Equilibrium Information Disclosure: Grade Inflation and Unraveling*

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Abstract

This paper explores information disclosure in matching markets, e.g. the informativeness of transcripts given out by universities. We show that the same amount of information is disclosed in all equilibria. We then demonstrate that if universities disclose the equilibrium amount of information, unraveling does not occur; if they reveal more, some students will find it profitable to contract early.

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1 Introduction

The words "grade inflation" suggest that the inflation of grades is similar to the inflation of currencies. The analogy is very imprecise. A period of financial inflation merely adds zeros to prices, without making a long-term impact on the economy. Not so with grade inflation: years of grade inflation make grades permanently less informative (if half of the class gets A's, a lot of information is thrown out, thus making grades more noisy). This begs a question about costs and benefits to a school from making grades less informative.[1] Thus, in order to understand grade inflation we should think systematically about how informative a school would like to make its transcripts. This is the question addressed in this paper. We consider the equilibrium amount of information that schools would like to convey in their transcripts and offer a framework for estimating long-term costs and benefits of reducing their informativeness. We show that even if there is no short-run benefit from inflating grades, schools may prefer to make transcripts less than perfectly informative. Thus, reducing the informativeness of grades (by means of grade inflation) may be an optimal response to a changing environment, even if employers can not be temporarily fooled by high grades.

We assume that the ability of each student and the distribution of students among schools is given exogenously. The ability of students is perfectly observed by schools but not by outsiders. Each school decides how much information to reveal in its transcripts in order to maximize the average desirability of placement of its alumni. Outsiders use transcripts to infer the expected ability of students and rank them solely according to their expected ability. The desirability of each position is common knowledge, and students rank positions based on desirability (thus all students have the same preferences and so do all recruiters). We assume that supply of placement slots of given desirability is fixed exogenously. This is a sensible assumption if we think of schools as high schools and placement positions as college slots. There is a "rent" attached to being admitted to a good college. The amount of "rent" captured by successful applicants probably has little relation to the quality of the application pool. We can also think of "schools" as colleges and placement slots as admission to professional schools[2], or "schools" as

[1] Clearly, less informative grades are good for bad students and bad for good students. The tradeoff, however, is not obvious.

[2] It is a reasonable approximation to assume that professional schools do not adjust tuition and the number of admission slots based on the ability of available applicants.

law schools and placement slots as clerkships in the US courts[3], or in general placement slots as any jobs that have rents associated with them.

Consider competition for admission to law schools. Strategically introducing noise into transcripts may enable a school to substantially increase placement into moderately desirable law schools at a cost of slightly reducing the number of students placed into top law schools. Essentiality, a school changes the distribution of placement desirability of its graduates by introducing noise into transcripts. Notice that the aggregate distribution of positions does not change as a result of noisy transcripts, and so the total desirability of placements is unchanged. However, we will see that in a broad range of situation noise is a necessary feature of equilibrium transcripts.

The relationship between the distribution of desirability and the informativeness of grades is often not transparent. For example, one may think that if the premium for excellence goes up, i.e. the world becomes more and more of a "winner takes all" environment, grades should become more informative to clearly indicate star students. However, this is not necessarily the case. Suppose over time the difference in desirability between law schools ranked 1 and 10 increases, but the difference in desirability between law schools ranked 10 and 20 increases even more (one may think of a law school's desirability as a fixed effect that it has on its students' subsequent wages). Then the equilibrium amount of noise in transcripts should increase, because it becomes more profitable to use pooling, thus "minting" more students of the top ten level. The number of students in the tenth-ranked law school does not change, rather the equilibrium expected ability of students going there increases.

The change in the *expected* ability of students in the tenth-ranked school is not due to the change in the distribution of *true* ability in the student population; the latter is exogenously fixed. Rather, it is due to schools varying the amount of noise in their transcripts, thus changing the distribution of *expected* ability. The more noise the transcripts contain, the less informative they are, and hence the more compressed the distribution of expected ability is. In Section 3 we show that the amount of information disclosure can be

[3]The pay of clerks can be viewed as determined exogenously. There is a more or less clear hierarchy of clerkships in terms of desirability. We can assume that post-clerkship wages of lawyers are a sum of ability and enhancement of human capital obtained during the clerkship. Then, the desirability of a clerkship can be defined as the enhancement of human capital obtained during clerkship net of costs associated with it. If the ability of clerks changes the desirability of clerkship need not change.

meaningfully quantified by the aggregate distribution of expected ability.

Theorem 1 establishes that both the equilibrium amount of information disclosure and the utility of every employer are the same in all equilibria. The equilibrium information disclosure is independent of how students are distributed among colleges. It is solely determined by the aggregate distributions of student ability and position desirability.

We also show that there is no unraveling under the equilibrium information disclosure. Intuitively, if a student finds it profitable to withhold some information by signing a contract earlier, his school would also find it profitable to exclude this information from its transcripts. Thus, if schools behave optimally and disclose the equilibrium amount of information, a student can not benefit by moving early and getting "insurance" against further information. Moreover, we show that if more than the equilibrium amount of information is disclosed (for example, if students can signal their ability using test scores not controlled by schools), some students do benefit from contracting early. These results allow us to better understand when unraveling can be explained by the demand for insurance.[4]

The rest of the paper is organized as following. In Section 2 we introduce the formal model of grade inflation. Section 3 defines equilibrium information disclosure and shows some of its properties. Section 4 considers unraveling and shows that it can not happen under equilibrium information disclosure. Section 5 concludes.

