Liquidity Supply and Demand in Limit Order Markets*

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Abstract

We model a trader's decision to supply liquidity by submitting limit orders or demand liquidity by submitting market orders in a limit order market. The best quotes and the execution probabilities and picking off risks of limit orders determine the price of immediacy. The price of immediacy and the trader's willingness to pay for immediacy determine the trader's optimal order submission, with the trader's willingness to pay for immediacy depending on the trader's valuation for the stock. We estimate the execution probabilities and the picking off risks using a sample from the Vancouver Stock Exchange to compute the price of immediacy. The price of immediacy changes with market conditions — a trader's optimal order submission changes with market conditions. We combine the price of immediacy with the actual order submissions to estimate the unobserved arrival rates of traders and the distribution of the traders' valuations. High realized stock volatility increases the arrival rate of traders and increases the number of value traders arriving — liquidity supply is more competive after periods of high volatility. An increase in the spread decreases the arrival rate of traders and decreases the number of value traders arriving — liquidity supply is less competitive when the spread widens.

Keywords: liquidity; limit orders; market orders; high frequency data; discrete choice
JEL Codes: G14, G15, C25, C41

1 Introduction

Market liquidity is used by exchanges, regulators, and investors to evaluate trading systems. In a limit order market, all traders with access to the trading system can supply liquidity by submitting limit orders or demand liquidity by submitting market orders. Market liquidity is determined by the traders' order submission strategies. Understanding the determinants of liquidity in a limit order market therefore requires understanding the determinants of the traders' order submission strategies.

A market order transacts immediately at a price determined by the best quotes in the limit order book: a market order offers immediacy. A limit order offers price improvement relative to a market order, but there are costs to submitting a limit order rather than a market order. The limit order may take time to execute and may not completely execute before it expires; we call the probability that the order executes the execution probability. Since the limit order may not execute immediately, there is chance that the underlying value of the stock changes before the limit order executes; we call the resulting risk the picking off risk. The best quotes and the price improvements, execution probabilities and picking off risks of limit orders determine the price of immediacy. A trader's optimal order submission depends on the price of immediacy, and the trader's willingness to pay for immediacy.

Why do traders' optimal order submissions vary? For example, the bottom panel of Table 3 in Harris and Hasbrouck (1996) reports that on the NYSE, 42% of the order submissions are market orders when the spread is \$1/8 and 30% of the orders submissions are market orders when the spread is \$1/4. The change in the order submission frequency depends on the change in the price of immediacy and the distribution of the traders' willingness to pay for immediacy. But we do not directly observe the price of immediacy, nor the traders' willingness to pay for immediacy. Instead, we only observe the traders' order submissions.

We model a trader's decision to supply liquidity by submitting limit orders or demand liquidity by submitting market orders. In our model, a trader's willingness to pay for immediacy depends on his valuation for the stock. Traders with extreme valuations for the stock lose more from failing to execute than traders with moderate valuations for the stock. Traders with extreme valuations therefore have a higher willingness to pay for immediacy than traders with moderate valuations. We interpret traders with extreme valuations as liquidity traders and traders with moderate valuations as value traders. A trader's valuation along with the price of immediacy determines whether the trader submits a market order, a limit order, or no order.

We use a sample from the Vancouver Stock Exchange to estimate the price of immediacy and we estimate the unobserved distribution of traders' valuations and the unobserved arrival rates of traders. We estimate the price of immediacy by estimating the execution probabilities and picking off risks for alternative order submissions under the identifying assumption that traders have rational expectations. We estimate the distribution of the traders' valuations and the arrival rates of the traders by combining the estimated price of immediacy with the traders' actual order submissions under the identifying assumption that traders make their order submissions to maximize their expected utility.

In our sample, when the proportional spread is 2.5%, approximately 37% of the orders submissions are market orders and when the proportional spread is 3.5%, approximately 30% of the order submissions are market orders. We use our estimates to compute the valuations for the traders who submit market orders in both cases. When the proportional spread is 2.5%, traders with valuations at least 4.9% away from the average valuation submit market orders, and when the proportional spread is 3.5%, traders with valuations at least 7.1% away from the average valuation submit market orders. The change in the spread changes the price of immediacy by changing the best quotes, and the execution probabilities and picking off risks for limit orders. The magnitude of the change in the price of immediacy exceeds the change in the spread because a limit order offers relatively more immediacy for the same price improvement when the spread is wider.

We also use our estimates of the price of immediacy to compute the expected utilities for liquidity and value traders in different market conditions. Traders can increase their expected utility by submitting different orders in different market conditions. Liquidity traders can increase their expected utility by up to 40% by submitting a limit order rather than a market order when the spread is wide and depth is low. Value traders can increase their expected utility by up to 10% by submitting a limit order rather than submitting no order when the spread is wide and the depth is low.

The idea that the price of immediacy and the traders willingness to pay for immediacy determine trading activity goes back to Demsetz (1968). In Glosten (1994), Seppi (1997) and Parlour and Seppi (2001), liquidity is provided by a large number of risk neutral value traders who are restricted to submit limit orders. The equilibrium price of immediacy is determined by a zero-expected profit condition for the value traders.

Sandås (2001) empirically tests and rejects the zero-expected profit conditions using a sample from the Stockholm Stock Exchange. Biais, Bisière and Spatt (2001) estimate a model of imperfect competition based on Biais, Martimort and Rochet (2000), finding evidence of positive expected profits before decimalization and zero afterward using a sample from the Island ECN. Both studies use models where multiple limit orders are first submitted, followed by a single market order submission. We focus instead on how the order book evolves in real time from order submission to order submission.

In our sample, value traders with a valuation within 2.5% of the average value of the stock account for between 32% and 52% of all traders. The value traders typically submit limit orders or no orders at all. The average expected time until the arrival of a value trader is approximately 23 minutes. The average time between orders submissions is 6 minutes. Profit opportunities for value traders are competed away slowly relative to the frequency of order submissions.

We allow for the possibility that any trader can submit a limit order in our model; liquidity traders may compete with the value traders in supplying liquidity. In this respect, our model is similar to the models in Cohen, Maier, Schwartz and Whitcomb (1981), Foucault (1999), Foucault, Kadan, and Kandel (2001), Handa and Schwartz (1996), Handa, Schwartz, and and Tiwari (2002), Harris (1998), Hollifield, Miller and Sandås (2002), and Parlour (1998). We extend Hollifield, Miller and Sandås (2002) to allow for a stochastic arrival process for traders and a non-zero payoff to the traders at order cancellation.

Several empirical studies document that traders' order submissions respond to market conditions. Biais, Hillion, and Spatt (1995) find that traders on the Paris Bourse react to a large spread or a small depth by submitting limit orders. Similar results hold in other markets. For example, see Ahn, Bae and Chan (2001) for the Stock Exchange of Hong Kong; Al-Suhaibani and Kryzanowksi (2001) for the Saudi Stock Market; Coppejans, Domowitz and Madhavan (2002) for the Swedish OMX futures market; and Chung, Van Ness and Van Ness (1999) and Bae, Jang and Park (2002) for the NYSE.

Harris and Hasbrouck (1996) measure the payoffs from different order submissions on the NYSE for a trader who must trade and for a trader who is indifferent to trading. For a trader who must trade, submitting limit orders at or inside the best quotes is optimal, while for a trader indifferent to trading, submitting no order is optimal. Griffiths, Smith, Turnbull and White (2000) measure the payoffs from different order submissions on the Toronto Stock Exchange, finding that limit orders submitted at the quotes are optimal submissions for a trader who must trade. Al-Suhaibani and Kryzanowski (2001) find similar results for the Saudi Stock Market.

A number of empirical studies examine the timing of orders. Biais, Hillion and Spatt (1995) document that traders submit limit orders in rapid succession when the spread widens on the Paris Bourse. Russell (1999) estimates multivariate autoregressive conditional duration models for the arrival of market and limit orders using a sample from the NYSE. Hasbrouck (1999) finds that the arrival rate of market and limit orders is negatively correlated over short horizons using a sample from the NYSE. Easley, Kiefer and O'Hara (1997) and Easley, Engle, O'Hara and Wu (2002) develop and estimate structural models relating the time between trades and the bid-ask spread to the arrival rates of informed and uniformed traders on the NYSE.

2 Description of the Market and the Sample

In 1989, the Vancouver Stock Exchange introduced the Vancouver Computerized Trading system. The Vancouver Computerized Trading system is similar to the limit order systems used on the Paris Bourse and the Toronto Stock Exchange. In 1999, after the end of our sample, the Vancouver Stock Exchange was involved in an amalgamation of Canadian equity trading and became a part of the Canadian Venture Exchange, which in turn was recently renamed the TSX Venture Exchange. The TSX Venture Exchange uses a similar trading system to the Vancouver Computerized Trading system.

Our sample was obtained from the audit tapes of the Vancouver Computerized Trading system. The sample contains order and transaction records from May 1990 to November 1993 for three stocks in the mining industry. Table 1 reports the stock ticker symbols, stock names, the total number of order submissions, and the percentage of buy and sell market and limit orders submitted in our sample.

The bottom panel of the table reports the mean and standard deviation of the percentage bid-ask spread, and the mean and standard deviation of the depth in the limit order book at or close to the best bid and ask quotes, measured in units of thousands of shares. The depth measure is calculated as the average of the number of shares offered on the buy and the sell side of the order book within 2.5% of the mid-quote.

Only the forty-five exchange member firms can submit market or limit orders directly into the system. A member firm may act as a broker submitting orders on behalf of its customers and as a dealer submitting orders on its own behalf. There are no designated market makers.

Limit orders in the order book are matched with incoming market orders to produce trades, giving priority to limit orders according to the order price and then the time of submission. Order prices must be multiples of a tick size. The tick size varies between one cent for prices below $3.00, five cents for prices between $3.00 and $4.99, and twelve and a half cents for prices at $5.00 and above. Orders sizes must be multiples of a fixed size which varies between 100 and 1000 shares.

Member firms can submit hidden orders where a fraction of the order size is not visible on the limit order book. A minimum of 1,000 shares or 50% of the total order size must be visible. The hidden fraction of the order retains its price priority, but loses its time priority. Once the visible part of the order is executed, a number of shares equal to the initially visible number of shares is automatically made visible. In our sample, few hidden orders are submitted.

The Vancouver Computerized Trading system offers a large amount of real time information. Member firms can view the entire limit order book including identification codes for the member firm who submitted a given order. Customers who are not members of the exchange can buy order book information from commercial vendors, including the five best bid and ask quotes with the corresponding order depth and the ten best individual orders on each side of the market, but not the identification codes that match orders to member firms.

We reconstruct individual order histories and the time-series of order books. A record is generated for every trade, cancellation, or change in the status of an order. Each record includes the time of the original order submission. Combining the changes with the limit order book at the open of each day we reconstruct the changes in the limit order book. We extract individual order histories, including the initial order submission and every future order execution or cancellation, and the corresponding order books. For less than one percent of the orders there are inconsistencies between the inferred order histories and the trading rules. We drop such orders from our sample.

We have detailed information, but there are limitations. First, we cannot separate the trades that a member firm makes on its own behalf from those it makes on behalf of its customers. Second, we cannot link different orders submitted by the same customer or member firm at different times. Third, we do not observe the identification codes the member firms observe. The first limitation causes us to focus on how a representative trader makes order submission decisions.

Table 2 reports the mean order size for buy and sell limit orders and market orders. The mean depth reported in Table 1 corresponds to a little more than three times the mean order size for all three stocks. The second row in each panel of Table 2 reports t-tests of the null hypothesis of equal mean order sizes for market and limit orders, with p-values in parentheses. The test rejects the null hypothesis for six out of nine pairs of means. Despite evidence of statistically significant differences between market and limit order sizes, the economic significance of the differences is small. The relative difference between the mean order size for market and limit orders reported in the last column of the table is between one-half and four percent.

To determine if traders' order submission decisions change in systematic ways as conditions change, we estimate models to predict the timing and type of order submissions, using conditioning variables reported in Table 3. We divide the conditioning variables into five groups: book, activity, market-wide, value proxies, and time dummies.

The book variables measure the current state of the limit order book, and include the bid-ask spread, and measures of depth close to the quotes and away from the quotes.

Biais, Hillion, and Spatt (1995) and Engle and Russell (1998) document that in the Paris Bourse and the New York Stock Exchange, periods of high order submission activity are likely to be followed by periods of high order submission activity, and similarly for periods of low order submission activity. We include the number of recent trades, the sum of the duration of the last ten order book changes, and the volatility of the mid-quote over the last ten minutes to capture such effects.

We include market-wide conditioning variables to capture any market-wide effects on order submissions. We use the absolute values of the changes in the market-wide variables to proxy for their volatility. Because of data availability, all of our market-wide conditioning variables are computed at a daily frequency. Changes in the Toronto Stock Exchange (TSE) market index measure the overall information flow into the market. We use the TSE mining index to capture any industry effects. The change in the Canadian overnight interest rate is included because frictions such as margin requirements depend on the overnight interest rate. The change in the Canadian/US dollar exchange rate is a proxy for news about the Canadian economy.

We include the absolute value of the lagged open to open mid-quote return of each stock to measure realized stock volatility. We compute a centered moving average of the mid-quotes over a twenty minute window as a proxy for the underlying value of the stock. We use a moving average to reduce any mechanical price effects arising from market orders using up all liquidity at the best quotes and changing the mid-quotes. We include the distance between the current mid-quote and the centered moving average as a measure of temporary order imbalances in the order book. We also include six hourly dummy variables to capture any deterministic time effects.

Table 4 reports the results from estimating a Weibull model for the hazard rate of order submissions:

$$\Pr_t\left(\text{Order submission in } [t, t+dt)\right) = \exp\left(\gamma' z_{t_i}\right)\alpha(t-t_i)^{\alpha-1}dt, \tag{1}$$

where the subscript t denotes conditioning on information available at t, t_i is the time of the previous order submission, and z_{t_i} is a vector of conditioning variables.[1] The point estimates of α are all less than one; the conditional probability of an order submission is decreasing in the length of time since the previous order submission.

The parameters on the spread are negative — a wider spread predicts a longer time to the next order submission. The depth variables have mixed effects on the predicted time to the next order submission. The parameters are positive for recent trades and negative for duration: short time between order submissions predicts short time between order submissions in the future. The signs of the parameters on market-wide variables vary from stock to stock and many are not

[1] Equation (1) is interpreted as

$$\lim_{\Delta t \downarrow 0} \frac{\Pr_t\left(\text{Order submission in } [t, t+\Delta t) \mid \text{previous order submission at } t_i\right)}{\Delta t} = \exp\left(\gamma' z_{t_i}\right)\alpha(t-t_i)^{\alpha-1}.$$

statistically different from zero. The parameters on lagged return are all positive; periods of high stock volatility predict shorter time between order submissions in the future. The parameters on the hourly dummies indicate that in general, the time between order submissions is longer in the first three hours of the day than during the last few hours of the day.

To determine whether the conditioning variables predict the time between order submissions, we report chi-squared tests of the null hypothesis that all parameters are jointly equal to zero. The test statistic is reported below each group of conditioning variables with the corresponding p-value in parenthesis. Except for the market-wide variables for BHO, we reject the null in all cases.

Table 5 reports the estimation results for six ordered probit models of buy and sell order submissions. We condition on the variables in Table 3 but use only close depth on the opposite side of the order, and include the log of order size. We model the traders' choice between three types of orders: a market order, a limit order at one tick from the best quote, and a limit order at two or more ticks from the best quote. The dependent variable is zero for a market order, one for a limit order at one tick from the best quotes, and two for all other limit orders.

The parameters on the spread are positive: traders are more likely to submit limit orders when the spread is large. The parameters for the close ask depth for sell orders and close bid depth for buy order are both negative; traders are less likely to submit limit orders when the depth on the same side as the order is high. The parameters on order size indicate that traders submitting larger orders are more likely to submit limit orders than traders submitting smaller orders. The last row of each panel reports chi-squared test statistics for a test of the null hypothesis that the estimated parameters on the conditioning variables are jointly equal to zero. The null hypothesis is rejected for all groups of conditioning variables but the market-wide variables. For the market-wide variables we reject the null for sell orders for BHO and for buy and sell orders for ERR. Overall, the conditioning variables predict the traders' decisions to supply liquidity by submitting limit orders or to demand liquidity by submitting market orders, as well as the timing of the order submissions.

3 Model

We model the traders' order submission strategies. Traders arrive sequentially and differ in their valuations for the stock. The probability that a trader arrives is

$$\Pr_t \left(\text{Trader arrives in } [t, t+dt)\right) = \lambda_t dt. \tag{2}$$

The subscript t denotes conditioning on information available at time t. Information available at time t includes the time since the last order submission, the history of order submissions, general market conditions, and the current limit order book.

Once a trader arrives, he can submit a market order for q shares, a limit order for q shares, or no order. Although we assume a fixed order size, we condition on the observed order size in our empirical work to allow for the possibility that the optimal order submission depends on q. The decision indicator variables $d_{s,t}^{sell}$ for $s = 0, 1, \ldots, S$; $d_{b,t}^{buy}$ for $b = 0, 1, \ldots, B$; and d_t^{NO} denote the trader's decision at t. If the trader submits a sell market order, $d_{0,t}^{sell} = 1$; if the trader submits a sell limit order at the price s ticks above the bid quote, $d_{s,t}^{sell} = 1$; if the trader submits a buy market order, $d_{0,t}^{buy} = 1$; if the trader submits a buy limit order b ticks below the ask quote, $d_{b,t}^{buy} = 1$; and if the trader does not submit any order, $d_t^{NO} = 1$.

