x_{Gt}$, x_{Jt} satisfies (10) and (11) (ensuring that the wage maximises the Nash bargaining solution, and implying that x_{Gt} is within a similar interval), prices are set to maximise profits (as given by (13)), and finally the aggregate demand function (12).

The equilibrium of the overall economy can be summarized in the following result (Bhaskar, 1990, derives a similar result).

Result 2: For a given value of the nominal money stock m_t, there exists a range of overall symmetric equilibria to the economy, as characterized as follows. Let $x^L_t = m_t - \ln(\mu) - y^L$ and $x^H_t = m_t - \ln(\mu) - y^H$, where $x^H_t < x^L_t$. Then any wage level x_t in the interval $[x^H_t, x^L_t]$ is a symmetric perfect forecast equilibrium, with associated output level $y_t = m_t - \ln(\mu) - x_t$ in the interval $[y^L, y^H]$.

Note that the equilibria are Pareto rankable, as profits and employment are increasing in output, whereas the real wage is independent of output (from (13), $x_t - p_t = -\ln(\mu)$). Thus, some agents gain from higher output, whereas no agents lose from higher output. However, in a large economy with a vast number of small agents, it is not clear that agents will be able to coordinate on the best equilibrium. If some agents set low wages x^H_t to ensure the Pareto optimal equilibrium with y^H, they run the risk of getting lower wages than others, with associated loss of utility.

An interesting possible solution to the coordination problem would apply if one had a fully credible price or inflation target. If the central bank could credibly announce a price target p^E, ensuring that all agents indeed expected p^E to be realized, the Pareto optimal, high employment equilibrium could be realized by setting $m = y^H - p^E$.[7] However, in the sequel we will focus attention on a situation without a credible price or inflation target.

[6] As with equation (3), this is a standard outcome in models with Dixit-Stiglitz preferences over consumption goods and either a cash-in-advance constraint or real balances in the utility function. See Blanchard and Kiyotaki (1985) for one derivation.

[7] In independent work, McDonald and Sibly, 2001, discuss a similar idea.

Overlapping wage contracts

Now consider an overlapping contracts version of the model: Each group set wages for two periods, one group in odd periods and the other in even periods, as in the standard Taylor model. Let x_t denote the wage set in period t. The constraints derived from the wage setting now reads (replacing x_{Gt} with $(x_{t-1} + E_t x_{t+1})/2$ in (10) and (11), as well as using the definitions of y^L and y^H)

$$(14)\qquad x_t \leq \frac{1}{2}\left(x_{t-1} + E_t x_{t+1}\right) + \frac{\gamma_0}{2}(y_t - y^L)$$

$$(15)\qquad x_t \geq \frac{1}{2}\left(x_{t-1} + E_t x_{t+1}\right) + \frac{\gamma_0}{2}(y_t - y^H)$$

(14) and (15) can be rewritten as constraints on the nominal wage growth

$$(16)\qquad \Delta x_t \leq E_t \Delta x_{t+1} + \gamma_0 (y_t - y^L)$$

$$(17)\qquad \Delta x_t \geq E_t \Delta x_{t+1} + \gamma_0 (y_t - y^H)$$

As before, the wage and price setting do not uniquely pin down the dynamics of output and inflation. Although Equations (16) and (17) restrict wage growth to lie between bounds, the multiplicity of equilibria implies that, otherwise, both output and inflation depend on workers' expectations. This implies that agents cannot deduce other agents' behavior logically from the assumption that they behave rationally. In this situation it seems reasonable to assume that agents base their beliefs regarding wage growth on the past behavior of wage growth. This basic premise is common to a variety of approaches to expectation formation. Evans and Honkapohja (2001) advocate adaptive learning as a selection mechanism in situations with multiple rational expectations equilibria. Experiments on games with a multiplicity of equilibria also show that agents learn from the past behavior of other agents (Ochs, 1995). At the more general level, observing other people's behavior and making inferences on this basis is indeed how we form expectations about other people's behavior every day. If agents share this way of forming expectations, it will work as a focal point or coordination mechanism for agents' expectations.

Consider the following wage equation, representing a stylized version of existing empirical wage equations

$$(18) \qquad \Delta x_t = \beta \Delta x_{t-1} + (1-\beta)\Delta x_{t-2} + \gamma_1(y_{t-1} - y^*), \qquad \gamma_1 > 0.$$

(For convenience, we specify (18) to only include two lags, but will allow for more lags in the empirical work.) Assuming that agents have observed wage inflation to adhere to (18) in the past, it seems reasonable that they would expect wage inflation to follow (18) in the future also, as long as this is consistent with the rational expectations equilibrium of the model, ie. it satisfies the constraints given by (16) and (17). In other words, (18) would work as a focal point for the wage setting behavior. Given that agents have these beliefs, they would be self-fulfilling and thus both *ex ante* and *ex post* rational. In a situation where agents set wages on the basis of (18), realization of another equilibrium would require all agents to simultaneously switch to a different behavior. If one disregards such simultaneous switches, the unique equilibrium outcome in this situation would be that agents continue to set wages according to (18).

