The Equivalence of the Social Security's Trust Fund Portfolio Allocation and Capital Income Tax Policy

Kent Smetters

The Wharton School,
University of Pennsylvania

Stanford Economics Department (Visiting)

and NBER

Revised: July 8, 2001

* September 1, 2000 - June 31, 2001: Department of Economics, Stanford University, Stanford, California, 94305-6072. Otherwise: The Wharton School, University of Pennsylvania, 3641 Locust Walk, CPC 302, Philadelphia, PA 19104-6218. 215-898-9811. 215-898-0310 (Fax). <smetters@wharton.upenn.edu>.

I benefitted from helpful comments and conversations with Andy Abel, Alan Auerbach, Douglas Bernheim, Timothy Besley, Henning Bohn, Peter Diamond, Jon Gruber, Peter Hammond, Olivia Mitchell, Agnar Sandmo, and participants of seminars held at Berkeley, MIT, Santa Barbara, and Stanford. Suggestions by Antonio Rangel were especially helpful.

The Equivalence of the Social Security's Trust Fund Portfolio Allocation and Capital Income Tax Policy

[Revised June 8, 2001]

Abstract

This paper proves that the stock-bond portfolio choice of the public social security trust fund is Arrow-Debreu equivalent to the tax treatment of private capital income by the non-social security part of government: the state-contingent consumption of every agent is the same under both policies. A larger [smaller] share of social security's portfolio invested in stocks is equivalent to a larger [smaller] symmetric linear tax on the risky portion of capital income returns received on assets held by private agents. The tax change causes agents to adjust their private portfolios so that the total (private plus public) levels of demand for equities and bonds are the same under both policies.

At first, it would seem that this equivalency requires that there are no pre-existing tax distortions or market frictions. However, the equivalency is shown to be quite general. First, the initial tax rate on private capital can be non-zero, and the initial tax can take any form (hence, possibly different form than the new tax). Although the amount of revenue collected at the initial tax rate (i.e., not including the rate change) is disrupted after private agents alter their portfolio, this pre-existing tax distortion does not undo equivalency. Second, since private trading with the unborn is impossible, trading markets between generations are allowed to be missing. It follows that the social security trust fund's stock-bond portfolio choice is *not neutral* since risk is transferred across non-trading generations. But policy *equivalency* still holds. Third, an arbitrarily large share of agents (short of everyone) can even be borrowing constrained. Borrowing constrained people do not even have a portfolio of assets that can respond to taxes. Still, while consumption levels vary by state and agent type, they are identical under both policies, even for the constrained.

To the extent that trust fund investment in equities improves risk sharing in the context of missing or incomplete markets, as shown in some previous papers, the equivalent capital income tax rate can be interpreted as a Lindahl (corrective) tax. This tax gives a decentralized way of achieving the same potential risk sharing outcomes as the government directly owning part of the capital stock. The decentralized approach to improving risk sharing might be more palatable to those who fear direct government intervention in financial markets (e.g., Greenspan, 1999).

General-equilibrium simulation results are presented using an overlapping-generations model with aggregate uncertainty. The model incorporates a fully endogenous equity return distribution, and several other features that have been taken as exogenous in previous models. The model is used to produce policy-equivalent tax rates along the non-neutral ex-post mean transition path from the initial stochastic steady state (before the trust fund is invested in equities) to the final stochastic steady state (after the trust fund is invested in equities). The results suggest that investing the entire US Social Security trust fund in equities is equivalent to a 4 percent tax on risky capital income tax returns. This equivalent tax rate is fairly constant along the mean transition path.

JEL Codes: H3,E6,G0,H2

I. Introduction

The classic equivalency between the level of unfunded social security wealth and government debt, as emphasized in Feldstein (1974), Barro (1974), and Miller and Upton (1974), revolutionized how economists view the government's budget constraint.[1] Debt is no longer viewed in isolation of the rest of a nation's unfunded liabilities. This equivalency is the motivation behind 'generational accounting' literature that unifies all liabilities.[2] This equivalency also explains why a debt-financed transition to a funded social security system, in which each generation only pays the debt service, has no impact on unfunded liabilities, as reemphasized by Geanakoplos, Mitchell and Zeldes (1998).

This paper extends the equivalency literature to include risk. It proves that an equivalence also exists between a nation's tax treatment of risky private capital income returns and how assets in its defined-benefit social security trust fund are allocated between risky capital and government bonds. By "equivalence," the strong (Arrow-Debreu) form is meant: state-contingent consumption levels are the same under both policies. Hence, these policies are also ex-ante equivalent.

The equivalency result proven herein, though, is more subtle than that between debt and unfunded social security.[3] Debt and social security are equivalent because they generate the same private budget constraints and, hence, the same private demand for capital. This private budget constraint equivalence led Auerbach, Gokhale, and Kotlikoff (1994) to refer to debt and social security as "arbitrary labels." In contrast, the equivalency proven herein requires distorting the after-tax rate of return to capital that is received by private agents, *changing* their demand for capital.

As an example, suppose that the social security trust fund is currently invested in bonds (as

---

[1] The Ricardian equivalence debate, which emerged from these papers, focused on whether debt was also *neutral*. Although this debate is still active, the *equivalency* between debt and social security is now well accepted.

[2] See Auerbach, Gokhale, and Kotlikoff (1994).

[3] Conversations with Alan Auerbach, Peter Diamond, and Antonio Rangel helped me clarify this exposition.

in the US) and the government wants to re-balance the trust fund's portfolio in order to hold some equities. At existing prices, this open market operation (i) increases the total (public plus private) demand for equities; (ii) decreases the total demand for bonds; and (iii) changes payroll tax rates in most future states of the world. The total demand for equities and bonds must change since the risks in a defined-benefit system are borne by future taxpayers, as benefits remain defined by law. The change in the future payroll tax rate is needed in most future states in order to buffer the trust fund's new exposure to capital income shocks, so that social security has enough revenue to pay benefits.

This paper shows that the economic outcomes of this open market operation can be replicated by simply creating a positive symmetric linear tax on the risky portion of asset returns earned by *private* agents.[4] The new tax reduces the after-tax variance of risky returns, which entices private agents to increase their demand for equities and to decrease their demand for bonds by the same amount. The total (public plus private) levels of demand for equities and bonds, therefore, can be made the same under both policies if the new tax is chosen at the right rate. Hence, a tax on risk can replicate the outcomes (i) and (ii) of the trust fund's open market operation listed above. But full equivalency also requires generating the same state-contingent payroll tax rates, or outcome (iii). To see how that works, note that, under the government's budget constraint, wage taxes must also adjust to buffer loss and gains in capital income tax revenue. As proven below, the capital income tax rate that generates outcomes (i) and (ii) is the same rate that generates outcome (iii). In words, the capital income tax rate that generates the same total demand for equities and bonds as the trust fund's operation exactly equals the capital income tax rate that places the same amount of risk on the wage tax base as the trust fund's operation.

[4] In converse, suppose that the trust fund is currently invested in some stocks (as in Canada) and that the government wants to re-balance the trust fund's portfolio in order to hold more bonds. This operation can be perfectly replicated in general equilibrium by instead *decreasing* the tax rate on risky capital income (possibly going negative).

At first, it would seem that this full equivalency requires that there are no pre-existing tax or market frictions. However, equivalency is proven to be quite general. First, the initial tax rate on private capital income can be non-zero. To be sure, the presence of this pre-existing distortion would seem to disrupt equivalency because more revenue is collected at the *initial* capital income tax rate after agents shift their portfolio. This revenue is in addition to the revenue collected by the *change* in the tax rate itself. Hence, the size of the tax rate change must *depend* on the size of the initial tax rate in order to recognize the new level of revenue collected at the initial tax rate. But, it is also true that the amount of private saving that must be shifted between capital and bonds, in order to replicate the trust fund's operation, is *independent* of the initial capital income tax rate. So it would seem that a single capital income tax rate cannot clear the financial markets *and* generate the same state-contingent wage tax rates as the trust fund's open market operation. But, as shown below, full equivalency holds even with this pre-existing tax distortion. Moreover, the initial tax need not take the same form as the new tax being implemented.

Second, since private trading with the unborn is impossible, trading markets between generations are allowed to be missing. Hence, the social security trust fund's portfolio choice is not *neutral* since portfolio risks are shared with future taxpayers, as benefits remain defined by law. But, *equivalency* still holds, i.e., both policy choices are *equivalently non-neutral*.[5]

Third, an arbitrarily large share of agents (short of everyone) can even face endogenously binding borrowing constraints. To be sure, borrowing constrained people do not hold a portfolio of assets that can respond to taxes. So, it would seem that equivalency has no chance of holding. In particular, while investing the trust fund in equities shifts some of payroll taxes paid by borrowing-constrained agents from bonds to stocks, these agents cannot be enticed, via taxes, to mimic this

[5] Similarly, debt and unfunded social security wealth are non-neutral (unless agents are Ricardian) even though they are equivalent fiscal policy instruments.

operation on their own. Still, this paper shows that full Arrow-Debreu equivalency still holds. While consumption levels obviously vary by agent type and state, the consumption levels are the same under both policies for each agent, including constrained agents. Moreover, the equivalent capital income tax rate depends only on aggregate labor income and capital: the fraction of the constrained population is immaterial. So, the optimal tax rate can be calculated using data that can typically be found in a nation's NIPA and Flow of Funds Accounts.[6]

The equivalency of both policies is not only quite general, the policy-equivalent capital income tax can also take on an interpretation as a Lindahl (corrective) tax. As reviewed in Section II, previous papers have shown that investing the trust fund in equities could improve risk sharing between generations (e.g., Bohn 1997, 1999; Social Security Advisory Council, 1997; Diamond, 1997), or if some agents are borrowing constrained (Diamond and Geanakoplos, 1999; Abel, 2001). The model herein nests both types of market incompleteness. A Lindahl tax gives a decentralized way of achieving the same efficiency gains.

A decentralized approach to redressing market inefficiencies might be preferred by those who are concerned about the potential for 'additional costs,' in the form of political risks, associated with the government directly owning assets. These fears have taken on a heightened relevance in light of the recent debate on social security reform. In his nationally televised 1999 State of the Union Address, President Clinton proposed investing part of the US Social Security trust fund in equities. His proposal re-ignited a debate about the government's role in capital markets.[7] Alan Greenspan (1999) voiced opposition to Clinton's plan, citing political danger. Political risks may include a

[6] In the US, total wage income is computed by the Bureau of Economic Analysis and the value of the capital stock is computed by the Board of the Federal Reserve Bank.

[7] The debate is not new: the idea of the government holding private equities was hotly debated among policymakers even before the passage of the 1935 US Social Security Act (Shoven and Schieber, 1999).

home-bias of stock choices (e.g., not investing in foreign auto makers), excluding controversial stocks (e.g., tobacco) from the government's portfolio, a lax anti-trust policy (e.g., would the government sue Microsoft if it owned it?), using stock voting rights to conduct social policy (e.g., quota hiring), and – if the government forfeited it's voting rights – principle-agent problems associated with having a large passive investor that is insensitive to performance. Evidence in White (1996) and Iglesias and Palacios (2000) shows that governments investing directly in capital markets have tended to earn inferior rates of returns.

However, the fact that political risks exist with government ownership of equities does not necessarily imply that the government should not do so. Political risks exist with many government activities. A military contract might be given to a large campaign donor; public work projects might be located in a powerful politician's precinct; etc. Still, the provision of national defense and public works are generally viewed as important public goods. Similarly, the political risks associated with the government holding capital could be outweighed by improved risk sharing.

This paper shows, though, that this tradeoff need not exist: the same potential risk sharing benefits can be achieved without government ownership.[8] So, for example, President Bush's opposition to investing part of the US Social Security trust fund in equities does not mean that the US must forego a risk sharing opportunity, as one might think. Indeed, if trust fund investment in equities enhances efficiency, it can be replicated with a simple tax on risky private capital income.

