

# Asset Pricing Implications of Firms' Financing Constraints

Joao F. Gomes, Amir Yaron, and Lu Zhang\*

July 2001

## **Abstract**

We ask whether firms' financing constraints are quantitatively important in explaining stock returns. We show that, for a large class of theoretical models, firms' financing constraints have a parsimonious representation amenable to empirical analysis. Quantitative experiments suggest that financing constraints can help match the volatility of stock returns but at the cost of reducing the model's ability to match stocks' return correlation structure. This latter effect makes financing constraints unsuccessful in improving the overall statistical ability of investment returns as a factor pricing model.

## **Preliminary and Incomplete**

---

\*Finance Department, The Wharton School at the University of Pennsylvania, Philadelphia, PA 19104-6367. We have benefited from helpful discussions with Andy Abel, Ravi Bansal, Michael Brandt, Tom Tallarini, and comments of participants of Macro Lunch seminar at Wharton and SED meetings. All remaining errors are of course our own. E-mail: gomesj@wharton.upenn.edu, yarona@wharton.upenn.edu, and zhanglu@wharton.upenn.edu.

# 1 Introduction

Several authors have examined the role of financing constraints in determining the optimal investment behavior of firms, while many others have incorporated these frictions into aggregate models to study their implications for typical macroeconomic phenomena.<sup>1,2</sup> Unfortunately, in spite of this enormous interest, research on their asset pricing implications has been, by and large, neglected. This is an important oversight since fluctuations in asset prices often play a crucial role in the dynamic behavior of these models. In addition, asset prices can also provide important additional information above and beyond the restrictions imposed by the behavior of typical macroeconomic aggregates.

In this paper we ask the question whether financial frictions are quantitatively important in explaining asset market phenomena. To do this we begin by showing that for a large class of models, firms' financing constraints have a common general representation amenable to empirical analysis. Although these models can differ substantially on the foundations of the financial frictions (such as asymmetric information, costly state verification, "lemon problems" with issuing stocks and so on), they all share a common general structure for the firm's optimal investment decision and the returns to physical investment — a restriction we then use in our empirical analysis.

Our empirical analysis is divided into two parts. First, we examine the implications of the models with financial frictions for the unconditional properties of investment returns and compare these with the empirical properties of stock market data. Our results here

---

<sup>1</sup>Some of the earlier studies on the impact of financing constraints on firm behavior include Fazzari, Petersen and Hubbard (1988), Hayashi and Inoue (1991), Hoshi, Kashyap and Scharfstein (1991), Blundell, Bond, Devereaux and Schiantarelli (1994), Kashyap, Lamont and Stein (1994) Gertler and Gilchrist (1994) and Kaplan and Zingales (1997).

<sup>2</sup>The aggregate implications of models with firm based financing constraints have been explored by, among others, Bernanke and Gertler (1989), Bernanke, Gertler and Gilchrist (2000), Cooley and Quadrini (1999, 2000), Den Haan, Ramey, and Watson (1999), Kiyotaki and Moore (1997), and Holmstrom and Tirole (1997).

indicate that adding financial frictions can significantly raise the volatility of investment based returns to a level comparable to the volatility of stocks, something that would be difficult to accomplish with physical adjustment costs alone. However, due to the countercyclical properties of the profits to investment ratio, financing constraints also reduce the contemporaneous correlation between stock and investment returns.

In the second part of our analysis we investigate whether an investment return factor pricing model holds, i.e., whether a stochastic discount factor based on the returns to accumulation of physical capital (generated from the model) prices assets correctly. In particular, we are interested in examining to what extent the presence of financing constraints improves the ability of such a model to price the cross-section of asset returns. Specifically, we use the Generalized Method of Moments (GMM) to formally test the asset pricing restrictions of financing constraints. By parameterizing the stochastic discount factor in the economy as a linear function of physical investment returns, the effects of financing constraints are now incorporated into the pricing kernel.

Our GMM analysis shows, as in Cochrane (1991, 1996), that investment based models can account well for asset returns. More importantly however, our results strongly suggest that the role of financing frictions in pricing asset returns is quite negligible. Without exception, all our model specifications deliver economically insignificant values for the level of financing frictions.

Examining the effects of financing constraints we find that they produce a significantly lower market price of risk. These findings are also confirmed by the model's implied beta representation of excess returns, where the introduction of financing costs consistently increases the pricing errors on the cross-section of expected returns. Our findings are also robust to alternative formulations of our model such as allowing for time variation in the

degree of financing frictions or restricting attention to only the stock returns of small firms.

Our findings cast some doubt on whether fluctuations in asset prices, induced by the presence of financial frictions, can provide a realistic channel for the propagation mechanism in macroeconomic models. While these constraints may indeed help generate more interesting dynamics for the typical macroeconomic aggregates, they seem to strain the model's ability to match financial data.

Our work is most closely related to earlier research by Cochrane (1991, 1996) that first addressed the issue of constructing and testing production based asset pricing models, and to work by Restoy and Rockinger (1994) who generalize some of results in Cochrane (1991) to an environment with investment constraints and taxes.

More recently, Lamont, Polk, and Saá-Requejo (2000), using an index of financing constraints as a pricing factor in a reduced form model of returns, document that while financing constraints may impact unconditional returns, there is no evidence that they react to macroeconomic conditions. Thus these authors conclude that the cyclical fluctuations in asset returns do not appear to be linked to financial frictions.

The remainder of this paper is organized as follows. Section 2 shows that much of the existing literature on firms' financing constraints can be characterized by specifying a simple dynamic problem for the firm. Using this canonical representation, Section 3 derives the optimal investment policy and obtains expressions for returns to physical investment that can be used to evaluate the asset pricing implications of the model. The next two sections use financial market data to examine both the performance of the model and the role of financing constraints. In particular, Section 4 studies their implications for unconditional returns. Formal GMM tests are contained in Section 5. Finally, we offer some concluding remarks in Section 6.

## 2 A General Representation of Financing Frictions

In this section we show that a majority of the existing literature on firms' financing constraints can be characterized by a fairly simple canonical problem that firms face.

Consider the following firm's value maximization problem:

$$V_0 = \max_{D_t, B_{t+1}, N_t} E_0 \left[ \sum_{t=0}^{\infty} M_{0,t} [D_t - W(N_t) N_t] \right]$$

$$\text{s.t.} \quad D_t + I_t = \Pi_t + N_t + B_{t+1} - R(B_t) B_t \quad (1)$$

$$D_t, N_t \geq 0 \quad \forall t \quad (2)$$

where  $M_{0,t}$  is the stochastic discount factor (of the owners of the firm) between time 0 and time  $t$ .  $N_t$  denotes issues of new shares, which reduce the value of the firm to existing shareholders, by an amount of  $W(N)$  per share. The firm also needs to repay last period debt  $B_t$ , with (gross) interest rate  $R(B_t)$  and may acquire new debt  $B_{t+1}$ . These resources, in conjunction with current period profits  $\Pi_t$ , can be allocated to dividends,  $D_t$ , and investment,  $I_t$ . Note that we allow debt to be negative, in which case the firm will accumulate liquid assets.

Regarding the properties of the functions  $W(\cdot)$  and  $R(\cdot)$  we make the following assumptions:

$$W(N_t) > 1, \quad W'(N_t) > 0, \quad \forall N_t > 0 \quad (3)$$

$$E_t[M_{t,t+1} R(B_{t+1})] > 1, \quad R'(B_{t+1}) > 0, \quad \forall B_{t+1} > 0 \quad (4)$$

$$R(B_{t+1}) = 1/E_t[M_{t,t+1}], \quad R'(B_{t+1}) = 0, \quad \forall B_{t+1} \leq 0 \quad (5)$$

where  $M_{t,t+1} = M_{0,t+1}/M_{0,t}$ . Assumption (3) on the function  $W(\cdot)$  captures not only the direct reduction in value for the existing shareholders, associated with the new issues, but

also the transaction costs and possible informational premium associated with new equity issues. Assumption (4) on the risky rate of interest,  $R(\cdot)$ , reflects the fact that debt financing has a higher cost than that contained implicitly in retained earnings to the extent that it always exceeds the riskless rate of interest,  $R_{f,t+1} = 1/E_t[M_{t,t+1}]$ . Finally, assumption (5) imposes that this riskless rate will be the return earned by liquid assets (negative debt).

Several researchers have provided various theoretical arguments for each of our aforementioned assumptions. For our empirical analysis, it suffices to directly assume a “financing hierarchies” structure proposed by Myers (1984). That is, we directly assume that internal funds are the least costly source of financing. Whether debt or new equity is the most expensive source of funds is essentially irrelevant for our purposes and we hence make no assumptions regarding their relative costs.

The optimality conditions for the above problem are:

$$\mu_t = 1 + \alpha_{1t} \tag{6}$$

$$\mu_t = W'(N_t)N_t + W(N_t) + \alpha_{2t} \tag{7}$$

$$\mu_t = E_t [M_{t,t+1}\mu_{t+1} (R'(B_{t+1})B_{t+1} + R(B_{t+1}))] \tag{8}$$

where  $\mu_t$  denotes the multiplier on the resource constraint (1) and  $\alpha_{1t}$  and  $\alpha_{2t}$  are the (non-negative) multipliers on the inequality constraints on dividends and new shares issues, respectively.

It follows immediately from equations (6)–(8) that it is never optimal for firms to pay positive dividends, when either  $N_t$  or  $B_{t+1}$  is positive. For example, suppose that both  $D_t$

and  $N_t$  are positive, (6) and (7) become:

$$\begin{aligned}\mu_t &= 1 \\ \mu_t &= W'(N_t)N_t + W(N_t) > 1\end{aligned}$$

where the last equation follows from the fact that  $W(N_t) > 1$  and  $W'(N_t)N_t > 0$ .

Contradiction.

Now suppose that both  $D_t$  and  $B_{t+1}$  are positive, we obtain from (6) and (8) that:

$$1 = E_t [\mu_{t+1} (M_{t,t+1}R(B_{t+1}) + M_{t,t+1}R'(B_{t+1})B_{t+1})] > E_t [\mu_{t+1} M_{t,t+1}R(B_{t+1})] \quad (9)$$

since  $M_{t,t+1}R'(B_{t+1})B_{t+1} > 0$  when  $B_{t+1}$  is positive.

Next, observing that:

$$E_t [\mu_{t+1} M_{t,t+1}R(B_{t+1})] = \text{cov}_t (\mu_{t+1}, M_{t,t+1}R(B_{t+1})) + E_t[\mu_{t+1}]E_t[M_{t,t+1}R(B_{t+1})]$$

we obtain:

$$\begin{aligned}E_t[\mu_{t+1}] &= \frac{E_t [\mu_{t+1} M_{t,t+1}R(B_{t+1})] - \text{cov}_t (\mu_{t+1}, M_{t,t+1}R(B_{t+1}))}{E_t[M_{t,t+1}R(B_{t+1})]} \\ &< E_t [\mu_{t+1} M_{t,t+1}R(B_{t+1})] - \text{cov}_t (\mu_{t+1}, M_{t,t+1}R(B_{t+1})) \\ &< E_t [\mu_{t+1} M_{t,t+1}R(B_{t+1})]\end{aligned} \quad (10)$$

where the first inequality follows from the assumption that  $E_t[M_{t,t+1}R(B_{t+1})] > 1$  and the second follows from the fact that higher debt increases both  $R(\cdot)$  and  $\mu_{t+1}$  (the marginal value of one additional unit of internal funds).