The trader is risk neutral and has a valuation per share for the stock of v_t, equal to the sum of a common value and a private value:

$$v_t = y_t + u_t. \tag{3}$$

The common value, y_t, is the trader's time t expectation of the liquidation value of the stock. The common value changes as the traders learn new information. Traders who arrive at $t' > t$ therefore have more information about the common value than a trader who arrives at t.

The private value, u_t, is drawn *i.i.d.* across traders from the continuous distribution

$$\Pr_t \left(u_t \leq u\right) \equiv G_t \left(u\right), \tag{4}$$

with continuous density g_t. The distribution is conditional on information available at t, with a mean of zero.

Once the trader arrives, his private value is fixed until an exogenous random resubmission time $t + \tau_{resubmit} > t$. At $t + \tau_{resubmit}$, the trader cancels any unexecuted limit orders and receives a fixed utility of V per share for any unexecuted shares, where V is the expected utility of a new order submission at $t+\tau_{resubmit}$. The trader does not know the realization of the resubmission time when he arrives at the market. The resubmission time is bounded by $t + T$ where the constant T satisfies $T < \infty$.

Suppose that a trader with valuation $v_t = y_t + u_t$ submits a buy order b ticks below the ask quote at price $p_{b,t}$: $d_{b,t}^{buy} = 1$. Define $0 \leq Q_{t+\tau} \leq 1$ as the cumulative fraction of the order executed by time $t + \tau$, and

$$dQ_{t+\tau} \equiv Q_{t,t+\tau} - Q_{t+\tau-} \tag{5}$$

as the fraction of the order that executes at time $t+\tau$. If the order is canceled at time $t+\tau_{resubmit}$,

$$dQ_{t+\tau} = 0, \text{ for } \tau \geq \tau_{resubmit}. \tag{6}$$

Ignoring the cost of submitting the order and the utility of any resubmission, the utility that the trader receives from executing $dQ_{t,t+\tau}$ shares at $t + \tau$ at price $p_{b,t}$ is

$$(y_{t+\tau} + u_t - p_{b,t})\, dQ_{t+\tau} = (v_t - p_{b,t})\, dQ_{t+\tau} + (y_{t+\tau} - y_t)\, dQ_{t+\tau}. \tag{7}$$

Here, $y_{t+\tau}$ is the common value at $t + \tau$; $(v_t - p_{b,t})\, dQ_{t+\tau}$ is the utility from executing $dQ_{t+\tau}$ with the common value unchanged; and $(y_{t+\tau} - y_t)\, dQ_{t+\tau}$ is the utility from any common value changes between t and $t + \tau$.

Integrating over the possible execution times for the order, including the resubmission utility and the cost of the submission, the realized utility from submitting the order is

$$U_{t,t+T} = \int_{\tau=0}^{T} (v_t - p_{b,t})\, dQ_{t+\tau} + \int_{\tau=0}^{T} (y_{t+\tau} - y_t)\, dQ_{t+\tau} + V\,(1 - Q_{t+T}) - c. \tag{8}$$

Define

$$\psi_{b,t}^{buy} \equiv E_t \left[Q_{t+T} \,\middle|\, d_{b,t}^{buy} = 1 \right] \tag{9}$$

as the execution probability for the order. For a market order, the execution probability is one. Further, define

$$\xi_{b,t}^{buy} \equiv E_t \left[\int_{\tau=0}^{T} (y_{t+\tau} - y_t)\, dQ_{t+\tau} \middle| d_{b,t}^{buy} = 1 \right] \tag{10}$$

as the picking off risk for the order. The picking off risk is the covariance of changes in the common value and the fraction of the order that executes. For a market order, the picking off risk is zero.

The trader's expected utility from submitting a buy order at price $p_{t,b}$ is the expected value of equation (8), conditional on the trader's information, which using the definitions of the execution probability and picking off risk is equal to

$$E_t \left[U_{t,t+T} \middle| d_{b,t}^{buy} = 1, v_t \right] = (v_t - p_{b,t})\, \psi_{b,t}^{buy} + \xi_{b,t}^{buy} + V \left(1 - \psi_{b,t}^{buy} \right) - c. \tag{11}$$

Similarly, the expected utility of submitting a sell order at $p_{s,t}$ is

$$E_t \left[U_{t,t+T} \middle| d_{s,t}^{sell} = 1, v_t \right] = (p_{s,t} - v_t)\, \psi_{s,t}^{sell} - \xi_{s,t}^{sell} + V \left(1 - \psi_{s,t}^{sell} \right) - c. \tag{12}$$

The trader's order submission strategy maximizes his expected utility,

$$\max_{\{d_{s,t}^{sell}\},\{d_{b,t}^{buy}\},d_t^{NO}} \sum_{s=0}^{S} d_{s,t}^{sell} E_t \left[U_{t,t+T} \middle| d_{s,t}^{sell} = 1, v_t \right] + \sum_{b=0}^{B} d_{b,t}^{buy} E_t \left[U_{t,t+T} \middle| d_{b,t}^{buy} = 1, v_t \right] + d_t^{NO} V, \tag{13}$$

subject to:

$$d_{s,t}^{sell} \in \{0,1\},\ s = 0, ..., S,\ \ d_{b,t}^{buy} \in \{0,1\},\ b = 0, ..., B,\ \ d_t^{NO} \in \{0,1\}, \tag{14}$$

$$\sum_{s=0}^{S} d_{s,t}^{sell} + \sum_{b=0}^{B} d_{b,t}^{buy} + d_t^{NO} = 1. \tag{15}$$

Equation (15) is the constraint that at most one submission is made at t.

Let $d_{s,t}^{sell*}(v)$, $d_{b,t}^{buy*}(v)$, $d_t^{NO*}(v)$ be the optimal strategy, describing the trader's optimal order submission as a function of his information and valuation. Lemma 1 shows that the optimal order submission strategy is monotone in the trader's valuation.

Lemma 1 *Suppose that a buyer with valuation v optimally submits a buy order at price $b \geq 0$ ticks below the ask quote, so that $d_{b,t}^{*buy}(v) = 1$.*

If the execution probabilities are strictly decreasing in the distance between the limit order price and the best ask quote,

$$b < b+1 \textit{ implies that } \psi^{buy}_{b,t} > \psi^{buy}_{b+1,t}, \textit{ for } b = 0, \ldots, B-1, \tag{16}$$

then a trader with valuation $v' > v$ submits a buy order at a price $p_{b'}$ weakly closer to the ask quote:

$$\psi^{buy}_{b',t} \geq \psi^{buy}_{b,t} \textit{ and } b' \leq b. \tag{17}$$

Similar results hold on the sell side.

Lemma 1 implies that all traders whose valuations are in the same interval submit the same order. We assume that sell market orders, sell limit orders between 1 and S_t ticks above the bid quote, buy market orders, and buy limit orders between 1 and B_t ticks below the ask quote are all optimal submissions for the trader depending on his valuation. The assumption holds if the thresholds defined below form a monotone sequence.

Define the threshold valuation $\theta^{buy}_t(b, b')$ as the valuation of a trader who is indifferent between submitting a buy order at price $p_{b,t}$ and a buy order at price $p_{b',t}$

$$\theta^{buy}_t(b, b') = p_{t,b} + V + \frac{\left(p_{b,t} - p_{b',t}\right)\psi^{buy}_{b',t} + \left(\xi^{buy}_{b',t} - \xi^{buy}_{b,t}\right)}{\psi^{buy}_{b,t} - \psi^{buy}_{b',t}}. \tag{18}$$

The threshold valuation for a buy order at price $p_{b,t}$ and not submitting an order is

$$\theta^{buy}_t(b, \mathrm{NO}) = p_{b,t} + V - \frac{\xi^{buy}_{b,t} - c}{\psi^{buy}_{b,t}}. \tag{19}$$

The threshold valuation for a sell order at price $p_{s,t}$ and a sell order at price $p_{s',t}$ is

$$\theta^{sell}_t\left(s, s'\right) = p_{s,t} - V - \frac{\left(p_{s',t} - p_{s,t}\right)\psi^{sell}_{s',t} + \left(\xi^{sell}_{s,t} - \xi^{sell}_{s',t}\right)}{\psi^{sell}_{s,t} - \psi^{sell}_{s',t}}. \tag{20}$$

The threshold valuation for a sell limit order at price $p_{s,t}$ and not submitting any order is

$$\theta_t^{sell}(s, \mathrm{NO}) = p_{s,t} - V - \frac{\xi_{s,t}^{sell} + c}{\psi_{s,t}^{sell}}. \tag{21}$$

The threshold valuation for a sell order at price $p_{s,t}$ and a buy order at price $p_{b,t}$ is

$$\theta_t(s, b) = \frac{\left(p_{b,t}\psi_{b,t}^{buy} + p_{s,t}\psi_{s,t}^{sell}\right) + V\left(\psi_{b,t}^{buy} - \psi_{s,t}^{sell}\right) - \left(\xi_{b,t}^{buy} + \xi_{s,t}^{sell}\right)}{\psi_{s,t}^{sell} + \psi_{b,t}^{buy}}. \tag{22}$$

Traders with high private values submit buy orders with high execution probabilities and prices. Traders with low private values submit sell orders with high execution probabilities and low prices. Traders with intermediate private values either submit no order or submit limit orders if the execution probabilities are high enough and the picking off risks are low enough.

Define the marginal thresholds for sellers and buyers as

$$\begin{aligned} \theta_t^{buy}(\text{Marginal}) &= \max\left(\theta_t(S_t, B_t), \theta_t^{buy}(B_t, \mathrm{NO})\right), \\ \theta_t^{sell}(\text{Marginal}) &= \min\left(\theta_t(S_t, B_t), \theta_t^{sell}(S_t, \mathrm{NO})\right). \end{aligned} \tag{23}$$

If the marginal threshold for the buyers is equal to the marginal threshold for the sellers, all traders find it optimal to submit an order. Otherwise, there are traders who find it optimal not to submit any order.

Proposition 1 *The optimal order submission strategy is*

$$\begin{aligned} d_{s,t}^{sell*}(y_t + u_t) &= 1, \ \textit{if} \begin{cases} s = 0, \ \textit{and} \ -\infty \leq y_t + u_t < \theta_t^{sell}(0,1), \\ \textit{or} \\ s = 1, ..., S_t - 1 \ \textit{and} \ \theta_t^{sell}(s-1, s) \leq y_t + u_t < \theta_t^{sell}(s, s+1) \\ \textit{or} \\ s = S_t, \ \textit{and} \ \theta_t^{sell}(S_t - 1, S_t) \leq y_t + u_t < \theta_t^{sell}(Marginal), \end{cases} \\ &= 0, \ \textit{otherwise.} \end{aligned} \tag{24}$$

$$
\begin{aligned}
d_{b,t}^{buy*}(y_t+u_t) &= 1,\ if \begin{cases} b=0 \ and\ \theta_t^{buy}(0,1) \le y_t+u_t < \infty, \\ or \\ b=1,...,B_t-1 \ and\ \theta_t^{buy}(b-1,b) \le y_t+u_t < \theta_t^{buy}(b,b+1), \\ or \\ b=B_t \ and\ \theta_t^{buy}(Marginal) \le y_t+u_t < \theta_t^{buy}(B_t-1,B_t), \end{cases} \\
&= 0,\ otherwise. \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
d_t^{NO*}(y_t+u_t) &= 1 \quad if\ \theta_t^{sell}(Marginal) \le y_t+u_t \le \theta_t^{sell}(Marginal), \\
&= 0,\ otherwise. \qquad (26)
\end{aligned}
$$

Figure 1 provides a graphical representation of the trader's order submission problem. Here, buy market, one tick and two tick buy limit orders, sell market orders, and one tick and two tick sell limit orders are optimal for a trader with some valuation. The continuation value is equal to zero. The expected utility as a function of the trader's valuation from submitting different sell orders are plotted with dashed lines and the expected utility from submitting different buy orders are plotted with dashed-dotted lines. From equations (11) and (12), the trader's expected utility from submitting any particular order is a linear function of his valuation, with slope equal to the execution probability for that order. The dark solid line is the maximized utility function.

Geometrically, the thresholds are the valuations where the expected utilities intersect. For example, the threshold for a sell market order and a one tick sell limit order is $\theta_t^{sell}(0,1)$; a trader with a valuation less than $\theta_t^{sell}(0,1)$ submits a sell market order. The thresholds associated with submitting any particular order and submitting no order are the valuations where the expected utilities cross the horizontal axis. Here, $\theta_t^{sell}(2,\mathrm{NO}) < \theta_t(2,2)$, and $\theta_t^{buy}(2,\mathrm{NO}) > \theta_t(2,2)$, so that if the trader's valuation is between $\theta_t^{sell}(2,\mathrm{NO})$ and $\theta_t^{buy}(2,\mathrm{NO})$, the trader does not submit any order.

The threshold valuations measure the price of immediacy. Consider the threshold valuation for two buy orders given in equation (18). A lower execution probability for a buy order at $p_{b',t}$ implies a decrease in the threshold valuation. For the same price improvement and picking off risk, the higher priced buy order, $p_{b,t}$, now offers relatively more immediacy than the lower price buy order; the relative price of immediacy is lower. The traders willingness to pay for immediacy is determined by their valuations. Traders with valuation equal to or above the threshold valuation in equation (18) submit buy orders at the price $p_{b,t}$ or higher. A lower price of immediacy as a result of a lower execution probability at $p_{b',t}$ implies that a larger fraction of the traders will submit buy orders at $p_{b,t}$ or higher.

Using the optimal order strategy given in Proposition 1, the distribution of traders' valuations and the arrival rates of traders, we compute the conditional probability of different order submissions. The conditional probability of a buy market order between t and $t+dt$ is the probability that a trader who arrives finds it optimal to submit a buy market order times the probability that a trader arrives:

$$\begin{aligned} \Pr_t\left(\text{Buy market order in } [t, t+dt)\right) &= \Pr_t\left(y_t + u_t \geq \theta_t^{buy}(0,1)\right)\lambda_t dt \\ &= \left[1 - G_t\left(\theta_t^{buy}(0,1) - y_t\right)\right]\lambda_t dt. \end{aligned} \tag{27}$$

Similarly,

$$\Pr_t\left(\text{Buy limit order in } [t, t+dt)\right) = \left[G_t\left(\theta_t^{buy}(0,1) - y_t\right) - G_t\left(\theta_t^{buy}(\text{Marginal}) - y_t\right)\right]\lambda_t dt, \tag{28}$$

$$\Pr_t\left(\text{Sell market order in } [t, t+dt)\right) = \left[G_t\left(\theta_t^{sell}(0,1) - y_t\right)\right]\lambda_t dt, \tag{29}$$

$$\Pr_t\left(\text{Sell limit order in } [t, t+dt)\right) = \left[G_t\left(\theta_t^{sell}(\text{Marginal}) - y_t\right) - G_t^b\left(\theta_t^{sell}(0,1) - y_t\right)\right]\lambda_t dt. \tag{30}$$

No orders may be submitted between t and $t+dt$ for two reasons. Either a trader does not arrive or a trader arrives and does not submit any order,

$$\begin{aligned} &\Pr_t\left(\text{No order submission in } [t, t+dt)\right) \\ &\quad = 1 - \lambda_t dt + \left[G_t\left(\theta_t^{buy}(\text{Marginal}) - y_t\right) - G_t\left(\theta_t^{sell}(\text{Marginal}) - y_t\right)\right]\lambda_t dt. \end{aligned} \tag{31}$$

Equations (27) through (31) show how the thresholds, the private values distribution and the arrival rate of the traders jointly determine the timing of order submissions. The arrival rate of traders and the distribution of traders' valuations determine the relative competitiveness of liquidity supply. Consider a stock with a high arrival rate of traders and many traders with private values close to zero. From equations (28) and (30), the model predicts a large probability of limit order submissions when the price of immediacy increases so that value traders find it optimal to submit limit orders. In this case, we expect the order book to offer profit opportunities for value traders only for short periods of time. Liquidity supply is therefore relatively competitive.

4 Empirical Results

The model described in the previous section provides a framework for our econometric approach. The model characterizes a trader's choice between submitting an order or not, and if he submits an order, what kind of order to submit. In our empirical work, we model the rate at which orders are executed and canceled parametrically. Our approach allows us to identify the unobserved arrival rate of traders and their distribution of valuations from the timing of market and limit order submissions in the sample.

We use the conditional probabilities of observing different order submissions in equations (27) through (31) to compute the conditional log-likelihood function for buy and sell market and limit order submission times. The log-likelihood function is conditional on the variables reported in Table 3 and the log of order size. The conditional log-likelihood function is reported in Appendix B. The conditional log-likelihood function depends on the common value at the time of order submission, the thresholds, the arrival rate of traders, and the private value distribution. The thresholds depend on the market and limit order prices; execution probabilities and picking off risks of market and limit order submissions; the expected utility of a resubmission; and the costs of submitting an order.

4.1 The Price of Immediacy

We assume that the traders have rational expectations about the execution probabilities and picking off risks. We therefore use all order submissions and the realized execution histories in our sample to form estimates of the execution probabilities and picking off risks for orders that the traders could have submitted. In forming estimates of the picking off risks, we use a twenty minute centered average mid-quote to proxy for the common value.

An order leaves the book when it is executed or when it is canceled. We use a Weibull independent competing risks model to estimate how long an order remains in the limit order book. Lancaster (1990) provides an introduction to the competing risks model. Suppose that a limit order is submitted at t_i. The order leaves the book at the minimum of the hypothetical cancellation time for the order and the hypothetical execution time for the order. The hazard rate for the hypothetical cancellation time is

$$\Pr_t\left(\text{Order submitted at } t_i \text{ cancels in } [t, t+dt)\right) = \exp(z'_{t_i}\gamma_c)\alpha_c(t-t_i)^{\alpha_c-1}dt, \tag{32}$$

and the hazard rate for the hypothetical execution time is

$$\Pr_t\left(\text{Order submitted at } t_i \text{ executes in } [t, t+dt)\right) = \exp(z'_{t_i}\gamma_e)\alpha_e(t-t_i)^{\alpha_e-1}dt, \tag{33}$$

where z_{t_i} is a vector of conditioning information known when the order is submitted at t_i.

