# Financial Intermediation with Heterogeneous Agents: Transitional Dynamics and Policy Responses ${ }^{1}$ 

Martin Berka and Christian Zimmermann ${ }^{2}$

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preliminary, discussion is incomplete

[^0]
#### Abstract

The paper studies loan activity in a context where banks have to follow Basle Accord type rules and need to find financing with the households. Loan activity typically decreases when investment returns of entrepreneurs decline, and we study which type of policy could revigorate an economy in bad shape. We find that active monetary policy increases loans even when the economy is in good shape, while introducing active capital requirement policy can be effective as well if it implies tightening of regulation in bad times.

Keywords: Bank Capital Channel, Capital Requirements, Basle Accord, Occupational Choice, Bankruptcy, Credit Crunch.


JEL Classification: E44, E22, G28, E58

## 1 Introduction

Traditionally, the literature on financial intermediation and credit channels, especially credit crunches, emphasized the relation between banks and entrepreneurs requiring credit, neglecting the funding of banks. With this paper, we want to be much more precise in this respect and study the impact of funding on credit. Indeed, regulation that has become world wide with the Basle Accord puts limits on the amount of loans banks can give, limits that are determined by the level of bank equity. How much equity the banks can issue depends in particular on how much households are willing to buy equity in addition to deposits.

In our model economy, households have heterogenous asset holdings because they have different labor histories and because only some of them get credit as entrepreneurs (and among those, the return on investment is stochastic). Non-entrepreneur households invest in bank deposits and bank equity, and banks maximize profits while following regulations. A central bank conducts the monetary policy and regulates the banks.

Therefore, when banks need to reduce their loan portfolio, the displaced entrepreneurs become new equity holders, thereby acting as "automatic stabilizers". However, banks typically cut loans as a consequence of their loan portfolio becoming too risky, and households may then want to hold less equity in banks that are now more risky. Whether banks have to tighten credit a lot or not now depends very much on the distribution of assets across households and their equity decisions.

We solve this very rich model using numerical methods, in particular for the transitional dynamics that may lead an economy into a credit crunch. We then look which policies may help the economy out of a trough. We find that the endogenous distribution of assets has strong implications that should not be neglected in future research. Also, monetary policy can have positive real effects if the central bank can commit to act in particular ways.

We find some evidence in our model that a credit crunch can arise in the presence of capital requirements. Numerical simulations show this it is not large. It is natural to look whether flexible capital requirements can have a positive impact. One would first think that loosening those requirements in a trough would expand the loan mass. It appears the contrary is indicated, as tighter capital requirements increase the demand for equity, and thus facilitate the financing of banks, sufficiently to offset the reduction of allowable loans for given equity. Again, this highlights the importance of household savings decisions.

Bernanke and Gertler (1995) highlight several channels through which monetary policy influences real activity. Two operate on lending. In the balance sheet channel, Fed policy affects the financial position of borrowers and hence there ability to post collateral or self-finance. In the bank lending channel, Fed policy shifts the supply of bank credit, in particular loans. They argue the important of the latter channel has declined with deregulation, as this channel relies on reserves. Van de Heuvel (2001) identifies another channel stemming specifically from Basel Accord like rules. The "bank capital channel" arises from maturity transformation through banks: higher short term interest rates depress profits, thus equity and capital adequacy. This model has a very detailed banking structure, but neglects the problems of households and firms. Our model has a simpler banking structure but emphasizes the source of financ-
ing (households) and the demand for loans (entrepreneurs) by modeling occupational choice, savings and bankruptcy. ${ }^{1}$

Chami and Cosimano (2001) identify a similar channel, there called "bank-balance sheet channel", introducing the concept of increasing marginal cost of external financing. As Van den Heuvel, they need market power in the banking industry to obtain the result. Our model has fully competitive banks. Bolton and Freixas (2001) find that capital requirements can be the origin of a credit crunch. Their model is very detailed on the lending market and asymmetric information. Our model puts more emphasis on the financing side and does not require asymmetric information.

The structure of this paper is as follows: section 2.2 analyzes the heterogenous behavior of households, sections 2.3 and 2.4 analyze the (homogeneous) financial sector and the central bank, section 2.6 defines and analyzes the equilibrium and section 3 presents the calibration of the model. Section 4 analyses bank lending and optimal monetary policy behavior following negative shocks. Section 5 concludes. Appendices give details about various aspects of the model and the solution strategy.

## 2 Model

### 2.1 Overview

There are four types of agents in the economy: households, banks, mutual funds, and a central bank. Given a shortage of internal financing, households that are in a productive stage of their lives must apply for external funds. Successful applicants become entrepreneurs and others stay workers. Each worker faces an idiosyncratic shock of becoming unemployed while entrepreneurs have risky returns on investment. All households in a productive stage of life (entrepreneurs, employed and unemployed workers) face a risk of becoming permanently retired, and all retirees face a risk of dying. New households are born to replace the dead ones.

When the households make their consumption-savings decision, savings are invested in a mutual fund company that decides on their optimal allocation between bank deposits and bank equity. Banks collect deposits and equity and provide loans to entrepreneurs and purchase riskfree government bonds in order to maximize their profits. Banks screen loan applications and accept them according to the level of household's net worth. Banks have to purchase deposit insurance and are subject to a capital adequacy requirement imposed by the central bank. The central bank also controls the government bond rate.

We now go through the model in more detail.

[^1]
### 2.2 Households

In the model economy, there is a continuum of measure one of households, each maximizing their expected utility by choosing an optimal consumption-savings path. A household can either be productive or retired, and the probability of a productive household retiring $\tau$ is exogenous ${ }^{2}$.

A productive household $i$ is endowed with one indivisible investment project of size $x_{t}^{i}$, which is always greater than the household's net worth $m_{t}^{i}$. We assume that the total investment is a fixed multiple of household's net worth: $x_{t}^{i}=\phi m_{t}^{i}$ where $\phi>1$. The project is indivisible, and so $(\phi-1) m_{t}^{i}$ has to be funded by the bank in order for a project to be undertaken $^{3}$. If a household receives a loan it becomes an entrepreneur and invests into a project, receiving a return $r_{t}^{i}$ drawn from a trinomial distribution. We assume that the distribution of returns is such that households always prefer investing into projects and becoming entrepreneurs to becoming workers ${ }^{4}$. The returns are drawn independently across households (i.e. projects) and time. The lowest of the returns is sufficiently negative with a positive probability to lead to bankruptcy, in which case a household is guaranteed a minimal amount of consumption $c_{\min }$ and starts next period with no assets.

When bank rejects a loan application, the household enters the work force and faces exogenous probabilities $1-u$ of becoming employed and $u$ of becoming unemployed. Workers receive wage income $y$. Unemployed workers receive unemployment benefits $\theta y$ where $\theta$ is the replacement ratio.

Labor supply is inelastic at an individual level. At the aggregate level, labor supply is determined by the flows between the pools of workers, entrepreneurs, unemployed and retirees. This assumption increases the role asset accumulation plays in the economy. We use aggregate labor input data on the average hours per worker to calibrate the labor demand. Therefore, the labor market clears implicitly at the level of the utility function. Because of the inelastic labor supply, we need to assume exogenous wages for the calibrated parameter values.