Together inequalities (9) and (10) imply that:

$$E_t[\mu_{t+1}] < E_t [\mu_{t+1} M_{t,t+1}R(B_{t+1})] < 1$$

contradicting the optimality condition (6) which requires that

$$\mu_{t+1} = 1 + \alpha_{1t+1} \geq 1$$

In summary, the firm's optimal financing-dividend policy can be usefully summarized as:

$$\begin{aligned} D_t > 0 & \quad \text{if} \quad N_t = B_{t+1} = 0 \\ D_t = 0 & \quad \text{if} \quad B_{t+1} > 0 \text{ or } N_t > 0 \end{aligned} \quad (11)$$

This framework is quite simple, but it effectively summarizes most, if not all, of the existing theoretical literature on the firms' financing constraints. For example, Fazzari, Hubbard, and Petersen (1988) and Gomes (2001) examine models where firms issue only new equity, i.e.,  $B_t = B_{t+1} = 0$ . In this case it is easy to see that (11) reduces to:

$$\begin{aligned} D_t = \Pi_t - I_t > 0 & \quad \text{if} \quad N_t = 0 \\ D_t = 0 & \quad \text{if} \quad N_t = I_t - \Pi_t > 0 \end{aligned} \quad (12)$$

and the value of the firm can be recursively defined by the equation

$$\begin{aligned} V_t &= \max_{N_\tau} E_t \left[ \sum_{\tau=t}^{\infty} M_\tau [\Pi_\tau - I_\tau - (W(N_\tau) - 1)N_\tau] \right] \\ &= \max_{N_t} \Pi_t - I_t - (W(N_t) - 1)N_t + E_t [M_{t,t+1} V_{t+1}] \\ &= \Pi_t - I_t - g(I_t - \Pi_t) + M_{t,t+1} V_{t+1} \end{aligned}$$

where the "financing cost" function  $g(\cdot)$  satisfies:

$$g(I_t - \Pi_t) = g(N_t) = (W(N_t) - 1)N_t \geq 0 \quad (13)$$

$$g'(N_t) = W'(N_t)N_t + (W(N_t) - 1) > 0 \quad \text{if} \quad N_t > 0 \quad (14)$$

Thus financing costs are non-negative and strictly increasing in the level of external finance.

Alternatively, in the work of Bernanke and Gertler (1989) and Cooley and Quadrini



(1999), where debt financing is the only available source of external finance, dividend policies satisfy:

$$\begin{aligned} D_t &= \Pi_t - I_t - R(B_t)B_t > 0 & \text{if } B_{t+1} &= 0 \\ D_t &= 0 & \text{if } B_{t+1} &= [I_t + R(B_t)B_t - \Pi_t] > 0 \end{aligned} \quad (15)$$

and the value of the firm is recursively defined by the equation:

$$\begin{aligned} \tilde{V}_t &= V_t + B_t = \Pi_t - (R(B_t) - 1)B_t - I_t + (1 - M_{t,t+1}R(B_{t+1}))B_{t+1} + M_{t,t+1}\tilde{V}_{t+1} \\ &= \tilde{\Pi}_t - I_t - g(I_t - \tilde{\Pi}_t) + M_{t,t+1}\tilde{V}_{t+1} \end{aligned}$$

where  $\tilde{\Pi}_t = \Pi_t - (R(B_t) - 1)B_t$  denotes profits net of interest payments and the function  $g(\cdot)$  now satisfies:

$$g(I_t - \tilde{\Pi}_t) = g(B_{t+1}) = (M_{t,t+1}R(B_{t+1}) - 1)B_{t+1} \geq 0 \quad (16)$$

$$g'(B_{t+1}) = (M_{t,t+1}R(B_{t+1}) - 1) + R'(B_{t+1})M_{t,t+1}B_{t+1} > 0 \quad \text{if } B_{t+1} > 0 \quad (17)$$

Again the financing costs are non-negative and strictly increasing in the level of external finance.

The similarities between (13) and (16) are quite obvious as they both imply that the financing costs have the following general representation:

$$\text{Financing Costs} = \text{External Premium} \times \tilde{g}(\text{Investment} - \text{Profits}) \quad (18)$$

In the first case the external premium is captured by the additional costs of issuing new equity, while in the second case, they are summarized by the default premium:

$$M_{t,t+1}R(B_{t+1}) - 1 > 0$$

Depending on the nature of the underlying structural model, this premium may well be state-dependent. Obvious examples include models with a counter-cyclical default spread and/or counter-cyclical premium on new issues of equity,  $W(\cdot)$ . In addition, in both cases the financing costs are increasing in  $I - \Pi$ . Figure 1 depicts a typical form of the unit cost of financing.<sup>3</sup>

Modeling financing costs through (18) is not only theoretically appropriate, as we just showed, but also empirically appealing. Figure 2 shows the (HP-filtered) quarterly series of investment to profit ratios along with the NBER recession dates between June 1951 and July 1999. It is quite evident from the figure that all recessions are characterized by a sharp drop in the investment-profit ratio. This pattern implies that investment fluctuations are much more pronounced than those on aggregate profits. By explicitly linking the relation between profits and investment to the costs of external finance, equation (18) delivers a simple, but powerful, framework to studying the role of financial frictions on firm behavior.

### 3 Investment Returns with Financing Frictions

#### 3.1 Investment Returns

In this section we derive the asset pricing implications of models with financing constraints. We use the results above to construct the optimal investment policy for a representative firm facing financing constraints. This, in turn, allows us to obtain expressions for returns to physical investment which then can be used to evaluate the asset pricing implications of the model.

We start by adding a little more structure to the firm's problem. Assume that firms

---

<sup>3</sup>Note that in the figure we assume that issue of new stocks is more expensive than issue of new debt. For the purpose of our paper, however, this ranking is immaterial as mentioned before.

accumulate capital,  $K_t$ , according to the following equation

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (19)$$

Defining  $\Pi(K_t)$  as the profit function which can be viewed as the outcome of a static optimization problem where all other inputs (e.g. labor) have been determined. We can now write the firm's problem as:

$$\max_{\{K_{t+1}, I_t\}} \left\{ \sum_{t=0}^{\infty} M_t [\Pi(K_t) - I_t - h(I_t/K_t)K_t - g(I_t, \Pi(K_t))] \right\} \quad (20)$$

subject to the capital accumulation rule (19). The function  $h(\cdot)$  captures the notion that capital accumulation is subject to physical adjustment costs. We assume that this function satisfies the following standard assumptions:

$$h_I(\cdot) > 0, \quad h_{II}(\cdot) > 0, \quad h_K(\cdot) < 0$$

As in Cochrane (1991, 1996) adjustment costs are necessary to provide for some time variation in optimal investment choices and in asset returns in the absence of financial frictions.

We now derive the optimal investment policy for the problem (20), which in turn allows us to obtain expressions for returns to physical investment that can be used to evaluate the asset pricing implications of the model. Denote  $q_t$  as the Lagrange multiplier associated with (19) and let the function  $\phi(I_t, K_t)$  capture both financing costs and the physical adjustment cost.

It is straightforward to show that the first-order conditions associated with this problem

are given by:

$$q_t = E_t [M_{t,t+1} (\Pi'(K_{t+1}) - \phi_K(I_{t+1}, K_{t+1}) + q_{t+1}(1 - \delta))] \\ q_t = 1 + \phi_I(I_t, K_t)$$

Combining these two equations we can obtain the expression for one-period investment returns as,

$$E_t [M_{t,t+1} R_{t+1}^I] = 1 \tag{21}$$

where

$$R_{t+1}^I \equiv \frac{\Pi'(K_{t+1}) - \phi_K(I_{t+1}, K_{t+1}) + (1 - \delta) [1 + \phi_I(I_{t+1}, K_{t+1})]}{1 + \phi_I(I_t, K_t)} \tag{22}$$

Given functional forms for the profit function  $\Pi(\cdot)$  and cost functions  $h(\cdot)$  and  $g(\cdot)$  this expression can be used to construct a series of firms' returns on capital accumulation that can then be compared with financial market returns.

### 3.2 Functional Forms

We begin by specifying the following profit function:<sup>4</sup>

$$\Pi(K_t) = A_t K_t \tag{23}$$

Physical adjustment costs are quadratic and equal to

$$h(I_t, K_t) = \frac{a}{2} \left( \frac{I_t}{K_t} \right)^2 K_t, \quad a > 0 \tag{24}$$

---

<sup>4</sup>This functional form of profit function can be obtained as long as the underlying technology exhibits constant returns to scale.

Financing costs follow:

$$g(I_t, K_t) = \frac{b}{2} \max \left( 0, \frac{I_t}{K_t} + \frac{a}{2} \left( \frac{I_t}{K_t} \right)^2 - c \frac{\Pi_t}{K_t} \right)^2 K_t, \quad b > 0, c \in [0, 1] \quad (25)$$

The specifications regarding  $\Pi(\cdot)$  and  $h(\cdot)$  are fairly standard and require little explanation. The functional form for the function  $g(\cdot)$  is consistent with the properties established in Section 2. It essentially says that financing costs are incurred when investment, inclusive of adjustment costs, exceeds profits. The imposition of quadratic costs, while not following from our earlier results necessarily, seems to be a natural first-order approximation.

The parameter  $c$  is introduced as a scaling factor, necessary for empirical purposes as the profits and investment data may not be strictly comparable.<sup>5</sup> Given the series on investment and profits, high values of  $c$  imply lower financing costs. In fact, it is immediate to see that there exists a threshold level of  $c$  at which financing costs disappear altogether.

Together (23)–(25) imply that total adjustment costs are described by the function:

$$\phi(I_t, K_t) = \left[ \frac{a}{2} \left( \frac{I_t}{K_t} \right)^2 + \frac{b}{2} \left[ \left( \frac{I_t}{K_t} \right) + \frac{a}{2} \left( \frac{I_t}{K_t} \right)^2 - c A_t \right]^2 \mathbf{1}_{\left\{ \left( \frac{I_t}{K_t} \right) + \frac{a}{2} \left( \frac{I_t}{K_t} \right)^2 - c A_t \right\}} \right] K_t \quad (26)$$

where  $\mathbf{1}_{\{x\}}$  is an indicator function that takes the value of one when  $\{x > 0\}$  and zero otherwise. (26) captures the idea that the firm incurs, besides the usual physical adjustment cost, certain extra “financing” cost when its investment is higher than a fixed proportion of its profits. Naturally, if either  $b = 0$  or  $\left( \frac{I_t}{K_t} \right) + \frac{a}{2} \left( \frac{I_t}{K_t} \right)^2 > c A_t$  the financing costs will be zero.

---

<sup>5</sup>In particular, the means of profits and investment series may differ to the point such that the financing constraints, as stated above, never bind. Since our focus is on cyclical fluctuations we are not interested in the average ratio of profits to investment, but on its business cycle properties. Appropriate scaling allows us to do this.

