Diversification as a Public Good: Community Effects in Portfolio Choice *

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Abstract

We examine the impact of community interaction on risk sharing, investments and consumption. We do this using a rational general equilibrium model in which agents care only about their personal consumption. We consider a setting in which, due to borrowing constraints, individuals who are endowed with local resources under-participate in financial markets. As a result, individuals "compete" for local resources through their portfolio choices. Even with complete financial markets (in the sense of spanning) and no aggregate risk, in all stable equilibria agents herd into risky portfolios. This yields a Pareto dominated outcome as agents introduce "community" risk that does not follow from fundamentals.

This framework allows us to examine the influence of behavioral agents on the equilibrium portfolio choices of other agents in the community. We show that when some agents are behaviorally biased, a unique equilibrium exists in which rational agents choose even *more extreme* portfolios and amplify the behavioral effect. This can rationalize the behavioral bias, as following the behavioral bias is optimal. A similar effect will result if some investors cannot completely diversify their holdings (for control or moral hazard reasons) and are biased towards a certain sector. Finally, we show that in our model, equilibrium Sharpe ratios can be high, even absent aggregate consumption risk. We also show that from a welfare perspective diversification has "public good" features. This provides a potential justification for policies that subsidize diversified holdings and limit trade in risky securities.

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1 Introduction

In this paper we examine how an individual's investment choices may be influenced by the investment choices of other members of his community. We show that when there are scarce local resources, competition for these resources leads investors to care about their relative wealth in the community. As a result, rational risk averse investors have an incentive to herd and choose a portfolio similar to the rest of their community.

These *community effects* have a number of important implications. First, they imply that even a small group of traders with a behavioral or other bias in their portfolio choice may have a large impact on equilibrium outcomes. Though standard intuition is that rational traders would trade in the opposite direction and offset the effects of such a bias, in our model these biased traders "pull" the entire community to trade in the same direction, amplifying the effects of the bias. In fact, in the resulting equilibrium, the bias itself can be rationalized – once the whole community is biased, it is optimal for each individual to follow.

Another important implication relates to welfare. While all agents would be better off if the community were diversified, it is in no individual investor's best interest to diversify on his or her own. Because of this externality, diversification has the features of a public good. This creates a role for public policies that subsidize diversified portfolios and limit agents' ability to trade in risky securities. In particular, it suggests that there may be social benefits to preventing individuals from holding undiversified stock portfolios in 401K or social security retirement portfolios.

Finally, we show that community effects also have implications for asset prices. When a community herds into an asset class, this can drive up the price of that asset class in a way that is unrelated to aggregate consumption risk. These "price bubbles" lead to high equilibrium Sharpe ratios that cannot be explained in the context of standard asset pricing models.

Note that all of the implications described above are inconsistent with standard general equilibrium asset pricing models. In standard models, allocations are efficient and investors do not hold undiversified portfolios in equilibrium. Portfolio choices are driven by the risk/return characteristics of the individual securities, without regard to the portfolio choices of other agents; there is no "herding" in portfolio choice.

Our model differs from the standard approach in that agents are segmented into different communities. Our interpretation of a community is that of a group of people who share similar tastes. Specifically, a community is defined by the presence of local resources that are valued only by its members. This may take a geographical interpretation as well as other demographic interpretations; for example, a community may stand for individuals of a certain age group.

Because of this segmentation, competition within each community will cause the price of scarce local resources to fluctuate with the wealth of the community. These local resources represent local real estate, local labor and services, or other community specific goods. The desire to hedge this price volatility then biases portfolio choice. This implies that individual investors care about the correlation of their portfolio returns with the returns of other investors in the community.

We examine a simple version of such an economy in which financial markets are complete.

The model has two periods; in period one agents trade in financial assets, while in period two they trade goods in the spot market and consume. In section 2.1 we examine a benchmark for our analysis. There we get the standard result that the equilibrium is unique and agents hold the market portfolio. While agents want to hedge against local price uncertainty, the trades of those whose endowments are short the local good offset the trades of those whose endowments are long the local good.

We modify this framework by assuming that agents face borrowing constraints in financial markets, and that local goods are not fully accepted as collateral.[1] This implies that there is asymmetric participation in financial markets: agents endowed primarily with local goods will be more constrained than agents endowed primarily with global goods (including financial assets).

Therefore, as a result of these borrowing constraints, financial markets are dominated by traders who wish to positively correlate their portfolio with the price of the local good. Since the local good price is increasing in the wealth of the community, this creates an externality in portfolio choice: if other investors hold portfolios with a high payoff in some state, then the local good price will be high in that state, and so each individual investor will also want their portfolio payoff to be high in that state. Due to this externality, when risk aversion is not too low multiple equilibria exist. While fully diversified portfolios can still exist in equilibrium, this equilibrium is not stable. The stable equilibria are ones in which investors in a given community tilt their portfolio away from a fully diversified portfolio, thereby taking on unnecessary risk.

We show that in these equilibria, agents are worse off than in a fully diversified equilibrium. The externality effect creates a kind of "prisoners dilemma": while all investors would be better off if all held diversified portfolios, it is not in any single investor's best interest to diversify on his or her own. This public goods aspect of portfolio diversification has important policy consequences. For example, restricting investors to hold well-diversified portfolios in retirement accounts may lead to welfare gains for all agents in the economy. On the contrary, adding new financial securities (through financial innovation) or new trading partners (through financial integration) may increase the risk investors can take in equilibrium and result in a welfare loss for all agents.

It is important to note that these results are derived in a setup for which agents have standard utility functions. That is, agents care only about their own personal consumption. Hence, the result that agents' care about other people's wealth is endogenous. One could also derive similar implications by exogenously assuming that agents care about their community's aggregate wealth or consumption. If this preference is similar to the indirect utility that we obtain then the implications are similar (though the welfare analysis may differ). We refer to this approach as an exogenous "preference for status," and discuss it in more detail in section 3.5. A weakness of the exogenous approach is that it is not clear how such preferences should be defined. As we will show, standard functional forms that have been used do *not* produce herding in equilibrium.

While the results thus far show the existence of stable equilibria in which agents choose undiversified portfolios, they still admit multiple equilibria. Specifically, stable equilibria require that investors in a given community tilt their portfolios in the same direction,

[1]For example, in the case of local labor, due to legal restrictions and frictions such as moral hazard, one cannot sell future labor services in advance.

but this direction is arbitrary. The reason for this is that the model thus far is perfectly symmetric in that there is no link between financial assets and the communities.

In section 4, we examine the impact of agents who are subject to a behavioral or other exogenous bias in their portfolio choice. We show that if there are enough such agents, the equilibrium is unique. In this unique equilibrium, rational investors tilt their portfolios in the direction of the behavioral bias, in many instances choosing even more extreme portfolios than the behavioral traders. This response by rational agents amplifies the behavioral bias and "rationalizes" it: the biased traders are behaving optimally in equilibrium. This stands in contrast to common wisdom that suggests that rational agents exploit the behavioral bias by trading in the opposite direction.

An alternative story to the behavioral bias is that some investors are constrained to hold undiversified portfolios due to corporate control or moral hazard considerations. (For example, workers in Silicon Valley receive much of their compensation through stock-based incentive schemes which cannot be fully diversified.) More generally, agents may possess non-tradable skills or human capital whose value is positively correlated with the productivity of some sector. Similar to the behavioral bias case this induces other agents to invest in the same sector.

In section 5, we demonstrate that our model's endogenously generated externality in portfolio choice can increase equilibrium Sharpe ratios, and that this can be related to the equity premia puzzle and pricing "fads." Finally, in section 6 we evaluate the robustness of our main modelling assumptions.

1.1 Related Literature

Related Empirical Literature: Recently there is a growing literature studying community effects and social interaction. Duflo and Saez (2000) show that co-workers tend to choose similar portfolios. Benartzi (2001) finds that employees tend to over-invest in their company's stock in the retirement account. Since this choice is often discretionary, it implies that investors knowingly choose undiversified portfolios. Hong, Kubik and Stein (2002) show that investment decisions are related to social interaction. Our interpretation of social interaction is of rational imitation as agents are worried to be left behind. Some papers focus more on geographical biases. Huberman (1999) (within the U.S.), Grinblatt and Keloharju (1999) (within Finland), and Feng and Seasholes (2002) (within China) show that investors are more likely to invest in firms that are geographically close to them. Coval and Moskovitz (1999, 2001) show that also US mutual fund managers exhibit a preference for local companies. Our model of community effects can generate these patterns, and predicts then uniquely if some traders are constrained to hold local stocks (either for moral hazard reasons or due to a behavioral bias such as "familiarity").

Also related is the well-known "home bias" puzzle in international finance. Lewis (1999) surveys a large literature documenting this phenomenon. We discuss in the last section to what extent our model may explain this bias. An even more perplexing (and arguably more important) puzzle in the international literature is the lack of consumption risk sharing across countries. As Lewis (1999) notes there is little consumption co-movement across countries, which is inconsistent with standard models yielding Pareto-optimal outcomes. The fact that our model yields inefficient consumption patterns may help in explaining this puzzle as well.

Related Theoretical Literature: A related theoretical literature studies tournaments. In a tournament agents are rewarded for their relative standing, and in many cases only the winner is rewarded. Lazear and Rosen (1981) and Green and Stokey (1983) study the effects of laborers having tournament-based compensation. Chevalier and Ellison (1999) argue that mutual fund managers are subject to tournament-based compensation and study the implications for their portfolio choice. Cole, Mailath and Postelwaite (1992, 2001) study saving and investment decisions in a matching game between men and women. This is a double-sided tournament in which, after investment returns are realized, men and women with equally ranked wealth are matched. Some of their conclusions are similar to ours. Our paper is different in that we develop a general equilibrium framework in which relative evaluation arises endogenously. Also, prices are endogenous in our model and are determined by aggregate demand.

The endogenous indirect utility function we derive in our model shows that an agent's utility is increasing in own wealth, but decreasing in the wealth of the community. This is related to models, such as Abel's (1990) "catching up with the Jonses" specification, which assume *exogenous* preferences related to relative consumption. In addition to endogenizing these preferences, our model differs in two dimensions. First, similar to Gali (1994), who analyzes the potential impact of consumption externalities on the equity risk premium, our agents care about consumption relative to current aggregate consumption per capita; as opposed to consumption relative to lagged aggregate consumption per capita.[2] Second, and more important, our agents are heterogeneous. Specifically, an agent cares about her consumption relative to per capita consumption in her community, not relative to per capita consumption in the whole economy. [3]

In more recent papers, Chue (2001), Reisman (1999) and Shore and White (2002) study a model that is related to our status interpretation. They examine a partial equilibrium model in which agents have an exogenous preference to mimic other people's consumption. This naturally implies that agents mimic each other's portfolio choice. Because these papers are in a partial equilibrium framework, they do not consider market clearing (which would potentially eliminate the bias) nor the effect on asset prices.

Finally our paper is related to some of the theoretical literature on the "home bias" in international markets. More specifically, our paper is mostly related to papers that examine the effect of non-tradable assets. Similar to our paper these papers make the distinction between goods that can only be consumed locally and goods that are traded and can be consumed by all agents. However, the implications of this assumption are quite different. Unlike our model these papers examine equilibria that are Pareto-optimal. This is a result of having each country be represented by a single representative agent, thereby precluding any within community effects. If there are complementarities between non-tradable and

[2] Abel (1999) generalizes the structure in Abel (1990) to allow for dependence both on current and on lagged aggregate consumption.