2 The Model

Consider a population of students, each with some level of ability—a real number a in the interval $[\underline{a}, \overline{a}]$. Each student attends one of I schools. We assume that the distribution $\lambda_i(a)$ of ability levels at school i is given exogenously and is common knowledge. Without loss of generality we assume that schools observe the true abilities of their students[5].

Each school decides how precise to make its transcripts. A school can

[4]See Li and Rosen (1998), Li and Suen (2000), Suen (2000) for the models of insurance-driven unraveling.

[5]Suppose nobody observes the true ability, but each school observes a signal regarding the true ability of each of its students. Based on this signal a school can form expectation about a student's ability. All results in the paper continue to hold if instead of "true ability" we use "expected ability based on information available to schools."

make transcripts completely informative, revealing the ability level of each student, or it can make them completely uninformative, or anything in between. For example, Harvard Business School has a policy that prohibits employers to ask students about grades. The only information the school reveals is whether a student is in the top decile of the class.

More formally, a school chooses a transcript structure, which consists of two parts. First, it chooses a set of all possible transcripts. Second, it chooses how to map abilities of students into these transcripts. This mapping may be stochastic, i.e. for each ability a there is a probability distribution over the set of transcripts that a student of this ability can get.

Definition 1 *A transcript structure TS is a pair $(T, f(.))$, where T is a set of possible transcripts, and $f(t|a)$ is a probability density function with which a student of ability $a \in A$ receives transcript $t \in T$.*[6,7]

On the other side of the market is a continuum of available positions. The desirability of each position, $q \in [\underline{q}, \overline{q}]$, is common knowledge. The distribution of desirability has density $\mu(q)$. We assume that the mass of positions is equal to the mass of students.[8]

After schools announce transcript structures and give transcripts to their students, employers can compute each student's expected ability conditional on his transcript.[9] Notice that generally the distribution of *expected* ability (conditional on transcripts) will be different from the distribution of *true*

[6] Here and elsewhere in the paper, in the interests of clarity we sacrifice some rigor by omitting such technical details as integrability of $f(t|a)$ on $T \times A$ (to be able to compute conditional expectation) and its treatment in the space of generalized functions (to allow for point masses).

[7] This definition is very similar to the definition of "information structure" in Bergemann and Pesendorfer (2001). That paper, however, considers information disclosure in a very different environment—a single-seller, single-object auction, whereas we consider a matching market.

[8] This assumption is not too restrictive because unemployment can be considered as a position of the lowest desirability, and because if the mass of positions is greater than the total mass of students, the same subset of positions gets assigned a student under any information disclosure.

[9] We assume that schools can commit to their transcript structures. However, even if they could not, the equilibrium information disclosure would still remain an equilibrium outcome of the resulting cheap-talk game (see Crawford and Sobel (1982) for a formal analysis of cheap-talk games). On the other hand, there are many other equilibria in the resulting cheap-talk game, in some of which even more information may be disclosed.

ability (unless transcripts are completely informative, i.e. any two students of different true abilities get different transcripts). In the Harvard Business School example above, the resulting distribution of *expected* ability is concentrated on two points (assuming that no information aside from the transcript is available to recruiters), even though *true* ability could have any distribution.

Students rank schools by desirability, and employers rank students by expected ability. The resulting rankings induce a unique (up to permutations of equally desirable positions) assortative stable matching between students and positions. Note that the assumption that employers are risk-neutral and rank students solely by expected ability allows for a wide variety of employer payoffs, including the ones commonly assumed in the matching literature—e.g. Suen (2000)—$F = \theta(a)\phi(q)$; Li and Suen (1998)—$F = aq$, $F = aq^2$; Haruvy, Roth, and Unver (2001)—$F = aq$.[10]

Each school selects a transcript structure to maximize the average desirability of positions obtained by its students. Each school is small relative to the labor market and is a "price taker"—its actions have no effect on placement of students of a given expected ability.[11]

The following example is an illustration of our model.

Example 1. Consider a simple setup: student abilities at each school are distributed uniformly on $[0, 100]$, and position desirabilities are distributed uniformly on $[0, 200]$. If all schools fully reveal student abilities (e.g. set $t \equiv a$), the resulting mapping from abilities to position desirabilities is linear ($Q(a) = 2a$) and no school can benefit by deviating. Thus fully informative transcripts form an equilibrium, and, as we later show in Corollary 2, no other equilibria exist.

Let us now see what can happen out of equilibrium. Suppose for an

[10]This is true because as long as the output of a worker is a function of his ability, we can find a rescaling of ability such that a particular firm is indifferent between having a worker of ability a_0 for sure and a worker of uncertain ability with expectation a_0. However, we do have to assume that this rescaling is the same for all firms. This does not preclude a possibility of complementarity between worker ability and position desirability: for example, it is consistent with our model if the output of a worker in a firm of desirability q is a product of his ability and some function of desirability, $g(q)a$, because $E_a[g(q)a] = g(q)E_a[a]$.

