Preliminary Draft

Optimal Tax Treatment of Private Contributions for Public Goods with and without Warm Glow Preferences*

Peter Diamond

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The US relies on tax-favored contributions for financing some public goods[1] as well as having direct government expenditures. There are a number of reasons why such reliance may be useful. From the political perspective, this approach shifts some decisionmaking from the legislative process to the

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[1]This includes covering fixed costs for private goods with low marginal costs.

decisions of individual donors (and the managers of charitable organizations). This includes religious organizations for which direct expenditures are constitutionally banned. From the economic perspective, this approach can be a useful part of optimal tax and expenditure policy. This paper explores the latter issue, using first a model with standard preferences and then a model with a warm glow of giving (Andreoni, 1990). While standard preferences are very distant from empirical reality, it seems useful to examine the economic mechanisms in this setting before examining a more realistic model with warm glow preferences.

The focus is on the interaction between an optimal nonlinear income tax and subsidized donations to finance public goods.[2] The model uses an additive preference structure so that the incentive compatibility constraint is not affected by the level of a publicly provided public good implying that optimal public provision in the absence of donations satisfies the Samuelson rule. The optimality of the Samuelson rule does not generally carry over to optimal provision with subsidized donations. In addition to showing the conditions for the rate of subsidized private provision, the paper considers

[2]This paper considers only resource-using public good provision. I plan to consider gifts to individuals and contributions to charities for the purpose of redistribution in a separate paper.

the pattern of optimal subsidization across earnings levels. While relevant for analysis of tax deductibility of charitable donations, the analysis does not get that far.

Analysis of optimal taxation with warm glow preferences is sensitive to the choice of preferences that are relevant for a social welfare evaluation. After considering optimal rules with formulations of social welfare which do and do not include warm glow utility, the paper considers the choice of normative criterion.

Conditions for the optimal level of publicly provided public goods have been studied with different types of taxation available (Atkinson and Stern (1974), Boadway and Keen (1993), Kaplow (1996)). Equilibrium with privately provided public goods has been studied by Bergstrom et al (1986). Analyses of tax expenditures for private contributions for public goods have been done by Feldstein (1980) and Roberts (1987). Closest to this analysis is that of Saez (2000b) that analyzed the tax treatment of voluntary donations in the case of a linear income tax. This paper focuses on private contributions with nonlinear income taxation. The issue arises in two contexts - as a substitute for public financing and when the government leaves the choice completely to the public.

There are two sides to the potential gain from subsidized donations. First, for a given level of public good provision, the presence of private donations with high earners donating more than low earners eases the incentive compatibility constraint for donors and so can raise social welfare. This follows since considering a lower-paid job includes a perception of a drop in public good provision. Second, private donation reduces consumption, easing the resource constraint. While this observation is readily shown in the two-types model used by Boadway and Keen to analyze public provision of public goods, the analysis in the text uses instead a more tractable model of income taxation where the hours of work are fixed for any job rather than being a choice variable for workers (Diamond, 1980, Saez, 2000a).[3]

Briefly considered is a setting with multiple public goods that must receive uniform subsidization. Like the earlier literature, this paper assumes that organizing private donations is costless while tax collection has a deadweight burden. Since private charitable fundraising is very far from costless, the paper is an exploration of economic mechanisms, not a direct guide to policy.

[3]In addition to considering a model where the incentive compatibility constraint comes from imitating a worker with slightly lower skill, this alternative model allows the possibility that the best alternative to the equilibrium earnings level is withdrawal from the labor force.

1 Standard Preferences

1.1 Optimal Public Provision

To begin, we consider a variant of optimal income tax model in Diamond (1980) to which we add a single public good (as in Boadway and Keen (1993)). In this model, labor hours are not adjustable, implying that a job involves nonvarying levels of disutility for workers with different skills. To focus on the primary issue of the contrast between public and private provision, we assume that individual preferences are additive in the utility from the public good. A worker of type n can only work at a job that has productivity n or lower. Disutility from work at a job with productivity m for a worker of type n $(m \leq n)$ is additive and denoted a_{mn}.[4] If a type n worker is holding down a type m job, then utility is $u[c_m] - a_{mn} + b_n v[G]$, where c_n is the consumption from after-tax earnings, G is the level of public good, with u and v both increasing, concave and twice differentiable and b_n a weight that can vary with skill level but does not vary across people with

[4]In the Mirrlees model, a worker can provide any number of hours of labor at a fixed wage per hour, with the wage depending on skill. In practice, the opportunities in the labor market have a more complex structure. In the model in Diamond (1980) each individual has two opportunities - a particular job or no work at all. Here we effectively assume that each person has two job opportunities, just as Mirrlees assumes the next best alternative is to provide the effective labor of a slightly lower-skilled worker. This changes the nature of the comparisons relevant for income taxation.

the same skill. We assume that this parameter is the same for everyone, $b_n = b$, although we preserve the more general notation as a reminder of an effect that particularly matters when considering multiple public goods. We also assume the Inada condition for u. If not working, we simply set labor disutility equal to zero and assume the same utility functions of public and private good consumption. There may be a type-0 worker who has no productivity and does not work.

We assume that lower skilled workers find any job more difficult: for $n_1 < n_2$ we have $a_{mn_1} \geq a_{mn_2}$. We make this convenient assumption even though it is not plausible once we consider the different nature of jobs with different productivities. Assume that the optimum allocates each type to its own skill-level job. I conjecture that sufficient conditions for the optimum to have this character is that the difference in disutilities decline with skill - that is, the higher the skill the less the increase in disutility for any step up in job productivity,[5] and that there are similar relative numbers of different workers, and that the economy be poor enough to need this much work. Thus, if a worker chooses some skill level, no more highly skilled worker would choose a lower skill job. But this disutility condition alone does not

[5] For $n' < n$, $m' < m$: $a_{mn'} - a_{m'n'} > a_{mn} - a_{m'n}$

rule out having no workers at some jobs and several types at others. With private donations, the analysis is potentially more complicated.

Restricting analysis to allocations where each type of workers is at the matching skill, social welfare maximization is:

$$\text{Maximize}_{c,G} \quad \sum N_n \left(u\left[c_n\right] - a_{nn} + b_n v\left[G\right]\right)$$

$$\text{subject to:} \quad E + pG + \sum N_n \left(c_n - n\right) \leq 0$$

$$u\left[c_n\right] - a_{nn} + b_n v\left[G\right] \geq u\left[c_m\right] - a_{mn} + b_n v\left[G\right] \text{ for } m < n \text{ for all } n \tag{1}$$

where E is other government expenditures and p is the cost per unit of the public good. Given the additivity of utility in the public good, the FOC for public good provision is

$$\sum N_n b_n v'\left[G\right] = \lambda p \tag{2}$$

With the next best alternative for any worker being the next job down in productivity, the consumption of each type appears in two incentive compatibility constraints, except for the highest type. Thus, the FOC for consumption levels for all but the highest type are

$$N_n \left(u'\left[c_n\right] - \lambda\right) = \left(\mu_{n+1} - \mu_n\right) u'\left[c_n\right] \tag{3}$$

with the same expression holding for the highest type without a Lagrangian for the nonexistent higher incentive compatibility constraint. Each of the incentive compatibility constraints is binding, implying rising consumption with skill (so that workers are willing to take the highest skill job for which they are able). That is, with public good utility dropping out of the incentive compatibility constraint, consumption rises with skill to offset the increase in labor disutility from a more difficult job:

$$u[c_n] - u[c_{n-1}] = a_{nn} - a_{n-1n} \tag{4}$$

the pattern of marginal taxes can be different from that in the Mirrlees model (Diamond, 1980, Saez, 2000b)

As shown by Boadway and Keen and Kaplow using the Mirrlees model, with this additive structure of benefits we obtain the Samuelson rule. The same holds here.

