A Simple Model of Firm Heterogeneity, International Trade, and Wages

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Abstract: In this paper, I develop a simple model to explore the connection between international trade, productivity, and the wage premium. The model recreates several important stylized facts concerning the within industry distribution of productivity, the propensity of the most productive firms to export, and the tendency of exporters to pay higher wages. I then show that when trade barriers between countries are reduced, productivity, both within and across industries, rises. In addition, freer trade increases the premium paid to the most highly skilled workers and reduces the premia paid to more moderately skilled workers. Hence, trade, even between identical countries, can cause a disappearance of "good manufacturing jobs, paying good wages."

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Causal observation suggests that firms within any given industry differ considerably in their abilities. Some firms grow and prosper while others do not. Some firms export to other markets while others do not. In recent years international trade economists have gone beyond the causal observation that firms that engage in international trade are in some sense better than those that do not. Using highly disaggregated plant level data, a number of recent papers have shown that even within narrowly defined industries not all firms participate in export markets and those that do tend to be larger, use more advanced technologies, appear to be more productive, and tend to pay higher wages than those that do not.[2]

This paper contributes to a growing theoretical literature that attempts to explain the systematic differences between exporters and non-exporters and to understand the economic implications of international trade in the presence of such heterogeneity. Much of the existing literature on this topic generates firm heterogeneity by assuming that firms are assigned their productivity levels randomly. Having been assigned their productivity levels, the economic environment then determines which of these firms exit, which produce for the local market, and which expand into international markets.[3] Our point of departure is quite different. Firms are identical when born, are free to produce with technologies that differ in their characteristics, and are free to hire workers who vary in their skill. Firm heterogeneity arises in our model not because of random productivity draws but because firms have made different choices in equilibrium. We show that our

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[2] See for instance, Bernard and Jensen (1997 and 1999). It should be noted that even within narrowly defined industries there is considerable technological variance so there is always the possibility that firm heterogeneity with respect to exports may reflect aggregation error and so might instead be consistent with comparative advantage. That this phenomenon appears in developing countries buttresses the argument that exporters really are different animals than non-exporters. Note also that this empirical finding is consistent with an older literature that multinational firms tend to be "better" than non-multinationals.

[3] For recent models fitting this description see Melitz (1999) and Bernard et al (2001).

model generates many of the same desirable predictions as models based on random productivity draws and delivers another set of predictions that relates international trade to changes in the wage distribution.

Our point of departure is to posit that firms in a monopolistically competitive industry can choose between two technologies that differ in terms of their fixed and unit cost and can choose among workers who vary continuously in terms of their skill. To fix ideas, suppose that a recent advance has lead to the creation of a technology that allows production at a lower unit cost relative to an older technology. Adopting this technology is costly, however, requiring a firm to incur a higher fixed to begin production. Further, suppose that there exists a distribution of workers that vary in terms of their skill. Of two workers with different levels of skill, the more skilled worker has an absolute advantage working with all technologies but a comparative advantage in the newer, low unit cost technology.[4] A possible equilibrium outcome in this framework is that there will coexist firms using the old technology and firms using the new technology with the numbers of each type of firms reflecting the scarcity of skill in the population.

Now suppose that international trade is costly, requiring both a fixed cost to enter and an iceberg transportation cost.[5] In the presence of such a fixed cost, only firms that can hope to sell a large quantity in the foreign market will see entry into the export market as desirable. In our model, these firms are exactly those that have chosen to use the low unit cost technology. Hence, firms that export will be larger, more productive, use more advanced technology, and pay higher wages than those that do not. These

[4] This assumption has empirical merit. See, for instance, Bartel and Lichtenberg (1987) and Bartel and Sicherman (1999) for evidence supporting this assumption.
[5] The existence of fixed costs to exporting has been well documented. See, for instance, Robert and Tybout (1997).

predictions are each consistent with the stylized facts and are wider in scope than those generated by models that rely solely on random productivity draws to generate firm heterogeneity.

In our setting, a reduction in transport costs between countries increases the incentive for firms to adopt the new, lower unit cost technology. As a result, a larger number of firms adopt the new technology while the absolute number of firms in the industry falls. In addition, total employment in the industry falls as the industry consolidates and the least skilled workers leave the industry for employment elsewhere, so that observed labor productivity, both within and across industries, rises as a result of this reallocation. Hence, the model yields many of the same predictions as those that are based on the random productivity draws.

Skill heterogeneity in the labor force, a distinguishing characteristic of our model, gives rise to yet another prediction: a reduction in trade frictions between countries, even two identical countries, will tend to raise the relative demand for skilled workers and the average skill level of workers across all industries. Thus, our model's additional implication is relevant to the literature on trade and wages, a topic on which many earlier models of trade and firm heterogeneity is silent.[6] The message that emerges from our model is that skill biased technical change and opportunities for international trade may interact and so cannot be considered mutually exclusive explanations for the growing disparity between the wages of skilled and unskilled workers.[7]

[6] A very notable exception is Bond (1987) who considered a Heckscher-Ohlin setting with a continuum of workers who vary in their skill. A major difference between our paper and his is that our imperfect competition setting allows us to distinguish between exporters and non-exporters.
[7] For a review of the literature on trade and the growing wage gap, see Slaughter (1998) and Feenstra and Hanson (2001).