Note also that if a share, however small, of the agents in the economy has adaptive expectations according to (18), this will serve as a coordination mechanism so that (18) is the unique strategy consistent with rational expectations (as also observed by Bhaskar, 1990).

Given (18), y* is the unique long run equilibrium rate of output. Output cannot remain above or below y*, as this would lead to consistently increasing or decreasing nominal wage growth. Note however that y* is inherently expectations based. y* should not be interpreted as the natural rate as given by other considerations like search behavior or efficiency wages; the equivalent to these considerations are already captured in the model described in Result 2, which had a range of equilibria. If agents' expectations change, for instance they believe that the labor market has changed so that stable nominal wage growth is consistent with higher output y** rather than y*, this would imply a change in the long run equilibrium to the new level y**.

The important role of expectations in determining y* suggests that one cannot expect to find a stable relationship between output and inflation. And this is indeed the case: Staiger, Stock and Watson (1997) find considerable imprecision in the estimates of the natural rate, and there has been considerable debate over the last decade in the U.S. over whether the decline in unemployment without a corresponding rise in inflation is evidence of a decrease in the natural rate. This noisy behavior is, however, consistent with our story. The structure of the labor market, and of price and wage setting, do not pin down a tight relationship between inflation and unemployment. In our model, expectations play a large role, and one is less surprised to find more noise and fluctuations, because expectations are likely to be more volatile than other features like preferences and technology.

The implications of the adaptive expectations focal point relationship (18) is well-known, the key novelty is the implications of the constraints (16) and (17). Before turning to this, let us add prices and money to the model. Specifically, we maintain the assumption that prices are a constant unit markup over wages so that the log of the price index in period t, p_t, is the average of the contract wages negotiated in period t and period t-1.

$$p_t = \ln \eta + \tfrac{1}{2}(x_t + x_{t-1}). \qquad (19)$$

First differencing of (19) yields

$$\Delta p_t = \tfrac{1}{2}(\Delta x_t + \Delta x_{t-1}). \qquad (20)$$

(13) in logs is

$$y_t = m_t - p_t. \qquad (21)$$

To explore the implications of the bounds, consider first a temporary positive money shock, implying that (17) binds in one period, while agents expect the future wage inflation to follow (18). $E_t\Delta x_{t+1}$ can be derived by leading (18) one period

(22) $E_t\Delta x_{t+1} = \beta\Delta x_t + (1\text{-}\beta)\Delta x_{t\text{-}1} + \gamma_1(y_t - y^*)$.

Substituting out for (22) in (17), and rearranging, we obtain

$$\Delta x_t \geq \beta\Delta x_t + (1-\beta)\Delta x_{t-1} + \gamma_1(y_t - y^*) + \gamma_0(y_t - y^H) \tag{23}$$

or

$$\Delta x_t \geq \Delta x_{t-1} + \frac{\gamma_1}{1-\beta}(y_t - y^*) + \frac{\gamma_0}{1-\beta}(y_t - y^H) \tag{24}$$

Comparing (24) and (18), we note that the coefficient in front of output is considerably larger in the former case. When the bounds bind because a temporary positive money shock takes the economy above the static upper bound y^H, the effect of output on wage growth is much stronger than it is within the bounds, where wage inflation follows the adaptive focal point behavior as represented by (18).

Second, the bounds can bind because of an expected future monetary expansion. To see this as simply as possible, assume that agents expect the positive money shock to take place in period t+1. Leading (24) one period, we see that this will imply that agents expect high wage inflation in period t+1. For a sufficiently large expected positive money shock in period t+1, expected wage inflation in period t+1 will be sufficiently large that the constraint (17) binds already in period t, even if no positive monetary shock has taken place in that period. The implication will be that wage growth increases, raising prices, thus involving a contractionary effect as money growth has yet to increase in period t.

Likewise, an anticipated future monetary tightening, taking place when the economy is close to the lower output bound y^L, will imply that (16) binds and dampens wage growth, with a temporary expansionary effect. In fact, the immediate effects of an expected future monetary tightening correspond to the expansionary effect of a disinflation shown by Ball (1994) to be a prediction of the Taylor model. Note however that this effect only takes place under much more restrictive circumstances than in the Taylor model. In the Taylor model an anticipated future monetary tightening will induce output to exceed its equilibrium level. Here, the temporary expansionary effect only takes place when the monetary tightening is expected to subsequently take output down to the lower bound. Thus, this effect does not prevent that the overall effect of the monetary tightening is to induce a recession.

More generally, the existence of the bounds (16) and (17) will imply that whenever they bind, variation in expected future wage inflation will induce variation in actual inflation. Thus, whenever the bounds bind, inflation will not be determined by the persistent and adaptive behavior specified in equation (18), but will fluctuate with variation in expected future inflation. Empirically, we would consequently expect inflation to be less persistent outside the bounds.