Lo, MacKinlay and Zhang (2001) estimate a Gamma model for the time to first execution and time to completion for limit orders, treating canceled orders as censored observations. Al-Suhaibani and Kryzanowski (2000) and Cho and Nelling (2000) estimate Weibull models for the time to execution of limit orders, also treating canceled limit orders as censored observations. Cho and Nelling (2000) compute the execution probabilities for limit orders based on the parameter estimates for the time to execution. We compute execution probabilities using the competing risks model, where we explicitly estimate the hazard rate of cancellations. Details of the computations of the execution probabilities are reported in Appendix C.

We condition on the variables in Table 3, including the log of order size. We track the orders for two-days, treating orders outstanding two days after submission as censored observations. We handle partial executions by assuming an order was an execution if at least 50% of the order size is executed, otherwise we treat the order as a cancellation. We estimate hazard rates for execution and cancellation for buy and sell limit orders submitted one tick from the best quotes and for marginal limit orders. We chose the marginal limit order so that approximately 95% of the limit order submissions are closer to the quotes than the marginal order at any price level. Tables 6 through 8 report the estimation results for the Weibull competing risks models for the execution and cancellation hazard rates for buy and sell limit orders at one tick from the best quotes and for marginal limit orders. The models are estimated by maximum likelihood.

The parameter estimates for the Weibull α parameters are between 0.544 and 0.817 for the execution hazard rates and between 0.468 and 0.652 for the cancellation hazard rates. The longer the order has been outstanding, the lower is the probability that the order either executes or cancels. The Weibull α parameter is lower for the cancellation hazard rate than for the execution hazard rate; the probability that a limit order leaves the book by cancellation rather than execution increases with the time the order spends in the book.

Increasing the spread increases the hazard for execution for all one tick orders and for most marginal orders. Depth on the same side decreases the hazard for execution for all orders. Depth on the opposite side increases the hazard for execution for all but two order types: the exceptions are the marginal orders for BHO. The marginal impact of increasing the depth on the same side is greater for the marginal orders than for the one tick orders. Larger order size decreases the hazard for execution for all orders and for all stocks.

Orders are executed and canceled more quickly following periods of frequent order submissions. For one tick orders, higher mid-quote volatility increases the hazard for execution and decreases the hazard for cancellation for all sell orders. The effect is reversed for buy orders. More recent trades increases the hazard for executions and cancellations and an increase in the durations for the last ten orders decreases the hazard for executions and cancellations. The market wide variables have small, but statistically significant, parameters on the hazards for execution and cancellation.

Lagged return increases the hazard for execution for all but one order type, although many of the parameters are not significantly different from zero. When distance to mid-quote is positive so that the common value is above the mid-quote, sell orders have higher hazards for executions and cancellations. When distance to mid-quote is negative so that the common value is below the mid-quote buy orders have higher hazards for executions and cancellations.

Most of the parameters on the hourly dummies are negative and statistically significant for the hazard for cancellation; orders submitted earlier during the day are less likely to be canceled. For execution times there is an offsetting effect for one tick away orders; orders submitted earlier in the day have a lower hazard for execution.

We forecast execution probabilities for one tick and marginal limit orders at every order submission in our sample using the parameter estimates from the competing risks model. We compute the probability that the order executes within two days. We use a two day cutoff because the majority of executions occur within two days. For BHO, the average execution probability for marginal sell limit orders is approximately 16%, for one tick sell limit orders 61%, for marginal buy limit orders 13% and for one tick buy limit orders 63%. The estimates for the other stocks are similar.

We also compute the elasticities of the execution probabilities with respect to the conditioning variables, evaluated at the means of the conditioning variables. For all stocks, an increase in spread leads to the largest estimated increase in the execution probabilities among all conditioning variables. When the spread is more than one tick a limit order submitted at one tick away from the quotes undercuts the prevailing quotes and moves to the front of the order queue. The wider the spread, the more the one tick order undercuts the quotes and lowers the price of immediacy for traders on the opposite side. As a consequence, one tick away limit orders have execution probabilities that are increasing in the spread. We find smaller effects on marginal orders from an increase in the spread.

Larger order size decreases the execution probabilities for all limit orders. Greater depth on the buy side of the book increases the execution probabilities for sell limit orders, and depth on the sell side of the book increases the execution probabilities for buy limit orders. Depth on the buy side of the book decreases the execution probabilities for buy limit orders, with a similar effect on the sell side.

Proposition 1 in Parlour (1998) predicts that in equilibrium the execution probabilities for buy limit orders are decreasing in the depth on the own side and increasing in the depth on the other side, and the sensitivity to own side depth is greater than the sensitivity to other side depth. The estimated execution probabilities for one tick buy and sell limit orders are consistent with her prediction. For all stocks, we find that the execution probability is decreasing in own side depth and increasing in the other side depth, but the absolute magnitudes of the sensitivities are only consistent with Proposition 1 in Parlour (1998) for ERR and for buy orders for WEM.

Increasing recent order submission activity as measured by increasing the number of recent traders, increasing mid-quote volatility and reducing lagged duration increases the execution probabilities for one tick and marginal sell limit orders, and decreases the execution probabilities for one tick and marginal buy limit orders. The market-wide variables have small effects on the execution probabilities. Lagged returns also have small effects on the execution probabilities. Execution probabilities are in general lower for the last hour of trading and they tend to decrease over the trading day although the trend is not monotone for all stocks.

Using the competing risks model to compute execution probabilities, we assume that either the order fully executes or fully cancels. We show in Appendix D that the assumption implies that the picking off risk is

$$\xi_{b,t_i}^{buy} = E_{t_i}\left[\int_{t_i}^{t_i+T} (y_{t_i+\tau} - y_{t_i}) dQ_{t_i+\tau} \middle| d_{b,t_i}^{buy} = 1, Q_{t_i+T} = 1\right] \psi_{b,t_i}^{buy}. \tag{34}$$

We assume that the expected change in the common value conditional on an execution is a linear function:

$$E_{t_i}\left[\int_{t_i}^{t_i+T} (y_{t_i+\tau} - y_{t_i}) dQ_{t_i+\tau} \middle| d_{b,t_i}^{buy} = 1, Q_{t_i+T} = 1\right] = z_{t_i}'\beta, \tag{35}$$

where z_{t_i} is information known at the time of the order submission.

We report the results of estimating equation (35) with ordinary least squares in Table 9. We condition on the variables in Table 3, also including the log of order size. Using the parameter estimates, we compute the expected change in the common value, conditional on the limit order executing. At the mean values of the conditioning variables, the expected change is approximately zero for one tick limit orders, minus four cents for marginal buy limit orders and four cents for marginal sell limit orders.

The expected change in the common value conditional on execution is decreasing in the spread for sell orders except for marginal sell orders for BHO and increasing in the spread for all buy limit orders. The only other conditioning variable with much impact on the expected change in the common value is the distance between the mid-quote and the common value. When the common value relative to the mid-quote increases, the expected change in the common value decreases for sell and buy limit orders, consistent with the distance capturing temporary imbalances in the order book.

We form estimates of the picking off risk by substituting our estimates of the expected change in the common value conditional on execution and the execution probabilities in equation (34). At the mean values the picking off risk is close to zero for one tick limit orders, and of the order of one cent for marginal limit orders. A change in the distance to the mid-quote has the largest effect on the picking off risk. When the common value is one standard deviation above its mean value, the picking off risk for a marginal sell limit orders drops roughly by one-half. Similar results hold on the buy side. The impact of the spread is mixed. A one standard deviation increase in the spread leaves the average picking off risk for a marginal sell limit order unchanged and approximately doubles the picking off risk for a marginal buy limit order. The other conditioning variables have smaller impact on the estimated picking off risks.

4.2 The Arrival Rate of the Traders and the Private Value Distribution

The arrival rate of the traders is parameterized as a Weibull distribution,

$$\lambda_t dt \quad = \quad \exp(\gamma_a' x_{t_i}) \alpha_a \, (t - t_i)^{\alpha_a - 1} \, dt, \tag{36}$$

with x_{t_i} denoting the market-wide variables and absolute value of the lagged return at t_i. The private value distribution is parameterized as a mixture of two normal distributions with standard deviations depending on the common value and market-wide variables,

$$G_t(u) = \rho \Phi \left(\frac{u}{y_t \sigma_1 \exp\left(\Gamma' x_t\right)} \right) + (1 - \rho) \Phi \left(\frac{u}{y_t \sigma_2 \exp\left(\Gamma' x_t\right)} \right), \tag{37}$$

where Φ denotes the standard normal cumulative distribution function; $\sigma_1 \neq \sigma_2$, $0 < \rho < 1$, and x_t denotes the market-wide variables and absolute value of the lagged return at t.

The mixture of normals allows both for more mass in the middle of the distribution and fatter tails than the normal distribution. We normalize valuations as percentages of the common value by parameterizing the standard deviation as proportional to the common value.

Table 10 reports the estimates of the discrete choice model, with associated standard errors reported in parentheses. The model is estimated by maximizing the conditional likelihood function. The likelihood function is relatively flat with respect to the order submission cost, for positive order entry costs. We therefore did not estimate the order entry cost but set it equal to 0.1 cents.

The first row of the top panel reports estimates of the Weibull parameter, α_a. The parameter estimate is less than one for all stocks: the longer the time since the last order submission, the lower the conditional probability that a new trader will arrive. The second through seventh rows of the top panel report estimates of the arrival rate parameters, γ_a. The conditioning variables are standardized by dividing by their sample standard deviations. Evaluated at the mean values of the conditioning variables, the expected time between traders arrivals is 148 seconds for BHO, 131 seconds for ERR and 396 for WEM. The mean time between order submissions in our sample are 367 seconds for BHO, 347 seconds for ERR and and 425 seconds for WEM. There are traders who arrive and optimally do not submit orders.

The estimated parameters on many of the market-wide variables are positive; increases in the market-wide variables tend to increase the trader arrival rate. For example, the arrival rate of traders increases following periods of higher market-wide volatility, as measured by the TSE market index. The parameters on the absolute value of the lagged return are positive and larger in magnitude than the other arrival rate parameters. Higher realized stock volatility predicts an increase in the arrival rates. The null hypothesis of constant arrival rates is rejected for all three stocks.

The second panel reports estimates of the valuation distribution parameters. For all stocks, the private value distribution is a mixture of two normal distributions, with approximately 85% weight on a distribution with a small standard deviation and 15% on a distribution with a large standard deviation. The estimates imply the standard deviation of private value distribution is 21% for BHO, 24% for ERR, and 21% for WEM. The estimated distributions differ from a normal. The kurtosis is 17 for BHO, 24 for ERR, and 18 for WEM; the extreme tails of the private values distribution are fatter than a normal distribution. The inter-quartile range is 9% for BHO, 6% for ERR, and 6% for WEM. The inter-quartile range is between one fifth to one third of what it would be for a normal distribution with the same standard deviation; there is more mass in the middle of the distributions than for a normal distribution with the same standard deviation.

The estimated parameters on many of the market-wide variables are statistically significant, although small. The parameters on the lagged return on all three stocks are negative — when realized stock return volatility is high, there tend to be more value traders. The hypothesis of constant variance for the valuation distribution is rejected for all three stocks.

For all three stocks, the continuation value is approximately 2% of the common value. The continuation value is approximately the same as one-half the mean spread.

4.3 Interpreting the Findings

Figures 2 through 7 provide plots of the effects of changing the conditioning information on the fitted probability of different order submissions conditional on an order being submitted and the expected time to an order submission. The probability of observing a sell market order is computed by substituting our estimates of the execution probabilities, the picking off risks, the order submission costs, the continuation values, the common value and the coefficients of the valuation distribution into equation (38) below:

$$\begin{aligned}&\Pr_t\left(\text{Sell market order in } [t, t+dt) \middle| \text{ Order submission in } [t, t+dt)\right)\\&\qquad = \frac{G_t\left(\theta_t^{sell}(0,1) - y_t\right)}{1 - G_t\left(\theta_t^{buy}(\text{Marginal}) - y_t\right) + G_t\left(\theta_t^{sell}(\text{Marginal}) - y_t\right)}, \end{aligned} \tag{38}$$

with the other probabilities computed similarly. The fitted expected time to an order submission is computed using the implied hazard rate for order submissions from the discrete choice model, evaluated at the parameter estimates.

The top left graphs in Figures 2 through 4 plot the probability of a sell market order submission and a sell limit order submission conditional on an order submission, as a function of the spread, keeping all other variables at their sample means. The top middle graphs in Figures 2 through 4 plot the conditional probabilities of buy market and buy limit order submissions conditional on an order submission, as a function of the spread, holding all other variables at their sample means. For all three stocks, a larger spread increases the probability that limit orders are submitted.

The top right graphs of Figures 2 through 4 plot the fitted expected time to the next order submission as a function of the spread, holding all other variables at their sample means. The expected time to the next order decreases for all stocks. According to our estimates, a wider spread implies a higher price of immediacy. A higher price of immediacy encourages value traders to supply liquidity rather than stay out of the market; the fitted expected time to the next order submission therefore decreases. But the estimates of the Weibull model for order submissions in Table 4 show that a wider spread predicts a longer expected time to the next order submission. The arrival rate of traders and the relative proportion of value traders to liquidity traders must decrease when the spread widens. There is less competition in supplying liquidity in wider spread markets.

The second row of Figures 2 through 4 plot the fitted probabilities of limit and market order submissions and the fitted expected time to the next order submission as a function of the bid depth, holding all other variables at their sample means. Higher bid depth leads to a higher execution probability for sell limit orders and a lower execution probability for buy limit orders; the price of immediacy is lower for sellers and higher for buyers. Sellers therefore are more likely to submit market orders and buyers are more likely to submit limit orders although the change is small compared with the change for the spread.

The third row of Figures 2 through 4 plot the fitted probabilities of limit and market order submissions and the fitted expected time to the next order submission as a function of the distance between the common value and the mid-quote, holding all other variables at their sample means. The distance between the common value and the mid-quote reflects temporary imbalances in the order book. When the common value is higher than the mid-quote the price of immediacy is high for sellers and low for buyers, leading sellers to submit limit orders and buyers to submit market orders. Similar effects hold on the buy side. For both the bid depth and the distance to the mid-quote, the effects on the expected time to the next order submission are mixed.

Figures 5 through 7 plot the order submission probabilities and the time to the next order changing the absolute value of the lagged stock return. The top left and middle plots of the figures show that as the absolute value of the lagged return increases, more traders are likely to submit limit orders, conditional on an order submission. The sole exception is the buy order for BHO.

In the second row, we plot the fitted probabilities and times holding the arrival rate and the valuation distribution at their mean values, while varying the estimated price of immediacy with the absolute value of the lagged return. In the third row, we plot the fitted probabilities and expected times holding the price of immediacy at its sample mean, while varying the valuation distributions and arrival rates to change with the absolute value of the lagged return. Comparing the top, middle and bottom plots for the conditional probability, both changes in the price of immediacy and changes in the valuation distribution causes the order submission probabilities to change. Changes in the expected time to the next order are mainly caused by changes in the arrival rate of traders.

We performed similar computations for the effects of changing the market-wide variables. Generally, we found small changes in the fitted probabilities and expected times from changing the market-wide variables despite the statistically significant impact of market-wide variables on the arrival rates and the valuation distributions reported in Table 10.

Table 11 reports the expected utilities for traders with six different private values across three different market conditions: a low liquidity state with a wide spread and low depth, a high liquidity state with a narrow spread and high depth, and an order imbalance state where the common value is above the mid-quote. For each private value, we report the expected utility from submitting a market order, one tick limit order, marginal limit order, or no order. The private values are 1.25%, 2.5%, or 5% higher or lower than the common value; the corresponding private values in cents are reported in the second row. The reported expected utility is a lower bound on the trader's expected utility since we do not compute the expected utility for limit orders between one tick from the quotes and marginal limit orders. The maximum utility is indicated for each private value and market condition with a box.

In the low liquidity state, the price of immediacy is high. A trader with a private value 2.5% from the common value optimally submits a marginal limit order and a trader with private value 5% from the common value optimally submits a one tick limit order. In the high liquidity state, the price of immediacy is lower than in the low liquidity state. A trader with a private value 1.25% from the common value optimally submits no order in BHO and ERR and optimally submits a limit order in WEM. A trader with a private value 2.5% from the common value optimally submits limit orders for BHO and ERR and submits market orders for WEM. A trader with a private value 5% from the common value optimally demands liquidity by submitting a market order for all stocks.

In the order imbalance state, the optimal order strategies are asymmetric. A high common value relative to the mid-quote implies that the price for immediacy is low for buyers and high for sellers. For traders with positive private values, the optimal strategy is therefore to submit market orders in ERR and WEM for all three valuations above the common value, picking off some sell limit orders. Traders who submit buy orders earn a higher surplus than they did in the other market conditions. For BHO, traders with private values 1.25% and 2.5% above the common value submit one tick limit orders rather than market orders.

Overall, the expected utility calculations reported in Table 11 show that traders can earn a higher expected utility by following the optimal order submission strategies rather than a naive strategy of always making the same order submission.

Table 12 reports summary statistics for the estimated optimal order submission strategies for five intervals for the private value. The first two rows in each panel report the mean and the standard deviation of the proportion of traders in each private value interval. The next five rows report the mean order submission probabilities for a sell market, a sell limit, a buy limit, a buy market, and no order. The bottom five rows report the standard deviations of the order submission probabilities.

