

# Inflation and Output Dynamics with State Dependent Pricing Decisions\*

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## Abstract

We study the effects of monetary policy on output and inflation in a dynamic general equilibrium model. We assume that firms face a fixed cost of changing their pricing plans: once a firm pays this fixed cost, it can choose both its current price and a plan specifying an entire sequence of future prices. We find that our model's predictions are qualitatively consistent with the conventional wisdom about the response of the economy to three widely studied monetary experiments. Allowing firms to choose a sequence of prices rather than a single price generates inflation inertia in the response of the economy to small changes in the growth rate of money. Allowing firms to choose when to change their pricing plan generates a non-linear response of inflation and output to small and large changes in the money growth rate. Our non-linear solution method allows us to quantify the range of changes in the growth rate of money for which time dependent models are a good approximation to state dependent models. This approach also reveals that the model generates an asymmetric response of output and inflation to monetary expansions and contractions.

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## 1. Introduction

There is a large literature that studies the effects of monetary policy on output and inflation in models with sticky prices. The models in the literature can be classified into two broad classes. In the first one, commonly referred as *time dependent* pricing models, the number of firms changing prices is fixed exogenously. Firms control only the degree to which they change their price once they have the opportunity to do so.<sup>1</sup> However, time dependent models are often viewed as an approximation of a more complicated firm behavior. As an alternative, a second class of models, commonly referred as *state dependent* pricing models, also endogenizes the number of firms changing prices. Typically, this extensive margin is modeled by assuming that firms face a fixed cost of changing their nominal price. Dotsey, King, and Wolman (1999) develop a tractable way to incorporate state dependent pricing models into a quantitative general equilibrium framework.<sup>2</sup>

In this paper, we study a dynamic general equilibrium model in which firms choose dynamic pricing plans. Once a firm pays a fixed cost, it can choose not only its current price, but also a plan specifying an entire sequence of future prices. Nominal rigidities arise because: (1) changing the plan is costly, and (2) prices in the plan can be made contingent on the current information set but cannot be made contingent upon future aggregate variables. This pricing behavior is consistent with the fact that costs of changing prices are broader than physical menu costs that have prevailed in the literature. They also include managerial costs such as decision making and communication costs, as documented by Zbaracky

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<sup>1</sup>Seminal papers include those by Barro (1972) and Calvo (1983), among many others. See Taylor (1998) for a comprehensive literature review on time dependent sticky price models.

<sup>2</sup>Other papers that study state-dependent pricing models include Caballero and Engel (1993), Caplin and Leahy (1991), Caplin and Spulber (1987) and Ireland (1997). These papers make simplifying assumptions to gain analytical tractability.

et. al. (2000).<sup>3</sup> <sup>4</sup> This pricing structure resembles Fischer's (1977) contracting model. Mankiw and Reiss (2001) and Calvo, Celasun and Kumhof (2001) are two recent papers that study related time dependent pricing models.<sup>5</sup>

Relative to the previous sticky-price literature, our model has two desirable properties. First, it generates inflation inertia in the response of the economy to widely studied policy experiments involving small changes in the growth rate of money. Conventional time dependent and state dependent pricing models in which firms choose a single price do not.<sup>6</sup> Second, the model is consistent with the view that large, persistent changes in the growth rate of money, have relatively smaller effects on output. In contrast, standard time dependent models in which the number of firms changing prices is constant, are not. Since we do not rely on linear approximations, we can study qualitative differences in the response of the model to changes in the growth rate of money of different magnitudes. In particular, we can address the question: how big does a change in the money growth rate have to be so that all firms adjust their pricing plan? Among other things, this allows us to assess when time dependent models cease to provide accurate approximations of state dependent models in the context of the monetary

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<sup>3</sup>In their case studies, Zbaracky et. al. (2000) find that only 4% of the costs associated to changing prices are related to physical costs as studied in the menu cost literature. Our model abstracts from information gathering costs, which are implicit in the analysis of Mankiw and Reiss (2001) and Woodford (2001a).

<sup>4</sup>Although actual business cycle examples are harder to find in low inflation environments like the US economy, seasonalities and varying week/weekend prices in restaurants are illustrations of this type of pricing behavior.

<sup>5</sup>In the model studied by Mankiw and Reiss (2001), firms have flexible prices but only a fraction of firms updates its information set every period. In the model studied by Calvo, Celasun and Kumhof (2001), pricing plans are constrained to consist of an initial price level and a constant growth rate of the price over time. Our model extends these papers by combining the ability to set a price plan, versus a single price, and the ability to choose when to change the plan itself. For small changes in the growth rate of money, we obtain similar results to both of these papers.

<sup>6</sup>See Chari, Kehoe and McGrattan (2001) and Mankiw and Reiss (2001) for a criticism of conventional sticky price models. See also Christiano, Eichenbaum and Evans (2001) and Dotsey and King (2001) for richer sticky price models that can account for the response of the US economy to small monetary shocks.

experiments we study.

In order to understand the aggregate implications of our model, we follow Mankiw and Reiss (2001) and focus on three monetary experiments. According to conventional wisdom:

1. Small temporary increases in the growth rate of money lead to a hump-shaped response of output and inflation. Large changes in the growth rate of money have relatively smaller effects on output.<sup>7</sup>
2. Permanent credible disinflations in low inflation environments generate a temporary contraction in output. Credible disinflations where the initial level of inflation is very high cause a relatively smaller decline in output.<sup>8</sup>
3. Pre-announced credible disinflations do not generate short term booms in economic activity. Output either falls or is unaffected.<sup>9</sup>

Our model is consistent with (1)-(3). The central features underlying its predictions are as follows. First, the fraction of firms changing their pricing decisions every period is endogenous. This drives the differential response of output and inflation to small and large changes in the growth rate of money, as in (1) and (2). In response to a small change, only a small number of firms decide to adjust their pricing decisions. But, after a large change, many firms find it optimal to adjust their pricing decisions. In contrast, the response of output and inflation in time

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<sup>7</sup>This conventional wisdom is not free of controversies, but roughly speaking it has been widely accepted. See for example Friedman (1967). For more recent studies, see for example Christiano, Eichenbaum and Evans (2001), Fuhrer and Moore (1995) and references therein for a discussion of small shocks to money growth rate in postwar US data. See Sargent (1982) for a discussion about the relation of money and prices during hyperinflations. See also Ravn and Sola (1996), Weise (1999), and Fischer, Sahay, and Vegh (2001) for other empirical evidence supporting an asymmetric response to small and large monetary shocks.

<sup>8</sup>See Gordon (1982), Ball (1994), Dornbusch and Fischer (1993) and references therein for a discussion of credible disinflations where initial inflation rates are relatively moderate. See Sargent (1982) for a discussion of credible disinflations where the initial level of inflation is very high.

<sup>9</sup>See Ball (1994) and Mankiw and Reiss (2001).

dependent models is roughly proportional to the size of the change in the money growth rate.

Second, firms face a fixed cost of changing their pricing plan rather than their current price level. This feature is key to understanding the response of inflation and output to small changes in the growth rate of money, as in (1)-(2), as well as to pre-announced changes in the money growth rate, as in (3). Suppose that firms expect nominal marginal costs to increase over time after an unforeseen monetary expansion. If a firm could only choose a single time non-varying price, it would choose a price that is higher than the current marginal cost. This is because, depending on the future costs of changing prices, it might not raise its price in the near future when higher marginal costs materialize. We refer to this type of forward looking behavior as ‘front-loading’. Other models in the literature, such as Nelson (1998) and Woodford (2001b), reduce front-loading incentives by imposing frictions on the firm’s problem so that the rate of change of its price is small. In contrast, firms in our model have smaller front-loading incentives because they have the freedom to choose a sequence of prices. Then firms can plan future price increases in advance without actually having to implement them today. As we discuss below, the fact that firms have no incentives to front-load their price in anticipation of increases in marginal cost is the key feature that allows our model to generate an inertial response of inflation to small changes in the growth rate of money.

In contrast to much of the literature, we do not use a linear approximation to solve the model.<sup>10</sup> For computational reasons, we focus on one-time, unanticipated changes in monetary policy. The advantage of working with non-linear solution methods is that we can establish the following two properties of the model. First, as the size of the growth rate of money increases, a larger fraction of firms adjusts their plans. This makes inflation more responsive and output less responsive to monetary shocks. In particular, we find that for increases in the quarterly growth rate of money that are smaller than 6.5%, traditional time dependent models

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<sup>10</sup>See for example Dotsey, King and Wolman (1999).

are a good approximation to state dependent models. For increases smaller than 6.5%, the ability of a time dependent model to approximate state dependent models deteriorates.<sup>11</sup> Second, our model economy responds asymmetrically to monetary expansions and contractions. In particular, the rise in output after a monetary expansion is smaller than the fall in output that occurs after a monetary contraction. This is because in our model, firms are more averse about having a relative price that is too low than too high. In the extreme, if the price is low enough, the firm can have negative profits. On the contrary, if the price is too high, profits will be low but never negative. As a result, monetary expansions are more likely to induce price adjustment than monetary contractions. This generates an asymmetric response in output to monetary expansions and contractions.<sup>12</sup>

The remainder of the paper is organized as follows. Section 2 lays out the basic model and two different pricing assumptions: sticky prices and sticky plans. Section 3 discusses the chosen parameter values, and describes some properties of the steady state. Section 4 presents the sticky price and sticky plan models' aggregate implications to three policy experiments. Section 5 discusses the size of temporary changes in the growth rate of money such that all firms adjust their plan, under different parameter configurations. Section 6 provides some conclusions and future extensions.

## 2. The model

The model is composed of a representative household, a government that follows an exogenous policy, a competitive sector producing the final consumption good, and a continuum of monopolistic producers of intermediate goods.

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<sup>11</sup>Taylor's (1999) claim that Dotsey, King, and Wolman '...find that money, output, and price dynamics resulting from their state dependent model are not too dissimilar from the dynamics of the purely time dependent model...' (pp. 40) is only supported in the context of small changes in the growth rate of money.

<sup>12</sup>See Maklem, Paquet and Phaneuf (1996) and references therein for evidence of this asymmetry in the US economy.

### *Household*

We consider a representative agent economy. Preferences are defined over sequences of consumption and hours worked by a time-separable utility function,

$$U = E_t \sum_{t=0}^{\infty} \beta^t u(C_t, N_t). \quad (2.1)$$

$N_t$  denotes the total amount of hours worked and  $C_t$  denotes consumption. The function  $u(.,.)$  is increasing in the first argument and decreasing in the second one, twice continuously differentiable, and concave in both arguments.  $\beta$  is the discount factor rate, with  $0 < \beta < 1$ .

The agent faces the following budget constraint in period  $t$ :

$$P_t^C(1 - \tau_t^C)C_t + B_t + M_t - M_{t-1} = W_t N_t + (1 + i_t)B_{t-1} + R_t K + \Pi_t + T_t \quad (2.2)$$

Here  $M_{t-1}$  and  $B_{t-1}$  denote the household's beginning of period  $t$  holdings of cash and nominal bonds. The latter pay a nominal interest rate equal to  $i_t$ .  $P_t^C$  represents the price of the aggregate consumption good,  $T_t$  denotes nominal lump sum transfers from the government and  $\tau_t^C$  denotes an ad-valorem subsidy to consumption purchases. We assume that households hold a constant amount of capital,  $K$ , which earns nominal rent  $R_t$  for its services in period  $t$ . Nominal profits are denoted by  $\Pi_t$ .

Finally, we assume that agents face the following cash in advance constraint on consumption purchases:

$$P_t^C C_t \leq M_t \quad (2.3)$$

The consumer maximizes (2.1) subject to (2.2) and (2.3).<sup>13</sup>

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<sup>13</sup>Given our representative agent assumption, an equilibrium condition is that  $B_t = 0, \forall t$ . In the recursive formulation of the problem, we suppress  $B_t$  from the consumer's problem in order to save on notation. We use the intertemporal Euler equation corresponding to the choice of  $B_t$  to pin down the nominal interest rate:

$$(1 + i_{t+1}) \beta E_t \{ U_{C,t+1} / [P_{t+1}^C(1 - \tau_{t+1}^C)] \} = U_{C,t} / [P_t^C(1 - \tau_t^C)].$$

### *The Government*

We denote by  $\mu_t$  the exogenous growth rate of money in period  $t$ , namely  $\mu_t = M_t/M_{t-1}$ . In order to isolate the effects of monetary policy that operate through sticky prices, we abstract from the costs of inflation that would arise in a flexible price version of this economy. Specifically, we assume the government chooses  $\tau_t^C$  so as to imply:

$$\tau_t^C = \frac{i_{t+1}}{1 + i_{t+1}}.$$

By subsidizing consumption, the government eliminates the distortions present in the two following margins: (1) consumption versus leisure, and (2) valuation of future versus present nominal profits. In addition, we assume the government chooses lump sum transfers,  $T_t$ , to satisfy the following balanced budget constraint:

$$M_t - M_{t-1} = T_t + \tau_t^C P_t^C C_t \tag{2.4}$$

### *Final Goods Producer Firms*

This sector is composed of competitive firms. The final good,  $Y_t$ , is produced with a constant returns to scale CES production function in intermediate inputs,  $Y_{it}$ :

$$Y_t = \left[ \int_0^1 Y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.5}$$

Final goods producers take the schedule of prices  $P_{it}$  as given and minimize the unit cost of producing a unit of  $Y$ ,

$$\min_{Y_{it}} \int_0^1 P_{it} Y_{it} di, \tag{2.6}$$

subject to  $Y_t = 1$  and (2.5). Individual demands for intermediate goods are given by:

$$Y_{it} = \left( \frac{P_{it}}{P_t^C} \right)^{-\varepsilon} Y_t. \tag{2.7}$$



In equilibrium, the price of the final good firms is equal to the unit cost of production:

$$P_t^C = \left[ \int_0^1 P_{it}^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \quad (2.8)$$

and profits are zero.