In sum, we expect to see a Phillips curve which:

- implies inflation persistence for moderate levels of unemployment
- implies different effects for contractionary and expansionary monetary policy at low levels and at high levels of unemployment.
- implies less inflation persistence and has a different slope for low and high levels of unemployment

3 Empirical Specification

To test the predictions, we adopt a levels version of Staiger, Stock and Watson (1997)'s specification:

$$(25) \quad \pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3\pi_{t-3} + \beta_1 u_{t-1} + \beta_2 u_{t-2} + \gamma Z_t$$
$$+ \alpha^H{}_0 I^H + \alpha^H{}_1 I^H \pi_{t-1} + \alpha^H{}_2 I^H \pi_{t-2} + \alpha^H{}_3 I^H \pi_{t-3}$$
$$+ \beta^H{}_1 I^H (u_{t-1} - u^H) + \beta^H{}_2 I^H (u_{t-2} - u^H) + \gamma^H I^H Z_t$$
$$+ \alpha^L{}_0 I^L + \alpha^L{}_1 I^L \pi_{t-1} + \alpha^L{}_2 I^L \pi_{t-2} + \alpha^L{}_3 I^L \pi_{t-3}$$
$$+ \beta^L{}_1 I^L (u_{t-1} - u^L) + \beta^L{}_2 I^L (u_{t-2} - u^L) + \gamma^L I^L Z_t + \varepsilon_t,$$

where $\pi_t \equiv p_t - p_{t-1}$, I^H is a dummy variable taking the value 1 when $u > u^H$, I^L is a dummy variable taking the value 1 when $u < u^L$, and Z represents a vector of proxies for aggregate supply shocks. Note that we have invoked an Okun's Law relationship to replace output with unemployment. This is a common practice in empirical Phillips curves, and has the advantage that it is not necessary to make assumptions concerning the stationarity of output. The interaction of the dummy variables with the inflation and unemployment terms above and below the bounds allows us to test the model's prediction that the short-run dynamics of inflation and unemployment differ for low and high levels of unemployment.

Aside from the inclusion of interaction terms to allow inflation dynamics to change outside the bounds, we depart from Staiger, Stock and Watson (1997) in two ways. First, as noted above, we write the equation in levels; this allows us to more easily compare our results with previous estimates of the Phillips curve and evaluate the behavior of inflation persistence. Second, we do not explicitly attempt to estimate a time-varying natural rate of unemployment.[8] While there in general is reason to believe that parameters change over time, allowing for a time-varying natural rate in addition to the bounds would presumably be too ask for too much from the data.

We include supply shock variables for two related reasons. First, they represent deviations from the inflationary dynamics implied by the other coefficients in the model, and thus need to be controlled for to prevent omitted variable bias. Second, in principle equation (25) represents only one equation in a two-equation system (the other equation being the aggregate demand curve with unemployment substituted for output). OLS estimates of (25) will therefore suffer from simultaneous equations bias. The bias in estimating the aggregate supply coefficients will depend on the relative variance of the aggregate supply shocks to the aggregate demand shocks. By trying to proxy for the largest aggregate supply shocks, we reduce the variance of the unexplained portion of the aggregate supply shocks, and thus reduce the amount of the bias.[9]

The bounds, u^H and u^L as derived from y^H and y^L, are determined by structural parameters of the model, including how the threat point depends on output and the size of the kink in preferences. Although in principle one could calibrate the size of the bounds, or, more simply, the size of $y^H - y^L$ by picking values for the parameters, it is not clear what reasonable values for some of the parameters are. Were the bounds known, (25) would be estimable via OLS. Although they are not known, it is possible to estimate them

[8] Although the presence of the interaction terms between the dummy variables and constants does implicitly allow for this possibility during time of high and low unemployment.
[9] An alternative but complementary approach would be to estimate (22) via instrumental variables using an instrument for exogenous variations in aggregate demand or supply.

endogenously. We follow the structural break literature[10] by reestimating (25) for different values u^H and u^L and picking the specification yielding the highest value for the log-likelihood.

4 Data and Estimation Results

We use the unemployment rate for all civilians age 16 and over, seasonally adjusted, monthly, and the CPI for all urban consumers, seasonally adjusted, monthly.[11] We average the data to obtain quarterly figures, and construct an inflation measure by multiplying the percent change in the CPI by 400.

Following Ball and Mankiw (1995)[12], we use three supply shock measures:

1. FOOD, constructed by taking the difference in inflation rates between the processed foods and feeds component of the PPI (series 1300) and PPI inflation
2. FUEL, constructed by taking the difference in inflation rates between the fuel and energy component of the PPI (series 1100) and PPI inflation
3. NIXON, a dummy for the wage and price controls in the Nixon and Ford administrations introduced by Gordon (1990).

---

[10] See Quandt (1958) and Maddala and Kim (1998)

[11] We have also tried the demography-adjusted unemployment rates created by Shimer (1998), which captures the idea that the natural rate of unemployment may change over time due to changes in demographic variables (since the young are more likely to be unemployed than the old). The coefficient estimates were generally little changed, and the fit in terms of adjusted R squared worse, so we stick to the model with the ordinary unemployment series.

