Corporate Finance and the Term Structure of Interest Rates

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Abstract

A dynamic equilibrium model of the term structure of interest rates is presented. The short-term interest rate is the price at which investors supply funds to the corporate sector. However, unlike neoclassical models, we assume that firms are run by managers whose interests conflict with those of their shareholders. Managers are empire-builders who prefer to invest all free cash flow rather than distributing it to shareholders. Shareholders are aware of this problem but it is costly for them to intervene to increase earnings payouts. Firms with more cash invest more. Aggregate investment and the short-term interest rate are highest at business cycle peaks, when corporate cash flow is high, while the term spread is lowest at these times. Procyclical movements in interest rates are driven primarily by changes in corporate earnings rather than by shocks to the expected marginal rate of transformation. The pricing kernel derived under this free-cash-flow friction mimics one in which investors are "debt-holders" on the productive sector. They bear downside risk, but do not share equally on the upside. This aspect of the model sheds light on empirical regularities concerning the pricing of risky securities.

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1 Introduction

One of the strongest empirical findings in Economics is the evidence from research on investment that firms with more cash on hand invest more.[1] This deviation from the Modigliani and Miller paradigm can be explained by corporate control problems. In this paper we integrate a corporate control problem with a standard neoclassical asset pricing model, that of Cox, Ingersoll, and Ross (CIR) (1985), to produce a tractable dynamic model of the term structure of interest rates. In the model, firms with more cash invest more and we show that this can account for some of the basic regularities of the term structure, namely, that real and nominal interest rates are high at business cycle troughs and low at peaks, that the slope of the yield curve is high at troughs and low at peaks, and that the slope of the yield curve is a leading indicator of economic activity (e.g., Harvey, 1988).

Corporate finance is fundamentally about the separation of ownership and control, first identified by Berle and Means (1932).[2] Managers are in control of the corporate assets and can make investment decisions. But, since the preferences and incentives of managers are not perfectly aligned with those of shareholders, there is a problem of corporate control. In our model, corporations are run by managers, who are empire-builders and devote all of their free cash flow to investment, as in Jensen (1986). Free cash flow is the total return from operations net of contractual payments to investors. Shareholders attempt to control managerial decision-making with costly monitoring. The short-term (real) interest rate equates the supply of savings from households with the demand for investment from the manager-controlled corporations. The driving force of the model is that, unlike in neoclassical models, these two sectors are distinct and driven by their own interests. This makes investment demand vary with free cash flow. The basic mechanism of our model is that when free cash flow is high, as in a cyclical peak, investment demand is high, and therefore real interest rates are high.

Our results contrast with those of a standard asset-pricing model. In a neoclassical

[1] The link between investment and measures of corporate cash or retained earnings is a strong empirical regularity. It was noted as early as Tinbergen (1939, p. 49), who finds "that the fluctuations in investment activity are in the main determined by the fluctuations in profits earned in industry as a whole some months earlier," or, in Meyer and Kuh (1957, p. 192), who conclude that "the investment decision is subject to a multiplicity of influences " but that, at business cycle frequencies, there was "a clear tendency for liquidity and financial considerations to dominate the investment decision in the short run." Fazzari, Hubbard, and Petersen (1988) provide more recent evidence, or see Hubbard (1998) and Stein (2001) for surveys. This link is not only true at the firm level, but also at the industry level and at the aggregate level. See Chirinko (1993) and Caballero (1991).

[2] Also see La Porta, Lopez-de-Silanes, and Shleifer (1999) who emphasize another control problem, namely, that minority shareholders often have no ability to control dominant shareholders. Stein (2001) surveys the literature.

model, such as CIR, all variation in interest rates is driven by changes in the expected marginal rate of transformation (or productivity). Thus, the CIR account of the procyclical interest rate, for example, is that it is driven by cyclical movements in productivity. It is hard to believe that a deep technological parameter such as productivity moves as much as the real interest rate at business cycle frequencies.[3] For example, in the current recession, the one-year nominal interest is 1.8 percent.[4] The ten-year rate on the U.S. Treasury inflation-indexed bond is close to 3 percent. In the summer of 1996, both the one-year interest rate from the inflation-indexed bonds, as well as the ten-year rate were close to 4 percent. It is plausible that the drop in the long-term rate is driven by technological factors, but it is hard to believe that the decrease in the short-term rate is completely due to the same factors. Our model delinks the short-term rate from productivity and ties it to investment demand in the corporate sector, which is related to managerial motivations to invest corporate cash.[5]

In our model, as in CIR, there is a constant-returns-to-scale production process with a stochastically evolving productivity parameter. However, unlike CIR, the production technology is not directly operable by consumers, and instead is run by an empire-building manager. We model the limitations on the ability of investors to control the firm's investment and payout policy. Plausibly, investors do have some ability to extract cash from the firm: otherwise the firm would never produce any returns to investors and the lending market would completely break-down. We assume a mechanism that is unrealistically simple, but delivers tractable results and captures the basic insight that investors have limited powers of control. We assume that requiring a payout is costly in proportion to the size of the payout: each period consumers transfer resources to the manager. They simultaneously hire some number of auditors, at a cost, to monitor the firm. The manager invests the resources, which yields some stochastic output next period. The auditors have a technology to seize this output and then transfer the resources back to the consumers. However, each

[3]The same issue arises in the real business cycle literature (of which CIR is one example). Large productivity shocks are needed at fairly short intervals to quantitatively account for business cycle fluctuations. Many macroeconomists have criticized this as unreasonable, and researchers have sought amplifications mechanisms whereby small shocks can have large effects. Mechanisms involving aspects of corporate net worth or retained earnings have been studied by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997).

[4]There is no one-year inflation-indexed bond, but with inflation positive, we can conclude that the real rate is less than 1.8 percent.

[5]The procyclical real interest rate, as well as behavior of the yield curve, may also be driven by countercyclical monetary policy. However, as is widely appreciated in the literature, monetary policy is endogenous and driven by expectations of private sector activity. At an extreme, our model might say that the monetary authority simply chooses the money supply so that the interbank rate aligns with our "real" interest rate.

auditor can be responsible for only a limited amount of output. Thus, if output is realized to be unusually high, the firm will retain some earnings. This free-cash-flow, along with any new infusions from consumers, will then be immediately invested in production. The next period, consumers again choose some number of auditors to hire, and so on. We refer to consumers as "shareholders" and to the hiring of auditors as the "payout policy." Thus, each period shareholders infuse resources into the firm and simultaneously specify a payout policy for next period.

Relative to CIR, shareholders have an imperfect lever to control investment. When free-cash-flow is high, investment may be higher than shareholders would optimally prefer. While shareholders are still able to extract resources from the firm, this will only occur in the future. The result is that shareholders can end up holding "too many" financial claims on the corporate sector and to induce them to do this, interest rates have to be high. Alternatively, investment demand is higher, leading to higher current interest rates. In addition to expected future productivity, the amount of cash retained in the corporate sector, which depends on past realizations of production, enters as a second state variable in the determination of the term structure of interest rates. Thus, the objection that productivity shocks are too small is not an impediment for the model in explaining the movements of short-term real interest rates.

The model articulates the following links between the level of corporate cash and real interest rates. When cash is high, as in a cyclical peak, investment is high, and interest rates will be high. However, the term structure is downward-sloping, as future payout policy will induce firms to release the cash back to consumers, so that corporate cash is expected to be lower in the future. In addition, since these actions will induce firms to cut back on investment, there is an expected drop in economic activity. The opposite is true when the level of corporate cash is low. Thus, interest rates are procyclical, and slope of the yield curve is countercyclical.

The distinctive feature of our model is that investors are effectively "debt-holders" on the productive sector. They bear downside risk, but do not fully share in the upside as in these states managers exercise control over free cash flow. The resulting pricing kernel rationalizes some empirical regularities in the pricing of risky securities. We show that it is possible to generate a higher risk premium on corporate bonds. This is because bond payoffs match the debt-like pattern of investors' consumption. We also show that equity-like securities exhibit mean reversion in returns, and that implied volatilities on these securities will resemble the empirically observed volatility "smirk."

The free-cash-flow theory modeled in this paper is compatible with the considerable evidence that corporate cash flow explains investment.[6] Many studies have also investigated traditional neoclassical investment models, in which investment is determined by optimal capital adjustment as a function of the user cost of capital (interest rate) and Tobin's q. While the interest rate and q are statistically significant (at least in some specifications) in explaining investment, as predicted by those models, they only explain a small portion of the variability in investment (Caballero, 1999). Smooth adjustment costs, such as a quadratic cost function, do not remedy this problem, but "lumpy investment" models achieve a better fit with the data (Caballero and Engel, 1999, and others). Like our model, they are compatible with the idea that small shocks can lead to large fluctuations. However, Thomas (2002) argues that when the lumpy investment model is considered within a general equilibrium asset-pricing model, rather than in isolation as a partial equilibrium investment equation, investment is considerable smoothed and this largely negates the effects of modeling the "lumpiness" of investment.