In building a model in which firms are equally productive ex ante, we do not mean to argue that random differences across firms are unimportant. Much of the variation in the productivity levels across firms probably defies any systematic explanation. We do believe, however, that some component of firm heterogeneity might plausibly be explained by choice of technology. Recent empirical research on international specialization suggests that productivity levels appear to vary considerably across countries. While the sources of these differences are the matter of much dispute, they are at least suggestive of the coexistence of different technologies even within the same industry. If technologies can coexist internationally it may also be plausible that they coexist domestically. Moreover, that changes in the trade environment might spur changes in technology choice has been a conjecture that has appears in the trade and wages literature.[8] Finally, these two explanations for firm heterogeneity, technology choice and random productivity draws, need not be mutually exclusive and could in principle be combined.

Before continuing, we should note that we are not alone in exploring the possibility that trade induces firms to change their choices of technology. A similar framework can be found in Ekholm and Midelfart-Kvarnik (2001) who consider the effect of trade and technology choice in a reciprocal dumping setting. They find, however, that the model is analytically intractable and are forced to rely heavily on simulations results that limit the extent to which they can flesh out the implications of their model. In contrast, our monopolistic competition setting with fixed costs to international trade is highly tractable.

[8] See for instance, Wood (1994). Lawrence (1999) considers at least part of this conjecture in an exercise that illustrates the difficulty of estimating the hypothesis that trade induces skill upgrading.

The remainder of the paper is organized into four sections. In the first, I outline the main features of the model. In the second, I characterize the model's equilibrium. In the third, I consider the effect of a reduction in international trading costs on the allocation of factors across firms and industries and derive the implications of expanding world trade for the wage distribution. The final section concludes.

I. The Model

The model is kept as simple as possible to highlight the role that trade has on adoption of technology, and its effects on resource allocation and factor prices. In the interest of tractability, I consider only trade between two identical countries where trade is motivated by economies of scale and monopolistic competition. I describe below only the characteristics of the representative country with it understood that the other country is identical.

I.1 Goods and Preferences

Consumers have identical preferences over two types of goods, X and Y. Good X is differentiated by variety while good Y is a homogeneous good, which will serve as the numeraire. They are given by

$$U = (1-b)\ln Y + b \ln X \;\; where$$
$$X = \left[\int_0^N x(i)^a \, di\right]^{\frac{1}{a}} \;\; and \;\; a = \frac{s-1}{s},\; s > 1. \tag{1}$$

Consumers have Cobb-Douglas preferences over Y and the composite good X, and Dixit-Stiglitz preferences over varieties of X. As usual, σ is the elasticity of substitution across varieties of X and the homogeneous good, Y, is the numeraire. Since I consider only trade between identical countries, only varieties of X will be traded internationally.

I.2 Workers

In each country there is a continuum of workers with mass, L. Workers are differentiated by their skill level, which I will index by Z. A larger value of Z corresponds to a more skilled worker. I assume that the allocation of skills in the population is described by the density function, $\mu(Z)$, which is defined over zero to infinity,[9] and satisfies

$$\int_0^{\infty} m(Z)dz = 1.$$

Each worker's unit of labor is inelastically supplied to the labor market, which I assume to be perfectly competitive.

I.3 Production Technology

The crucial feature of the model is that relatively skilled workers have a comparative advantage working with new technologies. Since there is only one factor in the model, I begin by describing the productivity levels of workers with different skill levels across goods. The amount of a good a worker of skill Z can produce is given by $a_i(Z)$ where the index "i" varies with the good and production technique used by a worker. A skilled worker is more productive than an unskilled worker, so $a_i(Z)$ is continuous and increasing in Z for all activities. Let the subscript H denote the new (or high) technology used in production of the X good and let the subscript L denote the old (or low) technology used in the production of the X good. The subscript Y denotes the technology used in the production of Y.

The assumptions over the productivity levels of a worker of skill Z are

$$\begin{gathered} a_H(0) = a_L(0) = a_Y(0) = 1 \\ \textit{and} \\ \frac{a'_H(Z)}{a_H(Z)} > \frac{a'_L(Z)}{a_L(Z)} > \frac{a'_Y(Z)}{a_Y(Z)} > 0. \end{gathered} \tag{2}$$

These assumptions are consistent with worker comparative advantage based on skill. Highly skilled workers have a comparative advantage in high technology production in the X sector relative to moderate and low skilled workers and moderately skilled workers have a comparative advantage in the X sector relative to low skilled workers. Also note that for all but the least skilled worker, the new technology (subscripted H) is lower unit cost than the old technology (subscripted L).

Firms are free to enter in both sectors, but in the X sector, they must first bear a fixed cost. A producer of a variety of X must invest a fixed cost θ to use the old technology, ϕ to use the new technology, and λ to export a good. The fixed cost to exporting is consistent with the need to learn about the characteristics of the foreign market and the potential need to alter a products characteristics to better suit that market.

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[9] It should be understood that this interval can be quite large but is finite at some upper bound.

I assume that $\phi>\theta$ so that the new technology is more costly to adopt than the old technology. All of these fixed costs are in terms of X output. A firm choosing to export abroad, after incurring the fixed cost, λ, must also bear a unit transport cost, which take the standard iceberg form. For one unit of a good to arrive, $\delta>1$ must be shipped.

II. Equilibrium

There are many steps to characterizing the equilibrium. We begin with the most difficult part, which is to explain the distribution of labor across activities in the economy and the associated factor prices. To do this we assume that all three firm types (X-sector firms using the new technology, X-sector firms using the old technology, and Y sector firms) exist in equilibrium and ask what is the labor market equilibrium that would correspond to the existence of all three firm types. We later derive the conditions under which this equilibrium would exist and characterize this equilibrium.