The relevant derivatives for the investment return equation (22) are given by:

$$\Pi'(K_t) = A_t = \frac{\Pi(K_t)}{K_t} = \left( \frac{\Pi(K_t)}{I_t} \right) \left( \frac{I_t}{K_t} \right) = \pi_t i_t \quad (27)$$

$$\phi_K(t) = \frac{\phi'(t)}{K_t} - ai_t^2 - b [i_t + ai_t^2] \left[ i_t + \frac{a}{2} i_t^2 - c\pi_t i_t \right] \mathbf{1}_{\{i_t + \frac{a}{2} i_t^2 - c\pi_t i_t\}} \quad (28)$$

$$\phi_I(t) = ai_t + b(1 + ai_t) \left[ i_t + \frac{a}{2} i_t^2 - c\pi_t i_t \right] \mathbf{1}_{\{i_t + \frac{a}{2} i_t^2 - c\pi_t i_t\}} \quad (29)$$

It is now clear that investment returns are completely driven by two fundamental factors in this model: the investment to capital ratio  $i_t \equiv (I_t/K_t)$  and the profit to investment ratio  $\pi_t \equiv (\Pi_t/I_t)$ .

It is also useful to define the quantities:

$$\theta_{1t} = (1 - \nu_t)\theta_t = \frac{a}{2} \left( \frac{I_t}{K_t} \right)^2 \frac{K_t}{I_t} = \frac{a}{2} i_t \quad (30)$$

and

$$\theta_{2t} = \nu_t \theta_t = \frac{b}{2} i_t [1 + \theta_{1t} - c\pi_t]^2 \mathbf{1}_{\{1 + \theta_{1t} - c\pi_t\}} \quad (31)$$

to denote the fraction of investment lost to physical adjustment costs and to financing costs, respectively. Thus  $\theta$  is the total fraction of investment spending due to adjustment costs, and  $\nu$  is the share of the total costs due to financing constraints.

## 4 Quantitative Effects of Financing Constraints

This section contains the first part of our empirical analysis and it is mostly designed to build intuition for understanding our results of GMM estimation in the next section. In particular, we compare the investment returns to the CRSP value-weighted portfolio returns deflated by the CPI. We begin by providing an overview of the data sources and the construction of the

series of investment returns from the available macroeconomic aggregates. We then discuss the implications of alternative parameter choices for the properties of investment returns.

## 4.1 Data

We require two main types of data: asset returns and macroeconomic aggregates suitable to construct the series of investment returns. Asset returns mimic closely those used in Cochrane (1996) and are obtained directly from CRSP. The construction of investment returns on the other hand requires two macroeconomic aggregates: profits and investment. Our sample period is restricted between the first quarter of 1952 and the last quarter of 1999. This is done to eliminate the earlier (1947–1951) data on investment and profits which is subject to excessive variations likely due to the chain weighting procedures.

Macroeconomic aggregates are all obtained from National Income and Product Accounts (NIPA). The investment series corresponds to the gross private domestic investment. Profits in our model correspond to output minus wages and thus are essentially constructed by removing labor income from national income. A detailed overview of this construction is provided in an Appendix. Given this information we can construct investment returns as follows. First, rewrite equations (19) and (22) to obtain:

$$R_{t+1}^I \equiv \frac{\Pi'(\pi_{t+1}, i_{t+1}) - \phi_K(\pi_{t+1}, i_{t+1}) + (1 - \delta) [1 + \phi_I(\pi_{t+1}, i_{t+1})]}{1 + \phi_I(\pi_t, i_t)} \quad (32)$$

$$i_{t+1} = \frac{\gamma_{t+1} i_t}{(1 - \delta) + i_t} \quad (33)$$

with  $\gamma_{t+1} \equiv I_{t+1}/I_t$  denotes investment growth. Given historical data on profits and investment we can easily construct empirical measures for investment growth,  $\gamma$ , and the profits to investment ratio,  $\pi$ . Reliable empirical estimates of the depreciation rate,  $\delta$ , are also available in the literature. With this information at hand we can then use (33) to

construct the series on the investment-capital ratio and use this to obtain investment returns from (32).<sup>6</sup> This simple procedure also has the great advantage of avoiding the use of any data on the capital stock, notoriously unreliable at quarterly frequency.

## 4.2 Properties of Investment Returns

Before presenting the results of our formal GMM tests, it is useful to discuss some of the main properties of the series on investment returns in order to build some intuition regarding the role of financial frictions. To do so we conduct a number of experiments and detail the results in Tables 1–5. These tables report the impact of alternative parameter choices on the mean, standard deviation, autocorrelation of investment returns, and its cross-correlation with stock returns. For completeness the same set of unconditional moments for the series on stock returns is also reported.

We start by examining a simple version of the model with no financial frictions, i.e., where  $b=0$ . Table 1 documents our findings, assuming a value of  $\delta=2.5\%$ , usually the most popular estimate in the literature. The second column, where  $a=0$ , shows the special case where no physical adjustment costs exist. It is clear that without them the model can not generate any volatility in investment returns. Moreover this series also shows high average returns, an extremely high autocorrelation, and a very low correlation with stock returns. This is not surprising since, without any adjustment costs investment return series (32) simplifies to

$$R_{t+1}^I \equiv \pi_{t+1} i_{t+1} + 1 - \delta = \frac{\Pi_{t+1}}{K_{t+1}} + 1 - \delta \quad (34)$$

thus essentially inheriting the properties of the profits series.<sup>7</sup>

---

<sup>6</sup>We also need an initial condition for  $i_0$ . We do so by assuming that it is equal to its long-run mean  $i^* = \gamma - (1 - \delta)$ .

<sup>7</sup>Note that our specification, contrary to Cochrane (1996), allows for the time variation of marginal product of capital. Therefore, investment returns are tied down to the time-varying profits even in the absence of financing constraints. In spite of this less restrictive feature, as discussed below, the model



Adding adjustment costs dramatically improves the ability of the model in fitting these moments. They provide a very effective way of matching average returns, which is not surprising since, by definition, they raise the costs of accumulating capital. In particular, when physical adjustment costs constitute about 14% of investment spending we can exactly match average stock returns.

Adjustment costs also add to the volatility of investment returns. By making firms less willing to adjust the capital stock, they induce larger volatility of underlying returns to capital accumulation. Nevertheless, it is clear that it is very difficult for physical adjustment costs alone to match the large values found in stock returns data: even with an implausibly large value of  $\theta$  around 100% we can only obtain around half of the amount of volatility found in the data. Moreover, there exists certain trade-off between matching the mean and variance since large values of adjustment costs lead to extremely low average investment returns.

We are more successful in the dimension of correlations, however. Higher physical adjustment costs lower the persistence in investment returns and raise its correlation with stock returns. The persistence numbers actually match those in stock returns rather well, for reasonable values of  $\theta$  (somewhat less than 10%). The cross correlation is much larger now; however, it stays around 0.42 and appears to change very little once  $\theta$  is above 5%.

Table 2 explores the role of financing constraints in this model. Our point of departure is column two, reproduced from Table 1, where the physical adjustment costs are set so that average returns match those of stocks. Let us first examine the case where the parameter  $c$  is sufficiently low so that the financing constraints always bind. We can accomplish this by assuming  $c=0.5$ .

---

without physical adjustment costs still has difficulty matching salient features of stock returns.

The contribution of these financing costs improves our results mostly by substantially adding to the variance of returns, while leaving the remaining moments mostly unaffected. Comparing with Table 1, we see that raising  $\nu$  for a fixed amount of financing costs (by increasing  $b$  and reducing  $a$ ) leads to pronounced increment in the variance with only negligible effects on average returns. Raising financing costs avoids the trade-off between matching the first and second moments faced when relying on physical adjustment costs alone. In particular, values of  $\nu$  around 0.40 (with  $\theta_1 = 14\%$ ) seem to be able to deliver both the first and second moments.

There is some deterioration on both the autocorrelation of the series, which is about 0.10 now, and, to a less extent, the cross-correlation with stock returns, which drops to about 0.40. Interestingly, neither of these two moments appear particularly sensitive to the exact level of the financing costs. In fact, out of the four moments only the variance seems to depend heavily on the value of  $\nu$ .

The lower cross-correlation between these two series can be explained by the countercyclical behavior of the profit-investment ratio  $\pi_t$  shown in Figure 2. Recall that the financing constraints bind when  $(1 + (a/2)i_t - c\pi_t) > 0$ . For the estimated  $a$ 's and the constructed  $i_t$  series in the model, the middle term is almost negligible. Hence, a countercyclical  $\pi_t$  implies that the financing constraints bind *less* during recessions. Given (32), this means that, relative to the model with only physical adjustment costs, financing constraints *increase* ex-post investment returns in downturns (as they bind less) and reduce them during expansions. This channel effectively reduces the contemporaneous correlation with stock returns

Table 3 repeats this experiment using a higher value of  $c = 0.75$ . This value is chosen so that the financing constraints now bind only about 50% of the time. In this case the models'

performance deteriorates very rapidly. Even relatively low financing costs ( $\nu=30\%$ ) lead to very large variances and extremely low serial and cross correlations.

Finally, Tables 4 and 5 reproduce the experiments in Tables 1 and 2 using a value of  $\delta$  of only 1.5%. Lowering the depreciation rate has an important impact on the effects of physical adjustment costs, because it also lowers the average level of the investment-capital ratio,  $i$ , given by (33). For a given level of adjustment costs, returns are now lower and this means that the tension between matching the first and the second moments becomes more apparent. Since we need an even lower value of  $\theta$  (0.0538) to match the average stock returns, the implied standard deviation of investment returns falls to around 1% per annum. On the positive side, however, this value of adjustment costs also matches the serial correlation in stock returns.

The costs and benefits of introducing financing costs remain much the same. Raising  $\nu$ , while keeping  $\theta$  at 10%, dramatically increases the variance of investment returns while significantly lowering its serial correlation and leaving its mean essentially unaffected.

To summarize, this exercise suggests that: (i) without adjustment costs the model is not capable of generating realistic moments for investment returns; (ii) adding physical adjustment costs can match the average stock returns and can also go a long way in generating realistic values for the serial correlation of investment returns and in raising the contemporaneous correlation with stocks; and (iii) adding financing costs is mostly successful along the volatility dimension by substantially raising the variance of returns to more realistic values, albeit at some cost in terms of the model's ability to match some of the other moments.

## 5 GMM Tests of Factor Pricing Models

Much of the exercise in the previous section was motivated by the assertion that the return on aggregate investment should behave in the same way as the return on the aggregate stock market. In this section we investigate whether an investment return factor pricing model holds, i.e., whether a stochastic discount factor based on the returns to accumulation of physical capital (generated from the model) prices assets correctly. In particular, we are interested in examining to what extent the presence of financing constraints improves the ability of such a model in pricing the cross section of asset returns.