Calculations reported below suggest that investing the entire US Social Security trust fund in equities is equivalent to a four percent tax on capital income returns. These results are derived using an overlapping-generations model with aggregate uncertainty, which incorporates many

[8] To be sure, taxes/tariffs are also influenced by politics. But it might be harder to dis-favor individual firms in the tax-tariff system, and it could even be illegal domestically or violate WTO rules. Indeed, opponents of government ownership of private assets have not voiced the same type of concerns regarding taxes.

features in the household sector, production sector and government sector that have been taken as exogenous in previous models. To the extent that the current capital income tax is already approximately a linear symmetric tax on the risky component of private returns (as, argued in Gordon, 1985, for the US), investing the US trust fund in equities can be replicated by simply increasing the existing tax rate. If a nation's tax rules deviate from these assumptions, the tax rules would also have to be adjusted for the *new* tax; however, the existing tax could remain unchanged.

This paper does not consider some related issues. It does not consider political-economy issues in overlapping-generations economies (Rangel, 2000). Nor does it re-derive the conditions under which trust fund investment in equities increases efficiency (Bohn, 1997, 1999; Diamond and Geanakoplos, 1999). These issues are not necessary for demonstrating equivalency. This paper also does not examine tax law provisions in different countries. As mentioned above, policy equivalency continues to hold even if the existing capital income tax is not symmetric or linear; these two tax provisions only have to be applied to the new tax. Finally, this paper does not consider individual accounts since they are already decentralized, and the risk is not shared inter-generationally.[9]

Section II reviews the literature. Section III presents an overlapping-generations model with incomplete markets between generations. Section IV derives a closed-form solution for the capital income tax rate that is equivalent to the trust fund's investment portfolio. Section V reports calculations. Section V adds endogenously binding borrowing constraints. Section VII concludes.

II. Literature Review

As illustrated in Figures 1 and 2, this paper bridges the literature on the taxation of risky capital income with the recent literature on the portfolio choice of the social security trust fund. In

---

[9] If the government guarantees a minimum return to the private account, there is only some limited inter-generational risk sharing since future taxpayers are exposed to downside risk but not upside potential.

particular, this paper focuses on demonstrating the equivalency between the policies connected by Line (A) in Figures 1 and 2. Two independent sets of previous research have focused on Lines (B) and (C), which compare two fiscal policy changes against the status quo.

Taxation of Risky Capital Income – Line (B)

The literature on taxing risky capital income started with Domar and Musgrave (1944) and was derived rigorously by Mossin (1968), Stiglitz (1969), Sandmo (1969, 1985), Gordon (1985), and Bradford (1995). That literature made two assumptions: (i) the government taxes only the difference between the risky return and the risk-free rate and (ii) the tax is symmetric in that returns above and below the risk-free rate are taxed. Assumption (i) is usually justified on tax law provisions, or the low historic return to debt relative to equities.[10] Assumption (ii) is usually based on loss offset rules imbedded in tax law. Gordon (1985) argues that these conditions approximately hold for the US.

This literature argued that a tax on capital income is neutral, i.e., the policies connected by Line (B) in Figures 1 and 2 *are* equivalent. Two types of models were used to prove neutrality. One type of model analyzed *compensated* tax changes. Gordon (1985), who was first to include the government's budget constraint, considered a single-agent two-period model. The agent bore, via a lump-sum tax rebate in the second period of his life, all of the risk associated with the capital income tax that he paid. Gordon proved that a tax on risk did not change the agent's private demand for capital or bonds. It follows that the agent's state-contingent level of consumption is unchanged.

The second type of model analyzed *uncompensated* tax changes. The papers in this literature abstracted away from the government's budget constraint and instead focused on the saving and portfolio choices. Mossin (1968) and Stiglitz (1969) considered one-period models. The seminal

[10] The average real return to US Treasury bills between 1926 and 2000 was 0.8 percent, while the average return to long-term US bonds was 2.7 percent. The equity premium is between 6 and 7 percent (Ibbotson, 2001).

work by Sandmo (1969) modernized the discussion to a two-period inter-temporal choice model with multiple assets. Bradford (1995) considered the taxation of risky capital income returns in the context of fundamental tax reform. In all of these papers, a capital income tax reduces the variance of after-tax equity returns received by agents. So agents respond by increasing their demand for stocks and decreasing their demand for bonds by the same amount. Portfolio re-balancing allows agents to achieve the same tradeoff between after-tax expected return and risk as before the tax. Portfolio re-balancing occurs because, unlike the compensated framework discussed above, agents are not the residual claimants of fluctuations in the government's capital income tax revenue. But, like the compensated framework, the state-contingent consumption of agents remains unchanged since the after-tax risk and expected return of the agent's portfolio are unchanged.

In contrast to this previous literature, capital income taxes are *not* consumption neutral herein, i.e., the policies connected by Line (B) are not equivalent. As in the compensated model of the previous literature, the government's budget constraint is enforced. But, unlike that model, generations are allowed to overlap herein so that risks are shared across non-trading generations through the tax system. Higher [lower] capital income taxes collected in the second period of life from generation t requires less [more] wage taxes to be collected in the first period of life from generation t+1 in order to satisfy the government's revenue needs. Some agents might also be borrowing constrained. Non-Ricardian sharing of risks (and ensuing g.e. effects) leads to portfolio re-balancing. But the after-tax expected return and risk are *changed*.

<u>Investing the Social Security Trust Fund in Equities – Line (C)</u>

Some studies have also focused on the effect of investing the social security trust fund in equities, or Line (C) shown in Figures 1 and 2. The early literature argued that trust fund investment is a neutral operation, i.e., the policies connected by Line (C) are equivalent. Similar to the

Modigliani-Miller theorem of corporate finance, agents rationally re-balance their private portfolios in response to a change in their 'corporate' (public) portfolio held in their name by the trust fund. If, for example, the trust fund holds more stocks, agents hold less stocks in order to achieve their original private-plus-public asset allocation. This argument, though, assumes that social security benefits absorb the trust fund's capital income risk, like with personal investment accounts. The argument also requires that agents face perfect capital markets.

The more recent literature, including the papers referenced in Section I and the current paper, recognizes that social security benefits remain defined by law, so that risk would be passed to future generations via payroll taxes. Some agents might also not hold any assets. Consumption neutrality, therefore, fails due to these multiple sources of market incompleteness.

III. Model

Households

Without any loss in generality, suppose that agents live for two periods. Generation-t consumers decide how much to save in bonds, s^B, and unleveraged capital, s^K, to maximize their expected lifetime utility over first-period consumption, c_1, and second-period consumption, c_2,

(1) $$\max_{s_t^B, s_t^K} E_t U\left(c_{1,t}, c_{2,t+1}\right) = u(c_{1,t}) + \beta E_t u(c_{2,t+1})$$

subject to the following budget constraints,

(2) $$c_{1,t} + s_t^K + s_t^B = w_t(1 - \tau_t^{SS} - \tau_t^W)$$

(3) $$c_{2,t+1} = s_t^K \cdot \left[1 + e_{t+1} - \left(e_{t+1} - r_{t+1}\right)\tau_{t+1}^K\right] + s_t^B \cdot \left(1 + r_{t+1}\right) + b_{t+1}$$

τ terms are taxes described below, w is the wage rate known at time t, e is the risky realized return to capital (equities), r is the risk-free return to government debt and b is the social security benefit. The function, $u(c)$, is standard: $\partial u(c)/\partial c > 0$, $\partial^2 u(c)/\partial c^2 < 0$, and $\lim_{c \to 0} \partial u(c)/\partial c = \infty$.

In equation (2), the sum of first-period consumption and saving in risky capital and bonds equal after-tax wages received in the first period. Workers face two taxes in their first period: a social security payroll tax, τ^{SS}, and a wage tax, τ^{W}, used to finance other government spending.

In equation (3), second-period consumption equals a worker's resources which is the sum of after-tax capital income, the return to bonds, and social security. Risky capital income is taxed at rate τ^{K} and is of the Domar-Musgrave type discussed earlier which taxes only the risky component of investments. Risk-free bond returns are also not taxed; this assumption is always immaterial since the full incidence would fall to the government under the no-arbitrage conditions derived below.

Social Security

Social security benefits are partly pay-as-you-go and partly funded. Let $n_{t+1} \equiv \left(L_{t+1} / L_t\right) - 1$ denote the population growth rate from period t to $t+1$, and let $g_{t+1} \equiv \left(w_{t+1} / w_t\right) - 1$ denote the wage growth rate. Let φ represent the fraction of a generation's social security payroll tax that goes to a trust fund, T, which is used to help pay for that same generation's second-period social security benefit (which is the purpose of funding). The fraction $(1-\varphi)$ of payroll taxes get paid out immediately as benefits for the previous generation. The per-capita social security benefit is,

$$
\begin{aligned}
b_{t+1} &\equiv \left[(1-\varphi)\tau^{SS}_{t+1} L_{t+1} w_{t+1} + \mathrm{T}_{t+1}\right] / L_t \\
&= \left[(1-\varphi)\tau^{SS}_{t+1}(1+n_{t+1}) w_t (1+g_{t+1})\right] + \left[\mathrm{T}_{t+1} / L_t\right]
\end{aligned}
\tag{4}
$$

The expression in the first bracket in the second equality is the stochastic wage-indexed pay-as-you-go portion of social security. The expression in the second bracket is the funded portion that does not get paid out immediately as benefits. It is immaterial whether Social Security pays the Treasury taxes on its investments since the defined-benefit liabilities remain unchanged.

The trust fund is modeled the same way as in Diamond (1997), Bohn (1997, 1999) and Abel (1999) who also consider a defined-benefit system. Let ϕ be the fraction of the trust fund at time

t+1, T_{t+1}, invested in equities; hence, $(1-\phi)$ is the fraction invested in government debt.

$$\mathrm{T}_{t+1} \equiv \varphi\tau_t^{SS} L_t w_t \left\{1+\left[\phi e_{t+1} + (1-\phi)\cdot r_{t+1}\right]\right\} + S_{t+1} \tag{5}$$

where S is the required subsidy from either general revenue or from changing payroll taxes,

$$\begin{aligned} S_{t+1} &\equiv \varphi\tau_t^{SS} L_t w_t \left\{r_{t+1} - \left[\phi e_{t+1} + (1-\phi)\cdot r_{t+1}\right]\right\} \\ &= -\varphi\phi\tau_t^{SS} L_t w_t \left(e_{t+1} - r_{t+1}\right) \end{aligned} \tag{6}$$

The subsidy, S, equals the difference between what the trust fund would have earned if invested in government debt and what the trust fund actually earns. Currently $\phi = 0$ and so $S = 0$. The subsidy is positive ($S > 0$) if realized equity returns are below the risk-free rate; negative ($S < 0$) otherwise. A negative subsidy to generation t+1 is expected along the mean growth path where $E_t\left(e_{t+1}\right) > r_{t+1}$.[11] But this expected benefit to generation t+1 comes at a cost of additional risk.

Equivalently, the sum $\tilde{\mathrm{T}}_{t+1} \equiv \mathrm{T}_{t+1} + S_{t+1}$ could be referred to as the value of the "trust fund" at time t+1. The labeling is arbitrary: the distinction is made herein only to emphasize that it does not matter whether gains/losses are credited to social security or to the rest of the government.