II.1 Labor Market Equilibrium

In this section, we propose an allocation of workers to the three different firm types if all three types of firms coexist in equilibrium, leaving to the next two sections the task of deriving the conditions under which these firms coexist. The first step is to show that firms in the X-sector that use the new technology hire the most skilled workers, firms in the Y sector hire the least skilled workers, and firms in the X sector using the old technology hire the moderately skilled workers in the interior of the distribution. In the process we also derive the wage distribution that must obtain given the coexistence of each firm type.

Consider an equilibrium in which Y sector firms, X sector firms using the low technology (subscripted L), and X sector firms using the new technology (subscripted H), each produce a positive quantity of output. To produce output these firms will need to hire a positive quantity of labor. Partition the distribution of laborers into three segments defined by thresholds Z_1 and Z_2 where $Z_2>Z_1$. Suppose that for $Z<Z_1$ workers are employed only by Y sector firms, that for $Z\in[Z_1, Z_2]$ workers are employed only by old technology X sector firms, and that for $Z>Z_2$ workers are employed only by new technology X sector firms.

In a competitive labor market equilibrium the wage paid to a worker of type Z, W(Z), must be set so that the unit cost of all firms using the same production technology must be the same. Define $C_Y(Z)=W(Z)/a_Y(Z)$ to be the unit cost of a firm using the Y-sector technology when employing a worker of skill Z. Since the unit cost of all Y sector firms must be the same, $C_Y(Z)=C_Y(0)$ for $Z<Z_1$. The wage that makes this so is

$$W(Z) = \frac{W(0)}{a_Y(0)} a_Y(Z) \equiv a_Y(Z) \ \ for\ Z \leq Z_1. \tag{3}$$

Note that because we normalize $P_Y=1$, $W(0)/a_Y(0)=1$ by our assumptions in (2).

Now define $C_L(Z)=W(Z)/a_L(Z)$ to be the unit cost of production for a firm using the X-sector old technology incurred from employing a worker of skill level $Z\in[Z_1, Z_2]$. Again, the unit costs must be the same for all workers using the technology, so $C_L(Z)=C_L(Z_1)$, where a worker of skill Z_1 is made to be indifferent to being employed by either an X-sector low technology firm or a Y sector firm. The wage that satisfies this no arbitrage condition is

$$W(Z) = W(Z_1)\frac{a_L(Z)}{a_L(Z_1)} = \frac{W(0)}{a_Y(0)}\frac{a_Y(Z_1)}{a_L(Z_1)}a_L(Z)$$
$$\equiv S(Z_1)a_L(Z) \;\; for \; Z \in [Z_1, Z_2]. \quad (4)$$

To obtain this expression we used (3) to find $W(Z_1)$ and defined $S(Z_1)=a_Y(Z_1)/a_L(Z_1)$. By our assumptions in (2), $S'(Z_1)<0$.

Finally, define $C_H(Z)=W(Z)/a_H(Z)$ to be the unit cost of production for a firm using the X-sector high technology incurred from hiring a worker of skill level $Z>Z_2$. The no arbitrage condition in this case is given by $C_H(Z) = C_H(Z_2)$. Using the definition, the no arbitrage condition, and the fact that the marginal worker must be indifferent between working for either type of X-sector firm, we find the equilibrium wage must be

$$W(Z) = W(Z_2)\frac{a_H(Z)}{a_H(Z_2)} = \frac{W(0)}{a_Y(0)}\frac{a_Y(Z_1)}{a_L(Z_1)}\frac{a_L(Z_2)}{a_H(Z_2)}a_H(Z)$$
$$\equiv S(Z_1)A(Z_2)a_H(Z) \;\; for \; Z \geq Z_2. \quad (5)$$

We obtain (5) by using (4) to find $W(Z_2)$ and by defining $A(Z_2) = a_L(Z_2)/a_H(Z_2)$. Note that $A'(Z_2)<0$ via our assumptions in (2).

If the labor market were in the equilibrium we have proposed, then the wage as a function of skill would be given by equations (3)-(5). It is continuous, increasing in Z, and its slope becomes steeper at every threshold. Finally, the wage function is completely characterized by the two thresholds, Z_1 and Z_2.

Proposition One: *If all three firm types produce positive levels of output in equilibrium, then there exists thresholds Z_1 and Z_2, where $Z_2>Z_1$. For $Z<Z_1$, all workers are employed in the Y sector, for $Z>Z_2$ all workers are employed in the X sector at firms using the new technology, and for $Z \in [Z_1, Z_2]$ all workers are employed in the X sector at firms using the old technology.*

Proof: Suppose the wage gradient is given by (3)-(5). We show that a profit-maximizing firm would never hire a worker outside its interval. Consider the case where X-sector firms using the old technology hires workers that are outside $[Z_1, Z_2]$. Let $Z' \in [Z_1, Z_2]$ and $\underline{Z}>Z_2$. Suppose a firm could lower its cost by hiring a worker corresponding to $\underline{Z}$ rather than a worker with skill Z' so that $C_L(\underline{Z})<C_L(Z')$. Using (4)-(5), $C_L(\underline{Z})<C_L(Z') \Rightarrow A(Z_2)<A(\underline{Z})$, but $A'(Z)<0$ so this cannot be true. Now suppose $\underline{Z}<Z_1$. $C_L(\underline{Z})<C_L(Z') \Rightarrow S(Z_1)>S(\underline{Z})$, which cannot be true because $S'(Z)<0$. Hence, given the wage gradient (3)-(5) a X sector firm using the old technology increases its production cost if it hires a worker with $Z \notin [Z_1,Z_2]$. The same analysis establishes that Y sector firms hire only $Z<Z_1$ and to X sector firms using the new technology hire only $Z>Z_2$.QED

Conditional on the existence of firms of all three types, Proposition One establishes that workers sort across firms according to their comparative advantage. Given that workers are indeed sorting themselves in the manner proposed, then the wage function must be given by (3)-(5) in order to satisfy the no arbitrage condition for the competitive labor market.