[3] A different dimension of heterogeneity is utilized in Chan and Kogan (2000); they use a catching up with the Joneses framework in order to characterize implications of cross-sectional heterogeneity in risk aversion on asset prices. While the agents in their paper have different risk aversion parameters, they still all use the same weighted average of past realizations of the aggregate consumption process as their benchmark (habit index).

tradable goods and the output of the non-tradable good is stochastic then the representative agent biases his portfolio. This bias is a hedge against local productivity shocks. These models have been empirically rejected, both because the volatility of local productivity is not sufficiently high, and these models predict greater consumption risk sharing than is observed. Our model does not depend on fluctuations in the supply of the local resource – in fact we will assume it is fixed. Also, as stated earlier, our model is consistent with a lack of consumption risk sharing across communities.

2 Basic Model

Our model begins with a standard, 2-period stochastic exchange economy: There is a set of investors who live for 2 periods. In period one, these investors trade securities and choose portfolios. In period 2 the state of nature is realized, which determines investors' endowment income as well as their portfolio payoff. Agents then trade goods in the spot market, and consume. Investors act to maximize their utility over final consumption.

An important feature of our model is the notion of investor "communities," which we define below.

Communities and Consumption Goods: There are two disjoint communities of investors. Community $j \in \{1, 2\}$ has members I_j, so that $I = I_1 \cup I_2$ is the set of all investors.

There are two types of consumption goods: (i) a global good that is consumed by all agents, labelled good 0, and (ii) local goods specific to each community, labelled as goods 1 and 2. Residents in community j consume both the global good 0 and their local good j.

This division into communities with distinct consumption sets is at the heart of our analysis. A natural interpretation is geographical communities (separate countries) with non-tradable goods such as local labor services, real estate, etc. More generally, communities can be thought of as social groups defined by their distinct tastes. For example, the communities of golfers versus skiers are defined by their consumption of country club memberships versus ski lift tickets. Finally, as we show in Section 3.5, the local goods may represent community "status." In this case, the communities are defined as peer groups, whose members are concerned about their status relative to the rest of the group.

Formally, the local nature of the goods is modelled through agents' preferences, as defined below.

Preferences: All agents maximize expected period 2 utility given a separable CRRA utility function:

$$u^i(x) = \frac{1}{1-\gamma} \sum_j \alpha_j^i x_j^{1-\gamma}$$

for some $\gamma > 0$. We use $\left\{\alpha_j^i\right\}$ to represent the weight that agent i places on consumption of good j. Without loss of generality, we normalize $\alpha_0^i = 1$ for all agents. To capture the notion of local goods, we let $\alpha_j^i = 0$ for all agents $i \notin I_j$. Finally, we assume that all agents are symmetric in the importance they attach to the local good; that is, for $j \in \{1, 2\}$, $\alpha_j^i = \alpha$ for all $i \in I_j$.

To summarize, each agent in the economy has CRRA utility which is separable over the global good and the local good of their home community. The relative importance of the local good versus the global good is given by the parameter α.

Note that our specification of agents' preferences explicitly precludes any complementarity in consumption of the local and global good. This distinguishes our model from others in the literature on home bias and non-tradables.

Assets and Uncertainty: There are two firms (or "Lucas trees") that produce output of the global good. The output of each firm is uncertain, and depends upon the state of nature, s, drawn from a finite set $\{1, \ldots, S\}$. We denote by $Y^1(s), Y^2(s)$ the output of the firms, and we assume they are not proportional (so that the assets are not redundant).

Note that at this point, beyond notation there is no formal association of firms with communities. However, we will later interpret firm j as being located in community j, and that this may be manifested as a bias in agents' initial endowments (see Section 4).

There are also $S-2$ additional zero-net supply securities, with payoffs $Y^l(s)$ of the global good for $l = \{3, \ldots, S\}$. The role of these securities is to provide complete markets, which we shall assume throughout. We denote by $Y(s) = (Y^1(s), \ldots, Y^S(s))$ the vector of all security payoffs.

Endowments: Each agent i is initially endowed with a portfolio $\bar{\theta}^i$, so that $\bar{\theta}^i_l$ is agent i's initial endowment of shares of security l. We let $\bar{\theta} = \sum_i \bar{\theta}^i$ be the aggregate endowment of shares, which we normalize so that $\bar{\theta}_l = 1$ for $l \in \{1, 2\}$ and $\bar{\theta}_l = 0$ for $l > 2$.

Agents may also be endowed with goods. Let column vector $\bar{x}^i(s)$ be the endowment of goods held by agent i in state s. We assume that for all s, $\bar{x}^i_j(s) = 0$ for $i \notin I_j$; agents are not endowed with the local good of the other community.

We denote the aggregate endowment in the economy by $\bar{X}(s)$, which is given by,

$$\bar{X}_j(s) = \begin{cases} \sum_i \bar{x}^i_0(s) + Y^1(s) + Y^2(s) & \text{for} \quad j = 0, \\ \sum_i \bar{x}^i_j(s) & \text{for} \quad j \in \{1, 2\}. \end{cases}$$

That is, the aggregate endowment consists of the endowments of each agent, together with the good 0 output of the firms. We assume that $\bar{X} > 0$, there is a positive supply of all goods

in each state.

Timing and Trade: Agents trade shares of the firms in period 1. We let q denote the vector of prices for shares, so that agent i's budget constraint in period 1 is given by

$$q(\theta^i - \bar{\theta}^i) = 0. \tag{1}$$

After forming portfolios in period 1, the state of nature s is realized and agents trade goods in period 2. We let $p(s) \in \Re^3_+$ denote the vector of spot prices for goods. Without loss of generality, we let the global good be the numeraire so that $p_0(s) = 1$ for all s. Agent i's budget constraint in period 2 can therefore be written

$$p(s)(x^i(s) - \bar{x}^i(s)) \le Y(s)\theta^i, \tag{2}$$

in each state s. That is, the agent's net expenditures cannot exceed his portfolio payoff.

Equilibrium: The standard notion of equilibrium in this setting is given by prices, portfolios and allocations $(q, p, (\theta^i), (x^i))$ such that

1. for each agent $i \in I$, (θ^i, x^i) maximizes $E[u^i(x^i)]$ subject to (1) and (2);
2. financial markets clear: $\sum_i \theta^i = \bar{\theta}$;
3. spot markets clear: for $s \in S$, $\sum_i x^i(s) = \bar{X}(s)$.

2.1 Aggregation and Diversification: The Benchmark Case

In this section we develop some standard results on aggregation and diversification for this economy. We show that equilibrium can be modelled as though there is a representative investor in each economy, and that these investors will hold the global market portfolio. These results will serve as a useful benchmark for our later analysis.

We begin by considering the spot market equilibrium in period 2. In this final period, the economy is a standard exchange economy, and we solve explicitly for equilibrium prices in terms of initial (start of period 2) endowments.

First, recall that at the start of period 2, agent i in community j has endowment $\bar{x}^i_j$ of the local good, and

$$z^i \equiv \bar{x}^i_0 + Y\theta^i$$

of the global good. Importantly, note that i has no endowment of the local good of the other community. This implies immediately that there is no trade between members of community 1 and 2 at this stage (both value the global good, but have nothing to exchange for it). Thus, we can solve for the exchange equilibrium within each community separately. In community j, there are two goods, 0 and j, which are traded. Recall that we let the global good be numeraire, $p_0 = 1$, so that p_j represents the relative price of the local good.

Given CRRA utility, it is easy to solve explicitly for the equilibrium price p_j. The necessary and sufficient first order condition for agent i is that the marginal rate of substitution equals the relative price:

$$p_j = \alpha(x^i_j/x^i_0)^{-\gamma}.$$

Equivalently,

$$x^i_j = (\alpha/p_j)^{1/\gamma} x^i_0, \tag{3}$$

which we can sum over $i \in I_j$ and solve for p_j as

$$p_j = \alpha \left(\sum_{i \in I_j} x^i_0 \right)^{\gamma} \left(\sum_{i \in I_j} x^i_j \right)^{-\gamma}$$

Using market clearing we have the following useful result:

Lemma 1 *The equilibrium price of the local good is given by*

$$p_j = \alpha \left(Z^j/\bar{X}_j \right)^{\gamma},$$

where $Z^j \equiv \sum_{i \in I_j} z^i$ *denotes the aggregate period 2 endowment of the global good in community* j*.*

Given the equilibrium prices in period 2, we can now derive agents' indirect utility functions for numeraire wealth in period 2. This utility function can then be used to determine portfolio preferences in period 1.

Lemma 2 *The indirect utility of agent* $i \in I_j$ *with numeraire wealth* w *is given by*

$$v^i(w) = \frac{1}{1-\gamma}\ w^{1-\gamma}\ (W^j/Z^j)^\gamma,$$

where $W^j = Z^j + p_j \bar{X}_j$*, the aggregate wealth of community* j*. The ratio of aggregate wealth to tradable wealth can also be written,* $(W^j/Z^j) = \phi(p_j) \equiv 1 + \alpha^{1/\gamma} p_j^{1-1/\gamma}$*, or alternatively as*

$$(W^j/Z^j) = h(Z^j/\bar{X}_j), \tag{4}$$

where $h(z) \equiv 1 + \alpha z^{\gamma-1}$*.*

Proof. Recall that the agent consumes only the global good and the local good j. Using the budget constraint and (3), we have

$$y = x_0^i + p_j x_j^i = x_0^i + p_j(\alpha/p_j)^{1/\gamma} x_0^i = x_0^i \phi(p_j).$$

From the definition of u^i and (3),

$$u^i(x^i) = \frac{1}{1-\gamma}\left[{x_0^i}^{1-\gamma} + \alpha {x_j^i}^{1-\gamma}\right] = \frac{1}{1-\gamma}\ {x_0^i}^{1-\gamma}\phi(p_j).$$

Combining these yields the expression for v^i in terms of ϕ. Finally, using Lemma 1,

$$\phi(p_j) = (1 + \alpha^{1/\gamma} p_j^{1-1/\gamma}) = (1 + p_j(\alpha/p_j)^{1/\gamma}) = (1 + p_j(\bar{X}_j/Z^j)) = (W^j/Z^j).$$

Similarly, h follows from ϕ by substituting for p_j using Lemma 1. ■

Given this indirect utility function, we can restate the first period investment problem for each agent $i \in I_j$ as follows:

$$\begin{aligned} \max_{\theta^i} \quad & E\left[v^i(x_0^i + p_j x_j^i + Y\theta^i)\right] \\ \text{s.t.} \quad & q(\theta^i - \bar{\theta}^i) \le 0. \end{aligned} \tag{5}$$

Thus, the first period problem looks like a standard, one-period, one-good investment problem with CRRA investors. There is one critical difference, however. The indirect utility function v^i depends upon the state variable $\phi(p_j) = W^j/Z^j = h(Z^j/\bar{X}_j)$. This "price dependence" of the utility function plays a key role in our analysis.