We interpret traders with valuations within 2.5% of the common value as value traders. Such traders represent 32% of the traders in BHO, 48% of the traders in ERR and 52% of the traders in WEM. The probability of the value traders submitting no order is 36% for BHO, 61% for ERR, and 12% for WEM. When value traders submit an order, it is typically a limit order. The probability of a market order submission for the value traders is between 0.7% and 3.6%. The value trader supply liquidity when the price of immediacy is high and do not submit orders when the price for immediacy is low.

Traders with valuations between 2.5% and 5% from the common value compete with the value traders in supplying liquidity. Such traders represent 26% of the traders in BHO, 28% of the traders in ERR and 26% of the traders in in WEM. They submit either market or limit orders most of the time; the probability that they submit no order is between 0.4% and 8.9%. Their order submissions are fairly evenly split between market and limit orders.

Liquidity traders with valuations more than 5% from the common value have the highest willingness to pay for immediacy. Such traders represent 42% of the traders in BHO, 23% of the traders in ERR, and 22% of the traders in WEM. Submitting no order is almost never an optimal strategy for liquidity traders: the probability of submitting no order is between 0.1% and 1.9%.

The probability of submitting a market order is roughly 80% for BHO, 81% for ERR, and 88% for WEM. The standard deviations of the optimal order submission probabilities for all types of traders show that their optimal order submissions vary with market conditions.

Using the estimated Weibull parameters from Table 10 and the proportion of the traders that are value traders, we compute the expected time between the arrivals of value traders for each stock. The average time across all three stocks is approximately 23 minutes while the average time between order submissions is approximately 6 minutes. Value traders compete in supplying liquidity relatively slowly.

5 Conclusions

We model a trader's decision to supply liquidity by submitting limit orders or demand liquidity by submitting market orders in a limit order market. The best quotes, and the execution probabilities and picking off risks of limit orders determine the price of immediacy. The price of immediacy and the traders valuation for the stock determine the optimal order submission. We estimate the execution probabilities and the picking off risks for a sample from the Vancouver Stock Exchange. We use a competing risks model to estimate the hazard rates for execution and cancellation. Both executions and cancellations are predictable. The predictable cancellations times suggest that one natural extension of our model would be to model the trader's decisions to cancel in more detail.

We use the estimates to compute the price of immediacy. The price of immediacy changes with market conditions; a value trader and a liquidity trader can increase their expected utility by changing their order submissions with market conditions. Increasing the spread increases the price of immediacy causing liquidity traders to switch from submitting market orders to submitting limit orders, and value traders to switch from submitting no orders to submitting limit orders.

We combine our estimates of the price of immediacy with the actual order submissions to estimate the unobserved arrival rate of the traders and the distribution of the traders' valuations. The arrival rate of traders and the relative proportion of value traders to liquidity traders increase following periods of high stock volatility. The arrival rate of traders and the relative proportion of value traders to liquidity traders decrease when the spread is wide. We leave the question of how value traders' decide to allocate resources to supplying liquidity in different stocks for future work.

Euronext, the recent amalgamation of the exchanges in Amsterdam, Brussels and Paris is a limit order market where liquidity providers supplement the liquidity in the limit order books for the less actively traded stocks. Liquidity providers are obligated to provide a specified level of liquidity in a stock, and in exchange their transaction fees are waived. Periodically Euronext evaluates whether or not the liquidity providers add enough liquidity to justify the loss in revenue from the waived transaction fees. Estimates of the arrival rates of the traders and the distribution of the traders' valuations provide useful information for such evaluations. We leave such computations for future work.

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A Proofs

Proof of Lemma 1

If it is optimal for a trader with valuation v to submit a buy order, then it is also optimal for traders with private values $v' > v$ to submit buy orders. Let s be an arbitrary sell order, and suppose that $d_{b,t}^{buy}(v) = 1$. Then,

$$\begin{aligned}
\psi_{b,t}^{buy}\left(v' - p_{b,t}\right) + \xi_{b,t}^{buy} + V(1 - \psi_{b,t}^{buy}) - c &> \psi_{b,t}^{buy}\left(v - p_{b,t}\right) + \xi_{b,t}^{buy} + V(1 - \psi_{b,t}^{buy}) - c \\
&\geq \psi_{s,t}^{sell}\left(p_{s,t} - v\right) - \xi_{s,t}^{sell} + V(1 - \psi_{s,t}^{sell}) - c \\
&\geq \psi_{s,t}^{sell}\left(p_{s,t} - v'\right) - \xi_{s,t}^{sell} + V(1 - \psi_{s,t}^{sell}) - c. \quad \text{(A1)}
\end{aligned}$$

The first line follows because $v' > v$; the second line follows because it is optimal for a trader with value v to submit a buy order at b; the third line follows because $v' > v$.

Let $p_{b',t}$ be the optimal buy submission for the trader with valuation v'. By optimality,

$$\psi_{b,t}^{buy}(v - p_{b,t}) + \xi_{b,t}^{buy} + V(1 - \psi_{b,t}^{buy}) - c, \geq \psi_{b',t}^{buy}(v - p_{b',t}) + \xi_{b',t}^{buy} + V(1 - \psi_{b',t}^{buy}) - c \quad \text{(A2)}$$

$$\psi_{b',t}^{buy}(v' - p_{b',t}) + \xi_{b',t}^{buy} + V(1 - \psi_{b',t}^{buy}) - c \geq \psi_{b,t}^{buy}(v' - p_{b,t}) + \xi_{b,t}^{buy} + V(1 - \psi_{b,t}^{buy}) - c. \quad \text{(A3)}$$

Subtracting equation (A3) from equation (A2) and rearranging

$$(v - v')(\psi_{b,t}^{buy} - \psi_{b',t}^{buy}) \geq 0. \quad \text{(A4)}$$

The result follows from equations (A4) and (16). Symmetric arguments hold on the sell side. ■

Proof of Proposition 1

The threshold characterization follows from the monotonicity in Lemma 1. ■

B The conditional log-likelihood function

Let t_i denote the time of the i^{th} order submission and I the total number of order submissions. Conditioning on the common value, order size, x_{t_i} and z_{t_i} the conditional log-likelihood function is

$$\begin{aligned}
\sum_{i=1}^{I} \Bigg\{ & d_{0,t_i}^{s} \ln\left(G_{t_i}\left(\theta_{t_i}^{sell}(0,1) - y_{t_i}\right)\lambda_{t_i}\right) \\
& + \left(\sum_{s=1}^{S_t} d_{s,t_i}^{sell}\right) \ln\left(\left[G_{t_i}\left(\theta_{t_i}^{sell}(\text{Marginal}) - y_{t_i}\right) - G_{t_i}\left(\theta_{t_i}^{s}(0,1) - y_{t_i}\right)\right]\lambda_{t_i}\right) \\
& + d_{0,t_i}^{buy} \ln\left(1 - G_{t_i}\left(\theta_{t_i}^{buy}(0,1) - y_{t_i}\right)\lambda_{t_i}\right) \\
& + \left(\sum_{b=1}^{B_t} d_{b,t_i}^{buy}\right) \ln\left(\left[G_{t_i}\left(\theta_{t_i}^{buy}(0,1), -y_{t_i}\right) - G_{t_i}\left(\theta_{t_i}^{buy}(\text{Marginal}) - y_{t_i}\right)\right]\lambda_{t_i}\right) \\
& - \int_{t_{i-1}}^{t_i} \left[G_t(\theta_t^{buy}(\text{Marginal}) - y_t) - G_t(\theta_t^{sell}(\text{Marginal}) - y_t)\right]\lambda_t dt \Bigg\}. \quad \text{(B1)}
\end{aligned}$$

The first line is contribution from the instantaneous probability of a sell market order at time t_i; the second line is the contribution from the instantaneous probability of a sell limit order; the third line is the contribution from the instantaneous probability of a buy market order; fourth line is the contribution from the instantaneous probability of a buy limit order; and the final line is the integrated hazard rate.

In our estimation, we assume that the common value y_t only changes when an order is submitted at t_i.

C Execution probabilities in the Weibull competing risks model

Suppose a limit order is submitted at time t_i. Let t_e be the hypothetical execution time for the order and t_c the hypothetical cancellation time for an order. Assume that the times are independent Weibull random variables, with cdf's F_e and F_c:

$$\begin{aligned} F_e(\tau) &= 1 - \exp\left(-\exp(\gamma_e' x_{t_i})(\tau - t_i)^{\alpha_e}\right), && \text{(C1)} \\ F_c(\tau) &= 1 - \exp\left(-\exp(\gamma_c' x_{t_i})(\tau - t_i)^{\alpha_c}\right), && \text{(C2)} \end{aligned}$$

The execution probability is defined as the probability that the order executes between t_i and t_i+T,

$$\begin{aligned} &\Pr\left(\tilde{\tau}_e \leq \tilde{\tau}_c, \tilde{\tau}_e \leq t_i + T\right) = \int_{t_i}^{t_i+T} \left(1 - F_c(t - t_i)\right) dF_e(t - t_i) \\ &= \int_{t_i}^{t_i+T} \exp\left(-\exp(\gamma_c' x_{t_i})(t - t_i)^{\alpha_c}\right) \exp(\gamma_e' x_{t_i}) \alpha_e (t - t_i)^{\alpha_e - 1} \exp\left(-\exp(\gamma_e' x_{t_i})(t - t_i)^{\alpha_e - 1}\right) dt. \end{aligned} \tag{C3}$$

We compute equation (C3) numerically with T equal to two trading days (48,600 seconds).

D The picking off risk

Assuming that the order either fully executes or fully cancels, so that $Q_{t+T} \in \{0, 1\}$, the picking off risk is

$$\begin{aligned} \xi_{b,t}^{buy} &= E_t\left[\int_{\tau=0}^{T} (y_{t+\tau} - y_t) dQ_{t+\tau} \middle| d_{b,t}^{buy} = 1\right] \\ &= E_t\left[E_t\left[\int_{\tau=0}^{T} (y_{t+\tau} - y_t) dQ_{t+\tau} \middle| d_{b,t}^{buy} = 1, Q_{t+T}\right] \middle| d_{b,t}^{buy} = 1\right] \\ &= E_t\left[\int_{\tau=0}^{T} (y_{t+\tau} - y_t) dQ_{t+\tau} \middle| d_{b,t}^{buy} = 1, Q_{t+T} = 1\right] \Pr_t\left(Q_{t+T} = 1 \middle| d_{b,t}^{buy} = 1\right) \\ &= E_t\left[\int_{\tau=0}^{T} (y_{t+\tau} - y_t) dQ_{t+\tau} \middle| d_{b,t}^{buy} = 1, Q_{t+T} = 1\right] \psi_{b,t}^{buy}. \end{aligned} \tag{D1}$$

The second line follows from the law of iterated expectations, the third line follows because $dQ_{t+\tau} > 0$ for some $t+\tau$ if and only if $Q_{t+T} = 1$ and the final line follows from the definition of the execution probability.

Table 1: **Summary Statistics**

	Barkhor Resources	Eurus Resources	War Eagle Mining Co.
Ticker	BHO	ERR	WEM
Number of order submissions	55,444	56,599	47,578
Percent of submissions that are:			
Sell limit orders	31.7	31.5	31.3
Sell market orders	21.6	23.7	19.9
Buy limit orders	28.5	27.9	32.0
Buy market orders	18.2	16.9	16.8
Mean percentage spread	3.9	2.8	3.4
Standard deviation of percentage spread	4.5	4.3	4.9
Mean depth	21.0	10.0	10.0
Standard deviation of depth	18.8	8.7	9.0

The sample period is May 1, 1990, to November 30, 1993. The depth is equal to the average of the bid and ask depth in the limit order book within 2.5% of the mid-quote, measured in units of 1,000's of shares.

Table 2: **Order Size**

	Sell orders		Buy orders		Buy and sell orders	
	Limit	Market	Limit	Market	Limit	Market
			BHO			
Mean	5.689	5.951	6.613	6.268	6.127	6.096
	(0.036)	(0.053)	(0.058)	(0.068)	(0.034)	(0.042)
T-test for equal means	-4.118		3.841		0.560	
P-value	(0.000)		(0.000)		(0.575)	
			ERR			
Mean	2.627	2.830	3.413	3.414	2.996	3.073
	(0.023)	(0.026)	(0.032)	(0.047)	(0.019)	(0.025)
T-test for equal means	-5.875		-0.029		-2.427	
P-value	(0.000)		(0.977)		(0.015)	
			WEM			
Mean	2.848	3.132	3.711	3.197	3.284	3.162
	(0.020)	(0.035)	(0.029)	(0.038)	(0.018)	(0.026)
T-test for equal means	-7.034		10.849		3.918	
P-value	(0.000)		(0.000)		(0.000)	

The mean order size in 1,000's of shares is reported for sell limit and market orders, buy limit and market orders, and for buy and sell limit and market orders. Standard errors are reported in parentheses. A test statistic with p-values in parentheses for a t-test of equal means is reported for each pair of means.

Table 3: **Conditioning Variables**

Name	Description
	Book variables
Spread	Bid-ask spread
Close ask depth	Log of ask depth at best ask quote
Far ask depth	Log of ask depth within 9 cents of best ask quote
Close bid depth	Log of bid depth at best bid quote
Far bid depth	Log of bid depth within 9 cents of best bid quote
	Activity variables
Recent trades	Number of traders in the last ten minutes
Lagged duration	Sum of last ten durations of order book changes
Mid-quote volatility	Volatility of the mid-quote over the last ten minutes
	Market-wide variables
TSE market index	Absolute value of the lagged close to close return on the TSE market index
TSE mining index	Absolute value of the lagged close to close return on the TSE mining index
Overnight interest rate	Absolute value of the lagged change in the overnight Canadian interest rate
Canadian dollar	Absolute value of the lagged change in the Canadian/US exchange rate
	Value proxies
Lagged return	Absolute value of the lagged open to open mid-quote return
Distance to mid-quote	Percentage deviation between mid-quote and moving-average mid-quote
	Time dummies
First hour	dummy variable for 6:30-7:30
Second hour	dummy variable for 7:30-8:30
Third hour	dummy variable for 8:30-9:30
Fourth hour	dummy variable for 9:30-10:30
Fifth hour	dummy variable for 10:30-11:30
Sixth hour	dummy variable for 11:30-12:30

Table 4: **Weibull Hazard Rate Model for Order Submissions**

	BHO	ERR	WEM
Power: α	0.586	0.567	0.561
	(0.003)	(0.002)	(0.002)
Constant	-3.486	-1.667	-2.031
	(0.157)	(0.130)	(0.216)
Spread	-2.287	-0.359	-1.196
	(0.418)	(0.092)	(0.222)
Close ask depth	-0.029	0.015	0.001
	(0.005)	(0.006)	(0.006)
Far ask depth	0.119	0.018	-0.002
	(0.009)	(0.007)	(0.008)
Close bid depth	-0.011	0.034	0.011
	(0.005)	(0.005)	(0.006)
Far bid depth	-0.034	0.023	-0.026
	(0.008)	(0.006)	(0.007)
χ^2_5: All parameters = 0	266.690	155.180	37.830
P-value	(0.000)	(0.000)	(0.000)
Recent trades	0.040	0.052	0.068
	(0.001)	(0.001)	(0.001)
Duration ($\times$1000)	-0.124	-0.109	-0.102
	(0.002)	(0.002)	(0.001)
Mid-quote volatility	0.526	-0.550	-0.152
	(0.086)	(0.074)	(0.127)
χ^2_3: All parameters = 0	8856.980	10326.160	12647.250
P-value	(0.000)	(0.000)	(0.000)
TSE market index	-0.002	0.032	0.003
	(0.005)	(0.004)	(0.005)
TSE mining index	-0.011	-0.005	-0.004
	(0.005)	(0.004)	(0.004)
Overnight interest rate	0.001	0.031	0.002
	(0.005)	(0.005)	(0.005)
Canadian dollar	0.000	-0.026	-0.033
	(0.005)	(0.004)	(0.005)
χ^2_4: All parameters = 0	6.230	135.340	52.080
P-value	(0.183)	(0.000)	(0.000)
Lagged return	0.088	0.025	0.043
	(0.005)	(0.005)	(0.005)
Distance to mid-quote	6.141	0.834	2.816
	(0.428)	(0.099)	(0.367)
χ^2_2: All parameters = 0	547.000	98.880	145.310
P-value	(0.000)	(0.000)	(0.000)
First hour	-0.249	-0.084	-0.295
	(0.017)	(0.016)	(0.017)
Second hour	-0.371	-0.154	-0.331
	(0.017)	(0.016)	(0.016)
Third hour	-0.202	-0.178	-0.213
	(0.016)	(0.015)	(0.016)
Fourth hour	-0.169	-0.080	-0.150
	(0.016)	(0.016)	(0.016)
Fifth hour	-0.137	-0.070	-0.141
	(0.016)	(0.016)	(0.016)
Sixth hour	-0.113	-0.041	-0.090
	(0.020)	(0.016)	(0.016)
χ^2_6: All parameters = 0	521.680	189.480	561.290
P-value	(0.000)	(0.000)	(0.000)
χ^2_{21}: Constant hazard rate	17399.970	12955.990	14301.930
P-value	(0.000)	(0.000)	(0.000)
N	56316	57441	48446

The table reports parameters estimates with asymptotic standard errors in parentheses for a Weibull model for the hazard for order submissions, and χ^2-tests of the null hypothesis of all parameters jointly equal to zero for each subset of conditioning variables and for all conditioning variables jointly with p-values in parentheses. The time until the first order each day is measured from the opening. The time elapsed from the last order each day until the close is included and treated as a censored observation.