[12] We choose these measures of food and energy aggregate supply shocks rather than the alternative, also PPI-based, measures used in Staiger, Stock and Watson (1997) because the energy price measure used there has become significantly more volatile and highly negatively autocorrelated since 1995, suggesting a change in definition of the series.

Following Staiger, Stock and Watson (1997), we begin the sample in 1955:I; we end it in 2000:IV. Beginning the sample in 1955 implies that we avoid the effects of wage and price controls imposed during the Korean War. [13] Ball and Mankiw (1995) also find evidence for previously unrecognized aggregate supply disturbances in the early 1950s.

Tables 1 provides the main empirical results. The first column of Table 1 reports the results of estimating (25) without any bounds. The coefficients on unemployment alternate in sign but do sum to -.213, so that the Phillips curve is downward sloping, as one would hope. Also as expected, the coefficients on lagged inflation are all positive and sum to 1.004, implying inflation is persistent.

The next three columns report the results of imposing the bounds, endogenously determined by the method described above. The first column reports the coefficients on output, inflation and the supply shocks between the bounds, the next columns the additional effects below and above the bounds. We find the bounds to be at 4.7 and 6.5 percent, which correspond to the 30^{th} and 70^{th} percentiles of observed unemployment.[14] Note that the more elaborate specification allowing all coefficients to take different values outside the bounds is supported by the data, as the restrictions that are involved by the regression without bounds (column 1) is rejected in a likelihood ratio test at the one percent level.

---

[13] An additional consideration is that since monthly CPS surveys, from which current unemployment figures are derived, did not start until 1967, it is not clear that monthly or quarterly unemployment before that date is as good, or is even the same series, as data after that date.

[14] Since our technique may also pick up any possible non-linearity in the Phillips curve, we restrict the bounds to lie above and below the median value of unemployment observed. If we relax this restriction, the estimated bounds lie at 9.9 and 10.1 percent, the third-highest and second-highest unemployment rates observed.

The third and fourth columns report the interaction terms describing how the coefficients change above the bounds. First, note that the coefficients on the lagged inflation interaction terms are almost all negative- implying that inflation is less persistent both below and above the bounds. Below the bounds, the interaction terms sum to -.775 and above to -.453, which are large in magnitude and statistically significant.

Below the bounds, the interaction terms on unemployment sum to –1.378, implying that the Phillips curve is more steeply sloped. Above the bounds, however, the unemployment terms sum to .587, which is close in magnitude to the value of .608 estimated between the bounds, implying a nearly-flat Phillips curve (although highly imprecisely determined).

Table 2 evaluates the predictions that the effective of contractionary and expansionary monetary policy disturbances are different at the bounds; one set shifts the Phillips curve, the other represents shifts along the Phillips curve

We use the measure of monetary policy derived by Bernanke and Mihov (1998) from a structural VAR model of the Federal Funds market. This measure essentially purges endogenous policy movements from the Federal Funds rate. From that variable, we construct a series consisting only of contractionary changes in policy and a series consisting only of expansionary changes. The variable is defined only over the period from 1966 to 1996, where the starting date is determined by the change of the Federal Reserve's policy instrument to the Federal Funds rate. For dates outside those years, we set the value for the contractionary and expansionary variables to zero.

The first column reports results not imposing any bounds. We see that monetary expansions have a small and statistically insignificant effect on the change in inflation and thus the position of the Phillips curve. By contrast, monetary contractions have a larger and statistically significant effect. It is possible that this may be due to the existence of the same phenomena underlying the price puzzle identified by Sims (1992). Monetary policy makers may react to aggregate supply disturbances not captured by our equation. In this case contractions in monetary policy are likely to precede expected increases in inflation. Hence it is quite possible that, even within the bounds, there will be some effect of monetary policy on inflation- but only to the extent that the identification assumptions underlying the estimation of (22) are not satisfied.

The remaining two columns report the results imposing the bounds. The bounds are estimated to be at unemployment rates of 4.7 and 6.5, unchanged from Table 1; the coefficients on lagged inflation and unemployment are also not much changed. The results are largely consistent with those of our model: monetary policy contractions at the upper bound (i.e. high unemployment) and expansions at the lower bound (i.e.low unemployment) shift the Phillips curve in economically significant ways; however only the former is statistically significant. Note that much of the effect seen in the regression with no bounds appears to be due to the effect of contractionary policy at the upper unemployment bound- the effects of monetary policy on the Phillips curve are smaller and statistically insignificant in most other cases.

5 Conclusion

Standard rational-expectations formulations of the aggregate supply curve, such as those of Fischer (1978), Taylor (1980) and Calvo (1983) are unable to explain the persistence of inflation observed in the data. We develop a wage-contracting model in a setting following Bhaskar (1990) in which workers care disproportionately more about being paid less than other workers than they do about being paid more than other workers. This model generates a continuum of equilibria over a range of output. We argue that as wage setters want to match the wage growth set by others, the wage behavior in the recent past will be a natural starting point for expectations. Within the range of output, such beliefs will create a self-fulfilling prophecy; and thus be consistent with rational expectations. These beliefs will combine the attractive features of both adaptive and rational expectations; they will be consistent with key features on actual inflation series, while at the same time not allowing for agents making systematic errors.

