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Hedonic estimation of depreciation for single family homes

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Abstract

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Using a database of 160,000 single family homes sold in the Washington-Baltimore area, we estimate hedonic models of transaction prices. Surviving houses depreciate only 0.09% per year. Using methods developed by Hulten and Wykoff to correct for censoring of discarded homes, estimates of depreciation rates vary within a wide range. The Hulten-Wykoff method is modified to allow discards to be estimated along with the hedonic price model and the results are still imprecise. The depreciation rate is found to be generally less than the current BEA rate of 1.14% while the average age at discard is close to BEA's estimate of 80 years. The results suggest that the discard pattern may peak to the left of the mean rather than the symmetric pattern of the Winfrey distributions.

The Hulten-Wykoff method is then examined in a Monte Carlo study. The Hulten-Wykoff method adjusts observed prices for discarded assets, but the average age of the sample is younger than the cohort age. When depreciation is not constant by age, a younger sample will misstate depreciation rates. The extent of the bias declines with the life of the asset and so their method should produce a trivial error in measuring the depreciation rate for homes. Finally, an estimating form is developed that allows for a distribution of asset lives rather than a distribution of retirements around a single asset life.

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In 1996 the Bureau of Economic Analysis (BEA) introduced, as part of its comprehensive revisions, new measures of economic depreciation based on analysis of studies as summarized in Fraumeni (1997). In this work, Fraumeni assembled the best available empirical evidence for the depreciation rate for individual assets. Much of this evidence for equipment and nonresidential buildings comes from a series of important papers by Hulten and Wykoff (1981a and 1981b) that measure age-price profiles for specific assets. For residential housing, Hulten and Wykoff (in Wykoff and Hulten (1979) relied on estimates developed by Leigh and Weston (1980). [1]

In 1998, the Bureau of Labor Statistics (BLS) applied all of the revised depreciation rates in conjunction with concave hyperbolic decay to its measures of capital stock and capital services save one.[2] For residential structures, BEA lowered the depreciation rate from 1.28% to 1.14%. Although with a low depreciation rate a substantial portion of the original value of an investment made in 1900 survives until the end of the 20th century, the impact of this investment on the total residential structures capital stock is very small. For example, under BEA's assumption of geometric decay, a

[1] Wykoff-Hulten cites 4 estimates from these studies. The simple average of these depreciation rates is 1.28%. BEA uses the declining balance method for many assets. For structures the declining balance rate is 0.91. The combination of a 1.3% depreciation rate and a 0.91 declining balance rate corresponds to a 70 year life. Switching to an 80 year life for residential structures, the declining balance method yields a 1.14% depreciation rate (0.91/80).

[2] Since 1997 BEA has used the empirical evidence on depreciation, together with estimates of vintage investments, a perpetual inventory model, and an assumption of geometric depreciation, to prepare its wealth estimates for fixed reproducible capital. In its productivity work, BLS estimates the productive services of capital, and related to that the deterioration of capital, as distinct from the wealth and depreciation associated with capital. Hall [1968] described the relationship between deterioration and depreciation. For vintage aggregation of capital services, BLS assumes that capital deteriorates slowly

third of the value of all investment in 1900 would still be in service in 1999, yet this represents only about one-tenth of one percent of the value of total capital stock in 1999. Under BLS's assumption of concave hyperbolic decay, the remaining value would be even higher, but again the investment represents a very small percentage of total capital stock in 1999. BLS analysts were skeptical of the combination of hyperbolic decay and the slower BEA depreciation rate and they decided to retain the older depreciation rate.[3] However, BLS had no evidence to support this rate.

This paper attempts to provide additional evidence on the deprecation rate of residential structures by estimating age-price profiles for a large sample of transactions in the Washington-Baltimore SMSA. We obtained selling price net of seller subsidies from the database of the Metropolitan Regional Information System (MRIS). The MRIS is the successor to the multiple listing service used by real estate agents to list homes for sale and to record sales. In addition to net selling price, the database contains an extensive collection of descriptive and itemized traits of the houses including their age.