[11]This can be reconciled with a finite number of schools using the standard general equilibrium approach—assume that there are I types and an infinite number of schools of each type.

exogenous reason all schools but one inflate grades and thus make their transcripts noisy. Namely, suppose a student of ability a receives transcript t—a random draw from $U[a, a+100]$. Then transcripts range from 0 (the least able student with the least lucky draw) to 200 (the most able student with the most lucky draw). The distribution of transcripts is triangular with the peak at 100. If $t \leq 100$, the true ability could be anywhere from 0 to t (distributed uniformly on this interval, since both noise and the underlying true ability are uniform and independent) and the expected ability is $t/2$. Similarly, if $t \geq 100$, the distribution of true abilities of students with transcript t is uniform on $[t-100, 100]$ the expected ability of a student conditional on his transcript is also given by $\hat{a}(t) = E[a|t] = \frac{(t-100)+100}{2} = \frac{t}{2}$. Thus $\hat{a}$ has triangular distribution F with the peak value of $\frac{1}{50}$ at 50. A student of expected ability $\hat{a}$ gets a position of the same rank as his own. This student is better than $\int_0^{\hat{a}} dF$ fraction of the student population, and thus gets a position of desirability $Q(\hat{a}) = 200 \int_0^{\hat{a}} dF$. Computing the integral, $Q(\hat{a}) = \frac{\hat{a}^2}{25}$ for $\hat{a} \leq 50$ and $200 - \frac{(100-\hat{a})^2}{25}$ for $\hat{a} \geq 50$; $Q(\hat{a})$ is S-shaped.

To solve for the best response of U-State (a representative school) to this form of grade inflation, notice that it will pool together students from the concave part of $Q(\hat{a})$ until the amount gained by the marginal added student a^*, $Q(\frac{a^*+100}{2}) - Q(a^*)$, equals the total amount lost by the pooled students of higher ability due to the decrease in the average pool ability, $(100-a^*) \cdot \frac{Q'(\frac{a^*+100}{2})}{2}$. Thus,

$$\frac{Q(\frac{a^*+100}{2}) - Q(a^*)}{\frac{100-a^*}{2}} = Q'(\frac{a^*+100}{2}). \tag{1}$$

Solving (1), we find that the best response of U-State is to reveal ability truthfully if it is less than a^* and pool together all students with ability above a^*, where $a^* = \frac{100}{7}(3-\sqrt{2})$. Notice that the best response to grade inflation of a better than average school that has a distribution of student types given by $U[50, 100]$ is to make the grades completely uninformative.

3 Equilibrium Information Disclosure

In our setup, the behavior of students and positions is straightforward—they get matched to the agent of highest quality available to them on the other side of the market (in the next section we give students and positions some flexibility by allowing them to sign contracts earlier). Thus, we focus on the actions of schools and the resulting transcript structures. Transcript structures form an equilibrium if no school can increase the average desirability of its students' placement by changing its transcript structure.

The main result of this section is that in all equilibria employers of a given desirability get workers of the same expected ability (even though the distributions of workers they get may be different). This result holds even if we allow the distributions of student abilities within schools to change, as long as the distribution of ability in the entire student population remains the same. Before we prove the result, we present several properties of an equilibrium.

3.1 Properties of an Equilibrium

In this subsection we establish several lemmas describing some interesting properties of an equilibrium. These lemmas are also used later to prove the main results. In the lemmas and all subsequent results we assume that density functions of position desirability and student ability in each school are atomless, continuous, and have full support on closed intervals. We also assume that if all abilities are revealed truthfully, function $Q_T(a)$ mapping ability level a to position desirability $Q_T(a)$ of the same rank does not switch from convexity to concavity infinitely often (i.e. there exists a finite sequence of ability levels a_i, starting at the lowest and ending at the highest true ability, such that Q_T is convex or concave on each interval $[a_i, a_{i+1}]$).

We first show that in equilibrium there is a one-to-one mapping from expected ability to position desirability, i.e. students of the same expected ability get equally desirable positions. This allows us to talk about this mapping as an invertible function $Q(\hat{a})$, with the inverse function $A(q)$. $A(Q(\hat{a})) \equiv \hat{a}$.

Lemma 1 *In equilibrium, any two students of the same expected ability $\hat{a}$ obtain equally desirable positions.*

Proof. See Appendix. ■

Function $Q(\hat{a})$ is monotonically increasing. This, however, does not necessarily mean that a student of a higher true ability will get matched to a better position than a student with a lower true ability: if a school gives out transcripts that are not fully informative, the lower ability student may receive a better transcript than higher ability student and thus get a better position. If in a given equilibrium better students always get better transcripts we say that the equilibrium is fair.

Definition 2 *An equilibrium is fair if for any two students the more able one secures a position that is at least as good as the position secured by the less able one.*

Definition 3 *An equilibrium is fair at a particular value of position desirability q if there is an ability level that is necessary and sufficient for receiving a position of this quality. More precisely, equilibrium is fair at desirability q if for any $\delta > 0$ there exists a positive $\epsilon < \delta$ and ability levels a_L and a_H such that all*[12] *students who are hired into positions of desirabilities $[q-\epsilon, q+\epsilon]$ have ability in the interval $[a_L, a_H]$ and among the students with ability in the interval $[a_L, a_H]$ nobody receives positions of desirability outside the interval $[h-\epsilon, h+\epsilon]$.*

It is straightforward to show that an equilibrium is fair if and only if it is fair at every position desirability.

Another property that we introduce is "connectedness." We will say that an equilibrium is connected if, roughly, there exists a sequence of schools, from best to worst, such that the first one produces some students of the highest expected ability in the population, the last one produces students of the lowest expected ability, and each school in the sequence produces a positive mass of students who are worse in expectation than the next school's best student. The following definition formalizes this idea.

