Admission, Tuition, and Financial Aid Policies in the Market for Higher Education*

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Abstract

In this paper, we present a general equilibrium model of the market for higher education. Our model simultaneously predicts student selection into institutions, financial aid, and educational outcomes. We show that the model gives rise to a strict hierarchy of colleges that differ by the educational quality provided to the students. We develop an efficient algorithm to compute equilibria for these types of models. To evaluate the model, we develop an estimation strategy that accounts for the fact that important variables are likely to be measured with error. We estimate the structural parameters using data collected by the National Center for Educational Statistics and aggregate data from Peterson's and NSF. Our empirical findings suggest that our model explains observed admission and price policies reasonably well. The findings also suggest that the market for higher education is quite competitive.

Keywords: higher education, peer effects, school competition, non-linear pricing, equilibrium analysis, empirical analysis.

JEL classification: I21, C33, D58

1 Introduction

Over the past several years, research has investigated normative and positive consequences of competition in primary, secondary and higher education, and the likely effects of policy changes including vouchers, public school choice, and changes in education financing.[1] Some of this research has relied on a set of general equilibrium models. Given the absence of large scale policy experiments, these models have been a primary tool to evaluate the impact of a variety of education reform measures. To date, the predictions of these models have been subjected to little formal empirical testing. This paper provides an integrated approach for estimation and inference based on this class of models. For this purpose, we focus on the market for higher education. Colleges and universities provide a promising environment for developing this approach because a variety of data sets collected by the National Center for Education Statistics and commercial companies such as Peterson's are available to researchers.

In the first part of the paper, we present a general equilibrium model of the market for higher education that extends earlier work on competition in the market for primary and secondary education. In our model, schools seek to maximize the quality of the educational experience provided to their students. The quality of the educational experience depends on peer ability of the student body, a measure of income diversity, and on instructional expenditures per student. If peer quality is an important component of school quality, students and their parents will seek out schools where the student body offers high quality peers.[2] Likewise, schools will attempt to attract students who contribute to improving peer

[1]Theoretical studies include Arnott and Rowse (1987), Bearse, Glomm, and Ravikumar (1996), Benabou (1996a, 1996b), Caucutt (2002), deBartolome (1990), De Fraja and Iossa (2002), Ehrenberg and Shermam (1984), Epple, Newlon, and Romano (2002), Epple and Romano (1998, 1999), Fernandez and Rogerson (1996, 1998), Manski (1991), Nechyba (1999, 2000), Rothschild and White (1995), and Stiglitz (1974). Empirical studies include Bergstrom, Rubinfeld, and Shapiro (1982), Cullen, Jacob, and Levitt (2000), Carlton, Bamberger, and Epstein (1995), Coleman (1966), Downes and Greenstein (1996), Dynarski (1999), Epple, Figlio, and Romano (1998), Fuller, Manski, and Wise (1982), Hoxby (1997, 1999, 2000a), Hsieh and Urquiola (2001), Rouse (1998), and Venti and Wise (1982).

[2]There is a large, growing, and controversial literature on peer effects by social scientists. Here we mention just some of the empirical studies by economists that are most closely related to our model. Most of the literature on peer effects on educational success concern primary and secondary education. Early studies are Coleman (1966), Henderson, Mieszkowski, and Sauvageau (1978), and Summers and Wolfe (1977). Manski

quality. In higher education, schools have the latitude to choose price and admission policies to attempt to attract a high quality student body. Our model thus yields strong predictions about the hierarchy of schools that emerges in equilibrium, the allocation of students by income and ability among schools, and about the pricing policies that schools adopt.[3]

In the second part of the paper, we provide an integrated approach for testing predictions from this class of models. The central idea of the estimation strategy is to match the admission and financial aid policies observed in the data to those predicted by our equilibrium model. The estimation is computationally intensive since it requires us to solve for the general equilibrium of the model at each step of the estimation algorithm. We develop an efficient computational algorithm to compute the equilibrium of our model. This allows us to estimate the structural parameters of the model using a Maximum Likelihood Estimator. We implement our estimation strategy using micro data collected by the National Center for Educational Statistics and school level data collected from a variety of other sources.[4]

Our empirical findings suggest that individuals sort based on abilities of their peer group. We also find that income diversity plays a significant role in the determination of school quality. One interpretation of this finding is that schools and students believe that the quality of a student's educational experience is enhanced by interacting with peers from diverse socioeconomic backgrounds. We also find that expenditure per student is a substantial component of school quality. Given schools' posted tuitions and financial

(1993) details the several difficulties in empirically identifying peer effects. Evans, Oates, and Schwab (1992) find no peer effects in predicting teenage pregnancy or school drop-out once selection is taken into account. Robertson and Symons (1996), Zimmer and Toma (1998), Hoxby (2000b), and Ding and Lehrer (2001) are more recent studies that find evidence of peer effects on educational success. Turning to research on peer effects in higher education, Sacerdote (2001) and Zimmerman (2000) find peer effects between roommates on grade point averages. Betts and Morell (1999) find that high-school peer groups affect college grade point average. Arcidiacono and Nickolson (2000) find no peer effects among medical students. Dale and Krueger (1998) has mixed findings and is discussed further below. This literature investigates how an outcome variable for an individual (e.g., school achievement) is affected by the individual's peers. We adopt a somewhat different but complementary approach, investigating whether college financial aid policies and the equilibrium allocation of students among colleges are consistent with predictions derived from a model in which peer effects are present.

[3] An insightful overview of the college quality hierarchy and its determinants is provided by Winston (1999).

[4] For empirical evidence on tuition policies see among others Hoxby (1997, 1999) and Epple, Romano, and Sieg (2002).

endowments, our model endogenously generates a distribution of expenditures per student among schools that matches the empirical distribution well. Our model also predicts the allocation of students across schools and the allocation of financial aid across students. The model replicates reasonably well both school admissions and the variation in financial aid packages received by students in the data.

In addition to contributing to understanding of the market for higher education, our paper attempts to advance the state of the art in unifying theory, computation of equilibrium, and estimation. We propose a theoretical framework characterizing equilibrium with quality competition, and we derive equilibrium implications of that theory. A counterpart computational model is then developed and solved for equilibrium. The model predicts sorting along two dimensions of the type space (household income and student ability), and entails equilibrium price discrimination with respect to both of these dimensions. Formulating and solving this computational model is a challenging undertaking. To set the stage for estimation, we obtained and merged three databases uniting student-level and college-level data. These data and our computational equilibrium model are then brought together for estimation of the parameters of the model. In the search for the maximum likelihood estimates, the computational model is solved for each set of parameters at which the likelihood function is evaluated. The result is, we believe, the first instance in which an equilibrium model with sorting and price discrimination along multiple dimensions has been estimated.

The paper is organized as follows. Section 2 lays out the general equilibrium model and derives a set of conditions that characterize the equilibrium allocation. The estimation strategy is explained in Section 3. Section 4 provides information about our data set, which is created by drawing together information from the National Center for Education Statistics, the National Science Foundation, and Peterson's. The empirical results are discussed in Section 5, and Section 6 presents the conclusions of the analysis and discusses future research.

2 A Theoretical Model of Higher Education

In this section, we sketch our theoretical model of provision of undergraduate education.

2.1 Preferences and Technologies

There is a continuum of potential students who differ with respect to their income, y , and their ability level, b. The joint distribution of income and ability is continuous with joint density $f(b, y)$. Each student chooses among a finite set of J schools. The quality of school j is given by

$$q_j = I_j^{\omega} \theta_j^{\gamma} \; d_j^{\psi} \quad \omega,\ \gamma,\ \psi > 0 \tag{2.1}$$

where θ_j is the peer-student measure, equal to mean ability level in the student body, I_j is the expenditure per student in excess of minimal or "custodial costs," and d_j is a measure of income diversity. As noted above, the data suggest that school quality includes an income diversity component. Denote mean income in school j by μ_j^y, and mean income in the population of all households by μ_p^y. When $\mu_j^y > \mu_p^y$ (as holds empirically), a simple but appealing diversity measure is the ratio of mean income in the population to mean income in school j:

$$d_j = \frac{\mu_p^y}{\mu_j^y} \tag{2.2}$$

The schooling cost function is

$$C(k_j, I_j) = F \; + \; V(k_j) \; + k_j \; I_j \qquad\qquad V', V'' > 0 \tag{2.3}$$

where k_j is the size of the school j's student body, and I_j is per student expenditure on quality enhancing inputs. Schooling costs include components that are independent of educational quality, the custodial costs mentioned above. In the empirical implementation of the model we assume that variable custodial costs are cubic in k_j:

$$V(k_j) = c_1\, k_j \; + \; \frac{c_2}{2}\, k_j^2 \; + \; c_3\, |k_j - k^*|^3 \tag{2.4}$$

where $k^* = \text{argmin} \frac{F+V(k)}{k}$ is the efficient school size.

