MARKET VALUATION AND MERGER WAVES

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ABSTRACT

Does valuation affect takeovers? The data suggests that periods of merger activity are correlated with high market valuations and that during these periods firms use stock in acquisitions. If bidders are simply overvalued then targets should not accept the offers. However, we show that private information on both sides can rationally lead to a correlation between stock merger activity and market valuation. We assume bidding firms have private information about the synergistic value of the target. All firms have a market price that is possibly over or under the true value of their firm as a stand alone entity. The target's and bidding firm's private information tells them whether they are over or under valued but not why (whether it is market (sector) or firm specific misvaluation). Thus, target firms cannot distinguish whether high bids are due to synergies, relative target under-valuation or bidder over-valuation. A rational target is unwilling to accept a takeover bid with expected value less than the true value of their firm. Consequently, the target uses all available information in an attempt to filter out the misvaluation from the bids. The rational target correctly filters on average but underestimates the market wide effect when the market is overvalued and over estimates the effect when the market is undervalued. Thus, the target rationally assesses high synergies when the market is overvalued or they are relatively undervalued and accepts more bids leading to merger waves. Furthermore, the market learns from watching the takeover market and slowly readjusts prices until they realign with fundamental value. Thus, we are able to explain a number of empirical puzzles with a simple fully rational model.

MARKET VALUATION AND MERGER WAVES

One of the puzzles in finance is why there are periods where mergers are plentiful and other periods where merger activity is much lower. For example, in the period 1963-64 there were 3,311 total acquisition announcements, while in 1968-69 there were 10,569 acquisition announcements. Similarly in both the period from 1979-1980 and from 1990-91 there were approximately 4,000 acquisition announcements while the late 80s and late 90s were much more active, with 9,278 announcements in 1999 alone.[1] These periods of high activity seem to be correlated with high market valuations, as shown by Jovanovic and Rousseau (2001). For example, 1998-2000 saw over $1.5 trillion in announced deals per year while 2001, after the market correction, saw half as much. Furthermore, the data suggest that firms tend to use stock in these high activity/high stock market periods as bidders use their stock as an "acquisition currency". In 1990 the percentage of stock as a fraction of total deal value was only 24%, while by 1998 the use of stock peaked at 68% of total deal value![2] Also, Martin (1996) shows that firms that use stock in acquisitions have lower book-to-market ratios than those that use cash. Stock deals were especially common in the high flying high tech sector where most takeovers involved securities. From 1996 to 2000 the Computer Software, Supplies & Services industry group accounted for 16.5% of all transactions, and ex post this industry is widely regarded as having been overvalued.[3] The classic example is America Online's which acquired Time Warner in a stock for stock deal. While the market value of American Online fell on announcement of the merger, the general view today is that America Online got an excellent deal as its stock was overvalued (ex-post the stock has fallen from $73.75 the day before the announcement to $27.28 on 3/11/02). The technology boom also saw companies like Cisco use stock aggressively as a way to undertake mergers.

The inability of financial theory to explain merger waves is noted by Brealey and Myers in their classic textbook *Principles of Corporate Finance*. In their concluding chapter, What We Do and Do Not Know About Finance, they pose the question, "How Can We Explain Merger Waves?" and they cite the need for "better theories to help explain these bubbles of financial activity." We propose that private information on both sides can lead rationally to increased stock merger activity that is correlated with market valuation.

Mergers involving securities are inherently different from cash takeovers as they involve a valuation problem. The target is offered shares in the bidding firm at some exchange ratio. Since the target firm receives shares, they are concerned whether the valuation of these bidder shares is appropriate.

[1] 2001 Mergerstat Review

[2] Source: JP Morgan M&A Research, Thomson Financial Securities Data Company Inc. Based on announced transactions.

[3] 2001 Mergerstat Review

Furthermore, the valuation of bidder shares often changes in response to announcements of the takeover itself. Valuation of bids is sometimes so contentious that courts are used to help determine if the highest bid was accepted.[4] Clearly, valuation is of great practical concern in takeovers and it is difficult to determine the true value of an offer. We build on the idea that targets attempt to value offers with limited information.

Our approach is based on a rational model of stock mergers. In our model, managers of bidding firms have private information about the stand-alone value of their firms and the potential value of merging with a target firm. Managers of targets have private information about the stand-alone value of their company. Both bidders and targets have market values that may not reflect the true value of their companies. Furthermore, the possible misvaluations have two components - a firm specific component and a market wide component. In equilibrium, stock bids reflect the expected level of synergies between the firms. However, the target has limited information about the misvaluations. Thus, the target perceives a bid to be high if the synergies are high, the bidding firm's stock is overvalued or the target is *relatively* undervalued.

The target management, on observing the bidders' fractional offers decides whether to accept or reject a bid. The bid contains no information about market over or under valuation because all firms are affected by market factors to the same extent. However, the target knows about their own misvaluation and is therefore not easily fooled. Thus, target management knows whether it's firm is over or undervalued but cannot distinguish whether this is a market (or sector specific) effect or a firm effect. Fiduciary responsibility requires the target management to accept any offer that, given management's information, yields more than the stand alone value. Hence, target management's decision is based on its assessment of synergies given the bids and its own information. A positive assessment of the offered synergies results in acceptance of the bid (and vice versa).[5] It is this assessment of synergies that is critical in our model.

Since the target's information and the bidder's bid are both positively related to the market wide component of the misvaluation, the target attempts to filter out the market (or sector) wide misvaluation effect. Consequently, the target's assessment of synergies is positively related to the bid but negatively related to his own reservation value (the target's true stand-alone value). This is because when the target is overvalued he expects that some of this overvaluation is due to a market wide effect and some to a firm specific affect. The rational target, is of course, right on average. However, the more the market is overvalued the larger the target's expectation of his firm specific

[4] See Paramount-Viacom A and B, University of Chicago, by Steve Kaplan. Hietala, Kaplan and Robinson (2001) also analyze the Paramount-Viacom merger in detail.

[5] Support for positive synergies has been found by every major study. See Andrade, Mitchell and Stafford (2001) for a summary of the literature. Recently, Hou, Olsson and Robinson (2000) find that synergies are positive even in a long run analysis.

misvaluation because he cannot tell these affects apart. Therefore, the target filters out of the bids too little of the market wide effect when the market is overvalued and too much when then market is undervalued (as mentioned, getting it correct on average). Therefore, the bids tend to look better when the market is overvalued. The target is not irrational, he can simply do no better given his information.

The opposite effect occurs when the target is overvalued because of firm specific reasons. Since the target cannot tell the difference between firm specific and market wide effects, the target expects that some of this overvaluation is due to each effect. Therefore, the more overvalued he is the larger his expectation of the market wide effect. Thus, when the target is relatively overvalued he filters too great a market wide effect out of the bids, making the bids look low.

Therefore, our theory predicts that mergers are more likely to occur in overvalued markets, and relatively undervalued targets are more likely to sell. We have not assumed that synergies are higher in boom times. Nor have we assumed that some managers are willing to sell their firm for less than it is worth. Nor is it the case that some managers have limited rationality. Instead there is a simple explanation: the target is concerned that instead of synergies, bidders have overvalued stock or that the target is relatively undervalued. Thus, the target uses all available information to get an expectation of the offered value. The target is on average correct and thus increases his firm's value by accepting those offers that exceed his reservation price in expectation. However, if the market is overvalued then the target is more likely to overestimate the synergies, and if only the target is overvalued then he is more likely to underestimate the synergies.

Thus, our theory is a Myers and Majluf (1984) set up such that overvalued bidders make high stock bids. The stock merger market does not collapse because some bidders have positive synergies. In addition, the target (buyer of the stock) has some noisy information about the bidder's (who is selling stock) valuation. This leads to mistakes that are correlated with valuation.

Throughout this analysis we allow multiple bidders. Multiple bidders provide the target with more information about synergies. This is because the target can filter, to some extent, the common misvaluation of the bidders. We show that the target's assessment of any bid decreases if the bids of other bidders increase, i.e., high bids by other bidders signals an increased likelihood of a high market wide misvaluation. The key limitation of the information from other bids is that synergies can be correlated across firms.

We extend our model in two ways. First we consider a sequence of mergers and show that following the acceptance of a bid the market assesses downwards the value of firms, while following the rejection of a bids the market assesses upwards the value of firms. This process can be slow because of the noise inherent in the information available to managers and the market. This suggests that the merger wave is self correcting as the increased activity alerts the market to the market-wide overvaluation which reduces prices and ends the wave. The opposite reaction occurs if the market is undervalued.

We also examine the market's reaction to the merger announcement. We assume that the target's stock holders only allow the target's management to accept a bid if the stock holders (the market) perceive that the bid is above the current value of the target. This effect reduces the ability of bidders to use cash in boom times. Management will still not accept an offer if it is below their stand alone value. Thus, cash bids are less likely to be accepted in boom times because of stock holders and management is less likely to accept a stock bid in bad times. This leads to waves of stock and cash use in mergers. This result is consistent with Andrade, Mitchell and Stafford (2001) who find a much larger positive announcement effect on the target for cash offers and a less negative effect on the acquirer. We suggest that it is not that cash mergers are better than stock mergers, but rather that cash mergers occur in undervalued markets. So, the rational market updates and increases stock prices.