Table 5: **Ordered Probit Models for Order Submissions**

	BHO		ERR		WEM	
	Buy	Sell	Buy	Sell	Buy	Sell
Spread	40.600	34.276	15.698	8.153	19.569	16.334
	(2.126)	(1.359)	(1.356)	(0.713)	(2.546)	(1.512)
Close ask depth	0.061	-0.142	-0.031	-0.106	0.018	-0.166
	(0.009)	(0.009)	(0.016)	(0.011)	(0.014)	(0.013)
Far ask depth	–	0.183	–	0.020	–	0.127
	–	(0.013)	–	(0.015)	–	(0.019)
Close bid depth	-0.145	0.049	-0.206	0.006	-0.140	-0.003
	(0.010)	(0.008)	(0.012)	(0.009)	(0.014)	(0.011)
Far bid depth	0.032	–	0.078	–	0.059	–
	(0.018)	–	(0.017)	–	(0.024)	–
Order size	0.168	0.064	0.145	0.071	0.213	0.018
	(0.010)	(0.010)	(0.011)	(0.008)	(0.012)	(0.011)
χ^2_5: All parameters = 0	888.860	1039.590	559.910	308.700	482.030	239.540
P-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Recent trades	-0.007	-0.005	-0.015	-0.019	-0.020	-0.018
	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)
Duration (×1000)	0.003	-0.010	0.014	-0.011	-0.005	-0.018
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Mid-quote volatility	-1.405	1.686	-0.447	2.160	-1.155	2.262
	(0.139)	(0.136)	(0.379)	(0.164)	(0.463)	(0.394)
χ^2_3: All parameters = 0	224.660	213.250	214.600	212.110	151.390	172.830
P-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
TSE market index	-0.001	-0.315	-0.473	-0.228	-0.041	-0.093
	(0.083)	(0.078)	(0.081)	(0.070)	(0.087)	(0.083)
TSE mining index	0.077	-0.032	0.136	0.036	-0.051	0.024
	(0.078)	(0.071)	(0.085)	(0.071)	(0.089)	(0.081)
Overnight interest rate	0.004	0.061	0.089	0.189	-0.153	0.050
	(0.079)	(0.073)	(0.083)	(0.070)	(0.088)	(0.083)
Canadian dollar	0.008	0.098	0.121	-0.002	0.021	-0.164
	(0.078)	(0.072)	(0.085)	(0.072)	(0.087)	(0.083)
χ^2_4: All parameters = 0	1.050	17.600	37.210	17.950	3.950	6.190
P-value	(0.902)	(0.002)	(0.000)	(0.001)	(0.413)	(0.186)
Lagged return (×10)	0.093	0.106	-0.057	0.126	-0.533	-0.156
	(0.081)	(0.076)	(0.080)	(0.070)	(0.088)	(0.081)
Distance to mid-quote	-16.597	14.291	-19.215	31.432	-16.888	21.768
	(0.858)	(1.317)	(2.822)	(1.225)	(3.278)	(1.243)
χ^2_2: All parameters = 0	377.170	118.660	48.240	661.670	54.250	309.110
P-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
First hour	0.212	0.223	0.229	0.268	0.320	0.197
	(0.030)	(0.028)	(0.031)	(0.028)	(0.033)	(0.032)
Second hour	0.219	0.187	0.201	0.199	0.329	0.203
	(0.029)	(0.027)	(0.030)	(0.027)	(0.032)	(0.029)
Third hour	0.153	0.122	0.210	0.199	0.199	0.119
	(0.029)	(0.027)	(0.031)	(0.027)	(0.034)	(0.030)
Fourth hour	0.081	0.141	0.121	0.187	0.203	0.056
	(0.030)	(0.028)	(0.031)	(0.027)	(0.031)	(0.029)
Fifth hour	0.102	0.172	0.081	0.158	0.165	0.042
	(0.029)	(0.028)	(0.032)	(0.028)	(0.032)	(0.030)
Sixth hour	0.083	0.094	0.045	0.131	0.143	0.108
	(0.030)	(0.028)	(0.032)	(0.028)	(0.030)	(0.029)
χ^2_6: All parameters = 0	82.940	81.410	93.370	106.130	141.510	72.440
P-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
χ^2_{20}: All parameters = 0	1504.130	1498.950	1319.090	1352.180	918.470	861.090
P-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Number of observations	25,904	29,540	25,345	31,254	23,199	24,379

The table reports parameter estimates for an ordered probit model of the choice between a market order, a one tick away limit order, and limit order more than one tick away from the best quotes. Standard errors are in parentheses. For each stock, a model is estimated for buy orders and for sell orders. The dependent variable is equal to zero for a market order, one for a one tick limit order and two for all other limit orders. A χ^2-test for the null hypothesis of all parameters jointly being equal to zero is reported for each subset of regressors and for the overall model.

Table 6: **Competing Risks Models for Execution and Cancellation Hazard Rates for BHO**

	Marginal sell		1 Tick sell		Marginal buy		1 Tick buy	
Variable	Execution	Cancellation	Execution	Cancellation	Execution	Cancellation	Execution	Cancellation
Power: α	0.655	0.625	0.580	0.512	0.700	0.615	0.613	0.524
	(0.031)	(0.015)	(0.009)	(0.017)	(0.044)	(0.016)	(0.009)	(0.010)
Constant	-15.689	-5.075	-10.420	-1.721	-4.310	-6.029	2.043	-6.928
	(1.513)	(0.890)	(0.420)	(0.854)	(1.851)	(0.630)	(0.473)	(0.733)
Spread	-1.530	-0.538	56.836	-9.189	16.289	3.457	68.731	2.203
	(2.690)	(1.477)	(2.360)	(4.214)	(2.783)	(1.685)	(2.670)	(4.327)
Close ask depth	–	–	-0.057	-0.008	0.000	0.007	0.201	0.053
	–	–	(0.023)	(0.025)	(0.098)	(0.037)	(0.023)	(0.025)
Far ask depth	-0.276	-0.154	–	–	–	–	–	–
	(0.081)	(0.040)	–	–	–	–	–	–
Close bid depth	-0.225	-0.007	0.138	0.002	–	–	-0.082	-0.013
	(0.066)	(0.030)	(0.021)	(0.022)	–	–	(0.022)	(0.025)
Far bid depth	–	–	–	–	0.049	-0.075	–	–
	–	–	–	–	(0.101)	(0.043)	–	–
Order size	-0.186	0.039	-0.221	-0.049	-0.498	0.025	-0.289	-0.115
	(0.075)	(0.039)	(0.027)	(0.030)	(0.101)	(0.041)	(0.027)	(0.031)
Recent trades	0.026	0.013	0.036	0.032	0.031	0.017	0.035	0.030
	(0.005)	(0.003)	(0.002)	(0.003)	(0.006)	(0.003)	(0.002)	(0.002)
Duration ($\times$1000)	-0.144	-0.108	-0.084	-0.022	-0.069	-0.081	-0.064	-0.048
	(0.026)	(0.010)	(0.006)	(0.004)	(0.026)	(0.009)	(0.006)	(0.006)
Mid-quote volatility	5.611	0.095	3.325	-1.090	-2.170	0.705	-4.309	1.985
	(0.796)	(0.487)	(0.235)	(0.496)	(1.084)	(0.364)	(0.273)	(0.430)
TSE market index	0.065	0.013	-0.001	0.022	0.053	-0.005	0.020	-0.003
	(0.054)	(0.028)	(0.023)	(0.025)	(0.077)	(0.031)	(0.023)	(0.027)
TSE mining index	-0.030	0.056	-0.058	0.032	0.010	0.009	-0.039	0.004
	(0.065)	(0.028)	(0.021)	(0.021)	(0.084)	(0.032)	(0.022)	(0.025)
Overnight interest rate	-0.027	0.003	-0.021	0.010	0.062	0.023	0.020	-0.031
	(0.072)	(0.031)	(0.020)	(0.021)	(0.087)	(0.034)	(0.021)	(0.023)
Canadian dollar	0.095	-0.020	0.015	0.022	-0.375	-0.058	-0.042	-0.048
	(0.059)	(0.030)	(0.022)	(0.022)	(0.101)	(0.032)	(0.023)	(0.025)
Lagged return	0.146	0.004	0.103	0.071	0.090	0.067	0.055	0.086
	(0.058)	(0.030)	(0.020)	(0.022)	(0.084)	(0.033)	(0.021)	(0.025)
Distance to mid-quote	26.973	0.224	38.506	21.246	-21.657	-2.826	-45.174	-11.919
	(2.755)	(2.082)	(1.825)	(2.186)	(3.453)	(1.730)	(1.977)	(2.120)
First hour	-0.679	-0.672	-0.569	-1.124	0.081	-1.041	-0.212	-1.006
	(2.755)	(2.082)	(1.825)	(2.186)	(3.453)	(1.730)	(1.977)	(2.120)
Second hour	-0.201	-0.664	-0.634	-0.963	-0.228	-0.870	-0.105	-1.111
	(0.265)	(0.115)	(0.087)	(0.084)	(0.393)	(0.129)	(0.089)	(0.093)
Third hour	-0.020	-0.368	-0.438	-0.902	-0.513	-0.764	-0.262	-1.002
	(0.273)	(0.118)	(0.085)	(0.082)	(0.425)	(0.136)	(0.089)	(0.086)
Fourth hour	0.057	-0.347	-0.405	-0.855	-0.278	-0.767	-0.189	-0.771
	(0.277)	(0.122)	(0.087)	(0.085)	(0.436)	(0.139)	(0.091)	(0.084)
Fifth hour	0.279	-0.049	-0.341	-0.644	-0.458	-0.429	-0.174	-0.891
	(0.277)	(0.121)	(0.088)	(0.082)	(0.478)	(0.143)	(0.090)	(0.084)
Sixth hour	0.396	0.133	-0.208	-0.463	0.003	-0.249	0.012	-0.653
	(0.290)	(0.123)	(0.089)	(0.083)	(0.485)	(0.153)	(0.093)	(0.088)
χ^2_{19}: Constant hazard rate	302.610	290.010	1731.080	576.140	188.960	247.370	1710.490	648.100
P-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Number of observations	1,748		4,498		1,353		4,105	

The table reports parameters estimates with asymptotic standard errors in parentheses for a competing risks model of hazard rates for executions and cancellations for marginal limit orders and one tick away limit orders. A χ^2-test for the null hypothesis of all parameters jointly being equal to zero is reported for each model with the p-value in parenthesis.

Table 7: **Competing Risks Models for Execution and Cancellation Hazard Rates for ERR**

	Marginal sell		1 Tick sell		Marginal buy		1 Tick buy	
Variable	Execution	Cancellation	Execution	Cancellation	Execution	Cancellation	Execution	Cancellation
Power: α	0.652	0.615	0.544	0.499	0.817	0.585	0.638	0.538
	(0.022)	(0.012)	(0.011)	(0.011)	(0.029)	(0.012)	(0.015)	(0.015)
Constant	-12.974	-6.642	-10.345	-5.602	-3.862	-4.232	3.927	-2.764
	(0.886)	(0.979)	(0.584)	(1.210)	(1.229)	(0.657)	(1.855)	(0.807)
Spread	3.463	2.205	22.510	-5.585	3.650	2.710	25.817	0.209
	(1.287)	(0.706)	(1.565)	(2.220)	(1.195)	(0.630)	(1.810)	(2.702)
Close ask depth	–	–	-0.284	0.000	0.105	-0.050	0.071	-0.050
	–	–	(0.036)	(0.036)	(0.061)	(0.034)	(0.038)	(0.044)
Far ask depth	-0.210	-0.060	–	–	–	–	–	–
	(0.055)	(0.032)	–	–	–	–	–	–
Close bid depth	0.091	-0.014	0.195	0.019	–	–	-0.079	-0.023
	(0.054)	(0.030)	(0.033)	(0.033)	–	–	(0.037)	(0.040)
Far bid depth	–	–	–	–	-0.261	-0.074	–	–
	–	–	–	–	(0.056)	(0.031)	–	–
Order size	-0.115	0.144	-0.017	-0.025	-0.248	0.005	-0.188	-0.018
	(0.048)	(0.029)	(0.031)	(0.032)	(0.057)	(0.033)	(0.036)	(0.041)
Recent trades	0.011	0.016	0.018	0.014	0.055	0.030	0.014	0.038
	(0.006)	(0.004)	(0.005)	(0.006)	(0.007)	(0.005)	(0.007)	(0.006)
Duration ($\times$1000)	-0.150	-0.056	-0.088	-0.007	-0.099	-0.058	-0.079	-0.043
	(0.022)	(0.008)	(0.015)	(0.008)	(0.018)	(0.007)	(0.014)	(0.011)
Mid-quote volatility	4.047	1.041	3.682	1.530	-2.525	-0.078	-4.958	-0.242
	(0.466)	(0.558)	(0.322)	(0.705)	(0.700)	(0.381)	(1.084)	(0.464)
TSE market index	0.175	0.050	0.025	0.004	0.001	0.044	-0.027	0.046
	(0.040)	(0.024)	(0.030)	(0.030)	(0.047)	(0.027)	(0.032)	(0.036)
TSE mining index	-0.156	-0.003	0.007	0.005	0.010	-0.049	-0.017	-0.124
	(0.051)	(0.026)	(0.033)	(0.033)	(0.048)	(0.026)	(0.031)	(0.039)
Overnight interest rate	-0.089	0.049	-0.080	0.047	0.081	-0.002	-0.024	0.046
	(0.046)	(0.023)	(0.036)	(0.032)	(0.042)	(0.026)	(0.045)	(0.047)
Canadian dollar	-0.102	-0.031	-0.066	-0.044	-0.072	-0.019	-0.006	-0.075
	(0.049)	(0.025)	(0.035)	(0.034)	(0.049)	(0.026)	(0.036)	(0.044)
Lagged return	0.010	-0.003	0.002	-0.060	0.074	-0.030	0.026	0.007
	(0.040)	(0.022)	(0.035)	(0.041)	(0.034)	(0.029)	(0.029)	(0.030)
Distance to mid-quote	36.361	9.413	49.488	25.914	-45.869	-9.096	-101.778	-31.003
	(4.648)	(2.932)	(4.639)	(5.050)	(4.455)	(2.469)	(6.043)	(6.374)
First hour	-0.548	-1.070	-0.347	-0.822	-0.287	-1.092	0.061	-0.719
	(4.648)	(2.932)	(4.639)	(5.050)	(4.455)	(2.469)	(6.043)	(6.374)
Second hour	-0.553	-0.966	-0.475	-1.030	-0.196	-1.020	-0.168	-0.805
	(0.198)	(0.095)	(0.124)	(0.113)	(0.242)	(0.102)	(0.131)	(0.127)
Third hour	-0.415	-0.914	-0.337	-0.973	-0.294	-0.943	-0.130	-1.088
	(0.200)	(0.096)	(0.127)	(0.118)	(0.245)	(0.104)	(0.130)	(0.136)
Fourth hour	-0.098	-0.611	-0.105	-0.817	-0.120	-0.769	-0.328	-1.023
	(0.202)	(0.097)	(0.121)	(0.113)	(0.252)	(0.108)	(0.132)	(0.130)
Fifth hour	-0.112	-0.770	-0.110	-0.631	-0.100	-0.635	-0.224	-0.771
	(0.203)	(0.102)	(0.125)	(0.111)	(0.262)	(0.110)	(0.140)	(0.134)
Sixth hour	-0.190	-0.381	-0.225	-0.486	-0.167	-0.366	-0.051	-0.770
	(0.217)	(0.098)	(0.127)	(0.107)	(0.276)	(0.105)	(0.134)	(0.130)
χ^2_{19}: Constant hazard rate	290.140	258.470	531.010	211.250	403.440	304.150	539.690	218.130
P-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Number of obervations	2,458		2,368		2,138		1,732	

The table reports parameter estimates with asymptotic standard errors in parentheses for a competing risks model of hazards for executions and cancellations for marginal limit orders and one tick away limit orders. A χ^2-test for the null hypothesis of all parameters jointly being equal to zero is reported for each model with the p-value in parenthesis.