Substantial financial aid to many undergraduates in the form of grants, loans, and work-study funding is provided by the federal government and to a lesser extent by other entities that are also independent of the student's school. We refer to such aid as non-institutional aid. Much of this aid is based on the federal government's calculation of the family's ability to pay. Let p_j denote the tuition at school j. We presume that the value of non-institutional aid to the student at school j, denoted a_j, can be written as:

$$a_j \quad = \quad a_j(b, y, p_j) \tag{2.5}$$

Most noninstitutional aid is need based, implying it depends on income and tuition of the school attended. Some noninstitutional aid seems to be meritorious, so we will allow it to depend on ability as well.[5]

We assume that the decision to attend college is made by the student's household. Household utility depends on numeraire consumption and educational attainment (T) of the college-aged member. Educational attainment is a function of quality and ability: $T = T(q_j, b_j)$. Specifically, household utility from attendance at school j is given by:

$$U(y - p_j + a_j, T(q_j, b_j)) \quad = \quad (y - p_j + a_j)\; q_j\; b_j^{\beta} \tag{2.6}$$

Let $r_j = r_j(q_j, b, y)$ denote a student's reservation price for attending a school of quality q_j. That is, r_j, satisfies:

$$U(y - r_j, q_j, b_j) \quad = \quad U_j^A(b, y) \tag{2.7}$$

[5] More detail on types of aid is provided below, when we empirically estimate the non-institutional aid formula.

where $U_j^A(b, y)$ is the maximum alternative utility to school j that type (b, y) can attain in equilibrium. Households choose among schools (or no school) taking as given school qualities and their tuition and admission policies.

2.2 School Optimization

Schools are assumed to maximize their quality. Schools take types' (b, y) utilities as given. They must satisfy a profit constraint, with revenue equal to the sum of all tuition from students plus earnings on exogenous endowment. Denote the latter earnings R_j. R_j also includes any other non-tuition revenues like state subsidies. We assume that there are J schools with differing endowment earnings: $R_J > R_{J-1} > ... > R_1 \geq 0$.[6] While schools will condition tuition on student characteristics, we presume that school j charges a maximum tuition denoted p_j^m. We do not have an explicit theory to explain or determine the magnitude of p_j^m so we treat it as exogenous. Our motivation for introducing price caps is empirical. We interpret the price maximum as the school's posted tuition, with lower tuition framed as financial aide, a scholarship, or, perhaps, a fellowship.[7]

Given the school's objective function, it will be optimal for j to set tuition p_j^r such that:

$$r_j = p_j^r - a_j \tag{2.8}$$

if p_j^r satisfying (2.8) is below p_j^m. This is simply because schools always want to maximize revenue for a given student body. School j's optimization problem may be written:

$$\max_{\alpha_j(b,y), p_j(b,y), k_j, \theta_j, \mu_j^y, d_j} q_j \;=\; \theta_j^\gamma \; I_j^\omega \; d_j^\psi \tag{2.9}$$

[6]The number of schools and their endowments are taken as exogenous. We do not allow for entry and restrict attention to cases where all J schools can cover their costs.

[7]Having a price cap below the maximum tuition chosen by an unconstrained quality maximizer might help a school market itself. We assume, however, that households observe all prices relevant to them, so our argument for price caps is somewhat incomplete. One can also conceive of the self-imposed price cap as reflecting some limit on revenue making, whether motivated by altruism or, again, related to marketing.

such that:

$$
\begin{aligned}
[y - r_j]\, \theta_j^{\gamma}\, I_j^{\omega}\, d_j^{\psi}\, b^{\beta} &= U_j^a(b,y) \quad \forall\, (b,y) && (2.10)\\
p_j^r(b,y,q_j) &= r_j(b,y,q_j) \;+\; a_j(b,y,p_j^r(b,y,q_j)) \quad \forall\, (b,y) && (2.11)\\
p_j(b,y,q_j) &= \min\{p_j^m, p_j^r(b,y,q_j)\} \quad \forall\, (b,y) && (2.12)\\
F \;+\; V(k_j) \;+\; k_j\, I_j &= \int\!\!\int p_j(y,b,q_j)\, \alpha_j(y,b)\, f(b,y)\, db\, dy \;+\; R_j && (2.13)\\
\alpha_j(y,b) &\in [0,1] \;\; \forall\, (b,y) && (2.14)\\
k_j &= \int\!\!\int \alpha_j(y,b)\, f(y,b)\, dy\, db && (2.15)\\
\theta_j &= \frac{1}{k_j} \int\!\!\int b\, \alpha_j(y,b)\, f(y,b)\, dy\, db && (2.16)\\
\mu_j^y &= \frac{1}{k_j} \int\!\!\int y\, \alpha_j(y,b)\, f(y,b)\, dy\, db && (2.17)\\
d_j &= \frac{\mu_p^y}{\mu_j^y} && (2.18)
\end{aligned}
$$

where here and henceforth integrals are over the support of (b, y) unless otherwise indicated. Constraint (2.10) defines, again, the reservation price, and (2.11) and(2.12) the implications for the school's optimal price given noninstitutional financial aid and the price cap constraint. (2.13) is the budget constraint, which will obviously hold with equality as specified at the school's quality maximum. The function $\alpha_j(y, b)$ is an admission function that indicates the proportion of type (b, y) in the population that school j admits.[8] In equilibrium, admission sets and attendance sets must coincide. The constraints (2.15), (2.16), (2.17), and (2.18) define school size, the peer group measure, the mean school income, and the diversity measure respectively.

[8]Since $\alpha_j = 0$ is feasible, it is innocuous to have specified that (2.12) holds for all students, i.e. , including those that will not be admitted.

The optimal admission criterion may be written:[9]

$$\alpha_j(b,y) \begin{pmatrix} = 1 \\ \in [0,1] \\ = 0 \end{pmatrix} \quad \text{as} \quad p_j \begin{pmatrix} > \\ = \\ < \end{pmatrix} \quad \text{EMC}_j(b,y) \tag{2.19}$$

where

$$\text{EMC}_j(b,y) \;=\; V'(k_j) \;+\; I_j \;+\; \frac{\partial q_j/\partial \theta}{\partial q_j/\partial I}\,(\theta_j - b) \;+\; \frac{\partial q_j/\partial \mu_j^y}{\partial q_j/\partial I}\,(\mu_j^y - y) \tag{2.20}$$

Equation (2.20) defines the "effective marginal costs (EMC)" of admitting a student of type (b, y) to school j. EMC is the sum of the marginal resource cost of educating the student, the cost of maintaining quality due to the student's impact on the peer group, and the cost associated with the diversity criterion. The peer effect is captured by the third term on the right-hand side in (2.20), which equals the peer measure change from admitting a student of ability b, multiplied by the expenditure change that maintains quality. Note that this term is negative for students with ability above the school's mean, and EMC_j itself can be negative. The diversity effect is captured by the last term in (2.20), which equals the diversity measure change from admitting a student of income y, multiplied by the expenditure change that maintains quality. Note that this term is positive for students with income above the school's mean, assuming the mean school income is higher than the mean income in the population.