1.2 Increasing Social Welfare by Subsidized Private Provision

In the equilibrium occurring with optimal taxation and public provision, no worker would make a voluntary contribution to the public good. Since, by assumption, everyone has a nonnegative marginal utility from increases in public good provision, the Samuelson rule for public good provision ensures that no single individual would have a marginal rate of substitution large enough to warrant a voluntary contribution.[6] We begin by showing that social welfare can be improved from this allocation by inducing individuals with the highest productivity to make subsidized contributions. The gain from private donations has two sides. One is that a contribution to financing the public good by the highest type substitutes their consumption for government resources (without violating the incentive compatibility constraint), which is a gain for social welfare. The second is that a highest type worker considering switching to a lower paid job would then perceive a drop in public good provision. Thus, compared with completely public provision, there is a weakening in the incentive compatibility constraint. This can be used to tax

[6]We assume throughout the section without warm glow preferences that in the absence of subsidization there are no private donations, even though the Samuelson rule may not hold.

the highest workers more heavily (net of contributions), freeing up valuable resources. Since no other workers can earn this much, this opportunity for the highest earners does not change the equilibrium for other workers. With preferences the same (apart from labor disutility) across skill levels, if those with the highest income are just willing to contribute a little to the public good, those with lower incomes are not willing to contribute, so we do not need the restriction that only the highest earners have access to subsidized donations for this argument. The two sides of this effect are shown in two separate proofs of this welfare gain.

We begin by considering the level of private donations. For later use we use general notation, although in this subsection we restrict the subsidy to the highest earners. Denote by s_n the fraction of the public good contribution by a worker of skill n that is financed by the government. We denote earnings net of tax by x_n and the addition to the public good financed by the donation by g_n, so that we have

$$c_n = x_n - (1 - s_n)\, p g_n \tag{5}$$

Given the opportunity to contribute, a worker of type n choosing a job of type n makes the donation decision to maximize utility which can be written:

$$u\left[x_n - (1 - s_n)\, p g_n\right] - a_{nn} + b_n v\left[G_{\text{-}n} + g_n\right] \tag{6}$$

where we have introduced the notation $G_{\text{-}n}$ equal to the aggregate level of public good financed by contributions by other workers and the government as perceived by a worker of type n. In equilibrium, own contribution plus perceived contributions of others will sum to the level of supply, G.

The contributions for workers who make positive contributions satisfy

$$(1 - s_n)\, p u'\left[c_n\right] = (1 - s_n)\, p u'\left[x_n - (1 - s_n)\, p g_n\right] = b_n v'\left[G_{\text{-}n} + g_n\right] = b_n v'\left[G\right] \tag{7}$$

We turn now to an argument that the social welfare optimum with only public provision can be improved. This argument uses the assumption that all workers of the highest type have the same preferences, thereby making the gain from weakening the incentive compatibility constraint straightforward. Below we will consider the same issue in a setting with diverse public good preferences among the highest type. Starting with the equilibrium with the optimal taxes and government provided public good supply above, denoted by adding a * to variables, consider raising the after-tax income of the highest type enough to finance all of the public good given the subsidy rate needed for them to be willing to contribute to provide the same level of public good as at the optimum. This combination of transfer and donation subsidy leaves the real allocation totally unchanged but, as we will see, weakens the incentive compatibility constraint, thereby allowing a welfare improvement by increasing the taxation (lowering the consumption) of the highest type. Denote the skill level of the highest workers by n^H. In order to induce them to donate enough to provide the same level of public good while having the same consumption level, we must use a subsidy level that satisfies the individual first order condition:

$$(1 - s_{n^H})\, pu'\left[c^*_{n^H}\right] = b_{n^H} v'\left[G^*\right] \tag{8}$$

Net of tax income must be sufficient to finance both consumption and donation:

$$x^*_{n^H} = c^*_{n^H} + (1 - s_{n^H})\, g_{n^H} = c^*_{n^H} + (1 - s_{n^H})\, G^*/N_{n^H} \tag{9}$$

To see that we have weakened the incentive compatibility constraint (and so have the potential to further raise welfare by increasing the taxation of high earners and decreasing the taxation of all lower earners) we can compare the incentive compatibility constraints with public add private provision. Note that by choosing x and s the government can select c and g. This is the place where we use the uniformity of preferences among the highest type workers. The incentive compatibility constraint with government provision

$$u\left[c^*_{n^H}\right] - a_{n^H n^H} + b_{n^H} v\left[G^*\right] \geq u\left[c_m\right] - a_{m n^H} + b_{n^H} v\left[G^*\right] \text{ for } m < n^H \quad (10)$$

changes to the following with subsidized private provision

$$u\left[c^*_{n^H}\right] - a_{n^H n^H} + b_{n^H} v\left[G^*\right] \geq u\left[c_m\right] - a_{m n^H} + b_{n^H} v\left[G - g_{n^H}\right] \text{ for } m < n^H \quad (11)$$

Thus the perceived drop in public good supply if switching jobs weakens the incentive compatibility constraint, allowing an increase in social welfare as long as this constraint is binding. The same argument can be seen from a dual perspective. With public provision, a worker can contribute to the public good, but faces a price p when doing so. With a subsidy, the price falls to $(1 - s_{n^H})\, p$, thereby raising utility if there is a positive donation. Switching to a lower job decreases income and therefore decreases the utility gain from the decline in the price of the public good since less (zero) will be donated at this price. Having different subsidies for different earnings levels permits the government to exploit this opportunity, although such differentiation in pricing is not needed for the argument used here.