II.2 Equilibrium Conditions in the X sector

We now turn our attention to the decisions facing firms in the X sector. These firms first decide whether to enter, and then, if they enter, they must choose which technology to use. Finally, firms must decide whether to export. Note that because the two countries are identical, we need only consider the equilibrium conditions in one of the two countries.

As is well known, consumer maximization of (1) yields the following demand function for each of the two countries

$$x(i) = bEp(i)^{-s}\ P_X^{s-1},\ where\ P_X = \left[\int_0^N p(i)^{1-s}\ di\right]^{\frac{1}{1-s}}. \qquad (6)$$

The demand for an individual variety is increasing in expenditure on X goods, E, decreasing in its own price, p(i), and increasing in the industry price index, P_X. Note that E represents the expenditure in only one country. Also well known is that firms maximizing profits subject to (6) will, if they are small relative to the market, charge $p(i)=C(i)/\alpha$.

We now turn to the zero profit conditions. Using the fact that a firm's profits net of fixed cost is proportional to its revenue, we can obtain the following four free entry conditions for the four possible firm types:

$$X_L - (\sigma-1)\theta \leq 0 \qquad (7a)$$

$$X_L(1+\delta^{1-\sigma}) - (\sigma-1)(\theta+\lambda) \leq 0 \qquad (7b)$$

$$X_H - (\sigma-1)\phi \leq 0 \qquad (7c)$$

$$X_H(1+\delta^{1-\sigma}) - (\sigma-1)(\phi+\lambda) \leq 0 \qquad (7d)$$

where X_L indicates the domestic sales of firms using the old technology and X_H indicates the domestic sales of firms using the new technology. Sales in the identical foreign market are $X_L\delta^{-\sigma}$ and $X_H\delta^{-\sigma}$ for firms using the old and new technology respectively. The level of sales of old and new technology firms differ because the firms face different unit costs. Using (6), the constant mark-up, the definition of unit cost, and the wage distribution (3)-(5), it can be shown that

$$\frac{X_L}{X_H} = \left(\frac{C_H(Z_1, Z_2)}{C_L(Z_1)} \right)^s = A(Z_2)^s < 1 . \qquad (8)$$

Looking back at the zero profit conditions in (7), it is clear that firms that use the old technology and do not export (7a) have the lowest fixed costs but also enjoy the lowest sales. Firms that use the old technology and export (7b) have higher fixed costs but sell more than old technology firms that serve only the local market. Firms that use the new technology but serve only the local market (7c) have higher domestic sales than old technology firms that serve only the local market but have higher fixed cost. Finally, firms that adopt the new technology and also sell in the foreign market (7d) have the largest sales and the largest fixed costs.

Compare the increase in profits associated with exporting to the foreign market relative to serving only the local market. This magnitude of this increase (7b minus 7a versus 7d minus 7c) depends on whether the firm is producing with the old or new technology. The magnitudes are compared in the following expression:

$$X_L d^{1-s} - (s-1) l < X_H d^{1-s} - (s-1) l .$$

Because firms that adopt the new technology sell more than firms that do not, they gain more from incurring the fixed cost of exporting than do firms that do not adopt the new technology. This means that if there are firms that use different technologies in equilibrium, then exporting by old technology firms necessarily implies that new technology firms are also exporting. The converse is not true, however.

It can easily be confirmed that new technology firms export and old technology firms do not if the following condition is met [10]

$$f > l d^{s-1} > q \; . \tag{9}$$

We have assumed that new technology firms have higher fixed costs than old technology firms so $\phi>\theta$. If the fixed cost of exporting and/or the transport cost are high, then old technology firms do not to export. If the fixed cost of entry with the new technology is sufficiently high relative to the trade costs, both fixed and variable, then all firms that adopt the new technology also export.[11] We summarize this finding in the following proposition.

Proposition Two***:*** *If (9) holds, then only firms that adopt the new technology export in equilibrium.*

That firms with new technology export while old technology firms do not is intuitively sensible. With a fixed cost of exporting, firms must make enough profit from selling abroad to justify the additional expense. Firms using the new technology can profitably sell more units and make the fixed cost worth incurring. We assume for the

[10] To derive this condition set the variable profit gain to exporting to be greater than the fixed cost for new technology firms and less than the fixed cost for old technology firms, and then plug in the free entry conditions from (7a) and (7b).

remainder of the paper that (9) holds. Henceforth, we refer to firms that adopt the new technology as exporters and firms that do not as non-exporters.