We start by assuming the existence of a (positive) stochastic discount that correctly prices the observed cross-section of stock market returns (see Harrison and Kreps (1979) and Hansen and Richard (1987) for a detailed discussion). As in Cochrane (1996) we start by specializing the stochastic discount factor,  $M_{t,t+1}$ , to be linear in investment returns,

$$M_{t,t+1} = \sum_k l_k R_{k,t+1}^I = l_0 + l_1 R_{t+1}^I = \mathbf{f}'_{t+1} \mathbf{l} \quad (35)$$

where  $\mathbf{f}_{t+1}$  is the vector of pricing factors (e.g. investment returns), and  $\mathbf{l}$  are the factor loadings.<sup>8</sup> This discount factor is also assumed to satisfy the moment conditions

$$E_t [M_{t,t+1} \mathbf{R}_{t+1}] = \mathbf{p}_t \quad (36)$$

where  $E_t$  is the conditional expectation,  $\mathbf{R}_{t+1}$  can be any asset or investment return and  $\mathbf{p}_t$  is the corresponding price (one for gross returns or zero for excess returns). Following the factor pricing tradition, we then estimate the loadings of investment returns in the stochastic

---

<sup>8</sup>The linear specification can be viewed as an approximation. For example, if preferences are logarithmic and there is full depreciation, then  $R_{t+1}^I = \frac{1}{\beta} \frac{c_{t+1}}{c_t} = 1/M_{t,t+1}$ . Thus  $M_{t,t+1} \approx 1 - (R_{t+1}^I - 1)$ , that is the stochastic discount factor is approximately linear in  $R^I$ . Alternative approaches modeling nonlinear pricing kernels have been advanced in the literature (e.g., Bansal and Vishwanathan (1993), Brandt and Yaron (2001), and Chapman (1997)).

discount factor,  $l_k$ , as free parameters. More importantly, we also estimate the technological parameter,  $a$  and the financing cost parameters  $b$ , and  $c$ .

We consider three sets of moment conditions in implementing (36). We look first at the relatively weak restrictions implied only by the unconditional moments. The second set focuses on the conditional moments by adding instruments to the returns, while the third set allows for time variation in the factor loadings. We now provide some more details about the estimation of each of these.

## 5.1 Moment Conditions

### Unconditional Factor Pricing Models

We use the standard GMM procedure to estimate the parameters  $\mathbf{l}$  to minimize a weighted average of the sample moments (36). Letting  $\sum_T$  denote the sample mean we can rewrite the sample moments, denoted  $\mathbf{g}_T$  as:

$$\mathbf{g}_T \equiv \sum_T [M\mathbf{R} - \mathbf{p}] = \sum_T [(\mathbf{R}\mathbf{f}')\mathbf{l} - \mathbf{p}]$$

where  $g(\cdot)$  is a function of the parameter vector  $\kappa \equiv \{a, b, c, l\}$ , and the data  $i_t$ ,  $\pi_t$  and  $R_t$ .

One can then choose  $\kappa$  to minimize a weighted sum of squares of the pricing errors across assets:

$$J_T = \mathbf{g}_T' \mathbf{W} \mathbf{g}_T \tag{37}$$

where  $\mathbf{f} = R^I(a, b, c; i, \pi)$ . Note that a convenient feature of our problem is that given  $a, b$ , and  $c$  the criterion function above is linear in  $l$  — the factor loading coefficients.<sup>9</sup>

---

<sup>9</sup>To see this note that  $a, b$ , and  $c$  determine investment returns,  $R^I$ . Given  $R^I$ , however, the rest of the problem is linear. Assuming that at least one element of  $\sum_T(\mathbf{p})$  is not equal to zero, and conditional on  $R^I$ , the first order conditions of (37) yield  $\hat{\mathbf{l}} = (\mathbf{D}'\mathbf{W}\mathbf{D})^{-1} \mathbf{D}'\mathbf{W} \sum_T(\mathbf{p})$ , where  $\mathbf{D} \equiv \partial \mathbf{g}_T / \partial \mathbf{l} = \sum_T [\mathbf{R}\mathbf{f}'] = \sum_T [\mathbf{R}\mathbf{R}^{I'}(\mathbf{a}, \mathbf{b}, \mathbf{c})]$ .

Standard  $\chi^2$  tests on over-identifying restrictions follow from this procedure. This also provides a framework to assess whether certain factors,  $\mathbf{l}$ , or financing constraints ( $b$  and  $c$ ) are significant for pricing assets.

### Conditioning Information

It is straightforward to include the effects of conditioning information by scaling the returns and/or scaling the factors by instruments. The essence of this exercise lies in extracting the conditional implications of (36), since as Hansen and Richard (1987) note, the conditional moment implications for a time-varying conditional model may not be captured by a corresponding set of unconditional moment restrictions.

To test the conditional predictions of (36) we expand the set of returns to include returns scaled by instruments and then proceed as before; that is, we use the moment conditions:

$$E[\mathbf{p}_t \otimes \mathbf{z}_t] = E[M_{t,t+1} (\mathbf{R}_{t+1} \otimes \mathbf{z}_t)]$$

where  $\mathbf{z}_t \in I_t$ , is some instrument and  $I_t$  is the information set at time  $t$ .<sup>10</sup>

A more direct way to extract the potential non-linear restrictions embodied in (36) is to let the stochastic discount factor be a linear combination of factors with weights that vary over time. That is, the vector of factor loadings  $\mathbf{l}$  is a function of instruments  $\mathbf{z}$  that vary over time:<sup>11</sup>

$$M_{t,t+1} = \mathbf{l}(\mathbf{z}_t)' \mathbf{f}_{t+1}$$

Therefore, to estimate and test a model in which factors are expected only to price assets conditionally, we simply expand the set of factors to include factors scaled by instruments.

---

<sup>10</sup>As noted in the finance literature the scaled returns  $R_{t+1}z_t$  can be interpreted as returns on managed portfolios in which the portfolio manager invests more or less according to the signal  $z_t$ .

<sup>11</sup>With sufficiently many  $z$ 's raised to powers, the linearity of  $l$  can accommodate nonlinear relationships.

The stochastic discount factor utilized in estimating (36) is then,

$$M_{t,t+1} = \mathbf{1}'(\mathbf{f}_{t+1} \otimes \mathbf{z}_t)$$

## 5.2 GMM Results

In order to implement the estimation procedure we require returns other than the CRSP value-weighted portfolio returns. The other asset returns to be priced by the investment return factor model include the ten portfolios of NYSE stocks sorted by market value (CRSP series DECRET1 to DECRET10), the real three-month Treasury-bill return, as well as the investment return itself. Since the investment return is based on quarterly average investment, we transform the asset returns to quarterly average returns rather than use end-of-quarter to end-of-quarter returns.<sup>12</sup> In order to focus on risk premium, all moment conditions, except the level of the risk free rate, utilize excess returns. Excess returns are defined as the premium over the three-month Treasury-bill return. Table 6 reports the summary statistics of the asset returns used in our GMM implementation.

We construct two instruments: the term premium, defined as the yield on long-term government bonds less that on three-month Treasury bills, and the dividend-price ratio of the equally weighted NYSE portfolio.<sup>13</sup> The dividend-price ratio is based on CRSP EWRETD and EWRETX, the equally weighted portfolio returns with and without dividends. The returns are cumulated for a year to avoid the seasonal in dividends; thus,  $d/p = (\text{annual EWRETD}/\text{annual EWRETX}) - 1$ . To avoid overlap with the average return series, we lag the instruments twice so that an instrument used for the return from the first to second quarter is known by the last day of December. We limit the number of moment

---

<sup>12</sup>For details see Cochrane (1996).

<sup>13</sup>In the first-stage estimation, the moments corresponding to scaled returns are treated equally with the nonscaled returns, so it is convenient that the scale of the two is comparable. Following Cochrane (1996), we also use  $1 + 100[(d/p) - 0.04]$  in place of the raw dividend-price ratio.

conditions and scaled factors in three ways: (1) we do not scale the Treasury-bill return by the instruments since we are more interested in the time-variation of risk premium than that of risk-free rate. (2) We scale the variable factors by the instruments but we do not include the instruments themselves as factors. (3) We only use deciles one, two, five, and ten in the conditional estimates (return times instruments).

Table 7 reports the results of our GMM estimation when the depreciation rate is set to 2.5%.<sup>14</sup> Each panel in these tables corresponds to one of the three models tested (unconditional, conditional, and scaled factor). The message from all the panels is uniform! The point estimate of  $b$  is either zero or the corresponding  $\nu$  is effectively zero when  $b$  is slightly different from zero<sup>15</sup>. This result is also robust to the use of first (not reported) or second stage GMM estimates.<sup>16</sup> Table 8 replicates these results for the case where  $\delta = 1.5\%$ , showing that these findings are independent of the level of the depreciation rate.

Overall, the model with physical adjustment costs only is, for the most part, not rejected by the data at standard 5% significance levels.<sup>17</sup> Moreover the estimates of physical adjustment costs ( $\hat{\theta}$ ) required to match asset returns are not excessive (in all cases around 20% of investment spending)<sup>18</sup>. The factor loadings  $\mathbf{l}$  also show very similar patterns across the two non-scaled models (a positive intercept and a negative loading on the investment return). In the scaled model, that has two additional loadings, the conditional loading on the term-premium is significant while that on dividend yield is only marginally significant.

---

<sup>14</sup>Our estimation criterion is quite non-linear in the dimensions of  $a$ ,  $b$ , and  $c$ , which leads us to initially search over the parameter space using a very fine grid. Therefore, for each of the models, we searched over  $a$  in a set that was constrained to be three standard deviations in both directions of the  $a$ 's that delivered  $E(R^{vwret}) = E(R^I)$  in Tables 1 and 4, respectively. In no case do the optimal  $a$ 's reside on the boundaries.

<sup>15</sup>Estimates of  $c$  are undetermined when  $b = 0$ .

<sup>16</sup>The only case when  $b$  is not literally zero is the second stage estimation of the scaled factor model with conditional estimates. But even in that case the implied in-sample  $\hat{\nu}$  is less than 0.5%.

<sup>17</sup>Our  $p$ -values are computed based on estimating only parameter  $a$  and the loadings  $\mathbf{l}$ . We do this since the  $b$ 's are zero and estimating a pure physical adjustment costs model would deliver such  $p$ -values.

<sup>18</sup>These results resemble those in Cochrane (1996) although our  $p$ -values are somewhat higher than his due to the fact that we allow for time variation in the marginal product of capital.



## The Effect of Financing Constraints

The interesting question however is why financing constraints do not seem to be priced, or, alternatively, why they do not appear relevant for the construction of the investment based stochastic discount factor? This is especially surprising given the promising evidence in section 4 about the effects of financing constraints on the volatility of investment based returns. To better understand the effects of the financing frictions on returns it is useful to look at two additional pieces of evidence implied by the model: (i) the volatility bounds for the discount factor (Hansen and Jagannathan (1991)); and, (ii) the results for the beta representation of asset returns.

Figure 3 plots the Hansen-Jagannathan (1991) bounds for (unconditional) asset returns against the implied market price of risk for the model with various levels of financing costs. For completeness, we also plot the consumption based pricing kernel using standard CRRA preferences.<sup>19</sup> Clearly adding financing costs only moves the model farther away from the data.<sup>20</sup> This happens because financing constraints effectively lower the market price of risk  $\sigma(M)/E(M)$ . Table 9 presents the estimated market price of risk  $\sigma(M)/E(M)$  and the correlation between the pricing kernel and value-weighted returns for all three models. While this correlation does not, in general, move very much, in all cases without exception, financing constraints significantly decrease the market price of risk and thus deteriorate the performance of the pricing kernel.