The other side of the role of private donations can be seen in the following argument supporting the same conclusion that subsidized donations by the highest type can raise social welfare. This argument is more complex, but has the virtue of carrying over to settings where the public good preferences of the highest type might vary. As noted above, at the optimum with public provision, no worker would make an unsubsidized donation to add to the public good. Consider the maximal subsidy for the highest type that results in no contribution, denoted s_{nH}. Consider the impact on the Lagrangian expression from a small increase in this subsidy level. This adds to public good provision by the amount of change in public good provision (which is a general equilibrium derivative). The impact on utility is the utility gain from the increase less the utility loss of the highest type from the consumption they give up for the unsubsidized share of the public good increase. This impacts the resource constraint only by the subsidized share of the cost of the increase in public good provision. This does not affect the incentive compatibility constraints.

$$\frac{\partial L}{\partial s_{n^H}} = \sum N_n b_n v' [G] \frac{dG}{ds_{n^H}} - (1 - s_{n^H}) pu' (c_{n^H}) \frac{dG}{ds_{n^H}} - \lambda s_{n^H} p \frac{dG}{ds_{n^H}} \quad (12)$$

Using the FOC for the consumption of the highest type and for public good provision, we can write this as

$$\begin{aligned} \frac{\partial L}{\partial s_{n^H}} &= \left(\lambda p - (1 - s_{n^H}) p \left(\lambda - \frac{\mu_{n^H}}{N_{n^H}} u' (c_{n^H}) \right) - \lambda s_{n^H} p \right) \frac{dG}{ds_{n^H}} \\ &= (1 - s_{n^H}) p \frac{\mu_{n^H}}{N_{n^H}} u' (c_{n^H}) \frac{dG}{ds_{n^H}} > 0 \end{aligned} \quad (13)$$

The positivity of this impact follows from the gain the the highest type perceive from their own contributions to the public and the low marginal utility of their consumption as a consequence of their being the best paid workers in the economy.

This argument rests heavily on the assumption that there is no cost of fund-raising. As stated in the introduction, this is an exploration of economic mechanisms, not a discussion that is directly policy relevant.

1.3 Optimal Subsidized Private Provision with Two Types of Workers

Next let us consider optimizing the allocation with the possibility of subsidizing the contributions of workers with different earnings at different rates. Since the choice of subsidy rate is equivalent to the choice of contribution level, we can let the contributions be the controls. Let us assume there are only two types. As we will see the optimum will have only the higher type contributing (or an equivalent allocation).

To set this up formally, we have

$$\text{Maximize}_{c,G,g} \quad \sum N_n \left(u\left[c_n\right] - a_{nn} + b_n v\left[G\right]\right)$$

$$\begin{aligned} \text{subject to:} \quad & E + pG + \sum N_n \left(c_n - n\right) \leq 0 \\ & u\left[c_2\right] - a_{22} + b_2 v\left[G\right] \geq u\left[c_1\right] - a_{12} + b_2 v\left[G - g_2 + g_1\right] \\ & G \geq \sum N_n g_n; \;\; g_n \geq 0 \text{ for all } n \end{aligned} \tag{14}$$

The optimum will have one of two forms - either both have the same consumption and (generically) the incentive compatibility constraint is not binding, or the optimum will have the incentive compatibility constraint binding. Both seem possible even when the constraint is binding with public provision.

Since individual donations enter only the incentive compatibility constraint, if that constraint is binding, the optimum will have only the higher type contributing $g_2 > 0, g_1 = 0$. In this case, the consumption allocation is similar to that above, with $c_2 > c_1$, although c_2 is lower relative to c_1 in the incentive compatibility constraint and the Samuelson rule may no longer hold. If the incentive compatibility constraint is not binding, then both skill types have the same consumption - higher skill workers are paid enough more to make their contributions to the public good. They are willing to undertake a more arduous job because of the increase in the public good that they perceive from the higher contribution they make when holding a higher paying job. That is, if the workers care enough about the public good it is not necessary to give higher consumption in order to induce employment at a more difficult job.

In both cases, we have an optimum with $g_1 = 0$ and $g_2 = G/N_2$. If the incentive compatibility constraint does not bind, other allocations of public good contributions are also optimal as long as the incentive compatibility constraint continues not to bind. In the allocation with $g_1 = 0$, allowing type 1 to contribute with the same subsidy rate as type 2 does not change the equilibrium since type 1 will not contribute (same v', equal or higher u' at $g_1 = 0$). Note that the allocation could have the property that the highest type receive a net-of-tax, gross-of-contribution income which exceeds their productivity.

Note that the subsidy rate for type 1 can exceed the subsidy rate for type 2 and still support the optimum. Using the maximal subsidy for type 1 that still leaves a zero contribution, we have:

$$(1 - s_1)\, pu'[c_1] = b_1 v'[G] \tag{15}$$

$$(1 - s_2)\, pu'[c_2] = b_2 v'[G] \tag{16}$$

With $b_1 = b_2$, we have $s_1 \geq s_2$ since $c_1 \leq c_2$ with strict inequality if the incentive compatibility constraint is binding.

In the case that the incentive compatibility constraint continues to bind, the constraint becomes

$$u[c_2] - a_{22} + b_2 v[G] = u[c_1] - a_{12} + b_2 v[G - G/N_2] \tag{17}$$

Contrasting this with a binding incentive compatibility constraint without donations, we note that in the presence of donations c_2 is lower relative to c_1

because of the term $v[G - G/N_2]$ rather than $v[G]$ on the right hand side.

The FOC for the public good satisfies:

$$\sum N_n b_n v'[G] = \lambda p - \mu b_2 (v'[G] - v'[G(1 - 1/N_2)](1 - 1/N_2)) \qquad (18)$$

This may or may not satisfy the Samuelson rule and can deviate in either direction depending on the shape of the public good utility function, that is the sign of $(v'[G] - v'[G(1 - 1/N_2)](1 - 1/N_2))$.

With more than two types there are different types of equilibria along the same lines, with the possibility of some incentive compatibility constraints binding and others not binding.

In this setting it does not matter whether the government has the ability to directly contribute to the public good or not, but this does not extend to multiple public goods. If public good preferences vary across skills, we might now have lower skill workers contributing to different public goods than higher skill workers. In the Appendix we explore the case with three types of workers and the case with uniform subsidization.

1.4 Diverse Preferences with Two Types of Workers

We now drop the assumption that all workers of a given skill have the same preferences and assume that a fraction f_n of workers of type n have no utility from the public good. The second argument above that welfare can always be improved above the optimum with public provision by subsidizing donations by the highest skill type carries over to this case. Since those with and without public good concern receive the same pay gross of donations, there is a further constraint on the ability of the government to control both consumption and donations. The incentive compatibility constraints will differ for the workers with different preferences. We examine that before considering optimization. A type-2 worker who does not value the public good will choose a type-2 job under the condition

$$u\left[x_2\right] - a_{22} \geq u\left[x_1\right] - a_{12} \tag{19}$$

The condition is exactly the same for a type-2 worker who does value the public good if there are no private donations. If there are private donations, then the constraint becomes

$$\text{Max } u\left[x_2 - (1-s_2)\,pg_2\right] - a_{22} + b_2 v\left[G_{\text{-}2} + g_2\right] \geq \text{Max } u\left[x_1 - (1-s_1)\,pg_1\right] - a_{12} + b_2 v\left[G_{\text{-}2} + g_1\right] \tag{20}$$

Assuming that type-1 workers do not receive a donation subsidy, this becomes

$$\text{Max } u\left[x_2 - (1-s_2)\,pg_2\right] - a_{22} + b_2 v\left[G_{\text{-}2} + g_2\right] \geq u\left[x_1\right] - a_{12} + b_2 v\left[G_{\text{-}2}\right] \tag{21}$$

Subtracting the incentive compatibility constraint for those who do not value the public good from that of those who do, under these circumstances the former constraint will imply the latter provided

$$\text{Max } u\left[x_2 - (1-s_2)\,pg_2\right] + b_2 v\left[G_{\text{-}2} + g_2\right] \geq u\left[x_2\right] + b_2 v\left[G_{\text{-}2}\right] \tag{22}$$

which is true as a maximization.