A popular motive for investing the trust fund in equities is to help maintain the statutory benefit level without raising taxes (SSAC, 1997). Specifically, *statutory* benefits are defined by a formula that is a function of previous wages adjusted for wage growth: $\tilde{b}_{t+1} = R\left(w_t \cdot \left(1+g_t\right)\right)\cdot w_t$, where $\tilde{b}_{t+1}$ is the statutory benefit level and $R(\cdot)$ is the 'replacement rate.'[12] However, the *actual* benefit paid must satisfy the resource constraint (4), b_{t+1}. Denote $\tilde{\tau}_{t+1}^{SS}$ as the payroll tax rate such that $b_{t+1}\left(\tilde{\tau}_{t+1}^{SS}\right) = \tilde{b}_{t+1}$. In the US and many other countries, say currently at time t, the ratio of retirees to workers at generation t+1, L_t / L_{t+1}, will increase. Hence, the payroll tax rate at time t+1 that is needed to produce present-law statutory benefits at time t+1 will exceed the current payroll

[11] This inequality is guaranteed by the production technology shown below.

[12] In the US and many other countries, $\partial R / \partial w_t < 0$ but $\partial b_{t+1} / \partial w_t > 0$.

tax rate if the trust fund holds no equities ($\phi = 0$): $\tilde{\tau}^{SS}_{t+1} > \tau^{SS}_t$. But if the trust fund holds equities *and* if equities at time t+1 pay a return exceeding the risk-free rate ($e_{t+1} > r_{t+1}$) as expected, then $S_{t+1} < 0$ and so $\tau^{SS}_{t+1} < \tilde{\tau}^{SS}_{t+1}$, i.e., payroll taxes won't have to be raised as much.[13] If, however, $e_{t+1} < r_{t+1}$ then $\tau^{SS}_{t+1} > \tilde{\tau}^{SS}_{t+1}$, i.e., payroll taxes will be even higher. Hence, investment in equities is a gamble, but it also shares risk across generations – a potential efficiency gain investigated in earlier papers.

First-Order Conditions

The first-order conditions for the demand for bonds and equities for given tax parameters are,

(7) $$\beta E\left[\frac{u'(c_{2,t+1})}{u'(c_{1,t})}\left(1+r_{t+1}\right)\right] = 1 \quad ,$$

and,

(8) $$\beta E\left\{\frac{u'(c_{2,t+1})}{u'(c_{1,t})}\left[1+e_{t+1}-(e_{t+1}-r_{t+1})\tau^{K}_{t+1}\right]\right\} = \beta E\left[\frac{u'(c_{2,t+1})}{u'(c_{1,t})}\left(1+e_{t+1}\right)\right] = 1.$$

Equation (7) is the standard "intertemporal" condition governing resource allocation over time. Equation (8) is the "portfolio" condition governing the allocation of saving between bonds and equities.[14] The second equality in equation (8) follows after some algebra and using condition (7). The capital income tax rate falls out of equation (8) due to tax symmetry around the risk-free rate.

Production

Net output at time t takes the Cobb-Douglas form and is produced using capital, K, and labor, L, and is also determined by the economy's level of productivity, A, and the depreciation rate, δ:

(9) $$f\left(k_t\right) = A_t k_t^{\alpha} - \delta_t k_t$$

where $k_t \equiv K_t / L_t$. Both A and δ are stochastic to allow for an imperfect correlation between wage

---

[13] The model specification is flexible enough to allow for any tax collected at time t+1 to be changed. However, as explained below, this paper focuses on changes in payroll tax rates in order to be consistent with most plans.

[14] Both equations must, in general, be solved simultaneously for s^B and s^K, except in special cases.

and capital returns, as in Bohn (1999). Let $A_t = (1+a_t)A_{t-1}$ where $a \sim F(\lambda, \underline{a}, \overline{a}, \ldots)$, $\underline{a} < a < \overline{a}$, with mean λ. $\delta = \hat{\delta} + \Delta$ where $\hat{\delta}$ is a constant and Δ is i.i.d. with mean zero.

Stochastic factor prices for wages and the net return to risky capital are neoclassic,

(10) $$w_t = A_t(1-\alpha)k_t^{\alpha}$$

(11) $$e_t = A_t \alpha k_t^{\alpha-1} - \delta_t$$

The neoclassical specification implies that the conditional equity return distribution, $e \sim \Xi(\lambda, k, \ldots)$, as well as the risk-free rate, must be solved jointly with the saving and portfolio decisions. The inequality, $-1 < a$, guarantees positive productivity. This Cobb-Douglas specification, although used in the numerical results below, is not necessary for the analytical results. The main requirement is that capital is not stochastically dominated by bonds in equilibrium and, therefore, some capital is held. For CD technology, $e \to \infty$ as $k \to 0$. Hence, some capital must be held in equilibrium.

Rest of Government

Government debt as a fraction of the capital stock evolves as follows:

(12) $$\frac{D_{t+2}}{K_{t+2}} = \frac{\left\{G_{t+1} + S_{t+1} - \left[\tau^W_{t+1} L_{t+1} w_{t+1} + \tau^K_{t+1} \cdot \left(e_{t+1} - r_{t+1}\right) \cdot L_t s^K_t\right]\right\} + \left(1 + r_{t+1}\right) D_{t+1}}{L_{t+2} k_{t+2}} \equiv \tilde{d}_{t+2}$$

where $G_t = G_0 \cdot \left(f(k_t) \big/ f(k_0) \right)$ is non-social security government spending. Period $t = 0$ represents some fixed date, maybe the start of a policy change. Scaling government spending to net output is required to prevent the debt-capital ratio from diverging. Non-social security tax rates must also adjust to prevent the debt-capital output from diverging even at small values of government spending in the presence of outstanding debt. Without any loss in generality, we assume that tax rates adjust to maintain a constant capital-debt ratio, $\tilde{d}_t = \overline{\tilde{d}}$. This restriction is imposed only at a low generational frequency and all equivalency results hold if we allowed for a Keynesian debt policy.

Tax revenue at time $t + 1$, excluding social security contributions, therefore equals the sum of non-social security government spending, any subsidy to social security, and debt service:

$$\begin{aligned}\tau^W_{t+1} L_{t+1} w_{t+1} + \tau^K_{t+1} \cdot (e_{t+1} - r_{t+1}) L_t s^K_t &= \\ G_{t+1} + S_{t+1} + (1 + r_{t+1}) \cdot \overline{\tilde{d}} \cdot L_{t+1} k_{t+1} &- \overline{\tilde{d}} \cdot L_{t+2} k_{t+2}\end{aligned} \tag{13}$$

Equation (13) requires that wage taxes and/or capital income taxes are state contingent. The exact combination does not matter for demonstrating policy equivalency. It can be shown that trust fund investment in equities is actually *neutral* (i.e., the policies connected by Line (C) in Figures 1 and 2 are equivalent) if capital income taxes fully adjust to offset changes in the subsidy, S (just like adjusting benefits). But if capital income taxes do not fully adjust to offset changes in S, at least some of the policy risk is passed to future generations through wage taxes, generating non-neutrality.

As in Diamond (1997), Bohn (1997, 1999) and Abel (1999), it is natural, therefore, to assume that only wage taxes adjust to the trust fund's capital income shocks. This assumption increases the inter-generational risk sharing potential. This assumption is also consistent with adjusting social security payroll tax rates to shocks in the value of the trust fund. Hence, social security still appears to be a pension system, the approach advocated by many proponents of trust fund investment in equities, including six members of the 1994-1996 Social Security Advisory which formed the basis of President Clinton's proposal. Since benefits remain defined by law, one does not need to distinguish between payroll taxes and other wage taxes. The state contingent wage tax rate equals

$$\tau^W_{t+1} = \frac{G_{t+1} + S_{t+1} + (1 + r_{t+1}) \cdot \overline{\tilde{d}} \cdot L_{t+1} k_{t+1} - \overline{\tilde{d}} \cdot L_{t+2} k_{t+2} - \tau^K_{t+1} \cdot (e_{t+1} - r_{t+1}) L_t s^K_t}{L_{t+1} w_{t+1}} \tag{14}$$

where the capital income tax rate at time t+1, τ^K_{t+1}, is an exogenous policy parameter. It is shown below how this tax rate can be chosen to replicate an open-market operation by the trust fund.

Market Clearing

Market clearing requires that the capital stock held at time *t* is equal to the sum of capital held by private agents and the Social Security Administration. Similar is true regarding government debt.

(15) $$k_{t+1} = \frac{s_t^k + \phi\varphi\tau_t^{SS} w_t}{1 + n_{t+1}}$$

(16) $$D_{t+1} = L_t s_t^B + (1-\phi)\varphi\tau_t^{SS} L_t w_t$$

By Walras' Law, the goods-market condition also clears. It can be shown (á la Wang [1993]) that the model produces a globally unique and stable non-degenerate stochastic stationary equilibrium since capital saving at time *t*, s_t^K, is concave in the capital stock, k_t, conditional on $a_{t+1} \in \{\underline{a}, \overline{a}\}$.

IV. Equivalence Between Trust Fund Investment and Capital Income Taxes

We start with the following lemma.

Lemma 1. *Let $s_t^{K^*}$ be the per-capita level of capital saving by generation-t agents in the first period of their lives when the capital income tax they face at period two equals $\tau_{t+1}^{K^*}$. Suppose the government changes the capital income tax rate to $\tau_{t+1}^{K^{**}}$. At the pre-reform equity price distribution, $\Xi(\lambda, k_t^*, \ldots)$, and risk-free rate, r_{t+1}^*, the new desired level of capital saving equals*

(17) $$s_t^{K^{**}} = s_t^{K^*}\left[\frac{(1-\tau^{K^*})}{(1-\tau^{K^{**}})}\right]$$

and the new level of saving in the risk-free asset equals

(18) $$s_t^{B^{**}} = s_t^{B^*} - \left(s_t^{K^{**}} - s_t^{K^*}\right) .$$

The proof of Lemma 1 is similar to that derived in the previous literature and so it is relegated to the Appendix.[15] Lemma 1 is intuitive. For example, suppose the government increases the capital income tax rate from zero to 50 percent, reducing the variance in each agent's after-tax return to equities. Agents respond by "doubling their bets" in equities while decreasing their bond holdings an equal amount, allowing agents to obtain their original tradeoff between expected return and risk.

The following theorem derives the value of the capital income tax that is equivalent, in general equilibrium, to investing some or all of the social security trust fund in equities.

<u>Theorem 1.</u> *Suppose the government invests some or all of the trust fund in equities (i.e.,* $\phi > 0$*) and let* $s_t^{K^*}$ *and* w_t *equal the level of private saving and the wage rate, respectively, in the economy at time t under this policy. This policy can be replicated in general equilibrium by instead increasing the current value of the capital income tax from* $\tau_{t+1}^{K^*}$ *to the following value,*

$$\tau_{t+1}^{K^{**}} = 1 - \frac{s_t^{K^*} \cdot \left[1 - \tau_{t+1}^{K^*}\right]}{s_t^{K^*} + \phi\varphi\tau_t^{SS} w_t} \quad . \tag{19}$$

<u>Proof of Theorem 1</u>

To prove equivalency in general equilibrium, both policies must generate the same equilibrium sequence of capital holdings, bond holdings, and state-contingent wage taxes.

(i) *Capital*. If $\phi > 0$ fraction of the trust fund is invested in equities at time *t*, $k_{t+1} = \dfrac{s_t^{K^*} + \phi\varphi\tau_t^{SS} w_t}{1 + n_{t+1}}$ by equation (15). Suppose instead that $\phi = 0$ fraction of the trust fund invested is in equities, and that the capital income tax rate is raised from $\tau_{t+1}^{K^*}$ to the value shown in equation (19). Then, by equation (17), private saving in capital at time *t* increases from $s_t^{K^*}$ to

$$s_t^{K^{**}} = s_t^{K^*} + \phi\varphi\tau_t^{SS} w_t \tag{20}$$

[15] Sandmo (1977, 1985) shows that this type of result extends to an arbitrary number of risky assets. In particular, he shows that $\frac{\partial s_j}{\partial \tau_j} = \frac{s_j}{1 - \tau_j}$ and $\frac{\partial s_i}{\partial \tau_j} = 0$ ($i \neq j$), where *i* and *j* are two different assets.