We investigate asset prices in a framework in which investment and the characteristics of firms play the primary role. It is well understood that the failures of the standard consumption based asset pricing model could be due to the difficulty of identifying the representative consumer and measuring its marginal rate of substitution. The literature has pointed to a number of problems in this regard, e.g. measurement issues with consumer data, the invalidity of using aggregate consumption as a proxy, limited market participation, etc. For empirical work, data from the production side may be better and may offer a substantive alternative to the purely consumption based models. Cochrane (1991), Berk, Green, and Naik (1999), and Holmstrom and Tirole (2001) are examples of other papers in the growing asset pricing literature which have taken this approach. We are closest in spirit to the papers of Holmstrom and Tirole (2001) and Gomes, Yaron, and Zhang (2001) in that we study the asset pricing implications of a corporate financing friction.

[6] The main question the empirical literature has tried to answer is whether the correlation is because cash flow proxies for investment opportunities, or if it is due to managerial agency problems. Blanchard, Lopez-de-Silanes and Shleifer (1994) empirically examine 11 firms that received a windfall of cash (that does not change its investment opportunity set), to see whether they return the money to shareholders. "The evidence supports the agency model of managerial behavior, in which managers try to ensure the long-run survival and independence of the firms with themselves at the helm." The original Fazzari, Hubbard, and Petersen (1988) (cited earlier) paper was not derived from any model; it was somewhat informal in its approach. More recent studies use more formal Euler-condition approaches, in which there is an explicit allowance for the possibility of a binding finance constraint (Gilchrist and Himmelberg, 1998). Bond and Meghir (1994), Hubbard, Kashyap, and Whited (1995), and Whited (1992) split the sample based on some financial criteria and find that firms identified a-priori as facing agency problems in credit markets differ from neoclassical firms in their investment behavior. Chirinko and Schaller (1995) examine 212 Canadian firms over 1873-1986 and conclude, "the main empirical finding is that liquidity matters more for firms that find it difficult to credibly communicate private information."

2 Real and nominal interest rates over the business cycle

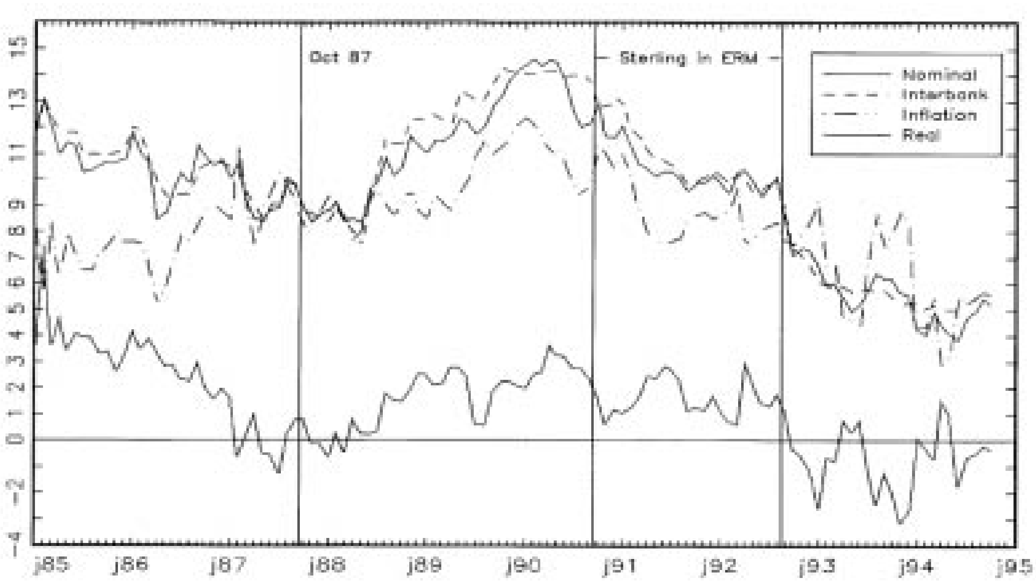


Figure 1: UK Estimated real and nominal 3-month interest rates.

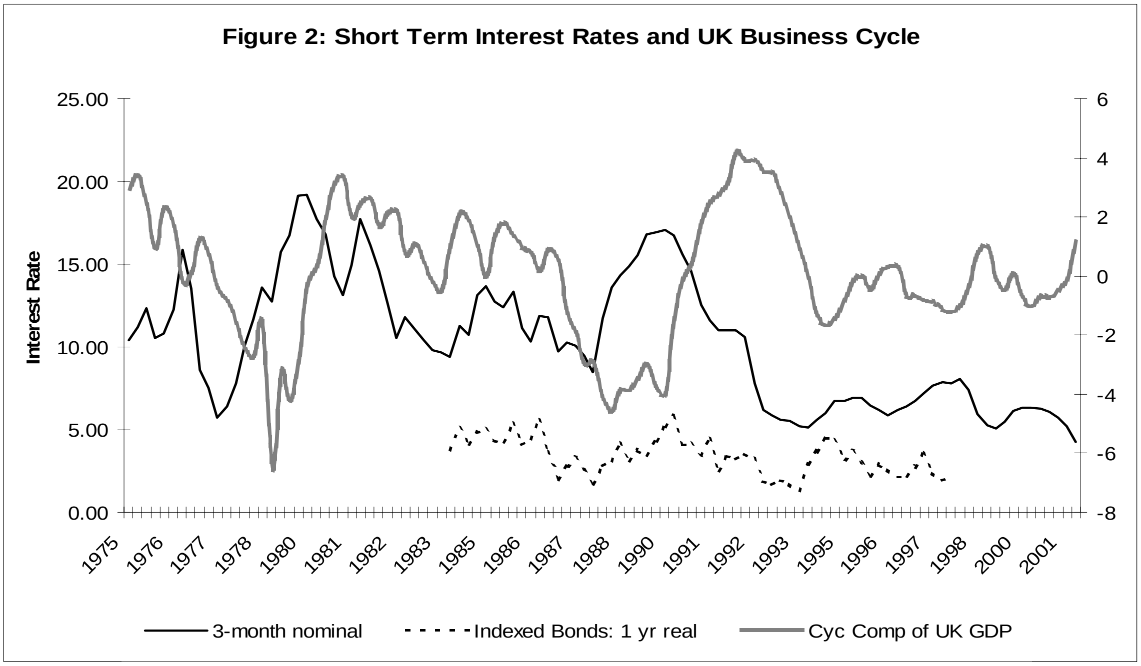
Source: Campbell and Barr (1997, Figure 2)

Figures 1, 2 and 3 illustrate the variation in short and long term interest rates over the business cycle. The UK government introduced inflation-indexed bonds in 1984, whose prices allow for a direct estimate of the market real interest rate. Figure 1 is extracted from Campbell and Barr's (1997, Figure 2) study of the term structure of real interest rates in the UK. The figure graphs the nominal 3-month interest rate as well as the estimated 3-month real interest rate. The figure illustrates that the short term real and nominal interest rates moved together over this period.

Figure 2 graphs the 3 month nominal interest rate from the UK 3 month Treasury Bill as well as the cyclical component of UK GDP, extracted using the Hodrick-Prescott filter. The sample period is extended back to 1975 in order to allow for more business cycle variation. One can see in the figure that the nominal interest rate is procyclical. The dashed-line in the figure is the estimated 1-year real interest rate from the inflation-indexed bonds. Although for a shorter sample period, the real interest rate also seems to be procyclical.[7]

Finally, Figure 3 plots the 1-year and 10-year real rates estimated from the inflation-

[7]The real interest data is provided by Steve Shaefer. See Brown and Shaefer (1994) for estimation details. See either Brown and Shaefer or Campbell and Barr (1997) for a description of the UK Index-linked Gilt market.



indexed bonds for the shorter sample period. The interest rate series are plotted versus the cyclical component of UK GDP.

The two stylized facts we note from these figures are: (1) Short term real and nominal interest rates are procyclical; and (2) The short term real rate is far more variable than the long term real rate.

3 The model

3.1 Preferences and Technology

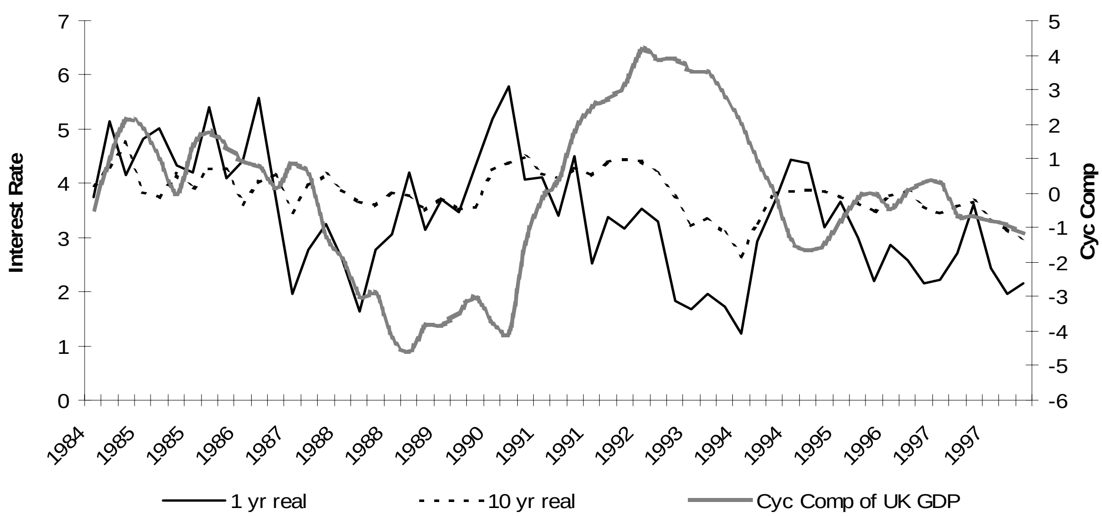
We consider an infinite horizon discrete time economy with a single consumption good. There is unit measure of consumers with preferences at time t of,

$$\sum_{s=t}^{\infty} \beta^{s-t} log C_s \tag{1}$$

There is a constant returns to scale production technology available each period that converts one unit of consumption good at time t into z_{t+1} units of goods at time $t+1$. z_{t+1} is a random variable that is distributed as follows:

$$log z_{t+1} = log \rho_{t+1} + \epsilon_{t+1} \qquad \text{where} \qquad \epsilon_{t+1} \sim \mathcal{N}(0, \sigma^2) \qquad \text{and is i.i.d across } t. \tag{2}$$