Table 8: **Competing Risks Models for Execution and Cancellation Hazard Rates for WEM**

	Marginal sell		1 Tick sell		Marginal buy		1 Tick buy	
Variable	Execution	Cancellation	Execution	Cancellation	Execution	Cancellation	Execution	Cancellation
Power: α	0.644	0.616	0.580	0.494	0.786	0.652	0.586	0.468
	(0.028)	(0.014)	(0.014)	(0.013)	(0.038)	(0.014)	(0.014)	(0.013)
Constant	-5.933	-4.505	-26.407	1.111	1.221	-3.551	12.912	-9.605
	(1.603)	(0.910)	(1.942)	(1.764)	(2.012)	(1.087)	(2.005)	(1.960)
Spread	3.982	3.535	19.346	3.637	3.219	2.109	35.598	-7.160
	(2.133)	(1.084)	(1.426)	(3.024)	(1.681)	(0.826)	(2.528)	(3.948)
Close ask depth	–	–	-0.018	-0.077	0.020	-0.015	0.050	0.043
	–	–	(0.044)	(0.045)	(0.081)	(0.037)	(0.046)	(0.046)
Far ask depth	-0.429	-0.145	–	–	–	–	–	–
	(0.080)	(0.044)	–	–	–	–	–	–
Close bid depth	0.017	-0.051	0.168	0.080	–	–	-0.197	-0.079
	(0.067)	(0.036)	(0.040)	(0.040)	–	–	(0.041)	(0.041)
Far bid depth	–	–	–	–	-0.488	-0.092	–	–
	–	–	–	–	(0.072)	(0.036)	–	–
Order size	-0.196	0.057	-0.130	-0.059	-0.303	-0.045	-0.201	-0.101
	(0.079)	(0.041)	(0.045)	(0.045)	(0.074)	(0.034)	(0.040)	(0.044)
Recent trades	0.040	0.039	0.044	0.044	0.064	0.033	0.039	0.038
	(0.010)	(0.005)	(0.007)	(0.008)	(0.008)	(0.005)	(0.007)	(0.007)
Duration ($\times$1000)	-0.078	-0.054	-0.062	-0.025	-0.070	-0.045	-0.035	-0.029
	(0.018)	(0.008)	(0.010)	(0.009)	(0.020)	(0.007)	(0.010)	(0.010)
Mid-quote volatility	0.279	-0.173	12.892	-2.736	-5.368	-0.947	-10.441	3.840
	(0.914)	(0.518)	(1.137)	(1.031)	(1.182)	(0.635)	(1.181)	(1.150)
TSE market index	-0.040	-0.014	0.041	0.007	0.074	0.010	0.000	0.026
	(0.058)	(0.029)	(0.035)	(0.035)	(0.058)	(0.027)	(0.036)	(0.036)
TSE mining index	-0.260	0.012	-0.036	0.065	0.102	0.000	0.046	-0.013
	(0.069)	(0.028)	(0.039)	(0.035)	(0.059)	(0.028)	(0.038)	(0.041)
Overnight interest rate	0.020	0.029	0.005	-0.002	0.064	-0.027	0.088	0.001
	(0.054)	(0.027)	(0.037)	(0.036)	(0.055)	(0.029)	(0.034)	(0.036)
Canadian dollar	-0.005	-0.069	-0.047	-0.007	-0.142	-0.063	0.027	-0.030
	(0.057)	(0.031)	(0.046)	(0.042)	(0.067)	(0.028)	(0.041)	(0.045)
Lagged return	-0.046	0.006	0.102	0.002	0.152	0.038	0.102	0.072
	(0.060)	(0.027)	(0.033)	(0.036)	(0.061)	(0.029)	(0.037)	(0.038)
Distance to mid-quote	25.431	7.155	42.530	31.780	-31.161	-5.006	-81.301	-12.414
	(2.616)	(2.543)	(3.719)	(3.914)	(5.153)	(2.731)	(5.880)	(5.566)
First hour	-0.916	-1.019	-0.807	-0.957	0.051	-1.029	-0.197	-0.624
	(2.616)	(2.543)	(3.719)	(3.914)	(5.153)	(2.731)	(5.880)	(5.566)
Second hour	-1.025	-1.013	-0.778	-0.802	0.021	-0.685	-0.102	-1.073
	(0.224)	(0.105)	(0.140)	(0.127)	(0.316)	(0.104)	(0.136)	(0.151)
Third hour	-0.484	-0.730	-0.477	-0.873	0.387	-0.712	-0.266	-0.683
	(0.222)	(0.108)	(0.130)	(0.126)	(0.319)	(0.112)	(0.132)	(0.119)
Fourth hour	-0.059	-0.742	-0.441	-0.726	-0.204	-0.483	-0.207	-0.554
	(0.214)	(0.113)	(0.133)	(0.127)	(0.342)	(0.108)	(0.133)	(0.121)
Fifth hour	-0.141	-0.503	-0.459	-0.562	0.629	-0.330	-0.191	-0.847
	(0.236)	(0.117)	(0.139)	(0.127)	(0.332)	(0.116)	(0.136)	(0.132)
Sixth hour	-0.083	-0.328	-0.130	-0.407	0.451	-0.118	-0.072	-0.487
	(0.223)	(0.108)	(0.127)	(0.119)	(0.330)	(0.107)	(0.130)	(0.118)
χ^2_{19}: Constant hazard rate	218.200	273.520	501.500	188.440	244.620	273.480	493.200	201.660
P-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Number of observations	1,679		1,793		1,792		1,689	

The table reports parameter estimates with asymptotic standard errors in parentheses for a competing risks model of the hazards for executions and cancellations for marginal limit orders and one tick away limit orders. A χ^2-test for the null hypothesis of all parameters jointly being equal to zero is reported for each model with the p-value in parenthesis.

Table 9: **Picking Off Risk Regressions**

	BHO				ERR				WEM			
	Sell orders		Buy orders		Sell orders		Buy orders		Sell orders		Buy orders	
Variable	Marginal	1 Tick	Marginal	1 Tick	Marginal	1 Tick	Marginal	1 Tick	Marginal	1 Tick	Marginal	1 Tick
Constant	7.303	-2.834	-6.664	-2.650	21.697	0.724	10.638	-1.209	3.208	-6.783	-11.826	-9.236
	(2.732)	(0.629)	(4.865)	(0.501)	(6.643)	(1.219)	(5.058)	(1.183)	(4.787)	(1.828)	(7.347)	(1.879)
Spread	3.657	-10.137	3.598	6.231	-11.324	-13.703	9.076	16.850	-15.297	-12.394	16.575	5.711
	(6.265)	(2.353)	(6.272)	(1.667)	(3.257)	(2.244)	(3.849)	(2.296)	(4.629)	(3.244)	(3.775)	(2.968)
Close ask depth	–	0.013	-0.081	-0.007	–	0.116	-0.124	0.052	–	0.043	0.025	-0.020
	–	(0.015)	(0.200)	(0.012)	–	(0.050)	(0.146)	(0.040)	–	(0.039)	(0.198)	(0.033)
Far ask depth	0.281	–	–	–	-0.025	–	–	–	0.307	–	–	–
	(0.160)	–	–	–	(0.135)	–	–	–	(0.139)	–	–	–
Close bid depth	-0.098	0.001	–	0.006	-0.130	-0.168	–	-0.061	-0.296	-0.116	–	0.005
	(0.116)	(0.011)	–	(0.009)	(0.124)	(0.042)	–	(0.040)	(0.136)	(0.039)	–	(0.029)
Far bid depth	–	–	-0.264	–	–	–	0.231	–	–	–	-0.069	–
	–	–	(0.192)	–	–	–	(0.158)	–	–	–	(0.171)	–
Order size	0.006	-0.029	0.205	0.004	-0.177	0.000	0.207	0.147	-0.173	-0.051	0.188	0.011
	(0.150)	(0.014)	(0.234)	(0.016)	(0.132)	(0.041)	(0.133)	(0.039)	(0.173)	(0.046)	(0.177)	(0.033)
Recent trades	0.000	0.004	0.018	0.000	0.000	-0.002	0.003	0.001	-0.026	-0.003	0.066	-0.002
	(0.009)	(0.002)	(0.015)	(0.001)	(0.025)	(0.007)	(0.028)	(0.007)	(0.020)	(0.006)	(0.023)	(0.006)
Duration (×1000)	-0.056	0.003	0.156	0.003	-0.029	-0.018	0.048	-0.010	-0.031	-0.004	0.016	0.008
	(0.033)	(0.002)	(0.029)	(0.002)	(0.047)	(0.011)	(0.039)	(0.008)	(0.020)	(0.006)	(0.031)	(0.005)
Mid-quote volatility	-2.342	1.790	1.530	1.470	-8.852	-0.088	-9.082	0.616	0.105	4.237	4.847	5.419
	(1.424)	(0.344)	(3.000)	(0.304)	(3.784)	(0.710)	(3.184)	(0.696)	(2.802)	(1.067)	(4.444)	(1.106)
TSE market index	-0.046	-0.013	0.102	0.014	-0.135	0.073	0.208	0.030	0.281	0.152	0.095	0.029
	(0.101)	(0.013)	(0.145)	(0.011)	(0.106)	(0.036)	(0.095)	(0.034)	(0.129)	(0.046)	(0.106)	(0.020)
TSE mining index	0.358	0.006	-0.015	-0.007	-0.288	-0.055	-0.233	0.001	-0.103	-0.016	-0.052	-0.048
	(0.116)	(0.010)	(0.162)	(0.009)	(0.131)	(0.031)	(0.087)	(0.030)	(0.142)	(0.031)	(0.135)	(0.025)
Overnight interest rate	-0.253	-0.019	0.254	0.005	-0.102	0.052	-0.013	-0.027	0.071	0.000	-0.105	-0.030
	(0.118)	(0.008)	(0.113)	(0.008)	(0.103)	(0.036)	(0.091)	(0.038)	(0.141)	(0.026)	(0.170)	(0.026)
Canadian dollar	-0.020	0.014	-0.149	0.016	-0.024	-0.046	-0.068	-0.021	0.177	0.047	-0.039	0.010
	(0.119)	(0.010)	(0.170)	(0.010)	(0.112)	(0.039)	(0.103)	(0.029)	(0.119)	(0.027)	(0.116)	(0.015)
Lagged return	-0.101	0.000	-0.258	-0.009	-0.059	0.003	0.011	-0.016	0.074	-0.058	-0.265	-0.014
	(0.145)	(0.009)	(0.140)	(0.008)	(0.078)	(0.034)	(0.062)	(0.017)	(0.123)	(0.026)	(0.130)	(0.026)
Distance to mid-quote	-57.052	-7.500	-42.051	-5.310	-128.705	-33.594	-149.037	-45.025	-54.864	-20.177	-55.172	-11.241
	(6.722)	(1.323)	(12.228)	(0.953)	(16.583)	(5.503)	(18.288)	(5.585)	(14.627)	(5.795)	(22.796)	(4.549)
First hour	-0.616	-0.115	0.810	-0.034	-0.841	-0.118	-0.210	-0.255	0.560	-0.017	-1.174	-0.132
	(0.703)	(0.039)	(1.006)	(0.043)	(0.596)	(0.161)	(0.498)	(0.138)	(0.467)	(0.157)	(0.540)	(0.120)
Second hour	0.201	-0.027	0.766	-0.013	-0.719	-0.312	-0.047	-0.332	0.721	-0.081	-1.038	-0.060
	(0.655)	(0.043)	(1.012)	(0.041)	(0.599)	(0.147)	(0.500)	(0.133)	(0.467)	(0.135)	(0.550)	(0.090)
Third hour	-0.049	0.026	0.874	0.041	-0.553	0.012	-0.511	-0.102	0.766	-0.243	-0.625	-0.060
	(0.662)	(0.038)	(0.981)	(0.042)	(0.573)	(0.148)	(0.519)	(0.128)	(0.416)	(0.130)	(0.499)	(0.085)
Fourth hour	-0.626	-0.020	0.885	0.007	-0.586	0.167	-0.405	-0.096	0.535	-0.205	0.146	0.076
	(0.638)	(0.038)	(1.095)	(0.039)	(0.608)	(0.156)	(0.514)	(0.124)	(0.444)	(0.137)	(0.579)	(0.072)
Fifth hour	-0.060	-0.003	0.468	0.011	-0.361	-0.362	-0.408	-0.299	-0.313	-0.034	0.068	0.021
	(0.655)	(0.039)	(1.023)	(0.041)	(0.588)	(0.147)	(0.521)	(0.123)	(0.459)	(0.139)	(0.553)	(0.079)
Sixth hour	-0.557	0.010	0.075	0.063	-0.438	0.119	-0.523	-0.108	0.203	-0.256	-0.365	-0.027
	(0.687)	(0.039)	(1.038)	(0.044)	(0.601)	(0.153)	(0.690)	(0.129)	(0.420)	(0.124)	(0.643)	(0.072)
R^2	0.328	0.128	0.444	0.089	0.283	0.140	0.399	0.203	0.351	0.196	0.290	0.094
N	293	2388	170	2261	565	1194	484	955	327	892	281	858

The table reports the parameter estimates with standard errors in parentheses for the picking off risk regressions. Four regressions are estimated for each stock; one for marginal limit orders and one for one tick away limit orders, for both buy and sell orders.

Table 10: **Estimation Results for the Discrete Choice Model**

	BHO	ERR	WEM
Trader arrival rate			
Power: α_a	0.450	0.465	0.453
	(0.002)	(0.002)	(0.002)
Constant	-2.146	-2.077	-2.554
	(0.018)	(0.019)	(0.024)
TSE market index	0.029	0.018	0.032
	(0.005)	(0.005)	(0.005)
TSE mining index	0.032	-0.000	0.011
	(0.005)	(0.005)	(0.005)
Overnight interest rate	-0.114	-0.000	0.066
	(0.006)	(0.005)	(0.005)
Canadian dollar	0.105	0.061	-0.038
	(0.005)	(0.005)	(0.005)
Lagged return	0.271	0.152	0.185
	(0.005)	(0.007)	(0.006)
χ^2_5: Constant arrival rate	3871.950	680.940	985.280
	(0.000)	(0.000)	(0.000)
Private value distribution			
Mixing probability: ρ	0.849	0.891	0.842
	(0.040)	(0.034)	(0.048)
σ_1	0.058	0.040	0.037
	(0.001)	(0.001)	(0.001)
$\sigma_2 - \sigma_1$	0.472	0.682	0.483
	(0.035)	(0.064)	(0.045)
Time-varying variance			
TSE market index	-0.000	0.025	-0.047
	(0.007)	(0.006)	(0.007)
TSE mining index	-0.032	-0.018	-0.045
	(0.006)	(0.007)	(0.007)
Overnight interest rate	0.072	-0.051	-0.032
	(0.006)	(0.006)	(0.007)
Canadian dollar	-0.041	-0.022	0.026
	(0.007)	(0.006)	(0.007)
Lagged return	-0.104	-0.135	-0.116
	(0.006)	(0.005)	(0.008)
χ^2_5: Constant private value variance	434.550	814.080	369.700
	(0.000)	(0.000)	(0.000)
Continuation value	0.021	0.024	0.011
	(0.001)	(0.001)	(0.001)

The table reports parameter estimates for the discrete choice model. Asymptotic standard errors are reported in parentheses.

Table 11: **Expected Utility by Trader Valuation**

	BHO Private value						ERR Private value						WEM Private value					
Percent	-5.00	-2.50	-1.25	1.25	2.50	5.00	-5.00	-2.50	-1.25	1.25	2.50	5.00	-5.00	-2.50	-1.25	1.25	2.50	5.00
Cents	-4.25	-2.12	-1.06	1.06	2.12	4.25	-9.68	-4.84	-2.42	2.44	4.84	9.68	-6.75	-3.38	-1.69	1.69	3.38	6.75
Low liquidity state: wide spread, low depth																		
Buy market	-6.31	-4.19	-3.13	-1.00	0.06	2.18	-13.65	-8.81	-6.39	-1.55	0.87	5.71	-9.94	-6.57	-4.88	-1.51	0.18	3.55
Buy 1 tick	-4.33	-2.45	-1.51	0.38	1.32	$\boxed{3.20}$	-8.99	-5.08	-3.13	0.77	2.73	$\boxed{6.63}$	-7.12	-4.26	-2.84	0.02	1.45	$\boxed{4.31}$
Buy marginal	1.31	1.54	1.65	$\boxed{1.88}$	$\boxed{2.00}$	2.22	3.46	3.93	4.16	$\boxed{4.63}$	$\boxed{4.86}$	5.33	0.97	1.27	1.42	$\boxed{1.72}$	$\boxed{1.87}$	2.17
No order	1.79	1.79	1.79	1.79	1.79	1.79	4.61	4.61	4.61	4.61	4.61	4.61	1.54	1.54	1.54	1.54	1.54	1.54
Sell marginal	2.47	$\boxed{2.04}$	$\boxed{1.83}$	1.40	1.18	0.75	5.72	$\boxed{5.02}$	$\boxed{4.66}$	3.96	3.61	2.90	2.67	$\boxed{2.11}$	$\boxed{1.83}$	1.28	1.00	0.44
Sell 1 tick	$\boxed{3.16}$	1.28	0.35	-1.52	-2.46	-4.33	$\boxed{6.17}$	2.05	0.00	-4.12	-6.17	-10.29	$\boxed{3.64}$	1.64	0.64	-1.36	-2.36	-4.36
Sell market	2.18	0.06	-1.00	-3.13	-4.19	-6.31	4.56	-0.28	-2.70	-7.54	-9.96	-14.80	3.56	0.18	-1.51	-4.88	-6.57	-9.95
High liquidity state: narrow spread, high depth																		
Buy marginal	-4.36	-2.24	-1.18	0.95	2.01	$\boxed{4.13}$	-11.70	-6.86	-4.44	0.40	2.82	$\boxed{7.66}$	-7.23	-3.86	-2.17	1.20	$\boxed{2.89}$	$\boxed{6.26}$
Buy 1 tick	-0.27	0.55	0.95	1.77	$\boxed{2.17}$	2.99	1.28	2.35	2.89	3.97	4.51	5.58	-0.91	0.11	0.62	$\boxed{1.64}$	2.15	3.17
Buy marginal	1.38	1.54	1.61	1.77	1.84	2.00	3.37	3.84	4.07	4.54	$\boxed{4.78}$	5.25	1.11	1.28	1.37	1.55	1.64	1.81
No order	1.79	1.79	$\boxed{1.79}$	$\boxed{1.79}$	1.79	1.79	4.61	4.61	$\boxed{4.61}$	$\boxed{4.61}$	4.61	4.61	1.54	1.54	1.54	1.54	1.54	1.54
Sell marginal	2.12	1.88	1.76	1.51	1.39	1.15	5.40	$\boxed{4.80}$	4.50	3.91	3.61	3.02	2.28	1.85	1.63	1.20	0.99	0.55
Sell 1 tick	2.81	$\boxed{2.05}$	1.67	0.91	0.53	-0.23	5.23	4.48	4.10	3.35	2.97	2.22	3.94	2.48	$\boxed{1.75}$	0.29	-0.44	-1.90
Sell market	$\boxed{4.13}$	2.01	0.95	-1.18	-2.24	-4.36	$\boxed{8.81}$	3.97	1.55	-3.29	-5.71	-10.55	$\boxed{6.27}$	$\boxed{2.89}$	1.20	-2.17	-3.86	-7.23
Moving market state: common value proxy higher than the mid-quote																		
Buy market	-3.69	-1.52	-0.44	1.73	2.81	$\boxed{4.98}$	-5.12	-0.08	2.44	$\boxed{7.48}$	$\boxed{10.00}$	$\boxed{15.04}$	-6.04	-2.60	-0.87	$\boxed{2.57}$	$\boxed{4.29}$	$\boxed{7.73}$
Buy 1 tick	-0.77	0.44	1.04	$\boxed{2.25}$	$\boxed{2.86}$	4.07	3.72	4.12	4.32	4.72	4.92	5.32	-0.77	0.38	0.96	2.11	2.68	3.83
Buy marginal	1.50	1.63	1.70	1.83	1.90	2.03	4.24	4.38	4.45	4.58	4.65	4.79	1.25	1.40	1.47	1.62	1.70	1.84
No order	1.79	1.79	$\boxed{1.79}$	1.79	1.79	1.79	4.61	4.61	$\boxed{4.61}$	4.61	4.61	4.61	1.54	1.54	$\boxed{1.54}$	1.54	1.54	1.54
Sell marginal	$\boxed{2.50}$	$\boxed{1.97}$	1.71	1.18	0.92	0.40	$\boxed{6.22}$	$\boxed{4.65}$	3.87	2.30	1.52	-0.05	2.53	$\boxed{1.85}$	1.51	0.83	0.49	-0.19
Sell 1 tick	2.35	0.77	-0.03	-1.61	-2.40	-3.99	2.48	-1.08	-2.86	-6.42	-8.20	-11.75	$\boxed{2.84}$	0.99	0.06	-1.78	-2.71	-4.56
Sell market	1.51	-0.66	-1.74	-3.91	-4.99	-7.16	-0.87	-5.91	-8.43	-13.47	-15.99	-21.03	2.36	-1.08	-2.81	-6.25	-7.97	-11.41

The table reports the expected utility from different order submissions for traders with different private values. The expected utilities are computed for three different states; a low liquidity state where the bid-ask spread is one standard deviation above its mean and the depth variables are one standard deviation below their means, a high liquidity state where the bid-ask spread is one standard deviation below its mean and the depth variables are one standard deviation above their mean values, and a moving markets state where the common value proxy divided by the mid-quote is one standard deviation above its mean. All other variables are at the in-sample mean values. All utilities are reported in cents per share.