Hence, $k_{t+1} = \dfrac{s_t^{K^{**}}}{1+n_{t+1}} = \dfrac{s_t^{K^*} + \phi\varphi\tau_t^{SS} w_t}{1+n_{t+1}}$ by equation (15), the same value as when $\phi > 0$.

(ii) *Bonds*. If $\phi > 0$ fraction of the trust fund is invested in equities at time *t*, $D_{t+1} = L_t s_t^{B^*} + (1-\phi)\varphi\tau_t^{SS} L_t w_t$, by equation (16). Suppose instead that $\phi = 0$ fraction of the trust fund invested is in equities, and that the capital income tax rate is raised from $\tau_{t+1}^{K^*}$ to the value shown in equation (19). Then, by equation (18), private saving in bonds at time *t* decreases from $s_t^{B^*}$ to $s_t^{B^{**}} = s_t^{B^*} - \phi\varphi\tau_t^{SS} w_t$. Hence, $D_{t+1} = L_t s_t^{B^{**}} + \varphi\tau_t^{SS} L_t w_t = L_t s_t^{B^*} + (1-\phi)\varphi\tau_t^{SS} L_t w_t$.

(iii) *State-contingent wage taxes*. If $\phi > 0$ fraction of the trust fund is invested in equities at time *t*, equations (6) and (14) imply

(21) $$\tau_{t+1}^W = \frac{G_{t+1} + S_{t+1} + (1+r_{t+1})\cdot \bar{\tilde{d}} \cdot L_{t+1}k_{t+1} - \bar{\tilde{d}} \cdot L_{t+2}k_{t+2} - \tau_{t+1}^{K^*}\cdot(e_{t+1}-r_{t+1})L_t s_t^{K^*}}{L_{t+1}w_{t+1}}$$

where

(22) $$S_{t+1} = -\varphi\phi\tau_t^{SS} L_t w_t \left(e_{t+1} - r_{t+1}\right) \quad .$$

If instead $\phi = 0$ fraction of the trust fund invested is in equities, and that the capital income tax rate is raised from $\tau_{t+1}^{K^*}$ to the value shown in equation (19). Then $S_{t+1} = 0$ and equation (14) gives,

(23)

$$\tau_{t+1}^W = \frac{G_{t+1} + (1+r_{t+1})\cdot \bar{\tilde{d}} \cdot L_{t+1}k_{t+1} - \bar{\tilde{d}} \cdot L_{t+2}k_{t+2} - \tau_{t+1}^{K^{**}}\cdot(e_{t+1}-r_{t+1})L_t s_t^{K^{**}}}{L_{t+1}w_{t+1}}$$

$$= \frac{G_{t+1} + (1+r_{t+1})\cdot \bar{\tilde{d}} \cdot L_{t+1}k_{t+1} - \bar{\tilde{d}} \cdot L_{t+2}k_{t+2} - \left[1 - \dfrac{s_t^{K^*}\cdot\left(1-\tau_{t+1}^{K^*}\right)}{s_t^{K^*} + \phi\varphi\tau_t^{SS} w_t}\right]\cdot(e_{t+1}-r_{t+1})L_t\left(s_t^{K^*} + \phi\varphi\tau_t^{SS} w_t\right)}{L_{t+1}w_{t+1}}$$

$$= \frac{G_{t+1} + S_{t+1} + (1+r_{t+1})\cdot \bar{\tilde{d}} \cdot L_{t+1}k_{t+1} - \bar{\tilde{d}} \cdot L_{t+2}k_{t+2} - \tau_{t+1}^{K^*}\cdot(e_{t+1}-r_{t+1})L_t s_t^{K^*}}{L_{t+1}w_{t+1}}$$

The second equality in equation (23) stems from substituting equations (19) and (20) into the first equality in (23). The third equality in equation (23) comes re-arranging the second equality and using equation (20). Notice that the third equality is the same expression as in equation (21). Q.E.D.

Understanding Theorem 1

The equivalency result shown in Theorem 1 demonstrates that a simple tax on capital income can replicate all state-contingent market quantities, pre-tax prices and wage tax rates stemming from investing the trust fund in equities. Only the after-tax return to capital is different.

Notice that policy equivalency holds even if the initial tax rate on capital, $\tau_{t+1}^{K^*}$, is not zero. This result is interesting because the size of the private portfolio shift from bonds to equities, required to replicate investing the trust fund in equities, is *independent* of the initial tax rate on capital income, $\tau_{t+1}^{K^*}$.[16] But, in order to equalize the state-contingent wage tax rates between the two policies, the change in the capital income tax rate, $\tau_{t+1}^{K^{**}} - \tau_{t+1}^{K^*}$, must be *decreasing* in the initial capital income tax rate, $\tau_{t+1}^{K^*}$, since extra revenue is collected at $\tau_{t+1}^{K^*}$ after agents hold more equities.[17] This extra revenue is, in addition, to that collected by the tax change itself, $\tau_{t+1}^{K^{**}} - \tau_{t+1}^{K^*}$. So the question is, how can a single change in the capital income tax rate simultaneously generate the same total demand for stocks and bonds as the trust fund's open market operation, *and* generate the same state-contingent wage tax rates? The answer is subtle: an agent's portfolio response to a change in the capital income tax rate is also more *sensitive* at a higher initial tax rate.[18] Hence, at a higher initial capital income tax rate, a smaller change in the tax rate is required to both equalize the state-contingent wage tax rates and to generate the same given portfolio shift by private agents.

To show this result formally, consider the capital income tax revenue at time t+1,

$$rev_{t+1}^{K}\left(e_{t+1}\right) = \tau_{t+1}^{K} s_t^{K} \cdot \left(e_{t+1} - r_{t+1}\right),$$

---

[16] I.e., each agent must increase his demand for capital saving (and decrease his demand for bonds) by the amount of his payroll tax deposited into the trust fund, $\phi\varphi\tau_t^{ss} w_t$, which is independent the initial capital income tax rate.

[17] Indeed, it is easy to verify that $\partial^2 \tau_{t+1}^{K^{**}} \big/ \partial\left(\phi\varphi\tau_t^{ss} w_t\right)\partial\tau_{t+1}^{K^*} < 0$.

[18] Note that $ds_t^K \big/ d\tau_{t+1}^K = \dfrac{s_t^K}{1-\tau_{t+1}^K} \Rightarrow ds_t^K = \dfrac{s_t^K}{1-\tau_{t+1}^K} d\tau_{t+1}^K$, i.e., the saving response is increasing in τ_{t+1}^K.

which is conditional on equity returns. Differentiate by the capital income tax rate to get:

$$\frac{d\left[rev_{t+1}^K\left(e_{t+1}\right)\right]}{d\tau_{t+1}^K} = \overbrace{s_t^K \cdot \left(e_{t+1} - r_{t+1}\right)}^{\text{change-in-tax-rate effect}} + \overbrace{\tau_{t+1}^K \frac{ds_t^K}{d\tau_{t+1}^K} \cdot \left(e_{t+1} - r_{t+1}\right)}^{\text{initial-tax-rate effect}}$$

$$= \left[s_t^K + \tau_{t+1}^K \cdot \frac{s_t^K}{1-\tau_{t+1}^K}\right] \cdot \left(e_{t+1} - r_{t+1}\right)$$

$$= \left[\frac{s_t^K}{1-\tau_{t+1}^K}\right] \cdot \left(e_{t+1} - r_{t+1}\right) = \frac{ds_t^K}{d\tau_{t+1}^K} \cdot \left(e_{t+1} - r_{t+1}\right)$$

where we have used the fact, $ds_t^K \big/ d\tau_{t+1}^K = \frac{1}{1-\tau_{t+1}^K}$. The increase in the revenue is decomposed into two parts: the increase in revenue coming from the change in the tax rate itself, as well as the increase in revenue stemming from the fact that the initial tax rate collects more revenue when agents save more. After some algebraic reduction, it is shown that the net responsiveness of the revenue to the capital income tax rate is directly proportional to the responsiveness of private saving to the tax rate. In other words, the revenue and portfolio responses are co-monotonic in the capital income tax rate.[19]

Finally, equivalency holds even if the original capital income tax does not take the same form as the new tax. In this case, the new capital income tax, which is equivalent to investing the trust fund in equities, is levied on the difference between the *original after-tax* return to equities and the risk-free rate. See the Appendix for the most general construction of the equivalent tax rate, which nests equation (19) for the special case in which the original tax takes the same form as the new tax.

V. Numerical Calculations

This section reports the capital income tax rate that is equivalent to investing the entire US trust fund in equities. *If* the social security trust fund was already invested in equities, equation (19)

---

[19] To put it yet another way, notice that $d\left[rev_{t+1}^K\left(e_{t+1}\right)\right] = ds_t^K \cdot \left(e_{t+1} - r_{t+1}\right)$. This revenue change equals the extra revenue received in the state e_{t+1} if the government instead invested an amount ds_t^K of the *trust fund* in equities.

gives the policy-equivalent tax rate. But, the US trust fund currently holds only bonds, and observable variables reflect this policy. Hence, if we want to calibrate the model to observable variables, computing an equivalent long-run tax rate requires us to also consider the non-neutral effects from going from no investment by the trust fund in equities to full investment (Line (C) in Figure 1). To be thorough, I report the equivalent tax rate in each period along the mean transition path from the pre-reform stochastic steady state to the post-reform stochastic steady state. As explained below, the tax rate along the mean transition path is always very close to its final stochastic steady state value, even though the rest of the economy takes longer to converge.

Solving the Model

Several variables must be determined simultaneously in general equilibrium. The equity return distribution, $\Xi(\lambda, k, ...)$, and the risk-free rate must be determined jointly with the saving and portfolio decisions of agents in order to satisfy conditions, (15) and (16). The state-contingent tax wage tax rate, (14), must also be part of this equation set since the level of debt passed to future generations depends on the amount of private capital saving. The system of equations must also include the return to the pay-as-you portion of social security since benefit payments are stochastic because they are indexed to random wage growth (equation (4)), which, in turn, is dependent on the amount of endogenous capital saving. The rate of return to the funded portion of social security must also be included in this equation set because its return is a linear combination of bond and stock returns, both of which also depend on the amount of endogenous capital saving.

The numerical solution to the model herein is further complicated by the fact the model allows for shocks to both productivity and depreciation in order to allow for an imperfect correlation between wage and capital returns. Since social security benefits are waged indexed, agents are exposed to productivity shocks through their pay-as-you-go social security benefits. That exposure affects their portfolio demand and, in particular, tends to reduce their demand for stocks.

The model is algebraically reduced and solved using a multi-variate Newton method. If more periods of life were added to the model, this technique could not be used and an exact solution would not be possible. The previous papers on trust fund investment in equities referenced earlier also used two-period models. To be sure, life-cycle models with many periods of life seem common these days. But most of those models only include idiosyncratic risk, and so future prices are perfectly predictable. As a result, bonds and stocks are perfect substitutes and so the equity premium is zero: the trust fund's portfolio is irrelevant. Some recent life-cycle portfolio choice models allow for random prices, but assets are not aggregated.[20] Aggregation is required herein in order to incorporate the general-equilibrium effects associated with a non-neutral fiscal policy change. Adding more periods to the model herein would soon run into a "curse of dimensionality" since aggregation would be required over a very large state space. Recent ad-hoc approximation methods allow for more periods, but cannot generate a realistic equity premium. To the best of my knowledge, the model herein is first to exactly solve a stochastic OLG life-cycle model with a fully endogenous equity return distribution. The other endogenous factors mentioned above add to the model's richness.