After retirement, the household earns income from its savings and pension (which equals unemployment benefit payments). Retirees face a probability $\delta$ of dying. They are then replaced by agents with no assets and any remaining assets are lost (no bequests).

The households make their consumption-savings decision to maximize their expected lifetime utility. The momentary utility function is a CRRA type:

$$
U(c)_{o c}=\frac{\left(l_{o c}^{\sigma} c^{1-\sigma}\right)^{1-\rho}-1}{1-\rho}
$$

where $o c \in\{W, U, E, R\}, l$ denotes leisure, $c$ consumption and $\rho$ is a risk-aversion parameter. As mentioned above, the labor supply is inelastic and the values $l_{o c}$ represent market-clearing values for leisure.

[^2]Let $V_{o c}$ denote the value functions and $m^{*}$ be the minimum net worth eligible for external financing. A worker with a net worth $m\left(<m^{*}\right)$ faces probability $(1-u)$ of being employed, following which he receives labor income $y=\left(1-l_{W}\right) w$ and interest income $R^{d} m$, consumes a desired level and deposits his net worth $m^{\prime 5}$ with a mutual fund. If unemployed, he receives unemployment benefit payment $\theta y=\theta w$ and makes a similar consumption-savings decision. In the next period, depending on the level of $m^{\prime}$, a worker may either become an entrepreneur (borrower) or remain a worker (depositor).

For an employed worker:

$$
\begin{array}{r}
V_{W}\left(m^{i}\right)=\max _{c^{i}, m^{i^{\prime}}}\left\{U_{W}\left(l_{W}, c^{i}\right)+\beta\left[( 1 - \tau ) \left[(1-u) V_{W}\left(m^{i^{\prime}}\right)+\right.\right.\right. \\
\left.\left.\left.u V_{U}\left(m^{i^{\prime}}\right)+E_{r^{\prime}} V_{E}\left(m^{i^{\prime}}, r^{i^{\prime}}\right)\right]+\tau V_{R}\left(m^{i^{\prime}}\right)\right]\right\} \tag{1}
\end{array}
$$

s.t.

$$
c^{i}+m^{i^{\prime}}=\left(1+r^{d}\right) m^{i}+y
$$

For an unemployed worker:

$$
\begin{array}{r}
V_{U}\left(m^{i}\right)=\max _{c^{i}, m^{i^{\prime}}}\left\{U_{U}\left(l_{U}, c^{i}\right)+\beta\left[( 1 - \tau ) \left[(1-u) V_{W}\left(m^{i^{\prime}}\right)+\right.\right.\right. \\
\left.\left.\left.u V_{U}\left(m^{i^{\prime}}\right)+E_{r^{\prime}} V_{E}\left(m^{i^{\prime}}, r^{i^{\prime}}\right)\right]+\tau V_{R}\left(m^{i^{\prime}}\right)\right]\right\} \tag{2}
\end{array}
$$

s.t.

$$
c^{i}+m^{i^{\prime}}=\left(1+r^{d}\right) m^{i}+\theta y
$$

An entrepreneur $i$ invests in a project, earns a net return $r^{i}$ and labor income $y=\left(1-l_{E}\right) w$ and pays the borrowing cost $r^{l}\left(x^{i}-m^{i}\right)$, while making a consumption-savings decision to maximize his expected utility. Because the net wealth is constrained to be non-negative, significant project losses drive the entrepreneur into bankruptcy. When bankrupt, an entrepreneur defaults on the portion of the debt he can not repay less a minimal consumption allowance $c_{\text {min }}$ which has to be granted by the bank. Upon default, entrepreneur starts the next period as a household with no assets and no liabilities. The returns on project $r_{i}$ are drawn independently across time and individuals and follow a trinomial distribution with mean $r$ and variance $\sigma_{r}^{2}$. The lowest of the returns is sufficiently negative to bring entrepreneur into bankruptcy. For an entrepreneur:

$$
\begin{array}{r}
V_{E}\left(m^{i}, r^{i}\right)=\max _{c^{i}, m^{i^{\prime}}}\left\{U_{E}\left(l_{E}, c^{i}\right)+\beta\left[( 1 - \tau ) \left[(1-u) V_{W}\left(m^{i^{\prime}}\right)+\right.\right.\right. \\
\left.\left.\left.u V_{U}\left(m^{i^{\prime}}\right)+E_{r^{\prime}} V_{E}\left(m^{i^{\prime}}, r^{i^{\prime}}\right)\right]+\tau V_{R}\left(m^{i \prime}\right)\right]\right\} \tag{3}
\end{array}
$$

s.t.

$$
\begin{gathered}
c^{i}=\max \left\{c_{m i n}, m^{i}+y+\left(1+r^{i}\right) x^{i}-r^{l}\left(x^{i}-m^{i}\right)-m^{i \prime}\right\} \\
x^{i}=\phi m^{i}
\end{gathered}
$$

[^3]To stress the effects of credit supply, we assume that ex-ante, households always prefer to apply for a loan. This implies a participation constraint for households in a productive stage of their lives:

$$
\begin{equation*}
E_{r} V_{E}\left(m^{*}, r\right) \geq(1-u) V_{W}\left(m^{*}\right)+u V_{U}\left(m^{*}\right) \tag{4}
\end{equation*}
$$

Every household faces an exogenous probability of retirement $\tau$. Once retired, the household collects retirement income $y_{R}=\theta w$ and manages its assets subject to the risk of death $\delta$.

$$
\begin{equation*}
V_{R}(m)=\max _{c^{i}, m^{i^{\prime}}}\left\{U_{R}\left(1, c^{i}\right)+\beta\left[(1-\delta) V_{R}\left(m^{i^{\prime}}\right)\right]\right\} \tag{5}
\end{equation*}
$$

s.t.

$$
c^{i}+m^{i^{\prime}}=\left(1+r^{d}\right) m+y_{R} .
$$

Because of their risk aversion, the agents smooth their consumption over time. The presence of heterogeneous risks of unemployment, retirement and death as well as the heterogeneity in project returns lead to a non-degenerate distribution of assets in the economy. Intuitively, the individual risks along these dimensions substitute for the uncertainty of income which is modeled as fixed. Without these risks, there would be no reason to save other than to invest, and the asset distribution would collapse along $m=0$ and $m=m^{*}$. This would not allow for financial intermediation due to lack of funds (no depositors). Any equilibrium in this binomial distribution is very unstable because all entrepreneurs can switch to zero assets following a shock. The distribution of assets plays a crucial role in determining the dynamics of the aggregate variables.