Perhaps more direct evidence on pricing errors is given in Table 10 where we regress

---

<sup>19</sup>The correct bounds should also include information on investment returns, but since these depend on the degree of financial costs they are omitted for the sake of clarity. Adding investment returns, however, would only sharpen the bounds and thus lead to stronger rejections of financing constraints.

<sup>20</sup>The plotted circles represent the pricing kernel estimates from the first stage GMM. The second stage GMM estimates lead to corresponding pricing kernels that are outside the bounds. However, after accounting for standard errors, the hypothesis that the investment based return with purely adjustment costs is inside the bound can not be rejected at conventional significance levels. Not surprisingly, this hypothesis is rejected for the consumption based model.

excess value weighted returns and the excess decile 1 returns on excess investment returns. Given the assumed structure of an investment based pricing kernel, we know that a beta representation of the form  $R_i - R_f = \alpha_i + \beta_i(R^I - R_f)$  also exists, with  $\alpha_i = 0$  (see discussion in Cochrane (2001) and citations there about linear factor models and beta representations). Therefore, large values of  $\alpha$  are evidence against the model.

Table 10 displays a clear pattern of increasing  $\alpha$  as we increase financing constraints. Indeed, while we can not reject that  $\alpha = 0$  for the benchmark case of pure physical adjustment costs, this hypothesis is rejected for most of the other parameter configuration.

The pricing properties of a factor model hinges on its covariance structure with returns. The overriding effect of increasing financing constraints is to increase the variance of the return on investment while at the same time leave its correlation with asset returns relatively unchanged. As displayed in Table 10, this effect basically leads to lower  $\beta$ 's on all returns and thus makes it more difficult for the model with financing constraints to price assets. Specifically, it leads to larger average pricing errors ( $\alpha$ 's) and in particular requires the estimated real risk free rate to rise. This phenomena, is confirmed in the lower estimates for the market price of risk, displayed in Table 9 and the circles depicting the estimated pricing kernels in Figure 3. The loadings on the more volatile return on investment, once financing constraints are introduced, are adjusted as to reduce the volatility of the pricing kernel. The underlying economics is that financing constraints add volatility but do not generate the covariance required for better asset pricing properties.

### **Small Firms Effects**

Most, if not all, studies on firm financing constraints emphasize that they are more likely to be detected when looking only at the behavior of small firms. To investigate this possibility, Table 11 shows the results of our GMM estimation that uses the lower 2 or 3

NYSE/AMEX/NASDAQ deciles. Since most firms on NASDAQ are small, adding NASDAQ stocks should in principle leave more room for financing constraints to play some role in pricing assets. Nevertheless, even when focusing mainly on these firms, we still can not find any evidence for a significant role of financing frictions.

### **Time Varying Financing Constraints**

The time series properties of financing constraints, as discussed above, can potentially influence the role they play in pricing assets. Our structure has thus far only considered constant values for the two coefficients governing financing costs,  $b$  and  $c$ . Time varying financing constraints can potentially affect the aforementioned important correlation structure. Moreover, from an economic perspective, it is quite likely that the unit costs associated with raising external funds are much more severe in recessions than in expansions. A plausible way of capturing this intuition is to allow for a more flexible specification for the financing costs where now:

$$b_t = b_0 + b_1 DF_t \tag{38}$$

where  $DF_t$  denotes default premium as measured by the difference of corporate BAA bond yields and AAA bond yields of corresponding maturity.  $DF_t$  is counter-cyclical and therefore can reduce the counter-cyclical measure of investment return induced by the counter-cyclical properties of the profits to investment ratio. However, as Table 12 documents that, even with this modification, our GMM tests confirm the negligible role played by financing constraints.

The parameter  $c$  captures to some extent the degree to which financing constraints are binding given investment and profit levels. It may well be the case that  $c$  is time varying as well. We, therefore, also report the results of allowing for time variation in the threshold,  $c$ .

Specifically we parameterize this as:

$$c_t = \frac{\exp[c_0 + c_1 DF_t]}{1 + \exp[c_0 + c_1 DF_t]} \quad (39)$$

thus restricting that its range is between zero and one. Again our results show that the estimated values for the share of financing constraints,  $\nu$ , is essentially zero.

### Reduced Form Factors

We construct a pricing kernel  $M$  by directly using  $i_t$  and  $\pi_t$  as two pricing factors. Table 13 report the results from these experiments. It is evident that the role of  $\pi$  is quite limited — lending support to the fact financing constraint is not being priced through the investment based returns.<sup>21</sup> On the other hand, these experiments also show that not forming the investment based returns,  $R^I$ , but rather directly using  $i_t$  as a factor, one may reject a priced factor.

### Alternative Channels

There are two potentially important aspects of our empirical implementation that may not be consistent with typical investment and its associated financing process. First, investment may have an important time to build component. In particular, the financing procedures may precede the actual investment by a quarter or two. The lending officer in that case may use lagged profit measures. In that case, our specification suffers from potential time aggregation problems. One remedy, albeit not perfect, is to look at annual returns. We do that as a check on our results, and find that they are again unaltered. Second, our specification leads to what one might consider as a firm liquidity constraint. That is the financing constraint is with respect to current profits and investments. One can alter the

---

<sup>21</sup>Note that the  $\chi^2$  measure of fit in Table 13 is not directly comparable to those in Table 7 as investment return is not priced and there are two factors directly entering the pricing kernel in Panel B of Table 13.

structure to make the financing constraint resemble a solvency constraint — a constraint that will depend on present value of expected future profits. This modification is beyond the scope of the current paper.

## 6 Conclusion

In this paper we ask the question to what extent financing constraints are quantitatively important for explaining stock returns. We first show that for a large class of theoretical models, the financing costs have a common general representation amenable to empirical analysis. Through some calibration exercises, we find that the model with financing constraints is more successful at generating volatile investment returns. In that dimension financing constraints help match the behavior of stock returns. However, when financing constraints are put into a linear factor pricing model, they seem to be invariably rejected as being important for pricing returns.

## References

- [1] Bansal Ravi, and S. Viswanathan, 1993, No Arbitrage and Arbitrage Pricing: A New Approach, *Journal of Finance*, 48, 1231–1261.
- [2] Bernanke, Ben and Mark Gertler, 1989, Agency Costs, Net Worth, and Business Fluctuations, *American Economic Review*, 79 (1), 14–31.
- [3] Bernanke, Ben, Mark Gertler, and Simon Gilchrist, 2000, The Financial Accelerator in a Quantitative Business Cycle Framework, in *Handbook of Macroeconomics*, Edited by Michael Woodford and John Taylor, North Holland.
- [4] Blundell, Richard W., Bond, Stephen R., Devereaux, Michael P. and Fabio Schiantarelli, 1992, Investment and Tobin’s Q: Evidence from Company Panel Data, *Journal of Econometrics*, 51, 233-257.
- [5] Brandt, Michael W., and Amir Yaron, 2001, Time-Consistent No-Arbitrage Models of the Term Structure, Working Paper, The Wharton School at the University of Pennsylvania.
- [6] Carlstrom, Charles T., and Timothy S. Fuerst, 1997, Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis, *American Economic Review*, 87 (5), 893–910.
- [7] Chapman, David A., 1997, Approximating the Asset Pricing Kernel, *Journal of Finance*, 52, 1383–1410.
- [8] Cochrane, John H., 1991, Production-Based Asset Pricing and the Link Between Stock Returns and Economic Fluctuations, *Journal of Finance*, XLVI (1), 209–237.

- [9] Cochrane, John H., 1996, A Cross-Sectional Test of an Investment-Based Asset Pricing Model, *Journal of Political Economy*, 104 (3), 572–621.
- [10] Cochrane, John H., 2001, *Asset Pricing*, Princeton University Press, Princeton, New Jersey.
- [11] Cooley, Thomas F., and Vincenzo Quadrini, 1999, Monetary Policy and The Financial Decisions of Firms, Working paper, Stern School of Business, New York University.
- [12] Cooley, Thomas F., and Vincenzo Quadrini, 2000, Financial Markets and Firm Dynamics, Working paper, Stern School of Business, New York University.
- [13] Den Haan, Wouter, Gary Ramey and Joel Watson, 1999, Liquidity Flows and Fragility of Business Enterprises, Working Paper 7057, National Bureau of Economic Research.
- [14] Fazzari, Stephen, R. Glenn Hubbard, and Bruce Peterson, 1988, Financing constraint and Corporate Investment, *Brookings Papers on Economic Activity*, 1, 141–195.
- [15] Gomes, Joao F., 2001, Financing Investment, forthcoming, *American Economic Review*.
- [16] Hansen, Lars Peter, 1982, Large Sample Properties of Generalized Method of Moments Estimation, *Econometrica*, 50, 1029–1054.
- [17] Hansen, Lars Peter and S. Richard, 1987, The role of Conditioning Information in Deducing Testable Restrictions Implied by Dynamic Asset Pricing Models, *Econometrica*, 55, 587–613.
- [18] Harrison, Michael and David Kreps, 1979, Martingales and Arbitrage in Multiperiod Securities Markets, *Journal of Economic Theory*, 20, 381–408.

- [19] Hayashi, Fumio, and Inoue, T., 1991, The Relation Between Firm Growth and Q with Multiple Capital Goods: Theory and Evidence from Japanese Panel Data, *Econometrica*, 59, 731-754.
- [20] Holmstrom, Bengt R. and Jean Tirole, 1997, Financial Intermediation, Loanable Funds, and the Real Sector, *Quarterly Journal of Economics*, CXII (3), 663-691.
- [21] Jermann, Urban J., 1998, Asset Pricing in Production Economies, *Journal of Monetary Economics*, 41, 257-275.
- [22] Kaplan, Steve and Luigi Zingales, 1997, Do Financing Constraints Explain why Investment is Correlated with Cash-Flow, *Quarterly Journal of Economics*, CXII (1), 168-216.
- [23] Kashyap, Anil, Lamont, Owen and Jeremy Stein, 1994, Credit Conditions and the Cyclical Behavior of Inventories, *Quarterly Journal of Economics*, CIX (3), 565-592.
- [24] Kiyotaki, Nobuhiro, and John Moore, 1997, Credit Cycles, *Journal of Political Economy*, 105 (2), 211-248.
- [25] Lamont, Owen, Christopher Polk and Jesús Saá-Requejo, Financial Constraint and Stock Returns, forthcoming, *Review of Financial Studies*.
- [26] Myers, Stewart C., 1984, The Capital Structure Puzzle, *Journal of Finance*, XXXIX (3), 575-592.
- [27] Newey, Whitney, and Kenneth West, 1987, Hypothesis Testing with Efficient Method of Moments Estimation, *International Economic Review*, 28, 777-787.



- [28] Restoy, Fernando, and G. Michael Rockinger, 1994, On Stock Market Returns and Returns on Investment, *Journal of Finance*, XLIX (2), 543–556.