Replacing output with unemployment, we estimate the model, including the bounds, on quarterly data over the period 1950-2000. We find that the dynamics of the Phillips curve do change at unemployment rates below 4.7 percent and above 6.5 percent. As predicted by our model, inflation seems less persistent outside the bounds. The prediction that inflation is more sensitive to changes in unemployment outside the bounds receives mixed results: we find stronger effects for low unemployment, but not for high unemployment. We also find that monetary policy contractions and expansions shift the position of the Phillips curve outside the bounds, as predicted by our model (but only significant for monetary contractions for high levels of unemployment).

At the more general level our story is perhaps easier to reconcile with the rather erratic relationship between inflation and unemployment that exists in the data than more traditional models. In such models, the erratic behavior is often explained as arising from a time-varying NAIRU. However, attempts to identify the structural determinants of the NAIRU are generally disappointing (see eg Staiger, Stock and Watson, 2001). In our model, expectations play a large role, and one is less surprised to find more noise and fluctuations, as expectations may well be more volatile than other features like preferences and technology.

We advocate coordination problems and multiple equilibria as the key to inflation persistence. In this connection one should note that similar coordination problems and multiple equilibria could also be derived in models with customer markets, and without assumptions on preferences for fair treatment. For example, Woglom (1982) showed the existence of a range of equilibria in a model where a price rise has larger negative effect on demand than the positive effect of a price reduction of the same magnitude. Non-differentiability or discontinuity of the profit function leads to multiple equilibria, and one can generate inflation persistence in a similar way to what is done here.

In our model, inflation persistence is generated as a focal point for agents' expectations, and it is not an inherent feature derived from preferences and technology. This implies that inflation persistence may weaken or disappear if another focal point becomes more prominent. Indeed, Ball (2000) showed that in the period from 1879 through 1914, when the US had a gold standard, the inflation process was close to a random walk. During this period, an even simpler expectation formation than the one presented here may be the most appropriate, namely that expected inflation was close to a constant (Ball, 2000).

In recent years, a possible candidate for a focal point for inflation expectations would be the inflation target of the central bank. If agents believe that the central bank will fulfill its inflation target, this can work as a coordinating device for expectations, as long as output remains without the equilibrium range. Somewhat speculatively, this suggests the following interpretation of why the high growth and falling unemployment in the US in the late 1990s did not lead to increasing inflation: the Federal Reserve commanded high credibility so that private agents expected the low inflation to continue, and thus set wages and prices according to this premise.

The use of preferences exhibiting loss aversion or other departures from standard assumptions has become commonplace in the study of consumption and asset pricing, and has been used to attempt to explain various empirical puzzles in those literatures. In this paper, we take a step towards applying preferences taken from behavioral economics to explain the empirical puzzle in the Phillips curve literature of inflation persistence. We show that a relatively minor departure from standard assumptions not only yields inflation persistence, but also sheds light on why the relationship between output and inflation is noisy and erratic.

References

Akerlof, George (1984). "Gift Exchange and Efficiency Wage Theory: Four Views." *American Economic Review Papers and Proceedings*, 74, pp 79-83.

Austin, William, Neil C. McGinn and Charles Susmilch (1980). "Internal Standards Revisited: Effects of Social Comparisons and Expectancies on Judgments of Fairness and Satisfaction." *Journal of Experimental Social Psychology*, 16, pp. 426-441.

Ball, Laurence S. (2000). "Near-Rationality and Inflation in Two Monetary Regimes." NBER Working Paper 7988.

----------- and Mankiw, N. Gregory (1995). "Relative-Price Changes as Aggregate Supply Shocks." *Quarterly Journal of Economics*, CV:2, pp. 161-193.

Bernanke, Ben S. and Ilian Mihov (1998). "Measuring Monetary Policy." *Quarterly Journal of Economics*, CVIII:3, pp. 869-902.

Bhaskar, V. (1990). "Wage relatives and the natural range of unemployment." *Economic Journal* 100, 60-66.

Blanchard, Olivier J. and Nobuhiro Kiyotaki (1985). "Monopolistic competition and the effects of aggregate demand." *American Economic Review*, pp. 647-667.

Binmore, Ken., Ariel Rubinstein, and Asher Wolinsky (1986)."The Nash bargaining solution in economic modelling." *RAND Journal of Economics*,17, pp176-188.

Calvo, Guillermo (1983). "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12:4, pp. 983-998.

Cooper, Russell W. (1999). *Coordination Games: Complementarities and Macroeconomics*. Cambridge University Press.

Evans, George W. and Seppo Honkaphohja (2001). *Learning and Expectations in Macroeconomics*. Princeton: Princeton University Press.