Calibration

Utility takes the constant relative risk averse form, $E_t U_t = \frac{1}{1-\gamma}\left[c_{1,t}^{1-\gamma} + \beta E_t(c_{2,t+1}^{1-\gamma})\right]$, where γ is the coefficient of relative risk aversion and β = 1/(1+ρ), where ρ is the time preference. Productivity is a two-state Markov process, $A_t = A_{t-1} \cdot (1+\lambda) \cdot (1+a_t)$, where λ is trend growth and a_t is a mean-zero stochastic shock, $a_t \in \{\chi, -\chi\}$, that can take the values χ and (-χ) with equal probability, $\chi < 1$. Depreciation is stochastic, $\delta = \hat{\delta} + \Delta$, with $\Delta \in \{\varepsilon, -\varepsilon\}$. Labor supply growth

[20] Instead, those models simply look at portfolio choice over the life-cycle for a given set of fixed price moments. Those models are not used to look at fiscal policy changes which would induce g.e. effects, as herein.

is constant, $n_t = n$. As in previous models, each period represents 30 years, or about one generation.

Calibrating the model requires choosing the parameter vector $\{k_0, A_0, \delta_0, \lambda, \gamma, \beta, \chi, \varepsilon, n, \varphi\}$ to match various baseline economic relationships shown in Table 1. The process entails "inverting" the above system of equations to express these parameters as a function of observable economic variables. The resulting parameter vector is unique. Additional details are provided in the Appendix.

Simulation Results

Table 2 reports the capital income tax rates for the next five generations (representing 150 years) that replicates investing the entire US Social Security trust fund in equities ($\phi = 1$). Table 2 also reports the impact of trust fund investment in equities on macroeconomic variables during the transition from the initial (pre-reform) stochastic steady state to the final stochastic steady state. State variables are updated between generations conditional on productivity and depreciation shocks taking their mean values ex post.[21]

Notice that trust fund investment can be replicated by increasing the capital income tax rate from its current value of 20 percent to 24 percent over the long run. The new tax rate incorporates the effects that the policy change has on the risk-free rate and the equity price distribution. The shift in the trust fund's portfolio toward more equities and less bonds increases the long-run annual risk-free rate by 20 basis points. As explained in Bohn (1997) and Abel (1999), this increase is required in order to entice private agents to hold bonds released by the trust fund. The annual equity premium falls by 30 basis points as the total (public plus private) demand for equity capital increases. As emphasized recently by Abel (1999), the negative subsidy S along the ex-post mean growth path causes a decline in wage tax rates. In the model herein, wages tax rates eventually decline by 1.2

[21] Each possible path of future shocks generates different equivalent tax rate paths. The earlier formulae and the equivalency of both policy reforms hold for all possible paths.

percentage points, which leads to a 2.9 percent increase in future after-tax wages.

VI. Endogenous Heterogeneity in Portfolio Choice and Saving

Almost half of US households are not exposed to stocks, either through retirement plans or other forms of saving. Most on these same households appear to hold very little non-housing wealth as well. This section introduces endogenous borrowing constraints by enforcing the legal restriction that prohibits borrowing against future social security benefits.[22] Two types of agents are considered: type-*L* agents with low first-period wages and type-*H* agents with high first-period wages. Type-*L* agents may endogenously become borrowing constrained in equilibrium. The equivalence between trust fund investment in equities and capital income taxation, though, still holds. Simulation results are also reported for this model.

<u>Extending the Model</u>

Consider L_t^L number of type-*L* laborers with low wages and L_t^H number of type-*H* laborers with high wages. As before, L_t is the total size of the labor force at time *t*, $L_t = L_t^L + L_t^H$. The model is extended to include agent heterogeneity by re-defining per-capita variables as *average* per-capita variables and then stating how average amounts are divided between different agents.

Let $\zeta = L_t^L / L_t$ represent type-*L*'s share of the labor force at time *t*. Hence, $(1-\zeta) = L_t^H / L_t$. The per-capita wage of agent type $i \in \{L, H\}$ is $w_t^i = \xi^i w_t$. The variable ξ^i represents agent *i*'s productivity relative to the average wage, $w_t = \zeta \cdot w_t^L + (1-\zeta) w_t^H$, which is given by equation (10). So then $w_t = \zeta \cdot \xi^L w_t + (1-\zeta) \cdot \xi^H w_t$, which implies $1 = \zeta \cdot \xi^L + (1-\zeta) \cdot \xi^H$, or,

(24) $$\xi^H = \frac{1-\zeta \cdot \xi^L}{1-\zeta} \quad .$$

---

[22] In the United States and some other countries, it is illegal to use social security benefits as collateral.

Notice that $\xi^H = \xi^L \Leftrightarrow \xi^L = 1$ (wage equality) and $\xi^H > \xi^L \Leftrightarrow \xi^L < 1$ (inequality).

Wage tax rates are allowed to be progressive. Denote the state-contingent wage tax rate of agent *i* as $\tau_t^{w^i} = \upsilon^i \tau_t^w$. The parameter υ^i represents agent *i*'s tax rate relative to the average wage tax rate, $\tau_t^w = \zeta \cdot \tau_t^{w^L} + (1-\zeta)\tau_t^{w^H}$, given by equation (14). It follows that,

$$\upsilon^H = \frac{1-\zeta \cdot \upsilon^L}{1-\zeta} \quad . \tag{25}$$

Note that $\upsilon^H = \upsilon^L \Leftrightarrow \upsilon^L = 1$ (linear tax rates) and $\upsilon^H > \upsilon^L \Leftrightarrow \upsilon^L < 1$ (progressive tax rates).

Agent *i*'s social security benefit equals $b_t^i = \eta^i b_t$. The variable η^i is agent *i*'s social security benefit relative to the average benefit, $b_t = \zeta \cdot b_t^L + (1-\zeta) b_t^H$, given by equation (4). Hence,

$$\eta^H = \frac{1-\zeta \cdot \eta^L}{1-\zeta} \quad . \tag{26}$$

The benefit received by agent *i* is proportional to previous payroll taxes paid by agent *i* if $\eta^i = \xi^i$. In this case, each agent receives the same replacement rate on their first-period pre-tax wages. Social security benefits are progressive if $\eta^H < \xi^H$, which by equations (26) and (24), implies $\eta^L > \xi^L$. In this case, type-*H* agents receive a replacement rate on their wages below type-*L*'s replacement rate.

The agent's optimization problem, with the agent index $i \in \{L, H\}$, now becomes,

$$\max_{s_t^{B^i}, s_t^{K^i}} E_t U\left(c_{1,t}^i, c_{2,t+1}^i\right) = u(c_{1,t}^i) + \beta E_t u(c_{2,t+1}^i) \tag{27}$$

subject to the following budget constraints,

$$c_{1,t}^i + s_t^{K^i} + s_t^{B^i} = \xi^i w_t (1 - \tau_t^{SS} - \upsilon^i \tau_t^W) \tag{28}$$

$$c_{2,t+1}^i = s_t^{K^i} \cdot \left[1 + e_{t+1} - (e_{t+1} - r_{t+1})\tau_{t+1}^K\right] + s_t^{B^i} \cdot (1 + r_{t+1}) + \eta^i b_{t+1} \tag{29}$$

where

$$s_t^{K^i} + s_t^{B^i} \geq 0 \quad .^{23} \tag{30}$$

Equation (30) enforces the legal restriction prohibiting borrowing against second-period social

[23] The capital income tax is modeled as proportional since the borrowing constrained don't face it anyway.

security benefits. The first-order conditions are

$$\beta E\left[\frac{u'(c^i_{2,t+1})}{u'(c^i_{1,t})}\left(1+r_{t+1}\right)\right]=\beta E\left[\frac{u'(c^i_{2,t+1})}{u'(c^i_{1,t})}\left(1+e_{t+1}\right)\right]=\tilde{\mu}^i_t \tag{31}$$

where $\tilde{\mu}^i_t \equiv 1-\mu^i \big/ u\left(c^i_{1,t}\right)$ and μ^i is the Lagrangian multiplier for restriction (30). $\tilde{\mu}^i<1$ if equation (30) binds for agent *i*; otherwise, $\tilde{\mu}^i=1$. Now interpreting s^K_t as the *average* per-capita saving in capital, $s^K_t=\zeta\cdot s^{K^L}_t+\left(1-\zeta\right)s^{K^H}_t$, and s^B_t as the *average* per-capita saving in bonds, the rest of the formulae shown in Section II, for the government sector and market clearing, remain the same.

Lemma 2. *Let* $\Omega=\left\{\left(\xi^L,\upsilon^L,\eta^L\right)\middle|\xi^L<1,\upsilon^L\leq 1,\eta^L\geq\xi^L\right\}$ *be the set of parameter tuples* $\left(\xi^L,\upsilon^L,\eta^L\right)$ *generating wage inequality along with non-regressive wage taxes and non-regressive social security benefits. Then, assuming homothetic preferences:*

(i) *Let* $\tilde{\Omega}_t=\left\{\sigma\in\Omega\middle|\tilde{\mu}^L_t<1\right\}$ *be the subset of* Ω *where, for each parameter tuple, the type-L agent is endogenously borrowing constrained at time t. Then* $\#\tilde{\Omega}_t>0$ *(i.e., the subset is not empty) under the Inada condition* $\lim_{c\to 0}\partial u(c)/\partial c=\infty$ *and with positive productivity,* $A>0$.

(ii) $\tilde{\mu}^H\left(\sigma\right)=1\,\forall\,\sigma\in\Omega$ *when* $\phi=0$ *, i.e., the type-H agent is not endogenously borrowing constrained for any parameter vector in* Ω *before the trust fund invests in equities .*

Proof. [Part i] Note that for any value $\eta^L\geq 1$, $\xi^L\to 0\Rightarrow c^L_{1,t}\to 0$ under the borrowing constraint shown in equation (27) while $c^L_{2,t+1}$ is strictly bounded above zero by the budget constraint (29). Hence, $\tilde{\mu}^L<1$ by equation (28). [Part ii] Consider case in which $\sigma\in\tilde{\Omega}_t$. Suppose $\tilde{\mu}^H\left(\sigma\right)<1$ and $\tilde{\mu}^L\left(\sigma\right)<1$. Then $\phi=0\Rightarrow k=0$ by equation (15) $\Rightarrow e\to\infty$ by equation (11), contradicting $\tilde{\mu}^i<1$ by equation (31). Now consider the case in which $\sigma\in\tilde{\Omega}^C_t$. Then $\tilde{\mu}^H\left(\sigma\right)<1\Rightarrow\tilde{\mu}^L\left(\sigma\right)<1\,\forall\,\sigma\in\Omega$ for homothetic preferences, contradicting $\sigma\in\tilde{\Omega}^C_t$. Q.E.D.

It follows that type-*L* agents are borrowing constrained if the parameter vector, $\sigma\in\Omega$, is in the non-empty set $\tilde{\Omega}_t$ at time *t*, before social security invests in equities (part i). But type-*H* agents will not be borrowing constrained (part ii). We now arrive at the following key theorem:

Theorem 2. *Let* $s_t^{K^*} = \zeta \cdot s_t^{K^{L^*}} + (1-\zeta) s_t^{K^{H^*}}$ *equal the average per-capita level of capital saving and let* $w_t^* = \zeta \cdot w_t^{L^*} + (1-\zeta) w_t^{H^*}$ *equal the average wage rate at time t after the trust fund is invested in equities. If type-L agents are endogenously borrowing constrained (* $\sigma \in \tilde{\Omega}_t$ *) or not (* $\sigma \in \tilde{\Omega}_t^C$ *), the trust fund policy to invest in equities can be replicated by instead increasing the value of the capital income tax from its current value to the value shown in equation (19) of Theorem 1.*

Discussion

The proof for Theorem 2 is same as that for Theorem 1 along with our interpretation of relevant lowercase variables as representing their per-capita average values. For a choice of the parameter vector $\sigma \in \tilde{\Omega}_t^C$, neither agent is borrowing constrained and so nothing of substance changes relative to Theorem 1. For $\sigma \in \tilde{\Omega}_t$, however, type-*L* agents are borrowing constrained before the trust fund is invested in equities and type-*H* agents are not constrained. Yet policy equivalency holds.