Our paper differs from the other approaches to merger waves. Jovanovic and Rousseau (2001, 2002) build on Gort (1969) and provide complete information models of merger waves that are based on technological change and the Q-theory. Mergers correspond to the purchase of used capital and merger waves occur when there is reallocation across sectors. Consequently, high Q-firms buy low Q-firms. We suggest it may also be that overvalued firms buy undervalued firms. Furthermore, they show that deregulation is not able to explain merger activity, and argue that deregulation is a result of the desire of firms to merge and not the other way around. Shleifer and Vishny (2001) provide a more behavioral story of merger waves where it is common knowledge that the market is mis-priced but will correct itself in the long run. When bidders are over-valued merger waves with stock occur because some managers care only for the short run market price (which does not adjust for the overvaluation of bidders) while others care for the long run value (they are essentially issuing cheap equity to get something valuable). Holmstrom and Kaplan (2001) summarize the research that suggests, and simultaneously argue, that corporate governance issues led to the merger waves of the 80s and 90s. Persons and Warther (1997) provide a symmetric information model of financial innovation where the value of the innovations is positively correlated and there is learning. Hence successful adoption by other agents increases one's estimate of the innovation leading to more adoption, i.e., clustering occurs. Further, a sequence of unsuccessful adoptions leads to a shutdown of the financial innovation. Although this is not a theory of merger waves it is a theory of sporadic activity.

Our theory shows that merger waves can occur solely because of valuation issues. However, we want to emphasize that our theory does not imply that the desire to merge could not be caused by innovation, deregulation, or corporate governance issues, etc. Rather, we suggest that valuation impacts mergers and merger waves regardless of the underlying motivation for the mergers. Furthermore, we demonstrate why any merger may involve cash versus securities in a rational framework.

The paper is organized as follows. Section I contains the general model. Section II demonstrates the equilibrium and considers how firms bid, how the target chooses the winner and the winner's payment. Section III examines the target's reservation price. Section IV shows how merger can occur in waves. Section V looks at the difference between short run and long run managers and the market's reaction. Section VI explores the possibility that some bidders can bid in cash. Section VII concludes.

I The Model

The basic model of a merger is an English (open-outcry ascending bid) auction. n risk-neutral firms with synergistic values for a target firm bid in the auction.[6] The risk-neutral target firm, firm T, considers the bids and decides whether to accept an offer. After the auction the market reacts (as anticipated by the bidders and target). Then, in the last period, the value of all firms, including the joint firm (if the merger has occurred) are realized.

A bidding firm, firm i, has a private value V_i for firm T. This is the true value of firm T, X_T, multiplied by a factor that represents the synergy $(1 + s_i)$,

$$V_i = X_T(1 + s_i).$$

$s_i > -1$, but may be positive or negative, so V_i may be greater than or less than the target's stand alone value X_T. Thus, merging the firms could add value (positive s_i) or destroy value (negative s_i). However, the bidding firm does not know the true value of the target, X_T or the synergy, s_i. Instead the firm only knows their own value for the target as a merger partner, V_i. All participants in the auction believe that the synergies and thus the merger values are independently and identically distributed and drawn from the distribution $F_s(s)$. Thus, this is an independent private value auction.[7]

The bidding firm also has private information about the value of their own assets, X, where firm i has value $X_i > 0$.[8] The target and the market do not know X_i, however, both see the current market value of the firm, M_i. M_i does not necessarily equal X_i because it is possible that the market has misvalued the assets. We assume that there are two forms of misvaluation: market (or sector) wide misvaluation and firm specific misvaluation. Thus,

$$X_i = M_i(1 - \varepsilon_i)(1 - \rho)$$

[6] $N = \{1, ..., n\}$ represents the set of n bidders.

[7] This assumption does not preclude a common component in the bidders' values that is known to all of the bidders. Section III will consider synergies that contain a common and firm specific component $(1 + s_i) = (1 + u)(1 + \omega_i)$.

[8] X_i could alternatively represent the bidder's beliefs about their true value.

where ρ represents the market wide misvaluation and is the same for every firm, and ε_i is the firm specific misvaluation. Thus, if ρ or ε_i are positive then the market is over valued, and if they are negative then the market is under valued. ρ, $\varepsilon_i < 1$ and are drawn iid zero mean from $F_\rho(\rho)$ and $F_\varepsilon(\varepsilon)$ respectively.[9] M_i is drawn iid from $F_M(M)$. ρ, ε_i and M_i are all independent. Therefore, $E[X_i \mid M_i] = M_i$, i.e., on average the market correctly prices the firms.

Target firms also have a stock or market value, M_T. This market value is known to the bidding firms. The target, however, also knows the true stand alone value of his assets, X_T. This value is different from the market value because of the same two forms of misvaluation that affect bidders: market wide misvaluation, ρ, and firm specific misvaluation, ε_T. Thus,

$$X_T = M_T(1 - \varepsilon_T)(1 - \rho)$$

where ρ is the same common component that affects the bidders, and ε_T is specific to the target. Since the target, firm T, has a stand alone value this value functions as a reserve price. Thus, firm T may not be sold.

Although each firm knows if they are under or over valued, they do not know ρ or ε_i. Bidders also have information through M_i, X_i, V_i, and M_T about the target's, and every other bidder's true value, but they do not know the true value of any other player. Therefore, they cannot risklessly arbitrage a market wide misvaluation by trading other firms' stock. We assume that some form of limited arbitrage allows equilibrium misvaluation. Management is simply not the marginal investor.[10]

Since the firms know V with certainty they are not concerned with the target's superior information about X_T. If the true value of the synergy depended on the target's information,[11] then the bidding firm would need to worry about whether the target accepts the offer. i.e. the target will accept if the offer is too high. This type of adverse selection is the focus of interesting papers by Fishman (1989) and Hansen (1987), but is not considered here.

To begin we assume that all firms must bid using only their own equity. However, in Section VI we consider the possibility of cash bids. The assumption of limited cash is justifiable if raising cash is costly. We might think that since firms are misvalued, the lemons problem prevents firms from selling their own stock for cash. This same problem does not collapse the merger market because some firms have large synergies and the target firm has information superior to the market. Section VI does consider the effect of some firms using cash, and shows why there may be waves of stock or cash mergers.

[9] Samuelson (1987) suggests the inclusion of a firm specific error in a comment on Hansen (1985).

[10] This is true even in their own stock. We assume that adverse selection and SEC insider information rules prevent managers from buying and selling their own under or over valued stock in large enough quantities to restore efficient pricing.

[11] For example, if s_i were known by the bidder, but not V_i.

An equity bid consists of an offer of fraction α_i of the joint firm. After the auction, if the bid is accepted the total firm value will be $X_i + V_i$. Thus, the winning firm will realize $(1 - \alpha_i)(X_i + V_i)$, and the payment to the target, firm T, will be $\alpha_i(X_i + V_i)$. However, since the target and the market know neither X_i nor V_i, they will rationally value any offer at $E[\alpha_i(X_i + V_i) \mid \Phi_T]$, and $E[\alpha_i(X_i + V_i) \mid \Phi_M]$, where Φ_T and Φ_M represent the Target's and Market's (differing) information set. The contents of this information set will be considered further below. Throughout the paper the players will rationally update the value of any offer and the market will update and condition values on all available information. Thus, even though bids may be misvalued, in expectation all prices will reflect expected values. Figure 1 provides an overview of the model. (Insert Figure 1 here)

After the target sees the bids, the target must decide which bid to accept if any. We assume that fiduciary responsibility rules require the target to accept only the offer with the highest expected value.[12] However, we consider three different rules that could be used to decide if even the best bid should be accepted. One assumption is that the target must reject (accept) any offer with long-run expected value less (greater) than X_T. Thus, we are assuming that managers are long run value maximizers. With this assumption we can generate merger waves. In an extension we consider short run managers who accept any offer with current market value above the current value of their stock, M_T. This could also be thought of as a hostile takeover rule if shareholders accept any offer (against managements' wishes) with market value above their current market value. Finally, we will consider restricted long-run managers who, for agency reasons, cannot accept an offer with current market value below the current market value of their stock, but will also not accept an offer unless its expected long run value is greater than X_T. This restricted manager will cause waves of the use of stock or cash in mergers.

II Equity Auction with Misvalued Stock

In an English auction with equity bids, firms bid by stating a fraction α of the joint firm which they will give to the owners of the target firm. The high bid is clearly not necessarily from the firm who states the highest fraction, α. This is because a firm with very substantial assets (IBM) must bid a lower α than a low asset firm (local computer vender) even if their values for firm T are the same. Furthermore, the target's incomplete information about the true value of the bidder's assets, X_i, means that the rule used to rank the bids will be a function of the bidder's stock market value, M_i, but not X_i. The target also has information about his own misvaluation and information from the other bids. Thus, the scoring rule is a function of the target's private information and all the firms'

[12] This assumption rules out revenue enhancing rules (when $F(\cdot)$is not regular) that require the seller to commit to accepting lower valued offers to encourage better types to bid higher.

bids,

$$Z_i \equiv g(\alpha_i, \Phi_T) \equiv g(\alpha_i, M_i, \alpha_j, M_j \; \forall j \neq i, M_T, X_T), \tag{1}$$

where the highest Z wins the auction.[13]

Since this is an English auction the winning firm only has to bid high enough to beat the second highest firm. Thus, the fraction the winner must pay does not depend on what they are willing to bid, but rather it depends only on what the second highest bidder bid. However, it is unnecessarily confusing to keep track of what firms actually have to bid to win. Instead we can use a standard auction trick. We will consistently focus on the fraction that bidder i is willing to bid. We will put this fraction into the scoring function to determine if bidder i will win. Separately, we can determine what fraction bidder i *actually* has to bid, $\widehat{\alpha}_i$ to beat the next largest Z-score. This is helpful because many results only require us to know who wins and only a few results require us to know what they actually had to bid to win.