### *Intermediate Goods Producers*

The intermediate goods sector is composed of a continuum of monopolistic producers indexed by  $i \in [0, 1]$ . These goods are produced with capital ( $K_{it}$ ) and labor ( $N_{it}$ ) using a constant returns to scale Cobb-Douglas production function:

$$Y_{it} = A (N_{it})^\alpha (K_{it})^{1-\alpha}. \quad (2.9)$$

Profits at time  $t$  for firm  $i$  are:

$$\Pi_{it} = P_{it}Y_{it} - W_tN_{it} - R_tK_{it} - \text{cost of changing price scheme} \quad (2.10)$$

Our assumption that the production function is homogenous of degree one implies that every monopolist faces the same marginal cost of production, equal to  $\kappa W_t^\alpha R_t^{1-\alpha}$ , where  $\kappa = 1/[\alpha^\alpha(1-\alpha)^{1-\alpha}A]$ . Intermediate producers set prices in the way described below, and meet demand at these prices.<sup>14</sup> Our assumption of constant elasticity of substitution in demand implies that, in a flexible price world, each monopolist would set price equal to a fixed markup over marginal cost.

Adjusting the price scheme is costly. As in Dotsey, King and Wolman (1999), we assume that each period a firm draws a random labor cost  $\xi$ . Costs of changing the price scheme are either  $\xi W_t$  in case the firm decides to change it, and zero otherwise. This cost is i.i.d. across time and firms, with c.d.f  $G(\xi)$  that has support

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<sup>14</sup>In equilibrium, intermediate firms' profits are positive. A simple way to generate non-positive steady state profits without explicitly modeling entry is to assume a fixed cost of operation. In this version of the paper we will abstract from this issue.

$[0, \bar{\xi}]$ .<sup>15</sup> Two particular functional forms of  $G(\cdot)$  that have received considerable interest in the literature are:

$$\text{Calvo Pricing: } \xi = \begin{cases} 0 & \text{with probability } q \\ \infty & \text{with probability } 1 - q \end{cases}$$

$$\text{Ss Pricing: } \xi = \bar{\xi} \text{ with probability } 1$$

We study two particular environments regarding the nature of these adjustment costs. In the first one, denoted as *sticky prices*, the period's fixed cost is incurred in order to change the current price. This is the menu cost interpretation that has prevailed in the literature. In the second environment, which we denote as *sticky plans*, the period's fixed cost is incurred in order to choose a time-varying path of predetermined prices rather than a single fixed price. The decision to change the plan is state contingent. Prices in the plan can be made conditional on the current information set, but cannot be made contingent upon future aggregate variables. These costs are those associated with decision making and communication as discussed by Zbaracki et al. (2000). While reality includes a mixture of firms that choose fixed price plans and time-varying price plans, in this paper we study the aggregate implications of both extreme settings.

We assume a positive steady state growth rate of money. To make the environment stationary, it is convenient to normalize prices with respect to the current money supply. We will denote by lower case letters those nominal variables that have been rescaled by the current money supply. For example,  $p_t^C = P_t^C/M_t$ .

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<sup>15</sup>Willis (2000) considers adjustment costs that follow an autoregressive log-normal process. He estimates the model using aggregate US data and finds that the persistence parameter is 0.49. We use the i.i.d assumption to gain numerical tractability. Note that if the fixed cost was constant for each firm, then changes in monetary policy would not generate interesting dynamic effects. This is because there would be no option value to waiting and drawing a lower cost in the future. So, firms would make their only adjustment decision in the period after the change in monetary policy

## Recursive formulation with Markov policies

The aggregate state of the economy is  $s = (\theta, \mu)$ .  $\mu$  denotes the growth rate of money, evolving according to the Markovian transition  $P(\mu, \mu')$ .<sup>16</sup>  $\theta$  denotes the cumulative distribution of firms over rescaled price  $p$ . Let  $S$  be the state space that contains all possible realizations of  $s$ .<sup>17</sup> The law of motion of  $s$  is given by the transition function  $Q^A$ . The consumer's recursive problem and the recursive competitive equilibrium are defined in the appendix. Here we will focus on the formulation of the firm's problem. We first describe the firm's problem in the sticky price environment. We then briefly discuss the changes to this problem in the sticky plans environment.

### *Firm's decision problem with sticky prices*

Let  $v(p, \xi; s)$  be the real value of a firm with rescaled price  $p$  that has drawn a cost of changing price  $\xi$ , when the aggregate state is  $s$ . Let  $v^0(s)$  denote the real value of a firm that changes its price when the aggregate state is  $s$ , excluding costs of price adjustment. Then  $v(p, \xi; s)$  and  $v^0(s)$  must satisfy the following Bellman equations:

$$v(p, \xi; s) = \max \left\{ \begin{aligned} &\pi(p; s) + \beta \int_{S \times [0, \bar{\xi}]} d(s, ds') v(p/\mu', \xi'; s') G(d\xi') Q^A(s, ds'), \\ &v^0(s) - \xi w(s) \end{aligned} \right\}$$

$$v^0(s) = \max_{p^*} \left[ \pi(p^*; s) + \beta \int_{S \times [0, \bar{\xi}]} d(s, ds') v\left(\frac{p^*}{\mu'}, \xi'; s'\right) G(d\xi') Q^A(s, ds') \right] \quad (2.11)$$

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<sup>16</sup>In the case of pre-announced disinflations, the money growth process is not a Markovian process. We solve this case using a sequential representation of the equilibrium.

<sup>17</sup> $S$  is equal to  $M \times \Lambda$  where  $M$  is the set of possible realizations of  $\mu$  and  $\Lambda$  is the space of all probability measures over  $\mathfrak{R}^+$ .

where

$$\begin{aligned}\pi(p; s) &= \max_{n, k} py - w(s)n - r(s)k, \text{ subject to} \\ An^\alpha k^{1-\alpha} &= \left(\frac{p}{p^C(s)}\right)^{-\varepsilon} Y(s)\end{aligned}$$

Functions  $w(s)$  and  $r(s)$  denote rescaled factor prices,  $p^C(s)$  denotes the rescaled aggregate price of consumption, and  $d(s, s')$  denotes the discount factor with respect to future state  $s'$ , all as a function of the current aggregate state. Firms take these functions as given. The choice of  $n$  and  $k$  are functions of the rescaled price and the aggregate state  $s$ , but not of the  $\xi$  realization. So, we can define functions  $n(p; s)$  and  $k(p; s)$  that solve the static maximization in (2.11). In addition, the optimal price is independent of the initial  $p$  and the  $\xi$  realization, so we can define the function  $p^*(s)$ . This price is a fixed markup over expected real marginal costs, weighted by the current and future levels of demand and the probability of price adjustment in future prices.

It is simple to see that  $v(p, \xi; s)$  is decreasing in  $\xi$ . For firms with rescaled price  $p$ , there is a threshold function  $\hat{\xi}(p; s)$  such that only those that draw a cost  $\xi$  lower than  $\hat{\xi}(p; s)$  change their price. This threshold function is defined as follows:

$$v^0(s) - \hat{\xi}(p; s) w(s) = v(p, \hat{\xi}(p; s), s)$$

It is also useful to define the hazard function for price changes.  $h(p; s)$  is the fraction of firms with rescaled price  $p$  that adjust their price when the aggregate state is  $s$ . It is defined as:<sup>18</sup>

$$h(p; s) = G(\hat{\xi}(p; s))$$

Finally, we define the function  $n_M(p; s)$  to denote labor used in changing prices by  $p$  firms when the aggregate price is  $s$ :

$$n_M(p; s) = \int_0^{\hat{\xi}(p; s)} \xi dG(\xi)$$

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<sup>18</sup>Two standard hazard functions in the literature are: (1) Calvo Pricing,  $h(p; s) = \bar{h}$ , and (2) Ss Pricing,  $h(p; s) = \begin{cases} 0 & \text{if } p \in (\bar{s}, \bar{S}) \\ 1 & \text{otherwise} \end{cases}$

In equilibrium, the transition function  $Q^A$  is defined by the law of motion  $\mu' \sim P(\mu, d\mu')$  and  $\theta'(p) = \omega(p\mu')$ , where:

$$\omega(p) = \begin{cases} [1 - h(p; s)] \theta(p) , & \text{for all } p \neq p^*(s) \\ \theta(p^*) + \int_0^\infty h(z; s) \theta(dz) , & \text{for } p = p^*(s) \end{cases} .$$

*Firm's problem with sticky plans*

The state of a firm at the beginning of period  $t$  is  $x$ .  $x = \{x_j\}_{j=0}^\infty$  is a plan of nominal prices for each future period  $j$  normalized by the current money supply. We assume  $x_j$  is not state contingent, so it cannot be indexed to aggregate variables such as the money supply. We define  $x'$  as the continuation of plan  $x$  from next period onwards.  $x_0$  is the first element of  $x$ . The aggregate state of the economy is  $s = (\tilde{\theta}, \mu)$ , where  $\tilde{\theta}$  denotes the cumulative distribution of firms over plan  $x$ . The firm's problem is

$$v(x, \xi; s) = \max \left\{ \begin{array}{l} \pi(x_0; s) + \beta \int_{S \times [0, \bar{\xi}]} d(s, ds') v(x'/\mu', \xi'; s') G(d\xi') Q^A(s, ds') , \\ v^0(s) - \xi w(s) \end{array} \right\} \quad (2.12)$$

$$v^0(s) = \max_{x^*} \left[ \pi(x^*; s) + \beta \int_{S \times [0, \bar{\xi}]} d(s, ds') v(x_0^*/\mu', \xi'; s') G(d\xi') Q^A(s, ds') \right]$$

and

$$\begin{aligned} \pi(x_0; s) &= \max_{n, k} x_0 y - w(s) n - r(s) k , \text{ subject to} \\ y &= A n^\alpha k^{1-\alpha} \\ y &= \left( \frac{x_0}{p^C(s)} \right)^{-\varepsilon} Y(s) . \end{aligned}$$

The aggregate price level is defined as

$$p^C(s) = \left[ \int (x_0)^{1-\varepsilon} \omega(dx) \right]^{\frac{1}{1-\varepsilon}} ,$$

The transition function  $Q^A$  is defined by the law of motion  $\mu' \sim P(\mu, d\mu')$ , and  $\theta'(x') = \omega(x\mu') \forall x_0$ , where  $\omega(x)$  is the analogous of  $\omega(p)$  in the sticky prices environment.

### The experiments

We assume that the economy is initially in a non-stochastic steady state with  $M_t$  growing at a constant rate  $\mu_0$  at  $t = 0$ . At  $t = 1$ , there is an unanticipated change in the path of  $M_t$ . After this shock, agents know with perfect foresight the realizations of the future money path. As in Mankiw and Reiss (2001), we focus on the three following experiments:

Experiment 1: Temporary increase in the growth rate of  $M$

$$\mu_t = \left\{ \begin{array}{l} \bar{\mu} \text{ for } t = 1 \\ \mu_0 \text{ for } t > 1 \end{array} \right\}$$

Experiment 2: Permanent credible disinflation

$$\mu_t = \{\bar{\mu} \text{ for } t \geq 1\}$$

Experiment 3: Pre-announced credible disinflation

$$\mu_t = \left\{ \begin{array}{l} \mu_0 \text{ for } t < \bar{T} \\ \bar{\mu} \text{ for } t \geq \bar{T} \end{array} \right\}$$

### 3. Parameter values and steady state

We now discuss how values were assigned to the model's parameters. The length of the period is a quarter. The discount factor is chosen so that the real interest rate is 1.6 percent annually. We use the following momentary utility:<sup>19</sup>

$$U(C, N) = \log \left( C - \frac{\phi_0}{1 + \phi} N^{1+\phi} \right)$$

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<sup>19</sup>This utility function is the indirect utility function in an environment that also includes home production. In order to be consistent with balanced growth, the key condition is that the home production function shifts at the same constant rate as the intermediate good technology.

In order for the sticky price model to have the best chance of generating inertial response of inflation to a money growth shock, real marginal costs should be fairly insensitive to fluctuations in output. In our framework, this can be achieved by choosing a high labor supply elasticity and a relatively low share of capital in production. Concretely,  $\phi$  is chosen so as to imply an infinite Frisch labor supply elasticity as in Hansen (1985), and  $\alpha$  is set to 0.8. We will see that, even with our extreme specification, the sticky price model is unable to generate a hump-shaped response of inflation. We view our simple specification as a first step in the direction of a richer model that includes additional elements such as investment, variable capital utilization and sticky wages, which can dampen movements in the real marginal costs.