Given this structure of incentive compatibility constraints, in looking for an optimum there are two structures, both of which seem to be possible optima for different fractions of the highest skill types with different preferences. In one optimum all workers of the highest skill work at the highest job, implying that the incentive compatibility constraint on the workers who donate does not bind (since it is strictly implied by the incentive compatibility constraint of nondonators). In the other case the workers of the highest skill who do not value the public good drop down to a lower job and the incentive constraint is binding on the remaining highest skill workers at the highest job. We present the FOC in both cases after reviewing the optimum without donations.

With public provision we have a slightly changed optimization:

$$\text{Maximize}_{c,G} \quad \sum N_n \left(u\left[c_n\right] - a_{nn} + f_n b_n v\left[G\right]\right)$$

$$\text{subject to:} \quad E + pG + \sum N_n \left(c_n - n\right) \leq 0 \tag{23}$$

$$u\left[c_n\right] - a_{nn} \geq u\left[c_m\right] - a_{mn} \text{ for } m < n \text{ for all } n$$

Since we have assumed that choice of job does not affect the utility from the public good (whether positive or zero) the incentive compatibility constraint is not affected by the preference diversity. Thus, the allocation has the same structure as above, apart from the presence of f_n. That is, the allocation is as if everyone were the same with public good preferences $f_n b_n v\left[G\right]$.

We turn now to the optimum with possible donations, assuming two types of workers. Type 1 workers will work at type 1 jobs (assuming enough of a resource need) and will not make donations (as above). Type 2 workers who care about the public good will hold type-2 jobs (again assuming enough of a resource need) and will make donations. There are two possibilities for type 2 workers who do not value the public good - they can hold type-1 jobs or type-2 jobs. If they hold type-2 jobs, their incentive compatibility constraint (weakly) implies the incentive compatibility constraint for type-2 workers who do value the public good (and strongly so if the latter make donations). Thus one candidate for the best allocation is the one that can be achieved subject to all type-2 workers holding type-2 jobs. In this case, type-2 workers making donations are overcompensated for holding type-2 jobs. Another possibility is that social welfare could be higher by giving up the extra output from type-2 workers who don't value public goods by having them hold type-1 jobs and thereby having a weaker incentive compatibility constraint for type-2 workers who do value making donations. The two types of optima are both continuous in the fraction of type-2 workers who do not value the public good. Presumably, the choice between the two candidates for this optimum depends on the fraction of the highest workers who value the public good.

For completeness, we set out the expressions for social welfare in these cases.

The best allocation with no type-2 workers holding type-1 jobs is the solution to the following problem:

$$
\begin{aligned}
\text{Maximize}_{x,G,g,s} \quad & N_1 \left(u\left[x_1\right] - a_{11} + f_1 b_1 v\left[G\right]\right) \\
& +N_2 \left(f_2 u\left[x_2 - \left(1 - s_2\right) p g_2\right] + \left(1 - f_2\right) u\left[x_2\right] - a_{22} + f_2 b_2 v\left[G\right]\right) \\
\text{subject to:} \quad & E + pG + N_1 \left(x_1 - n_1\right) + N_2 \left(x_2 - n_2\right) - f_2 N_2 \left(1 - s_2\right) p g_2 \leq 0 \\
& u\left[x_2\right] - a_{22} \geq u\left[x_1\right] - a_{12} \\
& \left(1 - s_2\right) p u' \left[x_2 - \left(1 - s_2\right) p g_2\right] = v' \left[G\right] \\
& G \geq \sum f_n N_n g_n; \;\; g_n \geq 0 \text{ for all } n
\end{aligned} \tag{24}
$$

In this case resources are saved by having some type-2 workers lower their consumption to donate. There are three possibilities for the optimal donations - providing all of the public good, providing the amount that maximizes their forgone consumption, and providing enough to lower their consumption to the socially most valuable level. To see this let us consider a new variable, $z \equiv \left(1 - s_2\right) p g_2$, equal to the amount of consumption given up for donations. It is only this variable that appears in the maximization problem, with the choice of subsidy rate and financed public good determined to equal this variable over an allowable range. One possibility is that the consumption given up is at the extreme of the range - that is as large as possible - either the maximum that can be induced or the maximum consistent with the total level of public good provision. Otherwise, z only appears in the following portion of the Lagrangian, $N_2 f_2 u\left[x_2 - z\right] + \lambda f_2 N_2 z$. Thus, with an interior solution, we have the FOC.

$$u'\left[x_2 - z\right] = \lambda \tag{25}$$

In contrast, the best allocation with some type-2 workers holding type-1 jobs is the solution to the following problem (setting g_1 equal to zero):

$$
\begin{aligned}
\text{Maximize}_{x,G,g,s} \quad & N_1 \left(u\left[x_1\right] - a_{11} + f_1 b_1 v\left[G\right] \right) + \left(1 - f_2\right) N_2 \left(u\left[x_1\right] - a_{12} \right) \\
& + f_2 N_2 \left(u\left[x_2 - \left(1 - s_2\right) p g_2\right] - a_{22} + b_2 v\left[G\right] \right) \\
\text{subject to:} \quad & E + pG + \left(N_1 + \left(1 - f_2\right) N_2\right)\left(x_1 - n_1\right) + f_2 N_2 \left(x_2 - n_2 - \left(1 - s_2\right) p g_2\right) \leq 0 \\
& u\left[x_2 - \left(1 - s_2\right) p g_2\right] - a_{22} + b_2 v\left[G\right] \geq u\left[x_1\right] - a_{12} + b_2 v\left[G - g_2\right] \\
& u\left[x_1\right] - a_{12} \geq u\left[x_2\right] - a_{22} \\
& \left(1 - s_2\right) p u'\left[x_2 - \left(1 - s_2\right) p g_2\right] = v'\left[G\right] \\
& G \geq \sum f_n N_n g_n; \quad g_n \geq 0 \text{ for all } n
\end{aligned} \tag{26}
$$

When some type-2 workers hold type-1 jobs, the optimum can occur with the incentive compatibility constraint binding or not binding on the remaining type-2 workers.

Contrasting the two problems, we see that we have reversed the constraint for type-2 workers who do not value the public good and have different conditions applying to those who do care about the public good.[7]

[7] Another approach to the trade-off between those who do and do not value the public good could be modeled by having a distribution of disutilities for each skill type (as in Diamond, 1980). Then there would be a marginal worker at each type with different margins for those with and without donations and so there would be some effect on labor supply from private contributions.

1.5 No Government Contribution

The model has a slightly different character if the government lacks the ability to directly contribute to a public good. At the equilibrium level of private supply without subsidy (which might be zero), some subsidy will be worthwhile, with a corresponding adjustment of net-of-tax earnings. The change in the maximization problem is an equality, rather than an inequality, in the definition of G, thereby ruling out public contributions. This is only relevant when at the optimum private donations do not cover all of the public good. I conjecture that this can happen when the model has diverse public good preferences across skill levels (as well as within a skill level) and all of the highest types hold the highest job, as in some of the examples immediately above. This becomes a more serious issue with multiple public goods and uniform subsidization across goods.