The remainder of this section will determine the equilibrium in the auction and the necessary assumptions. The reader who is less interested in the formation of the equilibrium is invited to skip to Section III.

To examine the equilibrium we will first consider the bids, then the probability of winning, and then the expected payment.

II.1 Bids

To determine who wins the auction and what they pay, we must first determine how high bidders are willing to bid. When the target is not accurately informed about the asset value of the bidding firms, the natural inclination is to think that firms who are over valued by the market bid a much larger fraction α than firms that are correctly valued. Surprisingly, this section will show that under a large class of reasonable scoring rules this is not the case. In fact, all bidders will bid the true largest fraction that they would ever be willing to pay, $\frac{V_i}{X_i+V_i}$.[14]

In order to determine the equilibrium bids, we must put some more structure on the scoring function. The following Lemma shows the condition that is both necessary and sufficient for bidders to bid up to $\frac{V_i}{X_i+V_i}$.

[13] We assume that every bidder is scored in a symmetric fashion.

[14] Remember, we mean that all bidders are *willing* to bid up to the largest fraction they would ever be willing to pay, $\frac{V_i}{X_i+V_i}$. In the actual auction they will stop bidding when they have beaten the second highest bidder (who did actually bid up to $\frac{V_j}{X_j+V_j}$).

Lemma 1 *In an English auction wherein bidders pay what they say, whenever two bidders are tied, i.e.,*

$$g(\alpha_i, \Phi_T) \equiv g(\alpha_i, M_i, \alpha_j, M_j, \alpha_k, M_k\ \forall k \neq j, M_T, X_T) \quad (2)$$
$$= g(\alpha_j, M_j, \alpha_i, M_i, \alpha_k, M_k\ \forall k \neq j, M_T, X_T) \equiv g(\alpha_j, \Phi_T)$$

if $g(\cdot)$ is continuous in every bidder's α and

$$\frac{\partial g}{\partial \alpha_i}(\alpha_i, \Phi_T) > \frac{\partial g}{\partial \alpha_i}(\alpha_j, \Phi_T)\ \forall\ \alpha, M, \quad (3)$$

then bidders are willing to increase α until

$$\alpha_i = \frac{V_i}{X_i + V_i}. \quad (4)$$

Furthermore, condition (3) is both necessary and sufficient for bidders to bid up to the truth.

Proof. See appendix. ■

The intuition is that firms are simply not willing to bid so that if they win they pay more than V. And while they will happily bid less than V they will not stop increasing their bid unless they win the auction, their payment exceeds V, *or* raising their bid lowers the chance that they win. It might seem that as long as $\frac{\partial g}{\partial \alpha} > 0$ then raising the bid always increases the chance of winning (this is true if X_i is known). However, firms also have to worry about the possibility that increasing their bid effects the scores of other bidding firms by altering the target's information about the market wide misvaluation. The only relevant moment occurs when a firm is currently tied with another firm.[15] Either tied firm only has the incentive to raise his bid as long as doing so increases his own score more than the other firm's score (or decreases his own score less than the other firm's score). If this is true, then firms will always continue to increase their bid until they reach $\alpha_i = \frac{V_i}{X_i+V_i}$.

Note that many odd scoring rules satisfy condition (3). Some targets may be tempted to rank bids as is often done in newspapers where the target's and buyer's market values are added to together and multiplied by α. This scoring rule satisfies condition (3) but it ignores a great deal of the target's information. The next section will focus on the scoring rule that chooses the highest bid.

II.2 Choosing the Winner

Thus, firms are willing to bid up to $\frac{V_i}{X_i+V_i}$ even though the market has misvalued their assets as long as condition (3) holds. We now focus on how the target will choose the winner, i.e. the equilibrium scoring rule. Once we have the target's equilibrium scoring rule we can find a slightly stronger but more primitive condition on the distribution of the misvaluation that will ensure that firm's bid up to $\frac{V_i}{X_i+V_i}$ and will allow us to demonstrate why merger waves occur.

[15] Continuity ensures that as α increases, bidders tie before they beat another bidder.

Throughout the paper we will use a standard auction theory trick to simplify the exposition. We will refer to a firm's bid as the highest amount he is willing to bid. This is not the amount a bidder actually has to bid to win. To win, a bidder only has to outbid the second highest bidder. However, given that we know the highest amount they would be willing to bid we can determine who would win and then find out what they actually had to bid by looking at the second highest bid. For example, in a standard cash auction we might determine that bidder 1 is willing to bid up to $12 and bidder 2 is willing to bid up to $14. We can then easily see that bidder 2 will win and will only pay $12. Thus, throughout the paper we will refer to a bidder's bid as the highest amount they would be willing to bid, so $\alpha_i = \frac{V_i}{X_i+V_i}$.

Even though each firm bids $\alpha_i = \frac{V_i}{X_i+V_i}$, the target cannot determine V_i because the target does not know X_i. Therefore, the target must award the firm to the highest score, $Z = g(\alpha, \Phi_T)$, which may not be the firm with the highest value. As we will see in a moment, this is true even when the target uses all available information.

Ex-ante the market's best estimate of the true asset value of a firm is the firm's market value. However, the target has information about his own misvaluation. Since part of the misvaluation is the same for every firm, the target has a better estimate of X_i than M_i. Even before the auction, the target's estimate of any firm's value is $E\left[X_i \mid M_i, X_T, M_T\right]$. Thus, if the target is overvalued he assumes (correctly on average) that part of this is due to market wide effect and part is due to a firm specific effect.[16]

After the bids, the target updates his expectation of X_i. A firm whose bid is scored high (not necessarily the same as a high α) is more likely to be over valued. The target must then decide the probability that the firm is overvalued versus the probability that the firm has a large synergy. For example, let bidder firm 1 have a market value $M_1 = \$100$. If he bids $\alpha_1 = 20\%$ then it might be the case that he values firm T at at least $25. Or it might be the case that the true value of his assets are less than $100. If his assets $X_1 = \$80$ then he only needs to value firm T at $20 in order to be willing to bid 20%. Thus, the question the target must ask is what is the probability that the bidder firm has a high synergy ($V_1 \geq \$25$) or that the bidder firm is overvalued ($M_1 > X_1$). If the probability of a high V_1 is low, then it is more likely that $M_1 > X_1$, and the target will revise down his expectation of X_1, and therefore he will also expect lower synergies.

Therefore, given the bid $\alpha_i = \frac{V_i}{X_i+V_i}$ the best estimate that the target has about the value of the firm's assets is

$$E\left[X_i \mid \alpha_j, M_j \ \forall j, X_T, M_T\right]. \tag{5}$$

The target can also determine an expected value that the bidding firm has for the target. The target

[16] Remember, we assumed some limits to arbitrage. Thus, managers do not have the ability to complete enough risky arbitrage trades to ensure the market is correctly valued.

once again uses all of his information

$$E\left[V_i \mid \alpha_j, M_j\ \forall j = 1..n, X_T, M_T\right] = X_T E\left[(1 + s_i) \mid \alpha_j, M_j\ \forall j, X_T, M_T\right]. \tag{6}$$

Fiduciary responsibility and/or a lack of commitment require the target to accept the highest offer. This assumption tells us that the largest score should be assigned to the offer with the highest expected value. Therefore, the only equilibrium scoring rule is any monotonic transformation of

$$Z_i = g(\alpha_i, \Phi_T) = E\left[\alpha_i(V_i + X_i) \mid \alpha_j, M_j\ \forall j, X_T, M_T\right]. \tag{7}$$

Remember, we are using a scoring rule that asks who would win *assuming* that each bidder bids the largest fraction he is willing to bid. We will show in a moment that given this scoring rule, it is incentive compatible for each bidder to do so.[17]

It may seem that the target should not be concerned with the true value of the offer as Equation (7) suggests, but rather the target should simply accept the offer with the highest stock value. After all, if the target can immediately sell their stock then shouldn't the target just accept the highest stock offer rather than worry about the true value? Yes. However, with a fully rational market we will show in a moment that these two scoring rules yield the same outcome, i.e., that each rank the same bid as the highest bid.

Lemma 1 tells us that as long as condition (3) holds then the firms will bid $\alpha_i = \frac{V_i}{X_i + V_i}$. We will refer to this fraction as the 'truth' since it is the largest amount that the bidder would actually be willing to pay. If the firms bid the truth then the rational updating scoring rule, Equation (7), becomes

$$Z_i = g(\alpha_i, \Phi_T) = E\left[\alpha_i\left(V_i + \frac{1-\alpha_i}{\alpha_i}V_i\right) \mid \alpha_i, \Phi_T\right] = E\left[V_i \mid \alpha_i, \Phi_T\right]$$

Therefore, since bidders are bidding the truth, the target is attempting to choose the firm with the highest positive synergy value.

Thus, it would seem that all of the information that improves accuracy of the expectation of the synergy, $1 + s_i$, improves the scoring rule. However, the following lemma shows that this is not the case, and $\frac{\alpha_i}{1-\alpha_i}M_i$ is sufficient to rank the bids!