Fehr, E. and K. M. Schmidt. (1999).A Theory of Fairness, Competition, and Cooperation. *Quarterly Journal of Economics CXIV*, pp. 769-816.

Fischer, Stanley. (1977). "Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule." *Journal of Political Economy* 85:2, pp. 191-205.

Friedman, Milton (1968). "The Role of Monetary Policy." *American Economic Review*, 58:1, pp. 1-17.

Fuhrer, Jeffrey and Gerald Moore (1995). “Inflation persistence.” *Quarterly Journal of Economics* ,CX, pp. 127-160.

Gali, Jordi and Mark Gertler (1999). “Inflation Dynamics: A Structural Econometric Analysis.” *Journal of Monetary Economics 44*, 195-222.
Goeree, J.K and C. A Holt. “Asymmetric Inequality Aversion and Noisy Behavior in Alternating-Offer Bargaining." *European Economic Review 44*, pp. 1079-1089.

Gordon, Robert J. (1990). “What is New-Keynesian Economics?” *Journal of Economic Literature*, XXVIII:3, pp. 1115-1171.

Kahneman, Daniel and Amos Tversky (1979). “Prospect Theory: An Analysis of Decision under Risk.” *Econometrica*, 47, pp 263-291.

Loewenstein, George F., Leigh Thompson, and Max H. Bazerman. (1989). Social Utility and Decision Making in Intermpersonal Contexts. “ *Journal of Personality and Social Psychology LVII*, pp 426-441.

Lye, I, I. M. McDonald and H. Sibly (2001). “An Estimate of the Range of Equilibrium Rates of Unemployment for Australia.” *Economic-Record 77*, 35-50.

Maddala, G. S. and In-Moo Kim (1998). *Unit Roots, Cointegration and Structural Change.* Cambridge University Press, Cambridge UK.

Mankiw, N. Gregory (2000). “The Inexorable and Mysterious Tradeoff Between Inflation and Unemployment.” NBER Working Paper Number 7884.

McDonald, Ian (1995). “Models of the range of equilibria.” In Rod Cross (ed). *The Natural Rate of Unemployment: Reflections on 25 years of the hypothesis*. Cambridge: Cambridge University Press.

McDonald, Ian (2001). “Reference Pricing, Inflation Targeting and the Non-inflationary Expansion.” Mimeo, University of Melbourne.

Ochs, Jack (1995). “Coordination Problems”. In John H. Kagen and Alvin Roth (eds). *Handbook of Experimental Economics*. Princeton: Princeton University Press.

Ordonez, Lisa D., Terry Connolly and Richard Coughlan (2000). “Multiple Reference Points in Satisfaction and Fairness Assessment.” *Journal of Behavioral Decision Making*, 13, pp. 329-244.

Phillips, A. W. (1959). “The Relation Between Unemployment and the Rate of Change of Money Wages in the United Kingdom 1861-1957.” *Economica*, 25:2, pp. 283-299.

Quandt, Richard E. (1958). "The Estimation of the Parameters of a Linear Regression System Obeying Two Separate Regimes." *Journal of the American Statistical Association*, pp. 873-880.

Roberts, John. (1998). Inflation expectations and the transmission of monetary policy. Board of Governors of the Federal Reserve System.

Sbordone, Argia M. (1999). "Prices and Unit Labor Costs: A New Test of Price Stickiness." Mimeo, Rutgers University.

Shimer, Robert (1998). "Why is the U.S. Unemployment Rate So Much Lower?" *NBER Macroeconomics Annual*, pp. 11-61.

Sims, Christopher A. (1992). "Interpreting the Macroeconomic Time Series Facts: The Effects of Monetary Policy." *European Economic Review*, 36(4), pp. 975-1011.
Staiger, Douglas, James Stock and Mark Watson (1997). "How Precise Are Estimates of the Natural Rate of Unemployment?" In Christina D. Romer and David H. Romer (eds). *Reducing Inflation: Motivation and Strategy*, Chicago University Press.

---------------- (2001). "Prices, Wages and the U.S. NAIRU in the 1990s". Mimeo, Kennedy School of Government, Harvard University.

Taylor, John. (1980). "Aggregate dynamics and staggered contracts." *Journal of Political Economy* LXXXVIII, 1-24.

--------------- (1999). "Staggered wage and price setting in macroeconomics." Chapter 15 in J. B. Taylor and M. Woodford (eds). *Handbook of Macroeconomics.* North-Holland.

Woglom, Geoffrey (1982). "Underemployment Equilibrium with Rational Expectations." *Quarterly Journal of Economics 9,* 89-107.