1.6 Two Public Goods

Returning to the setting where all workers of a given skill have the same utility functions, we examine some complications arising from multiple public goods.

We begin by considering the level of private donations. Given the opportunity to contribute, a worker of type n choosing a job of type n makes the donation decision to maximize utility which can be written:

$$u\left[x_n - \sum (1 - s_n) p g_{kn}\right] - a_{nn} + \sum b_{kn} v_k \left[G_{\text{-}kn} + g_{kn}\right] \tag{27}$$

where we have added a subscript k to distinguish public goods. We denote by s_n the fraction of the public good contribution financed by the government, which is assumed to be the same for all public goods. When different types of workers have different relative valuations of the different public goods, workers of the highest type will only contribute to one public good since they will not generally have the same marginal utility of the different public goods. (This is one of the arguments for the empirical irrelevance of this model of donations.) Thus there is a role for public contributions when preferences vary by good across skills. Whether the government can contribute would affect the pattern of subsidies by earnings level.

2 Warm Glow Preferences

The failure of standard preferences to make sense of the pattern of donations has been widely noted (e. g., references). In particular, some people spread their donations widely across charitable organizations even though their contributions are small relative to organizational budgets. Standard theory would direct all donations to the charity with the highest marginal utility of public good consumption until that had been reduced to equal the next highest. With small donations, such a move is unlikely to happen. To explain this pattern, a natural assumption is that of warm glow preferences (Andreoni (1990)).

With warm glow preferences, behavior is modeled as if it maximized utility that depends not just on the final allocation of resources but also on the process that results in that allocation, with a utility gain from one's own donation, denoted $w[g]$, and no similar per se utility gain from the donations of others or public contributions.[8] In a context of multiple public goods and public subsidies, there are a variety of ways in which such preferences could be modeled, which we do not explore.[9] Nor do we consider the context of charitable solicitation and the nature of such preferences, simply taking a concave function, $w[g]$.

[8]More complicated relative to the latter two assumptions seem the efforts of some to get others to donate or to lobby for more public provision. Thus the warm glow seems to come from activities to change public good provision, beyond just one's own donations.

[9]The pattern of giving across charities is complex. For example, prefernces might be written as $u[c_n] - a_{nn} + \sum v_k[G_k] + V[\sum w_k[g_{nk}, G_k]]$

That behavior is describable in this way does not necessarily imply that social welfare should be defined in the same way. That is, assume, as above, that the government switches from public provision to private provision, with the level of public good unchanged. Should the warm glows from private provision be part of a gain in social welfare from undertaking this project? Perhaps there is resentment at the need to provide privately what is seen as a government obligation. Perhaps the "warm glow" comes from decreasing the disutility coming from pressure to donate, suggesting that the utility level from warm glow is negative, although with a positive derivative. This would matter in an equilibrium model where the degree of charitable solicitation depends on government policy as to public provision and subsidization of private donations. Below we will discuss the normative issues in this choice. First, we consider optimization with and without incorporation of warm glow in social welfare. For this purpose, we set up the optimization with a parameter, θ, that can be set to zero or one.

2.1 Social Welfare Optimization

Compared to the problem in Part 1, the objective function may or may not be the same, the resource constraint is unchanged, and the incentive compatibility constraints are changed by the presence of warm glow. Moreover, assuming that contributions can be subsidized but not taxed, there is an inequality constraint that they be at least as large as the endogenous level without any subsidy (rather than simply being constrained to be nonnegative). We now assume that the preferences that determine behavior can be described by maximization of utility written as $u\left[c_n\right]-a_{nn}+b_n v\left[G\right]+w\left[g_n\right]$.

The donation level with a zero subsidy can be written as a function of net-of-tax earnings and the public goods level or as a function of the level of consumption (after donation) and the public goods level - allowing choice of the function that is most convenient for analysis. Note that it is the same function for all if b_n is the same for all, as we assume. The minimum donation function satisfies the FOC for donations at the government determined x and G:

$$pu'\left[x-pg\right]=bv'\left[G\right]+w'\left[g\right] \tag{28}$$

Differentiating, with x and G as control variables we have

$$\frac{\partial g}{\partial x}=\frac{pu''}{p^2u''+w''},\ \frac{\partial g}{\partial G}=\frac{-bv''}{p^2u"+w''}. \tag{29}$$

Since $c=x-pg$, we can also write the implicit equation for the minimum donation as a function of government determination of c and G, $g[c, G]$. Differentiating with c and G as control variables we have

$$\frac{\partial g}{\partial c}=\frac{pu''}{w''},\ \frac{\partial g}{\partial G}=\frac{-bv''}{w''}. \tag{30}$$

Subject to the minimum donation constraint, we can continue to write the social welfare maximization in terms of consumption and donations. Considering the two-types model with one public good, we can write the general formulation allowing warm glow to enter ($\theta = 1$) and not enter ($\theta = 0$) social welfare.

$$\text{Maximize}_{c,G} \quad \sum N_n \left(u\left[c_n\right] - a_{nn} + bv\left[G\right] + \theta w\left[g_n\right]\right)$$

$$\begin{aligned} \text{subject to:} \quad & E + pG + \sum N_n\left(c_n - n\right) \leq 0 \\ & u\left[c_2\right] - a_{22} + bv\left[G\right] + w_2\left[g_2\right] \geq u\left[c_1\right] - a_{12} + bv\left[G - g_2 + g_1\right] + w_2\left[g_1\right] \\ & G \geq \sum N_n g_n; \quad g_n \geq g\left[c_n, G\right] \text{ for all } n \end{aligned} \tag{31}$$

Differentiating we have lots of FOC:

$$\frac{\partial L}{\partial c_1}=N_1\left(u'\left[c_1\right]-\lambda\right)-\mu u'\left[c_1\right]-\xi_1\frac{\partial g\left[c_1,G\right]}{\partial c}=0 \tag{32}$$

$$\frac{\partial L}{\partial c_2} = N_2 \left(u'\left[c_2\right] - \lambda\right) + \mu u'\left[c_2\right] - \xi_2 \frac{\partial g\left[c_2, G\right]}{\partial c} = 0 \tag{33}$$

$$\frac{\partial L}{\partial G} = \sum \left(N_n b v'\left[G\right] - \xi_n \frac{\partial g\left[c_n, G\right]}{\partial G} \right) - \lambda p + \mu b \left(v'\left[G\right] - v'\left[G - g_2 + g_1\right]\right) + \nu = 0 \tag{34}$$

$$\frac{\partial L}{\partial g_1} = N_1 \theta w'\left[g_1\right] - \mu \left(b v'\left[G - g_2 + g_1\right] + w'\left[g_1\right]\right) - \nu N_1 + \xi_1 = 0 \tag{35}$$

$$\frac{\partial L}{\partial g_2} = N_2 \theta w'\left[g_2\right] + \mu \left(b v'\left[G - g_2 + g_1\right] + w'\left[g_2\right]\right) - \nu N_2 + \xi_2 = 0 \tag{36}$$

We consider the FOC separately for the values of θ of 0 and 1.