The parameter  $\phi_0$  is calibrated so that, conditional on the assigned values for the other parameters, agents work 25% of their time endowment. The capital stock is normalized to unity. The elasticity of substitution between intermediate goods is chosen so that the implied flexible price markup is 20%, as suggested by Hornstein (1993). In the benchmark model, the steady state quarterly growth rate of money is set equal to 1.5%, which is consistent with the average growth rate of money in post-war US.

Our experiments suggest that the dynamics in the response of output and inflation to monetary shocks are sensible to the shape of  $G(\cdot)$ . In this paper we use the beta distribution, which is very flexible as a function of only two parameters.<sup>20</sup> In order to make our analysis comparable to Dotsey, King and Wolman's work, our benchmark model uses  $\gamma_1 = 0.3$  and  $\gamma_2 = 0.1$ , so that the implied fixed-costs distribution is similar to the one they work with. This specification is consistent with microeconomic data on price adjustment that suggest that hazard rates are an increasing function of the absolute difference between  $p$  and  $p^*$  estimated by Engel and Caballero (1993) and Willis (2000).<sup>21</sup>  $\bar{\xi}$  is chosen so that in the steady

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<sup>20</sup>I want to thank Alexander Wolman for suggesting me to use this family of cost distributions.

<sup>21</sup>Caballero and Engel (1999) estimate positively sloped hazard curves in the context of firm investment dynamics.

state of the sticky price economy, prices are on average 2 quarters old (the average hazard rate is 0.23). This implies that expected labor costs of price adjustment for an individual firm are 4.5% of its average steady state employment level. Weighed by the fraction of firms changing their price, total labor used in changing pricing schemes is only 0.35% of total employment. This corresponds to 0.25% of average revenues, or 0.3% of average costs spent in labor for price adjustment. This is smaller than Levy et al (1997) and Dutta et al (1999)'s estimates of physical costs of price adjustment, which average 0.70% of store revenues in their supermarket and drugstore samples. In the steady state of the sticky plans model, the fraction of firms changing their pricing scheme is zero. Our parametrization of  $G(\cdot)$  in this environment is such that the response of the average hazard rate to a 1% temporary increase in the growth rate of money is the same as in the sticky price environment. The baseline parameter values are summarized in table 1. In section we study how the model's implications regarding the fraction of firms changing their pricing decisions depend on the values of certain parameters. In order to solve the model, we use an iterative non-linear method which is summarized in appendix B.<sup>22</sup>

| Table 1: Parameter values<br>in the Benchmark Economy |       |
|-------------------------------------------------------|-------|
| $\beta$                                               | 0.984 |
| $N^{SS}$                                              | 1/4   |
| $\phi$                                                | 0.01  |
| $\phi_0$                                              | 0.89  |
| $\varepsilon$                                         | 6     |
| $\alpha$                                              | 0.8   |
| $\mu_0$                                               | 0.015 |
| $\xi$                                                 | 0.015 |

### Steady state

In the sticky plans model, inflation does not affect steady state allocations. This is because all firms run the same plan, with prices increasing at the steady

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<sup>22</sup>This algorithm is developed with more detail in Burstein and Werning (2001).



state money growth rate. No labor is used in adjusting plans, and the allocations are the same as in a flexible price economy. Before getting into the model's dynamics, we briefly discuss some of the properties of the steady state in the sticky price model. Figure 1, Panel A, displays the hazard function  $h(p; s^{SS})$  and distribution of prices  $\theta(p)$  in steady state. The hazard rate function is V shaped around  $p^*$ . This is due to the fact that firms dislike real prices that are too low or too high relative to  $p^*$ , and are thus more willing to pay higher costs when the gap between  $p$  and  $p^*$  is higher. Very low or very high  $p$ 's are associated with hazard rates equal to 1. Recall that the Calvo model assumes that the hazard rate function is horizontal at the exogenous hazard rate.

Given our parametric assumptions, there are 7 vintages of firms in the steady state: every firm changes its price no more than 7 periods after the previous price change. In the figure, firms that have changed their price 7 periods ago are in the section of the hazard function where the probability of adjustment is equal to 1. The resulting average hazard rate is 0.23, and the average price is 2 quarters old. Note that  $\theta(p)$  is decreasing for lower  $p$ 's because the hazard function is positive for  $p < p^*$ .<sup>23</sup> If there was steady state deflation, then  $\theta(p)$  would lie to the right of  $p^*$ . The effects of an increase in the money supply, as described below, are: (1) a leftward shift in  $\theta(p)$  because  $p = P/M$  falls for any given  $P$ , and (2) a change in  $p^*$ .

In order to understand some of the basic forces in the model, we carry out two comparative static exercises. Panels B and C in Figure 1 display the steady state hazard function and distribution of prices for two alterations of the benchmark parameters: (B) a higher elasticity of substitution between intermediate inputs, such that the flexible price economy markup equals 3%, and (C) a higher quarterly steady state inflation rate, equal to 2.5%. With lower average markups, firms are less willing to let their real price erode because profits can easily become negative.

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<sup>23</sup>The S-s literature has mostly concentrated in the special case of a uniform  $\theta(p)$  in steady state. Assuming a degenerate cost distribution  $G(\cdot)$  with a mass equal to one for  $\xi = \bar{\xi}$  generates a uniform  $\theta(p)$ .

Therefore, the average hazard rate is 0.28. In addition, the hazard function is asymmetric around  $p^*$ . To understand this, suppose an extreme case in which the elasticity of substitution between intermediate goods is infinite and steady state inflation is 0. Then,  $p^* = p^C = mc$ , and profits are 0. A firm with  $p < p^*$  would have profits equal  $-\infty$ , so it would change its price with probability 1. Now take a firm with  $p > p^*$ . Its profits equal 0 if it doesn't change the price. Profits at  $p = p^*$  are also 0, excluding the costs of price adjustment. So, firms with  $p > p^*$  do not change their price unless  $\xi = 0$ .<sup>24</sup> Let's now consider case (C). With higher steady state inflation, the hazard function becomes flatter. Firms choose a higher  $p^*$  but also let their price erode by more before changing with probability one, as proved by Sheshinski and Weiss (1977) in a simpler model. The fact that the relative price erodes faster with a higher inflation implies that, even though the range of prices is wider, the number of vintages falls from 7 to 5. As a result, the average hazard rate increases to 0.29.

#### 4. Three policy experiments

In this section we study the response of the model in the context of the three monetary policy experiments described above. For each of them, we discuss the difference between the sticky price and the sticky plan model, and we assess their potential to account qualitatively for the conventional view about the outcome of these experiments in actual economies.

##### **Experiment 1: Temporary increase in the money growth rate**

###### *Sticky prices*

Figure 2 displays the response of inflation, output, and the average hazard rate to a 1% increase in the money supply above the steady state money growth path (i.e:  $\mu_0 = 1.5\%$  and  $\bar{\mu} = 2.5\%$ ). On impact (i.e:  $t = 1$ ), inflation increases by

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<sup>24</sup>The presence of fixed costs would shift down the profit function without affecting its asymmetry.

0.22% and output increases by 0.78%. Panel D shows that the fraction of firms adjusting their price increases from 23% to almost 25%. It is useful to decompose changes in the aggregate price level into an intensive margin inflation rate (broken line in panel B) and an overall inflation rate (solid line in panel B). The intensive margin inflation rate fixes the vintage hazard rates at their steady state level and computes the aggregate price level using  $p^*$  from the new equilibrium path.<sup>25</sup> Out of the 0.22% increase in the inflation rate, 0.15% is generated by the intensive margin and the remaining 0.07% is due to the inclusion of the extensive margin.

Panels E and F display the response of output and inflation to a 1% shock in a sticky price economy with time dependent pricing á la Calvo. Specifically, the hazard rate is exogenous and set equal to the steady state average hazard rate in the state dependent sticky price economy (i.e.  $h(p) = 0.23 \forall p$ ). Inflation on impact increases by 0.12% versus 0.22% in the state dependent model, implying that the impact effect on output is only 0.10% lower than in the state dependent model.

The time dependent model with Calvo pricing does not generate a hump-shaped response of inflation to a monetary expansion (panel F). Firms that adjust, front-load their price in anticipation of future increases in marginal costs. We discuss this with more detail when we compare the sticky price and the sticky plan environments. The state dependent model produces a short-lived hump-shaped response of inflation: the maximum inflation rate is at  $t = 2$  (panel B). This is because the hazard rate is higher for firms that have not changed their price for a longer period. So, given the accumulated increase in the nominal marginal cost at  $t = 2$ , firms adjusting at this point require a larger increase in their price than firms adjusting at  $t = 1$ . In fact, a modified time dependent model in which the hazard rate is increasing in the number of periods without adjustment, also displays this small hump-shaped response of inflation. Endogenizing the extensive margin reduces the hump because the largest change in the fraction of adjusting

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<sup>25</sup>Note that this measure of inflation differs from the one obtained in a time dependent model with Calvo pricing because hazard rates are different for each vintage.

firms occurs at  $t = 1$ .

Figure 3 displays the impact effect on inflation, output and the average hazard rate (y axis) as a function of the size of the temporary change in the growth rate of money (x axis). Solid and dotted lines trace the behavior of these variables under state dependent and time dependent pricing, respectively. Figure 4 displays the same information as figure 3, under the assumption of 0 steady state inflation. For small changes in the growth rate of money, the response of output is similar across both models. This is not the case for large changes in the growth rate of money. In the time dependent model, the relative response of output to changes in the growth rate of money is roughly constant (i.e. the impact curves are linear). In the state dependent model, as the size of the change gets larger, the fraction of firms adjusting prices increases and so does the implied inflation rate. Consequently, the increase in output relative to the size of the change in the money growth rate becomes smaller. With positive steady state inflation, the response of output is negative for changes larger than 5%.<sup>26</sup> Adjusting firms start with very low prices, so inflation can be larger than the increase in the money stock. Real money balances fall, and so does output. This does not take place when steady state inflation rate is 0%. Figure 4 illustrates that once the change in the money growth rate is larger than 8%, every firm adjusts, and prices increase by the magnitude of the monetary expansion. Real allocations are unaffected, except for labor employed in price adjustment, which is very small as a fraction of total employment.<sup>27</sup> This illustrates one of the non-linearities of this model: as the size of the change in the growth rate of money increases, the relative response is higher for inflation and lower for output.<sup>28</sup>

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<sup>26</sup>The average quarterly change in M1 in post-war US is 1.2% and the standard deviation is 1.7%.

<sup>27</sup>The results are almost unaffected if  $N^M$  is removed from the resource constraints.

<sup>28</sup>Larger shocks also reduce the persistence of output to the shock. This is consistent with Kiley (2000) and Fischer, Sahay and Vegh (2001), who present evidence that the autocorrelation of output is lower in high inflation countries. In order to make this link in our model, note that the mean and standard deviation of inflation are highly correlated in the data.

Another source of non-linearities is the asymmetric response of the economy to monetary expansions and contractions. The rise in output after a monetary expansion is smaller than the fall in output after a monetary contraction. This can be seen in figure 3. For example, a decline of 3% in the growth rate of money reduces output by 2.2% . An increase of 3% in the growth rate of money increases output by only 1.17%. There are two sources of asymmetries. First, the hazard function is asymmetric around  $p^*$ . This is because, as discussed above, the profit function of a firm is asymmetric. This effect is more important when markups are lower, as we saw in figure 1, panel B. Second, under positive steady state inflation, firms' prices are lower than or equal to  $p^*$ . The distribution  $\theta(p)$  is in the section of the hazard function that is decreasing in  $p$ . A monetary expansion implies a leftward shift in  $\theta(p)$ , so the average hazard rate increases. A monetary contraction implies a rightward shift in  $\theta(p)$ , so the average hazard rate declines if the money contraction is small. Many firms do not pay the fixed cost to reduce their price because the positive inflation rate erodes it costlessly.<sup>29</sup>

### *Sticky plans*

The broken lines in figure 5 display the dynamics of various variables in the sticky plans environment to a 1% increase in the money supply relative to the initial steady state.<sup>30</sup> The hump in inflation is much more pronounced compared to the sticky price model, with inflation now reaching a peak 12 periods after the shock.<sup>31</sup> The key difference is that in the sticky plan model, incentives to front-load prices are reduced. Firms expect the marginal cost to increase over time, and they can plan future price increases in advance without actually having to

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<sup>29</sup>In contrast to Ball and Mankiw (1994), our model also generate asymmetric response to monetary shocks under zero steady state inflation. A related asymmetry is studied by Kahn and Thomas (2001) in the context of a model with lumpy investment.

<sup>30</sup>The reason we now assume 0% steady state inflation is to facilitate the comparison with the sticky price model. Note that the allocations in the sticky plan model are neutral to  $\mu_0$ .

<sup>31</sup>The persistence of output (defined as the half-life in the response to a shock) is roughly equivalent in the sticky price and sticky plans models.

set today these higher prices. In particular, each period the price is set equal to a constant markup over marginal cost.