### 2.3 Financial Sector

### 2.3.1 Banks

Banks maximize their expected profits, taking the asset distribution in the economy as given. Profits equal asset returns less the funding costs, deposit insurance payments and the expected loan losses and liquidation costs. Bank's choice variables are loans $L$, bonds $B$, equity $E$ and deposits $D$. Because the banks take the distribution of assets as well as all returns as given, the choice of loan volume is identical to the choice of a threshold level of net worth $m^{*}$. Formally, the problem can be stated as:

$$
\begin{equation*}
\max _{L, B, D, E} r^{l} L+r^{b} B-r^{d} D-r^{e} E-\delta\left(\frac{D}{E}\right)^{\gamma} D-\left(1+l_{c}\right) \epsilon L \tag{6}
\end{equation*}
$$

subject to

$$
\begin{align*}
B+L & =D+E=M  \tag{7}\\
\frac{E}{L} & \geq \alpha  \tag{8}\\
D+E & \geq L \tag{9}
\end{align*}
$$

where $M$ is the total amount of loanable funds that are exogenous from the point of view of the $\operatorname{bank}^{6}, \delta$ is a per-unit deposit insurance cost parameter, $\epsilon$ is an expected share of loan losses and $l_{c}$ is a liquidation cost parameter. Equation (7) is the usual balance sheet constraint, (8) is a simplified version of a regulatory requirement on capital adequacy and (9) is a non-negativity constraint on bond holdings. The profit function (6) is non-linear due to the inclusion of deposit insurance costs which are an increasing function of the deposit/equity ratio. Because profits increase in loans for any given asset distribution, one and only one of the constraints (8) and (9) will bind at any time ${ }^{7}$. The solution of the profit maximization is described in the appendix.

### 2.3.2 Mutual fund company

The mutual fund company acts as a sole intermediary for savings between the households and the bank's liabilities ${ }^{8}$. It aggregates the savings and decides on a portfolio split between bank deposits and bank equity. The mutual fund maximizes a risk-adjusted return on portfolio ( $r^{\text {port }}$ ) by choosing an optimal deposit/equity investment ratio, taking the aggregate amount of deposits (M) as given. Formally:

$$
\max _{\omega_{r}} r^{p o r t}-\frac{1}{2} \lambda \sigma_{P O R T}^{2}
$$

where $r^{\text {port }}=r^{e} \frac{E}{M}+r^{d}\left(\frac{D}{M}\right)=\omega_{r} r^{e}+\left(1-\omega_{r}\right) r^{d}, \omega_{r} \equiv E / M$ is a weight on the risky (equity) investment, $\lambda$ is a risk-aversion parameter and $\sigma_{X}^{2}$ is a variance of $X$. Because bank deposits carry no risk ( $\sigma_{D}^{2}=0$ ), the mutual fund maximizes:

$$
\max _{\omega_{r}} \omega_{r} r^{e}+\left(1-\omega_{r}\right) r^{d}-\frac{1}{2} \lambda \omega_{r}^{2} \sigma_{E}^{2}
$$

which yields the optimal share of equity $\omega_{r}^{*}=\frac{r^{e}-r^{d}}{\lambda \sigma_{E}^{2}}$. This in turn defines the demand for equity (and implicitly for deposits):

$$
\begin{equation*}
\frac{E}{M}=\frac{r^{e}-r^{d}}{\lambda \sigma_{E}^{2}} \tag{10}
\end{equation*}
$$

[^4]
### 2.4 Central bank

The central bank determines the bond interest rate $r^{b}$ at which it inelastically supplies (government) bonds. In addition, central bank determines the capital asset ratio parameter $\alpha$. Therefore $\alpha$ and $r^{b}$ are the only monetary policy instruments it has at hand. In the simulation section 4 we show how different paths of monetary policy variables influence the behavior of the different types of households, banks and mutual fund companies, and ultimately what effect they have on the welfare.

### 2.5 Market clearing

On the financial side, markets for loans, bonds, equity and deposits must clear. Bond market clears automatically because of an inelastic supply of bonds ${ }^{9}$. The remaining market clearing conditions are:

$$
\begin{gather*}
D^{S}=D^{D}=\sum_{m^{i}<m^{*}} m^{i}\left(1-\omega_{R}\right)  \tag{11}\\
E^{S}=E^{D}=\sum_{m^{i}<m^{*}} m^{i} \omega_{R}  \tag{12}\\
L=\sum_{m^{i} \geq m^{*}}(\phi-1) m^{i}  \tag{13}\\
M=\sum_{m^{i}<m^{*}} m^{i}=D+E=B+L=\sum_{m^{i} \geq m^{*}}(\phi-1) m^{i} \tag{14}
\end{gather*}
$$

Moreover, expected losses of the bank must in equilibrium equal the realized loan losses:

$$
\epsilon=\sum_{m^{i} \geq m^{*}} \max \left\{0,(1+\nu)\left[r^{L}(\phi-1) m^{i}-\phi m^{i}\left(1+r^{i}\right)\right]+c_{\min }\right\}
$$

where $\nu$ are the auditing costs.The market clearing equations (11) - (14) connect the homogeneous and heterogenous parts of the model. The sum of individual demands for deposits, equity and loans on the right-hand sides must equal the supply levels decided on an aggregate level.

Equity market clearing implicitly defines return on equity $r^{E}$ as a function of all other returns. In the case of an interior solution, equations (21) and (10) imply:

$$
\begin{equation*}
\frac{1}{\delta}\left(r^{E}-r^{D}\right)^{3}-\left[\frac{1}{\alpha \delta}\left(r^{L}-r^{D}-\left(1+l_{c}\right) \epsilon\right)+1\right]\left(r^{E}-r^{D}\right)^{2}+2 \lambda \sigma_{E}^{2}\left(r^{E}-r^{D}\right)-\lambda^{2} \sigma_{E}^{4}=0 \tag{15}
\end{equation*}
$$

In the case of a corner solution, equations (24) and (10) imply:

$$
\begin{array}{r}
r^{E^{3}}-r^{E^{2}}\left[2 r^{D}+r^{L}-\left(1+l_{c}\right) \epsilon+1\right]-r^{E}\left[r^{D^{2}}+2 r^{D}\left(r^{L}-\left(1+l_{c}\right) \epsilon+1\right)+2 \lambda \sigma_{E}^{2}\right] \\
-\left[r^{D^{2}}\left(r^{L}-\left(1+l_{c}\right) \epsilon+1\right)+2 \lambda \sigma_{E}^{2} r^{D}+\delta \lambda^{2} \sigma_{E}^{4}\right]=0 \tag{16}
\end{array}
$$

[^5]To illustrate the functioning of the equity market, it is useful to undergo a following thought experiment. Consider a case of an increase in the lending interest rate $r^{L}$, possibly because of an increase in the demand for loans. As long as the ratio of expected losses as a proportion of loans $\epsilon$ rises less than $r^{L}$, bank's profit margin on each new loan goes up, which prompts the bank to lend more. To do so, bank has to raise more equity (it starts with no excess: $E=\alpha L$ ), which is why the equity supply equation (21) is increasing in the loan profit margin. The demand for equity (10) is unaffected by the return on loans, and so to raise more of equity, $r^{E}$ has to increase. Note that because the deposit rate is exogenous and bank can not choose the size of it's balance sheet $M, r^{E}$ plays an important role in bank's liability management: its rise will lead to an increase in the total amount of equity raised and to a more-than-proportional increase in the $E / D$ ratio for any size of the balance sheet $M^{10}$.