Figure 3: Short and Long term Interest Rates from UK Index-Linked Bonds



ρ_{t+1} is a state variable that represents the expected productivity of investment. It is known at time t, i.e. when investment occurs. The productivity state variable evolves as follows,

$$log\rho_{t+1} = log\rho_t + \nu_t, \tag{3}$$

where,

$$\nu_t \sim \mathcal{N}(0, \sigma_\rho^2) \qquad \text{i.i.d across } t \text{ and independent of } \epsilon_t.$$

We introduce a free cash flow friction to arrive at asset pricing implications of the separation of ownership and control. Managers, not consumers, control corporate assets and, in particular, make investment decisions. The preferences of the manager are those of an "empire-builder," in the spirit of Jensen (1986). That is, the manager prefers to invest all available corporate resources so as to maximize firm size. Shareholders have only one lever to control this investment. When they invest in the firm at time t, they simultaneously choose a number of "auditors" to hire for the upcoming period. Auditors have a technology to observe goods as they are produced and then to ring fence these produced goods so they do not fall under the control of the empire-building manager. However, each auditor can only be responsible for a maximum of one unit of the good. Moreover, hiring an auditor is costly, as auditors consume a fraction $1 - \gamma$ of the goods they seize.

Hiring more auditors means that, in expectation, investors will receive higher payouts next period. In effect, investors choose the auditors depending on how large a payout they would like next period. For this reason, we refer to the hiring of auditors as the *payout policy.*

At date $t-1$ the investors hand over some resources to the manager. The manager adds this to retained earnings from past production and, given his empire-building tendencies, invests all of it in production (amounting to I_{t-1}). Simultaneously the investors hire $d_t I_{t-1}$ auditors for the next period.

At t, production returns $z_t I_{t-1}$. If $z_t \geq d_t$, all of the auditors are occupied in observing and seizing output goods. Thus investors are returned γd_t goods per unit. The firm retains earnings of $E_t = (z_t - d_t)I_{t-1}$. If $z_t < d_t$, only $z_t I_{t-1}$ auditors are occupied, and investors are returned γz_t goods, while the firm retains no earnings.

On the one hand if investors chose to set d_t to infinity, E_t will be zero, investors will receive $(1-\gamma)z_t I_{t-1}$, and thus investors will have full control of corporate investment for period t. On the other hand, the investors will be spending too many resources on auditing. For suppose that investors were planning to infuse some resources into the firm in period t to fund new investment. In this case, investors would be better off by choosing a smaller d_t (in period $t-1$) so that the firm retained positive retained earnings to fund this new investment. Doing this avoids paying $1-\gamma$ to the auditors on the funds which in any case would have been returned to the firm to fund new investment. In this modeling, $1-\gamma$ parameterizes the extent of the corporate control problem.

Our strategy for characterizing asset prices is three-fold: (1) In the next two sub-sections we determine (C_t, d_{t+1}) as solutions to a planning problem; (2) Then we briefly discuss how the planning solution may be decentralized; (3) Finally, the solution to the planning problem is used to derive a pricing kernel that will allow us to price the various assets of interest.

We consider a planner who only maximizes the welfare of the consumer, while being constrained to operate production under the friction of separation of ownership and control. The controls are (C_t, d_{t+1}) given state variables of past investment, I_{t-1}, expected productivity, ρ_t, and production returns, z_t.

We use the following notation in the paper: Capital letters such as I_t or C_t represent aggregate quantities; Lower case letters such as d_t or c_t are per unit of investment quantities.

3.2 Discussion

The CIR benchmark is achieved when we set $\gamma = 1$. In this case it is costless for investors to hire auditors and as a result they have full control of the firm's investment policies. Relative to CIR, we have made a significant simplification by eliminating the correlation between ν_t and ϵ_t. This removes some of the richness in their interest rate dynamics, but it helps focus purely on the cash flow effects.

Macroeconomics models of investment make the realistic assumption that capital depreciates only partially each period. On the other hand, in the CIR model capital depreciates fully each period. An alternative interpretation is that capital does not depreciate, but can by costlessly transformed back into consumption goods. We also make this assumption.

We adopt the perspective that the consumers (outside claimants) choose payout policy. In practice, managers do not just exercise their power by controlling investment. They also influence the firms' capital structure and dividend policy in ways that may not be aligned with shareholders' interests. Furthermore, there are states of the world such as bankruptcy where the debtholders and other stakeholders influence payout policy. We abstract from these other effects in order to capture some of the richness of corporate finance, but in a model that deviates minimally from the neoclassical CIR model.

3.3 The planning problem

Let the value function for this problem be $J(I_t, \rho_{t+1}, z_t)$, where I_t is investment used to produce output at date $t+1$, and ρ_{t+1} is the state of productivity. The Bellman equation is,

$$J(I_{t-1}, \rho_t, z_{t-1}) = E_t \left[\max_{C_t, d_{t+1}} \{u(C_t) + \beta J(I_t, \rho_{t+1}, z_t)\} | \rho_t \right]$$

where current investment is given by,

$$I_t = \max[z_t - d_t, 0] I_{t-1} + W_t - C_t.$$

W_t is this periods remunerations to consumers,

$$W_t = \gamma I_{t-1}(d_t 1_{z_t \geq d_t} + z_t 1_{z_t < d_t})$$

For future reference we define retained earnings per unit of capital as,

$$e_t = \max[z_t - d_t, 0]. \tag{4}$$

Thus it should be clear that new capital is just retained earnings from last period plus new injections of funds from investors.

This problem can be simplified substantially by noting that the key friction that has been imposed is that investors can only make positive infusions so as to increase resources available for investment. This is equivalent to saying that,

$$C_t \leq W_t.$$

The only case in which this constraint will bind will be when $z_t > d_t$ and there is too much retained earnings in the firm so that the investor would ideally like to have some of this back in terms of consumption. Focusing just on this situation, $W_t = \gamma I_{t-1} d_t$, and we can rewrite the constraint as,

$$C_t \leq \gamma I_{t-1} d_t.$$

Substituting the expression for W_t into the investment equation gives,

$$I_t = z_t I_{t-1} - (1-\gamma) \min[z_t, d_t] I_{t-1} - C_t.$$

Investment this period is equal to resources produced from last period's investment minus audit costs minus consumption. Finally, define

$$y_t = z_t - (1-\gamma) \min[z_t, d_t] \tag{5}$$

as a "transformed" version of actual production returns, z_t. It is the actual production returns minus a term representing the audit costs associated with the payout policy. Equivalently, it is the return from the point of view of the consumer.

The problem can be concisely written as,[8]

$$J(I_{t-1}, \rho_t) = E_{t-1} \left[\max_{C_t, d_{t+1}} \{u(C_t) + \beta J(I_t, \rho_{t+1})\} | \rho_t \right] \tag{6}$$

subject to the constraint that,

$$C_t \leq \gamma I_{t-1} d_t. \tag{7}$$

Where the investment equation is,

$$I_t = I_{t-1} y_t - C_t. \tag{8}$$

Proposition 1 *The solution to the planning problem is as follows. Investment and new capital are:*

$$I_t = I_{t-1}(y_t - \min[(1-\beta) y_t, \gamma \theta \rho_t]). \tag{9}$$

Consumption is:

$$C_t = I_{t-1} \min[(1-\beta) y_t, \gamma \theta \rho_t] \tag{10}$$

and total required payouts are:

$$I_{t-1} d_t = I_{t-1} \theta \rho_t \tag{11}$$

where θ is a constant, and y_t is defined in (5).

[8] Note that in writing this problem we have dropped z_t as a state variable. In our problem, z_t is summarized in I_t, so we can reduce the problem to having two state variables.