Students whose maximum feasible tuition exceeds EMC_j permit quality increases and are all admitted, and the reverse for students who cannot be charged a tuition that covers

[9] Beside the constraints, the remaining first-order condition regards the choice of I_j. Letting $\lambda > 0$ denote the multiplier on the revenue constraint (2.13), this condition may be written:

$$\int \frac{\partial p_j}{\partial q_j}\frac{\partial q_j}{\partial I_j}\,\alpha\, f\, db\, dy \;-\; k_j = -\frac{\omega q_j}{\lambda I_j}$$

This result establishes that school j elects to spend more than the Samuelsonian amount on educational inputs, because school j values quality per se.

their EMC$_j$. Define

$$\alpha_j^r(b,y) = \begin{cases} \alpha_j(b,y) & \text{if } p_j(b,y) = p_j^r(b,y) \\ 0 & \text{otherwise} \end{cases} \tag{2.21}$$

Let, then, $A_j^r = \{(b,y) \mid \alpha_j^r(b,y) > 0\}$ denote the set of students that attend school j and pay their reservation price, and $A_j^m = \{(b,y) \mid \alpha_j(b,y) > 0 \text{ and } (b,y) \notin A_j^r\}$ denote the remaining students that attend j. Let $A_j = A_j^r \cup A_j^m$ denote school j's admission and attendance sets of student types.[10]

2.3 Market Equilibrium

In market equilibrium, households choose among the J schools or choose no school, taking school qualities and tuition and admission policies as given. The J schools choose admission and tuition policies to maximize quality, taking as given their endowment and students' alternative utility possibilities. The assumption of utility taking is a generalization of price taking that has been utilized in the competitive club goods literature.[11] The model is closed with the market clearing condition: $\sum \alpha_j(b,y) \leq 1 \quad \forall\ (b,y)$, where types for whom the inequality is strict are attending no school. Given our assumptions, the following Proposition holds:

Proposition 1 *A market equilibrium satisfies the following three conditions:*

1. *There is a strict quality hierarchy of colleges in equilibrium. The hierarchy follows the endowment ranking.*

2. *There exists a locus $b_j(y)$ for each college which defines the minimum ability that a student with income y must have to be admitted to school j. This threshold function is implicitly defined by the minimum b satisfying $EMC_j(b,y) = p_j(b,y)$.*

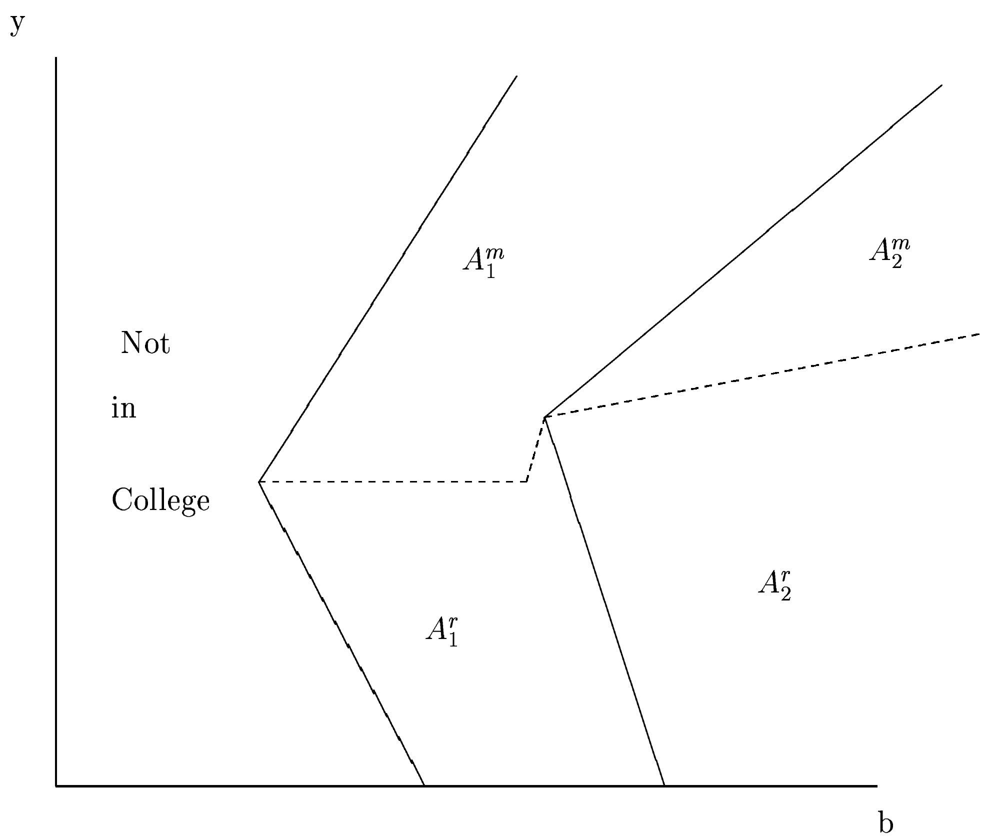
[10] The market-clearing condition presented below can be used to show that schools' attendance sets do not overlap with positive measure in the support of (b,y).

[11] See, for example, Gilles and Scotchmer (1997) and Ellickson, Grodal, Scotchmer, and Zame (1999).

3. *Choosing among the set of schools* $\widehat{J}(b,y) = \{j \in J | EMC_j(b,y) \leq p_j^m\}$ *and no college, student-type (b,y) attends the school that would maximize utility if* $p_j = EMC_j(b,y)$ *for all* $j \in \widehat{J}(b,y)$*. School pricing in excess of* EMC *is to take away consumer surplus (constrained by the price cap for some students).*

A proof of Proposition 1 is in Appendix A.

Figure 1: An Example of College Admission Spaces



This figure illustrates the boundaries of college admission sets.

Figure 1 shows an example of how type space is partitioned into schools, assuming just two schools. The tipped-L solid lines, or "boundary loci," separate types into schools and no college. Those to the right of the right-most boundary locus attend the higher-quality school 2. Those between the boundary loci attend school 1, with the rest not attending college. [12] The upward-sloping part of each boundary locus satisfies $p_i^m = EMC_i(b, y) \leq p_i^r$, and the downward-sloping part satisfies $p_i^r = EMC_i(b, y) \leq p_i^m$, where i is the number of the school to the right of the locus. The value of p_i^r depends on the student's type, including the options available to the student.

Consider students attending college 2 in this equilibrium. They all have access to college 1 at $EMC_1(b, y)$, since this is below p_1^m for all these students.[13] Their reservation price for college 2 is then determined by the utility they could obtain at college 1 at tuition $EMC_1(b, y)$. All students in college 2 not on the downward-sloping part of the boundary locus would strictly prefer college 2 at tuition equal to $EMC_2(b, y)$. College 2 charges all those in A_2^r their reservation price, which is below p_2^m, taking away their consumer surplus relative to attending college 1. Those in A_2^m have reservation price exceeding both $EMC_2(b, y)$ and p_2^m, so college 2 can "only" charge them p_2^m leaving them some surplus. Those in college 1 may or may not have access to college 2 (depending respectively on whether $EMC_2(b, y)$ is less than or greater than p_2^m), and those with access to college 2 may have it or no college as their preferred option. This somewhat complicates the determination of prices, but it is nevertheless straightforward to make the calculations. Note that students near the left-side boundary locus will pay tuition close to their $EMC(b, y)$ at their school, as will also students near the downward sloping portion of the right-side boundary locus (i.e., the latter being students who have access to the next best school.) Hence, the presence of a relatively large number of schools will squeeze attendance sets and lead to relatively competitive outcomes with tuition close to EMC for most students.[14]

While we can prove a number of theoretical properties of the model, additional insights can be gained by solving the model numerically and conducting comparative static exercises. Finding an equilibrium for the model is a classical fixed-point problem. To compute the fixed

[12] The no-college option is associated with given (high-school) educational quality q_0, an option for anyone at zero tuition. The example also assumes non-institutional financial aid is independent of ability as we find empirically (see below).

[13] To confirm the statement in the text, use that the upward-sloping line of the left-most boundary locus satisfies $p_1^m = EMC_1(b, y)$ and $EMC_1(b, y)$ is decreasing in b.

[14] In our model, $EMC_j(b, y)$ also equals the equilibrium social marginal cost of type (b, y)'s attendance at school j.

point, we must solve a $4 * J$ dimensional system of nonlinear equations. Solving this system of equations is challenging, since it requires computation of price and admission functions for each school. These are potentially highly nonlinear functions of ability and income. Appendix B provides a general algorithm which can be used to compute an equilibrium of the model.

In order to solve the model, we need to assign numerical values to the parameters of the model. Previous research has largely relied on calibrated parameters. Estimating the parameters is clearly more desirable, since little is known about reasonable magnitudes of most parameters of this model. Furthermore, estimation provides a more rigorous analysis of the empirical properties of the underlying equilibrium model. However, estimation is computationally intensive. The next section derives a general strategy which can be used to estimate the parameters of the type of models developed in this paper.