Appendix

Proof of Proposition 1
Using

$$\frac{d\Pi_{jt}}{dX_{jt}} = (1-\eta)\frac{\Pi_{jt}}{X_{jt}} \quad and \quad \frac{dV}{dX_{jt}} = \frac{(1+\alpha+D_{jt}\phi)V_{jt}}{X_{jt}}$$

(8) and (9) can be rewritten as

$$(26) \qquad (2+\alpha+\phi+\lambda-\eta)V_{jt} + (\eta-1)V_{0t} \geq 0$$
$$(27) \qquad (2+\alpha+\lambda-\eta)V_{jt} + (\eta-1)V_{0t} \leq 0$$

Substituting out for V_{jt} using (4), imposing equality, (26) and (27) can be solved for

$$(28) \qquad X^{-}{}_{t} = \left(\frac{\eta-1}{\eta-2-\alpha-\phi-\lambda}V_{0t}P_t X_{Jt}{}^{\alpha+\phi}X_{Gt}{}^{\lambda}\right)^{\frac{1}{1+\alpha+\phi+\lambda}}$$

and

$$(29) \qquad X^{+}{}_{t} = \left(\frac{\eta-1}{\eta-2-\alpha-\lambda}V_{0t}P_t X_{Jt}{}^{\alpha}X_{Gt}{}^{\lambda}\right)^{\frac{1}{1+\alpha+\lambda}}$$

To see that $X^{-}(X_{Jt}, X_{Gt}, Y_t, P_t) > X^{+}(X_{Jt}, X_{Gt}, Y_t, P_t)$, note that from imposing equality in (26) and (27), we obtain

$$(\eta-2-\alpha-\phi-\lambda)V_{jt}(X^{-}{}_{t}) = (\eta-1)V_{0t} = (\eta-2-\alpha-\lambda)V_{jt}(X^{+}{}_{t})$$

or

$$\frac{V_{jt}(X^{-}{}_{t})}{V_{jt}(X^{+}{}_{t})} = \frac{\eta-2-\alpha-\lambda}{\eta-2-\alpha-\phi-\lambda} \Rightarrow X^{-}{}_{t} > X^{+}{}_{t} \text{ as V is increasing in X.}$$

To conclude, we know that
either $X_{jt} = X_{Jt}$, and (8) and (9) both hold, so that $X^{-}_{t} \geq X_{jt} \geq X^{+}_{t}$,
or (8) holds with equality, in which case $X_{jt} = X^{-}_{t}$,
or (9) holds with equality, in which case $X_{jt} = X^{+}_{t}$. QED

Derivation of (10) and (11)

Using the same procedure as in the proof of Proposition 1, (8) and (9) can be rearranged to

$$(30) \qquad X_{jt} \le \left(\frac{\eta - 1}{\eta - 2 - \alpha - \phi - \lambda} V_{0t} P_t X_{Jt}{}^{\alpha+\phi} X_{Gt}{}^{\lambda} \right)^{\frac{1}{1+\alpha+\phi+\lambda}}$$

and

$$(31) \qquad X_{jt} \ge \left(\frac{\eta - 1}{\eta - 2 - \alpha - \lambda} V_{0t} P_t X_{Jt}{}^{\alpha} X_{Gt}{}^{\lambda} \right)^{\frac{1}{1+\alpha+\lambda}}$$

Imposing $X_{jt} = X_{Jt}$ and $P_t = \mu(X_{Jt})^{1/2}(X_{Gt})^{1/2}$, and rearranging, we obtain

$$(32) \qquad X_{Jt} \le X_{Gt} \left(\frac{(\eta - 1)\mu}{\eta - 2 - \alpha - \phi - \lambda} V_0(Y_t) \right)^{\frac{1}{1/2+\lambda}} .$$

$$(33) \qquad X_{Jt} \ge X_{Gt} \left(\frac{(\eta - 1)\mu}{\eta - 2 - \alpha - \lambda} V_0(Y_t) \right)^{\frac{1}{1/2+\lambda}} .$$

Using the log linear approximations

$$(34) \qquad -\theta^{-} + \frac{\gamma_0}{2} y_t = \ln \left(\frac{(\eta - 1)\mu}{\eta - 2 - \alpha - \phi - \lambda} V_0(Y_t) \right)^{\frac{1}{1/2+\lambda}} .$$

$$(35) \qquad -\theta^{+} + \frac{\gamma_0}{2} y_t = \ln \left(\frac{(\eta - 1)\mu}{\eta - 2 - \alpha - \lambda} V_0(Y_t) \right)^{\frac{1}{1/2+\lambda}} .$$