2.2 Warm Glow Preferences that do not enter Social Welfare

For warm glow preferences not entering the social welfare function, we set $\theta = 0$. This is the setting closest to that in Section 1 above. Warm glow affects the incentive compatibility constraint. Also the condition that donations not be subsidized may not be sufficient for them to equal zero. In this case, the impact of private consumption and public good level on donations will affect the optimal allocation. To consider this case, note that increasing the donation of the high type while lowering the donation of the low type weakens the incentive compatibility constraint while having no other effects, until the lower limit on g_1 is hit. Similarly, donations by the high type dominate public provision. Thus we know that $g_1 = g[c_1, G]$, and $\xi_2 = 0$, and we can write the FOC for the case $\theta = 0$:

$$\frac{\partial L}{\partial c_1} = N_1 (u'[c_1] - \lambda) - \mu u'[c_1] - \xi_1 \frac{\partial g[c_1, G]}{\partial c} = 0 \tag{37}$$

$$\frac{\partial L}{\partial c_2} = N_2 (u'[c_2] - \lambda) + \mu u'[c_2] = 0 \tag{38}$$

Thus the gain from limiting the donations of the lower type, $\xi_1 \frac{\partial g[c_1,G]}{\partial c}$, results in a higher marginal utility of consumption and so a lower consumption.

$$\frac{\partial L}{\partial G} = bv'[G] \sum N_n - \xi_1 \frac{\partial g[c_1, G]}{\partial G} - \lambda p + \mu b (v'[G] - v'[G - g_2 + g_1]) + \nu = 0 \tag{39}$$

$$\frac{\partial L}{\partial g_1} = -\mu (bv'[G - g_2 + g_1] + w'[g_1]) - \nu N_1 + \xi_1 = 0 \tag{40}$$

$$\frac{\partial L}{\partial g_2} = \mu (bv'[G - g_2 + g_1] + w'[g_2]) - \nu N_2 = 0 \tag{41}$$

Using () and () to eliminate ν we would have a similar structure to that in Part 1 above, with three differences, involving the response of contributions of the lower type to consumption and to public good level and the marginal utilities of warm glow. In addition, the incentive compatibility constraint (assumed to be binding) is different because of the warm glow, needing less consumption for the high type relative to that of the low type.

If, as is plausible, warm glow marginal utility is much larger than direct public good marginal utility (at an individual level), implying that $\partial g/\partial G$ is very small (at least for the lower type), and if v'' is very small, so that $v'[G] - v'[G - g_2 + g_1]$ is very small, the FOC become:

$$\frac{\partial L}{\partial G} = bv'[G] \sum N_n - \lambda p + \nu = 0 \tag{42}$$

$$\frac{\partial L}{\partial g_1} = -\mu w'[g_1] - \nu N_1 + \xi_1 = 0 \tag{43}$$

$$\frac{\partial L}{\partial g_2} = \mu w'[g_2] - \nu N_2 = 0 \tag{44}$$

implying

$$bv'[G] \sum N_n - \lambda p = -\mu w'[g_2] / N_2. \tag{45}$$

Thus the allocation is approximately as if the cost of the public good was $p - (\mu/\lambda N_2) w'[g_2]$.

Note that if the two types of workers care about different public goods, it remains the case that type 2 should be induced to contribute all of the public good provision, while type 1 should not contribute, with the public good publicly provided. This follows from a uniformity of any subsidy at a given earnings level, independent of which public good is being supported.

2.3 Warm Glow Preferences that do enter Social Welfare

By having contributions enter the social welfare function as well as the incentive compatibility constraint, marginal private donations do not directly affect social welfare. In this case the FOC become

$$\frac{\partial L}{\partial c_1} = N_1 \left(u'\left[c_1\right] - \lambda\right) - \mu u'\left[c_1\right] - \xi_1 \frac{\partial g\left[c_1, G\right]}{\partial c} = 0 \tag{46}$$

$$\frac{\partial L}{\partial c_2} = N_2 \left(u'\left[c_2\right] - \lambda\right) + \mu u'\left[c_2\right] - \xi_2 \frac{\partial g\left[c_2, G\right]}{\partial c} = 0. \tag{47}$$

$$\frac{\partial L}{\partial G} = \sum \left(N_n b v'\left[G\right] - \xi_n \frac{\partial g\left[c_n, G\right]}{\partial G}\right) - \lambda p + \mu b \left(v'\left[G\right] - v'\left[G - g_2 + g_1\right]\right) + \nu = 0 \tag{48}$$

$$\frac{\partial L}{\partial g_1} = N_1 w'\left[g_1\right] - \mu \left(b v'\left[G - g_2 + g_1\right] + w'\left[g_1\right]\right) - \nu N_1 + \xi_1 = 0 \tag{49}$$

$$\frac{\partial L}{\partial g_2} = N_2 w'\left[g_2\right] + \mu \left(b v'\left[G - g_2 + g_1\right] + w'\left[g_2\right]\right) - \nu N_2 + \xi_2 = 0 \tag{50}$$

It is no longer the case that the donations of the low type should never be subsidized. Substituting donations by the low type for those of the high type when only the latter is subsidized is a way of raising social welfare (since they differ in marginal warm glow) that does not change resource use and is attractive for that reason, although it weakens the incentive compatibility constraint and is unattractive for that reason. The dominance of donations of the high type over public provision remains true.

If donations of both types are subsidized ($\xi_1, \xi_2 = 0$), and if, as is plausible, warm glow marginal utility is much larger than direct public good marginal utility (at an individual level), the FOC become approximately

$$\frac{\partial L}{\partial c_1} = N_1 \left(u'\left[c_1\right] - \lambda\right) - \mu u'\left[c_1\right] = 0 \tag{51}$$

$$\frac{\partial L}{\partial c_2} = N_2 \left(u'\left[c_2\right] - \lambda\right) + \mu u'\left[c_2\right] = 0 \tag{52}$$

These FOC have the same form as in the problem with only public provision of the public good. The possibility that both types are subsidized allows an optimum which may not have the term $\xi_1 \frac{\partial g[c_1,G]}{\partial c}$ which was necessarily present in the case where warm glow does enter the social welfare function by the absence of the term .

Also assuming v'' is small relative to individual donations, the FOC for public good level and donations become approximately:

$$\frac{\partial L}{\partial G} = bv'[G]\sum N_n - \lambda p + \nu = 0 \tag{53}$$

$$N_1 w'[g_1] - \mu w'[g_1] - \nu N_1 = 0 \tag{54}$$

$$N_2 w'[g_2] + \mu w'[g_2] - \nu N_2 = 0 \tag{55}$$

Taking ratios, we have

$$\frac{(N_1 - \mu)\, u'[c_1]}{(N_2 + \mu)\, u'[c_2]} = \frac{N_1}{N_2} \tag{56}$$

$$\frac{(N_1 - \mu)\, w'[g_1]}{(N_2 + \mu)\, w'[g_2]} = \frac{N_1}{N_2} \tag{57}$$

Thus both types should have the same MRS between consumption and warm glow from donations, implying approximately the same rate of subsidization. Restoring the role of the marginal utility of the public good (the same for both types) and recognizing that the higher skilled type has higher consumption and so a lower marginal utility, the subsidy needs to be slightly larger for the lower type.