It is surprising that policy equivalency (but not neutrality) holds even when type-*L* agents are constrained. Under the centralized policy to invest the trust fund in equities, some payroll tax revenue coming from type-*L* agents is invested into equities. However, under the equivalent decentralized capital income tax approach, type-*L* agents are constrained and have no private investment income. A capital income tax, therefore, cannot be used to shift their own portfolio toward more equities.

To understand how policy equivalency holds even when type-*L* agents hold no private assets, consider first the equivalency of asset markets under both policies. In order to generate identical total demands for stocks and bonds, the policy-equivalent capital income tax has to be chosen to get type-*H* agents *alone* to clear the asset markets. Since type-*H* agents are being asked to carry the entire weight, it might seem, at first, that the equivalent capital income tax shown in Theorem 1 must be a function of the fraction of type-*H* agents in the economy, $(1-\zeta)$. However, that is not the case since the per-capita variables in Theorem 1 are, by definition, averaged across both types of agents.

Consider the following example. Suppose that the initial capital income tax rate is zero, type-*L* agents are borrowing constrained, and the aggregate holding of equities by type-*H* agents is $100. The government decides that it wants to invest the trust fund in equities by selling $100 worth of bonds and then buy $100 of equities. A capital income tax of 50 percent could replicate this open-market operation in a decentralized fashion: by Lemma 1, type-*H* agents would double their equity holdings to $200 and sell $100 worth of debt. Notice that the share of type-*H* agents, $(1-\zeta)$, is irrelevant for equivalency. To be sure, if the value of $(1-\zeta)$ is lower, each type-H agent holds more wealth for a given aggregate value of wealth (or, equivalently, for a given cross-sectional average). But only the aggregate value of wealth matters for computing the capital income tax rate that generates the same total demand for stocks and bonds under both fiscal policies.

Now let's discuss the equivalency of the wage tax policy functions under both fiscal policies. Although constrained type-*L* agents hold no wealth, they do pay taxes on their first-period wage income. The two fiscal policies are equivalent because, for each agent, the wage tax rates are the same under both policies in each state of the world. Returning to last example, under the government's budget constraint (14), the same amount of risk is passed to the wage base whether the trust fund holds $100 more of equities or whether type-*H* agents holds it. Of course, exactly how that wage base risk is distributed between both types of agents depends on the parameter υ^L, which determines the progressivity of the wage tax rates. But provided that υ^L is not changed between the experiments, the state-contingent progressivity of wage taxes are the same under both policies. In other words, the wage taxes faced by type-*L* agents might differ from type-*H* agents in every state, but those differences are the same under both fiscal policies.

In sum, increasing the capital income tax generates the same pre-tax and post-tax income and wealth distributions as investing the trust fund in equities. These two policies are fully equivalent – and not just equivalent for an "average" agent – despite the fact that type-*L* agents don't hold assets.

Indeed, the equivalence of trust fund investment in equities and capital income taxes is quite general. It does not matter if $\sigma \in \tilde{\Omega}_t$ and $\sigma \in \tilde{\Omega}_{t+1}^C$, i.e., if type-*L* agents are borrowing constrained at time *t* and not constrained at time *t*+1. In other words, the equivalency result does not require that the economy move along some mean path, or even be inside a stochastic steady state.

Moreover, the borrowing constraint shown in equation (30) does not rule out that type-*L* agents might want to hold no equities, or even a short position in equities, along with a long position in bonds – provided that the total value of assets is non-negative. A type-*L* agent might want to hold a non-positive amount of stocks in the presence of productivity shocks which cause their wage-indexed social security benefits to be correlated with stock returns. This correlation has not been captured in previous models, which have instead relied on large ad-hoc fixed costs or equivalent mechanisms to limit market participation.[24]

Simulation Results

Type-*L* agents are assumed to compose half of the economy: ζ = ½. For most parameter vector choices in the stationary set $\tilde{\Omega}_t$ along the mean path, including type-*L* agents into the model had little impact on the numerical calculations. Intuitively, the importance of including type-*L* agents approaches zero as their wage share, ξ^L, approaches zero. To establish an upper bound on the role of type-*L* agents, the parameter ξ^L, therefore, was chosen equal to unity (i.e., equal wages between

[24] The fixed cost approach will result in different macroeconomic outcomes. The reason is that with large fixed investment and utility costs (as in Abel, 2001), trust fund investment in equities generates wealth effects. These wealth effects do not occur when agents freely choose no stock holdings due to a correlation with other retirement resources.

the agent types), the maximum value possible before the type-*L* agent switches pre-tax resource rankings with the type-*H* agent. The parameters υ^L and η^L were also set at unity, their maximum possible values before type-*L* and type-*H* agents switch rankings on a post-fisc basis (i.e., after taxes and transfers). These parameter choices maximize the relevancy of type-*L* agents but, with homothetic preferences, also imply that type-*L* agents are not borrowing constrained unless they are less patient than type-*H* agents. To force type-*L* agents to be endogenously borrowing constrained throughout the transition, their time preference is set higher than that of type-*H* agent.[25]

Calibrating to the same macro economy as before requires type-*H* agents to hold more of both assets in order for the economy to achieve the same capital-labor ratio as before and, hence, the same average per-capita saving, as averaged across both types of agents. The inclusion of borrowing constrained type-*L* agents adds to the size of the labor force without a proportional increase in the size of the capital stock, and so type-*H* agents must each save more in order to get the same capital-labor ratio as before. The re-calibrated value of β (for type-*H* agents) increased slightly while the re-calibrated value of γ decreased slightly (Appendix). Both initial economies are otherwise identical.

As shown in Table 2, the policy-equivalent capital income tax rates reported in Section IV do not change much when type-*L* agents are added to the model, although the changes in the capital stock, national income and risk-free rate are larger. The largest difference between the two model versions is in the risk-free rate which increases by 120 basis points in the long run after the trust fund is invested in equities, versus by just 17 basis points with the previous single-agent model. Let's first discuss the relative changes in the macro variables before discussing the tax rates.

The differences in changes in macroeconomic variables between the two models reflect that

---

[25] Although changing type-*L*'s time preference rate is a mechanical way of generating the maximum impact of endogenously binding borrowing constraints, this approach also has *a priori* merit. Indeed, one reason why some people might be poor is due to their relative impatience in accumulating physical capital. Social security, therefore, plays a tangible role herein by providing the poor with retirement income. I am grateful to Peter Diamond for this insight.

the fact that constrained type-*L* agents have a non-trivial wage income when $\xi^L = 1$, but they hold no capital. This equal wage assumption means that the same amount of debt is released by Social Security in both models after the trust fund is invested into equities. But, relative to the single-agent model, unconstrained type-*H* agents now compose only half of the economy. Hence, they require a larger increase in the risk-free rate in order to hold all the newly released debt. Their portfolio shift, stemming from trust fund investment, does not crowd out as much of their capital saving either, allowing for a larger net increase in aggregate capital. The capital stock is now 8½ percent larger in the long run versus just 4½ percent in the single-agent model. National income, therefore, now increases by 2½ percent instead of by 1.2 percent.

A surprise emerges from Table 2. Wage tax rates increase over time despite a negative subsidy ($S < 0$) along the mean path from investing the trust fund in equities. In contrast, wage tax rates decreased over time in the single-agent model. The reason that they now increase is due to the sharp increase in interest rates, which, in turn, increases the government's cost of debt service.

Notice that the capital income tax rates that replicate investing the trust fund in equities are almost identical in both the single-agent model and heterogenous-agent model. This result can be understood via equation (19) which gives the policy-equivalent tax rate in both models, where the wage and saving variables are interpreted as cross-sectional means. Rewrite equation (19) as,

(19')
$$\tau_{t+1}^{K^{**}} = 1 - \frac{s_t^{K^*} \cdot \left[1 - \tau_t^{K^*}\right]}{s_t^{K^*} + \phi\varphi\tau_t^{SS} w_t} = 1 - \frac{1 - \tau_t^{K^*}}{1 + \phi\varphi\tau_t^{SS} \cdot \left(\frac{w_t}{s_t^{K^*}}\right)} .$$

Notice that the tax rates differ between the two models only if the ratio of the average per-capita wage divided by the average per-capita capital saving differs much between the two models. Although these variables differ *individually* a fair amount between the two models, their *ratio* is only slightly different. In fact, with Cobb-Douglas production and homothetic preferences, the only reason that the ratio differs at all between the two models is due to changes in prices, causing inter-temporal substitution. A larger interest rate in the heterogenous-agent model generates a slightly smaller wage-saving ratio which, in turn, generates a slightly smaller equivalent capital income tax rate in the heterogenous-agent model relative to the single-agent model. Also, notice that the equivalent tax rate changes very little along the transition path, despite larger changes in other variables. The reason, again, is that the wage-saving ratio is not affected much by price changes along the transition path.

VII. Conclusions

This paper proves that the policy of investing some or all of a social security trust fund in equities can be replicated with a simple linear tax on risky capital income. This tax entices private agents to shift their equity-bond portfolios so that the total (public plus private) demand for equities and bonds are the same under both policies. The policy *equivalence* is very general. It holds in the presence of pre-existing taxes and market distortions, including (i) a non-zero initial capital income tax rate (whether or not the initial tax takes the same form as the new tax); (ii) missing markets between generations; and (iii) borrowing constraints which may be endogenously binding for an arbitrarily large share of the population, short of everyone. To the extent that investing the trust fund in equities improves risk sharing in the presence of missing and incomplete markets, the equivalent tax rate can be interpreted as a Lindahl tax. This tax generates a decentralized way of achieving the same potential risk sharing opportunities as the command economy approach. The tax approach to improving market efficiency might be more palatable to those who fear direct government ownership of assets. Future papers can examine the political economy aspects of taxation versus direct government asset ownership.

References

Abel, Andrew. "The Social Security Trust Fund, the Riskless Interest Rate, and Capital Accumulation." NBER Working Paper #6991, 1999. Forthcoming in Risk Aspects of Investment-Based Social Security Reform, Campbell, John and Martin Feldstein, eds., Chicago: University of Chicago Press.

Abel, Andrew. "The Effects of Investing Social Security Funds in the Stock Market When Fixed Costs Prevent Some Households from Holding Stocks." American Economic Review, 91, 1, March 2001: 128-148.

Auerbach, Alan J., "Tax Reform, Capital Accumulation, Efficiency, and Growth," in Henry J. Aaron and William G. Gale, eds., Economic Effects of Fundamental Tax Reform, The Brookings Institution: Washington, D.C., 1996, 29-81.

Auerbach, Alan J.,Laurence Kotlikoff, Jagadeesh Gokhale. "Generational Accounting: A Meaningful Way to Evaluate Fiscal Policy." *Journal of Economic Perspectives*, (8), 1 (Winter 1994): 73-94.

Barro, Robert J. "Are Government Bonds Net Wealth?," *Journal of Political Economy*, 82, 6, 1974: 1095 - 1117.

Bohn, Henning. "Social Security Reform and Financial Markets." in Sass, Steven A.; Triest, Robert K., eds., *Social Security Reform: Conference Proceedings: Links to Saving, Investment, and Growth*, 1997: 193-227.

Bohn, Henning. "Should the Social Security Trust Fund hold Equities? An Intergenerational Welfare Analysis." *Review of Economic Dynamics*, vol.2, no.3, July 1999, 666-697.

Bradford, David. "Consumption Taxes: Some Fundamental Transition Issues." NBER Working Paper No. 5290, 1995.

Congressional Budget Office. "The Economic and Budget Outlook: An Update" July 1, 1999.

Diamond, Peter. "Macroeconomic Aspects of Social Security Reform." *Brookings Papers on Economic Activity*; 0(2), 1997, pages 1-66.

Diamond, Peter and John Geanakoplos. "Social Security Investment in Equities I: Linear Case." NBER Working Paper #7103, 1999.