We begin with an aggregation result, which is standard given CRRA utility. Note, however, that we can only aggregate investors within a single community:

Lemma 3 *If we replace a subset of investors of community j, $\hat{I}_j \subset I_j$, with a single aggregate investor with endowment $\sum_{i\in\hat{I}_j} \bar{x}^i$ and initial shareholdings $\sum_{i\in\hat{I}_j} \bar{\theta}^i$, then equilibrium prices and allocations (for the remaining agents) are unchanged.*

Proof. This is the standard aggregation result for CRRA utility functions. Here we have state dependent utility, but as the multiplicative factor is the same for all agents within community j, it can be treated as a change of measure, and the usual proof of aggregation applies. Note that one condition for aggregation is that endowments are traded. In our setting this is equivalent to

$$\bar{x}_0^i + p_j \bar{x}_j^i \in \text{span}(Y),$$

for all $i \in I_j$, for which complete markets is sufficient (though this can be weakened). ■

Thus, we can without loss of generality think of there being a single investor in each community. We will let investor $i = 1$ (2) be the representative investor for community 1 (2).

Note that in equilibrium, representative investor j has second period wealth

$$W^j = Z^j + p_j \bar{X}_j.$$

Given the indirect utility function from Lemma 2, the marginal utility of income for j is

$$(W^j)^{-\gamma}(W^j/Z^j)^{\gamma} = (Z^j)^{-\gamma}. \tag{6}$$

That is, in equilibrium the representative investor behaves as though he has CRRA utility directly over consumption of the global good. This leads immediately to the following important benchmark result.

Theorem 1 *In equilibrium, the consumption of the global good is perfectly correlated in the two economies ($Z^1 = \lambda Z^2$). If the global good endowments are zero ($\bar{x}_0^j = 0$), then each representative investor holds the market portfolio ($\theta_1^j = \theta_2^j$, $\theta_l^j = 0$ for $l > 2$).*

Proof. With complete markets, the marginal utility of income must be proportional for all investors. Thus, $(Z^1)^{-\gamma} = \hat{\lambda}(Z^2)^{-\gamma}$. The theorem then follows with $\lambda = \hat{\lambda}^{-1/\gamma}$. Since there are no redundant assets and $Z^j = \bar{x}_0^j + Y\theta^j$, if $\bar{x}_0^j = 0$ then $\theta^1 = \lambda\theta^2$. This plus market clearing ($\theta^1 + \theta^2 = \bar{\theta}$) implies the result. ■

The preceding result is not surprising, and confirms that absent market imperfections, we should not observe a "bias" in communities' investment portfolios.[4] Note from the proof that what is critical for this result is that the aggregate wealth of each representative investor fluctuates in a way that offsets the price dependence of the indirect utility function. In particular, it is essential that aggregate investor wealth is equal to aggregate community

[4]It is not the case, however, that each individual agent will hold the market portfolio, since individual endowments of the local good will differ.

wealth. When they are equated, we have a standard representative agent framework, and the equilibrium is Pareto optimal. In the next section, we introduce frictions that break the equality between investor and community wealth, and show that this leads to the possibility of suboptimal equilibria in which communities herd into undiversified portfolios.

3 Local Labor and Borrowing Constraints

A natural interpretation for the local good is local services, real estate and other local resources. To simplify the presentation, in this section we focus on local labor services, but discuss other possibilities in Section 6. A key feature of local labor services is that endowments consist of human capital. Due to moral hazard constraints, it is reasonable to assume that these agents cannot use this endowment as collateral for trading assets in the first period.

Formally, we assume that the community is composed of two distinct types of agents, $I_j = I_j^I \cup I_j^L$. Agents in I_j^I are **investors**; these agents are endowed with shares of firms which they trade to construct portfolios in period 1. Specifically, for $i \in I_j^I$, the goods endowment is zero: $\bar{x}_0^i = \bar{x}_j^i = 0$. They are endowed with shares $\bar{\theta}^i$, and we assume that $Y\bar{\theta}^i \geq 0$.

The second group of agents, I_j^L, we refer to as **laborers**. These agents are only endowed with the human capital that produces units of the local good in period 2. That is, for $i \in I_j^L$, the global good and share endowment is zero, $x_0^i = 0$ and $\bar{\theta}^i = 0$. Only the local good endowment x_j^i is non-zero.

In period 1, both groups of agents face the budget constraint,

$$q(\theta^i - \bar{\theta}^i) \leq 0.$$

In addition, we impose the "collateral constraint" that

$$Y\theta^i \geq 0. \tag{7}$$

That is, agents cannot borrow in the securities markets.

This collateral or borrowing constraint affects the two types of agents differently. Since investors have no endowment of goods, the constraint (7) is necessary in order to have positive consumption in all states. Thus, (7) does not bind for the investors, but is a natural consequence of their utility maximization.

On the other hand, (7) prevents laborers from using their endowment income in period 2 as collateral to trade securities in period 1. Since they also have no shares to trade, any non-trivial portfolio that satisfies the budget constraint and (7) represents an arbitrage opportunity, which cannot occur in equilibrium.[5]

We summarize this below:

Lemma 4 *In equilibrium, the constraint (7) does not bind for $i \in I_j^I$. For $i \in I_j^L$, constraint (7) implies that $\theta^i = 0$.*

[5]The assumption that laborers do not participate at all in financial markets is obviously extreme and made for simplicity. We relax this assumption in section 6.2.

Proof. If $\gamma > 0$, the marginal utility of consumption is infinite at zero. In equilibrium, agents consumption is therefore strictly positive. Thus, the constraint (7) does not bind for $i \in I_j^I$. For $i \in I_j^L$, the budget constraint implies $q\theta^i \leq 0$. This together with (7) implies an arbitrage opportunity unless $Y\theta^i = 0$. Given the non-degeneracy of the asset payoffs, this implies $\theta^i = 0$. ■

Thus, each community is composed of investors, who trade in period 1, and laborers, who are constrained from trading in period 1. Applying Lemma 3, we represent the set of investors I_j^I as a single aggregate investor in period 1. This aggregate investor has period 2 wealth,

$$Y\theta^j = Z^j.$$

This differs from the community wealth, which includes the endowment of the laborers,

$$W^j = Z^j + p_j \bar{X}_j.$$

This contrasts with the standard case consider previously in Section 2.1. Their, investor wealth and community wealth coincided. Here, investor wealth differs from community wealth ($Z^j \neq W^j$), leading to the expression for marginal utility shown below:

Theorem 2 *In equilibrium, the marginal utility of income for the representative investor of community j is given by*

$$(Z^j)^{-\gamma}(W^j/Z^j)^{\gamma} = (Z^j)^{-\gamma}\phi(p_j)^{\gamma} = (Z^j)^{-\gamma}h(Z^j/\bar{X}_j)^{\gamma}, \tag{8}$$

where $\phi(p_j) \equiv 1 + \alpha^{1/\gamma} p_j^{1-1/\gamma}$, *and* $h(z) \equiv 1 + \alpha z^{\gamma-1}$.

Proof. Immediate from the discussion above and Lemma 2. ■

Relative to the standard case considered in Section 2.1, Theorem 2 reveals that when laborers are constrained from participating in the asset market, the marginal utility of community j investors is altered. Comparing (6) with (8), we see that the nature of the effect depends critically on the magnitude of the risk aversion parameter γ. When $\gamma > 1$, the functions ϕ and h are increasing. Thus, the marginal utility of income is higher when the price p_j of the local good is higher, or equivalently when the global good is in relatively greater supply in the community. In this case, the agent has a desire to hedge and hold assets that payoff more when local prices are high. The effect is reversed if $\gamma < 1$. In that case, the agent exploits the price variability by holding assets that payoff when local prices are low. Finally, in the special case $\gamma = 1$, the effect disappears, and we have the following:

Corollary 1 *If $\gamma = 1$ (log utility), then the equilibrium coincides with that in Theorem 1.*

Before solving for the equilibrium for the case with $\gamma \neq 1$, we first introduce another specification of the model that leads to the same effect.

3.1 Equilibrium with Local Labor

In this section we analyze equilibrium portfolio choices in the presence of the frictions introduced in the previous section. Of interest is whether agents may choose to hold under-diversified portfolios in equilibrium.

To simplify the analysis we make the following two assumptions:

1. initially endowments of the global good are only through shareholdings, $\bar{x}_0^i = 0$;

2. there is no aggregate risk, $\bar{X}_1 = \bar{X}_2 = 1$ and $\bar{X}_0 = 2$.

Note that for item (2), we normalize the aggregate supply of each good to one per community. This normalization is without loss of generality.

Note that given these assumptions, there is no aggregate risk in the economy. Thus, the Pareto Optimal allocation is obvious – each investor should hold a fully diversified riskless portfolio. This is the unique equilibrium that corresponds to the conclusion of Theorem 1. This setting is therefore ideal for identifying any biases due to local labor effects.

First we show that even with local labor, full diversification remains as a possible equilibrium outcome:

Theorem 3 *Full diversification ($\theta_1^i = \theta_2^i$) is always a competitive equilibrium. Moreover, if $\gamma \leq 2$, this equilibrium is unique.*

Proof. With complete markets, the equilibrium condition is that agents' marginal utilities of income are proportional. If agents fully diversify, then Z^j is constant for each community j. Thus, the marginal utility of income is constant, and this is supported as an equilibrium. In this equilibrium, the price of each asset is equal to its expected payoff.

The equilibrium condition that marginal utilities are proportional implies that the marginal utility of income in community 1 is increasing with the marginal utility of income in community 2. Using Theorem 2, the marginal utility of income is monotone in $h(z)/z = 1/z + \alpha z^{\gamma-2}$, which is decreasing for $\gamma \leq 2$. If the marginal utility of income is decreasing in income, this implies that Z^1 is increasing in Z^2. Since $Z^1 + Z^2 = \bar{X}_0$ a constant, this implies that Z^1 and Z^2 are constant as well. Thus, both communities must fully diversify. ■

Thus, if agents are not particularly risk averse, full diversification is the unique equilibrium even with the frictions we have introduced. However, when agents are sufficiently risk averse, this is no longer the case. In the remainder of this section, we identify conditions such that equilibria exist in which agents fail to diversify completely. Since our model of local labor can be nested as a particular case of status, we focus our attention on that interpretation of the model.

We begin by introducing the following further simplifying assumptions:

1. There are two equally likely states, $s \in \{1, 2\}$,

2. Each firm pays
$$Y^j(s) = \begin{cases} 1+d & \text{if} \quad s = j \\ 1-d & \text{if} \quad s \neq j, \end{cases}$$
for some $d \in (0, 1]$,

3. Communities are symmetrically endowed, $\bar{\theta}_1^1 = \bar{\theta}_2^2$.

Given two states, markets are complete with trading in only the shares of the two firms. The two firms have identically distributed payoffs, but are perfectly negatively correlated. Given symmetrically endowed communities, it is natural to consider a symmetric equilibrium in which the two securities have equal prices, $q_1 = q_2$.

To solve for such an equilibrium, note first that if communities are symmetrically endowed and the securities are equally priced, then the budget constraint for each community is simply

$$E[Z^j] = 1.$$

Thus, we can represent the consumption of community 1 by its "volatility" σ. That is, if community 1 consumes $1+\sigma$ in state 1, it must consume $1-\sigma$ in state 2, where $\sigma \in [-1, 1]$. By market clearing, community 2 therefore consumes $1-\sigma$ and $1+\sigma$ in states 1 and 2, respectively.