we obtain (10) and (11) in the main text

Table 1
Phillips Curve Regressions, Quarterly Data, 1955:I-2000:IV

Dependent Variable: π_t							
Without Bounds		With Bounds					
		Between Bounds		Below Bound		Above Bound	
Const.	1.268**	Const.	2.956	I_L*Const.	.646	I_H*Const.	1.766**
	(.428)		(1.827)		(.627)		(.585)
π_{t-1}	.510**	π_{t-1}	.438**	$I_L*\pi_{t-1}$	-.114	$I_H*\pi_{t-1}$	-.083
	(.070)		(.127)		(.272)		(.156)
π_{t-2}	.111	π_{t-2}	.348**	$I_L*\pi_{t-2}$	-.363	$I_H*\pi_{t-2}$	-.387*
	(.079)		(.123)		(.275)		(.163)
π_{t-3}	-.384**	π_{t-3}	.413**	$I_L*\pi_{t-3}$	-.297	$I_H*\pi_{t-3}$	.018
	(.067)		(.130)		(.247)		(.151)
u_{t-1}	-1.821**	u_{t-1}	-1.967**	$I_L*(u_{t-1} - u_L)$	.857	$I_H*(u_{t-1}-u_H)$	.363
	(.314)		(.532)		(1.46)		(.689)
u_{t-2}	1.608**	u_{t-2}	1.358**	$I_L*(u_{t-2} - u_L)$	-2.23	$I_H*(u_{t-2}-u_H)$	.225
	(.306)		(.422)		(1.46)		(.590)
Food	.046**	Food	.042*	I_L*Food	.030	I_H*Food	.000
	(.015)		(.020)		(.043)		(.033)
Fuel	.011	Fuel	-.005	I_L*Fuel	.015	I_H*Fuel	.032
	(.009)		(.013)		(.021)		(.020)
Nixon	1.807	Nixon	-.498				
	(2.889)		(2.851)				
Sum on inflation	1.004**		1.198**		-.775**		-.453**
	(.040)		(.0526)		(.227)		(.083)
Sum on unemp.	-.213**		-.608		-1.378		.587
	(.073)		(.332)		(.907)		(.374)
Bounds	N/A			u_L	4.7	u_H	6.5
Adjusted R^2	.810		.841				
LogL	-311.30		-286.19**				
# Obs.	184		184				

Note: Inflation is measured by the (annualized) quarterly percent change in the seasonally-adjusted CPI for all urban consumers. The unemployment rate is that for all civilians over age 16. ‘Food’ is the relative PPI inflation rate for processed foods and feeds, and ‘Fuel’ is the relative inflation rate for energy, both lagged one period. ‘Nixon’ is a dummy for wage and price controls due to Gordon (1990). I_H and I_L are dummy variables for periods when lagged unemployment is outside the bounds u_H and u_L described in the text. Thus, the total effect of the RHS variables below (above) the bound, is given by the sum of the coefficient between bounds and the coefficient below (above) bounds.

* Denotes statistical significance at the 5% level
** Denotes statistical significance at the 1% level

Table 2
Phillips Curve Regressions, 1955:I-2000:IV
With Monetary Policy Indicator

Dependent Variable: π_t							
Without Bounds		With Bounds					
		Between Bounds		Below Bound		Above Bound	
Const.	1.125*	Const.	1.690	I_L*Const.	.785	I_H*Const.	1.190
	(.498)		(1.868)		(.685)		(.645)
π_{t-1}	.442**	π_{t-1}	.341*	$I_L*\pi_{t-1}$	-.011	$I_H*\pi_{t-1}$	.020
	(.068)		(.131)		(.274)		(.160)
π_{t-2}	.048	π_{t-2}	.352**	$I_L*\pi_{t-2}$	-.351	$I_H*\pi_{t-2}$	-.446**
	(.077)		(.125)		(.275)		(.165)
π_{t-3}	.388**	π_{t-3}	.459**	$I_L*\pi_{t-3}$	-.359	$I_H*\pi_{t-3}$	-.061
	(.063)		(.128)		(.242)		(.150)
u_{t-1}	-1.531**	u_{t-1}	-1.479**	$I_L*(u_{t-1} - u_L)$	.544	$I_H*(u_{t-1}-u_H)$	.118
	(.324)		(.573)		(1.525)		(.731)
u_{t-2}	1.354**	u_{t-2}	1.127**	$I_L*(u_{t-1} - u_L)$	-1.817	$I_H*(u_{t-2}-u_H)$	.111
	(.299)		(.431)		(1.502)		(.596)
Food	.036*	Food	.043*	I_L*Food	.024	I_H*Food	-.004
	(.015)		(.019)		(.044)		(.033)
Fuel	.001	Fuel	.001	I_L*Fuel	.012	I_H*Fuel	.027
	(.001)		(.013)		(.021)		(.020)
Nixon	.181	Nixon	-.010		N/A		N/A
	(2.776)		(3.068)				
Monetary Expansion	.0774 (.060)	Monetary Expansion	-.205 (.127)		.556 (.712)		.335* (.154)
Monetary Contraction	-.254** (.057)	Monetary Contraction	-.063 (.077)		-.017 (.197)		-.595** (.222)
Sum on inflation	.877** (.049)		1.152** (.072)		-.721** (.252)		-.487** (.103)
Sum on unemp.	-.176 (.097)		-.351 (.343)		-1.273 (1.085)		.228 (.403)
Bounds	N/A			u_L	4.7	u_H	6.5
Adjusted R^2	.828		.851				
LogL	-301.32		-276.91**				
# Obs.	184		184				

Note: 'Monetary Contractions' represents the value of the Bernanke and Mihov (1998) indicator for monetary policy when that indicator is negative, and 'Monetary Expansions' the value of that indicator when the indicator is positive. All other notation as in Table 1.

* Denotes statistical significance at the 5% level
** Denotes statistical significance at the 1% level