It follows that when bank increases the share of loans in its portfolio, it has to fund the higher equity holdings at an ever-increasing price. Eventually, the original profit margin disappears and a new optimal loan level is achieved. Two cases can occur: if the total amount of new loans is less than the new balance sheet level, loan market clearing conditions are satisfied and constitute an equilibrium candidate. However, if the total amount of new loans exceeds the new balance sheet volume $M$ (what we defined earlier as a corner solution), loan market does not clear and the banks ration some of the eligible loan applicants. Because there is no asymmetric information problem in this model (hence no adverse selection), an increase in the price of loans does not affect its quality and a higher $R^{L}$ is needed to clear the market. Therefore we have a choice of focusing on market-clearing equilibria which rule out corner solutions and equity "hoarding", or allowing credit rationing when multiple equilibria may arise and excess equity is kept as a backup in case the total amount of loanable funds $M$ increases. For simplicity, we only focus on the market-clearing equilibria, and equation (16) becomes irrelevant. One of the implications is that we will never observe banks hold excess equity in equilibrium. Therefore, regulatory changes in capital adequacy ratio $\rho$ will have direct effect on the loan volume.

The market clearing condition (15) defines a return on equity as function of all other returns and some parameters: $r^{E}=r^{E}\left(r^{L}, r^{D}, \sigma_{E}^{2}, \lambda, \alpha\right)$. The above cubic equations can be solved analytically but does not determine the $R^{E}$ uniquely. Depending on the parameter values, one or two out of three roots may be complex numbers which we disregard.

We now have a recursive system. Conditional on $M$, equation (21) determines the optimal level of equity $E$, equation (23) determines the optimal level of deposits $D$, equation (14) determines the optimal level of bonds $B$ and equation (22) determines the optimal level of loans $L$. We therefore have $\left\{r^{E}, r^{B}, E, D, L, B\right\}$ as a function of $\left\{r^{L}, M\right\}$ and exogenous variables.

[^6]
### 2.6 Equilibrium

A recursive equilibrium in this model economy is a set of decision rules $\left\{g_{o c}^{m}(m, s, r), g_{M}^{d}(s), g_{M}^{e}(s), g_{B}^{m^{*}}(s), g_{B}^{r l}(s)\right\}$ where oc $\in\{E, W, U, R\}$, government policies $\left\{\alpha(s), r^{b}(s)\right\}$, prices $\left\{r^{d}(s), r^{\text {port }}(s), r^{e}(s)\right\}$, aggregate asset levels $\{L, D, B, E\}$, and a function $\Psi(\mu)$ such that, for all aggregate states $s$ :

1. decision rules $g_{o c}^{m}(m, s, r)$ solve each household's optimisation problem with the associated value functions $V_{o c}(m, s, r)$.
2. decision rules $g_{M}^{d}(s)$ and $g_{M}^{e}(s)$ solve the mutual fund's optimisation problem.
3. decision rules $g_{B}^{m^{*}}(s)$ and $g_{B}^{r^{l}}(s)$ solve the bank's optimisation problem.
4. loan, equity and deposit markets clear:

$$
\begin{align*}
L(s) & =\int_{r, m \geq m^{*}}(\phi-1) m \mathrm{~d} \mu(m, s, r)  \tag{17}\\
E(s) & =\frac{r^{e}-r^{d}}{\gamma \sigma_{E}^{2}} \int_{r, m<m^{*}} m \mathrm{~d} \mu(m, s, r)  \tag{18}\\
D(s) & =\left(1-\frac{r^{e}-r^{d}}{\gamma \sigma_{E}^{2}}\right) \int_{r, m<m^{*}} m \mathrm{~d} \mu(m, s, r) \tag{19}
\end{align*}
$$

5. the distribution of households is the fixed point of the law of motion $\Psi$ :

$$
\begin{aligned}
\mu^{\prime}\left(M_{0}, R_{0}\right) & =\Psi\left(M_{0}, R_{0}\right)\left(s^{\prime}, \mu, s\right) \\
& =\int_{M_{0}, R_{0}}\left\{\int_{M, R, o c} 1_{m^{\prime}=g_{o c}^{m}(m, s, r)} \pi\left(s^{\prime} \mid s\right) \mathrm{d} \mu\right\} \mathrm{d} m^{\prime} \mathrm{d} r^{\prime}
\end{aligned}
$$

## 3 Parametrization

To simulate the economy and obtain numerical results, we parametrize the model to the Canadian economy in the years of 1988 to 1992, in accordance with the available data on project return distributions. Indeed, these are the only years for which Statistics Canada published such data.

First we calibrate the household sector. Several parameters are set in accordance with the literature: $\rho=2.5, \beta=0.96$ and $\sigma=0.67$. In accordance with the models that include explicit leisure specification, $l_{E}=l_{W}=l_{U}=0.55$ while $l_{R}=1$, as a result of which the labor input of entrepreneurs and workers, and the search effort of unemployed are set to 0.45 . Wages are exogenous and while they completely characterize the labor income of entrepreneurs and workers, the incomes of unemployed and retired are determined by the ratio of unemployment insurance benefits to wages $\theta=0.2929^{11}$.

[^7]The probability of unemployment is set equal to the average Canadian unemployment rate for the considered period: $u=0.0924$. The probability of retirement $\tau$ and the mortality rate $\delta$ are set at 0.05 and 0.1 , so that the number of expected periods while worker and retiree are 20 and 10 , respectively. Longer expected lifetime horizon allows us to utilize the effect of savings over time more fully than in the usual 2-period models (e.g. Williamson (1987) and Bernanke and Gertler (1989).

Now we turn to the financial side. All rates are modeled and therefore calibrated in real terms. Following the calibration in Yuan and Zimmermann (1999), we set the real bond rate $r^{b}$ at $1 \%$, such that the deposit rate $r^{d}$ is about $0.9 \%$, which corresponds to an average of savings rates and guaranteed investment certificate rates. The parameter $\alpha$ of the capital adequacy constraint is taken to represent the tier-1 capital requirements imposed by the Basle Accord (1988) and set to $\alpha=0.08$. The deposit insurance parameter $\delta$ is calibrated using the premium rates of the Canadian Deposit Insurance Corporation for banks in 2000/2001 ( $0.0417 \%$ of insured deposits). This per-unit rate corresponds to $\delta=0.0000417$ for an average D/E ratio of 10 . The loan administration cost $l_{c}$ is assumed to equal 0 . The account flat fee $\xi$ is set at 0.0003 by trial and error in order to get the banks to break even. The parameters of the equity market that need to be calibrated are $\lambda$ and $\sigma_{E}^{2}$. The variance of returns on equity of the banks is calculated from the TSE monthly series on financial enterprises' returns on equity from September 1978 until December 2000, which are deflated by the CPI. Therefore, $\sigma_{E}^{2}=0.24$. The risk-aversion parameter of the mutual funds' objective function $\lambda$ is calibrated from the market clearing condition (15) using the observed average real deposit, lending and ROE rates. This implies $\lambda=16$.