Previewing some of our results, we find that depreciation for single family houses is not faster than BEA's rate although our estimates are imprecise. Similarly, the average age at discard is similar to the 80-year figure used by BEA and BLS. While symmetric Winfrey distributions are commonly used to overcome a sample censoring problem in these models, we modify the age-price model to include a Weibull distribution of discards and find that it may peak to the left of the mean.

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when new and more rapidly as it ages. BLS used Hall's relationship to adhere to BEA's depreciation rates while maintaining its deterioration profiles.

[3] BLS evidence on the shape of the age-efficiency function can be found in <u>Trends in Multifactor Productivity, 1948-81</u>, pp. 41-45.

The plan of the paper is as follows. Next, we lay out the hedonic model for surviving assets and then apply the Hulten-Wykoff method to housing including some extensions to allow for discards to be estimated along with the age-price model. In section 2, we describe the MRIS data and present summary statistics for two sets of data. A larger sample of houses is used in the age-price profiles for net transaction prices. Where reasonable tax assessments are available, we estimate the value of the structure only and present sample means for this smaller sample. In section 3, we estimate age-price profiles for surviving houses. In section 4, we estimate age-price profiles using Winfrey distributions of discards and the censoring-corrected methods of Hulten-Wykoff. In section 5, age-price profiles are estimated allowing discards to be an empirically estimated Weibull distribution. Because land does not depreciate, the age-price models are estimated with and without residual value. To further investigate the role of land, the model is also estimated without the value of land included in the selling price. In section 6 we conduct Monte Carlo experiments to show when the Hulten-Wykoff method of correcting censoring yields unbiased estimates of depreciation and when it may not. In section 7, an age-price model is developed that allows for an explicit distribution of asset lives, with individual assets depreciating along their own age-price paths which also determine the age at discard. Section 8 concludes the paper with a discussion of the results and limitations of the study and points to improvements and future work.

Section 1: A hedonic model with geometric depreciation

There is a huge literature on hedonic estimation of housing prices. Basu and Thibodeau (1998) succinctly describe the pricing space for single-family homes as depending on seven classes of traits: characteristics of the lot, characteristics of the improvement (the house), neighborhood amenities, accessibility, proximity externalities, land-use regulations, and time. The first three and last classes of traits are relatively obvious, but the next three may require further explanation. Accessibility refers to distances to work, shopping and schools as well as available modes of transportation. Proximity externalities include distances to non-conforming land use such as noise from a major highway or air pollution from a factory. Finally, land use regulation includes zoning that may affect density of the development or mixed use.

Our concern is primarily on the seventh and last class of variables including the time period of the study and especially the age of the house. Equation (1) is a stylized hedonic model for housing:

$$\text{Price} = f(X, \text{Age}) + \varepsilon \tag{1}$$

Price depends on X, which represents the first 6 classes of variables, and on the age of the house. ε is an i.i.d. error term. Frequently, equation (1) is estimated in semi-log or log-log form to deal with potential problems with heteroscedasticity.

The interpretation of the estimated parameter on age is the decline in price for a house as it ages holding other traits fixed. Price declines with time because of depreciation (changes in price due to age holding time constant) and revaluation (change in price across time holding age constant). The set of characteristics X is amended to include time to control for price revaluation.

Malpezzi, Ozanne and Thibodeau (1987), surveying the literature, find that estimated depreciation rates for rental or owner-occupied housing range from 2.2% to as low as 0.4% per year. The hedonic models generally produced slower depreciation rates, in the range of 0.4% to 1.2%. Their own study of 59 SMSAs found an average depreciation rate of 0.6% for rental housing and 0.4-0.9% per year for owner-occupied housing. It should not be surprising that estimated depreciation rates in models like equation (1) are small.