Definition 4 *Let $\hat{a}_L$ be the lowest and $\hat{a}_H$ the highest expected ability levels produced in an equilibrium. Then we say that the equilibrium is connected if for any point $\hat{a} \in (\hat{a}_L, \hat{a}_H)$ there exists a school that produces students of all expected abilities in some ϵ-neighborhood of $\hat{a}$.*

Connectedness is a mild restriction. Indeed, if at least one school gives out some transcripts implying the worst and the best possible expected abilities, and everything in between, this restriction is satisfied.[13]

Desirability mapping $Q(\hat{a})$ is defined on $[\hat{a}_L, \hat{a}_H]$ and is monotonically increasing. We are now ready to characterize it further. Suppose a school

[12] Up to a set of measure zero.

[13] In the appendix we show that a sufficient condition for the existence of a connected equilibrium is for schools to be symmetric.

produces students of expected abilities b and c. This could only be optimal for the school if by mixing students of these abilities it could not raise its payoff, i.e. if $\alpha Q(b)+(1-\alpha)Q(c) \geq Q(\alpha b+(1-\alpha)c)$ for any $\alpha \in [0,1]$. Since this reasoning can be applied to every pair of points, and in a connected equilibrium there is a school producing students in a neighborhood of any point, $Q(\hat{a})$ has to be convex.

On the other hand, if a school does mix in any neighborhood of some desirability ν_0, i.e. the equilibrium is not fair at that point, then it has to be the case that $Q(\hat{a})$ is concave there—otherwise the school could do better by producing students of expected abilities $\nu_0 - \epsilon$ and $\nu_0 + \epsilon$. If the equilibrium is unfair on an interval, $Q(\hat{a})$ thus has to be both convex and concave there, and therefore linear.

Conversely, if $Q(\hat{a})$ is strictly convex at a certain expected ability level $\hat{a}$, it is fair at $Q(\hat{a})$. Moreover, it is unprofitable for a school to mix students of abilities higher than $\hat{a}$ and lower than $\hat{a}$, and so all more able students get positions better than $Q(\hat{a})$ and all less able students get positions worse than $Q(\hat{a})$. The following three lemmas summarize these results.

Lemma 2 *In any connected equilibrium function $Q(\hat{a})$ is convex.*

Lemma 3 *Suppose for some range of desirability $[\nu_1, \nu_2]$, a connected equilibrium is not fair. Then $Q(\hat{a})$ is linear in $\hat{a}$ on$[A(\nu_1), A(\nu_2)]$.*

Lemma 4 *If $Q(\hat{a})$ is strictly convex at $\hat{a}$, then schools do not mix students of ability higher than a with students of ability lower than $\hat{a}$.*

Our final lemma in this section shows that the lowest expected ability produced by schools in the job market is equal to the lowest true ability. This is similar to the "lowest type not signalling" in a separating equilibrium of a signalling game.

Lemma 5 *In a connected equilibrium, let $\hat{a}_L$ be the lowest expected ability level, and a_L be the lowest true ability level. Then $\hat{a}_L = a_L$.*

Proof. See Appendix. ■

3.2 Uniqueness of the Equilibrium Amount of Information Disclosure

In this section we show that the equilibrium amount of information disclosure is unique. We start with the definition that makes the words "amount of information disclosure" precise. Let τ denote a profile of transcript structures and let F denote the distribution of expected abilities generated by τ. Note that if each school introduces more noise in its transcripts, the resulting distribution of expected abilities becomes more compressed.[14] This leads to a natural partial ordering on profiles of transcript structures.

Definition 5 *Profile of transcript structures τ is more informative than profile of transcript structures τ' if distribution F of expected abilities generated by τ is second-order stochastically dominated by distribution F' of expected abilities generated by τ'.*[15]

This partial ordering has two extreme elements: the completely uninformative profile, which has zero variance, and the profile revealing all student abilities, which has the highest possible variance.[16] Also notice that if two profiles produce different distributions of expected student abilities, the resulting desirability mappings $Q_1(\hat{a})$ and $Q_2(\hat{a})$ will also be different, and so identical desirability mappings can only be generated by identical distributions of expected abilities in the student population.

We are now ready to state and prove the main result of this section. It says that in all connected equilibria function $Q(\hat{a})$ is the same or, equivalently, the same amount of information is disclosed. The equilibrium amount of information is independent of how students are assigned to schools—only the aggregate distribution of student abilities and the distribution of position desirabilities matter.

[14] To add noise, school i creates new transcript structure $(T_i', f_i'(t'|a))$ where $f_i'(t'|a) \equiv f_i'(t'|t(a))$; $(T_i, f_i(t|a))$ is the old transcript structure. If τ' is the new profile of transcript structures, and F' is the distribution of expected abilities corresponding to τ', then F is a mean-preserving spread of F'. This is true because τ' is obtained by mixing students of different expected abilities under τ.

[15] Slightly abusing terminology, we will identify different transcript structures if they generate the same distributions of expected student abilities. For example, a transcript structure that gives the same grade to every student is, for our purposes, the same as a transcript structure that gives completely random grades to all students.

[16] It is clear that a more informative profile has a higher variance than a less informative one, since the former is a mean-preserving spread of the latter.

Theorem 1 *Suppose there is a connected equilibrium with desirability mapping $Q_1(\hat{a})$. Now suppose some students are exogenously moved to different schools*[17]*, and there is a new connected equilibrium with desirability mapping $Q_2(\hat{a})$. Then $Q_1(\hat{a}) = Q_2(\hat{a})$.*

Proof. See Appendix. ■

A straightforward corollary is that when there are two equilibria in a market, their desirability mappings coincide.