This mechanism can be better understood by comparing the response of the sticky price (solid lines) and the sticky plan (broken lines) economies. In order to facilitate the comparison of both models, we assume zero steady state inflation so that both economies start from the same initial steady state. The initial increase in nominal marginal cost is roughly equivalent in both economies. In the sticky plan case, firms paying the fixed cost change their current price one to one with changes in the nominal marginal cost. On the contrary, the increase of  $P^*$  under sticky prices is larger than the increase in nominal marginal cost (i.e:  $P^*$  increases by 0.6%, which is double the increase in marginal cost). The hump-shaped response of inflation in the sticky plans model is driven by the fact that firms changing their plan in later periods increase their price by the accumulated increase in the marginal costs, which is larger than the initial rise in the marginal cost. Clearly, this effect is partly offset by the fact that less firms adjust their plan in later periods. The two key necessary conditions that produce the hump shape in inflation are: (1) the initial increase in the nominal marginal cost is not too large: this depends on real marginal costs being fairly insensitive to output, and (2) the fraction of firms changing their plans in early periods is not too large. Figure 6 displays the response of the two environments under the assumption that the growth rate of money follows an AR1 process, where  $\rho = 0.5$  [see Christiano, Eichenbaum and Evans (2001)] Under this assumption, the sticky plans model generates a hump-shaped response in both inflation and output.

As discussed for the sticky price economy, the response of output is non-linear in the magnitude of the change in the money supply. This can be seen in figure 7. Monetary expansions smaller than 6.4% (quarterly) increase output. Monetary expansions larger than 6.4% do not affect output, since every firm adjusts its plan. In addition, for expansions smaller than 6.4%, the state dependent model is very well approximated by a time dependent model.<sup>32</sup> Finally, as in the sticky

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<sup>32</sup>In the time dependent model, we exogenously set  $h(x) = 0.23$  for all  $x$ .

price model, monetary expansions and contractions have asymmetric effects on output and inflation. For example, money contractions have to be larger than 7.5% (versus 6.4% for money expansions) in order for all firms to adjust their plans. In section 5 we study how the size of the change in the money growth rate such that all firms adjust their plan depends on some of the model's parameters.

## **Experiment 2: Permanent credible disinflation**

### *Sticky prices*

The solid lines in figure 8 display the pattern of output, inflation, average hazard rate, marginal cost and  $p^*$  when the quarterly money growth is unexpectedly permanently reduced from 1.5% to 0%. The rate of inflation falls abruptly at  $t = 1$ , and output increases permanently by 0.7%. This result is at odds with the conventional wisdom that small disinflations are contractionary. This result is not surprising because the state dependent sticky price model behaves similarly to the Calvo pricing model for small changes in the growth rate of money. The inability of the latter models to generate inflation inertia has been extensively discussed in previous work [see for example Ball (1994)]. It is explained by the fact that the price level is sticky, but the rate of inflation is not. Firms changing their price increase it by a lower rate relative to the initial steady state, because they anticipate that their price could be too high in the future if they draw high costs of adjustment. In fact, under our parametrization  $P^*$  falls by 4%, and this generates a reduction of 1.5% in the overall inflation rate.<sup>33</sup> The model displays a similar pattern for large disinflations, as can be seen in figure 9.<sup>34</sup>

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<sup>33</sup>Output increases by 0.8% across the two steady states. This is the result of: (1) a decrease in the average markup from 20.2% to 20%, (2) a reduction in the dispersion of prices, which implies that the ratio between actual and potential output increases by 0.23%, and (3) labor used in price adjustment as a fraction of total employment falls from 0.4% to 0%.

<sup>34</sup>King and Wolman (1996) study a time dependent sticky price model with money in the utility function that can generate a contraction of output after a credible disinflation. However, their model does not generate inflation inertia in response to this shock.

### *Sticky plans*

Turning over to the sticky plan economy, the broken lines in figure 8 show that this model is capable of generating a contraction in output after a permanent reduction in quarterly money growth from 1.5% to 0%. This is because only 17% of the firms pay the fixed cost of plan adjustment to reduce the growth rate of their individual prices. The remaining mass of firms that do not change their plan keep on increasing their prices at the old steady state's high inflation rate. In addition, front-loading incentives are reduced. This is because firms that do adjust can plan on gradually reducing the growth rate of future prices rather than currently reducing abruptly the growth rate of their selected price.<sup>35</sup> All this implies that inflation is above zero during 6 quarters after the policy implementation. The reduction in real money balances entails output losses that last 7 quarters.

Things are different when the initial level of money growth is higher. The broken lines in figure 9 show the response of the sticky plans economy when the quarterly money growth rate is permanently reduced from 10% to 0%. In this case, at  $t = 1$ , every firm pays the fixed cost and changes its pricing plan to the lower rate of money growth. Therefore, output is almost unaffected by this policy. This is consistent with conventional wisdom that suggests that large inflations can be stopped at low output costs.<sup>36</sup>

### **Experiment 3: Pre-announced credible disinflation**

We consider disinflations announced 4 quarters in advance ( $\bar{T} = 4$ ).

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<sup>35</sup>We can neutralize the first effect (i.e.: firms that do not change their plan increase their individual prices by the pre-disinflation inflation rate) by starting in a 0% inflation steady state. In this case, it is still true that the sticky plans model is able to generate inflation inertia, but the sticky price model is not. This reinforces the view that front-loading is the key mechanism that generate inflation inertia.

<sup>36</sup>Note that the effects are driven by the large money growth rate reduction rather than the initial money growth rate.



### *Sticky prices*

The solid lines in figure 10 show that under sticky prices, the rate of inflation falls and output booms before the disinflation takes place. Consider a firm that has drawn a low  $\xi$  realization in a period before  $\bar{T}$ . This firm understands that it may not change its price again for a long time under high  $\xi$  realizations. Choosing a high  $p^*$  can be very costly because its relative price would be too high if the firm has not readjusted again when the disinflation takes place. So, firms paying the fixed cost choose a rescaled price that is lower than  $p^*$ , and the rate of inflation drops immediately. But the money supply growth has not changed yet, so real balances rise, and output booms. This implication of sticky price models has been criticized in the literature as counterfactual.<sup>37</sup>

### *Sticky Plans*

The broken lines in figure 10 show that, in the sticky plans economy, output and inflation are unaffected before the disinflation actually takes place. This happens even when a fraction of firms pay the fixed cost to alter their plan before the reduction in money growth. The key difference with the sticky price model is that adjusting firms know that they will be able to change their price in the future without paying any additional cost. Therefore, there is no need for them to reduce their relative price in advance to the period when the disinflation takes place. At  $t = \bar{T}$ , not all firms have changed their plan. So, the rate of inflation is above 0 and output is below the steady state level for 5 periods after the actual disinflation. If the initial steady state inflation is higher, every firm will have adjusted their plan at  $t = \bar{T}$ . Therefore, a contraction in output is avoided.

## **5. Full adjustment in sticky plan model**

In this section we ask the following question in the context of the sticky plans model: how large do monetary expansions and contractions have to be (in the

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<sup>37</sup>See for example Ball (1994).

context of experiment 1) in order for all firms to adjust their plan? A related question is: when do time dependent models cease to become good approximations of state dependent models? The size of the temporary change in the growth rate of money such that all firms adjust their plan is one metric, among others, with which we can evaluate this question. Below we report results for this question under different assumptions on the following parameters of the model: (1)  $\bar{\xi}$ , the maximum cost of changing a plan (both under the assumptions  $\gamma_1 = 0.3, \gamma_2 = 0.1$  as in the benchmark calibration, and  $\gamma_1 = \gamma_2 = 1$  corresponding to a uniform distribution), (2)  $\varepsilon$ , which changes average markups, (3)  $\alpha$ , the share of labor in production, and (5)  $\phi$ , which affects the labor supply elasticity (for  $\alpha = 0.66$ ).

| <i>Changing maximum cost of changing plan</i> |             |           |
|-----------------------------------------------|-------------|-----------|
| $\xi/N_{SS}$                                  | Contraction | Expansion |
| 0.5%                                          | -2.0%       | 1.9%      |
| 1%                                            | -2.9%       | 2.6%      |
| 3%                                            | -5.5%       | 4.5%      |
| 6% (baseline)                                 | -8.6%       | 6.4%      |
| 12%                                           | -14.9%      | 9.4%      |
| 18%                                           | -22.2%      | 11.8%     |

| <i>Changing maximum cost of changing plan<br/>uniform distribution (<math>\gamma_1 = \gamma_2 = 1</math>)</i> |             |           |
|---------------------------------------------------------------------------------------------------------------|-------------|-----------|
| $\xi/N_{SS}$                                                                                                  | Contraction | Expansion |
| 0.5%                                                                                                          | -2.8%       | 2.5%      |
| 1%                                                                                                            | -4.0%       | 3.4%      |
| 3%                                                                                                            | -7.5%       | 5.8%      |
| 6% (baseline)                                                                                                 | -11.5%      | 8.0%      |
| 12%                                                                                                           | -18.9%      | 10.9%     |

| <i>Changing markup</i> |             |           |
|------------------------|-------------|-----------|
| Markup                 | Contraction | Expansion |
| 20% (baseline)         | -8.6%       | 6.4%      |
| 5%                     | -6.2%       | 3.3%      |
| 50%                    | -10.9%      | 9.1%      |

| <i>Changing share of labor</i> |             |           |
|--------------------------------|-------------|-----------|
| $\alpha$                       | Contraction | Expansion |
| 0.8 (baseline)                 | -8.6%       | 6.4%      |
| 0.6                            | -6.8        | 5.4%      |
| 0.4                            | -5.2        | 4.4%      |

| <i>Changing share of labor<br/>and labor supp. elasticity</i> |             |           |
|---------------------------------------------------------------|-------------|-----------|
| $\alpha = 0.66$                                               | Contraction | Expansion |
| $\varepsilon^L = 3$                                           | n.a.        | 5.1       |
| $\varepsilon^L = 5$                                           | n.a.        | 5.32      |
| $\varepsilon^L = 10$                                          | -7.95       | 5.33      |
| $\varepsilon^L = \infty$                                      | -7.3        | 5.7       |

We also extended the model to include an input-output structure in the intermediate goods sector. Specifically, we assumed as in Basu (1995) that production requires the use of output of other intermediate goods firms,

$$Y_{it} = A (N_{it})^{1-\alpha} (I_{i,t})^\alpha, \quad (5.1)$$

where  $I_{i,t}$  denotes the composite of intermediate inputs demanded by firm  $i$ . We assume that the composite intermediate input aggregates intermediate goods with the same technology as the final consumption good (2.5). So, the price of the composite intermediate input is  $P_t^C$ , and prices of other firms directly affect marginal costs. Basu showed that increasing the share of intermediate goods ( $\alpha$ ) in production endogenously increases price stickiness by reducing incentives to pay the fixed cost of price adjustment after a monetary expansion. He reached this conclusion assuming that other firms do not change their price. In our model we find, as Basu did, that increasing the share of intermediate goods increases stickiness for small monetary expansions. However, as the table below shows, neutrality kicks in for smaller expansions relative to a model without intermediate inputs. As the size of the increase in money supply gets larger and more firms change their plan, incentives to adjust its plan increases for the marginal firm. Therefore, real rigidities become real flexibilities for large monetary expansions.

| <i>Changing share of intermediate inputs, <math>\alpha</math></i> |           |
|-------------------------------------------------------------------|-----------|
|                                                                   | Expansion |
| 0%                                                                | 9%        |
| 20%                                                               | 8%        |
| 40%                                                               | 7%        |
| 60%                                                               | 6%        |
| 80%                                                               | 4.5%      |

## 6. Conclusions and future extensions

This paper studies a dynamic general equilibrium model in which firms face a fixed cost of changing their pricing plan. We argue that the model's predictions are consistent with the conventional wisdom about three monetary experiments that have been widely studied, as well as the notion that changes in the growth rate of money that are large enough have relatively smaller effects on output.

In the context of our monetary experiments, we saw that time dependent models are a good approximation of state dependent models in environments where the size of the change in the growth rate of money is not very large. While changes in money growth rate are small in US in the post-war period, monetary shocks tend to be larger in the context of developing countries. For example, in these countries exchange rates are very volatile and monthly depreciation rates can be in the order of 50%.<sup>38</sup> Time dependent sticky price models, often used in international economics, appear ill suited to study shocks of this magnitude. An open economy version of our model has the potential of explaining the response of various prices to both small and large devaluations. In addition, the model may be able to rationalize other empirical regularities that characterize open economies. For example, Alvarez, Atkeson and Kehoe (2000) find that high inflation countries also have a higher standard deviation in their nominal exchange rate relative to the standard deviation of the real exchange rate. Given that average inflation is highly correlated to the volatility of the nominal exchange rate, the open economy

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<sup>38</sup>See Burstein, Eichenbaum and Rebelo (2001).

application of our model would be consistent with the feature of the data. High inflation countries also have larger fluctuations in their nominal exchange rate, so a higher fraction of firms would adjust their price every period.

It would also be also interesting to use non-linear solution methods to study the stochastic equilibrium for this economy. The effects of a monetary shock of a given size will differ according to the history of shocks. Hence, non-linearities might play a larger role in explaining US inflation and output dynamics.

Our model also provides a number of interesting cross-sectional price implications that can be empirically tested.<sup>39</sup> One such implication is that in response to large and persistent change in the growth rate of money, there is a high fraction of firms that make relatively small adjustments to their pricing plans. In contrast, standard time dependent models imply that there is a small fraction of firms making large price adjustments.

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<sup>39</sup>Eden (2001) is a recent paper that uses microeconomic data to distinguish between different competing models.