Hulten-Wykoff (1981a and 1981b) realized that observed asset prices for a cohort are censored. Only those still standing have an observable market price. Yet for measuring depreciation, the entire cohort should be included in the sample. Hulten-Wykoff suggest that the scrap value of assets was approximately equal to disposal costs and so the price is zero. However the characteristics of the discarded assets are unknown. Their solution to the censored sampling problem was to assume the unobserved assets were otherwise identical to the observed assets and create a weighted-average price for the combination of observed and unobserved assets. In Hulten-Wykoff's study of commercial and industrial buildings, the observed structure prices of a given age and vintage is multiplied by the proportion of that vintage surviving to that age to obtain an effective cohort price. The difficulty is then to determine for a vintage the pattern of discards. Their solution for commercial buildings was to use a Winfrey distribution for an asset of 80 years. The Winfrey distribution reports the surviving proportion of a cohort by vintage. They found a much faster depreciation rate when censoring is accounted for.

$$\text{Price} * \text{probability(still in service)} = f(X, \text{Age}) + \varepsilon \tag{2}$$

Equation (2) is a representation of the Hulten-Wykoff method. They did not apply this model to residential structures. In the work that follows, we will apply their method to single family houses, but there are several issues with the Hulten-Wykoff method in general and in its application to residential structures. First, we don't know with much certainty the discard pattern. Since it appears that most of the depreciation comes from discarded houses rather than depreciation-in-place of surviving houses, the assumption of a specific Winfrey distribution is equivalent to assuming a depreciation rate. It would be better if the data could determine the discard pattern. Second, the Hulten-Wykoff method uses a zero price for discarded assets. While disposal costs may well equal scrap value for equipment and some structures, the price of a home is a combination of land and improvements and land retains its value. So the residual value of the property when the house is scrapped will be considerably more than zero. Typically, land comprises about 15-40% of the value of a single family home in our data. Finally, the estimation is inefficient. If 10% of all 150-year-old homes are still standing, then an observation on a 150-year-old home represents 1 surviving house and 9 discarded ones. Accordingly, the appropriate least squares estimator should weight observations by the square root of the inverse of the proportion of the cohort still remaining.

To address some of these issues, we modify equation (2) to allow the model to determine the discard pattern. Expressing the survival function as F and noting that it depends on the age of the home, observe that equation (2) in semi-log form becomes,

$$\text{Ln (Price * F(age))} = \alpha + \beta X + \delta^*\text{Age} + \varepsilon \qquad (3)$$

We assume that the probability a home survives follows a Weibull cumulative density function of the form:

$$F = \exp(-(\text{Age}/m)^{n}) \qquad (4)$$

In this specification, n determines the general shape of the function. For values less than 1, the derivative of (1-F), the discard rate, declines continuously. For values greater than 1, the discard rate rises with age, peaks and declines thereafter. Depending on the value, the curve can be skewed left or right or can be approximately symmetric. The parameter m scales the distribution. Equation (3) now becomes:

$$Ln(Price *(Exp(-(Age/m)^n) = \alpha + \beta X + \delta * Age + \varepsilon \quad (5)$$

Rearranging terms, equation (5) becomes

$$LnPrice = \alpha + \beta X + \delta * Age + (Age/m)^n + \varepsilon \quad (6)$$

Equation (6) still assumes no residual value for a discarded home. It may be more reasonable to assume that a property has positive scrap value. This is especially true for structures because land retains its value even as the structure depreciates. Homes may be retired early because the value of alternative uses (e.g. commercial use) may have risen to exceed the current value of the home or the value of the location may have risen so that the optimal size home for that location and lot size may have changed. In a second specification, we limit the Weibull c.d.f. to a fraction less than 1. This implies that when a home of a given vintage is discarded, the unobserved residual value is positive. The final parameter ρ is a measure of residual value. As age increases, the adjusted Weibull distribution approaches $\rho/(1+\rho)$ or the ratio of residual to initial value.

$$LnPrice = \alpha + \beta X + \delta * Age - \ln((\rho + (\exp(-(Age/m)^n)) /(1 + \rho)) + \varepsilon \quad (7)$$

Section 2: The Metropolitan Regional Information System Database

The Metropolitan Regional Information Service (MRIS) is the successor to the old multiple listing service. It is now a computerized database containing information on the selling price and seller subsidy, location and characteristics of the structure and land. The database contains transactions over a rolling 4-year period covering the Washington-Baltimore area (although some earlier data remain in the database). Geographic coverage extends to eastern West Virginia, southeastern Pennsylvania (excluding the Philadelphia area), northern and central Virginia and almost all of Maryland except for the Eastern Shore and Western panhandle.