Finally, $q_1 = q_2$ implies that the marginal utility of income for each investor is equated across the two states. Since the consumption of the two communities is symmetric, using Theorem 2 we have the following, single equilibrium condition:

$$\frac{h(1+\sigma)}{1+\sigma} = \frac{h(1-\sigma)}{1-\sigma}. \tag{9}$$

Note that $\sigma = 0$, full diversification, trivially satisfies (9) and therefore is always an equilibrium. We now show that when investors are sufficiently risk averse, equilibria with less than full diversification are also possible.

Theorem 4 *For $\gamma > 2$, there exists an equilibrium with income volatility $\sigma > 0$ if the importance α of the local good (status) satisfies*

$$\alpha = \frac{2\sigma}{(1-\sigma^2)[(1+\sigma)^{\gamma-2} - (1-\sigma)^{\gamma-2}]}. \tag{10}$$

Proof. Using the definition, $h(z) = 1 + \alpha z^{\gamma-1}$, and cross-multiplying, (9) can be rewritten,

$$(1-\sigma)(1+\alpha(1+\sigma)^{\gamma-1}) = (1+\sigma)(1+\alpha(1-\sigma)^{\gamma-1}),$$

and the result follows by solving for α. ■

This result demonstrates that for sufficiently risk averse agents, any level of income volatility can be supported as an equilibrium given appropriate importance of the local good. The intuition for this result is the following. Recall that the price of the local good (or the price of obtaining status) is increasing in community income. Thus, when community income is volatile, so is the cost of the local good. Each agent in the economy therefore wants to hold a portfolio that is positively correlated with community income in order to hedge this price uncertainty.

Equation (10) gives α as a function of σ. Of course, it would be more natural to solve for σ a function of α, but an analytic solution is not possible in general. Certain cases can be explicitly solved, however, as shown below.

Corollary 2 *A sufficient condition for the existence of an equilibrium with* $\sigma > 0$ *is* $\gamma > 2 + 1/\alpha$*. Also, we have the following explicit solutions:*

$$\gamma = 3: \quad \sigma = \sqrt{1 - 1/\alpha}, \qquad \gamma = 5: \quad \sigma = \sqrt{\sqrt{4 - 1/\alpha} - 1},$$
$$\gamma = 4: \quad \sigma = \sqrt{1 - 1/(2\alpha)}, \qquad \gamma = 6: \quad \sigma = \sqrt[4]{1 - 1/(4\alpha)}.$$

Proof. Define $\alpha(\sigma)$ by (10). Note that α is continuous in $\sigma \in (0, 1)$ and as $\sigma \to 1$, $\alpha \to \infty$, while as $\sigma \to 0$, $\alpha \to 1/(\gamma - 2)$. This establishes the first result. The rest follow from algebraic manipulation. ■

Figure 1 plots equilibrium income volatility σ as a function of both α and γ. Note that σ is increasing in both risk aversion as well as the importance of local consumption. Note also that the sufficient condition $\gamma > 2 + 1/\alpha$, while not strictly necessary, is nearly so. The exceptions occur for $\gamma > 6$ and $\alpha < .25$.[6]

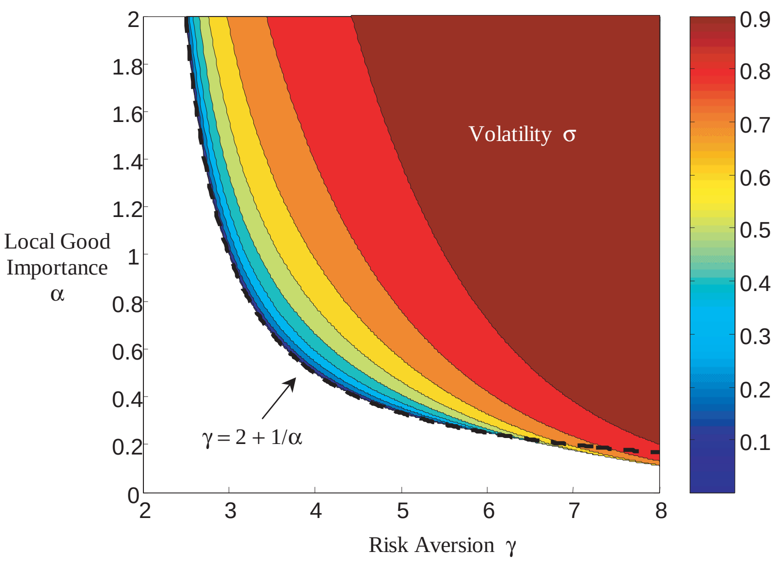

Figure 1: Equilibrium Volatility σ Given Risk Aversion γ and Importance of Local Good α

3.2 Investor Reaction Functions

The results of the previous section demonstrate the possibility of under-diversified equilibria. In this section, we develop a further understanding of these equilibria by examining the best response of an individual investor to the aggregate portfolio choice of his community.

[6]Finally, we remark that these results can be extended to general HARA class utility functions. In particular, if

$$u_j^i(x) \propto \alpha_j^i (A^i + x_j/\gamma)^{-\gamma},$$

(which is equivalent to the current model if $A^i = 0$, and equivalent to exponential utility with risk tolerance A^i if $\gamma = \infty$), then a sufficient condition for the existence of an undiversified equilibrium is

$$A \equiv \sum_i A^i < \frac{\alpha}{(1+\alpha)^2} \left[1 - \frac{2 + 1/\alpha}{\gamma} \right].$$

Consider the portfolio choice for an investor i in community j. The payoff of this portfolio can be decomposed as

$$z^i = \begin{cases} \bar{z}^i(1+\sigma^i) & \text{if } s=1 \\ \bar{z}^i(1-\sigma^i) & \text{if } s=2, \end{cases} \tag{11}$$

where $\bar{z}^i$ is the mean and σ^i is the volatility of i's portfolio choice. Recall also that with equally priced securities, the aggregate investment payoff Z^j of community j can be similarly written as $Z^j = 1 \pm \sigma$. We now address how the optimal volatility choice for agent i relates to the choice of his community.

Taking Z^j, or equivalently the community volatility σ, as given,[7] investor i chooses a portfolio to equate his marginal utility of income across states. Using Lemma 2, this implies that

$$\frac{h(1+\sigma)}{1+\sigma^i} = \frac{h(1-\sigma)}{1-\sigma^i}.$$

Solving for σ^i, we find that investor i's best response volatility choice is given by

$$\sigma^i = m(\sigma) \equiv \frac{h(1+\sigma)-h(1-\sigma)}{h(1+\sigma)+h(1-\sigma)}. \tag{12}$$

The best response function is illustrated for $\alpha = 1$ and $\gamma \in \{1/2, 1, 2, 3, 4\}$ in Figure 2. Since community volatility is the aggregate volatility of investors portfolios, an equilibrium is a fixed point $m(\sigma) = \sigma$; in the figure, this is where m crosses the 45° line. Thus, $\sigma = 0$ for all choices of γ, whereas $\sigma > 0$ is an equilibrium only for the case $\gamma = 4 > 3 = 2 + 1/\alpha$. Below we establish a number of properties of m which are evident from the figure.

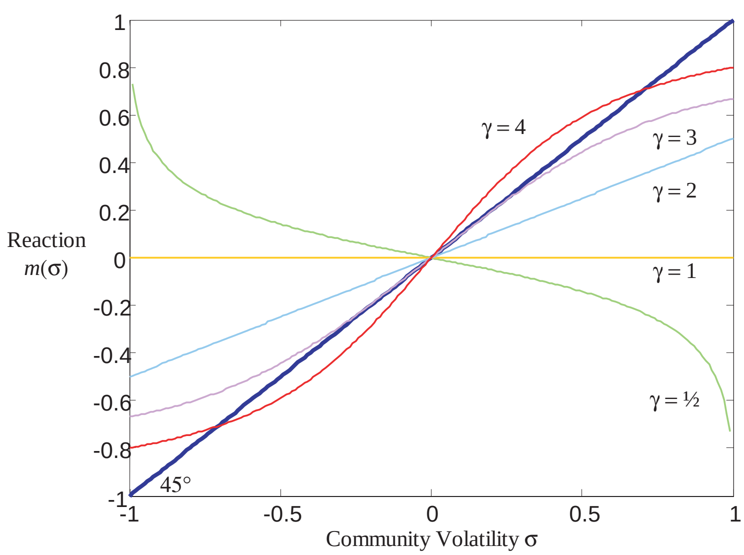

Figure 2: Optimal Portfolio Choice σ^i given Community Volatility σ

[7] Since Z^j maps to p_j, this is equivalent to taking the distribution of the local good price as given, as is standard in general equilibrium.

Lemma 5 *The best response function m satisfies*

1. *m is continuous in σ,*
2. *$m(0) = 0$,*
3. *$m(\sigma) = -m(-\sigma)$,*
4. *m is increasing (decreasing) for $\gamma > (<)1$,*
5. *$m(1) < 1$,*
6. *$m'(0) > 1$ if and only if $\gamma > 2 + 1/\alpha$,*

Proof. Property 1 follows since h is continuous and $h(z) > 1$. Properties 2 and 3 follow immediately from the definition of m. Property 4 follows from the monotonicity of h. Property 5 follows since $h(0) = 1$. For 6,

$$m'(\sigma) = 2\frac{h'(1+\sigma)h(1-\sigma) + h'(1-\sigma)h(1+\sigma)}{(h(1+\sigma) + h(1-\sigma))^2},$$

and so

$$m'(0) = \frac{2\alpha\,(\gamma-1)\,(2+2\alpha)}{(2+2\alpha)^2} = \frac{\alpha\,(\gamma-1)}{1+\alpha} \tag{13}$$

which implies that $m'(0) > 1$ if and only if $\gamma > 2 + \frac{1}{\alpha}$. ■

Property 2 above verifies our earlier result of Theorem 3 that full diversification ($\sigma = 0$) is always an equilibrium. That is, if the community portfolio is unbiased, it is optimal for each individual investor to fully diversify as well. Property 4 demonstrates the tendency to "herd" and choose a portfolio close to one's community when agents are more risk averse than log-utility. Property 3 implies that this tendency is symmetric across securities 1 and 2, as should be expected given the symmetry of the model.

For $\gamma > 2+1/\alpha$, properties 1, 5 and 6 establish the existence of undiversified equilibrium, consistent with Corollary 2. That is, since $m'(0) > 1$, for σ sufficiently close to zero, $m(\sigma) > \sigma$. But then $m(1) < 1$ and continuity implies that m must cross the 45° line for some $\sigma > 0$. The next result establishes that this is a complete characterization of the set of equilibria.

Theorem 5 *If $\gamma > 2 + 1/\alpha$, there exists a unique $\sigma^* \in (0,1)$ such that $m(\sigma^*) = \sigma^*$. Thus, the set of equilibria is given by $\{-\sigma^*, 0, \sigma^*\}$.*

Proof. Existence follows from the argument in the text. For a proof of uniqueness, see the appendix. ■

Figure 2 also provides further intuition for these undiversified equilibria. For $\gamma > 1$, investors hedge by choosing portfolios that payoff more when the price of the local good is high. That is, investors respond to a bias in the community portfolio by choosing a portfolio that is similarly biased (property 4). When investors are sufficiently risk averse, this effect is self-sustaining: in equilibrium, agents do not diversify because the rest of their community is not diversified.