Corollary 1 *Suppose there is a market that has two connected equilibria, with desirability mappings $Q_1(\hat{a})$ and $Q_2(\hat{a})$. Then $Q_1(\hat{a}) \equiv Q_2(\hat{a})$.*

Another corollary is that if truthful revelation of abilities, i.e. no grade inflation, is an equilibrium, then there are no other connected equilibria.

Corollary 2 *Suppose there are two connected equilibria in a market, and one of them is fair. Then the other one also has to be fair, and therefore the same.*

Proof. By Corollary 1, desirability mappings for these two equilibria have to be the same. Therefore, distributions of expected abilities generated in these equilibria also have to be identical (since they are uniquely determined by the mapping and the distribution of position desirability). But the fair equilibrium is strictly more informative than any unfair one, and so the second equilibrium also has to be fair, hence the same. ■

4 Unraveling

This section explores unraveling, i.e. contracting between students and positions before full information about the former is available. The literature on unraveling broadly divides the causes of early contracting into two categories: strategic considerations that only arise in discrete environments and the demand for insurance, which can arise in both discrete and continuous frameworks. We focus on the insurance aspect of unraveling.

There is a close, albeit not obvious, connection between information disclosure and unraveling. Theorems 2 and 3 show that if the equilibrium

[17] I.e. aggregate distribution of student abilities remains the same, but distributions of abilities within schools change.

amount of information is revealed, no unraveling occurs. Consequently, if schools can control the amount of information disclosed to potential employers, the insurance reason for unraveling disappears. The intuition is simple: in equilibrium, due to convexity of $Q(\hat{a})$, the expected position desirability that a student will get tomorrow is higher than the position desirability that he could get today.

That there is no unraveling under the equilibrium amount of information disclosure seems surprising—after all, imagine all positions have similar desirability except for a few which are terrible, e.g. unemployment. Then one might think that students would be eager to sign contracts earlier to avoid this outcome. However, as the following example shows, this does not happen. What happens instead is that equilibrium distribution of transcripts "mimics" the distribution of desirability—a small group of students gets very bad transcripts, and the rest get compressed transcripts with little information beyond being much better than the bad transcript.

Example 2. Suppose there are positions of five desirability levels: 0, 110, 120, 130, and 140. Each has mass .2. At each school, ability is distributed uniformly on $[0, 100]$. First, note that it is not an equilibrium for all schools to lump all students into one category. If they do, then a school can profitably deviate by separating a small fraction of the worst students into a new category. Second, providing fully informative signals is not an equilibrium either—in this case a school can achieve the average desirability of placement of 120 by pooling all students together instead of 100 by fully separating them. Thus, schools have to partially mix in equilibrium.

The following transcript structure is an equilibrium of the market. There are five possible transcripts—we'll call them A, B, C, D, and E, from best to worst. Entries in the following table are the probabilities of receiving a particular transcript for students of different ability levels.

	E	D	C	B	A
$a \leq 20$	1	0	0	0	0
$20 < a \leq 56$	0	$\frac{7}{11} \cdot \frac{5}{9}$	$\frac{18}{33} \cdot \frac{4}{9}$	$\frac{15}{33} \cdot \frac{4}{9}$	$\frac{4}{11} \cdot \frac{5}{9}$
$56 < a \leq 60$	0	0	1	0	0
$60 < a \leq 64$	0	0	0	1	0
$a > 64$	0	$\frac{4}{11} \cdot \frac{5}{9}$	$\frac{15}{33} \cdot \frac{4}{9}$	$\frac{18}{33} \cdot \frac{4}{9}$	$\frac{7}{11} \cdot \frac{5}{9}$

The expected ability of a student with the worst transcript, E, is equal to 10. $\hat{a}(D) = (7 \cdot 38 + 4 \cdot 82)/11 = 54$, $\hat{a}(C) = 58$, $\hat{a}(B) = 62$, and $\hat{a}(A) = 66$.

The resulting mapping $Q(\hat{a})$ is linear: $Q(\hat{a}) = \frac{5}{2}(\hat{a} - 10)$, and so a school can not increase the average desirability of placement by deviating from this transcript structure.

Notice that under this equilibrium there is no unraveling (or, more exactly, no incentive to unravel) since students become effectively risk-neutral. Consider a student whose first-year transcript indicates an expected ability level corresponding to a particular job desirability. This student can secure a job corresponding to his current expected ability or he can wait for second-year grades. In the absence of private information about ability the expected change in ability implied by the transcript must be zero. It is easy to see that the expected change in position desirability can not be negative as a result of arrival of new information.

Even if students did have private information, unraveling would still not occur, and the result would in fact become even stronger. In the absence of private information unraveling is a matter of indifference for both students and positions. If students do have private information, adverse selection works against unraveling, because the lowest ability students have higher payoff from unraveling than observationally equivalent students of higher ability. Essentially, only the lowest ability students are eager to unravel, and unraveling can not occur under equilibrium information disclosure except for a set of measure zero.[18]

In the remainder of the section, we first present a simple two-period model where no information is available in period 1, which is very similar to the model of Suen (2000). This similarity brings into focus the fact that the schools' ability to control information undermines the insurance reason for unraveling. We then move to a more general framework, where new information arrives continuously, and show a striking result—"no unraveling", i.e. contracting at the last possible moment, after all information becomes available, is an equilibrium only if the transcript structures eventually revealed by the schools form an equilibrium, and is not an equilibrium if more information is revealed.

[18] It is straightforward to make the above statement entirely formal.