We have extracted all sold homes that had a net selling price of at least $10,000 that report an age or year built and a zip code, and are sited on at least a thirtieth of an acre. Excluding co-ops and condominiums, we further limit the sample to single family homes with at least 1 bedroom and 1 full bath. Houses on more than 10 acres of land are also excluded. The usable sample contains more than 160,000 observations.

The MRIS contains extensive information on homes. Besides basic information on the number of bedrooms and bathrooms, more than 20 types of other rooms can be identified. Additional information on the style of kitchens, dining rooms, and entrance is also available. Other traits include the interior style, exterior style, roofing materials, ceiling style, heating and cooling fuels, air conditioning methods, basement type and garage spaces. Additional buildings on the property are noted as well as decking, fencing, hot tubs, swimming pools, the style of the home and whether it is attached or detached from other houses. Proximity to woods or water, public transportation and the general nature of the view from the house are classified.

Land characteristics include acreage, topography, water frontage, wooded acreage, county, state and 5-digit zip code of the property.

Real estate agents complete a computerized form to enter the information for a new listing. Some information is required while other information is optional. Furthermore, some information is calculated by the system based on other information. While there is no guarantee of accuracy, sellers are required to sign a statement verifying the accuracy of the information. Perhaps even more important, realtors can be fined for inaccurate information. Because agents regard their time as valuable, they do not wish to show a house that their client may not be interested in. As a result, there appears to be some reporting of inaccuracies by other agents and so there is some self-enforcement by agents to enter the data correctly and correct errors as soon as they are detected.

The MRIS system came on-line in 1998 and so houses sold before that date did not use the computerized form with its internal checks for consistency and reasonableness. Despite the possibility of more errors in the earlier data, we use all data from January 1994 through May 2000 in our analysis.

Some optional data are available only for a small subsample. A prime example is interior square footage. Other variables include separate tax assessments for land and improvements.

Two sets of housing prices were developed for this study. The first is the reported gross sales price less seller subsidies. This price represents the joint value of the structure and land. Seller subsidies are payments made by the seller to defray part of the cost of the purchase. Typically these include financing "points", taxes, state and county recording fees and other closing costs. Depending on how financing is arranged, closing costs can run as high as 8% in Maryland, but less in Virginia, West Virginia and the District of Columbia.

Since land does not depreciate, determining the depreciation rate for the structure alone requires an estimate of the land value. Tax assessments are available for a large subset of the sample. However, tax assessments may not accurately reflect the market value or even the relative market value of land and structure.

The second price, a structure only price, was defined as the net selling price less the assessed value of the land. The second set of prices was developed for those observations with tax assessments that approximate the net selling price. Land value is calculated as the product of the net sales price and the ratio of land to total assessed value. The sample is limited to those houses where the ratio of assessed value to net selling price ranged between 50% and 125%. Furthermore, the assessed value of land must range between 5 and 66% of the total assessed value. While these are wide ranges for assessments, 55 percent of total assessments were within 10% of net selling price and 86 percent of observations had land assessed between 15 and 40 percent of net selling price. In 4 counties, tax assessments were much lower than sales price. In order to include these observations, separate tax assessments for land and structures were scaled to match the average county net sales price. All of these restrictions remove about 40% of the sample.

Table 1 shows selected sample means for the full sample including those observations without tax assessments. The mean age of houses is 22.9, but varies from new to 300 years. Chart 1 presents the frequency distribution of sold homes by age. Several features are noteworthy. First, the number of new homes for sale exceeds 10,000, more than twice the number of any other age group. Not surprisingly, the number of very nearly new homes for sale declines sharply until age 2 and then increases until age 10. The number of homes then declines steadily until only a handful of homes (generally less than 15) of any given vintage over age 110 is for sale. While virtually invisible in the chart, there are 50 homes exactly 297 years old. Similarly, there are 10 or more homes for each age 195 to 200. These are probably misreported ages, but they did not affect the estimated parameters and so they were retained.

While home sales are reported from 4 states and the District of Columbia, the bottom of table 1 shows that nearly half of reported home sales are from just Fairfax and Montgomery counties, 2 suburbs of Washington. More than 80% of all sales are contained within 9 counties and Washington. While these are the most populous counties, home sales are still disproportionately concentrated.