This result highlights the fact that our model generates *portfolio externalities.* That is, the optimal portfolio choice of an investor depends upon the portfolio choices of his neighbors. When agents are sufficiently risk averse, this creates a "herding" effect: agents within a community choose portfolios that are highly correlated.

3.3 Equilibrium Stability

While undiversified equilibria exist, it is not clear that they are necessarily natural or plausible relative to the fully diversified one. One refinement criteria that has been used in the literature is that of dynamic stability. The definition of stability relies on an iterative procedure in which agents react

to the last period's outcome. A stable equilibrium can be thought as a limiting outcome of such a process. Hence this refinement has the view that an equilibrium is an outcome of a gradual process in which agents converge to an equilibrium strategy. Formally,

Definition 1 *An equilibrium σ is* **locally stable** *if for every σ' in a neighborhood of σ, the sequence $\{\sigma_n\}_{n=0}^{\infty}$ defined by $\sigma_0 = \sigma'$ and $\sigma_{i+1} = m(\sigma_i)$ converges to σ. An equilibrium σ is* **globally unstable** *if any sequence $\{\sigma_n\}_{n=0}^{\infty}$ for which $\sigma_{i+1} = m(\sigma_i)$ and $\sigma_0 \neq \sigma$ does not converge to σ.*

The next result shows that it is the undiversified equilibria which are stable.

Theorem 6 *If $\gamma > 2 + \frac{1}{\alpha}$ then:*

- *the full diversification equilibrium , $\sigma = 0$, is globally unstable,*
- *the undiversified equilibrium $\sigma^* > 0$ is a locally stable in the neighborhood $(0, 1]$ (as is $-\sigma^*$ in $[-1, 0)$).*

Proof. We first observe the fact that $m'(\sigma) > 1$ implies that σ is unstable. Hence, (i) follows from property 6 of m. Note that from property 5 that σ^* is the unique point in $(0, 1)$ such that $m(\sigma) = \sigma$. Then given properties 1, 5, 6, and 4, it must be that $m(\sigma) \in (\sigma^*, \sigma)$ for all $\sigma \in (\sigma^*, 1]$, and $m(\sigma) \in (\sigma, \sigma^*)$ for all $\sigma \in (0, \sigma^*)$. Thus, starting from any point in $(0, 1]$, the sequence converges monotonically to σ^*. The case of $-\sigma^*$ is symmetric. ■

The result of the theorem can be seen in Figure 2. For $\gamma = 4$, starting from σ arbitrarily close to but not equal to zero, investors choose progressively less diversified portfolios until the undiversified equilibrium is reached.

3.4 Welfare Analysis

We have seen that when risk aversion and the importance of the local good is sufficiently large, there exist multiple equilibria, and that in the stable equilibria investors hold undiversified portfolios. In this section we consider the efficiency properties of these equilibria. In doing so, we consider the welfare of both investors and of laborers.

Given that there is no aggregate risk in the economy, it is immediate that the fully diversified equilibrium is Pareto optimal, whereas an undiversified equilibrium cannot be (giving each agent the average consumption bundle that he consumes would make him better off). What is less clear is the welfare comparison of the two types of equilibria. While some agents must be worse off in an undiversified equilibrium relative to the fully diversified one, other agents might be better off (that is, the equilibria may be Pareto incomparable). The following result shows that this is not the case, and that in fact the full diversification equilibrium Pareto dominates the undiversified equilibria.

Theorem 7 *Every investor is worse off in the undiversified equilibrium than in the full diversification equilibrium. The same is true for every laborer as long as their endowments of the local good are uncorrelated with the payoffs of the firms.*

Proof. See Appendix. ■

The above theorem has important policy implications. First, it demonstrates that in this setting, restricting investors to invest in diversified portfolios can solve the coordination problem and make all agents strictly better off. In this sense, diversification has "public good" attributes. Alternatively, financial innovation that allows investors to hold riskier portfolios can reduce welfare by leading to less diversified equilibria.

Similar results apply to financial integration, which generally enhances welfare by allowing agents to better hedge against local risk factors. In our framework, however, financial integration creates new opportunities to trade risk with outsiders, and therefore creates the opportunity for agents to move towards a less diversified (rather than more diversified) equilibrium. For example, suppose $d < \sigma^*$ and compare the case in which the communities are separate autarkies to the case in which they are integrated. With autarky, each community will face the risk d of the production of the local firm. This equilibrium is obviously unique and stable, even if financial markets are complete. In contrast, if we integrate the communities, the only stable equilibria is the undiversified equilibrium, σ^*. Thus, financial integration will lead to increased risk in community consumption.[8] We formalize this with the corollary,

Corollary 3 *There exists a $\hat{d} > 0$ such that for $d \in [0, \hat{d}]$, every agent is better off under autarky than in the stable equilibrium if financial markets are integrated.*

3.5 Relative Consumption and Community Status

In this subsection we discuss an alternative approach in which one assumes that agents care explicitly about their community's wealth. As we shall see, while this approach may yield similar effects it depends crucially on the exact functional form. Relative utility functions that are commonly used such as Abel's (1990, 1999) notion of "catching up with Joneses" cannot generate herding behavior.

In the relative consumption framework there is only a single good. Individuals have utility directly over the consumption of this good, as before. In addition, however, agents also

[8]Newberry and Stiglitz (1984) show a somewhat similar result in a production economy.

care about how their individual consumption compares to the aggregate level of consumption in the community as a whole. We can interpret this concern for relative consumption as a concern for community "status." Each agent cares about his or her status, and so aggregate consumption appears directly in the utility function. To generate similar effects, assume that the utility of agent $i \in I_j$ is increasing in direct consumption x_0^i of the global good, and decreasing in $Z^j = \sum_{i \in I_j} x_0^i$, so that,

$$U^i(x_0^i, Z^j) = \frac{1}{1-\gamma}(x_0^i)^{1-\gamma} H(Z^j), \tag{14}$$

where H is a positive function and $H/(1-\gamma)$ is decreasing. That is, utility is increasing in own consumption, but decreasing in community consumption. If $\gamma > 1$ then H is increasing, so that the marginal utility of consumption is increasing with the level of community consumption – individuals value income more if their community is rich.

This functional form captures the idea of agents concern for their community status, yet preserves aggregation and allows us to model the community as a single aggregate investor. Exogenously assuming such preferences will therefore yield similar implications as our model; indeed, our model of local labor is equivalent to the case of $H = h^\gamma$. That is, when individuals must compete for scarce local resources, relative wealth matters. We view our model as way to "endogenize" individuals preferences regarding relative consumption. While the equilibrium outcomes are the same for either setting, the welfare implications and interpretation of the equilibrium will differ.

That said one should use caution when using a utility function that has a relative component. In many cases such utility functions do *not* yield herding. Consider for example a "catching up with the Joneses" utility function in which utility depends purely on an individual's share of aggregate community wealth. That is, suppose the utility function takes the form:

$$u(x_0^i/Z^j).$$

This specification does not support herding. Intuitively, if all agents choose the same undiversified portfolio, then there is no "relative" risk. However, each agent will find it profitable to deviate slightly towards a more diversified portfolio: in terms of relative wealth, it is profitable to give up a dollar when the community is rich and gain one when it is poor. This destroys a herding equilibrium.[9]

In general, to support herding in equilibrium it is necessary that in some instances individuals prefer to hold portfolios that are more extreme than the rest of the community (so that the reaction function has $m' > 1$). Many standard models of status do not produce this. In our setting, it occurs endogenously through the effect on relative prices.

4 Biased Traders

Thus far we have assumed that all investors are rational and unconstrained. We have established that, when they exist, the undiversified equilibria are the stable equilibria in

[9]Formally, marginal utility of income for the representative agent is $u'(1)/Z^j$, which is strictly decreasing in Z^j (and is identical to log utility). Thus, full diversification is the unique equilibrium follows as in the proof of Theorem 3.

this economy. There are two symmetric unstable equilibria, given by σ^* and $-\sigma^*$. In other words, investors in each community hold biased portfolios, but the bias can be towards either asset. This is natural since until now, there is perfect symmetry and there is no distinction between the assets. In this section, we consider the case in which some of the agents are subject to a behavioral bias, or otherwise constrained in their portfolio choice. We show that these biased traders can "pull" the community towards a particular equilibrium.

Suppose, for example, that a subset of the population with wealth ω is subject to a behavioral bias towards investing in "local" firms. As a result, they hold a portfolio with payoffs

$$\omega \begin{bmatrix} 1+\hat{\sigma} \\ 1-\hat{\sigma} \end{bmatrix}.$$

independent of behavior of other traders. That is, the hold portfolios with bias given by $\hat{\sigma}$. What effect does the presence of these behavioral investors have on the equilibrium portfolios of rational investors?

To preserve symmetry and simplify the analysis, we assume that the same decomposition applies to community 2, with $\hat{\sigma}$ replaced by $-\hat{\sigma}$, and look at equilibria in which securities are equally priced. In this case, given an aggregate (rational and behavioral) community volatility of σ, the reaction function $m(\sigma)$ gives the optimal portfolio volatility for rational investors. Since with equally priced securities $EZ^j = 1$, the rational investors have aggregate expected wealth $1-\omega$. This yields the equilibrium condition:

$$(1-\omega)m(\sigma) + \omega\,\hat{\sigma} = \sigma,$$

which can be rewritten as

$$m(\sigma) = \hat{\sigma} + \frac{\sigma - \hat{\sigma}}{1-\omega} \equiv f(\sigma|\omega, \hat{\sigma}).$$

This equilibrium condition is illustrated in Figure 3 with $\alpha = 1$ and $\gamma = 4$. Rather than an equilibrium being defined as the intersection of m with the 45° line, it is now the intersection of m with the line defined by f. The line f can be thought of as a rotation of the 45° line around the point $\sigma = \hat{\sigma}$ until it has slope $1/(1-\omega)$. This is illustrated with $\hat{\sigma} = 50\%$ and $\omega = 20\%$.

The following results can be seen easily from the figure:

Theorem 8 *For any $\omega > 0$, $\hat{\sigma} > 0$, full diversification is no longer an equilibrium. For any $\hat{\sigma} > 0$, there exists large enough ω such that there is a unique equilibrium. This equilibrium is stable and has $\sigma^* > 0$.*

Proof. The first statement follows immediately since $m(0) = 0$ and $f(0|\omega, \hat{\sigma}) < 0$. For the second result, we first observe that there exists a finite M such that $m'(\sigma) < M$. This follows from the definition of m plus the fact that $h(z) > 1$ and $h'(z)$ bounded for $z \in [0, 2]$. Since $m(0) = 0 > f(0|\omega, \hat{\sigma})$ and $m(1) < 1 \leq f(1|\omega, \sigma)$, there is at least one equilibrium with $\sigma > 0$. If $1/(1-\omega) > M$, then this equilibrium must be unique. ■

The figure also makes clear the following comparative statics properties of σ^*, which both follow from the fact that m is increasing:

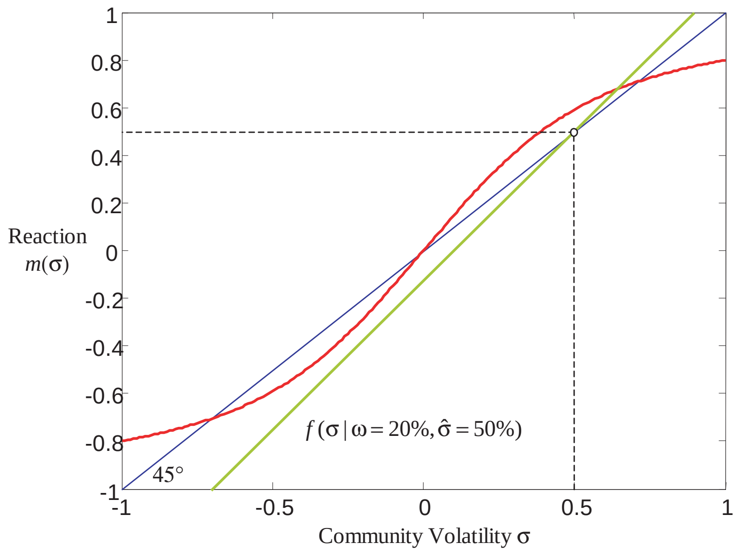

Figure 3: Equilibrium with Constrained Investors

1. As the wealth of the behavioral investors increases so does their influence on rational investors. That is, $|m(\sigma^*) - \hat{\sigma}|$ is decreasing in ω, and σ^* converges monotonically to $\hat{\sigma}$ as $\omega \to 1$.