4.1 Two-period Model, No Unraveling under Equilibrium Information Disclosure

Suppose students stay in school for 2 periods. In period 1 no information about them is known, and so for all students in school i expected ability in period 1 is the same, $\widehat{a_i}$. A student has no private information about his ability[19]. Suppose employers can hire a student in either year of study based on the information available in his transcript at that period, and the hiring contracts are binding.

Theorem 2 *If schools' transcript structures in period 2 form a connected equilibrium, then no position can increase the expected ability of its match by making an early offer.*

Proof. Take a student from school i in period 1. His expected ability in period 1 is $\widehat{a_i}$. If he waits until period 2, he will get a position of desirability $Q(\widehat{a_2})$. From the law of iterated expectations, $E_i[\widehat{a_2}] = \widehat{a_i}$. Desirability mapping $Q(\widehat{a_2})$ is convex, and therefore $E_i[Q(\widehat{a_2})] \geq Q(\widehat{a_i})$. Thus, a student will only accept an early offer from a position that is at least as desirable as $Q(\widehat{a_i})$. But positions of desirability $Q(\widehat{a_i})$ and higher get a student of expected ability at least $\widehat{a_i}$ if they wait until period 2, and so can not benefit form moving early. ■

4.2 Continuous-Time Model of Information Arrival

We now set up a continuous-time model of information arrival, and show a close connection between unraveling and equilibrium information disclosure.

Students are in school from time $t = 0$ until time $t = T$. At time 0 no information about a student is known except for the school he attends. While the student is at school, new information arrives continuously and is added to his transcript (we assume that information about students can not disappear). Namely, at each time t a potential employer can compute the student's expected ability $\hat{a}_t$ based on the current transcript. Since employers use Bayes' rule to form beliefs about a student's expected ability, the drift

[19] Allowing for private information does not change the result, except that the 0-mass of students and firms who are indifferent between contracting in periods 1 and 2 is of lower quality due to adverse selection.

term must be zero and the process is a martingale. Consequently, $\hat{a}_t$ follows a diffusion process

$$d\hat{a}_t = \sigma(\cdot)dz_t, \tag{2}$$

where we assume $\sigma(\cdot)$ to be a bounded continuous function of $\hat{a}_t$ and t, such that the process does not leave the interval $[a_L, a_H]$ and for all $\hat{a} \in (a_L, a_H)$ $\sigma(\hat{a}, T) > 0$. We also assume that for some $t < T$ function $\hat{Q}(\hat{a}_{t'}, t') = E[Q(\hat{a}_T)|\hat{a}_{t'}]$ is twice continuously differentiable for all $t' \in [t, T]$.[20] Whenever expected ability follows such diffusion process we will say that information arrives gradually.

Each position's desirability is common knowledge throughout the game, and any student-position pair can enter into a binding match any time during the game. Unraveling occurs if at some time $t < T$ there is a pair S and P that finds it profitable to sign such a contract.[21]

We now claim that it is an equilibrium for students and firms to sign contracts at time T without unraveling if transcript structures generated by schools on day T form an equilibrium. If more than equilibrium amount of information is disclosed, some students and employers will find it profitable to sign contracts earlier. The intuition for the first statement is the same as in Theorem 2. The intuition for the second statement is that if more information is revealed, some portion of the resulting mapping $Q(\hat{a}_T)$ will be concave, thus making some students effectively risk-averse.

Theorem 3 *Suppose that information about ability of students arrives gradually (see equation (2)). If at time T transcripts contain an equilibrium amount of information then it is an equilibrium for all students and positions to wait until time T to sign contracts. If at time T transcripts contain more than an equilibrium amount of information then some agents are strictly better off not waiting till time T to sign contracts.*

Proof. See Appendix. ■

[20]More formally, $\hat{Q}(\hat{a}_{t'}, t')$, is twice continuously differentiable on the set of points $\{(a, t')|t' \in [t, T]$, a is in the domain of $\hat{Q}(\cdot, t')\}$.

[21]It is profitable for the pair to sign such a contract if by waiting till time T P would get a student of expected ability no higher than the expected ability of S given the information available at time t; S, in expectation, would get a position of desirability no higher than that of T; and at least one of these two inequalities is strict.

5 Discussion

Our results hinge on the assumption that the supply of jobs of various desirability is fixed exogenously. However, as long as there are rents associated with at least some positions the model remains relevant. Suppose the desirability of some jobs is fixed exogenously (just like in the case of clerkships, where work conditions and wages are fixed and are not adjusted depending on the ability of the successful applicant.) Wages (and thus desirability) of other positions are chosen by employers in a profit maximizing manner—we can call these positions competitive market jobs. Productivity of workers in competitive market jobs is proportional to workers' ability level. In this case the distribution of desirability of jobs would adjust in response to any transcript structure so that the average ability of workers at any competitive market job is proportional to the wage. (If all jobs where competitive market jobs, any transcript structure is an equilibrium outcome and desirability is always linear in ability.) However, if some jobs are associated with rents the desirability may increase faster than linear in ability. In other words, our model remains relevant for jobs at the right tail of desirability distribution.[22]

Notice that even if schools compete for students, our results remain relevant. Indeed, in a two-stage framework (competition first, grade inflation second) the equilibrium amount of information disclosure is not affected, because it only depends on the distribution of ability in the aggregate student population[23]. Also note that we may observe noisy transcripts even if each student knows his ability before starting school and if schools can commit to fully informative transcripts. Of course, in the absence of transfers, if all students know their ability before entering schools pooling is no longer an equilibrium, however, if transfers are allowed, we should observe pooling because less able students are willing to pay to be pooled with able students. In fact, the equilibrium desirability schedule $Q(\cdot)$ remains unchanged if students could negotiate with each other who pools with whom. (Merit based fellowships might serve a role of transfer payments.)