Table 12: **Order Submission Probabilities by Trader Valuation**

	Private value				
	$(-\infty, -5\%]$	$(-5\%, -2.5\%]$	$(-2.5\%, +2.5\%)$	$[2.5\%, +5\%)$	$[+5\%, +\infty)$
			BHO		
			Probability of value in interval		
Mean	0.212	0.127	0.322	0.127	0.212
Standard deviation	0.026	0.007	0.039	0.007	0.026
			Mean order submission probabilities		
Sell market	0.735	0.247	0.014	0.000	0.000
Sell limit	0.244	0.642	0.247	0.003	0.000
No order	0.019	0.089	0.364	0.068	0.010
Buy limit	0.002	0.022	0.367	0.743	0.301
Buy market	0.000	0.000	0.008	0.187	0.689
			Standard deviation of order submission probabilities		
Sell market	0.300	0.357	0.070	0.010	0.000
Sell limit	0.286	0.393	0.201	0.042	0.006
No order	0.093	0.251	0.282	0.218	0.065
Buy limit	0.029	0.128	0.271	0.356	0.305
Buy market	0.000	0.004	0.050	0.316	0.309
			ERR		
			Probability of value in interval		
Mean	0.115	0.143	0.484	0.143	0.115
Standard deviation	0.018	0.009	0.049	0.009	0.018
			Mean order submission probabilities		
Sell market	0.851	0.316	0.011	0.001	0.000
Sell limit	0.147	0.656	0.164	0.001	0.000
No order	0.002	0.027	0.608	0.009	0.001
Buy limit	0.000	0.001	0.210	0.752	0.218
Buy market	0.000	0.000	0.007	0.238	0.781
			Standard deviation of order submission probabilities		
Sell market	0.233	0.351	0.059	0.026	0.008
Sell limit	0.231	0.355	0.116	0.030	0.011
No order	0.027	0.120	0.154	0.069	0.015
Buy limit	0.003	0.023	0.116	0.334	0.263
Buy market	0.000	0.000	0.039	0.330	0.264
			WEM		
			Probability of value in interval		
Mean	0.111	0.130	0.519	0.130	0.111
Standard deviation	0.017	0.012	0.055	0.012	0.017
			Mean order submission probabilities		
Sell market	0.921	0.483	0.036	0.000	0.000
Sell limit	0.079	0.512	0.413	0.003	0.000
No order	0.000	0.004	0.118	0.011	0.001
Buy limit	0.000	0.001	0.403	0.616	0.164
Buy market	0.000	0.000	0.030	0.370	0.835
			Standard deviation of order submission probabilities		
Sell market	0.157	0.413	0.090	0.001	0.000
Sell limit	0.157	0.411	0.171	0.047	0.008
No order	0.007	0.053	0.176	0.090	0.021
Buy limit	0.003	0.021	0.155	0.415	0.237
Buy market	0.002	0.007	0.089	0.414	0.238

The table reports the in-sample mean and the standard deviation of the probability of drawing a private value (u) from five different intervals. For each interval and stock the table reports the in-sample mean and standard deviation of the probability of a trader optimally submitting a sell market order, a sell limit order, no order, a buy limit order, or a buy market order.

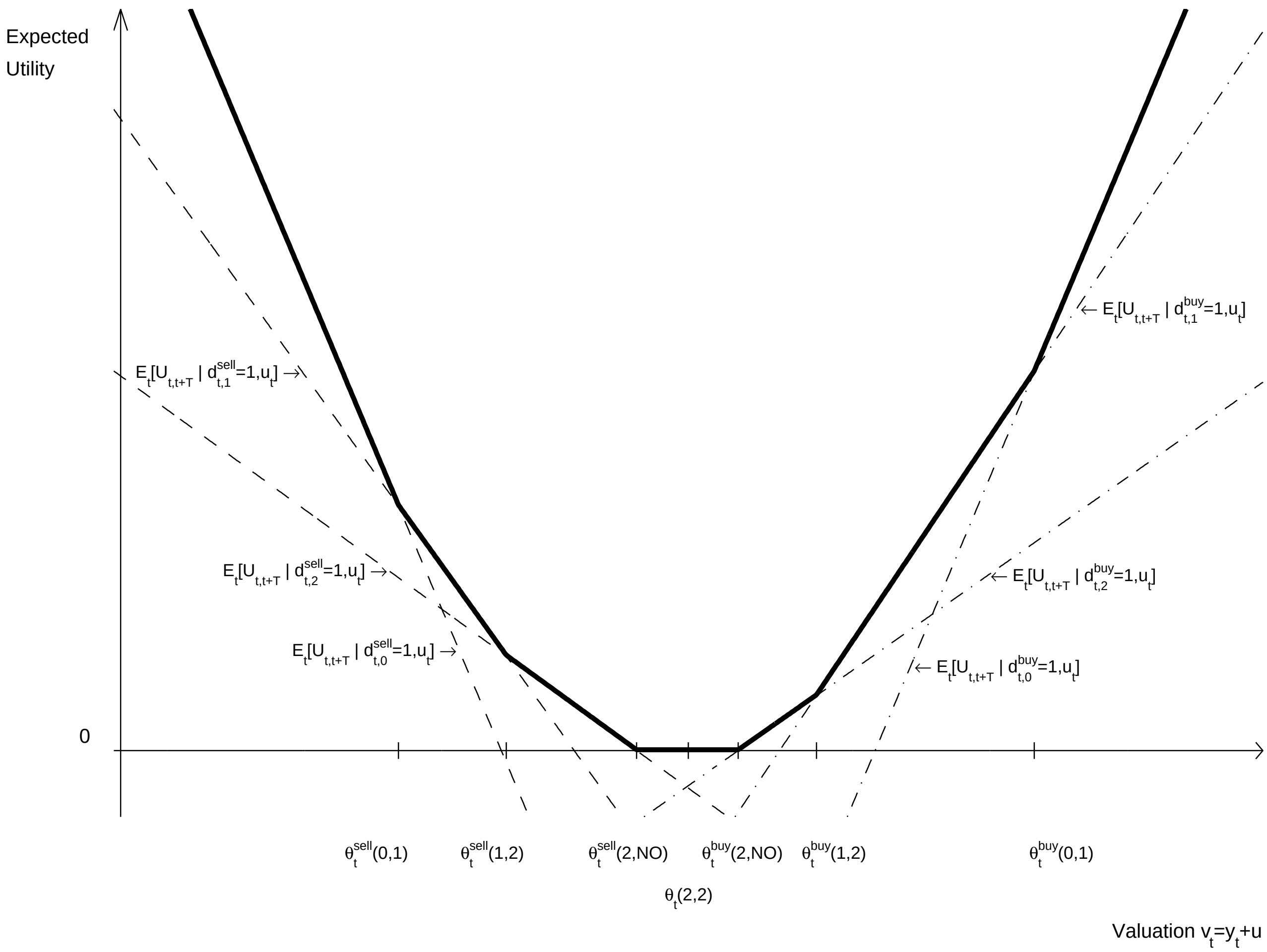

Figure 1: The graph provides an example of the traders optimal order submission strategy. The horizontal axis is the trader's valuation, and the vertical axis is the expected utility from various order submissions. The horizontal axis and the vertial axis have different scale. Sell orders are plotted with dashed lines (- - -) and buy orders are plotted with dashed-dotted lines (-.-.). The maximized utility function is plotted with a dark solid line (—). The continuation value is equal to zero.

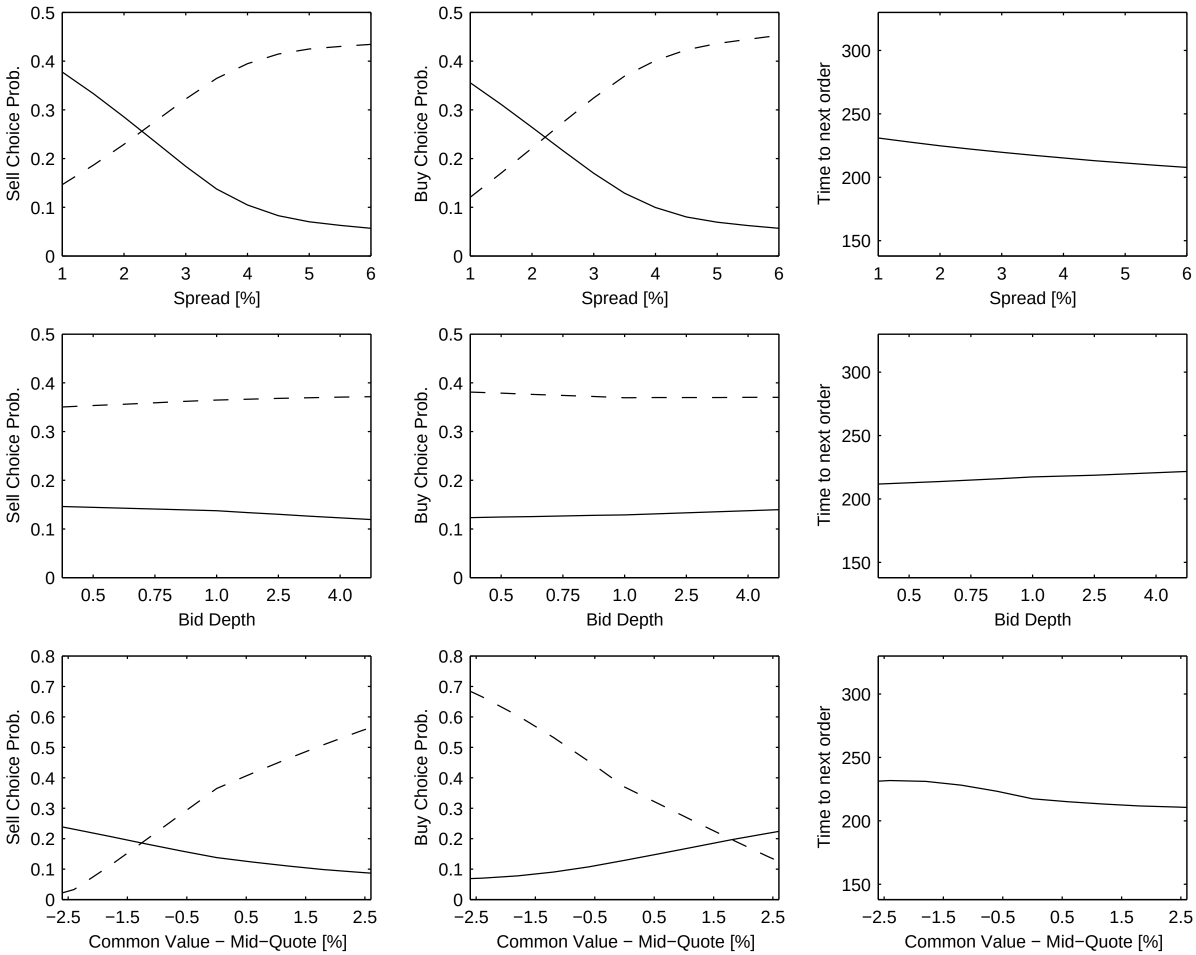

Figure 2: Comparative statics for BHO. The top left picture plots the probability of a sell market order (–) and the probability of a sell limit order (- -) conditional on an order submission as a function of the spread. The top middle picture plots the probability of a buy market order (–) and the probability of a buy limit order (- -) conditional on an order submission as a function of the spread. The top right picture plots the expected time to the next order submission as a function of the spread. The middle row and the bottom row of pictures plots the corresponding comparative statics for the choice probabilities and time to the next order for the bid side depth and the distance between the common value proxy and the mid-quote.

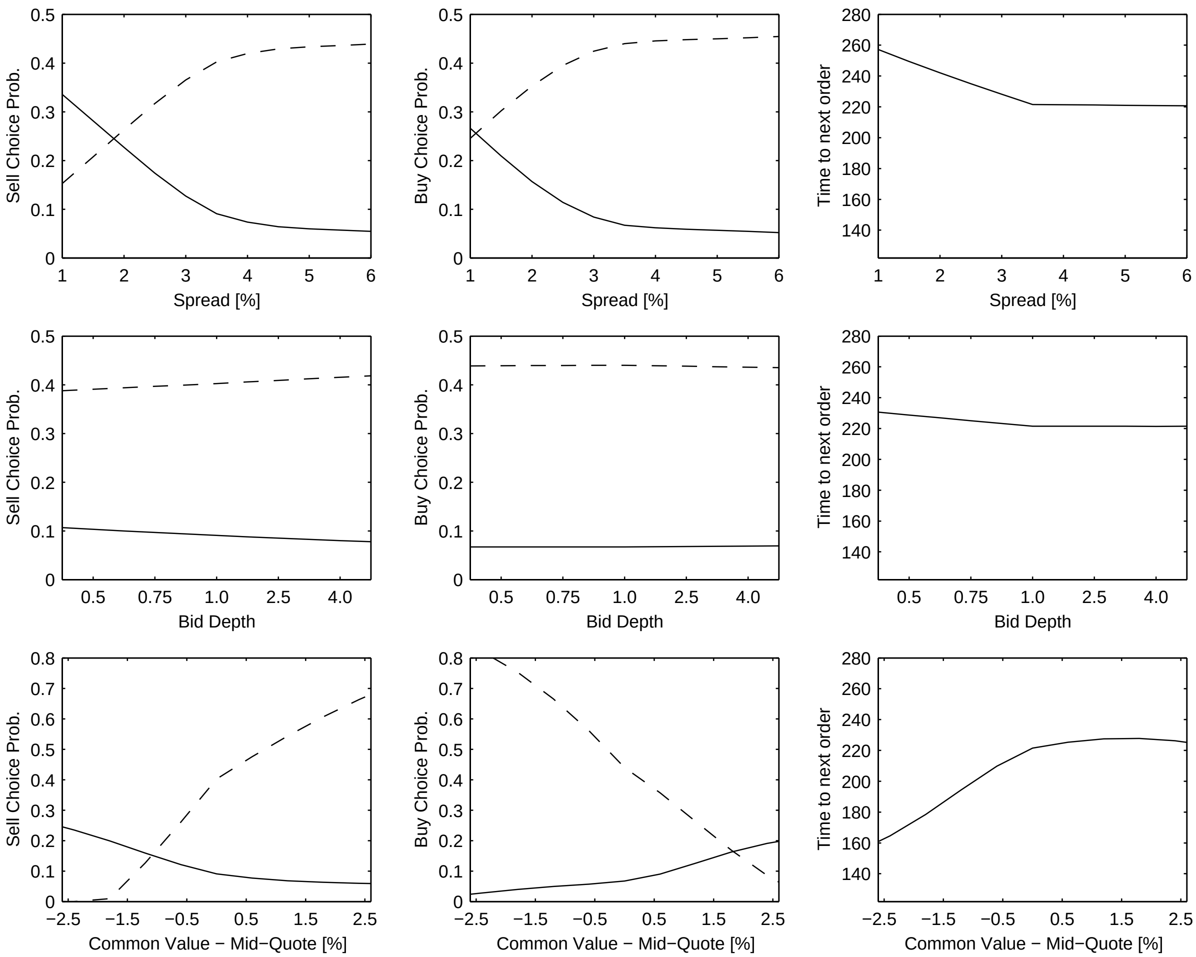

Figure 3: Comparative statics for ERR. The top left picture plots the probability of a sell market order (–) and the probability of a sell limit order (- -) conditional on an order submission as a function of the spread. The top middle picture plots the probability of a buy market order (–) and the probability of a buy limit order (- -) conditional on an order submission as a function of the spread. The top right picture plots the expected time to the next order submission as a function of the spread. The middle row and the bottom row of pictures plots the corresponding comparative statics for the choice probabilities and time to the next order for the bid side depth and the distance between the common value proxy and the mid-quote.