Lemma 2 *If condition (3) holds for scoring rule (7) then*

$$E\left[V_i \mid \alpha_i, \Phi_T\right] = E\left[V_i \mid \frac{\alpha_i}{1-\alpha_i}M_i\ \forall i, X_T, M_T\right] \tag{9}$$

[17] All of our results would hold if we kept track of the fact that the high bidder actually stops bidding at the second highest bid and the target used the scoring rule

$$E\left[\alpha_i(V_i + X_i) \mid \alpha > \alpha_i, \Phi_T\right],$$

where α is the unknown fraction $\frac{V_i}{X_i+V_i}$ and α_i is the fraction needed to win. However, the mathematics of the second price auction are simpler and it is logically equivalent to the English auction.

and $\frac{\alpha_i}{1-\alpha_i}M_i$ *is sufficient information to rank the bids.*

Proof.

$$E\left[V_i \mid \alpha_i, \Phi_T\right] = E\left[X_T(1+s_i) \mid \alpha_i, \Phi_T\right], \tag{10}$$

$$= X_T E\left[(1+s_i) \mid \frac{\alpha_i}{1-\alpha_i}, M_i\ \forall i, X_T, M_T\right] \tag{11}$$

$$= X_T E\left[(1+s_i) \mid \frac{\alpha_i}{1-\alpha_i}\frac{M_i}{M_T}, M_i\ \forall i, \frac{X_T}{M_T}, M_T\right]. \tag{12}$$

If condition (3) holds, then $\alpha_i = \frac{V_i}{X_i+V_i}$, so

$$\frac{\alpha_i}{1-\alpha_i}\frac{M_i}{M_T} = \frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)} \tag{13}$$

$$\frac{X_T}{M_T} = (1-\varepsilon_T)(1-\rho). \tag{14}$$

Therefore,

$$E\left[V_i \mid \alpha_i, \Phi_T\right] = X_T E\left[(1+s_i) \mid \frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)}, M_i\ \forall i, (1-\varepsilon_T)(1-\rho), M_T\right] \tag{15}$$

Furthermore, $\frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)}$, $(1+s_i)$ and $(1-\varepsilon_T)(1-\rho)$ are independent of M_i and M_T. Therefore,

$$E\left[V_i \mid \alpha_i, \Phi_T\right] = X_T E\left[(1+s_i) \mid \frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)}\ \forall i, (1-\varepsilon_T)(1-\rho)\right] \tag{16}$$

$$= X_T E\left[(1+s_i) \mid \frac{\alpha_i}{1-\alpha_i}\frac{M_i}{M_T}\ \forall i, \frac{X_T}{M_T}\right] \tag{17}$$

$$= E\left[V_i \mid \frac{\alpha_i}{1-\alpha_i}M_i\ \forall i, X_T, M_T\right] \tag{18}$$

Note that X_T and M_T do not depend on the bidder and bidders are scored symmetrically. Therefore, the only bidder specific information relevant for the expectation of V_i is $\frac{\alpha_i}{1-\alpha_i}M_i$. ■

This tells us that for the expectation of $(1+s_i)$, $\frac{\alpha_i}{1-\alpha_i}M_i$ is a sufficient statistic for α_i and M_i. Although, M_i and α_i do not add information above $\frac{\alpha_i}{1-\alpha_i}M_i$, this does not mean that the best estimate of V_i is $\frac{\alpha_i}{1-\alpha_i}M_i$, it is not. However, the target cannot tell the difference between a low V with an over valued stock (positive ε or ρ) and a high V with an undervalued stock. So, the target gives the same score to each. The additional information of α_i and M_i add no information because the errors, $(1-\varepsilon_i)$ and $(1-\rho)$, are orthogonal to the realization of the stock price and the α_i.

Lemma 2 tells us that any scoring rule needs only to be a monotonic transformation of $\frac{\alpha}{1-\alpha}M$. Although this may not seem intuitively obvious it is easy to understand why this works. Without misvalued stock, the target would be able to award the firm to the highest offer. However, with misvaluation the target may not be able to tell the difference between a firm with overvalued stock and a firm with truly high synergies, but an undervalued stock. With any offer by the firm, $\frac{\alpha}{1-\alpha}M$

equals the ratio of the offered synergies, V and the error in the stock price. Since everything is drawn iid this ratio provides all available information about the rank of the bid. Thus, the fact that $\frac{\alpha}{1-\alpha}M$ is a sufficient statistic just tells us formally that the target cannot tell between a high bid and overvalued stock and instead he knows only the ratio of the value and the error.

Thus, without synergies adverse selection would collapse the merger market. It may seems that the assumption of condition (3) which yields truthful bidding is assuming away adverse selection; it is not. Adverse selection arrises because at any given score the target must compare the probability that the synergies are high with the probability that the errors are high. If the expected synergies at a given score are negative then the market collapses as the target will not accept a bid with a negative value (the next section will further examine the targets decision). Condition (3) says that on the margin, raising α increases the expectation of V (even if the increase is only to a less negative V). Subsection II.4 will show that condition (3) is similar to but weaker than the standard assumption in auction theory of affiliation.

We asked earlier why the target cares about the true value of the offer. If the target can immediately sell their stock then shouldn't the target accept the highest stock offer rather than worry about the true value? Yes. However, the following corollary tells us that the market's expectation of the highest bid is the same as the target's management.

Corollary 1 *The markets perception of the highest bid and the targets perception of the highest bid are the same.*

Proof. This is a direct consequence of Lemma 2, which showed that $\frac{\alpha_i}{1-\alpha_i}M_i$ is sufficient information to rank the bids. This is still the case if the market evaluates the bids except the market's scoring rule is

$$= E\left[V_i \mid \frac{\alpha_i}{1-\alpha_i}M_i \; \forall i, M_T\right]. \tag{19}$$

■

We will see in a moment that the target's greater information does allow him to get a more accurate expectation of the value of the bid, but Corollary 1 shows that it does not affect the order of the bids.

Thus, this section has shown that the only relevant information available to the market is $\frac{\alpha_i}{1-\alpha_i}M_i$ $\forall i$, and the only additional useful information held by the target is X_T. This information will help us determine what the bidder must actually pay and it will help us show that the scoring rule does satisfy condition (3) as we assumed.

II.3 The firm's Payment

The Z-score tells us which firm will win the auction, but it does not tell us what they pay. Since bidding in an acquisition is like an English auction, the winning firm only has to out bid the second highest bidder. In equilibrium the second highest Z-score is

$$\frac{\alpha_2}{1-\alpha_2} E\left[X_2 \mid \alpha_2, \Phi_T\right] = \frac{\alpha_2}{1-\alpha_2} M_2 E\left[(1-\varepsilon_2)(1-\rho) \mid \alpha_2, \Phi_T\right], \tag{20}$$

where the subscript 2 represents the second highest Z-score, but not necessarily the second highest V. The winner, therefore, must pay a fraction $\widehat{\alpha}$ such that he just receives the same score as the second highest bidder,

$$\frac{\widehat{\alpha}}{1-\widehat{\alpha}} M_1 E\left[(1-\varepsilon_1)(1-\rho) \mid \widehat{\alpha}, \Phi_T\right] = \frac{\alpha_2}{1-\alpha_2} M_2 E\left[(1-\varepsilon_2)(1-\rho) \mid \alpha_2, \Phi_T\right], \tag{21}$$

where the subscript 1 represents the highest Z-score.[18]

However, Lemma 2 demonstrated that the only relevant information for the score is $\frac{\alpha}{1-\alpha}M$. Therefore, for firm 1 to get the same score as the second highest bidder

$$\frac{\widehat{\alpha}}{1-\widehat{\alpha}} M_1 = \frac{\alpha_2}{1-\alpha_2} M_2. \tag{22}$$

Rearranging shows that the highest bidder must bid

$$\widehat{\alpha} = \frac{\frac{\alpha_2}{1-\alpha_2} M_2}{M_1 + \frac{\alpha_2}{1-\alpha_2} M_2}. \tag{23}$$

Substituting $\frac{\alpha_2}{1-\alpha_2} = \frac{V_2}{X_2}$ and $X = M(1-\varepsilon)(1-\rho)$ into Equation (23) shows

$$\widehat{\alpha} = \frac{\frac{V_2}{X_2} M_2}{M_1 + \frac{V_2}{X_2} M_2} = \frac{\frac{V_2}{(1-\varepsilon_2)(1-\rho)}}{\frac{X_1}{(1-\varepsilon_1)(1-\rho)} + \frac{V_2}{(1-\varepsilon_2)(1-\rho)}} = \frac{V_2(1-\varepsilon_1)}{X_1(1-\varepsilon_2) + V_2(1-\varepsilon_1)}. \tag{24}$$

[18] Intuition may suggest that the target should account for the fact that the target does not get to see highest bidder's true alpha, α_i, because the highest bidder only has to out bid the second highest bidder to win. Therefore, the target can get a more accurate estimate of the winners bid and charge him a fraction, $\widehat{\alpha}$, that depends on this more accurate estimate and still just beats the second highest offer.

$$\begin{aligned} &\widehat{\alpha}\left[E\left[V_1 \mid \alpha_1 > \widehat{\alpha}, \Phi_T\right] + E\left[X_1 \mid \alpha_1 > \widehat{\alpha}, \Phi_T\right]\right] \\ &= \alpha_2\left[E\left[V_2 \mid \alpha_2, \Phi_T\right] + E\left[X_2 \mid \alpha_2, \Phi_T\right]\right]. \end{aligned}$$

However, this is not the result of an English auction where bidders must commit to pay what they say. To understand why, consider the case when X is known. Equation (7) is still the scoring rule but the expectations are not needed. Therefore, $\widehat{\alpha} = \frac{V_2}{X_1+V_2}$. If the target then took into account that the winner's V was greater than V_2 he could lower the winner's required fraction $\widehat{\alpha}$ until he paid in expectation V_2. However, the bidder already said a higher fraction so why would the target do this? This does not imply that the target cannot use a scoring rule that accounts for the fact that bidders are willing to bid higher than they say. For example, with the rule

$$\alpha_i E\left[V_i + X_i \mid \alpha > \alpha_i, \Phi_T\right]$$

bidders would still bid up to the truth.