Another assumption that we made is that a student's ability is exogenously fixed. If learning entails costly effort, noisy transcripts reduce the

[22]Note that in this model the payoff to employers that offer jobs with rent is not specified, all that matters that these employers prefer to hire candidates with the highest available expected ability.

[23]This argument hinges on the assumption that schools can not commit to informative grades and that there is a range of students of various abilities attending each school.

effort of at least some students (Becker 1982). However, the efficiency loss may be small, because loss in ability is partially compensated by saved effort. At least in theory, grade inflation does not necessarily reduce efficiency. If signaling high ability is merely a ticket to high-rent jobs, then noisy transcripts may be welfare improving. Our present model is not rich enough to analyze efficiency implications of grade inflation, and extending it is an interesting area for future research.

One can also consider the short-term benefits of inflating grades and costs of deflating them. If there are "seignorage" benefits from raising grades, a school may choose to overinflate and suffer the consequences later. Incorporating such short-run effects is an interesting topic for future analysis.

It is often said that "something should be done about grade inflation." Our analysis, however, shows that making transcripts more informative may make some schools worse off. This is the case whenever the highest expected ability under the equilibrium information disclosure is lower than the highest true ability in the population. Also note that if transcripts at the time of graduation are "too informative" unraveling will occur.

In some markets unraveling is a fact of life. For instance, it is well documented that the market for federal judicial law clerks in the US courts has unraveled (Avery, Jolls, Posner, and Roth 2001). Many market participants view unraveling as a serious problem.[24] In light of our model we believe that unraveling may occur in that market because law schools can not control the amount of information released about students abilities. Indeed, achievements like being an editor of a law review speak volumes about a student's ability, yet this information is not a part of the transcript and thus a school can not suppress this information.

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A Proof of Lemma 1

Suppose in equilibrium students of expected ability $\hat{a}$ get jobs of desirabilities from q_1 to q_2, $q_1 < q_2$[25] Let $\hat{q}$ be the average desirability that students of expected ability $\hat{a}$ get. $q_1 < \hat{q} < q_2$. Since there is a positive mass of students of expected ability $\hat{a}$, there must be at least one school producing a positive mass of such students. This school has to include some students of lower and some students of higher ability in this mass. Thus, it can select a small subset from the mass (say, ϵ-share of the mass) such that its expected ability is $\hat{a} - \delta$, where δ is also small. Then the remaining mass has expected ability higher than $\hat{a}$, and therefore all students there get positions of desirability q_2 or higher. For sufficiently small ϵ and δ, the net change in average desirability is positive, i.e. the school was able to improve upon its equilibrium transcript structure—contradiction.

B Existence and Connectedness of a Symmetric Equilibrium in a Symmetric Market — Sketch of the Proof

Theorem 4 *In a market where all schools have identical distributions of students, there exist a symmetric equilibrium in pure strategies, and this equilibrium is connected.*

Proof. Existence. Let S be the set of school's strategies. Let $B(s)$ be the best responce correspondence—the set of best responces for a school given that all other schools play s. We need to show that $B(s)$ has a fixed point.

Set S is the set of distributions that second-order stochastically dominate the underlying distribution of student abilities. S is convex (if each of two distributions dominates F, their average does too), compact, and the payoff function is continuous on S (the metric that we use is L_2; if the distance between two distributions is zero, they can be identified since all schools' payoffs are identical. Thus, by the generalization of Kakutani's Fixed Point Theorem (Glicksberg 1952) there exists distribution s^* such that $s^* \in B(s^*)$.

[25]It does not matter for the proof whether this is an open or closed interval. What's important is that all students of expected ability higher than $\hat{a}$ get a job of desirability q_2 or higher.

Connectedness. Now suppose this equilibrium is not connected. This implies that there is an interval (a, b) such that no school produces students of ability in this interval, but each (since the equilibrium is symmetric) produces positive masses of students on both sides of the interval, i.e. for any open interval containing a or b. But then the school can increase its payoff by mixing some students of ability slightly below a and some students of ability slightly above b so that the expected ability in this mix is equal to a. ■

C Proof of Lemma 5

It is clear that $\hat{a}_L \geq a_L$, since it is impossible to produce students of expected ability lower than the lowest true ability.

Suppose $\hat{a}_L > a_L$. Take a school that has students of true ability a_L (i.e. a positive mass of students of abilities $(a_L, a_L + \epsilon)$ for any positive ϵ). Since the school does not produce any students of ability below $\hat{a}_L$, it has to "bundle" students in the interval $(a_L, a_L + \epsilon)$ with higher ability students $(0 < \epsilon < \hat{a}_L - a_L)$. But then, since $Q(\hat{a})$ is increasing and convex, the school would increase the average desirability of placements of its students by "unbundling" these low ability students—contradiction.