The average gross selling price was $225,170 and the net selling price was $222,650. There are 2 very expensive homes (more than $5 million) in the sample and again their exclusion did not alter the estimates. Most homes had 3 or 4 bedrooms, 2 full baths and 1 half bath. Lot size averaged 0.42 acres. The dollar value of improvements is available, but since we don't know when the improvements were made nor if all agents reported improvements to the MRIS database, the variable though included in the model may not be meaningful.

About two-thirds of all houses were detached. Colonial is the predominant architectural style. Brick and/or siding are the most common materials used. Besides basic rooms, the most common additional rooms include a family room, den/library/study, kitchen level laundry room, recreation room, storage room and utility room. Homes averaged nearly 1 fireplace and 1 garage space. The foyer is the most common entrance style. Common exterior structures include decks, patios and sheds. About a third of homes are rear-fenced.

Table 2 presents a shorter list of summary statistics for the 95,000 homes that could estimate a structure-only price. The mean age of these homes is slightly older at 24.89 years because most new homes are removed. The average age of homes at least 1 year old is 24.82 years in the full sample compared to 25.06 years in the sample of houses using the structure-only price. Average gross selling price is $224,500 and the distribution of beds and baths are quite similar to the full sample. Lot sizes are slightly smaller and homes are more likely to be located in Maryland and less likely to be found in West Virginia or Pennsylvania than in the full sample. None of these differences appears significant.

Section 3: Simple OLS Estimates of Surviving Houses

We start by fitting a semi-log version of equation (1), the model that includes only surviving houses. It is important to note at the outset that geometric depreciation is imposed in all of the models. It is not that geometric is necessarily the correct description of depreciation, but our primary emphasis in this paper is to determine if BEA's best *geometric* average depreciation rate is reasonable or some other rate would be an improvement. We leave it for future work to employ a more general age-price profile.

Taking advantage of the great detail available in the database, this study creates dummy variables for the number of bedrooms, bathrooms, contract year and contract month. A large set of dummy variables is created to represent the presence of each type of other room including kitchen style, entrance style, construction materials, views, topography, other buildings, exterior features, heating and cooling systems, roofing materials, type of road fronting the property, proximity to water and public transportation, basement type, architectural style and ceiling heights. Geographic location dummies include state, county and zip code of the property.

Table 3 contains estimates for the full sample with decreasingly specific location dummies. While the estimated parameters are sensible, a few in the first column are worth mentioning. The premium for full bathrooms is consistently larger than for bedrooms. While other variables may be indicative of the size and cost of the house, bathrooms appears to be capturing some of this effect. Consistent with this theory, other rooms generally received little premium. Den/library/study, Florida room, lofts, master bedroom suite with sitting area and solariums added about 3 percent each to the price of a house. Detached houses sold for about 25% more while mobile homes sold for about 50% less. Guest houses and gazebos added about 10% to the price of the house. Each fireplace and garage space adds about 7% which seems high, but again may be indicative of the expense of materials or other unobserved features.

The parameter value for acreage is quite low. The value of land is allowed to vary by county. Washington D.C., the Northern Virginia counties of Fairfax and Arlington, and the cities of Alexandria and Falls Church received substantial premiums. Relatively affluent counties in Maryland, Montgomery, Howard and Anne Arundel received only small premiums. Zip codes, when present, also have large parameters in the traditionally exclusive and expensive neighborhoods. It appears that the model is unable to disentangle the value of land from a pure location effect and so we are not overly concerned with the low estimated land value in some locations.

Turning to estimates of depreciation, the estimated parameters largely confirm the expectations of the discussion above. Without accounting for censoring, OLS estimates indicate that depreciation is nearly zero for homes. Accounting only for the state of residence of the home, a home will *appreciate* 0.10% per year with age. Using dummy variables for county, homes depreciate a statistically significant 0.01% per year. Substituting zip codes, depreciation rates are 0.09% per year. Detailed locations are more closely correlated with age than county or state variables. The implication of these estimates is that standing houses either depreciate very little or unobserved valued traits are correlated with age. The Baltimore-Washington area may be atypical. Malpezzi, Ozanne and Thibideau find depreciation is not different from zero for Washington DC, but they do find a significant rate of depreciation for Baltimore. Most depreciation then comes when a house is discarded.