Proof: Our solution approach is to guess the value function, then use the guess to arrive at expressions for the optimal controls (C_t, d_{t+1}), and finally to substitute these controls back into the Bellman equation to verify the guess. Since preferences are log, we make the following guess for the value function:

$$J(I, \rho) = AlogI + B(\rho).$$

Let us also make the guess that $C_t = I_{t-1} \min[\hat{c}_t y_t, \gamma d_t]$, where $\hat{c}_t$ depends only on t. Thus,

$$I_t = I_{t-1}(y_t - \min[\hat{c}_t y_t, \gamma d_t]),$$

and

$$u(C_t)+\beta J(I_t, \rho_{t+1}) = logI_{t-1}+log \min[\hat{c}_t y_t, \gamma d_t]+\beta AlogI_{t-1}+\beta Alog(y_t-\min[\hat{c}_t y_t, \gamma d_t])+\beta B(\rho_{t+1}). \tag{12}$$

Since,

$$AlogI_{t-1} + B(\rho_t) = J(I_{t-1}, \rho_t) = E_{t-1}[u(C_t) + \beta J(I_t, \rho_{t+1})|\rho_t], \tag{13}$$

we can collect the terms on $logI_{t-1}$ and match coefficients to find that $A = \frac{1}{1-\beta}$.

Dropping the terms in (12) that do not contain $\hat{c}_t$, we can then maximize pointwise for each y_t, to find a solution for consumption. In the range where $\gamma d_t > \hat{c}_t y_t$, we solve,

$$\max_{\hat{c}}\{log\hat{c}y_t + \beta Alog(1-\hat{c})y_t\}$$

giving $\hat{c}_t = (1-\beta)$. Obviously for $\gamma d_t < \hat{c}_t y_t$, we don't need to solve for $\hat{c}_t$. Thus,

$$C_t = I_{t-1} \min[(1-\beta)y_t, \gamma d_t].$$

To arrive at the expression for the payout policy, return to (13) and substitute in the expression for A. The $logI_{t-1}$ terms drop out, and we are left with,

$$B(\rho_t) = E_{t-1}[log \min[\hat{c}_t y_t, \gamma d_t] + \beta Alog(y_t - \min[\hat{c}_t y_t, \gamma d_t]) + \beta B(\rho_{t+1})|\rho_t].$$

The RHS of this expression is the objective to maximize in choosing d_t. That is d_t is chosen at time $t-1$ so we need to take expectations. Substituting in for $\hat{c}_t$ and dropping the constant $B(\rho_{t+1})$, gives:

$$d_t = argmax\{E_{t-1}[log \min[(1-\beta)y_t, \gamma d_t] + \beta Alog(y_t - \min[(1-\beta)y_t, \gamma d_t])|\rho_{t-1}]\}.$$

Suppose we can write the solution for d_t as,

$$d_t = \theta\rho_t.$$

The payout policy solves:

$$\theta = argmax\{E[log \min[(1-\beta)\frac{y_t}{\rho_t}, \gamma\theta] + \beta Alog(\frac{y_t}{\rho_t} - \min[(1-\beta)\frac{y_t}{\rho_t}, \gamma\theta])]\}.$$

Then we note the following useful property:

Lemma 1

$$\frac{y_t}{\rho_t} = \left(\frac{z_t}{\rho_t} - (1-\gamma)\min[\frac{z_t}{\rho_t}, \gamma\theta]\right)$$

Since $\frac{z_t}{\rho_t}$ is an i.i.d. random variable (see (2), $\frac{y_t}{\rho_t}$ is also an i.i.d. random variable.

Given the lemma, θ is independent of time t state variables, which verifies our guess that d_t is linear in ρ_t. This completes the proof of Proposition 1. ■

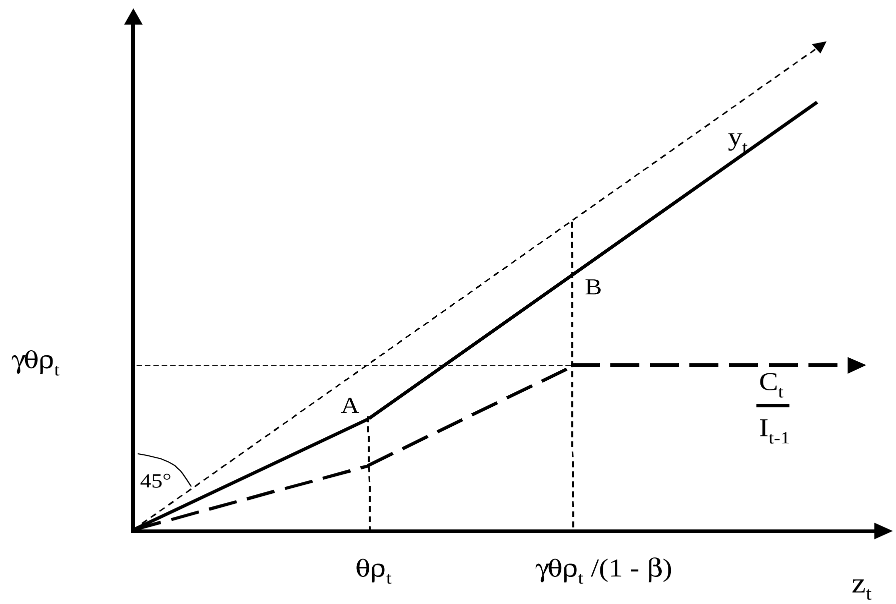


Figure 4

Figure 4 graphs consumption per unit of (past) investment as well as y_t as a function of z_t. Notice that there are three segments in the graph. The first region (to the left of A) is where z_t falls below d_t. In this case investors extract $z_t(1-\gamma)$ from the firm and consume a fraction $1-\beta$ of this amount. The second region (between A and B) is where returns exceed required payouts, however as retained earnings are small, investors choose to infuse some resources into the firm. The final region (to the right of B) is where retained earnings are high enough so that investors would ideally like to extract some more resources from the firm, but are unable to given the free cash flow friction. In this case investors simply consume their payouts of $\gamma I_{t-1}d_t$.

We refer to the region to the right of point B as the **over-investment region** and the region to the left of point B as the **optimal-investment region**. In the former region, relative to consumers' preferred choices, firms are over-investing because of the manager's empire building preferences. In the latter region, consumers control investment and investment is optimal.

Lemma 2 *(***CIR Case***)*
When $\gamma = 1$, the economy collapses to the CIR model. Investment and consumption are as follows:

$$\begin{aligned} I_t &= I_{t-1} z_t \beta, \\ C_t &= I_{t-1} z_t (1-\beta). \end{aligned}$$

Proof: When $\gamma = 1$, there is no cost of increasing θ, and thus claimholders may as well set $\theta \to \infty$. Then $y_t = z_t$ and from the equations of Proposition 1, $C_t = I_{t-1} z_t (1-\beta)$, while $I_t = I_{t-1} z_t \beta$. ■

3.4 Decentralization and the pricing kernel

One possible decentralization of the planning solution of the previous section is as follows. At date t, all claimholders of the firm have a meeting in which they decide on resources to inject into the firm for that period and on the payout policy/number of auditors to hire for the following period. Since the claimholders are identical, the policy will be approved unanimously. The manager then invests the firm's retained earnings plus any additional resources infused by the claimholders. Consumers are then free to trade a full set of contingent claims against all possible realizations of output. Between t and $t+1$, auditors observe and seize the designated amount of output goods, and then return them to claimholders. Finally, any contingent claims that are bought or sold between consumers are settled, and consumption takes place.

Since consumers can trade a full set of contingent claims against all output realizations, the pricing kernel can be defined in terms of the marginal rate of substitution of the representative consumer,

$$m_{t+1} = \frac{\beta u'(C_{t+1})}{u'(C_t)} = \beta \frac{I_{t-1}}{I_t} \frac{\min[(1-\beta)y_t, \gamma\theta\rho_t]}{\min[(1-\beta)y_{t+1}, \gamma\theta\rho_{t+1}]}. \quad (14)$$

4 Term structure

We begin our characterization of asset prices by computing the riskless short term interest rate, and then move on to describe the entire riskless yield curve. Our main interest is in describing the cyclical behavior of interest rates and term spreads and their correlation with aggregate investment and free-cash-flow.

4.1 Short rate

It is helpful to define,

$$\psi_t = \min((1-\beta)\frac{y_t}{\rho_t}, \gamma\theta) \tag{15}$$

and rewrite the pricing kernel as,

$$m_{t+1} = \beta\frac{I_{t-1}}{I_t}\frac{\rho_t\psi_t}{\rho_{t+1}\psi_{t+1}}. \tag{16}$$

From Lemma 1, we note that ψ_t is i.i.d. across time. Real interest rates are:

$$\frac{1}{1+r_t} = \beta\frac{I_{t-1}}{I_t}E_t\left[\frac{\rho_t\psi_t}{\rho_{t+1}\psi_{t+1}}\right].$$

Proposition 2 *(***Short rate***)*
There are two regimes depending on the relative magnitudes of $y_t(1-\beta)$ *and* $\rho_t\gamma\theta$*. In the case where* $y_t(1-\beta) < \rho_t\gamma\theta$ *– that is the optimal-investment region – the short rate is:*

$$log(1+r_t) = log\rho_{t+1} - log(1-\beta) - logE[\psi^{-1}]. \tag{17}$$

In the case where $y_t(1-\beta) \geq \rho_t\gamma\theta$ *– the over-investment region – the short rate is:*

$$log(1+r_t) = log\rho_{t+1} + log\frac{e_t}{\theta\gamma\rho_t} - log\beta - logE[\psi^{-1}]. \tag{18}$$

A little algebra verifies that interest rates coincide at the boundary between these two regions. The short rate has a simple expression then: it is only a function of e_t (realized retained earnings from (4)) and ρ_{t+1} (a proxy for expected production returns). If earnings are unexpectedly high, interest rates are high; if not they are just proportional to ρ_{t+1}, the expected production returns for the next period.