We now derive the number of exporters and non-exporters that enter in equilibrium as a function of the thresholds, Z_1 and Z_2. First, consider the number of firms in the X sector that adopt the new technology, N_H. We can solve for N_H by using the constraint that employment by exporters is a function of share of the labor force above the threshold, Z_2. This condition is given by

$$N_H\left(X_H(1+d^{1-s})+f+l\right)=L\int_{Z2}^{\infty}a_H(z)m(z)dz\equiv LH(Z_2)$$

where $H(Z_2)$ is decreasing in Z_2. The effective labor supply used by firms using the new technology (Right Hand Side) is equal to the number of those firms, multiplied by their output, $X_H(1+\delta^{1-\sigma})$, plus the fixed costs, which are in terms of X output. (Recall that δ must be shipped for one unit to arrive). Using (7d), this expression simplifies to

$$N_H=\frac{L}{s\left(f+l\right)}H(Z_2)\,. \qquad (10)$$

Repeating this exercise for the number of firms that enter the X sector but do not adopt the new technology, N_L, is

[11] Because $\phi>\theta$, if old technology firms export ($\theta>\lambda\delta^{\sigma-1}$), then new technology firms must export while the converse is not true.

$$N_L = \frac{L}{sq}\int_{Z1}^{Z2} a_L(z)m(z)dz \equiv \frac{L}{sq} J(Z_1, Z_2) \qquad (11)$$

where $J(Z_1,Z_2)$ is increasing in Z_2 and decreasing in Z_1 so that N_L is increasing in Z_2 and decreasing in Z_1.

Proposition Three*: Firms that export differ from that do not in that (1) they have higher sales, even in the local market, (2) they pay higher wages, and (3) their average labor productivity is greater.*

Proof: For part one, compare X_H and X_L, the local sales of each firm type. $X_H/X_L=(C_L/C_H)^{\sigma}=A(Z_2)^{-\sigma}>1$ from our assumptions in (2). Part two follows directly from the wage distribution (3)-(5). Every worker at an exporter is paid more than the best-paid worker at a nonexporting firm. To see that part three is true compare the average productivity per worker in exporting firms to nonexporting firms, which is by definition $N_H X_H(1+\delta^{1-\sigma})/L_H$ versus $N_L X_L/L_L$. L_H is the total number of workers at exporting firms and L_L is the total number of firms at nonexporting firms. Using (7a), (7d), (10), and (11) it can be shown that

$$\frac{N_H X_H \left(1+d^{1-s}\right)}{L_H} = a \frac{\int_{Z2}^{\infty} a_H(z)m(z)dz}{\int_{Z2}^{\infty} m(z)dz} = a \int_{Z2}^{\infty} a_H(z)\left(\frac{m(z)}{\int_{Z2}^{\infty} m(z)dz}\right)dz$$

$$\frac{N_L X_L}{L_L} = a \frac{\int_{Z1}^{Z2} a_L(z)m(z)dz}{\int_{Z1}^{Z2} m(z)dz} = a \int_{Z1}^{Z2} a_L(z)\left(\frac{m(z)}{\int_{Z1}^{Z2} m(z)dz}\right)dz$$

Hence, the labor productivity of each firm type is a weighted average of each worker employed in the sector. Since every worker at an exporting firm is more productive than any worker at a non-exporting firm, exporting firms must have higher average labor productivity than non-exporting firms. QED

Proposition three demonstrates that the model can reproduce the stylized facts that have emerged from the empirical literature on within sector heterogeneity and international trade. Exporters differ from nonexporters because they choose a lower variable cost technology. Because skilled workers are better able to use the new lower cost technology, exporters hire more highly skilled workers. Subsequently, these firms are more productive, produce more, and pay higher wages than nonexporters.[12]

Before concluding this section, we establish a condition under which both exporters and nonexporters exist in equilibrium. By dividing (7a) by (7d) and rearranging using the fact that $X_H/X_L=A(Z_2)^{-\sigma}$, we can establish the following condition that must hold in equilibrium if both firm types exist.

$$A(Z_2) = \left[\frac{\phi + \lambda}{\theta\left(1+\delta^{1-\sigma}\right)}\right]^{-\frac{1}{\sigma}} \tag{12}$$

The left hand side of (12) is decreasing and continuous in Z_2. Hence, if the right hand side is less than the left for $Z_2=0$ and exceeds the left hand side as Z gets large, then an equilibrium can exist in which both old tech firms coexist with new tech firms.[13] Moreover, (12) pins down Z_2, which is clearly increasing in δ, λ, and ϕ, and decreasing in θ. This establishes the following proposition.

Proposition Four*:* *A small decrease in the cost of exporting, either in terms of the fixed cost or in terms of the transport costs, shifts more of the labor force into X sector firms using the new technology.*

[12] Note that because of the competitive labor market, the distribution of labor productivity across firms within each firm type is random.
[13] Using (8), it can be shown that the crossing point must lie in $Z_2>0$.

Proposition Four follows from the fact that adopting the new technology and exporting are complementary activities. Given that the complementarity and reduction in the cost of exporting induces a larger number of firms to adopt the new technology. As more firms adopt the new technology, a larger share of the work force becomes employed at exporting firms. It is immediately obvious that this shift in employment to exporting firms is accompanied by an expansion of the volume of trade between the two countries.

II.3 Equilibrium in the Y Sector

I now complete the characterization of the equilibrium. The previous section tied down all we needed to know about the allocation of resources in the X sector so we can now close the model by specifying the Y sector.