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## 7. Appendices

### Appendix A: Recursive formulation of consumer's problem

The consumer takes the pricing functions as given and solves the following recursive problem:

$$W(s) = \max_{C, N} \left( U(C, N) + \beta \int_S W(s') Q^A(s, ds') \right) \quad (7.1)$$

subject to

$$p^C(s) (1 - \tau^C(s)) C + 1 - \frac{1}{\mu} = w(s) N + r(s) K + \pi(s) + \tau(s)$$

$$p^C(s) C \leq 1.$$

This problem defines functions  $C(s)$  and  $N(s)$ .

### Appendix B: Recursive competitive equilibrium

A recursive competitive equilibrium is:

- A set of policies for the firms,

$$\{n(p; s), k(p; s), p^*(s), h(p; s), n_M(p; s)\}$$

- A set of policies for the household,

$$\{C(s), N(s)\}$$

- A policy for the government,

$$\{\tau(s), \tau^C(s)\}$$

- Pricing processes,

$$\{w(s), r(s), p^C(s), i(s)\}$$

- Aggregate profits function and discount factor,

$$\{\pi(s), d(s, s')\}$$

- Aggregate quantity functions,

$$\{Y(s), N_M(s), N_P(s)\}$$

- An intraperiod operator,

$$\Psi(s)$$

- A transition function for the aggregate state,

$$Q^A(s, \cdot)$$

such that:

### 1. Optimality:

1. Firms: Given price functions  $\{w(s), r(s), p^C(s)\}$ , discount factors  $d(s, s')$ , and the transition function  $Q^A(s, \cdot)$ , firms' decision rules solve the maximization problem described in (??). Moreover,  $p^C(s)$  is given by

$$p^C(s) = \left[ \int p^{1-\varepsilon} \omega(dp) \right]^{\frac{1}{1-\varepsilon}},$$

$\pi(s)$  is given by

$$\pi(s) = \int \pi(p; s) \omega(dp) - N_M(s) w(s),$$

where

$$\omega = \Psi(s),$$

and  $r(s)$  is given by

$$r(s) = \frac{1-\alpha}{\alpha} w(s) \frac{N(s)}{K(s)}.$$

2. Household: Given price functions  $\{w(s), r(s), p^C(s)\}$ , profit function  $\pi(s)$ , government policy functions  $\tau(s)$  and  $\tau^C(s)$ , and the transition function  $Q^A(s, \cdot)$ , household decision rules solve the maximization problem described in (7.1). Moreover,  $w(s)$  is given by

$$w(s) = \frac{U_N(C(s), N(s))}{U_C(C(s), N(s))} p^C(s),$$

$d(s, s')$  is given by

$$d(s, s') = \frac{\beta U_C(C(s'), N(s')) / p^C(s')}{U_C(C(s), N(s)) / p^C(s)},$$

and  $i(s)$  is given by

$$i(s) = \int_S d(s, s') Q^A(s, ds').$$

2. Government policy: Given aggregate consumption function  $C(s)$ , the price of aggregate consumption function  $p^C(s)$  and the nominal interest rate function  $i(s)$ ,  $\tau(s)$  and  $\tau^C(s)$  satisfy:

$$1 - \frac{1}{\mu} = \tau(s) + \tau^C(s) C(s) p^C(s)$$

and

$$\tau^C(s) = \frac{i(s)}{1 + i(s)}$$

3. Aggregation:

$$N_M(s) = \int_0^\infty n_M(p; s) \theta(dp)$$

$$N_P(s) = \int_0^\infty n(p; s) \omega(dp)$$

$$K = \int_0^\infty k(p; s) \omega(dp)$$

where

$$\omega = \Psi(s)$$

4. Feasibility:

$$C(s) = Y(s)$$

$$N_P(s) + N_M(s) = N(s)$$

5. Consistency of individual and aggregate behavior: The intraperiod operator  $\Psi$  is such that  $\omega = \Psi(s)$  where

$$\omega(p) = \begin{cases} [1 - h(p; s)] \theta(p) , & \text{for all } p \neq p^*(s) \\ \theta(p^*) + \int_0^\infty h(z; s) \theta(dz) , & \text{for } p = p^*(s) \end{cases} ,$$

and the transition function  $Q^A$  is defined by the law of motion  $\mu' \sim P(\mu, d\mu')$  and  $\theta'(p) = \omega(p\mu')$ .

We briefly discuss the main changes to the previously described equilibrium conditions under the sticky plans environment. The aggregate price level is defined as

$$p^C(s) = \left[ \int (x_0)^{1-\varepsilon} \omega(dx) \right]^{\frac{1}{1-\varepsilon}} ,$$

The transition function  $Q^A$  is defined by the law of motion  $\mu' \sim P(\mu, d\mu')$ , and  $\theta'(x) = \omega(x'\mu')$ . The remaining equilibrium conditions are very similar to those in the sticky price environment.

### Appendix C: Numerical Algorithm

Given the nature of our policy experiments, it seems natural to think of the numerical solution method in terms of sequences of objects rather than time independent functions. We now proceed to lay the outline of the algorithm we use to solve the model. Details are in an appendix available upon request.<sup>40</sup>

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<sup>40</sup>In order to check the solution method, we also solved the model using a linearization algorithm similar to the one used by Dotsey, King, and Wolman (1999). For small shocks, the outcome of both techniques are almost equivalent. However, once the size of the shock increases, the linearization algorithm runs into the problem of occasionally binding constraints. Specifically, firms from older vintages want to change their price with probability one, so that the equality in the linearized threshold equation breaks up and cannot be used to compute the dynamic response of the economy.

1. Solve for the initial steady state and the new steady state of the economy after the shock (which can be different from the initial one in the case of permanent shocks). This involves solving a system of non-linear equations.
2. Compute  $v(p, \xi; s^{SS})$  and  $h(p, s^{SS})$ , where  $s^{SS}$  denotes the aggregate state in the steady state.
3. Guess that the economy is in the new steady state at time  $T$ .
4. Guess an initial sequence for  $w_t$ ,  $mc_t$  and  $Y_t$ :

$$\{w_t, mc_t, Y_t\}_{t=0}^T$$

The initial steady state is usually a good initial guess.

5. Using the aggregate variables guessed in (4), compute optimal price rules, hazard functions and factor demand functions between  $t = 1$  and  $t = T - 1$ . This requires solving for value functions in problem (??), which can be well approximated using splines. Given that we know the value functions at  $t = T$ , we start from  $t = T - 1$  and we move backwards up to  $t = 1$ .
6. Using these optimal firm responses, compute aggregate quantities and equilibrium prices from the consumer's maximization problem and market clearing conditions.
7. If the old and new aggregates are not the same, we re-start (4) with updated aggregate quantities and prices which are computed as a function of the old and new ones (usually with some weighting scheme).
8. Check that the economy has converged to the new steady state before period  $T$ . If not, choose a new  $T$  and re-start from (2).

Figure 1: Hazard functions  $h(p)$  and  $\theta(p)$

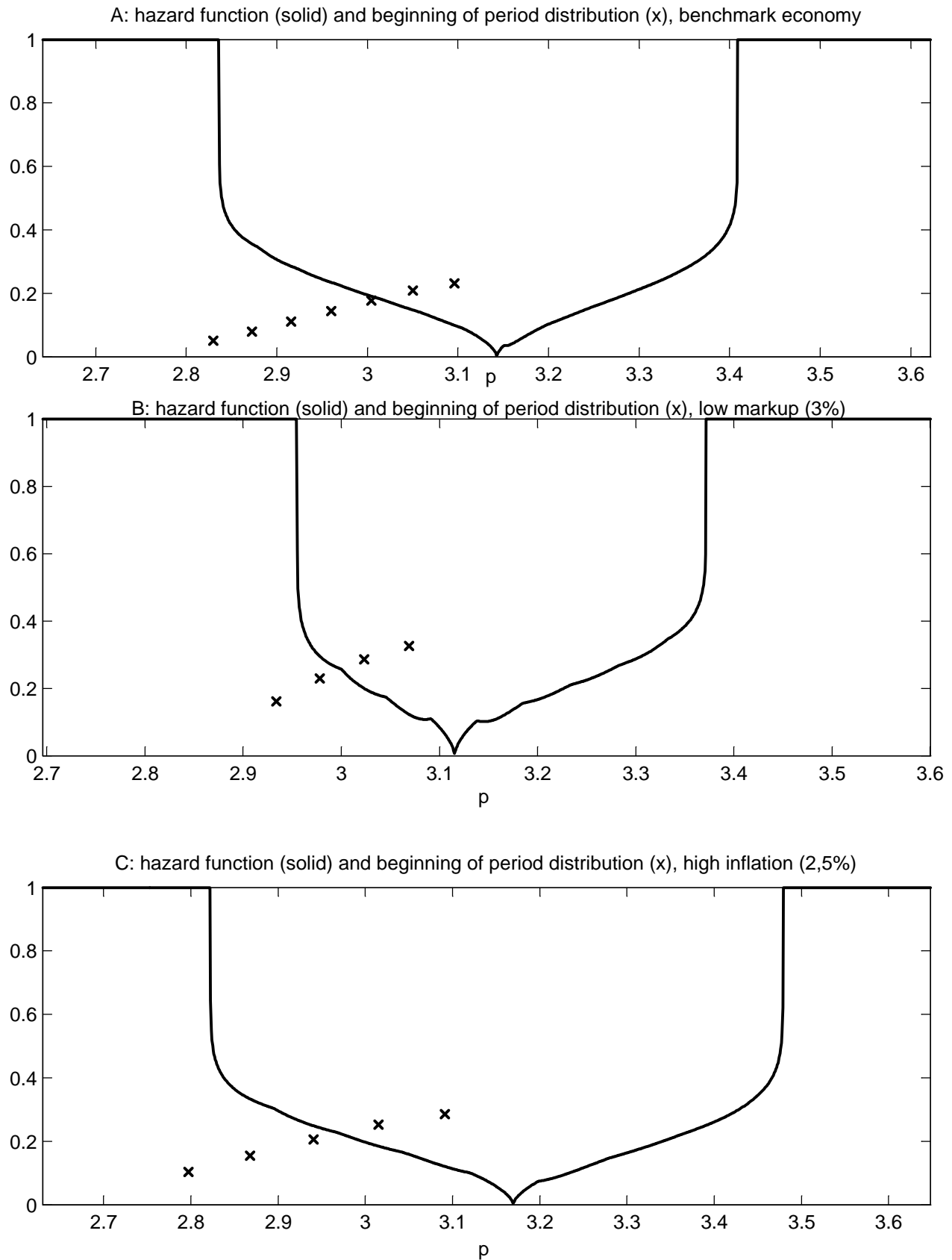




Figure 2, Experiment 1: Sticky Prices, 1% M increase

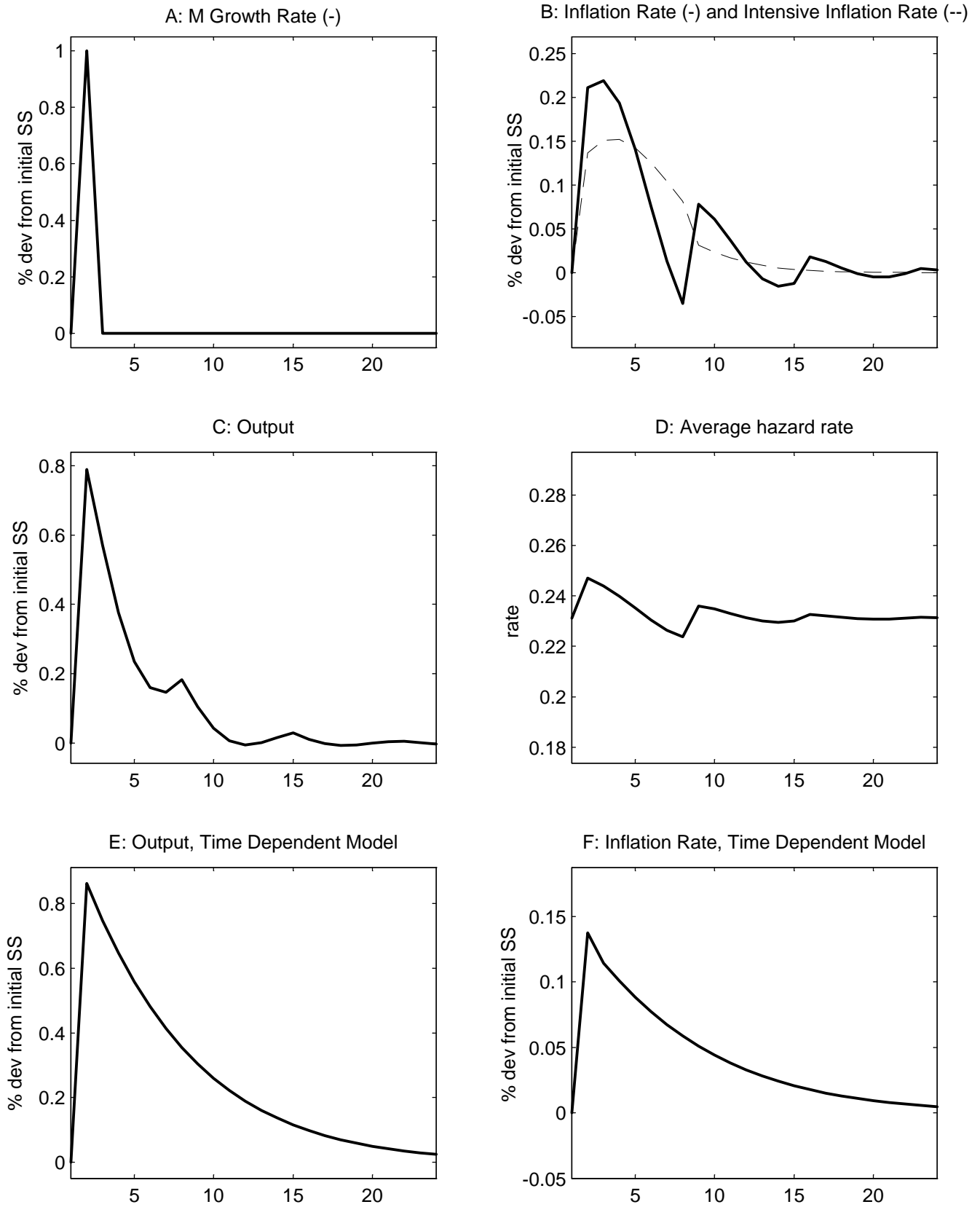


Figure 3, Exp. 1, Sticky Prices: state dep. (-) and time dep. (--), impact at t=1 for different size of shocks