2. The more volatile the bias is, the more volatile the portfolio of the rational investors. That is, σ^* and $m(\sigma^*)$ are increasing in $\hat{\sigma}$.

Thus, the existence of biased behavioral investors breaks the symmetry of the model and biases the equilibrium portfolio choice of the community.

In the above analysis we have assumed that behavioral investors *must* hold portfolios with bias $\hat{\sigma}$. Alternatively, we could consider a setting in which $\hat{\sigma}$ represents instead the *minimal* bias these investors can hold, but they are free to hold more biased portfolios. In this case, the equilibrium would be described by the intersection of m with the function

$$\hat{f}(\sigma) = \min(f(\sigma), \sigma).$$

That is, $\hat{f}$ coincides with f below $\hat{\sigma}$, and with the 45° line above $\hat{\sigma}$. As before, if there are enough such investors the unique equilibrium is one with a local bias. However, while the constraint has an effect on the equilibrium set, it need not bind in equilibrium. For instance, in the example of Figure 3, investors who are forced to hold a bias of at least $\hat{\sigma}$ would choose an even greater bias in equilibrium. Moreover, the behavioral bias is rational in the sense that if we "cure" a behavioral investor he would not change his portfolio. In other words, while the behavioral bias selects this equilibrium, in the resulting equilibrium *all* investors are behaving rationally!

The above analysis applies for the case when behavioral agents have a preference for local firms, and are unwilling to fully diversify (e.g., they hold at least 60% local stocks). This may result from a false sense of familiarity with local firms. An alternative story that yields to the same outcome is of constrained investors. Some agents may receive compensation that is directly tied to the performance of the local firms (e.g., option and bonus compensation, etc.). More generally, agents may hold skills or human capital whose value is positively correlated with the productivity of a certain sector. If these agents are unable to trade against this income, they will be constrained in their portfolio choice, affecting the equilibrium outcome. The result is the same in that unconstrained investors choose similar portfolios. The comparative statics are again similar in that the more constrained investors and the more biased their income the larger is their impact on the equilibrium. Still, the constraints may affect the outcome without being binding.

5 Price Effects

Thus far, we have considered economies in which the communities are symmetric and all securities have equal expected returns. In this section, we keep the same basic structure as before but make the model asymmetric. We then examine the potential distortions of prices and expected returns introduced by the effects that we have outlined.

In particular, we consider the case in which a small subset of the global population is subject to community effects. The remaining population cares only about the global good. If the small community "herds" into asset 1 and chooses an undiversified portfolio, then by market clearing, the remaining population will also be undiversified and hold a relatively higher share of asset 2. For this to occur in equilibrium, the return of asset 2 must exceed that of asset 1.

Formally, consider two communities as before, but now of uneven size. Let community 1 have share w of the aggregate global wealth; that is, $\bar{\theta}^1 = w\bar{\theta}$. This community also has an equivalent endowment of the local good $\bar{X}_1 = w$. We assume the local good has importance given by α, as before.

Community 2 has aggregate portfolio endowment $\bar{\theta}_2 = (1-w)\bar{\theta}$ and local good endowment $\bar{X}_2 = 1-w$. However, we assume that this community is not subject to the effects introduced by local goods. One way to achieve this is to assume that $\alpha_2^i = 0$ for $i \in I_2$; that is, the local good is unimportant in this community. Alternatively, we can leave preferences unchanged but remove the collateral constraint for community 2.

Given this specification, investors in community 1 are subject to the portfolio bias introduced by community effects, while those in community 2 are not. Thus, an undiversified equilibrium will affect equilibrium prices. Since markets are complete, we can describe asset prices in terms of state prices. Let π be the relative price of consumption in state $s = 1$ relative to consumption in state $s = 2$. Given π, the equilibrium condition for investors in community 2 implies that they will hold volatility σ_2 such that

$$\left(\frac{1/(1+\sigma_2)}{1/(1-\sigma_2)}\right)^{\gamma} = \pi. \tag{15}$$

If we denote by c the expected consumption of the global good by community 1, then by market clearing the expected consumption of community 2 is $1-c$, and since there is no aggregate risk,

$$c\sigma_1 + (1-c)\sigma_2 = 0. \tag{16}$$

The budget constraint for community 1 then implies

$$\pi c(1+\sigma_1) + c(1-\sigma_1) = \pi w + w. \tag{17}$$

Given w and σ_1, the three equations (15)-(17) can be solved for the unknowns, σ_2, π and c. We can interpret this as the "supply function" of community 2. That is, given community 1's demand for volatility σ_1, these equations give the marginal price π at which investors in community 2 are willing to supply it, and thus determine the expected level of consumption community 1 investors will be able to afford.

We can now determine the reaction function of a community 1 investor. That is, given aggregate holdings σ_1 in the community, and given state prices π above, what is the optimal risk choice σ^i for an individual investor i in community 1? Since community 1 investors are subject to community effects, the optimal portfolio choice for investor i satisfies

$$\left(\frac{h(c(1+\sigma_1)/w)/(1+\sigma^i)}{h(c(1-\sigma_1)/w)/(1-\sigma^i)}\right)^\gamma = \pi. \tag{18}$$

Combining this condition with the previous equations allows us to solve for the reaction function,

$$\sigma^i \equiv m_1(\sigma_1, w).$$

Again, an equilibrium corresponds to a fixed point of the reaction function m_1.

Note that if $w \to 0$, then from (17), $c \to 0$, and from (16), $\sigma_2 \to 0$. Thus, (15) implies $\pi \to 1$. In this case, (18) coincides with (12), noting that $c/w \to 1$. That is, if community 1 is negligible and has no price impact, then the reaction function is exactly the one we derived in the symmetric case. Also, $m_1(\sigma_1, w)$ is decreasing in w (see the proof of Theorem 9), so that as the community becomes larger, its price impact increases, leading investors in the community to reduce the scale of their positions. If $w = 1$, then of course $\sigma_1 = 0$ is a unique equilibrium; community 1 cannot hold a biased portfolio if there is no other community to trade with.

Figure 4 illustrates the reaction function for $\alpha = 1$, $\gamma = 4$ and $\omega \in \{0, 10\%, 25\%\}$. We have the following result regarding the existence of an undiversified equilibrium:

Theorem 9 *There exists an undiversified equilibrium if*

$$\gamma > 1 + \frac{1 + 1/\alpha}{1 - w}.$$

The largest undiversified equilibrium, σ_1^*, is decreasing in w.

Proof. See appendix. ■

In the undiversified equilibrium, σ_1^*, investors in community 2 hold risky portfolios. Thus, expected asset returns will no longer be equated and will contain risk premia. The importance of these risk premia can be measured by computing the maximal Sharpe ratio

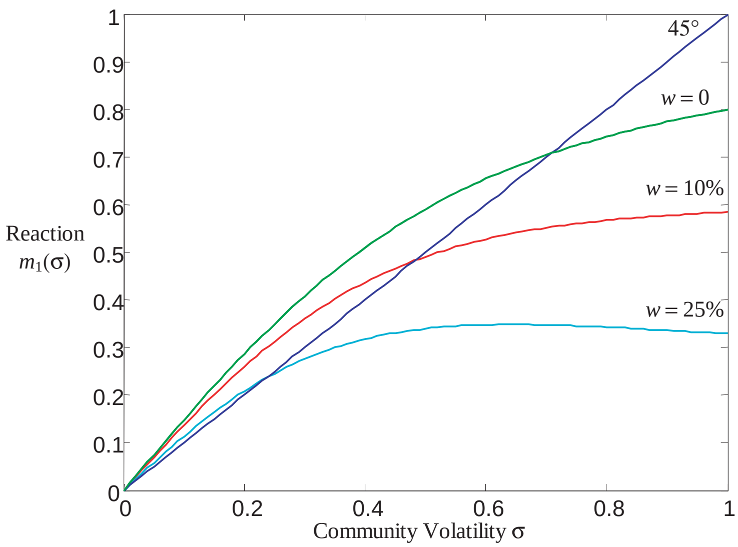

Figure 4: Equilibrium with Price Impact

of the assets:[10]

$$\rho = \max_R \left| \frac{E[R] - r_f}{\sigma(R)} \right| = \left| \frac{\pi - 1}{\pi + 1} \right|.$$

Figure 5 illustrates both σ_1^* and ρ for varying sizes w of the community, given $\gamma = 4$ and $\alpha = 1$. Consistent with Theorem 9, note that σ_1^* is decreasing with the size of community 1. The effect on the equilibrium Sharpe ratio ρ is non-monotonic, however. When community 1 is small, so is their price impact, and so returns are hardly affected. On the other hand, when community 1 is large enough, only the fully diversified equilibrium remains, and the Sharpe ratio is again zero. For intermediate sizes, however, the Sharpe ratio can be high even though aggregate consumption is *riskless*. Thus, our model produces an "equity premium puzzle." The resolution of the puzzle in the context of our model is that while aggregate consumption is smooth, individual consumption is very volatile due to the "herding" of community 1 investors.

6 Robustness – Local Goods and Participation

The analysis thus far has depended on two critical assumptions. First, we have assumed that local goods account for a large enough component α of utility. Second, we have assumed that agents endowed with the local good are constrained from participating in financial markets. In this section we evaluate the reasonableness of both of these assumptions.

[10]The second equality follows immediately from the Hansen-Jagannathan bound, interpreting π as the stochastic discount factor.