Domar, E. and R. Musgrave. "Proportional Income Taxation and Risk Sharing." *Quarterly Journal of Economics*, 1944, 58, 388 - 422.

Feldstein, Martin. "Social Security, Induced Retirement, and Aggregate Capital Accumulation." *Journal of Political Economy*, 85, 5, 1974: 905-926.

Geanakoplos, John, Olivia S. Mitchell and Stephen P. Zeldes. "Would a Privatized Social Security

System Really Pay a Higher Rate of Return?" Wharton FIC Paper #98-26, August, 1998.

Gordon, Roger. "Taxation of Corporate Capital Income: Tax Revenues versus Tax Distortions." *Quarterly Journal of Economics*, 1985, 100, 1 - 27.

Greenspan, Alan. "Testimony Before the Committee on the Budget, U.S. Senate." January 28, 1999.

Ibbotson, *Stocks, Bonds, Bills and Inflation – 2001 Yearbook*, Ibbotson Associates, Inc., 2001.

Inglesias, Augusto and Robert J. Palacios. "Managing Pension Reserves – Part I: Evidence from the International Experience." Pension Reform Primer Working Paper, The World Bank, 2000.

Miller, Merton and Charles Upton. Macroeconomics: A Neoclassical Introduction, First Edition, University of Chicago Press: Chicago, 1974.

Mossin, J. "Taxation and Risk-Taking: An Expected Utility Approach,"*Economica*,1968, 35, 74 - 82.

Poterba, James. "The Rate of Return to Corporate Capital and Factor Shares: New Estimates Using Revised National Income Accounts and Capital Stock Data." NBER Working Paper, 6263, 1998.

Rangel, Antonio. "Forward and Backward Intergenerational Goods: A Theory of Intergenerational Exchange." NBER Working Paper #7518, 2000.

Sandmo, Agnar. "Capital Risk, Consumption, and Portfolio Choice," *Econometrica*, 1969, 37, 586-599.

Sandmo, Agnar. "Portfolio Theory, Asset Demand and Taxation: Comparative Statics with Many Assets," *Review of Economic Studies*, 1977, 44, 369 - 379.

Sandmo, Agnar. "The Effects of Taxation on Savings and Risk Taking." in A.J. Auerbach and M. Feldstein, eds., Handbook of Public Economics, Vol I., 1985, 265 - 311.

Schieber, Sylvester and John Shoven. *The Real Deal: The History and Future of Social Security*. Yale University Press, 1999.

Social Security Advisory Council. *Findings, Recommendations, and Statements*. The United States Social Security Administration, Washington, DC, 1997.

Stiglitz, Joseph. "The Effects of Income, Wealth and Capital Gains Taxation on Risk-Taking." *Quarterly Journal of Economics*, 1969, 83, 262 - 283.

Wang, Y. "Stationary Equilibria in an Overlapping Generations Economy with Stochastic Production." *Journal of Economic Theory*, 61, 1993, 423 - 435.

White, Lawrence. 1996. "Investing the Assets of the Social Security Trust Funds in Equity Securities: An Analysis." *Investment Company Institute Perspective*, 2(4): 1-20.

Appendix

Derivation of Equation (8)

$$\beta E\left\{\frac{u'(c_{2,t+1})}{u'(c_{1,t})}\left[1+e_{t+1}-(e_{t+1}-r_{t+1})\tau_{t+1}^{K}\right]\right\}=1=\beta E\left[\frac{u'(c_{2,t+1})}{u'(c_{1,t})}(1+r_{t+1})\right] \text{ by (7) and (8)}$$

$$=> \qquad \beta E\left\{\frac{u'(c_{2,t+1})}{u'(c_{1,t})}\left[1+e_{t+1}-(1+r_{t+1})-(e_{t+1}-r_{t+1})\tau_{t+1}^{K}\right]\right\}=0$$

$$=> \qquad \beta E\left\{\frac{u'(c_{2,t+1})}{u'(c_{1,t})}\left[(1-\tau_{t+1}^{K})\cdot(e_{t+1}-r_{t+1})\right]\right\}=0$$

$$=> \qquad \beta E\left\{\frac{u'(c_{2,t+1})}{u'(c_{1,t})}\left[(1+e_{t+1})-(1+r_{t+1})\right]\right\}=0$$

$$=> \qquad \beta E\left\{\frac{u'(c_{2,t+1})}{u'(c_{1,t})}(1+e_{t+1})\right\}-1=0 \qquad \text{by (7).}$$

Proof of Lemma 1

The absence of the capital income tax in the first-order conditions implies that the first-period consumption and the ex-post second-period consumption are both invariant to the capital income tax rate, i.e., $s_t^{K^{**}} = s_t^{K^*} + \Delta$, $s_t^{B^{**}} = s_t^{B^*} - \Delta$ and $s_t^{K^*}\left[1+e_{t+1}-\left(e_{t+1}-r_{t+1}\right)\tau_{t+1}^{K^*}\right]+s_t^{B^*}\left(1+r_{t+1}\right) = s_t^{K^{**}}\left[1+e_{t+1}-\left(e_{t+1}-r_{t+1}\right)\tau_{t+1}^{K^{**}}\right]+s_t^{B^{**}}\left(1+r_{t+1}\right)$. Combining these three equations gives Lemma 1.

Proof of equivalency with a general initial tax on capital income

Let $\tilde{e}_{t+1} = e_{t+1} - z(e_{t+1})$ denote the after-tax return to equities, where $z(\cdot)$ is an arbitrary (possibly non-continuous) tax function. If, for example, $z(e_{t+1}) = \hat{\tau}^K(e_{t+1})\cdot\max(0,e_{t+1})$, for some arbitrary function $\hat{\tau}^K(\cdot)$, then the tax function is possibly non-linear and also does not allow for either symmetry or for the non-taxation of the risk-free part of risky capital income returns. With this tax in place, the consumer's budget constraints are straightforward and the FOCs become,

$$\text{(A1)} \qquad \beta E\left[\frac{u'(c_{2,t+1})}{u'(c_{1,t})}\left(1+r_{t+1}\right)\right] = \beta E\left[\frac{u'(c_{2,t+1})}{u'(c_{1,t})}\left(1+\tilde{e}_{t+1}\right)\right] = 1\,.$$

The government's budget constraint now becomes
(A2)

$$\tau_{t+1}^{W} = \frac{G_{t+1}+S_{t+1}+(1+r_{t+1})\cdot\bar{\bar{d}}\cdot L_{t+1}k_{t+1}-\bar{\bar{d}}\cdot L_{t+2}k_{t+2}-\tau_{t+1}^{K}\cdot(\tilde{e}_{t+1}-r_{t+1})L_t s_t^K-(e_{t+1}-\tilde{e}_{t+1})L_t s_t^K}{L_{t+1}w_{t+1}}$$

where τ_{t+1}^{K} is the Domar-Musgrave (DM) capital income tax that *does* allow for both symmetry and the non-taxation of the risk-free part of capital income returns. However, the DM tax is now levied over the difference between the after-*z*-tax return to equities, $\tilde{e}_{t+1}$, and the risk-free rate. Notice the addition of the rightmost term in the numerator of (A2), reflecting the tax revenue raised by the *z* tax. Since the *z* tax is allowed to be quite general, we don't explicitly need to consider a separate initial value for the DM tax, that is, $\tau_{t+1}^{K^*}$. But the notion is kept general in order to reuse earlier results.

<u>Theorem.</u> *Theorem 1 continues to hold in the presence of the capital income tax function z(·).*

<u>Proof of Theorem.</u> The sub-proofs demonstrating that the demand for capital and bonds are equal under both policy options (that is, investing the trust fund in equities versus changing the capital income tax rate to the value shown in Theorem 1) are very similar to the sub-proofs shown in Theorem 1; in both cases, the DM tax does not enter the consumer's first-order conditions. However, showing the equivalency of the government's budget constraint, (A2), requires more work.

Consider first the policy in which the government invests the fraction $\phi > 0$ of the trust fund in equities at time *t*. Then the government budget constraint becomes

(A4)

$$\tau_{t+1}^{W} = \frac{G_{t+1} + S_{t+1} + (1+r_{t+1})\cdot\tilde{\bar{d}}\cdot L_{t+1}k_{t+1} - \tilde{\bar{d}}\cdot L_{t+2}k_{t+2} - \tau_{t+1}^{K^*}\cdot(\tilde{e}_{t+1} - r_{t+1})L_t s_t^{K^*} - (e_{t+1} - \tilde{e}_{t+1})L_t s_t^{K^*}}{L_{t+1}w_{t+1}}$$

where

$$\begin{aligned} S_{t+1} &= -\varphi\phi\tau_t^{SS}L_t w_t\left(e_{t+1} - r_{t+1}\right) \\ &= -\varphi\phi\tau_t^{SS}L_t w_t\left(\tilde{e}_{t+1} - r_{t+1}\right) - \varphi\phi\tau_t^{SS}L_t w_t\left(e_{t+1} - \tilde{e}_{t+1}\right) \end{aligned}$$

Now suppose $\phi = 0$ fraction of the trust fund invested is in equities, and that the capital income tax rate is raised from $\tau_{t+1}^{K^*}$ to the value shown in equation (19). Then $S_{t+1} = 0$ and equation (A2) gives,

(A5)

$$\begin{aligned} \tau_{t+1}^{W} &= \frac{G_{t+1} + (1+r_{t+1})\cdot\tilde{\bar{d}}\cdot L_{t+1}k_{t+1} - \tilde{\bar{d}}\cdot L_{t+2}k_{t+2} - \tau_{t+1}^{K^{**}}\cdot(\tilde{e}_{t+1} - r_{t+1})L_t s_t^{K^{**}}}{L_{t+1}w_{t+1}} \\ &\qquad - \frac{(e_{t+1} - \tilde{e}_{t+1})L_t s_t^{K^{**}}}{L_{t+1}w_{t+1}} \\ &= \frac{G_{t+1} + (1+r_{t+1})\cdot\tilde{\bar{d}}\cdot L_{t+1}k_{t+1} - \tilde{\bar{d}}\cdot L_{t+2}k_{t+2} - \left[1 - \dfrac{s_t^{K^*}\cdot\left(1 - \tau_{t+1}^{K^*}\right)}{s_t^{K^*} + \phi\varphi\tau_t^{SS}w_t}\right]\cdot(\tilde{e}_{t+1} - r_{t+1})L_t\left(s_t^{K^*} + \phi\varphi\tau_t^{SS}w_t\right)}{L_{t+1}w_{t+1}} \\ &\qquad - \frac{(e_{t+1} - \tilde{e}_{t+1})L_t\cdot\left(s_t^{K^*} + \phi\varphi\tau_t^{SS}w_t\right)}{L_{t+1}w_{t+1}} \\ &= \frac{G_{t+1} + S_{t+1} + (1+r_{t+1})\cdot\tilde{\bar{d}}\cdot L_{t+1}k_{t+1} - \tilde{\bar{d}}\cdot L_{t+2}k_{t+2} - \tau_{t+1}^{K^*}\cdot(\tilde{e}_{t+1} - r_{t+1})L_t s_t^{K^*} - \left(e_{t+1} - \tilde{e}_{t+1}\right)L_t s_t^{K^*}}{L_{t+1}w_{t+1}} \end{aligned}$$

Notice that the last line in equation (A5) equals equation (A4). Q.E.D.

Calibration

The economy at time 0 to be targeted has the following characteristics, with each period representing 30 years. The expected annual depreciation equals 5 percent so that 79 percent of the capital stock is expected to be depreciated by the end of the 30-year period. The capital share, α, is set at 0.30. The arbitrary scaling parameter A_0 equals unity.