The distribution of returns follows a two-state Markov process calibrated such that the high state occurs $75 \%$ of the time. Specifically, a high state has a $75 \%$ chance of reoccuring the next period, while a low state can repeat itself with a $25 \%$ chance.

The distributions of project returns in both state are calibrated from firms' return on equity data. Statistics Canada (1994) reports the distribution of return on equity by non-financial enterprises from the fourth quarter of 1988 until the fourth quarter of 1992. Average returns in each quarter are reported for the top, middle and bottom tertile. Assuming the underlying distribution is normal, we find the returns and associated probabilities for trinomial distributions such that a) average returns are replicated, b) we have have two extreme returns, one implying bankruptcy. We compute two such distributions, one for the high aggregate state, corresponding to the average of the $75 \%$ best quarters in the sample, and the other for the low state. The returns and the associated distributions are the following:

$$
\text { High: }\left(\begin{array}{rrr}
-50 \% & 5.2 \% & 60 \% \\
0.71 \% & 98.48 \% & 0.81 \%
\end{array}\right) \text { Low: }\left(\begin{array}{rrr}
-50 \% & 2.57 \% & 60 \% \\
1.79 \% & 97.42 \% & 0.79 \%
\end{array}\right)
$$

The ratio of investment to net worth $(\phi-1)$ is calibrated to equal the average debt-equity ratio during the reference period, and so $\phi=2.2$. With a minimum return on investment of $-50 \%$, we have occasional bankruptcies. The auditing costs $\nu$ are assumed to equal 0.03.

## 4 Capital requirements, bank lending and monetary policy

In this section, we analyze a particular event history: one High aggregate shock followed by five Low and then by five High ones. Thus, the model economy wanders through a whole cycle, bottoming out in the middle. Note that this a particular history of shocks among many others, and that this history is not anticipated. In Figure 1, we show the behavior of various indicators in a benchmark economy, that is with no policy intervention from the central bank on bond rates or capital requirements.

First, we look at the two instrument variables of the banks. When an initial bad shock hits the economy, the lending rate jumps up, essentially to cover higher expected loan losses. As more bad news accumulate, the lending rate decreases as $m^{*}$ reacts and the households adapt their asset levels. Indeed, banks ration more and more as bad shocks accumulate, but revert to "normal" behavior as soon as good news come in. From peak to trough, the amount of loans decreases by $3.0 \%$, and $3.6 \%$ of all entrepreneurs are driven out. The consequence is that the size of an average loan increases by $0.6 \%$, corresponding to the empirically documented phenomenon that small businesses are hurt more when credit conditions worsen.

Do we have evidence of a credit crunch in this benchmark economy? Despite the fact that banks can increase the loan rate to compensate for higher rates, they have to decrease the total loan mass. The reason is the following. Facing increased risk, some entrepreneurs are forced to become workers, as the bankruptcy rate is higher. With more agents that save, the volume of assets in mutual funds increases. However, a smaller share of mutual funds' investments are channeled to bank equity because of its higher risk. The banks are then squeezed by the capital requirement and have to ration credit and invest more into "unproductive" government bonds. Without the capital requirement, banks could give more loans, in principle, by charging even higher loan rates, and entrepreneurs would still be ready to pay these rates. Although all agents behave optimally, we have a situation that can be described as a credit crunch, where marginal return and marginal costs of loans are not equal.

Capital requirements imply that changes in the compostition of banks' liabilities affect the amount of credit in the economy. An adverse productivity shock increases the number of depositors and lowers the number of borrowers. Yet risk averse depositors shy away from the highly risky bank equity which leads to a further credit decline (due to the capital requirements). However, the movements described above are relatively small.

### 4.1 Countercyclical monetary policy

The following experiments will help us understand what are the consequences of various policy actions. The first policy experiment, described in Figure 2, involves a 25 basis point reduction of the bond rate in the worst aggregate state (current shock Low, long history of Low shocks). ${ }^{12}$ Thus, the central bank reacts only after a prolonged decline in the economy. Note that the decisions of the banks are changed only in this specific state: $m^{*}$ and the lending

[^8]rate are unaffected when the central bank does not move, but when it does banks reduce the lending rate by the same margin and, more importantly, significantly relax their loan threshold. Thus the situation for entrepreneurs should improve noticeably: easier access to credit at better conditions. Loan activity is negatively affected, however, and equity is reduced compared to the benchmark. This is because workers decide to save slightly less (interest rates are lower) and put a smaller proportion into equity (risk is higher). Note that household decisions are affected even when the central bank has left the bond rate untouched, in anticipation of possible changes. Ultimately, the same number of entrepreneurs gets loans and the average loan is now smaller.

A one-time drop in the interest rate is therefore does not appear to be an effective policy. What now if the interest rate is gradually reduced by 5 points after each bad shock, and goes back to normal whenever a good shock comes by? This policy takes better into account the anticipations the households formulate. On Figure 3, we see that the outcome is quite different. Banks become much more generous to entrepreneurs in bad times, both in terms of lower lending rates in bad times (but higher in good ones) and quite significantly in terms of $m^{*}$. In all states, there are more entrepreneurs, loans, deposits and equity. While the average loan is larger in normal times compared to the benchmark, it is smaller in almost any other. This means that asset accumulation has increased for households: entrepreneurship is more interesting as monetary policy counterbalances the increased risk in bad times. Indeed, while firms face lower average returns and higher bankruptcy rates, monetary policy forces banks to offer better conditions. This has an impact on asset accumulation even in good times. We conclude that an active countercyclical monetary policy can have a significant positive impact. Note, however, that it can remove the cyclical nature of loans.

### 4.2 Procyclical monetary policy

If some policy of interest rate reduction may have negative consequences, one may naturally ask whether an interest rate increase can do some good. Indeed, higher bond rates mean higher returns on savings, and potentially more equity to satisfy the loan needs in the presence of capital requirements.

In Figure 4, we find that the model economy does not behave in a symmetric way, as compared to Figure 2. While the lending rate increases as expected, $m^{*}$ stays essentially put rather then shoot up. Consequently, loan activity does not change much as households barely change their decisons compared to the benchmark. The sum of all tiny changes results, however, in a noticeably decrease in the average loan size, but not as strong as in the opposite policy.

Comparing Figures 3 and 5, it appears that the same kind of asymmetry exists for a gradual policy. A gradual increase of the bond rate has a negative, but much smaller impact on the various assets.

An explanation of this asymmetry is as follows. Procyclical monetary policy induces a drop in $m^{*}$, leading to an increase in the loan volume as more smaller agents can become entrepreneurs. Moreover, a lower $m^{*}$ induces workers to save more (consumption drops) at
any given deposit rate because the entrepreneurship is more likely to be attained (this move is slightly offset by the distributional movements as there are fewer workers and more entrepreneurs). Because of such boom in banks' liabilities, asset sides of banks' balance sheets expand which reinforces the initial loan volume increse.

On the other hand, a countercyclical monetary policy induces a small rise in $m^{*}$. This is a strong disincentive for saving for workers who want to become entrepreneurs, and leads to a drop in the volume of deposits and equity. Such drop is partly offset by an increase of the pool of depositors and a rise in the deposit interest rate. These offsetting moves are behind the relatively small changes in the volume and the composition of banks' balance sheets.