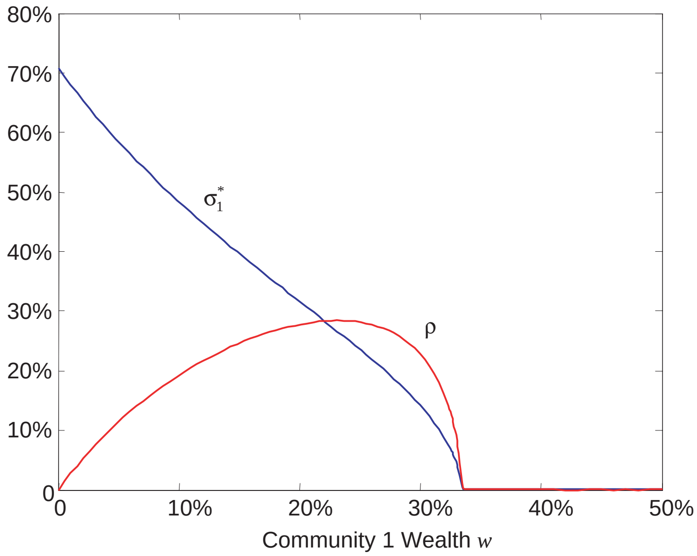

Figure 5: Equilibrium Volatility and Sharpe Ratio vs. Community Size

6.1 Empirical Evidence for Local Goods

The effects described in this paper depend upon the existence of goods that are consumed locally and are not traded across communities. To what extent is the existence of such goods justified empirically? At the international level, Stockman and Tesar (1995) and Kravis, Heston and Summers (1982) estimate that non-traded goods account for close to 50% of a country's output. Traditionally, these estimates include housing, health, education, construction, local transportation, electricity, etc. Taking this estimate as given suggest (using the full diversification benchmark) that α is close to 1.

However, it is important to note that the value of goods that are traditionally considered as tradable, such as manufactured retail goods, also include a significant non-tradable component. This is because their prices reflect the cost of labor, local transportation and real estate related costs related to their distribution. Burstein, Neves and Rebelo (2001) estimate that these (non-tradable) distribution costs represent close to 50% of the final price of (tradable) consumer goods. Taking this into account in our two good model would suggest $\alpha \approx 3$.

Alternatively, one can also interpret this data as a sign of complementarities. In particular, it suggests that individuals consume goods which are themselves "bundles" of both tradable and non-tradable goods. We can incorporate this in our current model by introducing a 3rd good, x_{0j}, in the utility function of agent's in community j, which is produced by combining g units of the global good and $1-g$ units of the local good. Figure 6 illustrates the effect on the reaction function when $g = 50\%$, $\gamma = 3$, and the weights for each good in the utility function are given by $\alpha_0 = \alpha_j = 1$, $\alpha_{0j} \in \{0, 1, 10\}$. Note that the production complementarity enhances the the effects described in this paper.[11] Intuitively, this arises

[11] Note also that this experiment only measures the importance of the production complementarity. It still leaves equal weight on the pure global and pure local goods. To be most consistent with the Burstein, et al. (2001) data, we should reduce the weight on the pure global good to close to zero. This would further

because the complementarity increases the importance of the local good.

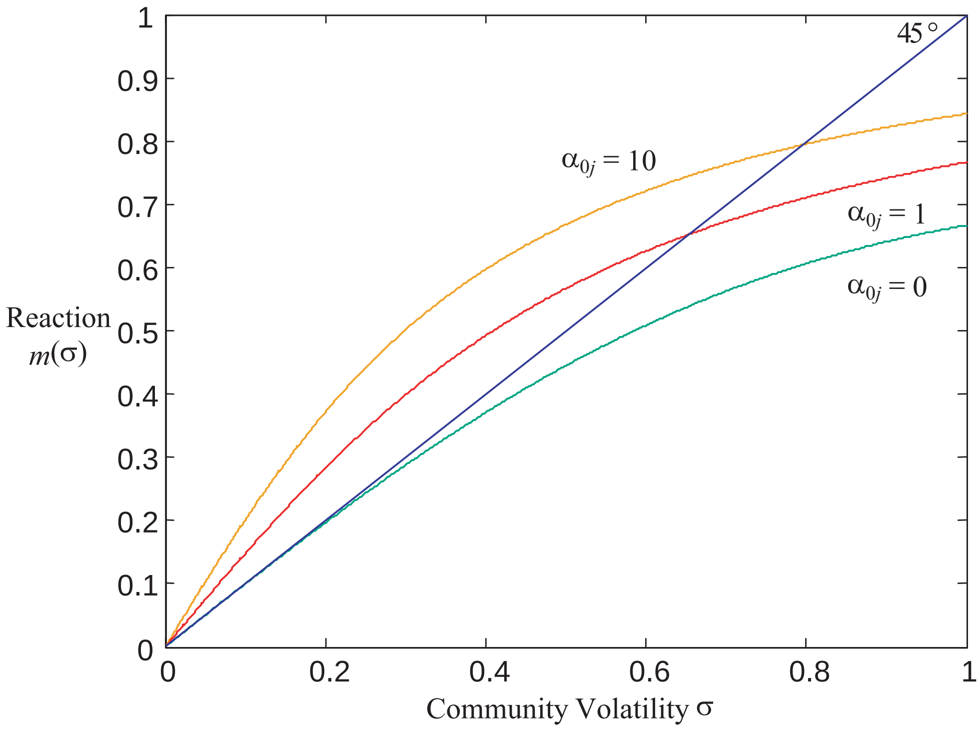

Figure 6: Equilibrium with Production Complementarities

Another issue that may affect our results is the elasticity of the supply of the local goods. We have assumed a fixed supply in our model. However, in the case of local labor, if there is some labor mobility than we would expect labor to migrate from the poor community to the rich community in period 2, reducing the volatility of the price of labor p_j. Alternatively, if the local good is a commodity that can be produced (e.g., build new housing), this will also mitigate the price volatility. Incorporating these possibilities will lower the investors' reaction function since the price effects are smaller. Still, however, an undiversified equilibrium will exist if the slope of the reaction function at full diversification is larger than 1. The conditions for this will be unchanged if there are any fixed costs associated with adjusting the supply of the local good. Hence, if local laborers have a cost of moving, then our sufficient conditions for undiversified equilibria are unchanged throughout the paper.

Recall also that in our model, local goods are defined in terms of tastes. Thus, "communities" in our sense may be "taste-based" rather than geographic. Given this interpretation, the community effects we describe depend on the existence of heterogeneous tastes across different groups of consumers. For example, retired consumers consume a distinct set of goods (e.g., retirement homes). As long as the distinguishing tastes are for goods in relatively inelastic supply, community effects in portfolio choice will emerge. This suggests interesting avenues for future empirical work.

enhance our results.

6.2 Participation Constraints

In the model we assumed that agents endowed with the local good are completely constrained from participating in the financial market. Given that participation rates in the stock market in the U.S. are still below 50% (and as recently as 1989 were close to 30%), and that participation is strongly correlated with wealth, it seems reasonable to assume that participation rates are relatively low for individuals in the local labor market. In addition, even if individuals endowed with local goods do participate in the financial market, it is likely that they are unable to fully collateralize the value of their future endowment – brokers typically do not accept future labor income, real estate, etc., as collateral for margin accounts.

Thus, it seems natural to assume that there are constraints that inhibit hedging the local price risk for owners of the local good. However, these constraints are unlikely to be as extreme as those imposed in the model. Here we show that we can relax these constraints somewhat without undermining the main results.

Suppose an agent endowed with 1 unit of the local good is permitted to participate in the financial market. Then, given community risk σ his optimal trade will be to adjust his portfolio risk to the optimal risk given by the reaction function $m(\sigma)$. That is, he will choose a portfolio that pays $\{-b, b\}$, where b satisfies:[12]

$$\frac{P(1+\sigma)-b}{P(1-\sigma)+b} = \frac{1+m(\sigma)}{1-m(\sigma)},$$

where $P(z)$ is the price of the local good given global community income z. Using Lemma 1 to compute P with $\bar{X}_j = 1$, solving for b yields:

$$b(\sigma) = .5\alpha[(1-m(\sigma))(1+\sigma)^{\gamma} - (1+m(\sigma))(1-\sigma)^{\gamma}].$$

Thus, if a fraction l of the endowment of local good is held by agents who are unconstrained, then we get the aggregate reaction function

$$m_l(\sigma) = m(\sigma) - lb(\sigma).$$

Figure 7 illustrates this reaction function for the case $\gamma = 4$, $\alpha = 2$ and $l \in \{0, 10\%, 25\%\}$.

Increasing l diminishes the equilibrium bias since the tendency of investors to herd is offset somewhat by the hedging of the holders of the local good.[13] However, in the example above as long as $l < 25\%$, undiversified equilibria still persist. Indeed, we have the following general result, which shows that our results do not depend on the extreme assumption that $l = 0$.

[12] Note that the trade $\{-b, b\}$ has cost zero (with symmetric prices) and so satisfies the budget constraint.

[13] In this framework we assumed that a fraction l is unconstrained and the remainder are fully constrained. However, one can also allow for partially constrained agents. A natural constraint, for example, is that the position is "capped" by some amount $\bar{b}$ which may depend on the equilibrium σ. In this case, $m_l(\sigma) = m(\sigma) - \min(b(\sigma), \bar{b}(\sigma))$. It is easy to show that the undiversified equilibrium persists in this alternative specification as well, as long as $\bar{b}(\sigma)$ is not too large. One possible choice is that $\bar{b} = P(1-\sigma)$, the amount of riskless borrowing a laborer can conduct.

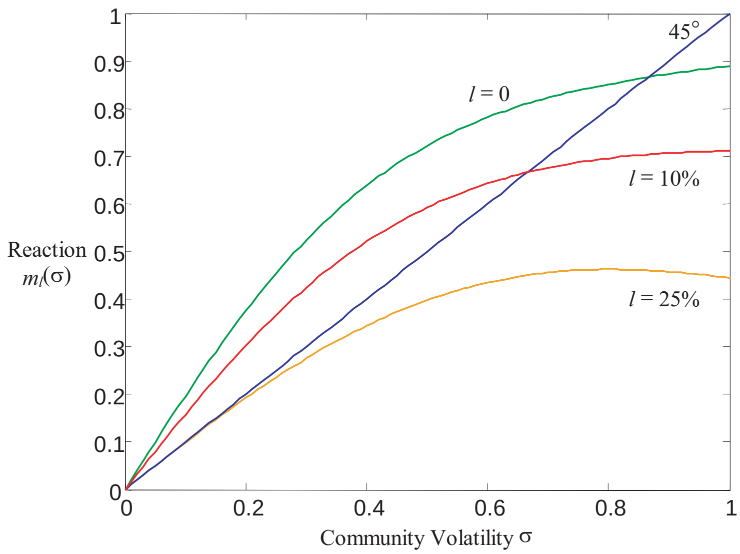

Figure 7: Equilibrium with Unconstrained Holders of the Local Good

Theorem 10 *There exists an undiversified equilibrium as long as*

$$l < \frac{\gamma - (2 + 1/\alpha)}{\gamma + \alpha}.$$

Proof. From (13) and the fact that $b'(0) = \alpha(\gamma - m'(0))$, this is the condition for $m_l'(0) > 1$. ■

7 Conclusion

Our paper provides an explanation for herding and lack of risk sharing. Indeed, we demonstrate that individuals may choose undiversified portfolios even in an environment with complete financial markets and no aggregate risk. We begin by showing that competition for local resources (such as local real estate, labor and other services) creates an externality so that individuals care about their relative wealth in the community. This effect has important consequences. If the local resources cannot be fully collateralized, and if investors are sufficiently risk averse, then individual investors will try to correlate their wealth with that of their community.