Therefore, winning firm's payment decreases if they are overvalued and increases if the second highest bidder is overvalued.[19]

Equation (24) also makes the following corollary to Lemma 2 very easy to understand.

Corollary 2 *Market wide misvaluation does not affect the equilibrium fraction that any firm is willing to offer, and therefore does not alter which firm offers the highest bid nor the amount they pay.*

Proof. Lemma 1, showed that the bids could be ranked by $\frac{\alpha}{1-\alpha}M$. Substituting for M and α shows that $\frac{\alpha}{1-\alpha}M = \frac{X_T(1+s_i)}{(1-\varepsilon_i)(1-\rho)}$. Thus, the score of every bid is affected by $(1-\rho)$ in the same way and the rank is preserved. Furthermore, Equation (24) shows that the bid fraction required to make the highest offer is unaffected by $(1-\rho)$. ■

Thus, possible firm specific misvaluation alters the firms' payments and who wins, but for given stock valuations, market wide misvaluation does not alter how a fully rational target ranks the bids. If every firm is currently over or under valued by some amount, then the fraction that any firm is willing to offer is unaffected. However, we will see in a moment that both types of misvaluation affect whether the target will accept any bid at all.

II.4 Affiliation

Condition (3) is the weakest condition which ensures firms bid the truth. We have assumed thus far that the target's scoring rule, Equation (7), satisfies this condition. In this section we demonstrate that we can make reasonable primitive assumptions about the distributions of the random variables to ensure that condition (3) holds.

The usual assumption in auction theory is that bidder values are affiliated. Since the firm synergy values are independent it would seem that affiliation is trivially satisfied. However, because of market wide misvaluation, the sufficient statistics used by the target are not independent. In fact, the target learns something about the synergies from looking at his own misvaluation and from looking at all of the bids. Lemma 2 shows if firms bid the truth, then in equilibrium

$$E\left[V_i \mid \alpha_i, \Phi_T\right] = X_T E\left[(1+s_i) \mid \frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)} \; \forall i, (1-\varepsilon_T)(1-\rho)\right] \tag{25}$$

or equivalently

$$= X_T E\left[(1+s_i) \mid \frac{(1+s_i)}{(1-\varepsilon_i)(1-\rho)}, \frac{(1+s_i)(1-\varepsilon_j)}{(1-\varepsilon_i)(1+s_j)} \; \forall j \neq i, \frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)}\right]. \tag{26}$$

[19] Ex-ante the expectation of both errors are zero. However, given that bidder 1 wins the auction $E[\varepsilon_1] > E[\varepsilon_2]$. Therefore,

$$\frac{V_2(1-\varepsilon_1)}{X_1(1-\varepsilon_2)+V_2(1-\varepsilon_1)} < \frac{V_2}{X_1+V_2}$$

and bidder 1's expected payment is lower than if there were no misvaluation. Thus, unsurprisingly, possible firm specific misvaluation lowers the target's expected revenue.

Appendix B shows that if the variables $\log(1-\varepsilon_i)$, $\log(1+s_i)$ $\forall i$ and $\log(1-\varepsilon_T)$ and $\log(1-\rho)$, which are mean zero, have log-concave densities then $(1+s_i)$, $\frac{(1+s_i)}{(1-\varepsilon_i)(1-\rho)}$, $\frac{(1+s_i)(1-\varepsilon_j)}{(1-\varepsilon_i)(1+s_j)}$ $\forall j \neq i$, and $\frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)}$ are affiliated. Thus, all of the sufficient statistics are affiliated with $(1+s_i)$. This essentially means that the expectation of $(1+s_i)$ increases with any of the sufficient statistics. See Milgrom and Weber (1982) for a formal definition of affiliation. Log-concavity is a standard although not completely trivial assumption. Examples of log-concave densities include the multivariate beta, Direchlet, exponential, gamma, Laplace, normal, uniform, Weibull and Wishart distributions. Milgrom and Weber (1982) and others have shown that in general little can be said without some form of the monotone likelihood ratio property.

With this assumption it is easy to show that condition (3) holds.[20]

III Mergers

We have now determined the equilibrium bid and the scoring rule. This section will show how the target's reservation price combined with misvaluation will lead to increased merger activity in overvalued markets.

Since the target has a stand alone value, X_T, the target is unwilling to (and has a fiduciary responsibility not to) accept any offer which delivers less than X_T. The scoring rule examined above demonstrated that the highest expected value offer is from the firm which bids the highest $\frac{\alpha}{1-\alpha}M$. However, the highest offer may have an expected value less than the target's stand alone value. This can occur when expected misvaluations are large relative to expected synergies. In this case the target will refuse the offer and no merger will occur. Thus, the target's acceptance rule is simply to accept any offer such that[21]

$$E\left[V_i \mid \alpha_i, \Phi_T\right] > X_T. \tag{27}$$

The following Theorem shows that this simple rule will cause merger waves.

Theorem 1 *Stock mergers are more likely to occur in overvalued markets than in undervalued markets.*

[20]We can see that this assumption is slightly stronger than condition (3). Affiliation ensures

$$\frac{\partial g}{\partial \alpha_i}(\alpha_i, \Phi_T) > 0 \;\forall\; \alpha, M,$$

since each statistic in the expectation of $(1+s_i)$ increases with α_i. And

$$\frac{\partial g}{\partial \alpha_i}(\alpha_j, \Phi_T) < 0 \;\forall\; \alpha, M,$$

since the expectation of $(1+s_j)$ decreases if α_i increases.

[21]It may seem that the target should condition the expectation of V on the fact that the high bidder would have been willing to bid more than the necessary stopping point. However, this would only allow bidders who would have been willing to bid higher to pay less or allow bidders with offers less than X_T to be accepted. See footnote 18

Proof. Lemma 2 and section II.4 tells us that the target's acceptance rule, equation (27), can be rewritten as

$$E\left[(1+s_i)\mid\frac{(1+s_i)}{(1-\varepsilon_i)(1-\rho)},\frac{(1+s_i)(1-\varepsilon_j)}{(1-\varepsilon_i)(1+s_j)}\;\forall j\neq i,\frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)}\right]>1. \tag{28}$$

The only term which depends on the market wide misvaluation is $\frac{(1+s_i)}{(1-\varepsilon_i)(1-\rho)}$, which is increasing in ρ. The assumption of affiliation ensures that the expectation of the synergy is increasing in $\frac{(1+s_i)}{(1-\varepsilon_i)(1-\rho)}$. However, we must remember that the bidder only had to bid high enough to beat the second highest bidder. Thus, we must make an assumption about the outcome when the second highest bid is not above the reservation rule, Equation (28). Two simple assumptions are that the merger occurs only if the second highest bidder bids above the reservation price or that the merger occurs as long as the high bidder was willing to bid above the reservation price. We are not interested in focusing on how the bargaining takes place and furthermore, it does not matter for our results. Either of the two assumptions mentioned will work as will any assumption such that the probability of a merger occurring increases if the target's reservation price decreases. Therefore, the more overvalued the market (the larger ρ is) the more likely that a bid exceeds the reservation price, and thus a merger occurs (and vice versa for undervalued markets). ■

This paper was designed to provide the simplest model which will yield a rational correlation between valuation and merger activity without assuming the result.[22] It is not the case that synergies are higher in boom times. It is not the case some managers are willing to sell their firm for less than it is worth. Nor is it the case that some managers have limited rationality. Instead there is a simple explanation: the target is concerned that any bidder has overvalued stock rather than a high synergy. Thus, the target uses all available information to get an expectation of the offered value. The target is on average correct and thus increases his firms value by accepting those offers which exceed his reservation price in expectation. However, if the market is overvalued then the target is more likely to overestimate the synergies because he underestimates the market wide misvaluation. Therefore, the target accepts more mergers in overvalued markets and less in undervalued markets.

This does not imply that the target losses money by accepting an offer. In an overvalued market, the target can expect his own stock to fall. Thus, accepting a merger proposal with a positive synergy will reduce the impact on the target when the market corrects. We will also see in a moment that this does not imply that the market has an arbitrage opportunity; the prices will correctly react to news of a merger.

[22] A simple thought experiment will show that each assumption is necessary to deliver this result. Clearly if there is no market wide misvaluation then there can be no market effect. If there is no firm specific error then the target can determine the market wide error with perfect precision. Therefore, the target never misestimates the synergies. Furthermore, there is no lemons problem since a market component effects every bidder in the same way. Thus, although it takes some work to deliver a well functioning equilibrium, the assumptions are basic and minimal.

Before we consider the market reaction, there are two corollaries which follow from theorem 1.

Corollary 3 *On average, overvalued firms or firms with large synergies win takeover battles and undervalued targets are purchased.*

Proof. Each term in equation (28) increases if firm i has a larger firm specific misvaluation, ε_i, of if firm i has a larger synergy, s_i. Therefore, firms with greater firm specific misvaluation or synergies are more likely to be over the reservation price. Furthermore, $\frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)}$ increases if the target specific error, ε_T, decreases. Therefore, targets who have smaller firm specific misvaluation (ε_T is smaller) are more likely to accept an offer. ■

This corollary elucidates the type of errors that are likely to occur. If the bidding firm has a large firm specific overvaluation then they are more likely to win because the target cannot distinguish between a large s_i and a large ε_i. This is easily seen because each relevant statistic in equation (28) is a function of $\frac{(1+s_i)}{(1-\varepsilon_i)}$, so large s_i and large ε_i have the same effect on every statistic.