3 Identification and Estimation

3.1 A Maximum Likelihood Estimator

As we have seen in the previous section, our model yields strong predictions regarding admission and pricing by income and ability of students. We would like to investigate whether these predictions are consistent with the observed regularities in the data. In this section we develop a strategy to estimate the underlying parameters of the model. We assume that the joint distribution of log-income and ability among students who attend college is bivariate normal:[15]

$$\begin{bmatrix} \ln(y) \\ b \end{bmatrix} \sim N \left[\begin{pmatrix} \mu_{\ln(y)} \\ \mu_b \end{pmatrix}, \begin{pmatrix} \sigma^2_{\ln(y)} & \rho\sigma_{\ln(y)}\sigma_b \\ \rho\sigma_{\ln(y)}\sigma_b & \sigma^2_b \end{pmatrix} \right] \tag{3.1}$$

[15]In our empirical analysis, we simplify somewhat relative to our theoretical model by assuming that there is no outside option to attending college. Thus, our empirical analysis allocates the students observed to attend college among the available set of colleges, assuming every college student has the option of attending the worst college at its price cap. We adopt the log-normal distribution as an approximation of the distribution of types that attend college.

Notice that the parameters of this distribution can be directly estimated from the data. The remaining parameters of the baseline model to be estimated are $\zeta = (\omega, \gamma, \psi, F, c_1, c_2, c_3)$.[16] In this section we discuss how to estimate these parameters based on observed data on prices, ability and income.

Given the structure of our model the probability of observing (y, b) in school j is given by:

$$f_j(y,b) = \begin{cases} f(y,b)\ /k_j & \text{if } (y,b) \in A_j \\ 0 & \text{otherwise} \end{cases} \tag{3.2}$$

where $f(\cdot)$ is the joint density function of (y, b) and $A_j = \{(b, y)|\ \alpha(b, y) > 0\}$ is the set of individuals admitted to school j. As above, let $p_j(y, b)$ denote the price predicted by our model, which is a deterministic function of (y, b).[17] Assuming the difference between the predicted price p and the observed price $\tilde{p}$ is independent of (y, b), then the joint likelihood of $(y, b, \tilde{p})$ is given by:

$$f_j(\tilde{p},y,b) = \begin{cases} g(\tilde{p} - p_j(y,b))\ f(y,b)\ /\ k_j & \text{if } (y,b) \in A_j \\ 0 & \text{otherwise} \end{cases} \tag{3.3}$$

where $g(\cdot)$ is the density of the measurement error for price.

This likelihood function assigns zero probability to observations $(y, b) \notin A_j$. Hence the likelihood function for any particular sample will not realistically be well defined. We therefore assume that income and ability are also measured with error.[18] Let b (y) denote unobserved ability (income), and $\tilde{b}$ $(\tilde{y})$ the observed ability (income) which includes measurement error. Let $h_b(\tilde{b}|b)$ and $h_y(\tilde{y}|y)$ be the corresponding density functions. The

[16] The parameter β is neither identified nor relevant to the properties of equilibrium in this study. This is a property of the Cobb-Douglas specification of the utility function.

[17] We suppress q_j as a an argument of $p_j(\cdot)$ henceforth.

[18] Our approach is thus similar in spirit to pioneering work on kinked budget constraints by Hausman (1985).

probability of observing $(\tilde{p}, \tilde{y}, \tilde{b})$ in school j is then given by:

$$\begin{aligned} f_j(\tilde{p}, \tilde{y}, \tilde{b}) &= \int_{A_j} f_j(\tilde{p}, y, \tilde{y}, \tilde{b}, b)\; db\; dy \\ &= \int_{A_j} g(\tilde{p} - p_j(y,b))\; h_b(\tilde{b}|b)\; h_y(\tilde{y}|y) f(y,b) \;/\; k_j\; db\; dy \end{aligned} \tag{3.4}$$

where the admission space A_j is implicitly defined by the admission policies $\alpha_j(b, y)$. We assume that measurement errors in log income, ability, and price are additive, distributed normally, and drawn independently with standard deviations $(\sigma^e_{\ln(y)}, \sigma^e_b, \sigma^e_a)$ respectively.

The integral on the right-hand side of equation (3.4) is evaluated numerically.[19] The log-likelihood function for a sample of N students is then simply given by

$$L(\zeta) = \sum_{n=1}^{N} \sum_{j=1}^{J} w_n\; d_{jn}\; \ln(f_j(\tilde{p}_n, \tilde{b}_n, \tilde{y}_n)) \tag{3.5}$$

where w_n is the weight associated with observation n and d_{jn} is equal to 1 if individual n attends school j and zero otherwise. Weights are necessary to reflect the sampling design of the NCES data set used in the analysis.

The parameters of the likelihood function ζ can be decomposed into the structural parameters of the equilibrium model ζ_1 and the parameters of the distributions of the measurement errors ζ_2. Maximization of the likelihood function is computationally intensive since we need to solve for the equilibrium of the model for each evaluation of the likelihood function. The estimation procedure consists of an outer loop, which searches over the parameter space; and an inner loop which computes the equilibrium, boundary loci and the choice probabilities, and evaluates the likelihood function for each parameter vector.[20]

[19] For a discussion of simulation in estimation, see, for example, Pakes and Pollard (1989), McFadden (1989) and Gourieroux and Monfort (1993).

[20] The main difference between the estimation procedure used in this paper and the ones outlined in Berry, Levinsohn, and Pakes (1995), Epple and Sieg (1999), and Epple, Romer, and Sieg (2001) is that we solve for equilibrium of the model in the estimation.

4 Data

4.1 The Distribution of Income and Ability

Our primary data source is the National Post-secondary Student Aid Study (NPSAS) obtained from the National Center for Education Statistics (NCES). The NPSAS contains extensive information for a sample of students. Of particular relevance for our work, the NPSAS contains the student's performance on standardized tests (either SAT or ACT), income of the student's family, and information about the financial aid received by the student. We have secured from the NCES a restricted-use version of the NPSAS that contains student-level data for 1995-96 and links each student in the sample to the school the student attended in academic year 1995-96. We study four-year private colleges and universities. For a given wave of the NPSAS survey, the NCES chooses a set of colleges and universities. It then selects a sample of students from within each of those institutions. Our sample consists of 1837 incoming freshman students attending 159 different colleges and universities.[21] The mean SAT score is 1040 with a standard deviation of 200.[22] Mean income is 58,753 with a standard deviation of 39,035. The correlation between income and SAT score is 0.24. [23]

[21] In selecting our sample of students we deleted observations for students with athletic scholarships, since their criteria for admission may not conform to the spirit of our analysis. We also deleted a small number of observations for students with reported income near zero.

[22] Dale and Krueger (1998) present evidence that future earnings increase in college quality. Their results cast doubt on use of mean SAT as a measure of college quality, but they find evidence of positive impacts on students from lower-income households and that increasing within-college SAT variance tends to favorably affect students' future earnings. They also find in one selection specification that mean SAT has a positive impact on graduation and obtaining an advanced degree. They observe that their findings about the lack of effect of mean SAT score on future earnings might be because "the average SAT score is a crude measure of the quality of one's peer group." Consistent with this, we find substantial measurement error in SAT as a measure of quality. We model quality as dependent upon mean SAT, expenditure per student, and income diversity, capturing three components of college quality. Our theoretical model can be extended to a peer measure that is the mean of an increasing function of individual student scores, allowing increased variability in scores to have a positive effect. Dale and Krueger's results point to the potential importance of extending our empirical analysis to encompass this generalization of the peer measure.

[23] The estimates for $\mu_{ln(y)}$ and $\sigma_{ln(y)}$ are 10.798 and 0.605. We use these estimates as the parameters of the distribution of income and ability in our computational analysis. We then estimate the model allowing for measurement error in both income and ability while treating the estimated moments from the data as characterizing the true distribution of income and ability. Technically, this is a valid procedure for the means of the distribution, but not for the variances because of the measurement error.

4.2 Market Structure and Aggregation

In addition to data for individual students, we use college level data. Peterson's conducts a survey of all colleges and universities, obtaining information on faculty resources, financial aid, the distribution of standardized test scores, and a host of other variables. We have supplemented this data set with information on educational expenditures and endowments from the NSF Web accessible Computer-Aided Science Policy Analysis and Research (WebCASPAR) database. The Peterson's database contains a total of 1868 four year colleges and universities within the United States. We view our model as being better suited to characterizing private than public institutions. We have therefore eliminated public universities and colleges from our sample.[24] We also do not consider private colleges that are highly specialized, do not have a regular accreditation and have missing values for key variables that are the focus of our analysis. This leaves us with a sample of 824 private universities and colleges.