## Appendix: Data Description

The measured value of capital income from fixed private capital is taken from the NIPA. There is some ambiguity about how much of Proprietor's income and some other smaller categories (specifically, the difference between Net National Product and National Income) should be treated as capital income. We define the measure capital income as follows. Let unambiguous capital income be defined as follows:

$$\text{Unambiguous Capital Income} = \text{Rental Income} + \text{Corporate Profits} + \text{Net Interests}$$

with Rental Income, Corporate Profits, and Net Interest from the NIPA (Table 1.14). We next allocate the ambiguous components of income according to the share of capital income in measured GNP, denoted  $\theta_p$ . Now define nominal capital income as follows:

$$\begin{aligned} \text{Nominal Capital Income} &= \text{Unambiguous Capital Income} + \theta_p(\text{Proprietors Income} \\ &\quad + \text{Net National Product} - \text{National Income}) \\ &\quad + \text{Consumption of Fixed Capital} \\ &= \theta \text{ GNP} \end{aligned}$$

where Consumption of Fixed Capital is taken from NIPA (Table 1.9). This equation can be solved for  $\theta_p$  as

$$\theta_p = \frac{(\text{Unambiguous Capital Income} + \text{Consumption of Fixed Capital})}{(\text{GNP} - \text{Ambiguous Capital Income})}$$

which, multiplied by GNP, gives us the measured value of nominal capital income.

To obtain real capital income, we need to deflate the nominal capital income. We use the investment deflator, defined as the ratio of nominal investment and real investment.

**Table 1 : Properties of Investment Returns Without Financing Costs and When  $\delta = 2.5\%$**

This table reports the mean, volatility, first-order autocorrelation of investment return series as well as its cross-correlation with stock returns. Investment returns are generated from the model without financing constraint ( $b=0$ ). In the table,  $\hat{\theta}$  denotes the average in-sample total adjustment cost (physical and financing costs) as a fraction of investment expenditure.  $\hat{\nu}$  denotes the average in-sample share of the total adjustment cost due to financing constraint.  $\rho(1)$  is the first-order autocorrelation of returns. corr is the cross-correlation between investment and stock returns. The means and volatilities are in annualized percent.

	$R^s$		$R^I$					
			In-sample Fractions of Costs					
$\hat{\theta}$	-	0.00	0.05	0.10	0.14	0.20	0.50	1.00
$\hat{\nu}$	-	-	0.00	0.00	0.00	0.00	0.00	0.00
			Moments of Returns					
mean	9.08	12.28	10.96	9.84	9.08	8.12	5.00	2.64
std	12.28	0.70	1.16	1.78	2.24	2.84	4.82	6.38
$\rho(1)$	0.33	0.96	0.46	0.29	0.24	0.21	0.17	0.17
corr	-	0.18	0.40	0.42	0.43	0.43	0.42	0.42

**Table 2 : Properties of Investment Returns With Financing Costs and When  $\delta = 2.5\%$  and  $c=0.50$**

This table reports the mean, volatility, first-order autocorrelation of investment return series as well as the cross-correlation between investment returns and stock returns. Investment returns are generated from the model with financing constraint and parameter  $c = 0.50$ . In the table,  $\hat{\theta}$  denotes the average in-sample total adjustment cost (physical and financing cost) as a fraction of investment expenditure.  $\hat{\nu}$  denotes the average in-sample share of the total adjustment cost due to financing constraint.  $\rho(1)$  is the first-order autocorrelation of returns. corr is the cross-correlation between investment and stock returns. The means and volatilities are in annualized percent.

	$R^s$		$R^I$					
			In-sample Fractions of Costs					
$\hat{\theta}$	-	0.14	0.20	0.25	0.35	0.50	1.00	
$\hat{\nu}$	-	0.00	0.30	0.44	0.60	0.72	0.86	
			Moments of Returns					
mean	9.08	9.08	8.08	7.76	7.56	7.64	8.16	
std	12.28	2.24	8.94	12.64	17.52	21.84	28.14	
$\rho(1)$	0.33	0.24	0.14	0.14	0.14	0.14	0.14	
corr	-	0.43	0.40	0.40	0.40	0.41	0.41	

**Table 3 : Properties of Investment Returns With Financing Costs and When  $\delta = 2.5\%$  and  $c = 0.75$**

This table reports the mean, volatility, first-order autocorrelation of investment return series as well as the cross-correlation between investment returns and stock returns. Investment returns are generated from the model with financing constraint and parameter  $c = 0.75$ . In this table,  $\hat{\theta}$  denotes the average in-sample total adjustment cost (physical and financing cost) as a fraction of investment expenditure.  $\hat{\nu}$  denotes the average in-sample share of the total adjustment cost due to financing constraint.  $\rho(1)$  is the first-order autocorrelation of returns. corr is the cross-correlation between investment and stock returns. The means and volatilities are in annualized in percent.

	$R^s$		$R^I$					
			In-sample Fractions of Costs					
$\hat{\theta}$	-	0.14	0.20	0.25	0.35	0.50	1.00	
$\hat{\nu}$	-	0.00	0.30	0.44	0.60	0.72	0.86	
			Moments of Returns					
mean	9.08	9.08	43.60	75.88	138.88	230.56	527.64	
std	12.28	2.24	100.78	163.32	286.84	473.04	1100.68	
$\rho(1)$	0.33	0.24	0.02	0.01	0.00	-0.02	-0.04	
corr	-	0.43	0.15	0.14	0.13	0.12	0.11	

**Table 4 : Properties of Investment Returns With  $\delta = 1.5\%$**

This table reports the mean, volatility, first-order autocorrelation of investment return series as well as the cross-correlation between investment returns and stock returns. Investment returns are generated from the model without financing constraint ( $b = 0$ ). In the table,  $\hat{\theta}$  denotes the average in-sample total adjustment cost (physical and financing cost) as a fraction of investment expenditure.  $\hat{\nu}$  denotes the average in-sample share of the total adjustment cost due to financing constraint.  $\rho(1)$  is the first-order autocorrelation of returns. corr is the cross-correlation between investment and stock returns. The means and volatilities are in annualized percent.

	$R^s$		$R^I$					
			In-sample Fractions of Costs					
$\hat{\theta}$	-	0.00	0.03	0.05	0.10	0.25	0.50	1.00
$\hat{\nu}$	-	-	0.00	0.00	0.00	0.00	0.00	0.00
			Moments of Returns					
mean	9.08	10.12	9.60	9.08	8.36	6.64	4.92	3.24
std	12.28	0.44	0.68	1.08	1.68	3.22	4.78	6.36
$\rho(1)$	0.95	0.53	0.31	0.22	0.18	0.17	0.17	0.17
corr	-	0.17	0.38	0.42	0.42	0.42	0.42	0.42

**Table 5 : Properties of Investment Returns With  $\delta = 1.5\%$**

This table reports the mean, volatility, first-order autocorrelation of investment return series as well as the cross-correlation between investment returns and stock returns. Investment returns are generated from the model with financing constraint and parameter  $c = 0.50$ . In the table,  $\hat{\theta}$  denotes the average in-sample total adjustment cost (physical and financing cost) as a fraction of investment expenditure.  $\hat{\nu}$  denotes the average in-sample share of the total adjustment cost due to financing constraint.  $\rho(1)$  is the first-order autocorrelation of returns.  $\text{corr}$  is the cross-correlation between investment and stock returns. The means and volatilities are in annualized percent.

	$R^s$		$R^I$				
			In-sample Fractions of Costs				
$\hat{\theta}$	-	0.05	0.10	0.25	0.50	0.75	1.00
$\hat{\nu}$	-	0.00	0.46	0.78	0.89	0.93	0.95
	Moments of Returns						
mean	9.08	9.08	8.24	8.20	9.12	9.96	10.60
std	12.28	1.08	8.50	20.10	28.08	32.36	35.26
$\rho(1)$	0.33	0.31	0.13	0.13	0.13	0.12	0.11
corr	-	0.43	0.42	0.40	0.40	0.40	0.39

**Table 6 : Summary Statistics of the Assets Returns in GMM**

This table reports the means, volatilities, Sharpe ratios, and first-order autocorrelations of excess returns of deciles 1–10, excess value-weighted market return ( $vwret$ ), and real t-bill rate ( $rtb$ ). These are the returns data used in our GMM estimation and tests. Means and volatilities are in annualized percent.

	Decile Returns										vwret	rtb
	1	2	3	4	5	6	7	8	9	10		
mean	12.57	10.05	9.55	9.83	8.86	9.01	8.21	8.60	7.71	6.92	7.42	1.87
std	19.73	17.60	16.86	16.21	15.60	15.29	14.57	13.82	12.93	11.45	11.97	1.33
Sharpe	0.64	0.57	0.57	0.61	0.57	0.59	0.56	0.62	0.60	0.60	0.62	–
$\rho(1)$	0.27	0.29	0.30	0.31	0.30	0.28	0.32	0.28	0.28	0.36	0.33	0.68

**Table 7 : GMM Estimates and Tests of Investment Return Factor Model ( $\delta = 2.5\%$ )**

This table reports results of GMM estimates and tests of investment return factor pricing model. Panel A reports the parameter estimates,  $t$ -statistics,  $\chi^2$ , and  $p$ -value for  $J_T$  test, as well as moments of investment returns generated using estimated parameters under unconditional model. Panel B reports the same set of results for conditional model and Panel C is from the scaled factor model. In the unconditional estimates,  $\mathbf{R}^e$  is the 10 CRSP size decile portfolio and one investment excess return and  $r_f$  is the *real* Treasury-bill return (12 moment conditions). The conditional estimates, in nonscaled and scaled model, use the decile 1, 2, 5, 10, and investment excess returns, scaled by instruments, and the real Treasury-bill return (16 moment conditions). Instruments are the constant, term premium ( $tp$ ), and equally weighted dividend-price ratio ( $dp$ ). The  $p$ -value is the probability of obtaining a  $\chi^2$  value as high or higher.

	parameter estimates							$J_T$ test		fractions		moments			
	$a$	$b$	$c$	$l_0$	$l_1$	$l_2$	$l_3$	$\chi^2$	$p$ -value	$\hat{\theta}$	$\hat{\nu}$	$E[R^I]$	$\sigma[R^I]$	$\rho[R^I]$	corr
Panel A: Unconditional Model															
params	13.88	0.00	–	39.56	-37.89	–	–	16.29	0.06	0.25	0.00	7.43	3.27	0.19	0.42
$t$ -stats	6.70			3.32	-3.25										
Panel B: Conditional Model															
params	9.92	0.00	–	101.80	-98.70	–	–	21.04	0.07	0.18	0.00	8.45	2.63	0.22	0.43
$t$ -stats	8.64			5.50	-5.45										
Panel C: Scaled Factor Model															
params	13.40	0.00	–	59.57	-57.55	-0.03	0.14	16.48	0.12	0.24	0.00	7.54	3.20	0.20	0.42
$t$ -stats	8.28			3.15	-3.12	-4.27	1.41								