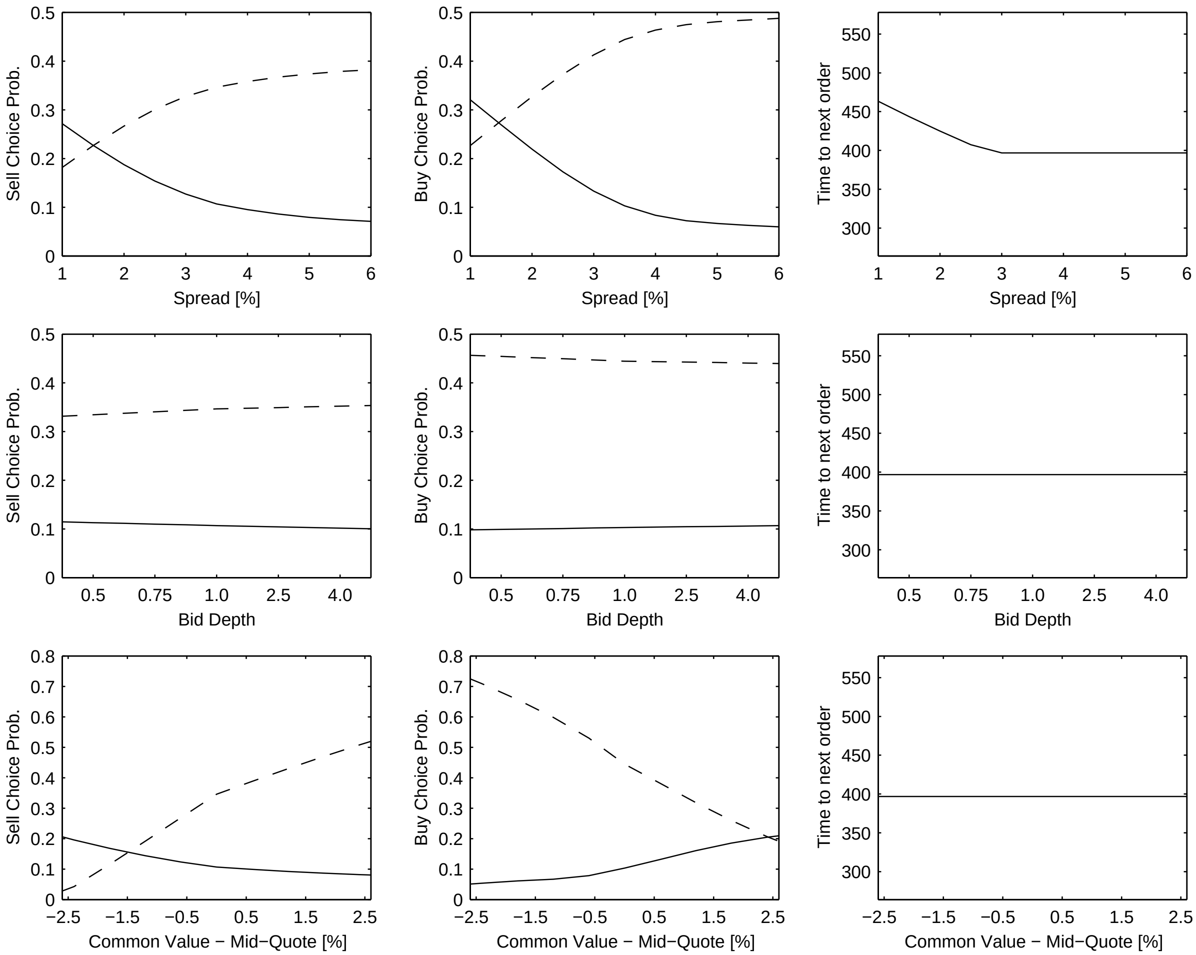

Figure 4: Comparative statics for WEM. The top left picture plots the probability of a sell market order (–) and the probability of a sell limit order (- -) conditional on an order submission as a function of the spread. The top middle picture plots the probability of a buy market order (–) and the probability of a buy limit order (- -) conditional on an order submission as a function of the spread. The top right picture plots the expected time to the next order submission as a function of the spread. The middle row and the bottom row of pictures plots the corresponding comparative statics for the choice probabilities and time to the next order for the bid side depth and the distance between the common value proxy and the mid-quote.

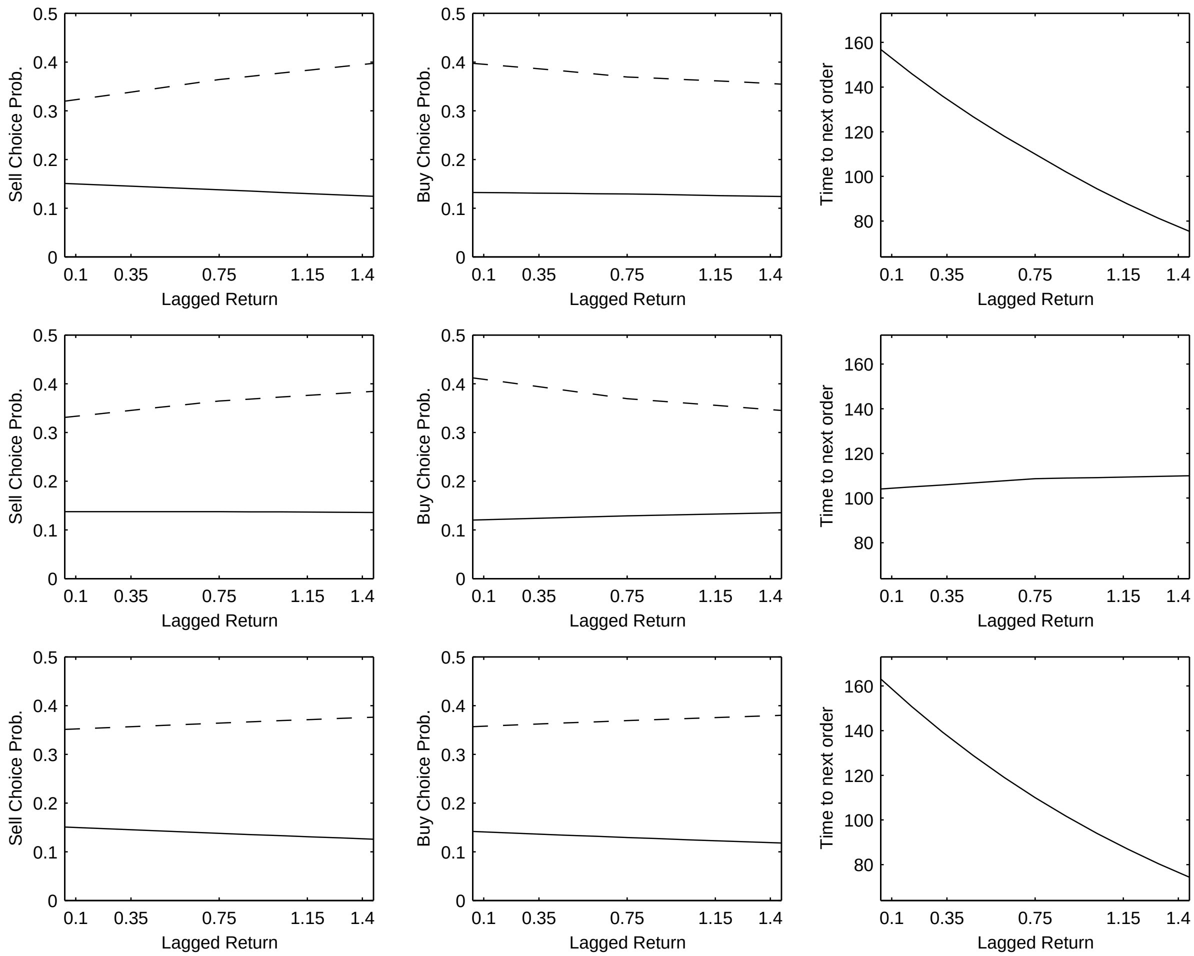

Figure 5: Comparative statics for BHO. The top left picture plots the sell market (–) and sell limit (- -) choice probabilities as a function of the absolute value of changes in the stock's lagged return. The top middle picture plots the buy market (–) and buy limit (- -) choice probabilities as a function of the absolute value of changes in the stock's lagged return. The top right picture plots the expected time to the next order submission as a function of the absolute value of changes in the stock's lagged return. The middle and the bottom row of pictures also plot the choice probabilities and time to the next order submission. In the middle row of pictures the valuation distribution and the arrival rate of traders are held constant at their mean values and in the bottom row of pictures the threshold valuations are held constant at their mean values.

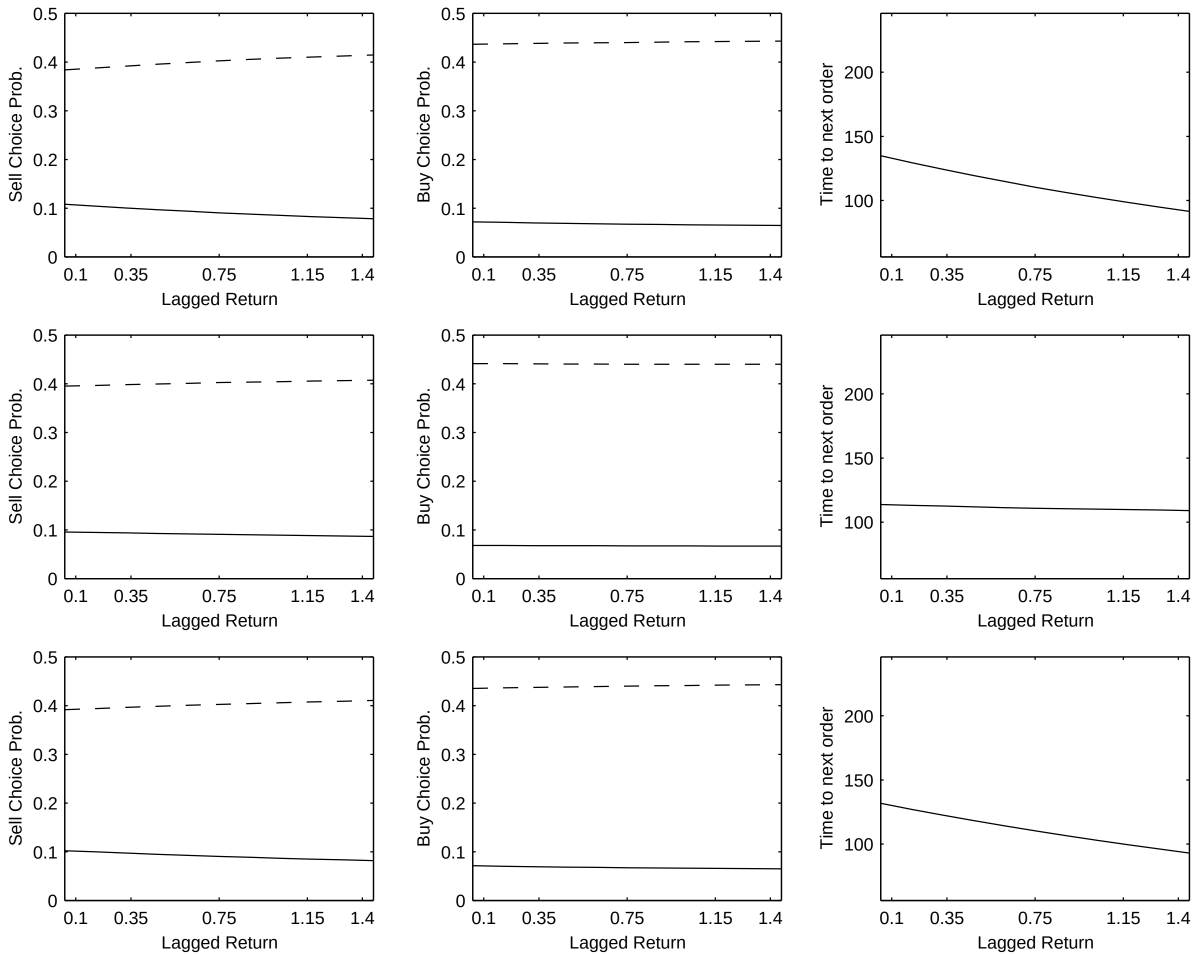

Figure 6: Comparative statics for ERR. The top left picture plots the sell market (–) and sell limit (- -) choice probabilities as a function of the absolute value of changes in the stock's lagged return. The top middle picture plots the buy market (–) and buy limit (- -) choice probabilities as a function of the absolute value of changes in the stock's lagged return. The top right picture plots the expected time to the next order submission as a function of the absolute value of changes in the stock's lagged return. The middle and the bottom row of pictures also plot the choice probabilities and time to the next order submission. In the middle row of pictures the valuation distribution and the arrival rate of traders are held constant at their mean values and in the bottom row of pictures the threshold valuations are held constant at their mean values.

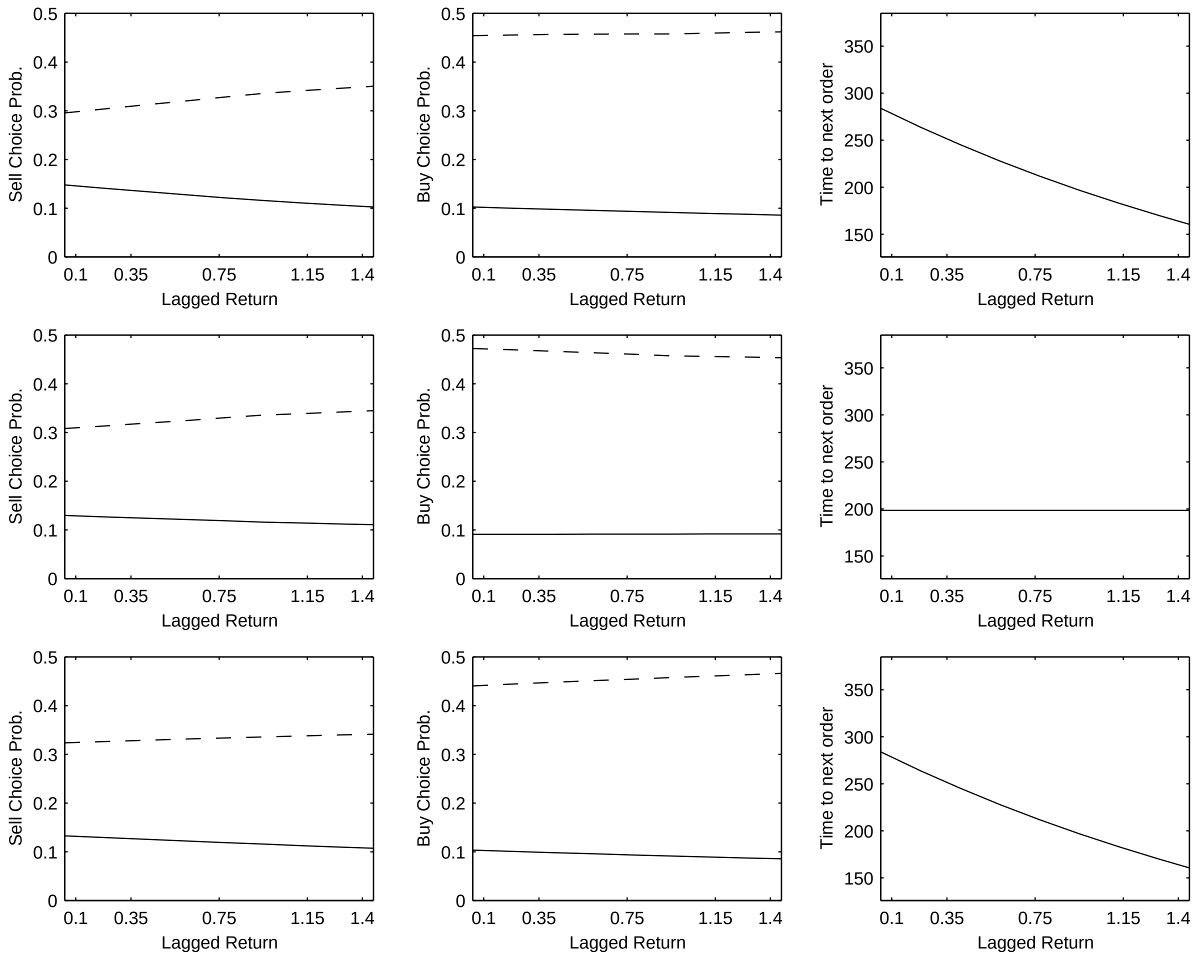

Figure 7: Comparative statics for WEM. The top left picture plots the sell market (–) and sell limit (- -) choice probabilities as a function of the absolute value of changes in the stock's lagged return. The top middle picture plots the buy market (–) and buy limit (- -) choice probabilities as a function of the absolute value of changes in the stock's lagged return. The top right picture plots the expected time to the next order submission as a function of the absolute value of changes in the stock's lagged return. The middle and the bottom row of pictures also plot the choice probabilities and time to the next order submission. In the middle row of pictures the valuation distribution and the arrival rate of traders are held constant at their mean values and in the bottom row of pictures the threshold valuations are held constant at their mean values.