Lemma 3 *(***CIR Short rate***)*
The short rate in the case where $\gamma = 1$ *coincides with the CIR short rate:*

$$log(1+r_t) = log\rho_{t+1} - \frac{\sigma^2}{2} \tag{19}$$

Proof: When $\gamma = 1$, $\theta \to \infty$. Thus, $y_t = z_t$ and $\psi_t = (1-\beta)\frac{z_t}{\rho_t}$. From Proposition 2, if θ is large, the short rate expression for the region to the left of point B (see (17)) applies:

$$\begin{aligned} log(1+r_t) &= log\rho_{t+1} - log(1-\beta) - logE[\psi^{-1}] \\ &= log\rho_{t+1} - logE[\frac{\rho_t}{z_t}] \\ &= log\rho_{t+1} - logE[e^{\epsilon_t}] \\ &= log\rho_{t+1} - \frac{\sigma^2}{2}. \end{aligned}$$

■

Comparison of the interest rate expressions in the Proposition 2 and Lemma 3 reveals precisely the difference between our model and CIR. First, in the optimal-investment region, the expressions for interest rates are similar. Expected productivity drives all stochastic variation in the interest rate. The only difference is that the value of $E[\psi^{-1}]$ is higher in the free-cash-flow model, which means the level of interest rates is slightly lower. In the over-investment region, shocks to both productivity as well as retained earnings drive interest rates.

We can illustrate these economic effects using a static loanable funds model. Consider first Figure 5, which represents the CIR model. On the horizontal axis is investment or loans. On the vertical axis is the real interest rate. Consumers' supply of loanable funds is upward-sloping in the interest rate. The demand for funds is perfectly elastic at a level determined by the expected productivity of investment (ρ_{t+1}). A shock to ρ_{t+1} shifts up both the demand and supply curves. In the log case, the shift is of the same amount so that investment remains unchanged (see Proposition 1), but the interest rate rises by the amount of the shock.

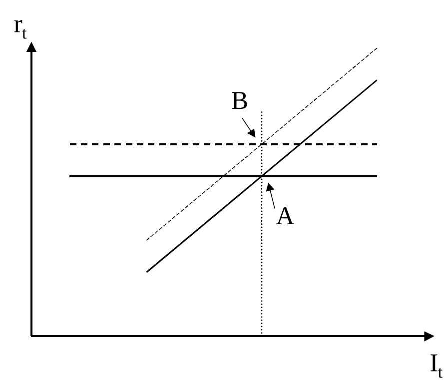


Figure 5: Productivity shocks in CIR

Figure 6 illustrates the effects of shocks to both productivity as well as retained earnings in the free-cash-flow model. The left-hand panel are shocks within the optimal-investment region. Note that the demand for funds is L-shaped. Firms will always invest their retained earnings, regardless of the interest rate. This sets a minimum amount of investment. At low interest rates, we are back in the CIR case: Firms find that their productivity is high relative to the interest rate and as such their investment demand is perfectly elastic. The L-shaped demand curve is the CIR case with a minimum investment level equal to retained earnings.

A shock to expected productivity moves both supply and demand curves up and shifts equilibrium from point A to point B. The net effect on the interest rate is the same as in the CIR case (Figure 5). There is also a shock to retained earnings. But it has no effect on the equilibrium in this region.

The right-hand panel illustrates the effect of these shocks when the retained earnings shock is large enough to take the economy into the over-investment region. At the margin, managers invest all of their retained earnings. Thus interest rates rise not only because of the rise in expected productivity, but also to induce investors to accept the higher required savings level.

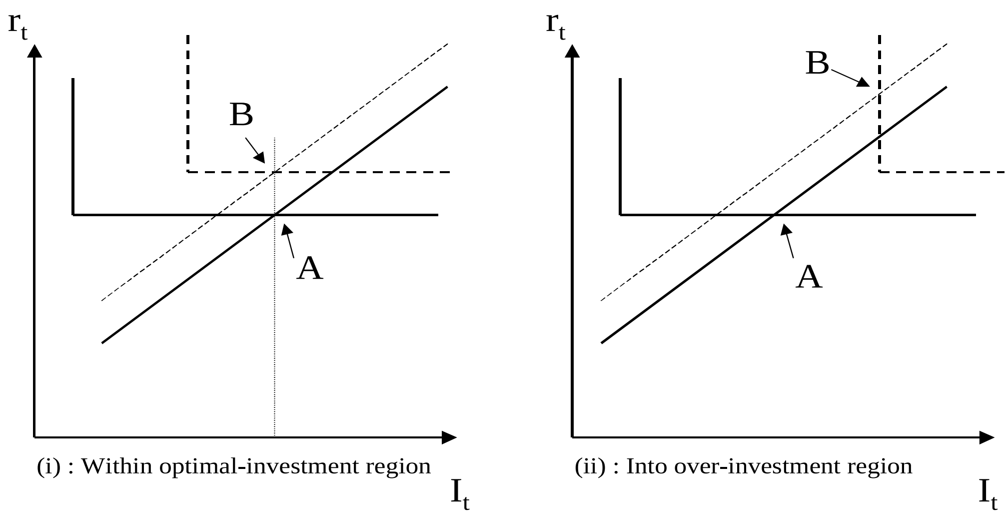


Figure 6: Simultaneous shocks to earnings and productivity

Plausibly, at business cycle frequencies, shocks to earnings are much larger than shocks to expected productivity. An asset pricing model such as CIR links all cyclical variation in interest rates just to variation in ρ_{t+1}, as can be seen in the diagram. However, in the over-investment region of the free-cash-flow model, large shocks to retained earnings have large effects on interest rates. Thus a success of the free-cash-flow model is that it links cyclical variation in retained earnings to the short rate: When retained earnings are high, as in a cyclical peak, the short rate is also high; When retained earnings are low, the short rate is low.

Our mechanism of retained earnings and investment demand as being the driving force for the procyclical interest rate is also in contrast to the standard consumption asset pricing model (e.g, the habit formation models of Constantinides, 1990, Sundaresan, 1989, and Campbell and Cochrane, 1999). There, the procyclical interest rate is often rationalized as follows. Fixing equity investment opportunities, if investors receive a negative income shock (i.e. cyclical trough), they will have higher risk aversion, consequently the risk premium will be higher and the interest rate will be lower in the trough.

4.2 τ-period rates

In order to study the slope of the yield curve, we now derive the expression for the τ-period interest rate.

$$\frac{1}{(1+r_{t,\tau})^{\tau}} = E_t\left[\frac{\beta^{\tau}u'(C_{t+\tau})}{u'(C_t)}\right]. \tag{20}$$

Lemma 4 *(***Long-rate***)*
The τ-period interest rate is:

$$\begin{aligned} log(1+r_{t,\tau}) = -log\beta \quad &+ \quad logE[(\frac{y}{\rho}-\psi)^{-1}] - \frac{1}{\tau}logE_t\left(\prod_{s=t+1}^{s=t+\tau}\rho_s\right)^{-1} \\ &- \quad \frac{1}{\tau}log\psi_t - \frac{1}{\tau}logE_t[\psi_{t+\tau}^{-1}]. \end{aligned} \tag{21}$$

Proof: From previous expressions we have that,

$$\begin{aligned} C_t &= I_{t-1}\rho_t\psi_t \\ C_{t+\tau} &= I_{t+\tau-1}\rho_{t+\tau}\psi_{t+\tau}. \end{aligned}$$

Now the investment equation is,

$$I_t = \rho_t I_{t-1}\begin{cases} \beta\frac{y_t}{\rho_t} & \text{if } \gamma\theta > (1-\beta)\frac{y_t}{\rho_t} \\ \frac{y_t}{\rho_t} - \gamma\theta & \text{if } \gamma\theta \le (1-\beta)\frac{y_t}{\rho_t} \end{cases}$$

which can be written more concisely as,

$$I_t = \rho_t I_{t-1}\left(\beta\frac{y_t}{\rho_t} + \max[(1-\beta)\frac{y_t}{\rho_t} - \gamma\theta, 0]\right) = \rho_t I_{t-1}(\frac{y_t}{\rho_t} - \psi_t),$$

where as we have noted before (see Lemma 1), $\frac{y_t}{\rho_t} - \psi_t$ is an i.i.d. random variable. Then,

$$I_{t+\tau-1} = I_{t-1}\left(\prod_{s=t}^{s=t+\tau-1}\rho_s\right)\left(\prod_{s=t}^{s=t+\tau-1}\frac{y_s}{\rho_s} - \psi_s\right).$$

In terms of consumption,

$$C_{t+\tau} = I_{t-1}\left(\prod_{s=t}^{s=t+\tau-1} \rho_s\right)\left(\prod_{s=t}^{s=t+\tau-1} \frac{y_s}{\rho_s} - \psi_s\right)\rho_{t+\tau}\psi_{t+\tau}.$$

Substituting all this back into the interest rate expression, (20), gives,

$$\frac{\beta^\tau u'(C_{t+\tau})}{u'(C_t)} = \beta^\tau \left(\prod_{s=t+1}^{s=t+\tau} \rho_s\right)^{-1} \left(\prod_{s=t}^{s=t+\tau-1} \frac{y_s}{\rho_s} - \psi_s\right)^{-1} \left(\frac{\psi_t}{\psi_{t+\tau}}\right).$$

Taking expectations and then logs we arrive at (21). ■

4.3 Slope of the yield curve

For τ large the long term interest rate from (21) is approximately,

$$\begin{aligned} log(1+r_{t,\tau}) &\approx -log\beta + logE\left[\left(\frac{y}{\rho} - \psi\right)^{-1}\right] - \frac{1}{\tau} logE_t \left(\prod_{s=t+1}^{s=t+\tau} \rho_s\right)^{-1} \\ &\approx log\rho_{t+1} - log\beta + logE\left[\left(\frac{y}{\rho} - \psi\right)^{-1}\right] - \frac{\tau}{3}\sigma_\rho^2. \end{aligned}$$

The last term is a constant that is linear in τ. Thus productivity is the primary determinant of long term interest rates. This is the same as in the CIR model.