The smaller a target's firm specific misvaluation, ε_T, the larger their estimate of every bidder's synergy. This is because the target only knows the total error, $(1-\rho)(1-\varepsilon_T)$. If ε_T is smaller then ρ must be larger. However, the target doesn't know they have a smaller firm specific component ε_T, so they under estimate the market wide component ρ. The target knows that the larger the market wide component the greater all of the bids will look. Thus, the expected value of an offer needs to be reduced more if the market wide component is larger. Thus, a smaller ε_T results in an under estimate of ρ and thus the expectation of the offer is not reduced by as much and therefore the expectation is more likely to be above the reservation price. Thus, the smaller a target's ε_T the more likely a merger occurs.

Corollary 4 *The larger the bids of the losing bidders the lower the probability of a merger occurring.*

Proof. In equation (28) the conditioning variables $\frac{(1+s_i)(1-\varepsilon_j)}{(1-\varepsilon_i)(1+s_j)}$ $\forall j \neq i$ all decrease if the bid of another firm $\frac{(1+s_j)}{(1-\varepsilon_j)(1-\rho)}$ increases. Affiliation ensures that this decreases the expectation of V_i and therefore decreases the probability of a merger. ■

The bids of the losing firms are relevant to the target because they provide information about $(1-\rho)$. If all of the bids are high then the target suspects that it is because ρ is large. Therefore, he thinks all bidders are overvalued and lowers his estimate of the synergies from the winning firm. Thus, more competing firms provide more information and increase the accuracy of the target. However, the following corollary shows that if the synergies have a common component, $(1+s_i) = (1+u)(1+\omega_i)$, then there is a limit to the information that can be learned from competing bids.

Corollary 5 *If the synergies have a common component then the bids of the losing firms are less informative about the synergies.*

Proof. If $(1+s_i) = (1+u)(1+\omega_i)$ then in equation (28) the conditioning variables $\frac{(1+s_i)(1-\varepsilon_j)}{(1-\varepsilon_i)(1+s_j)}$ $\forall j \neq i$ all become $\frac{(1+\omega_i)(1-\varepsilon_j)}{(1-\varepsilon_i)(1+\omega_j)}$ $\forall j \neq i$. Thus, the target is able to learn from the other bids about $\frac{1-\rho}{1+u}$, but cannot tell the difference between a high market wide synergy and a high market wide overvaluation. ■

Although increased competition reduces the information asymmetry and therefore the effects of market wide misvaluation, if the synergies have a common component then there is a limit to the information that can be gleaned from the competing bids.

The intuition for Theorem 1 and corollaries 4, 5 and 6 is that although the target is rational and thus correct on average, the noise in the model leads to different types of mistakes by the target. The target sees $\frac{\alpha_i}{1-\alpha_i}M_i = \frac{(1+s_i)}{(1-\varepsilon_i)(1-\rho)}$ from the bidder. If either the market wide or firm specific error, ρ or ϵ_i, is larger then $\frac{(1+s_i)}{(1-\varepsilon_i)(1-\rho)}$ is larger, but the target does not know if a larger $\frac{(1+s_i)}{(1-\varepsilon_i)(1-\rho)}$ is due to a larger synergy, $(1+s_i)$, or a larger error, ρ or ε_i. Thus, if $\frac{(1+s_i)}{(1-\varepsilon_i)(1-\rho)}$ is larger then the target assumes that the synergy, $(1+s_i)$, is somewhat larger and the misvaluation, $(1-\varepsilon_i)(1-\rho)$, is somewhat smaller.[23] The target also uses all of his information from the other bids and his own misvaluation to try to determine if the increase is due to a market wide effect, ρ. But the target's information is noisy. Thus, an increase in s_i, ρ, ε_i or $(1-\varepsilon_T)$ all increase the expectation of the synergy.

All of these results tell us that a simple lack of information can lead us to find exactly what our intuition would expect: merger activity increases in overvalued markets and overvalued firms buy undervalued firms.

IV Merger Waves

The fact that a merger occurs provides information about the true value of the target and the bidding firm. This section considers how the market reacts to the merger announcement and shows that the market's reaction actually brings a merger wave to an end.

For simplicity we will assume that the market learns of the merger after the auction.[24] Thus, we are interested in examining the change in market prices on the announcement day.

Corollary 6 *On the announcement of a stock merger...*

1) The target's market price could rise or fall.

2) The winning firm's market price could rise or fall.

[23] Affiliation ensures this is true.

[24] To model how the market values change throughout the auction is a paper in and of itself. At each point in the auction the bidder's market value depends on what he bids, what others bid and the differing probabilities about who will win! Thus, we consider the reaction post announcement.

4) If the target's reservation price did not bind then the market price of the second highest bidder falls.

Proof. Before the offer was accepted $E[\rho] = 0$, $E[\varepsilon_i] = 0\ \forall i$ and $E[\varepsilon_T] = 0$. However, conditional on an accepted offer Theorem 1 and corollary 3 tell us that $E[\rho] > 0$, $E[\varepsilon_1] > 0$, and $E[\varepsilon_T] < 0$ and since the second highest bidder bid the same as the highest bidder corollary 3 also tells us that $E[\varepsilon_2] > 0$ (where the subscript 1 and 2 represent the highest and second highest bidder). This is true because the market knows that the transaction was more likely if $E[\rho] > 0$, $E[\varepsilon_1] > 0$, and $E[\varepsilon_T] < 0$. Furthermore, since the target is rational $E[s_i] > 0$. See Appendix A for a formal proof.

Therefore, prices must adjust until the expectation of the three errors equals zero. $E[\rho] > 0$ pushes all prices down until $E[\rho] = 0$. $E[\varepsilon_1] > 0$ pushes the wining bidder's price down until $E[\varepsilon_1] = 0$. $E[\varepsilon_2] > 0$ pushes the second highest bidder's price down until $E[\varepsilon_2] = 0$. $E[\varepsilon_T] < 0$ pushes the target's price up until $E[\varepsilon_T] = 0$. $E[s_i] > 0$ pushes the winning bidder's price and the target's price up. ■

Thus, it is easy to see why empirical work finds that the winning firm's stock price falls and the target's stock price rises on a takeover announcement. This simply suggests that the market expects the winning firm to be overvalued, $E[\rho] > 0$ and $E[\varepsilon_1] > 0$, the target to be undervalued $E[\varepsilon_T] < 0$, and the the synergies to be small or that competition gave most of the synergies to the target. It is also interesting to note that the losing bidder should have a permanent negative change in their stock price.

If a takeover is rebuffed then target's price could fall if $E[\varepsilon_T] > 0$ is the largest effect. Furthermore the losing bidders stock prices should rise since $E[\rho] < 0$, $E[\varepsilon_1] < 0$.

Therefore, taken together the resulting data could look like takeovers destroy value. However all of our stock movements are the result of rational updating. Firms are attempting to create synergies in an environment with limited and asymmetric information. The stocks move not because any firm is destroying value by merging, but because in the attempt to create value they are revealing information about what their price should have been.

This understanding of the markets reaction leads directly to the following Theorem which shows that merger waves are self limiting.

Theorem 2 *Merger* ***waves*** *occur when the market is significantly overvalued. A merger wave is a self correcting phenomenon.*

Proof. Theorem 1 showed that mergers occur more often when the market is overvalued. Thus, mergers will happen with greater frequency as long as the market *stays* overvalued. However, Corollary 6 showed that after a merger $E[\rho] > 0$ so prices must correct. So, after a merger prices move to the point where $E[\rho] = 0$. Let us now assume that more than one merger can occur, but firms have only one chance to merge and mergers happen sequentially. Thus, each merger lowers the stock prices of all other firms making the next merger less likely. On average the rational market updates correctly so $E[\rho] = 0$ after any merger is announced.[25] However, if the original overvaluation realization was larger than expected *even conditional on a merger occurring*, then ρ will still be greater than zero after one merger. If $\rho > 0$ then another merger is more likely to occur. After each merger the market adjusts prices again until $E[\rho] = 0$. However, if ρ is significantly larger than the average ρ conditional on a merger occurring then many mergers are likely to occur before the overvaluation is eliminated. Eventually ρ will equal zero and the wave will end.[26] ■

Thus, increased activity alerts the market to a market-wide overvaluation. The market updates and reduces prices. If activity is still high then the market reduces prices again. This process eventually ends the wave. However, as corollary 5 elucidates, this process could be lengthy if synergies are also correlated.

On average after a merger firms are correctly priced. Hence, we should not expect a wave or any ex post drift in prices. In fact, a second merger is less likely than the first because of the market correction, but if the market is still overvalued then a merger is more likely than it would be in a correctly valued market. Thus, if ρ is *significantly* higher than expected, then we should see a merger wave and ex post downward drift. This is because each new merger reveals more about ρ and lowers all prices. Thus, if the data contains a few periods of significant unexpectedly high market overvaluation then the data could show significant negative drift even though a negative drift is not expected on average.

V Short-Run Managers (Hostile Takeovers)

This section considers the idea that management is sometimes more focused on the short run than the long run. In this case the manager does not care if the offer is above the true value, but only whether the market *perceives* the offer to be above the true value. This assumes that the managers can exit before the long run value is realized.