**Table 8 : GMM Estimates and Tests of Investment Return Factor Model ( $\delta = 1.5\%$ )**

This table reports results of GMM estimates and tests of investment return factor pricing model. Panel A reports the parameter estimates,  $t$ -statistics,  $\chi^2$ , and  $p$ -value for  $J_T$  test, as well as moments of investment returns generated using estimated parameters under unconditional model. Panel B reports the same set of results for conditional model and Panel C is from the scaled factor model. In the unconditional estimates,  $\mathbf{R}^e$  is the 10 CRSP size decile portfolio and one investment excess return and  $r_f$  is the *real* Treasury-bill return (12 moment conditions). The conditional estimates, in nonscaled and scaled model, use the decile 1, 2, 5, 10, and investment excess returns, scaled by instruments, and the real Treasury-bill return (16 moment conditions). Instruments are the constant, term premium ( $tp$ ), and equally weighted dividend-price ratio ( $dp$ ). The  $p$ -value is the probability of obtaining a  $\chi^2$  value as high or higher.

	parameter estimates							$J_T$ test		fractions		moments			
	$a$	$b$	$c$	$l_0$	$l_1$	$l_2$	$l_3$	$\chi^2$	$p$ -value	$\hat{\theta}$	$\hat{\nu}$	$E[R^I]$	$\sigma[R^I]$	$\rho[R^I]$	corr
Panel A: Unconditional Model															
params	16.96	0.00	–	42.29	-40.62	–	–	16.71	0.05	0.22	0.00	6.91	2.96	0.18	0.42
$t$ -stats	5.74			3.12	-3.06										
Panel B: Conditional Model															
params	12.36	0.00	–	110.34	-107.32	–	–	23.99	0.03	0.16	0.00	7.56	2.38	0.19	0.42
$t$ -stats	8.24			5.31	-5.28										
Panel C: Scaled Factor Model															
params	16.28	0.00	–	64.12	-62.11	-0.03	0.13	15.45	0.16	0.21	0.00	7.00	2.88	0.18	0.42
$t$ -stats	7.63			3.13	-3.10	-4.40	1.35								

**Table 9 : Properties of Pricing Kernels**

This table reports properties of pricing kernel, including mean ( $E[M]$ ), volatility ( $\sigma[M]$ ), market price of risk ( $\sigma[M]/E[M]$ ), the contemporaneous correlation between pricing kernel and real market return ( $\rho(M, R^s)$ ), along with the fraction of financing costs in total adjustment cost  $\nu$ . The physical cost parameters  $a$ 's used in generating investment returns are those corresponding ones reported in Table 7. Panel A reports the properties of the pricing kernels obtained using unconditional estimates. Panel B reports those of the pricing kernels from conditional estimates and Panel C reports the results for the scaled factor model. The assets returns used in the unconditional estimates are the 10 CRSP size decile portfolio, one investment excess return, and the real Treasury-bill return. The assets returns used in the conditional estimates, in both nonscaled and scaled model, are the decile 1, 2, 5, 10 and investment excess returns, scaled by instruments, plus the real Treasury-bill return. Instruments are the constant, term premium, and equally weighted dividend-price ratio. Finally, the market Sharpe ratio in the dataset we use is 0.60.

$b$	$c = 0.50$			$c = 0.75$		
	$\nu$	$\sigma[M]/E[M]$	$\rho(M, R^S)$	$\nu$	$\sigma[M]/E[M]$	$\rho(M, R^S)$
Panel A: Unconditional Model						
0	0.00	0.64	-0.42	0.00	0.64	-0.42
5	0.08	0.35	-0.41	0.01	0.43	-0.39
10	0.15	0.23	-0.41	0.02	0.32	-0.37
20	0.25	0.13	-0.41	0.03	0.21	-0.35
50	0.46	0.04	-0.41	0.07	0.09	-0.32
Panel B: Conditional Model						
0	0.00	1.26	-0.43	0.00	1.26	-0.43
5	0.08	1.91	-0.42	0.00	0.95	-0.38
10	0.15	1.63	-0.41	0.01	0.72	-0.34
20	0.26	0.32	-0.41	0.02	0.61	-0.30
50	0.46	0.07	-0.41	0.04	0.39	-0.26
Panel C: Scaled Factor Model						
0	0.00	1.10	-0.43	0.00	1.10	-0.43
5	0.08	0.68	-0.37	0.01	0.71	-0.43
10	0.15	0.73	-0.27	0.01	0.76	-0.35
20	0.25	0.73	-0.23	0.03	0.79	-0.29
50	0.46	0.72	-0.19	0.07	0.74	-0.24



**Table 10 : Jensen's  $\alpha$** 

This table reports intercepts ( $\alpha$ ) and slopes ( $\beta$ ) of the following regression:

$$R_i - R_f = \alpha + \beta(R^I - R_f)$$

where  $R_i$  is either value-weighted market stock return or Decile 1 return. The physical cost parameters ( $a$ ) used in generating investment returns are those corresponding ones reported in Table 7. The pricing error or Jensen's  $\alpha$  is in percent.

$c$	$b$	value-weighted return				decile 1 return			
		$\alpha$	$t$ -stat	$\beta$	$t$ -stat	$\alpha$	$t$ -stat	$\beta$	$t$ -stat
Panel A: Unconditional Model									
0.50	0	0.18	0.34	1.17	4.88	0.36	0.41	1.90	4.82
	5	0.72	1.60	0.82	5.38	1.26	1.69	1.31	5.22
	10	0.99	2.30	0.64	5.56	1.70	2.39	1.02	5.37
	20	1.24	2.98	0.47	5.72	2.09	3.05	0.75	5.51
	50	1.45	3.58	0.31	5.91	2.43	3.62	0.49	5.69
0.75	0	0.18	0.34	1.17	4.88	0.36	0.41	1.90	4.82
	5	0.65	1.38	0.82	4.90	1.14	1.46	1.33	4.78
	10	0.93	2.07	0.62	4.81	1.60	2.15	0.99	4.66
	20	1.20	2.79	0.41	4.66	2.05	2.86	0.65	4.48
	50	1.44	3.41	0.22	4.49	2.42	3.47	0.34	4.28
Panel B: Conditional Model									
0.50	0	-0.32	-0.51	1.29	4.45	-0.49	-0.47	2.12	4.42
	5	0.34	0.69	0.93	5.18	0.65	0.79	1.50	5.05
	10	0.71	1.58	0.72	5.43	1.25	1.68	1.15	5.26
	20	1.06	2.50	0.52	5.64	1.82	2.59	0.83	5.44
	50	1.37	3.35	0.33	5.86	2.30	3.41	0.52	5.64
0.75	0	-0.32	-0.51	1.29	4.45	-0.49	-0.47	2.12	4.42
	5	0.28	0.50	0.93	4.31	0.49	0.55	1.52	4.29
	10	0.68	1.35	0.68	4.08	1.15	1.39	1.12	4.06
	20	1.08	2.34	0.43	3.75	1.81	2.38	0.71	3.74
	50	1.43	3.27	0.20	3.34	2.39	3.31	0.33	3.34
Panel C: Scaled Factor Model									
0.50	0	0.13	0.24	1.18	4.84	0.27	0.30	1.93	4.78
	5	0.68	1.50	0.83	5.36	1.20	1.59	1.33	5.21
	10	0.96	2.23	0.65	5.55	1.65	2.32	1.03	5.36
	20	1.22	2.96	0.47	5.71	2.07	3.00	0.75	5.50
	50	1.44	3.55	0.31	5.90	2.42	3.60	0.49	5.68
0.75	0	0.13	0.24	1.18	4.84	0.27	0.30	1.93	4.78
	5	0.62	1.28	0.83	4.83	1.08	1.36	1.35	4.74
	10	0.91	2.00	0.62	4.72	1.55	2.07	1.00	4.61
	20	1.19	2.74	0.41	4.54	2.02	2.81	0.66	4.41
	50	1.44	3.40	0.21	4.34	2.42	3.45	0.34	4.20

**Table 11 : GMM Estimates and Tests Using Small Decile Returns**

This table reports results of GMM estimates and tests of investment return factor pricing model using small decile returns from NYSE/AMEX/NASDAQ. Panel A reports the parameter estimates,  $t$ -statistics,  $\chi^2$ , and  $p$ -value for  $J_T$  test, as well as moments of investment returns generated using estimated parameters under unconditional model. Panel B reports the same set of results for conditional model and Panel C is from the scaled factor model. In the unconditional estimates,  $\mathbf{R}^e$  is the excess returns of CRSP size decile 1, 2, and 3 portfolios and one investment excess return and  $r_f$  is the *real* Treasury-bill return (5 moment conditions). The conditional estimates, in nonscaled and scaled model, use the decile 1 and 2 and investment excess returns, scaled by instruments, and the real Treasury-bill return (10 moment conditions). Instruments are the constant, term premium ( $tp$ ), and equally weighted dividend-price ratio ( $dp$ ). The  $p$ -value is the probability of obtaining a  $\chi^2$  value as high or higher.

	parameter estimates							$J_T$ test		fractions		moments			
	$a$	$b$	$c$	$l_0$	$l_1$	$l_2$	$l_3$	$\chi^2$	$p$ -value	$\hat{\theta}$	$\hat{\nu}$	$E[R^I]$	$\sigma[R^I]$	$\rho[R^I]$	corr
Panel A: Unconditional Model															
params	13.06	0.00	–	58.60	-56.61	–	–	3.49	0.32	0.23	0.00	7.62	3.14	0.20	0.42
$t$ -stats				4.67	-4.61										
Panel B: Conditional Model															
params	10.06	0.00	–	122.13	-118.58	–	–	8.16	0.42	0.18	0.00	8.41	2.65	0.21	0.43
$t$ -stats				6.78	-6.76										
Panel C: Scaled Factor Model															
params	11.55	0.00	–	75.25	-72.85	-0.03	0.22	8.05	0.23	0.21	0.00	8.00	2.91	0.21	0.43
$t$ -stats				4.44	-4.40	-3.65	1.84								

**Table 12 : GMM Estimates and Tests On Time-Varying Financing Constraints**

This table reports results of GMM estimates and tests of investment return factor pricing model using time-varying parameters of financing constraints:  $b_t = b_0 + b_1 DF_t$  and  $c_t = \frac{\exp[c_0 + c_1 DF_t]}{1 + \exp[c_0 + c_1 DF_t]}$  where  $DF_t$  is the default premium. Panel A reports the results using time-varying  $b_t$  but  $c$  is still assumed to be constant through time, i.e., we restrict  $c_1 = 0$ . Panel B reports the results using time-varying  $b_t$  and  $c_t$ . In both panels, we report the parameter estimates,  $t$ -statistics,  $\chi^2$ , and  $p$ -value for  $J_T$  test, as well as moments of investment returns generated using estimated parameters under unconditional model, conditional model, and the scaled factor model. In the unconditional estimates,  $\mathbf{R}^e$  is the excess returns of CRSP size decile 1–10 portfolios and one investment excess return and  $r_f$  is the *real* Treasury-bill return. The conditional estimates, in nonscaled and scaled model, use the decile 1, 2, 5, and 10 and investment excess returns, scaled by instruments, and the real Treasury-bill return. Instruments are the constant, term premium ( $tp$ ), and equally weighted dividend-price ratio ( $dp$ ). The  $p$ -value is the probability of obtaining a  $\chi^2$  value as high or higher.