Banks' decision to change $m^{*}$ in an asymmetric way is just a reflection of the equilibrium nature of the problem. With procyclical monetary policy, banks' desire to give more loans requires a rise in their equity funding (capital requirements bind). Yet equity is more risky in bad states and households channel their savings away from equity and into deposits. Therefore, in order to expand their loans, banks must make the vision of entrepreneurship (a motivation for saving) highly desirable to get sufficient equity - hence a sharp drop in $m^{*}$. On the contrary, a countercyclical monetary policy motivates a loan volume drop which is achieved by an increase in $m^{*}$. Such increase can be small because for any amount of savings, risk-averse households prefer deposits in bad times anyway.

The heterogenous agent setup of this model highlights the effects of the changes in distribution of assets and bank financing on loan activity. In particular, it shows the asymmetric propagation of the monetary policy.

### 4.3 Countercyclical capital requirements

The interest rate is one of two instruments the central bank can use. The other is to modify the capital requirements, which in the benchmark economy are set at a $8 \%$ equity/loan ratio, as in the Basle Accord. As it appears capital requirements have an impact on the model economy, one may want to establish whether it can be used for cyclical purposes as well. In the first experiment, Figure 6, the equity/loan ratio is allowed to be reduced to $7 \%$ in the worst aggregate state only. While the banks can now offer more generous conditions, in this state only, households observe higher bank risk and shift from equity to deposits sufficiently to counterbalance and decrease the loan mass. As for a bond rate reduction, the average loan size decreases as the number of entrepreurs barely changes compared to the benchmark economy.

The next experiment involves a gradual decline of the capital requirements during the bad shocks, Figure 7. One would expect that the regulator allowing the banks to take more risks during a downturn may generate more loans. To the contrary, equity declines even more, resulting in a smaller loan mass. Interestingly, loans are lower even when the regulator does not intervene and has in fact slightly more stringent capital requirements to maintain the same average as in the benchmark. The reasons are the same as previously: households, through the mutual fund, shy away from banks when they take on more risk.

### 4.4 Procyclical capital requirements

If countercyclical capital requirements have adverse effects, maybe procyclical ones have a positive impact on lending ability. Figure 8 looks at the punctual policy, Figure 9 at the gradual one. Both policies have positive effects, locally and small for the first one, globally and massively for the second one. Thus it appears that tightening capital requirements is good for loan activity because it improves the financing of the banks. In this case the arguments are symmetric to the countercyclical policies.

Note that we have no informational problem in the model economy that would actually require the imposition of capital requirements. One can easily imagine that if the model would include this it would only reinforce the result: the presence of more entrepreneurial risk leads to a higher impact of asymmetric information and risk, thus furthering the need for regulation.

### 4.5 Credit crunch? What exactly happens in the model?

A negative aggregate shock lowers the expected project returns and increases their volatility. This affects the loan volume and the lending rate in four ways. First, both these effects decrease the expected value of risk-averse entrepreneurs ( $E_{r} V_{E}$ ) while the value functions of non-entrepreneurial households do not change. ${ }^{13}$ Therefore the incentive to accumulate assets in order to be eligible for a loan declines. This lowers the demand for credit because fewer agents save enough to pass the $m^{*}$ cutoff. Second, the risk-neutral banks only care about the expected return of projects. The relative net payoff of bonds versus loans rises and induces a substitution from loans to bonds. The loan supply drops and the lending rate $r^{L}$ increases to compensate for higher loan losses. This is the credit supply effect (i.e. the "crunch"). Third, an increase in $r^{L}$ further discourages loan applicants because their net return on investment declines, and the equilibrium credit level drops further. Therefore the post-shock equilibrium exhibits a higher lending rate and a lower level of loans which further propagates the shock. Note that the decline in the market-clearing volume of credit is partly demand-driven, and can not be only attributed to the the credit crunch behavior of the banks. Fourth, the mutual fund perceives more risk in the bank when entrepreneurial risk increases. It then shifts, on behalf of households, from equity to insured deposits, thus making it harder for banks to meet the capital requirements.

### 4.6 Does the equity market worsen or soften the credit decline?

The existence of equity market can either amplify or reduce the impact of a negative shock on a volume of credit. Only the second and fourth of the above mentioned four effects is directly affected by the existence of an equity market. The equilibrium condition (15) shows that only changes in $r^{L}$ and $\epsilon$ affect credit behavior through the equity channel, and they do so in an offsetting manner. An increase in $\epsilon$ (higher loan losses) increases the return on equity $r^{E}$, while

[^9]an increase in $r^{L}$ lowers it. We therefore distinguish two cases. (A) If $\mathrm{d}\left(r^{L}-\left(1+l_{c}\right) \epsilon\right)<0$, then a rise in $r^{E}$ increases the cost of funds to the bank which squeezes the profit margin further and leads to an additional substitution from loans to bonds ( $L$ drops) as well as an increase in $r^{L}$. At the same time $r^{P O R T}$ increases, making borrowing relatively less attractive (demand for credit drops). In this case, the presence of the equity market worsens the credit decline: a higher $r^{E}$ is only compatible with a lower amount of equity $E$ on the market ${ }^{14}$, which in turn requires an additional drop in loans due to a binding capital adequacy constraint (see equation (22)). Case (B) when $\mathrm{d}\left(r^{L}-\left(1+l_{c}\right) \epsilon\right)>0$ has the opposite implication - it softens the effects of financial accelerator.

According to the simulations (comparing peak and trough states), $\mathrm{d}\left(r^{L}-\left(1+l_{c}\right) \epsilon\right)=$ 0.0002 and we can conclude that the presence of the equity market softens the credit crunch.

## 5 Conclusion

We used a complex model to study the interaction of household saving decisions, project returns, Basle Accord type banking regulation and credit activity. We find that the Basle Accord has a noticeable impact on loans when project returns decline through the cycle. Active monetary policy through interest rate reductions in bad time is able to put loan activity at a higher level, but without removing its cyclical nature.

A relaxation of the Basle Accord capital requirements in bad times obtains negative results, as households shy away from the equity banks need to make loans. As in models with informational problems, of which there are none here, a tightening is in order. This calls therefore for regulatority policy to be active through the cycle, instead of the immuable policies currently in place.

Our results also emphasized that it is important to take into account the financing of banks. Given capital requirement, banks are limited in their lending by the bank equity households are willing to hold. As this decision is influence by interest rates, this gives rise to another channel of monetary policy. This channel has also been identified by Chami and Cosimano (2001) and van der Heuvel (2001). Unlike these papers, we do not require asymmetric information, market power in the banking industry or increasing marginal cost of financing.