Absent aggregate risk, there always exists an equilibrium in which all investors are fully diversified. While this equilibrium is Pareto optimal, we show that when agents are sufficiently risk averse, this equilibrium is not stable. In all stable equilibria, investors in a given community tilt their portfolio in the same direction, taking unnecessary risk. Each agent does not diversify because the rest of her community is not diversified. That is, each investor wants to hedge by choosing portfolios that yield a higher payoff when the price of local resources is high, and the price of the local resource is increasing in community wealth. As a result of this "herding" effect, agents are worse off than in a fully diversified equilibrium.

We use this to examine the impact of a behavioral bias. If some agents are subject to a behavioral bias then rational agents adopt this bias and amplify its effect. This in turn can rationalize the bias. A similar conclusion follows if some agents' income is tied to the productivity of a certain sector.

Finally, we also consider the implications of our model for asset returns. We show that the presence of a small subset of agents in the economy that are subject to community effects is sufficient to significantly impact returns. Specifically, equilibrium Sharpe ratios can be high, even though aggregate consumption is riskless. The intuition for this result is that propensity for individual communities to "herd" in their investment decisions implies that community consumption is much more variable than aggregate consumption.

Within our model, diversification is a public good. Any individual investor's failure to diversify will induce other investors in the same community to tilt their portfolios in the same direction, ultimately making the entire community worse off. One implication for this is a history dependence in portfolio choice. Prior to the development of financial markets, communities were likely unable to diversify many "local" risks. As markets have become more complete, one would expect investors to diversify their portfolios away from such risks. Our results make clear, however, that there is a coordination aspect to such diversification. As a result, the community is likely to remain in a stable equilibrium in which the local risk is still held.

This has obvious policy implications. For example, there is a role for social policies which subsidize investor diversification. There can be welfare gains from restricting investor portfolio choice in retirement accounts in a way that prevents them from holding undiversified positions. Indeed, our results imply that much of the policy implications related to public goods may also apply to investor diversification.

Our model predicts that the relative price of local goods in a community should be positively correlated with community wealth. In testing this, one important aspect is the choice of a time horizon. For example, Boudoukh and Richardson (1993) look at the relationship between stock market returns and inflation,[14] and find that while there is no positive correlation at high frequencies (e.g., monthly or quarterly), there is a strong positive correlation over longer horizons.[15] This might be a reflection of the speed of price adjustments for real goods. If so, then in testing our model a longer horizon is appropriate.

Early empirical support for our model includes Bodnaruk (2002). He shows that investors that move sell shares of companies located in their old residence and buy ones closer to their new home. This is consistent with our model, but also with the hypothesis that physical proximity facilitates information transmission. Evidence on the performance of stocks that movers sell compared with stock that they buy might help separate between the two explanations. If one considers the variant of our model where some investors are constrained so that their wealth is tied to local companies then there are additional predictions that can be tested. For example, holding all else equal, in cities where there is a

[14]Note that pure nominal inflation is not relevant in our model; what matters is the change in the *relative price* of the local goods, for which inflation might be a proxy if purchasing power parity holds more closely for traded goods.

[15]Not surprisingly papers such as Kaplanis and Cooper (1994) that use *monthly* international data do not find support to the idea that local stocks provide a better hedge against inflation.

small number of dominant employers ("company towns"), we would expect the portfolios of non-employees to be biased in the same direction. Moreover, this effect should be stronger the greater the volatility of the local firms. Observation on the part of the authors in their home towns (Silicon Valley and Austin) supports this prediction!

8 Appendix

8.1 Proof of Uniqueness for Theorem 5

From Equation (10), the condition for an equilibrium $\sigma^* > 0$ can be written as $\psi(\sigma^*) = \frac{1}{\alpha}$ where $n = \gamma - 2$ and,

$$\psi(\sigma) \equiv \frac{(1-\sigma^2)[(1+\sigma)^n - (1-\sigma)^n]}{2\sigma}.$$

Note that $\lim_{\sigma \downarrow 0} \psi(\sigma) = n$ and $\psi(1) = 0$, so that for $n = \gamma - 2 > 1/\alpha$, the continuity of ψ ensures that such σ^* exists. Rewriting the expression for ψ and simple differentiation yield:

1. $\psi(\sigma) = \nu(\sigma) + \nu(-\sigma)$ where $\nu(\sigma) = \frac{1-\sigma^2}{2\sigma}(1+\sigma)^n$,
2. $\psi'(\sigma) = -\kappa(\sigma)\nu(\sigma) + \kappa(-\sigma)\nu(-\sigma)$ where $\kappa(\sigma) = \frac{(n+1)\sigma^2 - n\sigma + 1}{\sigma(1-\sigma^2)}$,
3. $\psi''(\sigma) = -\left[\lambda(\sigma)\kappa(\sigma)\nu(\sigma) + \lambda(-\sigma)\kappa(-\sigma)\nu(-\sigma)\right]$ where $\lambda(\sigma) = \frac{2(n+1)\sigma - n}{(n+1)\sigma^2 - n\sigma + 1} - \frac{(n-2)\sigma^2 - n\sigma + 2}{\sigma(1-\sigma^2)}$.

We now argue that such σ^* is indeed unique. We focus on "critical points" $\hat{\sigma}$ which satisfy $\psi'(\hat{\sigma}) = 0$. We note that at a critical point, ψ'' reduces to:

$$\psi''(\hat{\sigma}) = -\left[\lambda(\hat{\sigma}) + \lambda(-\hat{\sigma})\right]\kappa(\hat{\sigma})\nu(\hat{\sigma}) = \frac{(n+1)n(1+\hat{\sigma})^n}{(1-\hat{\sigma}^2)((n+1)\hat{\sigma}^2 + n\hat{\sigma} + 1)}\left[n(1-\hat{\sigma}^2) - 4\right].$$

Consider two cases:

- <u>Case 1: $n < 4$:</u> the claim follows since $\psi'(\hat{\sigma}) = 0$ implies that $\psi''(\hat{\sigma}) < 0$. Combining with $\psi'(0) = 0, \psi''(0) < 0$ we conclude that ψ is strictly decreasing on $[0,1]$.
- <u>Case 2: $n > 4$:</u> $\psi'(0) = 0$ and $\psi''(0) > 0$ imply that ψ is increasing at zero. For it to satisfy $\psi(1) = 0$ it cannot be increasing on the whole $[0,1]$ interval. Hence, it hits first a local maxima $\hat{\sigma}'$ at which $\psi'(\hat{\sigma}') = 0$ and $\psi''(\hat{\sigma}') \leq 0$; it implies that $n(1-\hat{\sigma}'^2) - 4 \leq 0$. We conclude that for any other $\hat{\sigma} > \hat{\sigma}'$ for which $\psi'(\hat{\sigma}) = 0$ we have that $n(1-\hat{\sigma}^2) - 4 < 0$ and $\psi''(\hat{\sigma}) < 0$. Hence, the claim follows from ψ being strictly decreasing on $\left[\hat{\sigma}', 1\right]$.

Finally, the case $n = 4$ ($\gamma = 6$) is already resolved by the explicit solution in Corollary 2.

8.2 Proof of Theorem 7

From Lemma 2, the indirect utility for any agent in community j is given by

$$v(w) = \frac{1}{1-\gamma} w^{1-\gamma} h(Z^j)^\gamma$$

where w is the agent's numeraire wealth. First consider the investors. From 11 and the budget constraint, in equilibrium the wealth of investor i is given by

$$w^i = \bar{z}^i Z^j,$$

where $\bar{z}^i = \bar{\theta}_1^i + \bar{\theta}_2^i$. In the fully diversified equilibrium, $Z^j = 1$. Thus, since the existence of an undiversified equilibrium implies $\gamma > 2$ (by Theorem 3) and hence that utility is negative, investor i is worse off in the undiversified equilibrium if and only if

$$E\left[(Z^j)^{1-\gamma} h(Z^j)^\gamma\right] > h(1)^\gamma. \tag{19}$$

Now, the equilibrium condition for the undiversified equilibrium is that the investor's marginal utility of income is equated across states, or equivalently, $h(Z^j)/Z^j = c$, for some constant c. Using this plus the fact that $E[Z^j] = 1$, (19) is equivalent to

$$h(Z^j)/Z^j = c > h(1).$$

Suppose $c < h(1)$. Then multiplying by Z^j and taking expectations yields

$$E[h(Z^j)] < h(1),$$

which contradicts the convexity of h for $\gamma > 2$. Thus, $c > 1$, and every investor is worse off in the undiversified equilibrium.

Next consider the laborers. For $i \in I_j^L$, using Lemma 1,

$$w^i = p_j \bar{x}_j^i = \alpha \bar{x}_j^i (Z^j)^\gamma.$$

Thus, given $\bar{x}_j^i$ and Z^j are uncorrelated, laborer i is worse off in the undiversified equilibrium if and only if

$$E\left[(Z^j)^{\gamma(1-\gamma)} h(Z^j)^\gamma\right] > h(1)^\gamma.$$

Because $\gamma > 1$, a sufficient condition is

$$E\left[(Z^j)^{(1-\gamma)} h(Z^j)\right] > h(1),$$

which follows immediately since $z^{1-\gamma}h(z) = z^{1-\gamma} + \alpha$ is convex in z for $\gamma > 1$.

8.3 Proof of Theorem 9

Note that $m_1(0, w) = 0$. Differentiating (18) yields

$$m_1'(0, w) = \frac{\alpha}{1+\alpha}(\gamma - 1) - \frac{\pi'(0, w)}{2\gamma}.$$

From (15) and (16), and since $c = w$ for $\sigma_1 = 0$,

$$\pi'(0, w) = -2\gamma\sigma_2'(0, w) = 2\gamma \frac{w}{1-w}.$$

Combining these implies that $m_1'(0, w) > 1$ if and only if

$$\gamma > 1 + \frac{1 + 1/\alpha}{1 - w}.$$

Thus, the existence of an undiversified equilibrium follows by showing that $m_1(1, w) < 1$. But for $\sigma_1 = 1$, (18) can be written,

$$\frac{1-\sigma^i}{1+\sigma^i} = \frac{h(0)}{h(2c/w)}\pi^{1/\gamma}.$$

Because $\pi > 0$ and $h > 0$, $\sigma^i < 1$.

To show that σ_1^* is decreasing in w, it is sufficient to show that $m_1(\sigma_1, w)$ is decreasing in w for $\sigma_1 > 0$. Thus, fix $\sigma_1 > 0$ and note that (15) and (16) can be rewritten as,

$$\frac{1-c(1-\sigma_1)}{1-c(1+\sigma_1)} = \pi^{1/\gamma}.$$

This implies that $\pi > 1$ is increasing in c. Note also that (17) can be written as

$$c\left(1+\frac{\pi-1}{\pi+1}\sigma_1\right) = w.$$

This implies that $w > c$ is increasing in c, and that c/w is decreasing in c. Therefore, π is increasing in w and c/w is decreasing in w. Finally, (18) can be written as

$$\frac{h(c(1+\sigma_1)/w)}{h(c(1-\sigma_1)/w)}\pi^{-1/\gamma} = \frac{1+\sigma^i}{1-\sigma^i}.$$

Since $\frac{h(c(1+\sigma_1)/w)}{h(c(1-\sigma_1)/w)}$ is increasing in c/w, it is decreasing in w, as is $\pi^{-1/\gamma}$. Thus, the optimal response $m_1 = \sigma^i$ is also decreasing in w.

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