Note that with these assumptions, the approximate FOC can be interpreted as if the warm glow lowers the cost of public good production to equal the government cost:

$$\sum N_n b v' [G] = \lambda p - \nu = \lambda p - \frac{N_2 + \mu}{N_2} w' [g_2] \tag{58}$$

Note that the RHS differs from that with warm glow not in social welfare by the amount $w' [g_2]$. Substituting for λ from (), we can write this as

$$\sum N_n b v' [G] = \frac{N_2 + \mu}{N_2} p u' [c_2] - \frac{N_2 + \mu}{N_2} w' [g_2] \tag{59}$$

Using the FOC for donations, with v' small relative to w', we have

$$\sum N_n b v' [G] = \frac{N_2 + \mu}{N_2} s_2 p u' [c_2] = s_2 p \lambda \tag{60}$$

Thus, only the cost of subsidization enters the FOC

2.4 Warm Glow Preferences and the Formulation of Social Welfare

The fact that warm glows improve the description of individual behavior does not necessarily imply that social welfare should be defined including warm glows. That is, assume, as above, that the government switches from public provision to private provision, with the level of public good unchanged. Should the warm glows from private provision be part of a gain in social welfare from undertaking this project? I focus here on the argument relative to standard public goods, not donations to support redistribution.

One can argue the two sides of this issue in terms of underlying assumptions or in terms of the ethical appeal of outcomes given assumptions about the formulation of social welfare. The clear argument for inclusion is that these are the preferences that determine behavior and they should be respected by the social evaluation. The alternative argument (made by Hammond (1978) about altruism, but similarly applicable here) is that the use of a social welfare function incorporates the social interest inherent in the public good enjoyment of others (presumably a key part of the reason for a warm glow) and inclusion of warm glow utility is then a form of double counting. In terms of outcomes, inclusion of warm glow preferences calls for using resources to give people warm glows. Somehow this does not seem like a good use of resources.[10] Moreover it is unclear how preferences may be influenced by the process generating the opportunity/need for donations.

[10]This discussion is on the preferences to use for social evaluations assuming they are measurable. A critical issue is the degree of measurability of warm glow preferences, particularly if they are very sensitive to the context of charitable fund raising or hypothetical questioning.

One can not quite credit the possibility of someone being happy that the government underprovided a public good so that the opportunity to donate was present.

With a general equilibrium model, one tracks all uses of resources. If one were only tracking some uses and ignoring others, then the welfare implications of a policy might be distorted by the pattern of inclusion and exclusion in the analysis. Thus valid partial equilibrium analyses include a numeraire good reflecting the uses of resources that are not modeled in detail in the partial equilibrium setting. That people get pleasure from donating to finance public goods is shown by their pattern of donations. Similarly, people do volunteer work for charities to induce other people to donate. Thus there is a warm glow associated with the donations of others. But people also participate in the political process in a way that can not be explained solely by economic self-interest. This includes some donations to political campaigns and, of course, extends to voting. If someone gets a warm glow from work on an election that results in an increase in some public good level, then such a warm glow seems to be on the same footing as that from a charitable donation. One can argue that the political warm glow is related to final good outcome, so an increase in subsidy rate would be as appealing as an increase in direct provision. But then, should the warm glow from giving also be related to final outcomes? If the government cuts back direct provision and increases subsidies, should the warm glow of someone who cares about the public good necessarily increase from this aspect of the process.

The focus on warm glow has been on the marginal warm glow gain from an increase in donation. But some of this marginal warm glow is thought to come from decreasing the negative feelings associated with social pressure to donate. In this setting, the role of government policy in determining the amount of social pressure becomes a very important issue. If donation lowers a utility loss from pressure, public policy that lowers the pressure (for example by larger public provision and, perhaps, lower subsidies) may raise utility measured in this way.

The basic point here is that just as we need to pay attention to all of resource uses, so too if we want to count preferences based on process, we need to pay attention to all of the process, not just part of it. Paying attention to none of it may be a more valid measure even for those who think utility from process ought to be on the same welfare footing as utility from final resource use.

I do not intend to argue that the process of resource allocation is irrelevant for social welfare analysis, although this can be put in a different vocabulary. Thus coin tosses for fairness in allocation of an indivisible good has been argued to raise social welfare (Diamond, 1967) and denial of the ability to donate can be viewed as a violation of a basic freedom choice of a subsidy rate seems to be in a different category.

3 Fundraising

In the models above, the government can leave room for private contributions in order to alter the effect of income taxation. As modeled here and in the previous literature cited above, the use of private donations does not have any resource cost (unlike the marginal deadweight burden from financing public contributions and the subsidies of private contributions). This is factually inaccurate. Considerable sums are spent on fund-raising by organizations that provide public goods. Such an additional source of deadweight burdens should affect the analysis but is not explored here. A model with an endogenous level of resource use for fund raising as a function of both public provision and the level and pattern of donation subsidies would be interesting. Going from analysis of models without fundraising costs to policy recommendations seems very premature.

4 Concluding Remarks

This paper has taken the standard optimal tax approach, not recognizing issues in the definition of income as having independent relevance. Taking the latter approach, one could argue that donations reduce income over which a consumer maintains control and therefore ought not to be part of taxable income taken as measuring the latter. This would parallel the view that to the extent that medical expenditures are beyond the choice of a consumer, medical expenses ought to be deducted from the measure of income over which an individual has control. The roles of income definition in terms of the philosophy or the political economy of taxation are separate issues.

5 Appendix

Optimal Subsidized Private Provision with Three Types of Workers

With the standard model with only two types, relaxing the incentive compatibility constraint comes from having the public good contributions coming from the highest skilled type. With three types there is an issue of the allocation of contributions across the two higher types. We continue to analyze this in a setting where the degree of subsidy depends on income with complete flexibility. Since the choice of subsidy rate is equivalent to the choice of contribution level, we can let the contributions be the controls. Let us assume there are three types, with type 0 having no skill. Type 1 either works or job 1 or does not work. We continue to assume that the next best alternative for type 2 is to imitate type 1, with preferences such that if type 1 does not imitate type 0, then type 2 strictly prefers imitating type 1 to imitating type 0. We continue to assume that the optimum has each type working at the job that matches the worker's skill. We need to explore the allocation of donations between types 1 and 2.

$$\text{Maximize}_{c,G,g} \quad \sum N_n \left(u\left[c_n\right] - a_{nn} + b_n v\left[G\right]\right)$$

$$\begin{aligned}
\text{subject to:} \quad & E + pG + \sum N_n \left(c_n - n\right) \leq 0 \\
& u\left[c_1\right] - a_{11} + b_1 v\left[G\right] \geq u\left[c_0\right] + b_1 v\left[G - g_1 + g_0\right] \\
& u\left[c_2\right] - a_{22} + b_2 v\left[G\right] \geq u\left[c_1\right] - a_{12} + b_2 v\left[G - g_2 + g_1\right] \\
& G \geq \sum N_n g_n; \;\; g_n \geq 0 \text{ for all } n
\end{aligned} \tag{61}$$