Before we conduct our empirical and computational analysis, we need to define the appropriate choice set faced by households. A natural starting point of the analysis is to treat each college as a differentiated product. The relevant choice set is then the total number of colleges in our sample which is 824. This approach has some obvious limitations that arise due to the large number of potential choices. One of the drawbacks of this approach is that we would need to observe the complete choice set faced by the individuals in order to test the predictions of our model. The NPSAS, however, does not sample all colleges in the population, but only a representative sample of colleges. The most recent NPSAS sample only contains students of 146 colleges of the 824 colleges in the Peterson's sample.

Perhaps more importantly, we do not expect that the strong predictions of our underlying equilibrium model hold with great accuracy at the college level. There are many

[24]Given the presence of a substantial number of selective public institutions, this is an important simplification. Modifying our theoretical framework to reflect objectives and constraints of public institutions is an important task for future research. First, however, it seems prudent to investigate how well the framework we have developed is able to capture the admission and pricing decisions of private institutions.

Table 1: Descriptive Statistics by School Type

college number	number of colleges	number of students	price cap	endowment income	mean income	mean SAT score	instructional expenditure	institutional aid	non-institutional aid
1	200	239	8982	130	47721	851	4386	2346	4231
2	162	169	9900	163	50511	975	4498	2731	3490
3	154	326	12402	259	54053	997	5621	4282	4864
4	115	289	13330	420	62787	1045	5719	5099	3527
5	115	422	15588	847	63981	1139	9983	7271	3865
6	77	392	19674	4085	73616	1247	15947	7611	4917

The 6 college types are computed by partitioning the support of the empirical distribution of mean SAT scores in 6 clusters of equal size. There are 1560589 students in all private colleges in our Petersons' sample. The minimum cluster size equals 260098. Expenditures and endowment data are from the NSF WebCASPAR database. Endowment income is calculated assuming a 2 percent endowment draw per period is allocated to educational activities. Number of colleges is from Peterson's. All remaining variables are from the NCES.

idiosyncratic factors that influence individuals' college choices that are likely to be omitted in our relatively parsimonious equilibrium model. However, we expect that the model will be more successful in explaining patterns of choice, and admission and pricing behavior at a more aggregate level. The basic idea is that most of the idiosyncrasies are irrelevant on the aggregate level. For example, our model is better suited to explain whether a student with given income and ability attends a top private college or mediocre private college than whether a student attends Bowdoin or Middlebury College. By aggregating colleges with similar observed characteristics, we abstract from a number of factors such as regional preferences that may be important at a disaggregate level, but are likely to be less important at a more aggregate level. We rank the 824 private colleges by their mean SAT score and divide them into six groups each having the same number of students by partitioning the support of the mean SAT distribution accordingly.[25] Table 1 reports the student-weighted means for the six school types in our analysis.

Having assigned colleges to one of the six ranks, we then assign each student in the NPSAS sample to the same school rank to which his or her college is assigned. The results of this assignment are also shown in Table 1. We find that there is a hierarchy in mean income that follows the SAT ranking among schools. Mean income ranges from $47,721 in the lowest ranked school to $73,616 in the highest school. The same hierarchy holds for tuition, institutional aid per student, expenditure per student, and endowment per student. In particular, the average price caps (i.e., posted tuitions) range from $8,992 in the lowest ranked school to $19,674 in the highest ranked alternative. Students receive substantial amounts of financial aid from the institution they attend. Mean institutional financial aid ranges from $2,346 to $7,611. In addition, students receive large amounts of non-institutional aid as we discuss in detail below.

[25] We also explored K-means cluster analysis to aggregate colleges and found that it produced similar results to the ones reported in the paper (Epple et al., 2002).

4.3 Noninstitutional Financial Aid

To construct a computational general equilibrium analysis that incorporates the effects of federal and other non-institutional financial aid, we need a simple representation of the noninstitutional aid formula. Approximating this formula is challenging, however, because noninstitutional aid comes as federal aid, state aid or private aid, and aid from each source comes in a variety of forms. Noninstitutional financial aid takes the form of grants, loans, work study aid, and other forms. Federal grants include PELL grants, supplemental educational opportunity grants and other grants and fellowships. Federal loans include Perkins and Stafford loans and loans through the Public Health Service. Work study aid reflects aid under the Federal Work Study Program. Other federal grants include Byrd scholarships for undergraduates, and Bureau of Indian Affairs scholarships. There is also a multitude of state aid programs, such as the State Student Incentive Grants and state loans. Individuals may also receive private grants or loans that are not tied to a particular institution. Given the great many sources and forms of non-institutional aid, we estimate relationships that characterize the dependence of aid on student and college characteristics. Fortunately, the NPSAS data includes all noninstitutional aid received by the sampled students.

We define total noninstitutional aid, a_n, as a weighted sum of grants g_n loans l_n, work study aid w_n, and other forms, o_n.

$$a_n = g_n \; + \; o_n \; + \; 0.25 \; l_n \; + \; 0.5 \; w_n \tag{4.1}$$

The weights used in this formula are somewhat arbitrary, but are similar to ones used in the literature (Clotfelter, Ehrenberg, Getz, and Siegfried, 1991). To construct a noninstitutional aid formula, we need to express financial aid as a function of student and college characteristics. Most of the non-institutional aid is need-based. We would therefore expect that need-based non-institutional aid is primarily a function of income and tuition. Income is measured as total household income. Tuition is measured as the institution's price cap in 1995/96.

Table 1 provides some descriptive statistics of the main variables by school type.[26] We find that the vast majority of students in our sample receives some sort of federal financial aid. The fraction of students in each rank of schools receiving positive amounts ranges from 0.9 to 0.95. Mean financial aid amounts are similar across school types. The mean aid ranges form \$3,490 to \$4,917.

We estimate the financial-aid function directly based on observed financial aid payments and student characteristics.[27] Given that the vast majority of students receive positive aid, we use a simple regression framework.[28] Regressions for each school rank reveal that income has a negative coefficient ranging from -0.021 to -0.047. Ability measured by the SAT score is insignificant in 5 of the 6 regressions. Tuition is always positive with coefficients ranging from 0.083 to 0.223. Minorities only receive more financial aid in lower ranked schools. The coefficient ranges from 109 to 1721. We conclude that noninstitutional financial aid is primarily a function of income and tuition. In contrast to institutional financial aid, noninstitutional aid does not depend on ability, except for students in the highest ranked schools. Since it adds further complexity to an already challenging estimation problem, we reserve for future work the estimation of an extended model that includes minority status.[29] We therefore capture the noninstitutional aid in the computational equilibrium model by estimating a linear regression in income and the price cap, combining data for all students in our sample: $a_{jn} = 3912 - 0.025\, y_n + 0.132\, p_j^m$.

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[26] All empirical results reported in this paper use weights provided by the NPSAS to account for the sampling design used by the NCES.

[27] This approach is closely related to work in the welfare literature which focuses on the estimation of benefits functions for welfare recipients using data on welfare payments (Fraker, Moffitt, and Wolf, 1985) and in the income tax literature which estimates effective tax functions using observed income and tax payments (Sieg, 2000).

[28] We also estimated Tobit models and the results are similar.

[29] Epple, Romano, and Sieg (forthcoming) develops an extension of the framework here to incorporate minority status.

5 Empirical Results

5.1 Parameter Estimates

We estimate our model using full information MLE. Price caps and endowment income along the college hierarchy are first specified as approximately observed in the data. The parameter estimates and estimated standard errors are reported in Table 2.

Table 2: Parameter Estimates

	$\sigma^e_{ln(y)}$	σ^e_b	σ^e_a	$c1$	$c2$	$c3$	F	γ	ω	ψ
estimate	0.688	172.32	143.70	255.47	10755	3255208	169.91	0.761	0.089	0.054
std error	0.005	0.1326	0.1113	0.1667	4.657	2177	0.078	0.0005	0.0001	0.0001

.