Based on Poterba (1998) and Ibbotson data, the annual pre-tax (social) real rate of return to capital is 8½ percent per year, or 1,056 percent over 30 years, with a coefficient of variation equal to 0.87. The annual risk-free real return, r_1, equals 3 percent, or 143 percent over 30 years, based on returns to long-term government securities during the last few decades. (Note: the 30-year expected return to equities is about seven times that of bonds.) The annual expected rate of technological progress is set at 3 percent per year, the average growth rate of the total salaries and wage base since 1929, based on Bureau of Economic Analysis data. The point-estimate correlation between wage and stock returns at a 30-year frequency is about three-quarters.[26] The defended debt-capital ratio, $\tilde{d}$, is set at 0.25, close to the current ratio of government debt relative to the domestically-owned capital stock as measured in the Federal Reserve Board's Flow of Funds Accounts.

The initial tax rate on the generation-0 agent's second-period capital income, τ_1^K, is 0.20, following Auerbach (1996). The initial proportional tax rate on wage income, τ_0^W, is 0.15, which generates a plausible level of tax revenue derived from wages. The workforce size relative to retirees, averaged over the past decade and over projections for the next two decades, is constant. The Social Security payroll tax is set at 12 percent and the estimated ratio of contributions to the Social Security trust fund divided by benefits paid during the past and next few decades equals about 4 percent.

The calibrating vector needed to generate this baseline economy, $\{k_0, A_0, \delta_0, \lambda, \gamma, \beta, \chi, \varepsilon, n, \varphi\}$, equals {.0056, 1.0, 0.79, 0.860, 0.857, 0.27, 0.61, 6.07, 0.0, 0.04}. The value $\beta = 0.27$ corresponds to an *annual* rate of time preference equal to 4.4 percent. The value of $\gamma = 0.857$ reflects scaling the model to equity returns (rather than consumption) as well as both human capital depreciation in the second period and the correlation of wage-indexed pay-as-you-go Social Security returns with stock returns. When the model was re-calibrated to include type-L agents, $\{\beta, \gamma\}$ changed to {0.39, 0.67}.

This calibration generates additional plausible observable economic relationships. The implied net national saving rate equals 4.4 percent. The non-Social Security part of government spending equals 15.3 percent which is very close to the value of 15½ percent that the CBO (1999) reports for 1998. Capital income tax revenue equals 4.4 percent of GDP while wage income taxes, not including Social Security payroll taxes, compose 10½ percent of GDP.

[26] To be sure, this point estimate is associated with a large standard error. However, the equivalent tax rates estimated herein were fairly robust to changes in the correlation.

Figure 1
The Distinction Between Equivalence and Neutrality:
The Case of Investing the Trust Fund in Equities

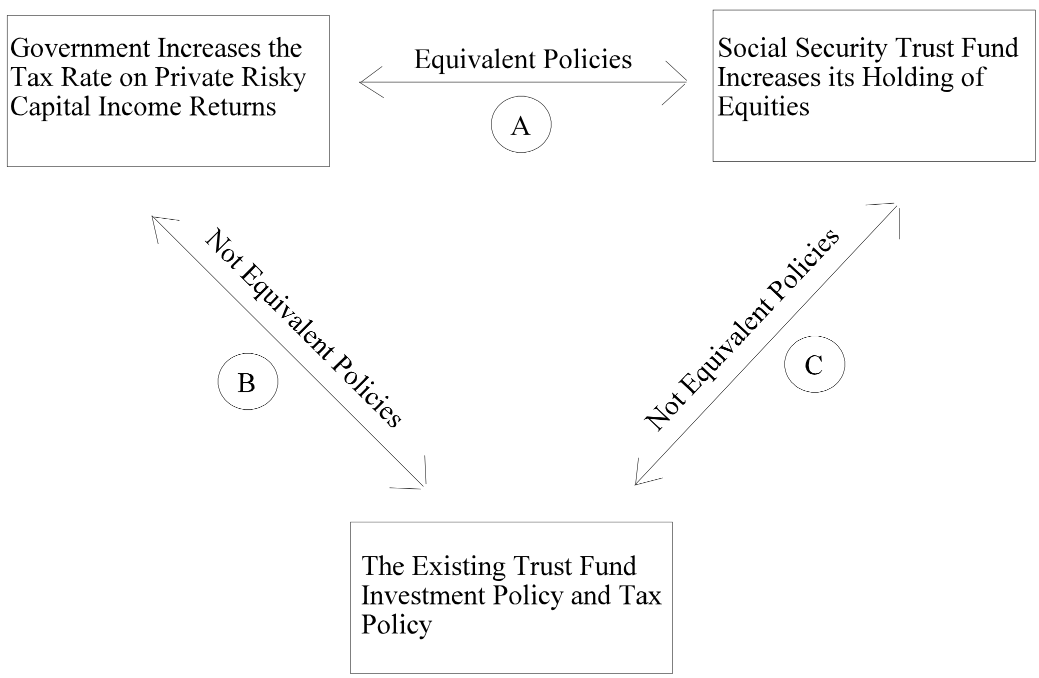

Figure 2
The Distinction Between Equivalence and Neutrality:
The Case of Divesting the Trust Fund of Equities

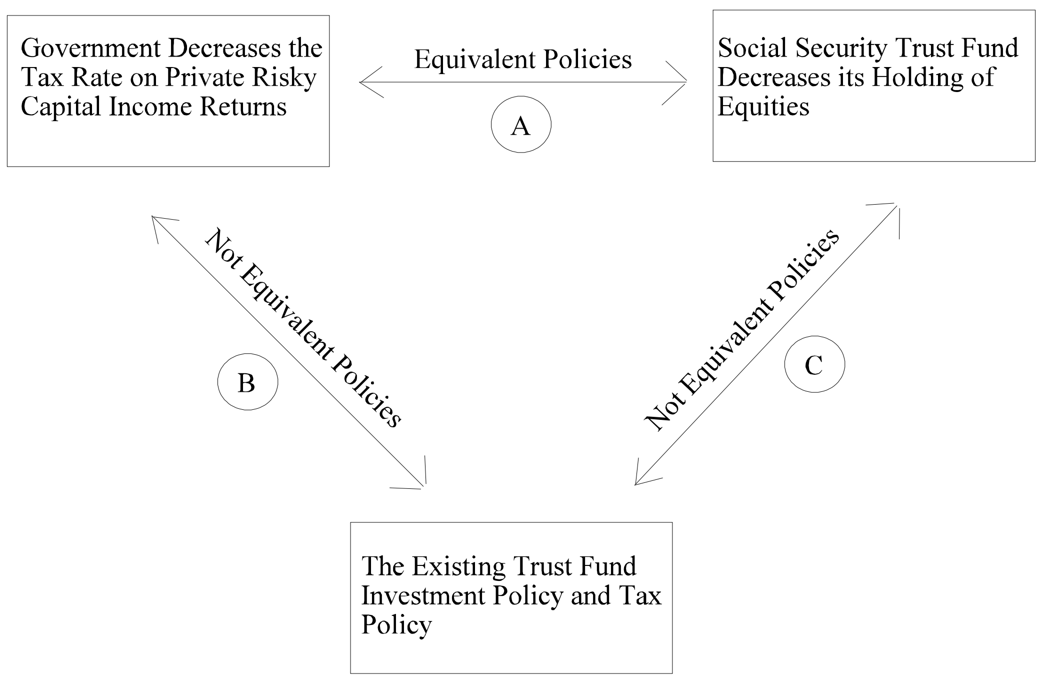

Table 1

Parameters and Implied Values along Mean Path in the Initial Stochastic Steady State
(I.e., Before the Social Security Trust Fund is Invested in Equities)

Variable Description	Value
Exogenous Parameters (same in all simulations, unless indicated otherwise)	
Average annual depreciation rate, $\bar{\hat{\delta}}_{annual}$	5 %
Capital share, α	0.30
Arbitrary Scaling of the Initial Productivity, A_0	1.00
Pre-tax 30-year return to equities on mean path, $\overline{E(e_1 \mid k_1)}$ (Corresponding annual return)	1,056 % (8.5 %)
Coefficient of Variation, $\bar{\sigma}_e / \overline{E(e_1 \mid k_1)}$	0.87
Pre-tax 30-year risk-free real return on mean path, r (Corresponding annual return)	143 % (3 %)
Rate of 30-year labor-augmenting tech. progress, λ (Corresponding annual return)	143 % (3 %)
Debt-capital ratio, $\bar{\tilde{d}}$	25 %
Tax rate on capital income, τ^K	20 %
Social Security pay-as-you-go liabilities tax rate, $\tau_{s\geq 0}^{SS,P}$	11.5 %
Social Security funded portion tax rate, $\tau_{s\geq 0}^{SS,F}$	0.5 %
Non-Social Security wage tax rate, τ^W	15 %
Implied Endogenous Variables (same in all simulations)	
Net national saving rate	4.4 %
"On Budget" Spending as a fraction of GDP, $G_0 / [A_0 k_0^{\alpha}]$	15.3 %
Capital income tax revenue as a fraction of GDP	4.8 %
Non-Social Security wage income tax revenue as a fraction of GDP	10.5 %
Exogenous Parameter (only for the benchmark)	
Correlation between capital income returns and wages	0.75

Table 2
Investing the Entire Social Security Trust Fund in Equities ($\phi = 1$) at Time 0:

Changes in Macroeconomic Variables and Equivalent Capital Income Tax Rates on the Mean Growth Path[1]

	Percent Changes				Levels (in percent)				
Generation Index[2]	Capital Stock	Pre-tax Wages	Post-tax Wages[3]	National Income	Wage Tax Rates	Risk-Free Rate (Annual)	Expected Return to Equities (Annual)	Equity Premium (Annual)[4]	Equivalent Capital Income Tax Rate[5]
Homogenous Agent Model (Section IV)									
0	0.0	0.0	0.1	0.0	14.9	3.0	8.4	5.4	24.19
1	4.2	1.2	2.3	1.2	14.3	3.5	8.4	4.9	24.23
2	4.4	1.3	2.8	1.2	13.9	3.2	8.4	5.2	24.23
3	4.4	1.3	2.9	1.2	13.8	3.2	8.4	5.2	24.23
4	4.4	1.3	2.9	1.2	13.8	3.2	8.4	5.2	24.23
5	4.4	1.3	2.9	1.2	13.8	3.2	8.4	5.2	24.23
Heterogenous Agent Model (Section V)									
0	0.0	0.0	0.3	0.0	14.8	3.0	8.3	5.3	24.01
1	8.6	2.5	0.9	2.4	16.1	4.4	8.3	3.9	24.11
2	8.7	2.5	1.3	2.4	15.9	4.3	8.3	4.0	24.12
3	8.7	2.5	1.4	2.4	15.8	4.3	8.3	4.0	24.12
4	8.6	2.5	1.5	2.4	15.7	4.2	8.3	4.1	24.12
5	8.6	2.5	1.5	2.4	15.7	4.2	8.3	4.1	24.12

Notes:
1. I.e., state variables updated between generations are conditional on all shocks (both productivity and depreciation) taking their mean values *ex post*.
2. Each generation represents 30 years. Generation 0 is the initial young at the time of the policy change. The timing is such that the policy change is announced before generation 0 optimizes. Hence, the equity premium faced by generation-0 agents immediately changes. Generation (-1) agents represent the elderly at the time of the reform whose saving and portfolio decisions and after-tax asset returns have already been determined by the time of the policy change.
3. I.e., after federal and Social Security taxes (the latter don't change for these simulations).
4. The equity premium equals 5.5 percent (annual) along the constant growth path before the policy change, reflecting a pre-reform expected return to equities of 8.5 percent (annual).
5. This is the capital income tax rate shown in Theorem 1 (homogenous-agent model) and Theorem 2 (heterogenous-agent model), applied at time $t + 1$ to generation-t's second-period capital income, that would exactly replicate investing the Social Security trust fund in equities. The tax rate was 20 percent along the mean growth path prior to the policy reform.