Since contributions by the lowest skilled can only hurt the incentive compatibility constraint, they are set to zero. Since contributions by the highest paid can only help the incentive compatibility constraint it follows that the government will not contribute and $G = \sum N_n g_n$. We continue to focus on the case where everyone has the same utility from the public good, the same value of b_n. To begin, we form a Lagrangian:

$$\begin{aligned}
L \;=\; & \sum N_n \left(u\left[c_n\right] - a_{nn} + b_n v\left[G\right]\right) - \lambda \left(E + pG + \sum N_n \left(c_n - n\right)\right) \\
& + \mu_1 \left(u\left[c_1\right] - a_{11} + b_1 v\left[G\right] - \left(u\left[c_0\right] + b_1 v\left[G - g_1\right]\right)\right) \\
& + \mu_2 \left(u\left[c_2\right] - a_{22} + b_2 v\left[G\right] - \left(u\left[c_1\right] - a_{12} + b_2 v\left[G - g_2 + g_1\right]\right)\right) \\
& + \nu \left(G - \sum N_n g_n\right) + \sum \xi_n g_n
\end{aligned} \tag{62}$$

Differentiating gives the FOC for consumption levels:

$$\frac{\partial L}{\partial c_0} = N_0 \left(u' \left[c_0\right] - \lambda\right) - \mu_1 u' \left[c_0\right] = 0 \tag{63}$$

$$\frac{\partial L}{\partial c_1} = N_1 \left(u' \left[c_1\right] - \lambda\right) + \left(\mu_1 - \mu_2\right) u' \left[c_1\right] = 0 \tag{64}$$

$$\frac{\partial L}{\partial c_2} = N_2 \left(u' \left[c_2\right] - \lambda\right) + \mu_2 u' \left[c_2\right] = 0. \tag{65}$$

Differentiating gives the FOC for public good level and donations (using the observation that the lowest type does not contribute):

$$\frac{\partial L}{\partial G} = \sum N_n b_n v' \left[G\right] - \lambda p + \mu_1 b_1 \left(v' \left[G\right] - v' \left[G - g_1\right]\right) + \mu_2 b_2 \left(v' \left[G\right] - v' \left[G - g_2 + g_1\right]\right) + \nu = 0 \tag{66}$$

$$\frac{\partial L}{\partial g_1} = \mu_1 b_1 v' \left[G - g_1\right] - \mu_2 b_2 v' \left[G - g_2 + g_1\right] - \nu N_1 + \xi_1 = 0 \tag{67}$$

$$\frac{\partial L}{\partial g_2} = \mu_2 b_2 v' \left[G - g_2 + g_1\right] - \nu N_2 + \xi_2 = 0 \tag{68}$$

Adding the last two equations and rearranging terms, we have

$$\mu_1 b_1 v' \left[G - g_1\right] = \nu \left(N_1 + N_2\right) - \xi_1 - \xi_2 \tag{69}$$

$$\mu_2 b_2 v' \left[G - g_2 + g_1\right] = \nu N_2 - \xi_2 \tag{70}$$

This problem can have different optima, depending on which incentive compatibility constraints are binding. One possibility is that the public good is so important that none of the incentive compatibility constraints are binding. Then all workers have the same marginal utility of consumption and the contributions are in the ranges that support the labor allocation. Another possibility is that $g_1 = 0$ and $g_2 = G/N_2$. This can happen with several possibilities of binding constraints.

If the donations of both types 1 and 2 are positive ($\xi_1 = \xi_2 = 0$), then if the incentive compatibility constraint for either type is binding,so too is that for the other. From the FOC:

$$N_0 \left(1 - \lambda / u' \left[c_0\right]\right) = \mu_1 = \frac{\nu \left(N_1 + N_2\right)}{b_1 v' \left[G - g_1\right]} \tag{71}$$

$$N_1 \left(1 - \lambda / u' \left[c_1\right]\right) = \mu_2 - \mu_1 = \frac{\nu N_2}{b_2 v' \left[G - g_2 + g_1\right]} - \frac{\nu \left(N_1 + N_2\right)}{b_1 v' \left[G - g_1\right]} \tag{72}$$

$$N_2 \left(1 - \lambda / u' \left[c_2\right]\right) = -\mu_2 = -\frac{\nu N_2}{b_2 v' \left[G - g_2 + g_1\right]} \tag{73}$$

Thus $\mu_2 \neq 0$ implies $\nu \neq 0$, implying $\mu_1 \neq 0$, and vice versa.

Uniform Subsidized Private Provision

We turn now to a setting with uniform subsidization, sufficient to induce some private contributions. Since we have assumed that everyone has the same preferences (apart from the disutility of work) and that everyone faces the same subsidy rate, the population falls into two categories – those who contribute, all of whom has the same level of private good consumption, and those who do not contribute, and have lower private good consumption. That is, among all those contributing, we have

$$(1-s)\,pu'\left[x_n-(1-s)\,pg_n\right]=v'\left[G\right] \tag{74}$$

For those not contributing, we have

$$(1-s)\,pu'\left[x_n\right]>v'\left[G\right] \tag{75}$$

What drives the pattern of contributions is the pattern of compensation. Since higher jobs must pay more if they are to be held, it follows that there is a critical value of skill such that everyone with higher skill contributes and has the same consumption and everyone with lower skill does not contribute. The incentive compatibility constraint (compared with the next lowest skill) for a contributor would be

$$u\left[x_n-(1-s)\,pg_n\right]-a_{nn}+v\left[G\right]\geq u\left[x_{n-1}-(1-s)\,pg_{n-1}\right]-a_{n-1n}+v\left[G-g_n+g_{n-1}\right] \tag{76}$$

With the next lower skill also contributing, since contributions continue until marginal utilities, and so private consumption levels, are equalized, we can write this as

$$u\left[x_n-(1-s)\,pg_n\right]-a_{nn}+v\left[G\right]\geq u\left[x_n-(1-s)\,pg_n\right]-a_{n-1n}+v\left[G-g_n+g_{n-1}\right] \tag{77}$$

or

$$u\left[x_n-(1-s)\,pg_n\right]-a_{nn}+v\left[G\right]\geq u\left[x_n-(1-s)\,pg_n\right]-a_{n-1n}+v\left[G-g_n+g_{n-1}\right] \tag{78}$$

or

$$-a_{nn}+v\left[G\right]\geq -a_{n-1n}+v\left[G-g_n+g_{n-1}\right] \tag{79}$$

Thus, where the incentive compatibility constraint is binding and donations are positive, we have a difference equation for contributions in equilibrium. (There are also equilibria where the incentive compatibility constraint does not bind.)

With everyone having the same utility of public and private consumption, we have a very peculiar equilibrium. Ranking people by after-tax income, we find that those with low incomes up to some level make no contributions to the public good. Above this level, contributions are so large that everyone has the same marginal utility of private consumption, and so the same level of private consumption.

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