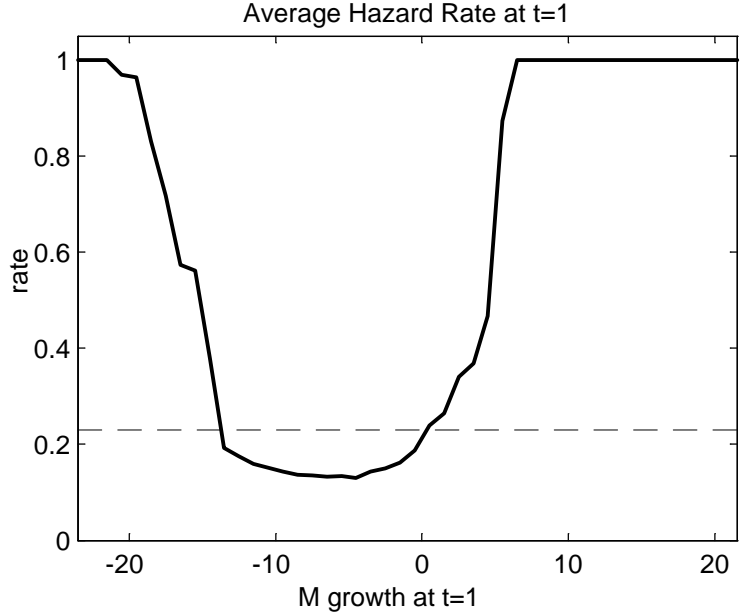
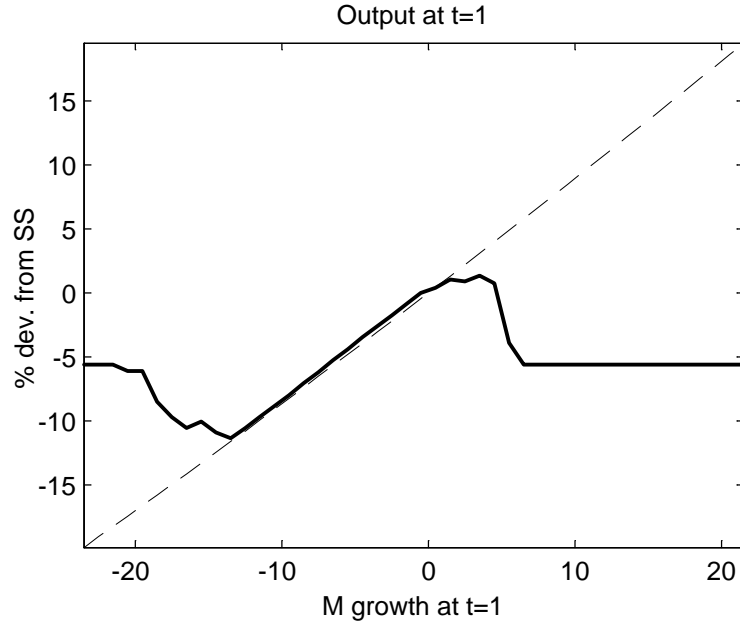
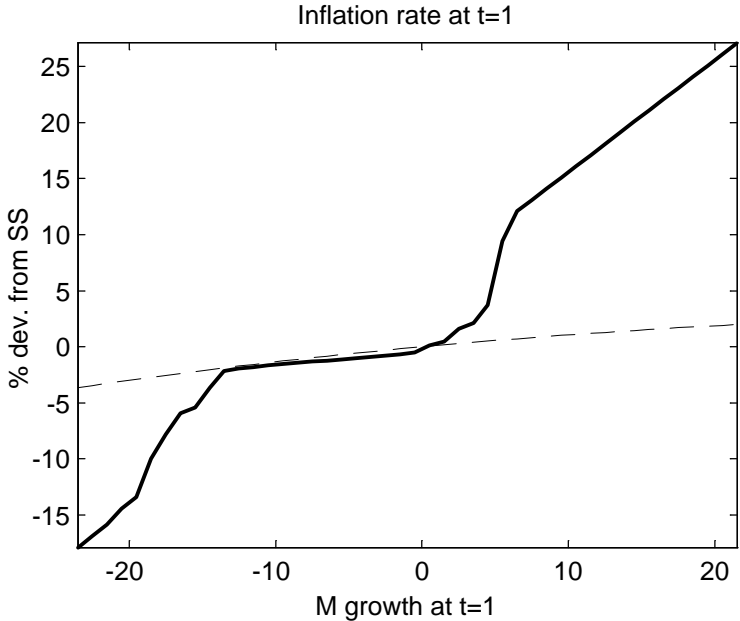


Figure 4, Exp. 1, Sticky Prices: state dep. (-) and time dep. (--), impact at t=1, diff. size of shocks,  $\mu_0=0\%$

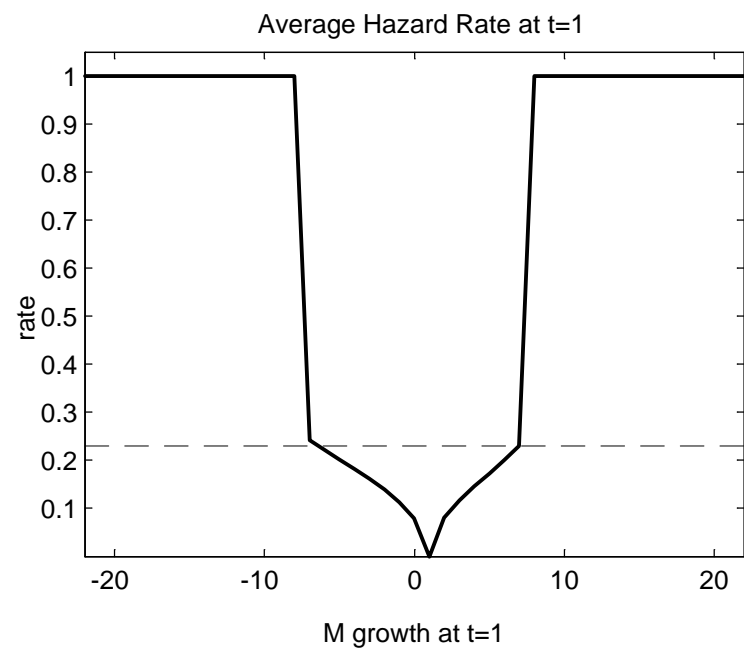
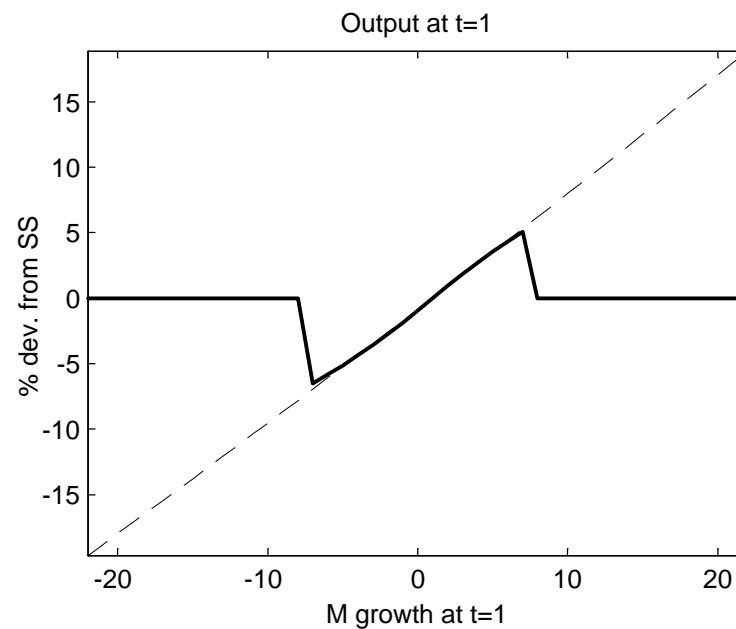
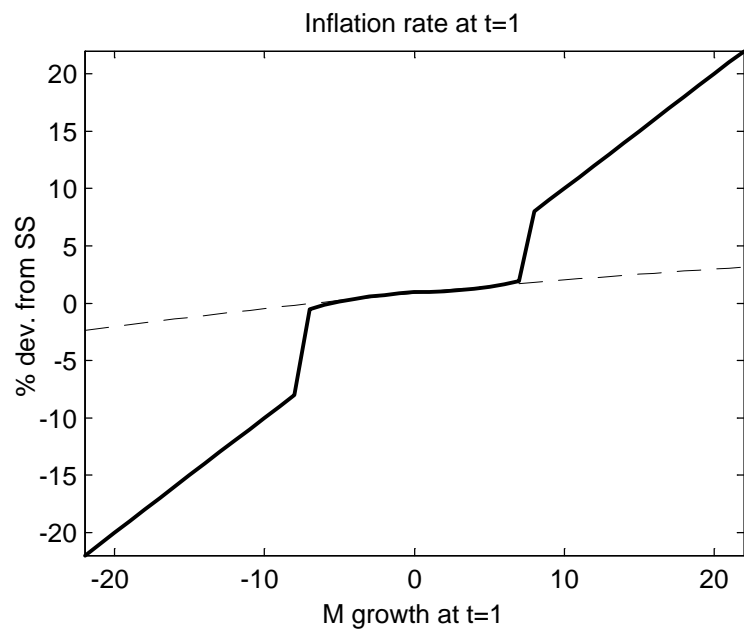


Figure 5, Experiment 1, Sticky Prices versus Sticky Plans,  $\mu_0=0\%$  SS Inflation