Proposition 3 *(Term spread)*
The term spread measured as (long - short) is: If $(1-\beta)y_t \le \rho_t\gamma\theta$ (optimal-investment region),

$$term = +log\frac{1-\beta}{\beta} + logE[\left(\frac{y}{\rho} - \psi\right)^{-1}] - logE[\psi^{-1}] - \frac{\tau}{3}\sigma_\rho^2;$$

If $(1-\beta)y_t > \rho_t\gamma\theta$ (over-investment region),

$$term = -log\left(\frac{e_t}{\theta\gamma\rho_t} - 1\right) + logE[\left(\frac{y}{\rho} - \psi\right)^{-1}] - logE[\psi^{-1}] - \frac{\tau}{3}\sigma_\rho^2.$$

Since changes in ρ_{t+1} have the same effect on both long and short rates, productivity changes have no effects on the term spread. In our free-cash-flow model, all movements in the term spread are driven by variation in retained earnings.

The retained earnings effect on the short rate lasts only one period. This is because we assume that in the period following a big retained earnings innovation, investors can hire sufficient auditors to reduce the free-cash-flow problem. Our intuition is that if the free-cash-flow problem took longer to resolve so that firms were over-investing for several periods, there would be persistence beyond one period.

Another mechanism to arrive at persistence is to suppose that capital did not depreciate fully and was irreversible. There is a large investment literature which makes the case for this friction at the level of an individual firm (see Caballero, 1999, Abel and Eberly, 1996, Dixit and Pindyck, 1994). We can imagine introducing this additional feature into our model. Initially, a high retained earnings shock leads firms to over-invest. If this leads to a persistently larger than optimal capital stock, the interest rate would have to remain high in order to induce investors to hold the larger capital stock. On the other hand, such persistence would also lead to zero net investment over a number of periods, which is clearly counterfactual. For this reason, the investment literature has only been concerned with micro-level irreversibility.

It is also instructive to compare our effect to what would arise in a model with physical irreversibility in capital investment, but without our free-cash-flow friction. This is interesting because our over-investment problem seems similar to a model of physical irreversibility, but with the following difference. In our model, the free-cash-flow problem means that investors are unable to reduce firm investment below the high level chosen by the manager. With physical irreversibility, investors are unable to reduce investment below zero.

Suppose that the CIR model was modified to have irreversible physical capital. In that model, the constraint will bind following a negative realization of output – as opposed to a positive realization of output, as in our free-cash-flow model. Agents would like to consume part of their physical capital, but could not. In order to make them hold the physical capital, interest rates would have to rise. Thus high interest rates would coincide with low output realizations (troughs). This is counterfactual.

5 Risk premia

We consider the implications of our kernel for the pricing of risky securities. The distinctive feature of our model is that the free-cash-flow friction leaves consumers as "debt-holders" on the productive sector. This helps us to rationalize high corporate bond spreads, as well as mean reversion in stock returns and the volatility smirk of equity index options.

To begin, we note that the debt and equity securities we price are "fictional" securities. We price derivative securities whose payoff is some function of y_t (i.e. GDP at date t). This is unusual because the equilibrium asset pricing literature usually identifies equity with the claim on the aggregate consumption stream, or equivalently, as the value of capital.

The reason we opt for pricing y_t-derivatives is that the value of capital in our model is always one. In the optimal-investment region, since investors lend consumption goods to be transformed into capital using a constant-returns-to-scale technology, the price of capital is fixed at one. In the over-investment region, this logic does fail. However, since investor preferences are log, the income and substitution effects exactly cancel and again the price of capital is one. Thus, equity defined as the value of capital will never show capital gains in our economy (although dividends will be state dependent). This is an unappealing feature of the model, but is the price we pay for simplicity. We discuss the issue further in the conclusion.

5.1 Corporate bond spreads

Let us consider pricing a one-period security whose payoff is as follows:

$$P_{t+1} = \min\left[1, \frac{y_{t+1}}{F}\right].$$

This is a corporate bond of one dollar of face on a company whose value next period is perfectly correlated with aggregate output. F is the default point on the bond. Its price today is just,

$$E_t[m_{t+1}P_{t+1}].$$

We can likewise price a riskless bond giving a price of $E_t[m_{t+1}] = \frac{1}{1+r_t}$. Define the spread between the corporate and riskless bonds as,

$$S_t = 1 - \frac{E_t[m_{t+1}P_{t+1}]}{E_t[m_{t+1}]}.$$

Note that this is somewhat unusual in that we look at the ratio of two bond prices as opposed to the difference in the reciprocals of the two prices. While in the data these two measures of the spread will be almost identical, in our analytical model this is the natural definition for analytical tractability.

We can rewrite the spread in a standard single-beta representation as follows (see Cochrane, 2001):

$$\begin{aligned} S_t &= E_t[m_{t+1}(1 - P_{t+1})]/E_t[m_{t+1}] \\ &= E_t[1 - P_{t+1}] + \frac{cov[m_{t+1}, 1 - P_{t+1}]}{E[m_{t+1}]} \\ &= E_t[1 - P_{t+1}] + \beta_{1-P,m}\left(\frac{var[m_{t+1}]}{E[m_{t+1}]}\right). \end{aligned}$$

where $\frac{var[m_{t+1}]}{E[m_{t+1}]}$ is the risk premium associated with holding the pricing kernel.

The spread is compensation for both the chance of default as well a risk premium for the loading of corporate bond defaults on the pricing kernel. Thus, the β is the coefficient on a regression of the corporate bond payoff on the pricing kernel:

$$\beta_{1-P,m} = \frac{\sigma(1 - P_{t+1})}{\sigma(m_{t+1})} corr[m_{t+1}, 1 - P_{t+1}].$$

If agents were risk neutral so that the pricing kernel was linear, then S_t would be a pure default premium (i.e. $1 - E_t[P_{t+1}]$). Since both $1 - P_{t+1}$ and m_{t+1} are decreasing functions of y_{t+1}, the correlation coefficient is positive. This means that the β is positive and the risk premium component of the spread is also positive. We have chosen to focus on a corporate bond whose payoff is only composed of aggregate risk, because this maximizes the risk premium component of the spread.

We are going to contrast this spread when bonds are valued under both the free-cash-flow pricing kernel and the CIR pricing kernel. The two pricing kernels are just,

$$m_{t+1}^{CF} = \beta \frac{I_{t-1}}{I_t} \frac{\rho_t \psi_t}{\rho_{t+1} \psi_{t+1}}, \tag{22}$$

and,

$$m_{t+1}^{CIR} = \beta \frac{I_{t-1}}{I_t} \frac{(1-\beta) y_t}{(1-\beta) y_{t+1}}. \tag{23}$$

m_{t+1}^{CIR} can also be obtained by setting θ to infinity in m_{t+1}^{CF}.