Demand for the good Y follows directly from the Cobb-Douglas preferences and from the fact that P_Y=C_Y=W(0)=1 so

$$(1-b)E = Y . \tag{13}$$

The amount of effective labor required to satisfy this level of demand is defined by

$$Y = L\int_0^{Z1} a_Y(z)m(z)dz \equiv LG(Z_1) . \tag{14}$$

All that remains to close the model is to solve for income, E, and then set (13) equal to (14). There are no profits so the value of expenditure is the sum of labor income or

$$E = L\int_0^\infty W(z)m(z)dz \qquad (15)$$

By rearranging (15), and by substituting in the wage distribution in (3)-(5), we obtain the following expression for expenditure.

$$E = L\left[G(Z_1) + S(Z_1)\big(J(Z_1, Z_2) + A(Z_2)H(Z_2)\big)\right]$$
$$\textit{where}$$
$$G(Z_1) = \int_0^{Z1} a_Y(z)m(z)dz \qquad (16)$$
$$J(Z_1, Z_2) = \int_{Z1}^{Z2} a_L(z)m(z)dz \ \textit{and}$$
$$H(Z_2) = \int_{Z2}^{\infty} a_H(z)m(z)dz$$

By substituting (16) into (13) and setting the resulting expression equal to (14), we obtain the labor market clearing condition for the Y sector. This function is

$$b\,\frac{G(Z_1)}{S(Z_1)} = (1-b)\big(J(Z_1, Z_2) + A(Z_2)H(Z_2)\big). \qquad (17)$$

This completes the specification. As noted before, (12) pins down Z_2 as a function of the parameters. Given Z_2, (17) pins down Z_1, which is unique because the left-hand of (14) is strictly increasing in Z_1 while the right-hand is strictly decreasing.

I assume that the Z_1 that satisfies (17) is less than the Z_2 given by (12).[14] In this equilibrium, the least skilled workers work in the Y sector, moderately skilled workers work in the "old tech" X sector firms, and the most skilled workers work for the "new tech" firms. Only "new tech" firms export so it is the exporting firms that are the most productive and pay the highest wages as given by (5). Y is not traded in equilibrium because there is no comparative advantage motivation for trade.

III. The Effect of a Reduction in Trading Costs between Nations

I now consider the effect of a reduction in trading costs between the two countries or a reduction in δ. Much of our work has already been done for us in the form of Proposition Four, which shows that Z_2 is decreasing with a decrease in δ. What remains to be done is to analyze how the rest of the endogenous variables respond.

Proposition Five: *A reduction in the cost of shipping goods across countries, d, reduces the share of the labor force employed in the X sector.*

Proof: Note that (17), the equilibrium condition for the Y sector is not directly a function of δ, so that a change in Z_1 caused by a change in δ must occur indirectly through Z_2. Totally differentiating (17) with respect to Z_1 and Z_2 yields

$$\frac{dZ_1}{dZ_2} = -(1-b)\frac{A'(Z_2)H(Z_2)}{J'_{Z1}(Z_1,Z_2) + bS'(Z_1)\dfrac{G(Z_1)}{S(Z_1)^2}} < 0\,.$$

Z_1 and Z_2 move in opposite directions. Since Z_2 falls with a reduction in δ as shown in Proposition Four, Z_1 must increase. An increase in Z_1 is a shift of workers out of the X sector. QED

[14] This is more likely to be true when β is large and must be true as β goes to one.

Since all the remaining variables of interest are functions of Z_1 and Z_2 and since we have shown that a reduction in transport cost increases Z_1 and decreases Z_2, Propositions Four and Five completely determine the effects of a reduction of transport costs on the equilibrium. All that remains to be done is to trace through the economy the effects on other variables of interest.

Proposition Six: *A reduction in the cost of shipping goods across countries (1) causes the number of exporting firms to expand and the number of nonexporting firms to contract, and (2) causes an increases average labor productivity in the X sector.*

Proof: Part (1) follows directly from equations (10) and (11). To prove part (2) we first calculate average labor productivity in X, given by $X_T/L_X=(N_HX_H(1+\delta^{1-\sigma})+N_LX_L)/L_X$, where L_X corresponds to all workers with $Z>Z_1$. Following the proof of proposition five, it can be shown that

$$\frac{X_T}{L_X}=a\frac{\int_{Z1}^{Z2}a_L(z)m(z)dz+\int_{Z2}^{\infty}a_H(z)m(z)dz}{\int_{Z1}^{\infty}m(z)dz}, \qquad (18)$$

Equation (18) states that average labor productivity in the X sector is a weighted average of average labor productivity in exporting and non-exporting firms. The reduction in δ has two effects, a decrease in Z_2 and an increase in Z_1. It is immediate that the decrease in Z_2 must increase average labor productivity by shifting workers from non-exporters to exporters. Less obvious is the effect of an increase in Z_1. An increase in Z_1 also raises average labor productivity because the workers cast out of the X sector are the least productive, which can be confirmed by differentiating (18). QED

The result presented in Proposition Six is similar to that in Melitz (2000) in that trade increases productivity and leads to a reduction in the number of relatively less productive firms. The mechanism is entirely different, however. While productivity differences between firms in the Melitz model are randomly drawn, the productivity differences in this model are due to the conscious decision of firms to adopt a new technology. Trade changes the incentives toward firms to adopt this technology and has a reallocative effect among the factors in the economy.