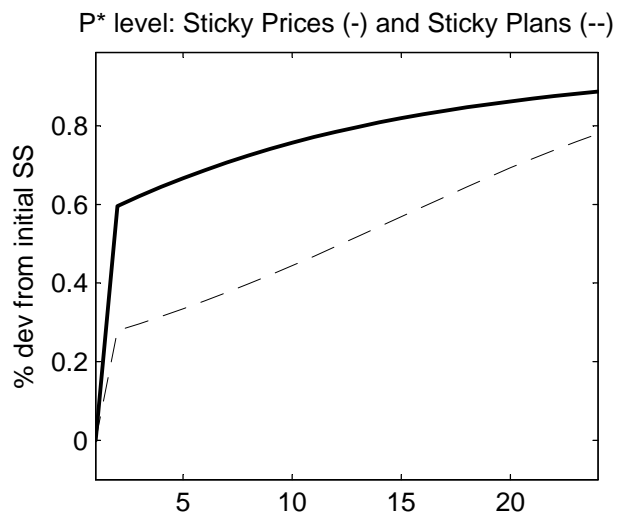
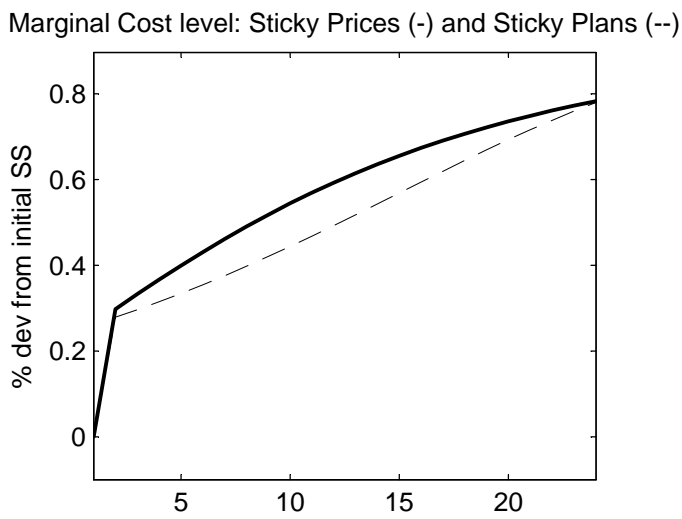
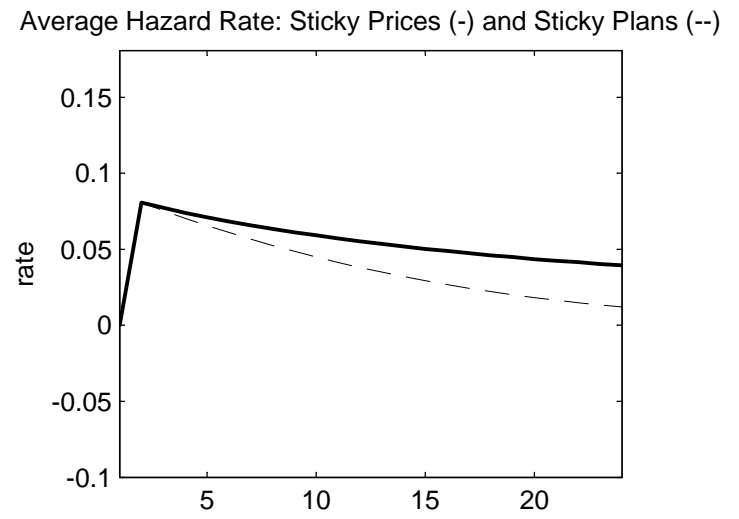
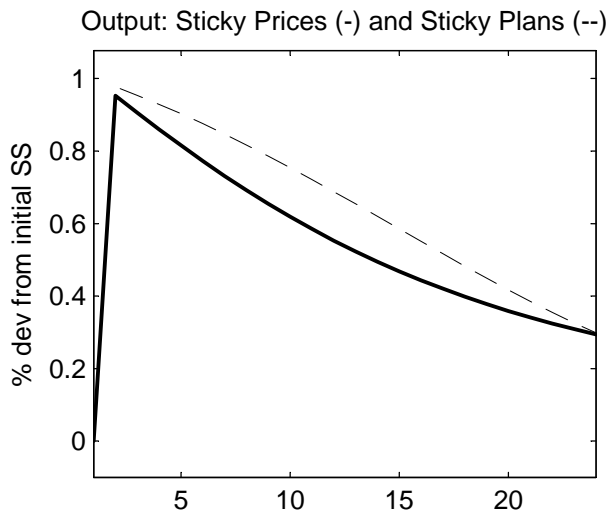
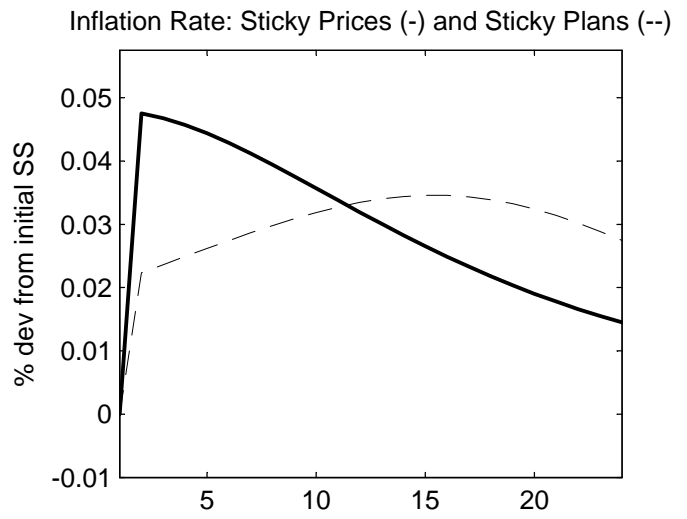
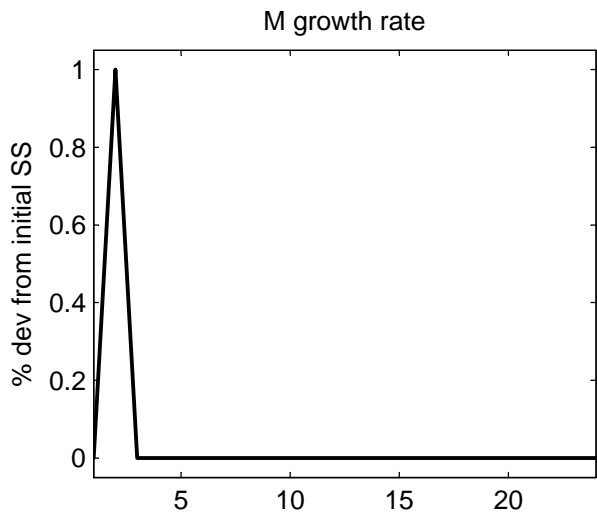


Figure 6, Experiment 1, Sticky Prices versus Sticky Plans,  $\mu_0=0\%$  SS Inflation, AR 0.5

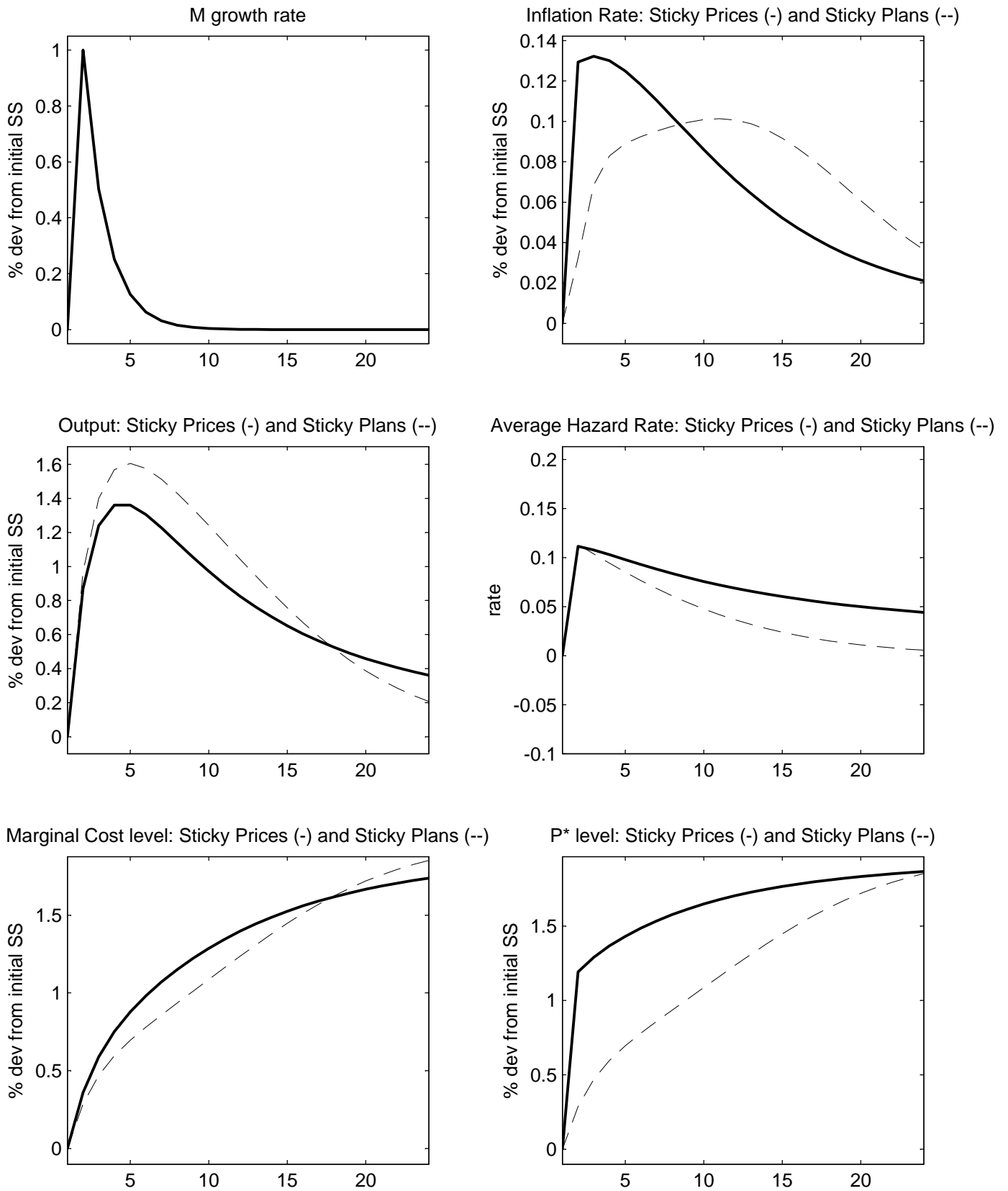


Figure 7, Exp. 1, Sticky Plans: State dep. (-) and time dep. (--), impact at t=1, diff. size of shocks

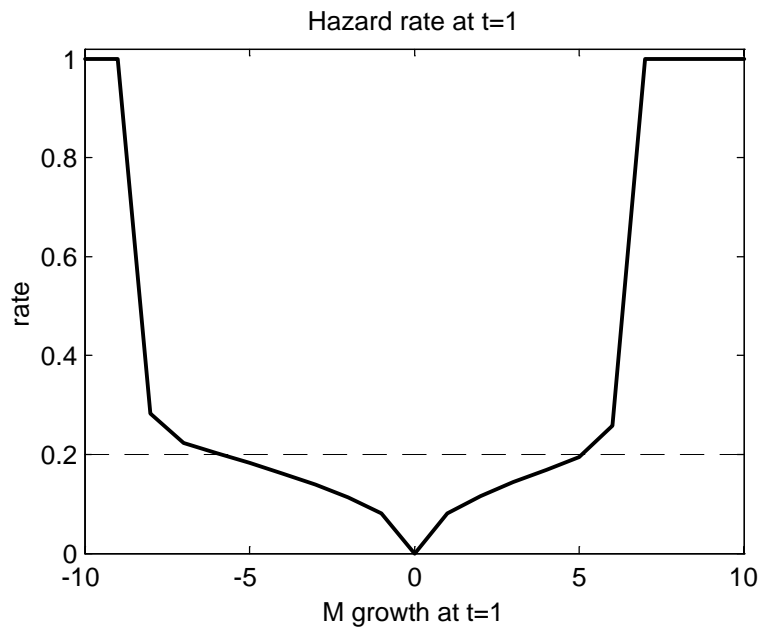
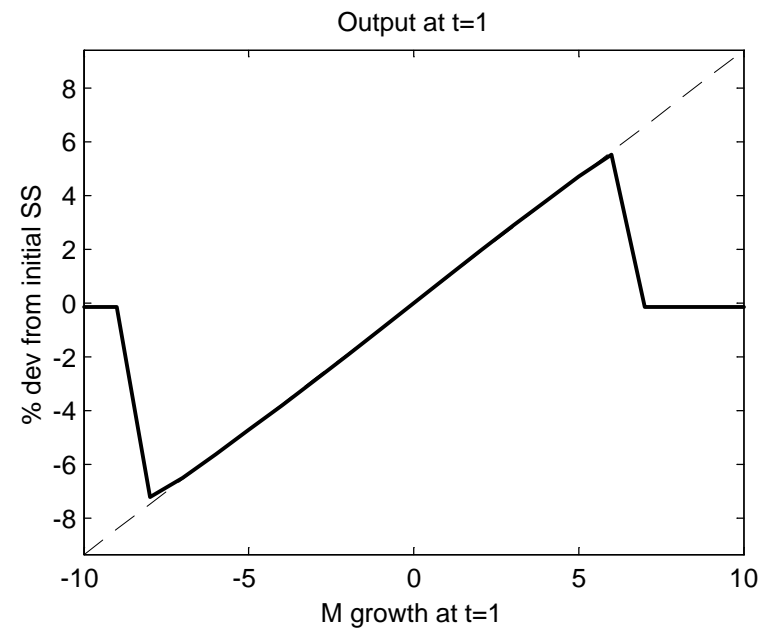
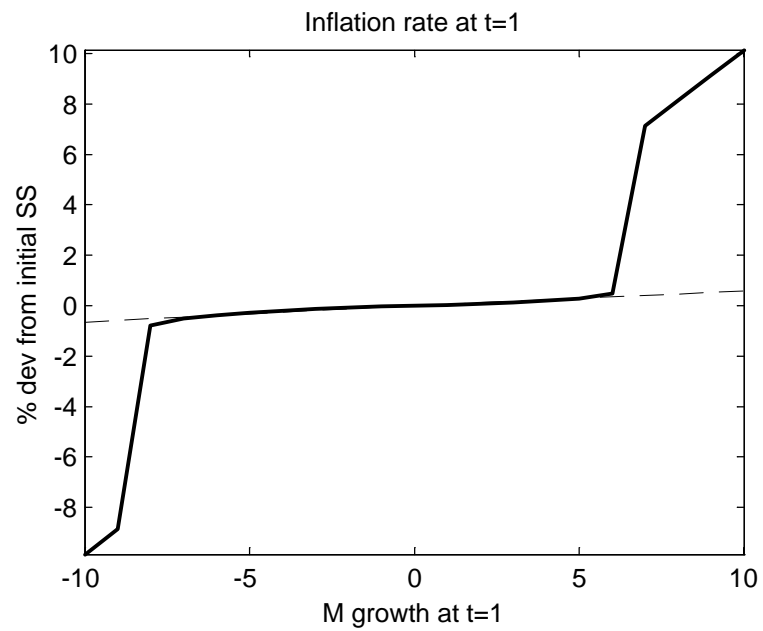


Figure 8, Experiment 2, Sticky Prices versus Sticky Plans,  $\mu_0=1.5\%$  SS Inflation