At time t all of the variables with the exception of $\rho_{t+1}\psi_{t+1}$ and y_{t+1} are known. Thus, the β-expression can be rewritten as,

$$\beta_{1-P,m} = \frac{\sigma(1-P_{t+1})}{\sigma(m_{t+1})} corr\left[\frac{1}{\rho_{t+1}\psi_{t+1}}, 1 - P_{t+1}\right] \tag{24}$$

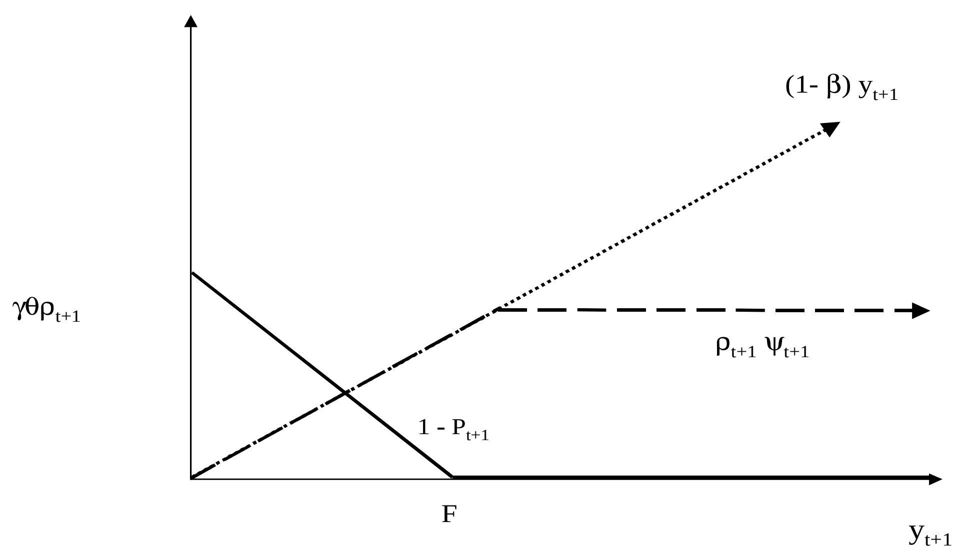


Figure 7: CIR and CF Pricing Kernels

Figure 7 graphs $\rho_{t+1}\psi_{t+1}$ and y_{t+1} along with $1 - P_{t+1}$. Note that $\rho_{t+1}\psi_{t+1}$ is equal to $(1-\beta)y_{t+1}$ in the optimal investment region, and uniformly less in the over-investment region. More importantly, $\rho_{t+1}\psi_{t+1}$ is more correlated with $1 - P_{t+1}$ than is $(1-\beta)y_{t+1}$. This implies that the correlation term is higher under the free-cash-flow pricing kernel than the CIR kernel.

From the figure we can also see that the volatility of the free-cash-flow kernel is lower than the CIR kernel. This implies that the β is higher under the free-cash-flow kernel than the CIR kernel.

Whether this translates into a higher corporate bond spread depends on what happens to the risk premium on the pricing kernel ($\frac{var[m_{t+1}]}{E[m_{t+1}}]$) across these two models. We have seen that $\sigma(m_{t+1})$ and the interest rate are lower under the free-cash-flow kernel. This means that the risk premium is actually lower under the free-cash-flow kernel. Thus, the end result on corporate bond spreads is ambiguous.

We will discuss the risk premium on the kernel in more depth below. However, if we fixed the risk premium across both the CIR and the free-cash-flow model, the β effect dominates. In a model where investors have log preferences, the return on the wealth portfolio (i.e. market return) is the reciprocal of the return on the pricing kernel (see, e.g., Cochrane, 2001). Thus the experiment of fixing the risk premium is close, but not exactly, to fixing the equity risk premium in the model.

Proposition 4 *(Corporate spreads)*
Fixing the risk premium of the pricing kernel, the corporate bond spread is higher in the free-cash-flow model than in the CIR model.

Intuitively, investors in the free-cash-flow world face a friction that means they don't fully share the upside of good production returns. The friction leaves them looking like "debt-holders" on the aggregate economy. In this environment, debt securities match their risk more closely and as a result investors demand a higher risk premium on these securities.

The largest effect will occur for bonds whose default point (F) is at the start of the over-investment region. Moreover the effect will have an inverse-U shape. This is a testable implication of the model. It says that a regression of the realized returns on corporate bonds versus the stock market, should have highest β's for bonds which default at the point where the yield curve changes from upward to downward sloping.

Collin-Dufresne, Goldstein and Martin (2000) find that firm-level variables have surprisingly little explanatory power for changes in corporate bond spreads. In contrast, macro variables, such as the return on the stock market, have much more explanatory power. Our previous result provides one explanation for this finding. Strictly speaking the result implies that only the stock market return should affect corporate bond spreads. However, adding an idiosyncratic component to P_{t+1} will change that. More importantly, if we move away from log preferences, the pricing kernel will include both the stock market return as well as aggregate free-cash-flow. Thus free-cash-flow should also have explanatory power for changes in corporate bond spreads. This remains to be checked.

5.2 Stock returns: Mean reversion, volatility smirk and skewness

We now consider equity-like securities whose payoff is linearly increasing in y_{t+1}.

We divide the time between t and $t+1$ into a series of infinitesimally small intervals. At each of these dates, agents can trade a full set of contingent claims against all output realizations at $t+1$. We take the limit, so that trading occurs continuously between t and $t+1$. Let $s \in [t, t+1]$ index time. Between t and $t+1$, the information flow concerning output realizations is described by a filtration generated by a standard Brownian motion process, which we denote as $\{Z_s\}$.

At each s, we consider pricing the security whose payoff at $s = t+1$ is $P_s = y_{t+1}$. Notice that consumption does not take place until $t+1$. Thus we price this risky security using the riskless bond (i.e. the security that pays one at $t+1$) as the numeraire.

We have seen that the free-cash-flow pricing kernel is given by (22). At time t, the only uncertainty in the pricing kernel of m_{t+1} is in y_{t+1}. For short, let us write this as,

$$m_{(s=t+1)} = \max\left[\frac{D}{y_{t+1}}, 1\right],$$

and derive the price of the risky asset at time s, using the bond as numeraire, as

$$P_s = \frac{E_s[m_{t+1} y_{t+1}]}{E_s[m_{t+1}]}.$$

Let $P_s^* = E_s[y_{t+1}]$ (i.e. the actuarially fair price of the risky asset). Then,

$$P_s = P_s^* \left(1 + \frac{cov_s(m_{t+1}, y_{t+1})}{E_s[m_{t+1}] P_s^*}\right) \equiv P_s^* h(P_s^*, s) \tag{25}$$

It should be clear that the value of $h(\cdot)$ is less than or equal to one, implying that the risky asset has a positive risk premium.

Proposition 5 *(Risk premium)*
$h(P_s^*, s)$ is monotonically increasing in P_s^*. For high values of P_s^*, since the pricing kernel is flat with respect to y_{t+1}, the value of $h(\cdot)$ approaches one.

Proof: Consider the derivative of $h(\cdot)$ with respect to P_s^*,

$$\frac{\partial h}{\partial P_s^*} = \frac{1}{E_s[m_{t+1}] P_s^*} \frac{\partial cov_s(m_{t+1}, y_{t+1})}{\partial P_s^*} - \frac{cov_s(m_{t+1}, y_{t+1})}{E_s[m_{t+1}] P_s^*} \frac{\partial E_s[m_{t+1}] P_s^*}{\partial P_s^*}.$$

Referring back to Figure 7, we see that the covariance of m_{t+1} with y_{t+1} is negative. Moreover, for high values of P_s^* (more precisely, y_{t+1}), the covariance is zero, while the covariance is most negative for low values. Thus, $\frac{\partial cov_s(m_{t+1}, y_{t+1})}{\partial P_s^*}$ is positive, while $cov_s(m_{t+1}, y_{t+1})$ is negative. It is fairly straightforward to show that $E_s[m_{t+1}]P_s^*$ is increasing in P_s^*.[9] ■

The result implies that good news about cash flows (P_s^*) raises P_s more than P_s^*, while bad news about cash flows causes P_s to fall more than P_s^*. It also implies that there is mean reversion in stock returns. News that pushes down the stock price will lead to a higher conditional stock return, so that the negative return will be followed by a positive return.

It is worth pointing out that in the CIR model with log investors, there would be no mean reversion. This is because $h(\cdot)$ will be constant, and not a function of P_s^*. Our result is due to the free-cash-flow friction we have assumed.

The result that stock prices are more sensitive to news can also be stated in terms of volatilities. Both P_s and P_s^* can be written as diffusions in Z_s ($\frac{dP_s}{P_s} = \mu_s dt + \sigma_s dZ_s$, and similarly for P_s^*). If we apply Ito's Lemma to equation (25) and retain only the dZ_s terms, we arrive at,

$$\sigma_s dZ_s = \left(1 + \frac{P_s^*}{h(P_s^*, s)} \frac{\partial h}{\partial P_s^*}\right) \sigma_s^* dZ_s. \tag{26}$$

σ_s and σ_s^* are the volatilities of P_s and P_s^*, respectively.

The term $1 + \frac{P_s^*}{h(P_s^*, s)} \frac{\partial h}{\partial P_s^*}$ represents the fraction of the volatility of the asset price that is due to the pricing kernel. That is, if m_{t+1} is constant, then $h(\cdot)$ will also be a constant (equal to one). This will mean that $\sigma_s = \sigma_s^*$, so that the volatility of the asset price is due solely to changes in the conditional expectation of its payoff (y_{t+1}).

Since we have show than that $\frac{\partial h}{\partial P_s^*}$ is positive, it is straighforward that $\sigma_s \geq \sigma_s^*$. More interestingly, since for high values of P_s^*, we have that $h(\cdot)$ is constant, it follows that σ_s

[9] The algebra is as follows. Since $P_s^* = E[y_{t+1}]$, we can write,

$$y_{t+1} = P_s^* x$$

where, x is a random variable that is positive, has mean of one, and is independent of the information filtration at time s. Denote its distribution function as $G(x)$. Then,

$$\begin{aligned}
\frac{\partial E_s[m_{t+1}]P_s^*}{\partial P_s^*} &= E_s[m_{t+1}] + P_s^* \frac{\partial E_s[m_{t+1}]}{\partial P_s^*} \\
&= \int_0^\infty \max\left[\frac{D}{P_s^* x}, 1\right] dG(x) + P_s^* \frac{\partial}{\partial P_s^*} \int_0^{D/P_s^*} \max\left[\frac{D}{P_s^* x}, 1\right] dG(x) \\
&= \frac{D}{P_s^*}\left(1 - G\left(\frac{D}{P_s^*}\right)\right).
\end{aligned}$$

equals σ_s^* for high values of P_s^*. This tells us that (subject to a caveat) volatility is higher when the stock price is low than when it is high.