D Proof of Theorem 1

Suppose $Q_1 \neq Q_2$. From Lemma 5 we know that they are defined on intervals starting at a_L, and $Q_1(a_L) = Q_2(a_L)$. Let a_D be the first point where these functions begin to differ, i.e. $a_D = sup\{a | \forall a' < a, a' \geq a_T \; Q_1(a') = Q_2(a')\}, a_D \geq a_T$. Notice that at least one of these equilibria (without loss of generality, the first one) has to be unfair at a_D (otherwise they would both be fair and thus identical), and so $Q_1(a)$ has to be unfair (and thus linear) on some interval (a_D, x) (If no such x exists, there must exist a decreasing infinite sequence $x_n \to a_D$ such that Q_1 is fair at x_n for every n. By assumption, Q_T does not switch from convexity to concavity infinitely often, and so for some n' Q_T has to be convex or concave on $[a_D, x_{n'}]$. If it is convex, then mixing is not profitable and the equilibrium has to be fair at a_D; if it is concave, then Q_1 can not be convex—it equals Q_T at $x_{n'}$, $x_{n'+1}$, and $x_{n'+2}$—contradiction). Let a_1 be the largest x such that $Q_1(a)$ is linear on (a_D, x). Take point b such that $Q_2(b) = Q_1(a_1)$.

There are three cases.

- Case 1. $b < a_1$.
- Case 2. $b > a_1$, Q_2 is linear at a_D. Then its slope has to be less than that of Q_1. Take point a_1'—the largest x such that Q_2 is linear on (a_D, x), and point b' such that $Q_1(b') = Q_2(a_1')$. Then $b' < a_1'$.
- Case 3: $b > a_1$, Q_2 is not linear at a_D. Then there exists a point a_1'' arbitrarily close to a_D at which Q_2 is fair and is less than Q_1. Take b'' such that $Q_1(b'') = Q_2(a_1'')$. $b'' < a_1''$.

The common feature of all three cases is that we were able to find a point at which ability is truthfully revealed in one equilibrium, and the corresponding wage is higher at another. We now get a contradiction for case 1; for other cases the proof is analogous.

Let $Q^* = Q_1(a_1) = Q_2(b)$. By construction, in the first equilibrium every position below Q^* is filled by someone of higher expected ability than in the second equilibrium. Thus, the total ability of people working at positions below Q^* is higher in the first equilibrium. But let us now look closely at these two populations. By Lemma 4, in the first equilibrium this population is just a group of least able students, since the equilibrium is fair at a_1. In the second equilibrium it is some mix of the least able students and more able students. But since the total masses of two populations are equal (they are equal to the share of positions below Q^*), the second one has average ability at least as high as the first one—contradiction.

E Proof of Theorem 3

Suppose the transcript structures form an equilibrium, and there is no unraveling. We then show that no student has an incentive to deviate, i.e. to sign a contract earlier than T. Consider an arbitrary school i. Let the interval of expected abilities of students at school i at time T be $[a_i, b_i]$.[26] By the law of iterated expectations, no student at school i can have expected ability outside of this interval at any time $t \leq T$. Take any time $t < T$ and any student from school i who has expected ability $\hat{a}_t$ inside the interval at time t. If he signs now, the best position he can get is of desirability $Q(\hat{a}_t)$. If he waits

[26] Proof is the same if the interval is open at one or both of the ends.

until time T, the expected desirability of position he gets is $E[Q(\hat{a}_T|\hat{a}_t)]$. By assumption, at time T the school produces a positive density of students on an interval, and transcript structures form an equilibrium—thus, $Q(\hat{a}_T)$ is convex on the interval. $E[\hat{a}_T] = \hat{a}_t$, and so $E[Q(\hat{a}_T|\hat{a}_t)] \geq Q(\hat{a}_t)$, and the student does not have an incentive to deviate.

Now suppose the transcript structures do not form an equilibrium, and there is information that a school (say, school i) would like to suppress. This means that by removing this information, and mixing students who are differentiated by it, the school would achieve a higher payoff. But this implies that there exist three points $\varphi_1 < \varphi_2 < \varphi_3$ inside the interval $[a_i, b_i]$ and weight α such that $\varphi_2 = \alpha\varphi_1 + (1-\alpha)\varphi_3$ and $Q(\varphi_2) > \alpha Q(\varphi_1) + (1-\alpha)Q(\varphi_3)$. But then there exists some point φ strictly inside the interval $[a_i, b_i]$ such that $Q''(\varphi) < 0$ (otherwise the function would be convex).

Since φ is strictly inside the interval $[a_i, b_i]$, there exists some $t_1 < T$ such that a positive mass of expected abilities is produced in a small ϵ-neighborhood of φ for any $t \in [t_1, T]$. By assumption, $\hat{Q}(a,t)$ is twice continuously differentiable; also, $\hat{Q}(a,T) = Q(a)$. Therefore, there exists $t_2 < T$, $t_2 \geq t_1$ such that $\hat{Q}(a,t)''_{aa}(\varphi, t) < 0$ for all $t \in [t_2, T]$. Finally, there exists $t_3 < T$, $t_3 \geq t_2$ such that $\sigma(\varphi, t) > 0$ for all $t \in [t_3, T]$.

By construction, $\hat{Q}$ is a martingale, and therefore $E[d\hat{Q}(a,t)] = 0$. By Ito's lemma, $0 = E[d\hat{Q}(a,t)] = \frac{1}{2}\sigma^2\hat{Q}''_{aa} + \hat{Q}'_t$. For $t \in [t_3, T]$, $\frac{1}{2}\sigma^2\hat{Q}''_{aa}(\varphi, t) < 0$, and so $\hat{Q}'_t(\varphi, t) > 0$. But this implies that $Q(\varphi) = \hat{Q}(\varphi, T) > \hat{Q}(\varphi, t_3)$, and so at time t_3 a student of expected ability φ strictly prefers unraveling and immediately matching with a position of desirability $Q(\varphi)$ to waiting until time T and getting, in expectation, $\hat{Q}(\varphi, t_3)$, while the employer is indifferent between unraveling and waiting until time T.