The short-run manager's reservation condition is similar to the long-run manager's rule from above, equation (27). The difference is that the short-run manager only cares about what the market thinks, and the market cannot use information about X_T. Therefore, the short run manger

[25] We have made no assumption about how much information is available to the market about the merger, i.e., all the bids, just the winning bid or just the fact that a merger occurred. This is because the rational market updates prices until $E[\rho] = 0$ no matter what their information.

[26] We could also model the stock prices and the errors as following a random walk, with mergers causing mean reversion. That is, at the end of each period a value is drawn that is added to whatever error is currently left in the prices. In each period a merger may or may not occur. Thus, prices could become worse at any time but mergers or their lack would cause prices to move toward the true value.

accepts an offer if

$$E\left[V_i \mid \alpha_i, \Phi_M\right] > M_T. \tag{29}$$

We assume that the market does not learn of an offer if the manger does not accept. Thus, if he rejects, then the market price stays at M_T.[27]

Corollary 7 *The short run manager's reservation price is unaffected by market wide misvaluation. But the short run manger is more likely to reject a bid if the target is overvalued relative to the rest of the market.*

Proof. Lemma 2 and Section II.4 tell us that the market's acceptance rule can be rewritten as

$$E\left[(1+s_i)X_T \mid \frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)}, \frac{(1+s_i)(1-\varepsilon_j)}{(1-\varepsilon_i)(1+s_j)} \; \forall j \neq i, M_T\right] > M_T, \tag{30}$$

Theorem 4 in Milgrom and Weber (1982) tells us that if one variable is removed from a set of affiliated variables then the others are still affiliated. Therefore, the assumption that the variables $\log(1-\varepsilon_i)$, $\log(1+s_i)$ $\forall i$ and $\log(1-\varepsilon_T)$ and $\log(1-\rho)$ have log-concave densities ensures that the expectation is increasing with $\frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)}$, and $\frac{(1+s_i)(1-\varepsilon_j)}{(1-\varepsilon_i)(1+s_j)}$ $\forall j \neq i$ and is non decreasing in M_T.

We can see that none of the conditioning variables are a function of $(1-\rho)$, therefore the market's reservation price is unaffected by market wide misvaluation.[28] However, $\frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)}$ increases with $(1-\varepsilon_T)$. Thus, the more overvalued the target is with respect to the rest of the market (ε_T is high) the more likely they are to reject the bid. ■

On average, because the market is rational, the short run manger will make a correct accept/reject decision, as does the long run manager. However, there will be greater error in the short run manager's rule because the market has less information. Furthermore, the errors will not occur in the same pattern as the long run manager's errors. Because the market does not know X_T they do not have any reference point to determine if the market is under or overvalued. Thus, they are just

[27] Allowing the market price to rise if the management rejects all bids, only ads complication and magnifies the effect below.

[28] **Proof.** Formaly, the market's acceptance rule can be rewritten as

$$E\left[(1+s_i)(1-\varepsilon_T)(1-\rho) \mid \frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)}, \frac{(1+s_i)(1-\varepsilon_j)}{(1-\varepsilon_i)(1+s_j)} \; \forall j \neq i\right] > 1,$$

$$E\left[\begin{array}{c} E\left[(1+s_i)(1-\varepsilon_T)(1-\rho) \mid \frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)}, \frac{(1+s_i)(1-\varepsilon_j)}{(1-\varepsilon_i)(1+s_j)} \; \forall j \neq i, (1+s_i)(1-\varepsilon_T)\right] \\ \mid \frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)}, \frac{(1+s_i)(1-\varepsilon_j)}{(1-\varepsilon_i)(1+s_j)} \; \forall j \neq i \end{array}\right]$$

$$= E\left[\begin{array}{c} (1+s_i)(1-\varepsilon_T)E\left[(1-\rho) \mid \frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)}, \frac{(1+s_i)(1-\varepsilon_j)}{(1-\varepsilon_i)(1+s_j)} \; \forall j \neq i, (1+s_i)(1-\varepsilon_T)\right] \\ \mid \frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)}, \frac{(1+s_i)(1-\varepsilon_j)}{(1-\varepsilon_i)(1+s_j)} \; \forall j \neq i \end{array}\right]$$

$$= E\left[(1+s_i)(1-\varepsilon_T) \mid \frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)}, \frac{(1+s_i)(1-\varepsilon_j)}{(1-\varepsilon_i)(1+s_j)} \; \forall j \neq i\right] > 1$$

■

as likely to value an offer above the current market price in bad times as good. However, this means that a short run manger is more likely to reject an offer when the target is relatively overvalued because the market has limited information about the target's overvaluation.

It is interesting to note that if the world is full of both short run and long run mangers then the market will have a much harder time ending a merger wave. This is because short run mangers are happy to merge in either good or bad times. Thus, the market learns nothing about the market wide misvaluation from a merger where the target had a short run manager. If the market does not know whether a manager is short or long run, then merger activity provides less information about overvaluation and thus it takes longer for the market to correct.

V.0.1 Market Constrained Long-Run Managers

Now we consider a combination of short and long run interests by assuming that mangers are beholden to stockholders. We assume that stockholders will only be pleased with a deal if the market perceives the offer to be greater than the current stock price. A simple principle agent problem can make the current stock holders worried that management might sell the company for less than it is worth and cut a side deal for themselves. See Callahan and Moeller (2002) for a perspective on the side deals cut in mergers. This constraint combines the features of the short run and long run manger. A manager who is constrained by their stock holders will not accept an offer with current value below the current value of their stock, M_T, but will also not accept an offer unless its expected long run value is greater than X_T.

Thus, a constrained manger will reject more offers than either the short run or long run manger. However, Theorem 1 and corollaries 3 and 7 tell us that with a constrained manager there will still be merger waves and overvalued firms are still likely to purchase undervalued targets.

We believe that this type of manager is most similar to an actual manager. We will use this type of manager in the next section where we add cash bids and complete the symmetry of the stock and cash merger waves.

VI Equity Versus Cash

Up until this point in the paper we have only allowed firms to bid with their own stock. If the firm knows that their stock is undervalued then they may prefer to switch to a cash bid. However, we now assume that only some firms have access to cash. We will show that when markets are overvalued mergers are more likely to occur and those that occur are more likely to use stock. While when markets are undervalued, mergers are less likely to occur and those that occur use cash.

If all firms had equal costless access to cash, then in equilibrium targets would only accept cash bids. With costless access to cash the only reason for a bidder to use stock is if it allows them to pay less than a cash offer. Therefore, any target would recognize that a stock offer must come from an over valued firm, because any firm whose stock was undervalued would use cash to improve the perceived value of his offer. Thus, the target would rationally update and reduce the value of any stock offer until the bidder switched to cash. This is true whether the manager is short or long run because the rational market would update in the same manner.

However, if we assume that only some firms can access cash, then the market for stock mergers does not collapse. For simplicity we assume that it is common knowledge which firms have access to cash. Thus, those firms with cash must always use cash and those firms without access must use stock.

Theorem 3 *If the manger is constrained to accept an offer only if the market perceives its value to be above the target's current stock price, M_T, and only if the offer's expected value is greater than the target's true value, X_T, but not all firms have access to cash, then, 1) stock mergers are more likely to occur in overvalued markets than in undervalued markets, and 2) cash mergers are more likely to occur in undervalued markets than in overvalued markets.*

Proof. We assume that some firms have access to an amount of cash larger than their bid and all firms can use their stock. However, it is common knowledge which firms have access to cash. Therefore, as discussed above, those firms that have access to cash will be unable to bid with stock as the target and market will lower the expected value of a stock offer until the bidder optimally switches to cash.

Any accepted bid must pass two hurdles: it must be perceived to be greater than M_T by the market, and greater than X_T by the management of the target.

Cash bidders will bid up to V_i which equals $X_T(1+s_i)$. Therefore, for a cash bid to be accepted

$$X_T(1+s_i) > M_T \text{ and } X_T(1+s_i) > X_T. \tag{31}$$

Since $X_T = M_T(1-\varepsilon_T)(1-\rho)$ these conditions reduce to

$$(1-\varepsilon_T)(1-\rho)(1+s_i) > 1 \text{ and } (1+s_i) > 1. \tag{32}$$

Management's rule (the second condition) is unaffected by misvaluation because they know their true value and thus ask only for positive synergies. The market, however, is less likely to accept a cash bid if the target or the market is overvalued, $\varepsilon_T > 1$ or $\rho > 1$. This is because the market cannot tell if the target is overvalued or the synergies are low since all they see is $(1-\varepsilon_T)(1-\rho)(1+s_i)$.

As above, equity bidders will bid up to the truth, $\alpha_i = \frac{V_i}{X_i+V_i}$. Corollary 7 tells us that the probability that market perceives an equity bid to be above M_T is unaffected by $(1-\rho)$. However, Theorem 1 tells us that the manager is more likely to perceive that a bid is greater than X_T if the market is overvalued, $\rho > 0$.