		parameter estimates						$J_T$ test			moments						
$a$	$b_0$	$b_1$	$c_0$	$c_1$	$l_0$	$l_1$	$l_2$	$l_3$	$\chi^2$	$p$ -value	$\hat{\theta}$	$\hat{\nu}$	$E[R^e]$	$\sigma[R^e]$	$\rho[R^e]$	corr	
Panel A: Time-Varying $b_t$ and Fixed $c$																	
Unconditional Model																	
params	13.88	0.00	1.02	0.82	-	42.17	-40.45	-	17.10	0.07	0.25	0.00	7.44	3.40	0.18	0.41	
$t$ -stats						6.95	-6.81										
Conditional Model																	
params	9.92	0.00	1.01	0.00	-	113.62	-110.38	-	12.47	0.57	0.20	0.00	8.10	2.80	0.11	0.29	
$t$ -stats						7.12	-7.08										
Scaled Factor Model																	
params	13.40	0.00	0.25	0.22	-	64.41	-62.28	-0.03	0.10	15.95	0.19	0.24	0.01	7.50	3.24	0.18	0.41
$t$ -stats						4.44	-4.40	-3.65	1.84								
Panel B: Time-Varying $b_t$ and $c_t$																	
Unconditional Model																	
params	13.88	0.00	5.26	-11.58	17.89	22.47	-21.13	-	14.73	0.14	0.26	0.05	7.12	5.31	0.19	0.41	
$t$ -stats						6.23	-6.00										
Conditional Model																	
params	9.92	0.00	0.56	6.72	-4.66	113.52	-110.20	-	9.63	0.79	0.18	0.01	8.41	2.63	0.17	0.34	
$t$ -stats						6.75	-6.72										
Scaled Factor Model																	
params	11.40	0.00	1.11	13.97	-9.83	38.75	-37.11	-0.04	0.11	14.01	0.30	0.25	0.02	7.48	3.62	0.10	0.25
$t$ -stats						5.63	-5.48	-5.50	1.14								

**Table 13 : Investment-Capital and Profit-Investment Ratios As Pricing Factors**

This table reports GMM estimation and testing using *ad hoc* pricing factors. Panel A uses investment-capital ratio ( $i$ ) as the single pricing factor and Panel B uses both investment-capital ratio ( $i_t$ ) and profit-investment ratio ( $\pi$ ) as two pricing factors. We scale the factor(s) using *term* (term premium) and *dp* (dividend-price ratio). For the one-factor model, the pricing kernel is:

$$M = 1_0 + l_1 f_1 + l_2 f_1 \times term + l_3 f_1 \times dp$$

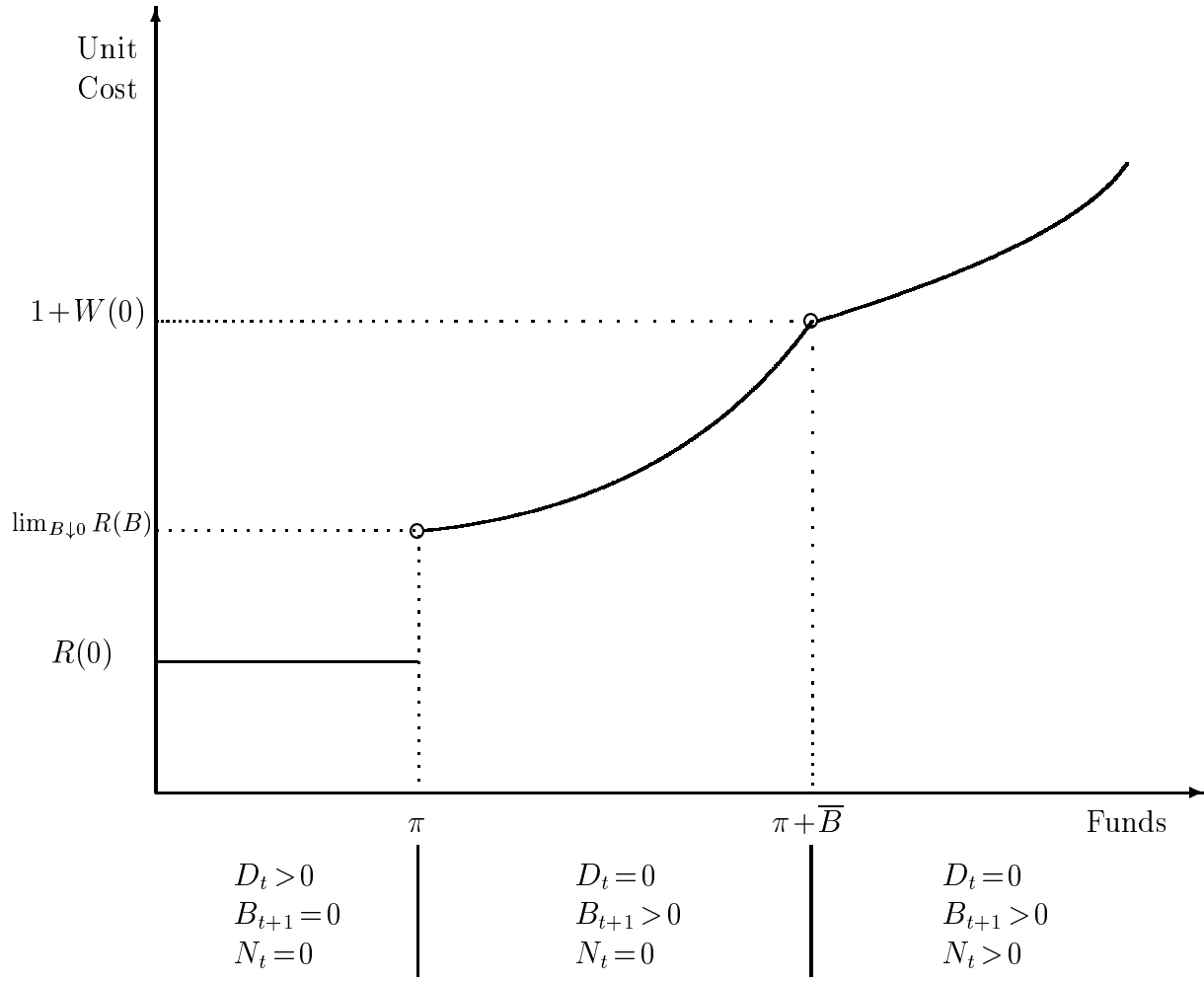
and for the two-factor model, the pricing kernel is thus:

$$M = 1_0 + l_1 f_1 + l_2 f_2 + l_3 f_1 \times term + l_4 f_1 \times dp + l_5 f_2 \times term + l_6 f_2 \times dp$$

The assets returns used in the scaled factor model are the decile 1, 2, 5, 10 scaled by instruments, plus the real Treasury-bill return.

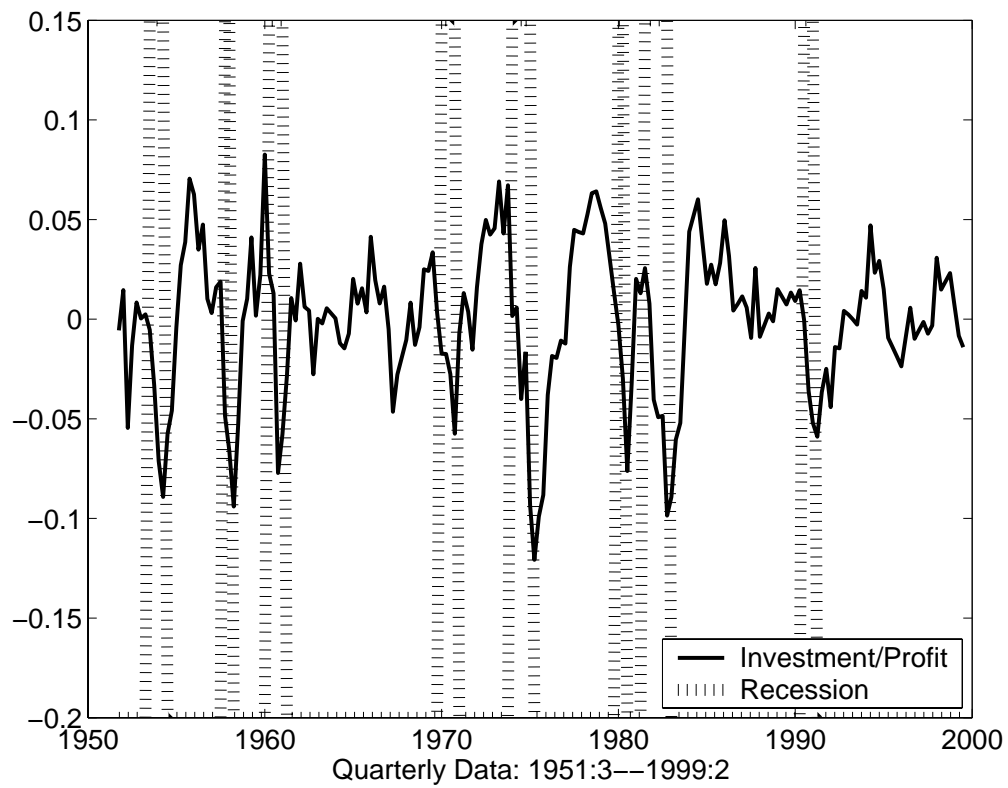
Panel A: $f_1 = i_t$ : Scaled Factor Model							
	$l_0$	$l_1$	$l_2$	$l_3$			
loadings	-1519	38549	107	3057			
<i>t</i> -stats	-5.71	5.61	5.17	3.63			
$\chi^2$	176.32						
<i>p</i> -value	0.00						
Panel B: $f_1 = i$ and $f_2 = \pi$ : Scaled Factor Model							
	$l_0$	$l_1$	$l_2$	$l_3$	$l_4$	$l_5$	$l_6$
loadings	1.12	-37.10	1.08	-1.72	-21.71	0.01	0.29
<i>t</i> -stats	0.08	-0.22	0.17	-0.39	-0.53	0.07	0.29
$\chi^2$	21.12						
<i>p</i> -value	0.00						

Figure 1: Hierarchical Structure of Financing



**Figure 2 : Investment-Profit Ratio Over Business Cycle**

This figure presents the HP-filtered quarterly Investment-Profit ratios (the solid line) and NBER recession dates (the dotted line) over the period: June 1951 to June 1999.



**Figure 3 : Hansen-Jagannathan Bound**

This figure presents the Hansen-Jagannathan Bound implied by the assets returns used in the unconditional model, i.e., the 10 CRSP size decile portfolios and the real Treasury-bill return. The circles are the maximal market price of risk generated using investment returns as the single pricing factor. Investment returns are generated using physical cost parameter  $a$  reported in Panel A of Table 7 and  $b$  parameter being 0, 1, 3, 5, and 10 with  $c = 0.50$ . The solid circle corresponds to the optimal parameter combination with  $b = 0$ . The triangles corresponds to the standard deviation-mean combinations of consumption based stochastic discount factor  $\beta(C_t/C_{t+1})^\gamma$  where  $\beta$  is set to be 0.9920 (quarterly) and  $C_t$  is aggregate consumption expenditure on nondurables and services in terms of 1992 dollars (GCNQF and GCSQF 1959:Q1–1999:Q4 from DRI) and  $\gamma$  is, from right to left corresponding to the triangles, 1, 3, 5, 10, 20, and 30.