Now notice that because Z_1 has risen, the least skilled workers have left the X sector and entered the Y sector. These workers, while low skilled compared to the X sector, are highly skilled compared with the Y sector. Hence, the average labor productivity in the Y sector has also risen. This effect is somewhat reminiscent of Feenstra and Hanson (1996) in that it is the changing composition of employment in activities that is of interest. Note, however, the mechanism is very different in their setting where outsourcing from developed to developing countries raises the average skill intensity in both countries. We summarize this result in the following proposition.

Proposition Seven: *A reduction in transport costs increases the observed labor productivity in all sectors, including the non-traded sector.*

The goods of the Y-sector are not traded in equilibrium because there is no comparative advantage motive for trade. Given that the good is not traded, we could attach an alternative interpretation that the Y sector is a non-traded service industry while X is a traded manufactured good. The reduction in transport costs reduces X-sector employment because it encourages firms to adopt the new technology. The surplus workers must then find employment in the less productive service sector.

Note that the skill composition of both the X and Y sector has risen as a result of the reduction in transport costs because Z_1 has increased. Hence, the effect of increased opportunities for international trade is to create the appearance of skill biased technological change. This phenomenon has been widely reported by empirical researchers seeking to understand the connection between trade and wages. Note, however, that while the average level of skill of workers in the X sector has risen, the average level of skill of exporters in the X sector has fallen. Observing this differential response of skill composition within the tradeable good sector would require data disaggregated by firm.

I now turn to effect of the reduction in transport cost on the wage-skill distribution as given by (3)-(5).

Proposition Eight: *A reduction in transport costs increases the wage of the most highly skilled members of society, $Z>Z_2$, and does not effect the wage of the least skilled workers, $Z<Z_1$. Moderately skilled workers, or those that remain employed in the X sector producing with the old technology or who have become employed in the Y sector, must see their wage fall.*

Proof: From (4), it is clear that for workers initially on $Z\in(Z_1,Z_2)$ that continue to employed in the X sector using the old technology must see their wage fall. This follows from the fact that $S'(Z_1)<0$. It is also clear from (3) that the wage of the least skilled in terms of the Y good is unchanged because (3) is not a function of the thresholds.

Now consider the wage of a worker with skill $Z\geq Z_2$. This wage is given by (5). Totally differentiating (5) yields

$$\frac{dW(Z)}{W(Z)}=\frac{S'(Z_1)}{S(Z_1)}dZ_1+\frac{A'(Z_2)}{A(Z_2)}dZ_2, \tag{19}$$

where the primed function are partial derivatives. We know from earlier comparative static that Z_1 is increasing and Z_2 is decreasing with a reduction in transport cost so (19) cannot be signed without additional information. To sign (19), we use our conditions for the Y sector. By (14), $dY/dZ_1>0$, so by (13), E must increase. Totally differentiating (16), the expression for E, setting it to be greater than zero, and rearranging the resulting expression yields:

$$dE>0\Rightarrow\frac{S'(Z_1)}{S(Z_1)}dZ_1>-\frac{A(Z_2)H(Z_2)}{J(Z_1,Z_2)+A(Z_2)H(Z_2)}\left[\frac{A'(Z_2)}{A(Z_2)}dZ_2\right].$$

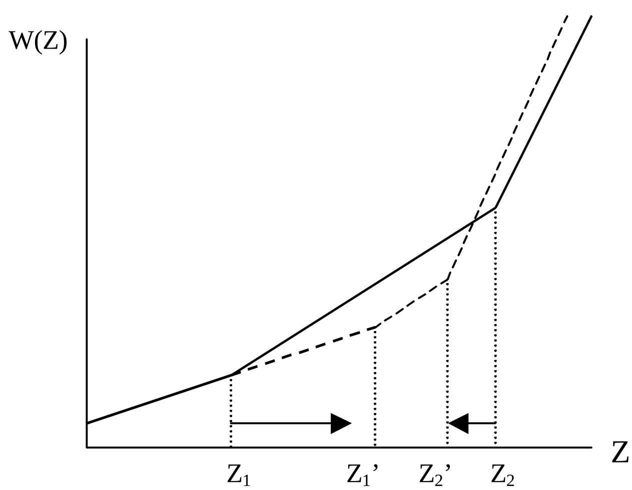
Adding $\frac{A'(Z_2)}{A(Z_2)}dZ_2$ to both sides and rearranging yields

$$\frac{S'(Z_1)}{S(Z_1)}dZ_1+\frac{A'(Z_2)}{A(Z_2)}dZ_2>\frac{J(Z_1,Z_2)}{J(Z_1,Z_2)+A(Z_2)H(Z_2)}\left[\frac{A'(Z_2)}{A(Z_2)}dZ_2\right]>0$$

The left-hand side of this expression must be positive so (19) must be positive. Hence, the wage of workers initially using the high technology in the X sector must see their wage rise. QED

The original wage gradient is shown in Figure 1 as the solid line. This is the wage of a worker of skill Z relative to the price of the numeraire Y good. An increase in skill is more valuable when a worker is using a more productive technology so that the slope of the wage as a function of skill changes at the threshold technologies.

Figure 1



A reduction in the cost of shipping goods across countries reduces the threshold Z_2 to Z_2'and increases the threshold Z_1 to Z_1'. This has the effect, as shown in the Figure 2, of changing the wage function from the solid to the broken line. Workers who were initially using the Y industry technology continue to use that technology after the reduction in transport costs and hence do not see their wage change relative to the least skilled member of society.