Therefore, when the market is overvalued, the market perceives cash bids to be low but the market's perception of stock bids is unaffected. However, management perceives stock bids to be more valuable but management's perception of cash bids is unaltered. The opposite occurs if the market is undervalued. Thus, stock bids are more likely to win in an overvalued market and cash bids are more likely to win in an undervalued market. ■

This theorem demonstrates why it is rational for mergers to occur in stock when the market is overvalued and in cash when the market is undervalued. Keep in mind that this is not as obvious as it sounds because we are not simply saying that bidders with overvalued stock would like to bid with stock. They would, but why would targets accept? Our point is that in any rational model the participants will choose every action correctly on average. Therefore, the target will correctly reject stock offers that are not valuable enough *on average*. And, the market will correctly reject cash offers that are not valuable enough *on average*. However, the target and market will make mistakes. The mistakes are correlated with market wide misvaluation, which neither the target nor market knows.

With increased stock mergers in good times and increase cash mergers in bad times, it is not clear when there will be more mergers. However, if we believe that cash is costly and thus few firms have easy access to cash, then a greater number of bidders will bid with stock. Therefore, not only should we see waves of stock mergers in overvalued markets, but in undervalued markets we should see less activity and the activity that we do see should be in cash.

VII Conclusion

The evidence that waves occur is clear. That we as of yet have been unable to fully explain them is also clear. There are a number of reasons why any given wave of mergers could occur. For example, deregulation could release pent up demand, or a new technology could require the redeployment of assets. However, we believe that these reasons are not the whole story. Furthermore, these ideas tell us nothing about why the medium of exchange is stock or cash. In this paper we lay out a valuation effect that we believe impacts all merger waves and we show that this effect can cause a wave even without deregulation or innovation.

Our idea is that even fully rational participants make mistakes. We focus on how these mistakes could be correlated with specific types of misvaluation. When the market is overvalued the target rationally reduces the expected value of a given stock offer and thus, the target values the offer correctly *on average*. However, the target is more likely to over value the offer the greater the market overvaluation. Thus, market overvaluation increases the chance that a merger occurs. Therefore, a wave can occur due to misvaluation even if there is no underlying reason for mergers. Furthermore, waves can be halted by undervaluation even if assets truly should be redeployed. Thus, we believe that the impacts of misvaluation are significant.

These mistakes also influence the medium of exchange. We believe that in most cases for a merger to occur the target's management must believe that the deal increases value, and the target shareholders must perceive that they receive value greater than their current stock price. Both management and the shareholders make errors when evaluating offers (although they get it right on average) but they make different kinds of mistakes. We show that the shareholders are more likely to reject a cash offer when the market is overvalued, and management is more likely to reject a stock offer when the market is undervalued. Since both groups must agree, the composition of accepted offers changes from stock to cash as the market moves from over to under valued.

We believe that valuation, or rather misvaluation, has a fundamental impact on all mergers. Valuation effects not only the likelihood that the merger occurs but also the medium of exchange. We show how merger waves and waves of cash and stock purchases can be at least partially driven by periods of over and under valuation of the stock market.

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Appendix A: Proofs

Proof of Lemma 1: The highest amount that firm i is willing to bid depends on this Individual Rationality constraint insuring he does not pay more than his value:

$$g^{-1}(Z_2, \Phi_T)(X_i + V_i) \leq V_i. \tag{A1}$$

where $\Phi_T = \alpha_j, M_j \ \forall j \neq i, M_i, X_T, M_T$, is the target's information set. A firm's choice of α may effect both his own score and the scores of the other firms. Thus, increasing α must not increase another player's score by more than his own (or must decrease the other players by more than his own).

$$\frac{\partial g}{\partial \alpha_i}(\alpha_i, \Phi_T) > \frac{\partial g}{\partial \alpha_i}(\alpha_j, \Phi_T) \ \forall \ \alpha, M. \tag{A2}$$

If this does not hold then a bidder may not increase his bid, therefore it is necessary. Conversely, if it holds and bidders are tied then bidder i will increase his bid, therefore it is sufficient.

Note that equation (A2) does not have to be globally true; it only has to be true if a firm is currently tied with another. This is because we are modeling an English (button) auction. That is, bidders are tied as long as they continue to hold down a button that causes their α to go up. Then, when the second to last bidder releases the button the auction stops and the high bidder wins. Therefore, when

$$\begin{aligned} g(\alpha_i, \Phi_T) &\equiv g(\alpha_i, M_i, \alpha_j, M_j, \alpha_k, M_k \ \forall k \neq j, M_T, X_T) \\ &= g(\alpha_j, M_j, \alpha_i, M_i, \alpha_k, M_k \ \forall k \neq j, M_T, X_T) \equiv g(\alpha_j, \Phi_T) \end{aligned} \tag{A3}$$

one firm will always increase his bid (i.e., continue to hold down the button) if doing so increases his own score more than the other firm's, and he does not pay more than his true value. Finally, the assumption of continuity ensures that changing α_i does not cause the score of some third bidder to jump to a score above α_i without at some point tying bidder i. Thus, given condition (3) firm i would be willing to raise his α which raises his Z until he bids $\alpha_i = \frac{V_i}{(X_i+V_i)}$.

Proof of Corollary 6: Note that acceptance of the merger implies that

$$E\left[(1+s_i) \mid \frac{(1+s_i)}{(1-\varepsilon_i)(1-\rho)}, \frac{(1+s_i)(1-\varepsilon_j)}{(1-\varepsilon_i)(1+s_j)} \ \forall j \neq i, \frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)}\right] > 1. \tag{A4}$$

Hence acceptance occurs if and only if

$$\begin{aligned} \frac{(1+s_i)}{(1-\varepsilon_i)(1-\rho)} &> c\left(\frac{(1+s_i)(1-\varepsilon_j)}{(1-\varepsilon_i)(1+s_j)}, \frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)}\right) \\ &\Longleftrightarrow S_i + u_i - m - \log \ d \ (S_i + u_i - (S_j + u_j), S_i + u_i - u_T) > 0 \end{aligned} \tag{A5}$$

where c() and d() are non-increasing functions. Hence

$$m < S_i + u_i - \log \ d\,(S_i + u_i - (S_j + u_j), S_i + u_i - u_T) \tag{A6}$$

$$\Longleftrightarrow \rho = 1 - e^m > 1 - e^{S_i + u_i - \log \ d(S_i + u_i - (S_j + u_j), S_i + u_i - u_T)} \tag{A7}$$

From this it directly follows that

$$E\left[\rho \mid \rho > 1 - e^{S_i + u_i - \log\ d(S_i + u_i - (S_j + u_j), S_i + u_i - u_T)}\right] > E[\rho] = 0 \tag{A8}$$

Appendix B: Affliation

To prove that the relevant variables are affiliated we make some assumptions on the distributions. To do we first define log concavity.

Definition: *A random variable x has log concave density $f(x)$ if $\log f(x)$ is concave.*

The assumption of log concavity is standard in economic problems where inference is involved. Caplin and Nalebuff (1991) discuss the origins and implications of this idea and list its applications in economics. Distributions with log concave densities include the multivariate beta, Direchlet, exponential, gamma, Laplace, normal, uniform, Weibull and Wishart distributions. Log concavity implies that the distribution is unimodal.

We note two implications of log concavity.

Implication 1: *If x has log concavity density, then so does $-x$.*

Implication 2: *If x and y have log concave densities, so does $x + y$.*

Assumption: *The random variables $\log(1-\rho)$, $\log(1-\varepsilon_T)$, $\log(1-\varepsilon_i)$ and $\log(1-s_i)$ for all i have log concave densities.*

We make the assumptions on the log of the variables as these variables have mean zero and correspond to distributions over the real line.

Lemma 2B: *The random variables $1+s_i$, $\frac{(1+s_i)}{(1-\varepsilon_i)(1-\rho)}$, $\frac{(1+s_i)(1-\varepsilon_j)}{(1-\varepsilon_i)(1+s_j)}$ $\forall j \neq i$, and $\frac{(1+s_i)(1-\varepsilon_T)}{(1-\varepsilon_i)}$ are affiliated.*

Proof. Define $S_i = \log(1+s_i)$,$u_i = -\log(1-\varepsilon_i)$, $u_T = -\log(1-\varepsilon_T)$and m =$\log(1-\rho)$.Then

$$\begin{aligned} & f(S_i, u_i, S_i + u_i - m, S_i + u_i - (S_j + u_j), S_i + u_i - u_T) \qquad \text{(B1)} \\ & = g(S_i, u_i) f(S_i + u_i - m, S_i + u_i - (S_j + u_j), S_i + u_i - u_T \mid S_i, u_i) \\ & = h(S_i) l(u_i) f(S_i + u_i - m \mid S_i, u_i) f(S_i + u_i - (S_j + u_j) \mid S_i, u_i) f(S_i + u_i - u_T \mid S_i, u_i). \end{aligned}$$

Consider the term $f(S_i + u_i - m|S_i, u_i)$,let $t = S_i + u_i - m$, $S_i = x$,$u_i = y$, and note that

$$\begin{aligned} f(t \mid x, y) &= f(t \mid x + y) \qquad \text{(B2)} \\ &= f_{-m}(t - x - y). \end{aligned}$$

Since m has a log concave density so does $-m$. Let $t' > t, x' > x$ and $y' > y$. The monotone likelihood property requires that

$$\begin{aligned} & f(t \mid x, y) f(t' \mid x', y') > f(t \mid x', y') f(t' \mid x, y) \qquad \text{(B3)} \\ & \Leftrightarrow \log f_{-m}(t' - x' - y') \; - \log f_{-m}(t - x' - y') > \log f_{-m}(t' - x - y) \; - \log f_{-m}(t - x - y) \\ & \Leftrightarrow \frac{d \log f_{-m}(t - x' - y')}{dt} > \frac{d \log f_{-m}(t - x - y)}{dt}, \end{aligned}$$