We find that the parameters of the utility function are estimated with good precision and have the expected signs. The estimate for γ, which measures the peer effect of ability, is positive and approximately 0.76. This finding supports our model's prediction that individuals sort based on the perceived quality of their peer group. We find, as expected, that ψ, the coefficient measuring income diversity, is positive. This implies that school quality increases with increasing income diversity. To attract students from lower-income backgrounds, schools give financial aid that is inversely related to income. We also find that the estimate of ω, the coefficient determining the demand for expenditures, is approximately 0.089. Expenditures per student are a substantial component of school quality, but with markedly lower elasticity than the peer measure.

The parameters of the custodial cost function influence the admission spaces in equilibrium in two ways. First, the costs of providing education affect the level of educational expenditures per student through the budget constraint. Second, marginal custodial costs are a key component of marginal costs. Cost function parameters also determine equilibrium college size. We have chosen a cubic cost function to provide flexibility in capturing these effects. The primary focus of our analysis is to characterize the allocation of students across colleges of differing qualities and the financial aid policies. Explaining choice of size by individual colleges is a secondary concern, and we have largely abstracted from that issue in our analysis by forming composite colleges. Rather than explaining the choice of size by individual institutions, our cost function parameters serve to match predicted and actual populations along the college quality hierarchy. Our estimates in Table 2 imply that total custodial costs at the optimal size, k^*, are approximately \$2167. The estimate of the parameter $c3$ is relatively large. This parameter penalizes deviations from the efficient scale and is key to matching predicted and actual populations across the college quality hierarchy. We expect that a large value of $c3$ would also arise at the school level. Peer effects and endowments create a strong force for schools to reduce size to increase student quality–in the limit maximizing quality by admitting a handful of brilliant students and lavishing the entire endowment on educating those students. The countervailing effect of scale economies is captured in our cost function primarily by the $c3$ term in the cost function.

5.2 Admission and Aid Policies in Equilibrium

The estimates reported in Table 2 imply an equilibrium in the market for higher education. Table 3 summarizes the most important features of this equilibrium.

Comparing the equilibrium outcomes in Table 3 with the outcomes in the data reported in Table 1, we find that our equilibrium model fits the data well. Our findings suggest that our model predicts aggregate mean income levels in all six school types reasonably well. The differences between observed mean income and predicted mean income are small for all six schools. Our equilibrium model also matches mean SAT score for most schools. It underpredicts the mean SAT score for the lowest school and overpredicts it for the highest school in the hierarchy.

To get a better understanding of how well the model fits the data we compare the observed admission decisions with the predicted decisions for each school. The results of this comparison are illustrated in Figures 2 and 3 which plot the observed data and a simulated sample generated from the equilibrium model. Note that the model fits the data

Table 3: Equilibrium

school	price cap	endowment income	school size	fraction at cap	income	ability	inputs	institutional aid	non-institutional aid
1	9000	0	0.176	0.45	41168	800	2945	3887	4436
2	10500	62	0.171	0.20	42904	910	3404	4981	4228
3	12500	186	0.170	0.17	52843	998	4247	5760	4178
4	13500	313	0.168	0.16	62891	1075	5352	6275	4125
5	15000	642	0.164	0.17	76800	1166	7101	6320	4379
6	19000	4140	0.150	0.15	79992	1338	11502	8977	4421

reasonably well. However, the estimates of standard deviations of the measurement errors are substantial, indicating that there is a significant amount of variation left in the data which is not explained by our model.

Our equilibrium model generates tipped L-shaped admission spaces. Figures 2 and 3 show that there is a reasonable congruence between admission spaces in equilibrium and the observed admission spaces. The panels show the observed (b, y) of the NPSAS sampled students in each consecutive rank of school (+'s) and a forecast of the same number of students in that school using our estimated model ($*$'s). We see that observed admission spaces are somewhat more spread out than those predicted by the model.

Table 4: Goodness of Fit

school number	number of students	correct predictions	prediction error less than or equal to one
1	239	0.456	0.715
2	169	0.189	0.675
3	326	0.206	0.577
4	289	0.183	0.606
5	422	0.230	0.664
6	392	0.515	0.745

We also compute goodness of fit statistics. Using the estimated parameters of the model and assuming that income and ability are measured without error, we assign the sampled students to the six schools. We then compute the fraction of students classified correctly into each school. These results are shown in Table 4. We find that for schools 1 and 6 we predict the admission of approximately 50 percent of the students in the sample correctly. For schools 2 through 5 the fraction ranges between 18 and 23 percent.[30] We also compute the fraction of students for whom our model's predictions are off by at most one rank.

[30]With random assignments, we would, of course expect to predict about 16.67 percent correctly.