The caveat is as follows. In the CIR model with log preferences, σ_s is equal to σ_s^*. As P_s^* approaches zero, our free-cash-flow model approaches the CIR model. Thus the volatility relation is non-monotonic. However, this non-monotonicity is due to the log preference assumption. In a range where the free-cash flow friction is the dominant concern, volatility will be decreasing in P_s^*.

Out-of-the-money equity index options command higher implied volatilities than out-of-the-money call options (the "volatility smirk.") That is, the volatilities implied by the Black-Scholes-Merton option pricing model vary across different strike prices, for the same underlying asset. This pattern has been widely documented in the literature. For example, see Dupire (1994) and Jackwerth and Rubinstein (1996). Bates (1996) provides a survey.

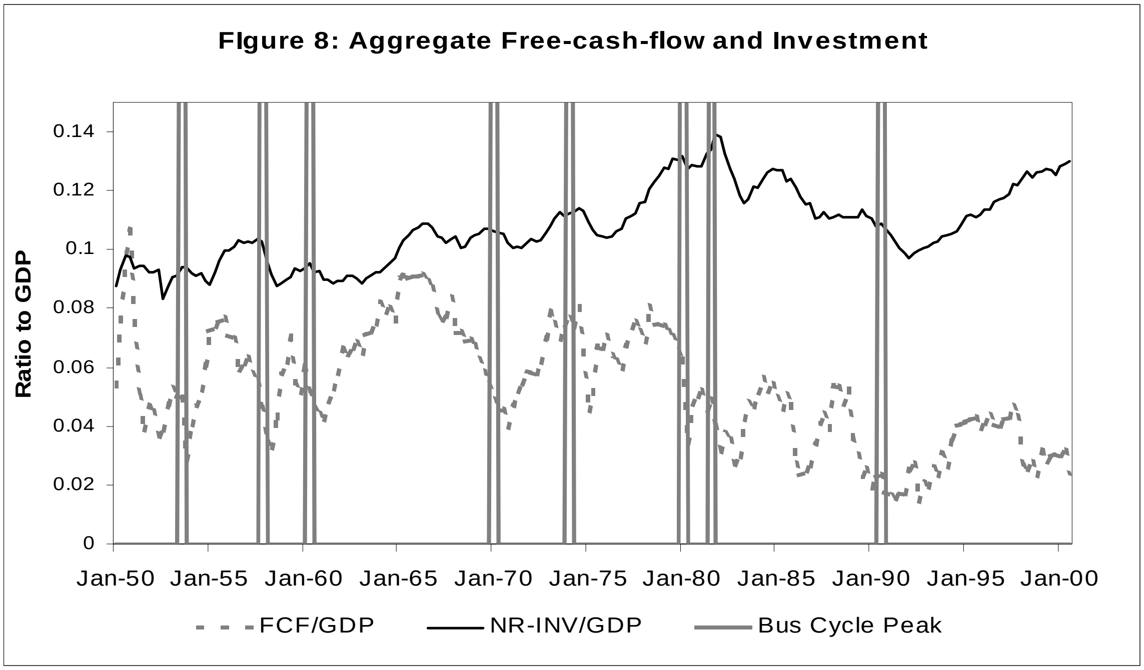
Our results provide an economic explanation for the smirk.[10] Referring back to Figure 7, we see that the Arrow-Debreu prices are high for low values of y_{t+1}, while they are equal to probabilities for high values of y_{t+1}. Alternatively, given investors debt-like claim on aggregate returns, they bear downside risk without sharing in upside risk. This implies that they have higher demand for put options than call options. Thus it is immediate that out-of-the-money puts will have higher implied volatilities than out-of-the-money calls.

6 Are movements in corporate free-cash-flow sizeable enough?

We measure free-cash-flow for the US by (1) aggregating retained earnings across the financial and non-financial sectors. Retained earnings is defined as after-tax-profits minus dividends, with an adjustment for changes in the values of inventory and capital. This measure of cash-flow is similar to that used in the investment literature (see, for example, Chirinko and Schaller, 1995, or Fazzari, Hubbard and Petersen, 1988). We then divide (1) by GDP for the period. The data is quarterly and is taken from the Bureau of Economic Analysis, National Income and Product Accounts.

This measure of free-cash-flow is admittedly very crude. In our model, free-cash-flow corresponds to the unexpected components of profits. That is, if the profits were expected to be high, then investors would intervene to make sure the profits were distributed. This distribution could be over the next period. However, in principle, it could also be over the

[10]The pattern may also be due to features of options markets. For example, Leland (1985) argues that transactions costs could induce a smile since at-the-money options have the highest gamma and would need the most frequent rebalancing. But, Constantinides (1998) argues that proportional transaction costs cannot account for the volatility smile of index options.



next few periods (e.g. via long-term debt). Thus, ideally we would like to get a measure of cash-flow that is unclaimed by investors in the current period.

Although crude, this measure of free-cash-flow helps to address the main question raised in the title of this section. Figure 8 graphs the ratio of our free-cash-flow measure to GDP versus non-residential fixed investment. We do not consider broader measures of investment, which would include consumer durables such as residential housing, in order to focus on the business investment which is subject to corporate control problems.

The investment series varies from 8.3% to 13.9% of GDP. The mean of the ratio is 10.7%. The free-cash-flow series varies from 1.4% to 10.7% of GDP, and the mean of this ratio is 5.3%. As another point of comparison, personal savings rates as a fraction of disposable income have averaged about 8% over the last 50 years (source: BEA). Thus the variation in corporate free-cash-flow is large in comparison to both actual investment and savings.

7 Conclusion

The introduction of corporate finance frictions into asset pricing models seems potentially fruitful. We investigated the effects of incorporating the leading corporate finance friction, the separation of ownership from control, into a simple asset pricing context. Managers in the model cannot be easily controlled by outside investors because of the auditing technology. Consequently, investors can only imperfectly control payout policy. This imperfection allows managers to use their discretion over free cash flow. They always invest as much as they can, consistent with the observation that firms with more cash invest more. Despite the simplicity of the preferences, the results are encouraging. The model can account for basic empirical regularities of the term structure of (real) interest rates. In particular, variation in the short real rate does not depend on an appeal to technology shocks. Moreover, real and nominal rates are high at business cycle troughs and low at peaks; the slope of the yield curve is high at troughs and low at peaks, and the slope is a leading indicator of economic activity.

While our goal was to produce a tractable model, the simplicity of the model does lead to some clear failings. As we have mentioned, the price of capital in our model is always one. On the one hand, we know exactly why this occurs in our model. In the CIR model, the price of capital is also fixed at one, and ours is a departure from the CIR model. On the other hand, this means we are unable to address the equity premium puzzle. Moreover, conditional on variables at time t, the Hansen-Jagannathan bound computed from our pricing kernel is tighter in the free-cash-flow model than in the CIR model. That is, $\frac{\sigma(m)}{E(m)}$ is lower under the free-cash-flow kernel. This means that the conditional market risk premium is lower in our model and is certainly a failure of the model. It seems clear that to address equity pricing we will have to depart from log preferences, perhaps to a successful specification such as habit formation.

On the bright side, it is well known that once production is added to an endowment economy, some frictions must be added to prevent consumption smoothing if the model is to address the equity premium (see, for example, Boldrin, Christiano, and Fisher, 2001). Our model provides a fairly simple and tractable way of doing this. Unconditionally, consumption is more volatile in our free-cash-flow model causing the ratio of $\frac{u'(C_{t+1})}{u'(C_t)}$ to be more volatile in our model than in CIR.

Within the context of the model, it is notable that our consumer/investors have no other way to store goods intertemporally other than through the corporate sector, which is subject to the agency problem. If there were a positive net supply riskless bond available, or another sector that was not subject to the agency problem, then there would be a demand for these securities. While it is clear that there would be a demand for such securities, the introduction of a government riskless bond raises other issues concerning, for example, Ricardian equivalence in this type of model.

Another leading corporate finance friction is the idea that external finance is costly, due to asymmetric information problems (see Myers and Majluf, 1984). At root the asymmetric information problems are also due to agency problems within the firm. However, ultimately this view of the firm sees hoarding of cash internally by the firm as a way of avoiding having to finance externally, and so also links investment to cash inside the firm. We remain agnostic on the question of whether costly external finance or the separation of ownership and control is more potent in explaining reality. In further research, we plan to investigate costly external finance.

Finally, an alternative view of the driving force behind the empirical interest rate regularities that we claim our model can explain is counter-cyclical monetary policy. Clearly, this could be the case. But since monetary policy is endogenous it could be the case that the monetary authorities are simply responding to the driving forces we have highlighted. The interaction between monetary policy and the forces in our model seems potentially interesting, but beyond the scope of this paper.

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