## A Appendix: Solving the banks' problem

Due to the inequality constraints, we have to use a Kuhn-Tucker approach and be careful about the corner solutions. The Lagrangean for this problem is:

$$
\begin{aligned}
\mathcal{L}= & r^{l} L+r^{b} B-r^{r} D-r^{e} E-\delta\left(\frac{D}{E}\right)^{\gamma} D-\left(1+l_{c}\right) \epsilon L \\
& +\lambda_{1}(D+E-B-L)+\lambda_{2}(E / L-\alpha)+\lambda_{3}(D+E-L)
\end{aligned}
$$

[^10]Then the first order conditions are:

$$
\begin{aligned}
r^{l}-\lambda_{1}-\lambda_{2} E / L^{2}-\lambda_{3}-\epsilon\left(1+l_{c}\right) & =0 \\
r^{b}-\lambda_{1} & =0 \\
-r^{d}-\delta(\gamma-1)\left(\frac{D}{E}\right)^{\gamma}+\lambda_{1}+\lambda_{3} & =0 \\
-r^{e}+\delta \gamma\left(\frac{D}{E}\right)^{\gamma+1}+\lambda_{1}+\lambda_{2} / L+\lambda_{3} & =0
\end{aligned}
$$

As noted above, there are two possibilities: either constraint (8) or constraint (9) bind. In terms of the Lagrangean we therefore need to consider two cases. The one where $\lambda_{2}>0$ and $\lambda_{3}=0$ (i.e. (8) binds while (9) does not) will be referred to as an "interior solution" because not all loanable funds are invested into loans. The opposite case where $\lambda_{3}>0$ and $\lambda_{2}=0$ will be referred to as a "corner solution". For simplicity, in what follows we assume $\gamma=1$.

## Interior solution

This is the case when bank holds just enough equity to satisfy the capital adequacy requirement $(E / L=\alpha$ and therefore $D+E>L)$. The above first order conditions can be combined into:

$$
\begin{align*}
r^{d} & =r^{b}-2 \delta \frac{D}{E}  \tag{20}\\
\frac{M}{E} & =1+\left[\frac{1}{\delta}\left(r^{e}-r^{d}\right)-\frac{1}{\alpha \delta}\left(r^{l}-r^{d}-\left(1+l_{c}\right) \epsilon\right]^{\frac{1}{2}}\right.  \tag{21}\\
L & =\frac{1}{\alpha} E  \tag{22}\\
D & =M-E \tag{23}
\end{align*}
$$

where (21) is an equity (or implicitly deposit) supply equation. Conditional on particular values of M and all levels of prices, equations (20) to (23) form a recursive system which uniquely determines all quantities.

## Corner solution

In a corner solution, bank holds more equity than required by the capital adequacy requirement $(D+E=L$ and therefore $E / L>\alpha)$. Now, $r^{b}>r^{d 15}$, and the above first order conditions can be combined into:

$$
\begin{align*}
\frac{M}{E} & =1+\left[\frac{r^{e}-r^{l}+\left(1+l_{c}\right) \epsilon}{\delta}\right]^{\frac{1}{2}}  \tag{24}\\
L & =M  \tag{25}\\
D & =M-E  \tag{26}\\
r^{l}-r^{b}-\left(1+l_{c}\right) \epsilon & =r^{b}-r^{d} \tag{27}
\end{align*}
$$

where (24) is again an equity supply equation. Note that now loans and equity supply decisions are disconnected. Equation (27) shows a wedge between the bond and deposit rates. The bond

[^11]"premium" on the right hand side equals the profit differential between net returns on loans and bonds that would equal zero in an interior solution.

## B Appendix: On the assumption of a single mutual fund

It may seem problematic to assume that a representative mutual fund allocates the portfolio in an identical manner for all households. Here we show that as long as the labor income remains the same across all depositors, the optimal splitting rules derived from their preferences will be identical across all of the households.

To prove this point, we use a simplified version of the problem. Households maximize $\max _{\left\{c_{t, i}, m^{i}, t+1, d_{t, i}, e_{t, i}\right\}} \mathrm{E}_{0}\left[\sum_{t=0}^{\infty} \beta^{t} U\left(c_{t, i}\right)\right]$ s.t. $c_{t, i}+d_{t, i}+e_{t, i}=m_{t, i}+y_{i}$, where $U\left(c_{t, i}\right)=$ $\frac{\left(l_{c c}^{\sigma} c_{t, i}^{1-\sigma}\right)^{1-\rho}-1}{1-\rho}, m_{i, t+1}=d_{t, i}\left(1+r_{t}^{d}\right)+e_{t, i}\left(1+r_{t}^{e}\right)$ and $e_{t, i}, d_{t, i}$ denote individual equity and deposit holdings, respectively. The Euler equations for this problem are:

$$
\begin{aligned}
& c_{t, i}^{\chi}=\beta \mathrm{E}_{\mathrm{t}}\left[c_{t+1, i}^{\chi}\left(1+r_{t+1}^{d}\right)\right]=\beta\left[\mathrm{E}_{\mathrm{t}}\left(1+r_{t+1}^{d}\right) \mathrm{E}_{\mathrm{t}}\left[c_{t+1, i}^{\chi}\right]\right] \\
& c_{t, i}^{\chi}=\beta \mathrm{E}_{\mathrm{t}}\left[c_{t+1, i}^{\chi}\left(1+r_{t+1}^{e}\right)\right]=\beta\left[\mathrm{E}_{\mathrm{t}}\left(1+r_{t+1}^{e}\right) \mathrm{E}_{\mathrm{t}}\left[c_{t+1, i}^{\chi}\right]+\operatorname{cov}\left[\left(1+r_{t+1}^{e}\right), c_{t+1, i}^{\chi}\right]\right]
\end{aligned}
$$

where $\chi=-(\sigma+\rho(1-\sigma))$. Solving with a method of undetermined coefficients, we make an educated guess that $e_{t, i}=\gamma_{e} m_{t, i}$ and $d_{t, i}=\gamma_{d} m_{t, i}$ and rewrite the Euler equations as:

$$
\begin{aligned}
& {\left[\left(1-\gamma_{e, i}-\gamma_{d, i}\right) m_{t, i}+y_{i}\right]^{\chi}=\beta\left(1+r^{d}\right) \mathrm{E}_{\mathrm{t}}\left[\left(1-\gamma_{e, i}-\gamma_{d, i}\right) m_{t+1, i}+y_{i}\right]^{\chi}} \\
& {\left[\left(1-\gamma_{e, i}-\gamma_{d, i}\right) m_{t, i}+y_{i}\right]^{\chi}=\beta \mathrm{E}_{\mathrm{t}}\left[\left(1+r_{t+1}^{e}\right)^{1 / \chi}\left[\left(1-\gamma_{e, i}-\gamma_{d, i}\right) m_{t+1, i}+y_{i}\right]\right]^{\chi}}
\end{aligned}
$$

The above two equations give determine the shares of equity $\gamma_{e, i}$ and deposits $\gamma_{d, i}$ in $m_{t+1, i}$ as functions of individual as well as aggregate variables. The important point for our argument is that if all agents have the same labor income $y_{i}=y \forall i$, then we can harmlessly assume that $y=0$ and these two equations collapse into:

$$
\begin{align*}
1 & =\beta\left(1+r^{d}\right) \mathrm{E}_{\mathrm{t}}\left[\left[\gamma_{d}\left(1+r^{d}\right)+\gamma_{e}\left(1+r_{t+1}^{e}\right)\right]\right]^{\chi}  \tag{28}\\
1 & =\beta \mathrm{E}_{\mathrm{t}}\left[\left(1+r^{e}\right)^{1 / \chi}\left[\gamma_{d}\left(1+r^{d}\right)+\gamma_{e}\left(1+r_{t+1}^{e}\right)\right]\right]^{\chi} \tag{29}
\end{align*}
$$

Note that in equations (28) and (29), $\gamma_{e}$ and $\gamma_{d}$ are independent of any individual variables. They are only functions of the rates of return and the parameters of the utility function. This way we have shown that as long as the agents have an identical labor income (and as long as their deposit and equity demands are linear in their asset holdings which can be proved for the case of $y=0$ ), the portfolio-splitting decisions can be assumed to be made by a single Mutual fund.