Figure 2: Admission Spaces

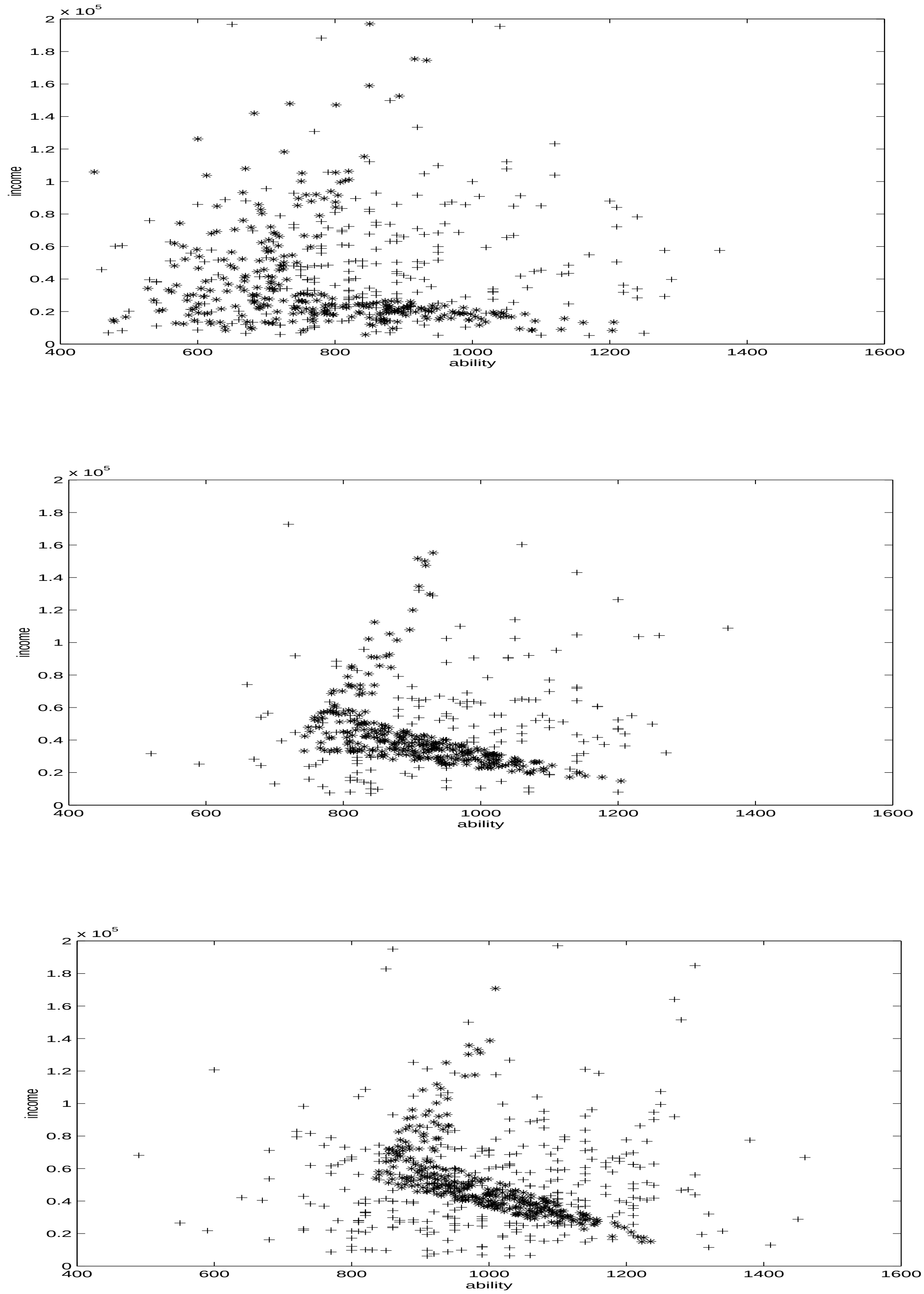
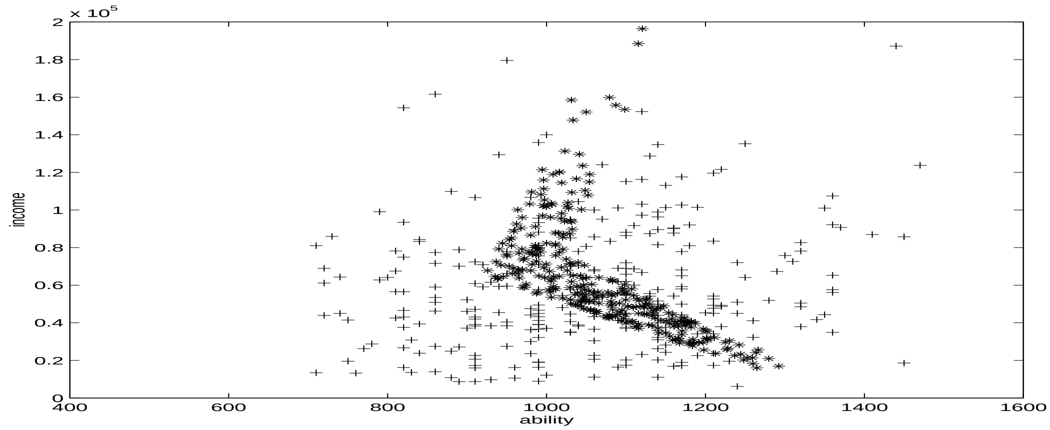
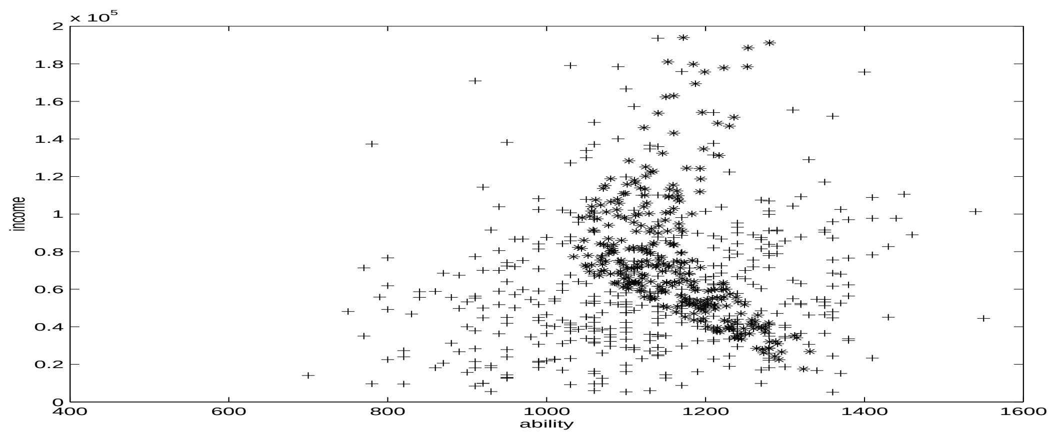
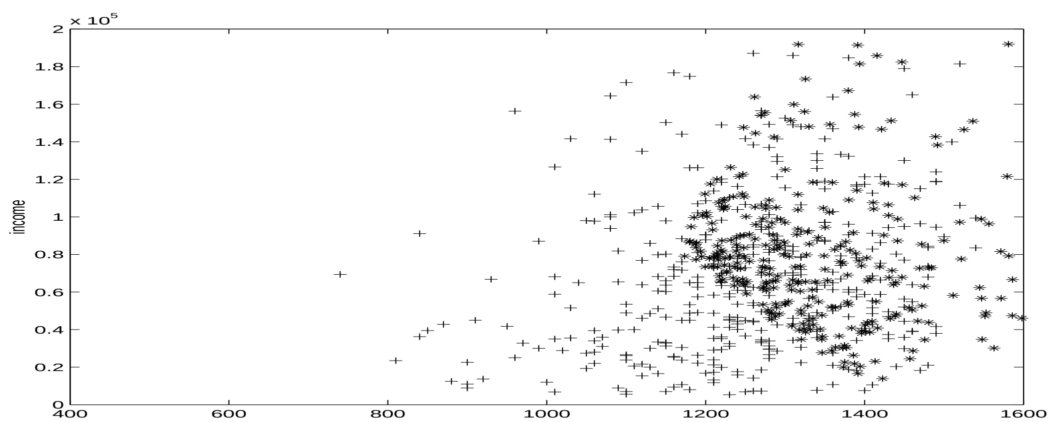


Figure 3: Admission Spaces







The results are reported in column 4. We find that our model's predictions are only off by one rank in the majority of observations in each school.[31] We conclude from this analysis that, on the whole, our model provides a reasonably good approximation for the observed admission policies in the sample.

We now turn our attention to financial aid policies. As explained in the previous section, the equilibrium of our model reflects noninstitutional financial aid received by students.[32] Table 1 suggests that individuals in our sample receive on average $4000 in noninstitutional aid. Table 3 shows that our model matches these levels of noninstitutional aid closely. The predicted mean of the distribution of noninstitutional aid is close to the observed mean in each school type.

The most challenging part of our analysis is to explain institutional financial aid policies. We observe in the data that a large fraction of students receives quite substantial amounts of institutional aid. We would like to know whether our model can replicate these generous financial packages. A comparison between Table 3 and Table 1 shows that our model explains observed financial aid well.

To get additional insights into the nature of price discrimination predicted by our model we compute the shadow prices for ability and income. Recall that equation (2.20) implies that the shadow price for ability is given by $\frac{\partial q_j/\partial \theta}{\partial q_j/\partial I}$. Similarly, the shadow price for income is $\frac{\partial q_j/\partial \mu_y}{\partial q_j/\partial I}$. Table 5 reports the estimates of the shadow prices for each school. We find that the shadow price for income ranges between -0.044 and -0.088. The estimated model predicts a $10,000 increase in household income raises tuition to students at the margin of switching schools by $440 and $880 at the bottom and top ends of the school quality hierarchy respectively. Our model thus predicts that schools will offer less financial aid to higher income students. Higher quality schools engage more aggressively in pricing by

[31] With random assignment, we would expect about one-third correct predictions for the bottom and top schools, and about one-half correct predictions for the middle schools.

[32] The model also predicts inputs and per student expenditure levels. If one adds in custodial costs to the input levels reported in Table 3, predicted per student expenditures along the school hierarchy are 5113, 5581, 6926, 7538, 9322, and 14163. Comparing these to the empirical expenditures in Table 1, we predict a flatter ascension. Obviously, the empirical expenditures depend heavily on the conventions used in their measurement.

Table 5: Shadow Prices for Ability and Income

college	shadow price of ability	shadow price of income
1	31.50	-0.044
2	32.01	-0.048
3	36.45	-0.049
4	42.60	-0.052
5	52.16	-0.056
6	73.59	-0.088

income. Our model also predicts quite significant financial aid for higher ability students. The shadow price for ability ranges between 31.5 and 73.6. The estimated model predicts 100 SAT points lowers tuition to students at the margin of switching schools by $3150 and $7360 at the bottom and top ends of the school hierarchy respectively.

We can also estimate linear financial aid regressions using simulated data from our equilibrium model. For the highest ranked school, we find that the coefficient of income is approximately -0.18. The estimated coefficient of ability is 45.1. Comparing these estimates with the corresponding shadow prices indicates that the highest ranked school has significant market power. The estimated coefficients for the other five schools do not differ as much from the estimated shadow prices. We thus conclude that highly ranked colleges exercise some market power over students that are located in the inside of the admission sets. These students receive less favorable financial aid packages than those on the boundary. Market power decreases substantially as one moves down in the quality hierarchy.

These predictions are qualitatively in line with reduced form estimates. The magnitudes of the marginal effects of income and ability on financial aid are, however, larger than those found in reduced form studies. There are at least two reasons why our estimates are larger. First, reduced form estimates often control for multiple measures of ability such as GPA in college and high school. More importantly, ability is most likely measured with error which biases reduced form estimates towards zero.[33]

Another measure of market power is obtained by calculating the ratio of each school's tuition revenue to the revenue it would obtain if all students were charged effective marginal cost. We find that this measure of market power ranges from 1.01 in the lowest ranked college to 1.72 in the highest ranked college. The measure ranges from 1.08 to 1.17 for the other four colleges.