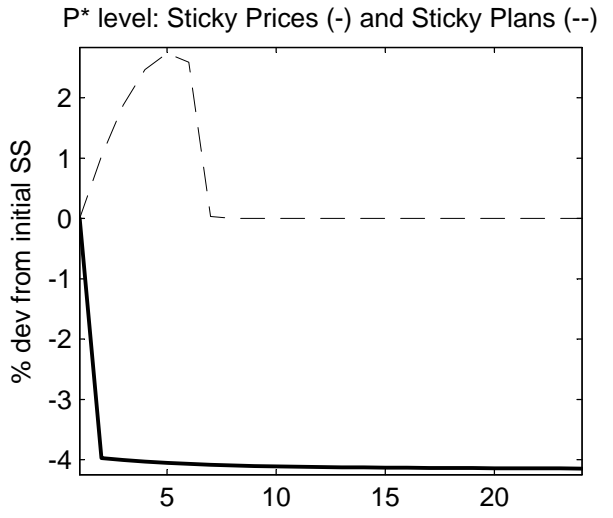
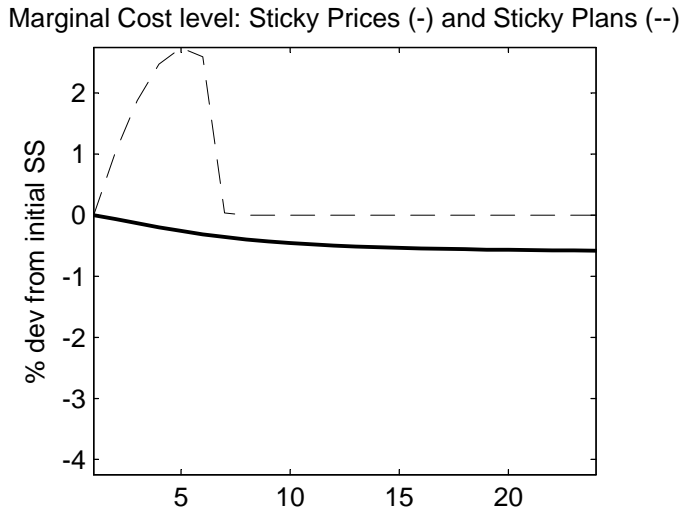
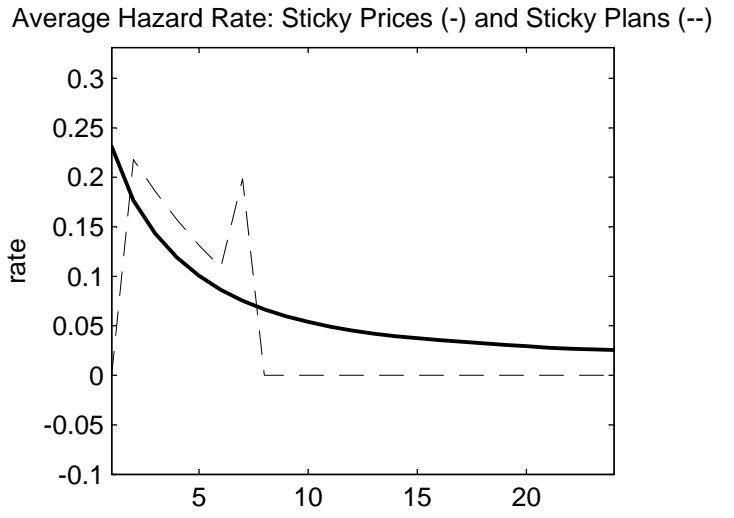
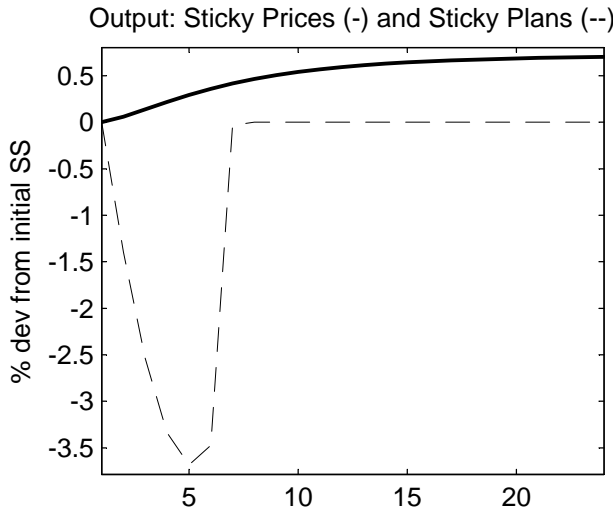
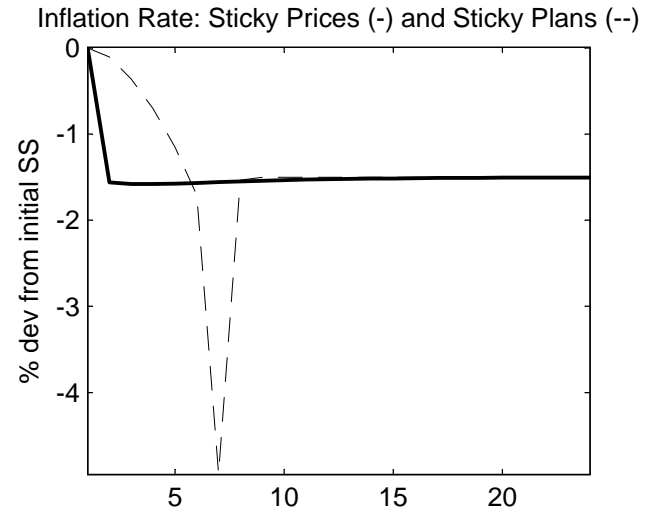
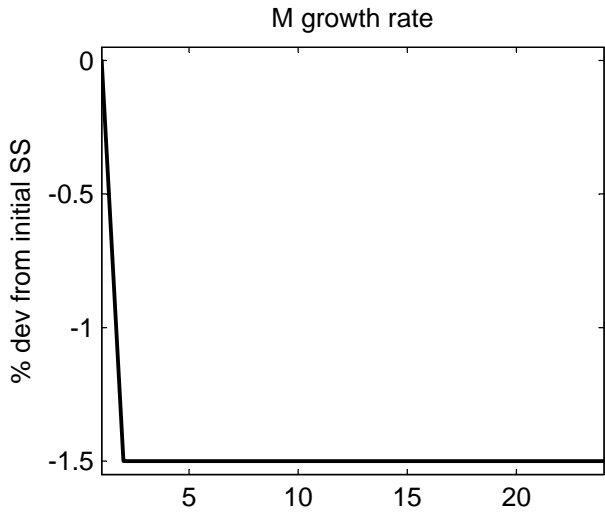


Figure 9, Experiment 2, Sticky Prices versus Sticky Plans,  $\mu_0=10\%$  SS Inflation

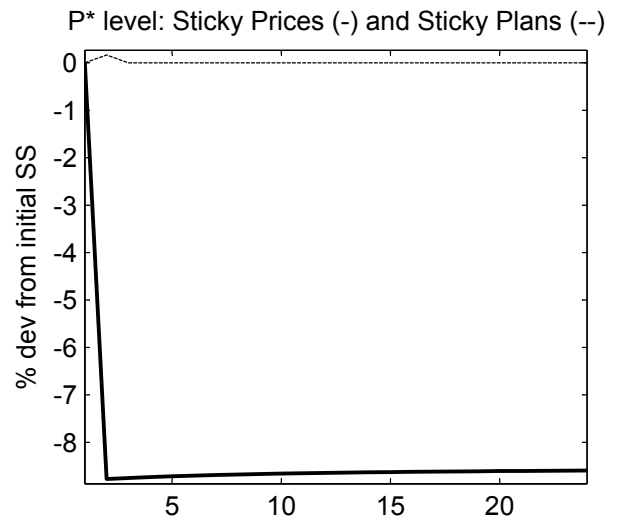
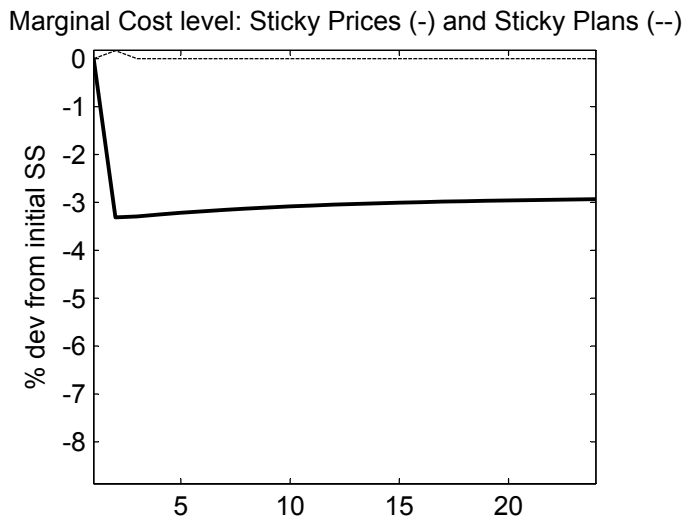
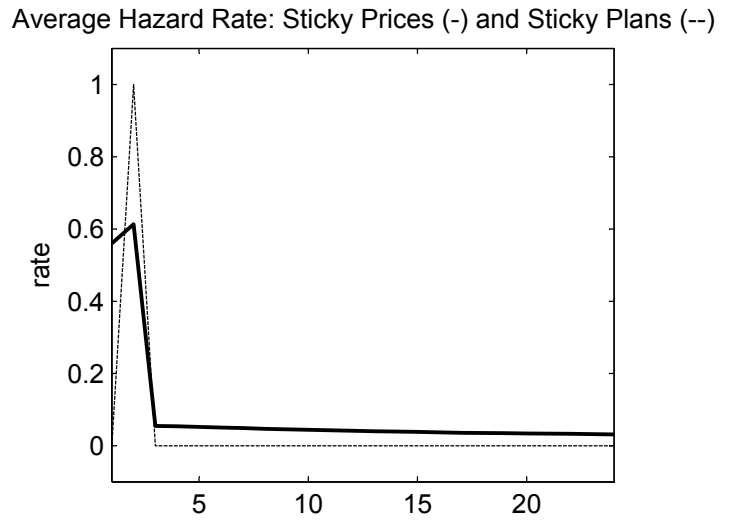
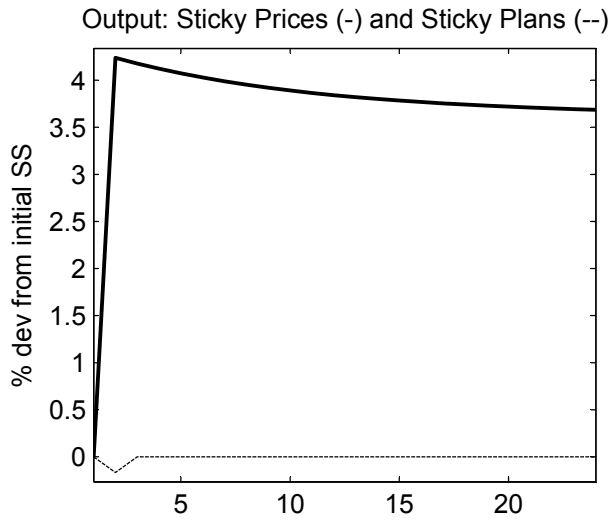
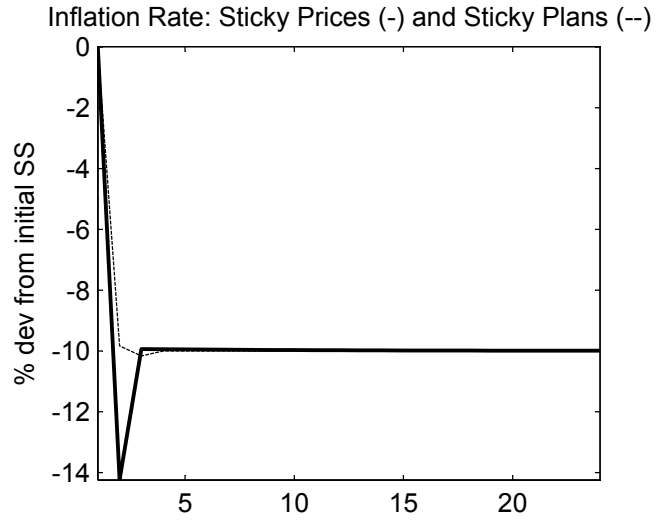
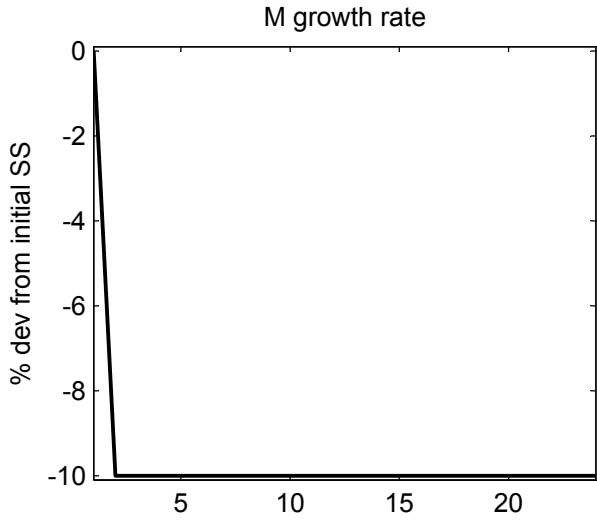




Figure 10, Experiment 3, Sticky Prices versus Sticky Plans,  $\mu_0=1.5\%$  SS Inflation