Because in this model we work with two types of depositors (workers and retirees/unemployed), the assumption of an identical labor income is only justified within these two groups. In the future research, we should therefore model the number of distinct mutual
funds equal to 2 . The optimality conditions will then present additional identification restrictions on the parameter values of Mutual fund's risk-aversion parameter $\lambda$. We ignore this in the current version of the paper due to the computational difficulty.

## C Appendix: The solution procedure

Heterogeneous agents models with aggregate shocks are difficult to solve because the distribution of agents is not invariant and becomes a highly dimensional state variable. The two main strategies to solve this problem is to either find a good way to summarize the distribution with very few variables, as Krusell and Smith (1998) demonstrate, or to work with linearization, as Cooley and Quadrini (1999) do. Unfortunately, neither is possible here due to some highly non-linear phenomena that are crucial in our model. For example, decision rules change abruptly in the vicinity of $m^{*}$. Finally, second degree effects appear to be quite important, and they are likely to vanish with linearization.

Our strategy uses the realization that aggregate shocks in a two-state Markov process lead to transitional states somewhere between two steady-states corresponding to repeated identical shocks. We therefore choose a sufficient number of aggregate states to represent a large proportion of actual aggregate states.

The aggregate state space is assumed two dimensional: one dimension is the current shock, High or Low, the other is a counter of how far from the the High steady-state the economy is. Specifically, this counter is incremented by one each time a Low shocks occurred in the previous period, or decreased by one if a High shock occurred. The minimum counter value is one, the maximum is chosen such that this state occurs infrequently. We choose a maximum of 5, implying with the transition probabilities of the Markov process that the economy will in any of the aggregate states $S_{s c} \%$ of the time, where

$$
S=\left(\begin{array}{rrrrr}
50.2 & 16.7 & 5.6 & 1.9 & 0.6 \\
16.7 & 5.6 & 1.9 & 0.6 & 0.2
\end{array}\right)
$$

We then solve this model economy with the standard tools for heterogeneous agent economies, that is value function iterations followed by iterations on the invariant distribution (defined over the aggregate states as well). The equilibrium is reached by finding the set of lending rates $r^{l}$ and loan eligibility rules $m^{*}$ that balance all markets and satisfy all constraints.

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## D Figures

Figure 1: Benchmark economy as it cycles through all possible aggregate states













Figure 2: Benchmark and policy with interest rate reduction in worst case only


Figure 3: Benchmark and policy with gradual interest rate reduction in bad return situations


Figure 4: Benchmark and policy with interest rate increase in worst case only


Figure 5: Benchmark and policy with gradual interest rate increase as aggregate states worsen


Figure 6: Benchmark and policy with relaxing of capital requirements in worst case only


Figure 7: Benchmark and policy with gradual relaxing of capital requirements as aggregate states worsen


Figure 8: Benchmark and policy with tightening of capital requirements in worst case only


Figure 9: Benchmark and policy with gradual tightening of capital requirements as aggregate states worsen



[^0]:    ${ }^{1}$ This work benefited from a lot of initial stimulus from Mingwei Yuan, as well as from discussions with Scott Hendry, Césaire Meh, Kevin Moran, Vincenzo Quadrini and seminar participants at the Universities of Alabama, Connecticut, Florida International and Montréal. The views expessed in this paper are those of the authors. No responsibility for them should be attributed to the Bank of Canada. Any errors or omissions are those of the authors.
    ${ }^{2}$ Berka: Department of Economics, University of British Columbia, \#997-1873 East Mall, Vancouver, B.C. V6T 1Z1 Canada; phone 1-604-822-5661; fax: 1-604-822-5915; email: mberka@interchange.ubc.ca
    Zimmermann: Bank of Canada, CREFE and UQAM. Contact address: Department of Economics, Université du Québec à Montréal, P.O. Box 8888, Montreal H3C 3P8, Canada; phone 1-514-987-3000 ext 0277\#; fax: 1-514-987-8494; email: zimmermann.christian@uqam.ca

[^1]:    ${ }^{1}$ The heterogeneity of firms we obtain is then nedogenous. Bernanke, Gertler and Gilchrist (1998) also have heterogeous firms, but they exogenously fix a share of firms to have easy access to credit.

[^2]:    ${ }^{2}$ Once retired, household can not become productive again.
    ${ }^{3}$ Therefore at a household level, demand for loans is uniquely determined by the net worth and so by the history of consumption-savings decisions and luck.
    ${ }^{4}$ Actually, a participation constraint is imposed, and it was always satisfied in our experiments.

[^3]:    ${ }^{5}$ A prime " $"$ " denotes variable values in the next period $t+1$.

[^4]:    ${ }^{6}$ The total amount of assets flowing through the financial sector is determined by households' decisions. Fraction $\phi$ of the total "financial" assets (the self-financed part of entrepreneur's project does not enter financial sector) has to equal total bank liabilities and therefore also bank's assets (see equation 14).
    ${ }^{7}$ The chances that both of them bind at the same time can be dismissed as arbitrarily low.
    ${ }^{8}$ The assumption of a single intermediary deciding on a portfolio split in the same manner for all households is not necessarily innocuous. As the Appendix B shows, as long as the households have the same labor income, their optimal splitting rule between equity and deposits is constant and identical for all households due to CRRA preferences. With labor income varying between workers and unemployed/retired, the optimal splitting rule may change. In future work, we therefore may consider modeling of a distinct mutual fund for the non-working population so as to maintain the current structure of the model.

[^5]:    ${ }^{9}$ One can think of banks depositing their non-loanable investments at the central bank which also sets the deposit rate in this model.

[^6]:    ${ }^{10}$ This follows from the fact that $\frac{E}{D}=\frac{\omega_{R}}{1-\omega_{R}}$ and $\omega_{R}$ increases in $R^{E}$.

[^7]:    ${ }^{11}$ This measure is based on the replacement rate of Hornstein and Yuan (1999).

[^8]:    ${ }^{12}$ Note that all experiments are designed such that the average $r^{b}$ or $m^{*}$ stay at the same level.

[^9]:    ${ }^{13}$ There is only a second order effect coming from expectations to be an entrepeneur in the future.

[^10]:    ${ }^{14}$ This is because households are risk averse while banks are risk neutral.

[^11]:    ${ }^{15}$ A lower demand for bank's financing by deposits (relative to equity) depresses their price.