Our equilibrium model matches observed admission and tuition policies relatively well. We can, in principle, use it to perform policy experiments and evaluate education reform measures. Admittedly, our model relies on a number of strong assumptions. More serious policy analysis would clearly demand improvements in the model. We will discuss these improvements in the next section. To illustrate the mechanics of our equilibrium model and to gain some additional insights into the properties of the model, we consider a simple policy experiment. Policy makers are interested in the impact of financial aid on admission, pricing, and educational outcomes. We analyze a 10 percent change in the noninstitutional aid formula, by multiplying the coefficients in the estimated financial aid formula by 1.1. Table 6 summarizes our main findings.

We find that a 10 percent increase in financial aid does not affect the admission policies in equilibrium by much. College expenditures increase on average by roughly $40 per student which indicates that approximately 10 percent of the increase in financial aid can be extracted by the colleges through higher tuition. We also find that institutional aid decreases on average by approximately 1 percent which suggests that noninstitutional aid only slightly crowds out institutional aid. Hence, students are better off in equilibrium largely because of the income effect. The results of the policy experiment thus indicate that the market is highly competitive. The expenditure increase in schools is on the order of what students want given an income elasticity of 1 and a 10 percent increase in aid. Schools having a preference for expenditure per se and thus would like to spend more, but

[33]Using simple regression and Tobit models, we find that much of institutional aid in our data set is not related to ability, income or minority status. Institutional knowledge of financial aid policies suggest that these reduced-form results are not plausible.

Table 6: Equilibrium After Policy Change

school	price cap	endowment income	school size	fraction at cap	income	ability	inputs	institutional aid	non-institutional aid
1	9000	0	0.176	0.465	41681	800	2988	3844	4878
2	10500	62	0.171	0.209	42905	911	3447	4939	4651
3	12000	186	0.170	0.178	52869	998	4290	5718	4596
4	13500	313	0.169	0.163	62769	1076	5394	6234	4542
5	15000	642	0.164	0.170	76664	1166	7146	6276	4381
6	19000	4140	0.150	0.154	79650	1339	11538	8943	4873

the market will not allow this. Our analysis, thus, reinforces the importance of general equilibrium effects in evaluating large scale policy changes.

6 Conclusions

In this paper, we have developed an equilibrium model of the market for higher education. We have shown that this model has strong predictions regarding sorting of individuals by income and ability. The model gives rise to a strict hierarchy of schools that differ by the educational quality provided to the students. To evaluate the model, we have developed an estimation strategy that accounts for the fact that important variables are likely to be measured with error. We estimate the structural parameters of the model using a combination of micro and aggregate data. The findings suggest that our equilibrium model can replicate many of the empirical regularities observed in the data reasonably well.

To conduct the analysis, we had to make a number of hard choices and rely on some restrictive assumptions. Some of these assumptions could be relaxed to improve the fit of the model and to conduct more serious policy analysis. Future research should address the question of how robust the results of this study are to different specifications of the utility function. For example, one could introduce unobserved heterogeneity in tastes for education along the lines suggested in Epple and Sieg (1999). Introducing taste heterogeneity into the demand side of the model yields more realistic predictions about individual sorting. It would also allow us to construct a well-behaved likelihood function without relying on measurement errors. However, solving the colleges' optimization problems and computing equilibria in such models is a major extension of the analysis presented in this paper.

It would also be desirable to control for additional sources of observed heterogeneity. An extension of our model which controls for minority status is feasible. Epple et al. (2002) discuss different strategies of incorporating minority status into a similar equilibrium model. Equilibrium of the model then depends on whether and how racial diversity measures enter the objective functions of colleges and the different types of households. Incorporating racial diversity measures into the analysis is a necessary prerequisite for analyzing the impact of affirmative action policies on admission and financial aid.

Another focus of future research should be to analyze jointly the markets for public and private education. The benefits from modelling private and public colleges in equilibrium would be substantial from the perspective of policy analysis. It is not particularly difficult to include a public school sector as an outside option into our model as discussed, for example, in Epple and Romano (1998). However, explaining stratification in both the public and the private sector within an equilibrium model is clearly more challenging and may require significant modifications of the underlying theory.

The computational analysis provided in this paper is pushing the boundaries of what has been achieved in general equilibrium analysis of markets for differentiated products such as higher education and estimation of these models. The extensions discussed above are desirable and seem to be feasible. However, these extensions are nontrivial. Implementing them in a meaningful empirical framework will be computationally challenging.

Finally, our research has important implications for research on differentiated products outside of public economics. Most previous empirical work in health economics and industrial organization treats product characteristics as exogenous and ignores price discrimination which can be quite prevalent in many markets. In contrast, our framework endogenizes important product characteristics and offers a new approach for analyzing price discrimination. We, therefore, view the methods developed in this paper and our main empirical results as quite promising for future research.

A Proof of Proposition 1

1. Suppose that $R_i > R_j$. We show a contradiction to either $q_i = q_j$ or $q_i < q_j$. In either of the latter conjectured equilibria, let college i follow the admission and tuition policy of college j and spend on inputs so as to have a balanced budget. College i would attract the same student body as college j since i would be spending more on inputs than j and thus be of higher quality. Hence, we have a contradiction to quality maximization by college i in the conjectured equilibrium.

2. For all types admitted to college j, $p_j(b, y) \geq EMC_j(b, y)$, by the admission criterion (2.19). To show a contradiction, suppose that $p_j(b, y) > EMC_j(b, y)$ for student of minimum ability b' for given y. Since both $p_j(b, y)$ and $EMC_j(b, y)$ are continuous in b, there exists types with $b < b'$ and $p_j(b, y) > EMC_j(b, y)$. All these types would be admitted to college j, a contradiction.[34]

3. Only schools in $\hat{J}(b, y)$ would admit type (b, y) by (2.12) and (2.19). Now, to generate a contradiction, suppose that a student attends school i such that $U(y + a_i - EMC_i, q, b) < U(y + a_j - EMC_j, q_j, b)$for colleges i and j in $\hat{J}(b, y)$. The student's equilibrium utility $U_i = U(y + a_i - p_i, q, b) < U(y + a_j - EMC_j, q_j, b)$ since $p_i \geq EMC_i$ by (2.19). Then $p_j = \min[p_j^r, p_j^m] \geq EMC_j$, implying $\alpha_j(b, y) = 1$ is optimal for college j. The latter and attendance at college i contradicts market clearance: $\sum_j \alpha_j \leq 1$. An analogous argument precludes choice of no college when attendance at tuition equal to EMC_j for $j \in \hat{J}$ would yield higher utility.

[34] Equilibrium requires that admission and attendance sets coincide, or market clearance will be violated.

B Computation of Equilibrium

Below is a sketch of an algorithm which can be used to compute approximate equilibria. To implement this algorithm one needs a random number generator to simulate from the underlying distribution of income and ability and an algorithm to solve a system of nonlinear equations. Based on our experience, we suggest to use a variation of Broydn's method (Press, Teukolsky, Vetterling, and Flannery, 1988). The algorithm to compute equilibria is then as follows:

1. Given starting values for θ_j, I_j, k_j and μ_j^y.
2. Generate random draws (b, y) from $h(b, y)$.
3. Admission is as if $p_j = EMC_j(b, y)$
4. Choice set of a household (b, y) is given by all schools such that: $EMC_j(b, y) \leq p_j^m$.
5. For each school in choice set compute utility:
   $U_j \; = \; (y - EMC_j + a_j(EMC_j)) \; q_j \; b^\beta$.
6. Rank schools by utility and determine first best (fb) and second best (sb) choices for (b, y).
7. Compute utility of best alternative (second best choice):
   $U^A(b, y) \; = \; (y - EMC_{sb} + a_{sb}(EMC_{sb})) \; q_{sb} \; b^\beta$.
8. Compute reservations price, p_{fb}^r:
   $(y - p_{fb}^r + a_{fb}(p_{fb}^r)) \; q_{fb} \; b^\beta \; = \; (y - EMC_{sb} + a_{sb}(EMC_{sb})) \; q_{sb} \; b^\beta$.
9. Compute price by comparing reservation price with price cap:
   $p_{fb}(b, y) \; = \; \min\{p_{fb}^r, p_{fb}^m\}$.
10. Update values for θ_j, I_j, k_j and μ_j^y.
11. Iterate until convergence.

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