It is the moderately skilled people who see their status in society eroded. Workers who are thrown out of the X sector are less productive in the Y sector and hence earn less relative to the least skilled member of society than they had before the increase in trade. The increase in Z_1 reduces the wages of those who remain in employed with non-exporting X sector firms that use the old technology because the marginal worker cannot be paid more than that worker is worth in sector Y.

Proposition Eight is interesting because it is consistent with the commonly heard complaint that good manufacturing jobs are disappearing and that the displaced workers can only find lower paying jobs in the service sector. The result is also consistent with the common complaint that the workers who are most likely to be affected are those that are moderately skilled, such as those with only a high school education. Finally, the story told here is also consistent with the observation that workers increasingly require at least rudimentary analytic skills to be employed in manufacturing.

While it is clear that the economic "status" of workers of different skill levels has changed, we have not yet said anything about their real income. To learn about the effects of increased opportunities on real income we need to know how prices of goods in the X sector have changed. The price index of X goods in terms of the Y good can be written

$$P_X = \frac{1}{a}\left\{N_H(1+d^{1-s})C_H^{1-s} + N_L C_L^{1-s}\right\}^{\frac{1}{1-s}}. \quad (20)$$

By substituting for N_H and N_L, using (10) and (11), and using (12), (3), and (17), we can obtain the following expression for the price index

$$P_X = \left[\frac{bL}{(1-b)sq} \right]^{\frac{1}{1-s}} \frac{S(Z_1)^{\frac{s}{s-1}} G(Z_1)^{\frac{1}{1-s}}}{a} . \qquad (21)$$

It is clear from (21) that the relative price of X goods is decreasing in Z_1, because $S'(Z_1)<0$ and $G'(Z_1)>0$. Since a reduction in δ leads to an increase in Z_1, a reduction in transport costs leads to a fall in the relative price of X goods.

Proposition Nine: *A reduction in transportation costs unambiguously makes the least ($Z<Z_1$) and most skilled ($Z>Z_2$) workers better off. Moderately skilled workers will only benefit from a reduction in transport costs if they value X sector goods highly relative to Y sector goods.*

Proof: From our earlier analysis, we know that the wage relative to the Y good of the least skilled workers does not change while the wage of the most highly skilled rises. Only the wages of the moderately skilled workers fall relative to the Y good. By differentiating the wage function (3)-(5) and the price index (21) the following observation can be made summarizing the changes in wages and price of the X good in terms of the Y good.

$$(A) \quad \frac{dP_X}{P_X}\frac{1}{dZ_1} < 0 = \frac{dW(Z)}{W(Z)}\frac{1}{dZ_1} \; if \;\; Z < Z_1$$

$$(B) \quad \frac{dP_X}{P_X}\frac{1}{dZ_1} < \frac{dW(Z)}{W(Z)}\frac{1}{dZ_1} < 0 \; if \;\; Z \in [Z_1, Z_2]$$

$$(C) \quad \frac{dP_X}{P_X}\frac{1}{dZ_1} < 0 < \frac{dW(Z)}{W(Z)}\frac{1}{dZ_1} \; if \;\; Z > Z_2$$

The least and most highly skilled workers must be better off because their wage does not fall while the price of the X good falls, (A) and (C) respectively. The case of the moderately skilled workers, given by (B), is not clear, however. Their wage has risen relative to the price of X but has fallen relative to the price of Y. If workers consume little Y, then they are better off, while if they consume little X, then they are worse off. QED

The reduction in transport costs represents a real gain to society because more resources are available for producing final consumption. The distribution of these gains across factors can only be analyzed using general equilibrium analysis.

IV. Conclusion

The model presented in this paper achieved three tasks. First, using very few, plausible assumptions over the costs of international trade, the comparative advantage of skilled workers using new and old technologies, and the adoption costs of these technologies we showed in proposition three that exporting firms are more productive, are larger, and pay higher wages than firms that do not export. These results are consistent with the stylized facts have been observed in detailed microdata (see Bernard and Jensen, 1997, 1999 and 2001). Second, the model showed that a reduction in trade frictions between countries can lead to an expansion of trade volumes, an increase in observed aggregate productivity, an increase in the wage premia paid to the most highly skilled workers and a decrease in the wage premia paid to moderately skilled workers. Hence, the model succeeded in establishing a link between the literature on trade and productivity, associated with Melitz (2000) and Bernard et al (2001), and the return to skill. Finally, the model suggests that observed skill biased technical change could find its roots in trade in differentiated goods between identical economies. Hence, it is entirely plausible that increased opportunities for international trade are behind at least some of the apparent skill biased technical change.

Note that while we have framed the analysis in terms of choice of technology there is another plausible interpretation that could be given to the model. In place of choice of technology, one could alternatively consider choice of quality. If higher quality goods require a higher fixed cost to produce than low quality goods, and if skilled workers have a comparative advantage at producing higher quality goods, then the same qualitative results would follow. The only difference in this case would be that observed TFP differences reflect unobserved quality differences across firms.

Finally, the model presented here was kept as simple as possible to highlight the issues. One element of this simplicity was that the model is static. A dynamic model could also be proposed in which firms choose their technology at the beginning of a period, produce in the middle of the period, and find that their technology becomes standardized at the end of the period. While the qualitative results would be unchanged, a dynamic framework might allow additional heterogeneity to be built into the analysis, which would have the advantage of making the individual firm a more important unit of observation. If there are sunk costs to exporting, for instance, then it seems likely that firms that choose to export today are also more likely to adopt new technologies more quickly and to export in subsequent periods. We leave this extension to future work.

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