Table 4 repeats the exercise using only the structure value. In these estimates houses appreciate regardless of the choice of location variables. Again, finer geographic detail reduces the parameter on age.

The estimates in Table 4 are nevertheless a bit disturbing. Parameter estimates are lower and implausibly low for bedrooms and some other variables. Furthermore, we expected the estimated depreciation would be faster once land, which does not depreciate, was removed. Two possibilities occur. The estimate of the land value from the tax assessment data may be too high and too little value remains for structures. The tax assessments indicate land comprises about 31% of the total assessment, a relatively high figure. Alternatively, if land value can be separated from structure value, the value of land in the full sample may not have been entirely captured by acreage and location variables. Given that once a house is built, separating bundled purchases into constituent parts may prove difficult. However, the point of a hedonic model is to do just that. While biased parameters on housing traits other than age are merely troublesome, the second interpretation should lead to caution in accepting depreciation estimates.

Before accepting the near zero depreciation rate for surviving houses, it is useful to review the specification and interpretation of the model. Fraumeni (1997) succinctly describes common sampling limitations of depreciation studies. In addition, there are several hundred descriptive variables spanning a number of characteristics of the structure and land. Not all of these are significant or correctly signed. While the model could have been reduced to only those variables that improve the fit and interpretation, that list is not strictly the same in every specification. While there are risks in including extraneous characteristics, every specification we considered indicated a low depreciation rate for surviving houses.

A few variables deserve comment. A dummy variable for new construction is included. It can readily be argued that the dummy variable represents a portion of the depreciation rate. However, the premium is about 10%, much more than the depreciation rate experienced in a year. Our interpretation is that some buyers prefer new homes to young homes because they can customize the design, paint to their taste or simply prefer new to used much like a car buyer. Regardless, when the new home dummy is dropped from the model with zip codes, the depreciation rate is a faster but still slow 0.15%.

The MRIS database includes agent assessments of the condition of the home. Ratings can vary from "shows well" to "fixer-upper" to "shell". These variables were not included because they seem likely to reflect cumulative depreciation. Adding these variables would have reduced the depreciation rate even closer to zero.

Houses in the MRIS database have changed in style and size over time. Our data can identify if any of nearly two dozen types of rooms are present in the house, but not the size of the rooms. The database also indicates styles for master bedrooms, kitchens, living rooms and dining rooms. Stylistic changes and the presence of certain types of rooms (e.g. media room, office) may be indicative of vintage. As a result, some of the premium for young houses may be captured by these variables rather than age.

Finally, we have treated age as a measure of depreciation, but age of the house may also be a taste or quality variable. While some may prefer new construction, others may value older homes for their unique style, mature landscape, micro-location (below the zip code level), and sense of community. For example, two older style of homes, Tudor and Victorian, command a 12% and 5% premium respectively. Construction quality may also vary across time. If older homes are better built, they should command a higher price, ceterus paribus. If age is a valued trait, then this model may not be able to disentangle the depreciation rate from the premium that some buyers place on older homes.

Despite these qualifications, the depreciation rate for surviving houses appears to be extremely low. Near zero depreciation is consistent with a substantial portion of a cohort surviving into the next millennium. Why then do we find this result? While maintenance costs may well rise with age, potential buyers are aware of this and should price houses accordingly. Professional home inspections are nearly universal in this area and so buyers are aware of important defects. However, maintenance expenses are lumpy and may be timed both over the life of the home and at particular events such as selling a home. Older houses are much more likely to need an updated kitchen or bath. Young homes simply do not need new roofs or furnaces. If these expenditures "restore" the value of an older house, the age-price profile will appear relatively flat when older homes are included.[4] Finally, seller subsidies can only include those items with a readily identifiable price. Other terms such as low interest rate seller financing and additional items such as furniture that may convey with the house may not be included in the seller subsidy and therefore in the net price. Why these items should be correlated with age is unclear.

Buyers may reasonably expect that real services from houses would rise over time. As cities grow, central locations become increasingly valuable. While the structure may in fact be deteriorating, the value of the land and location are rising. This implies that even though the house is deteriorating, the service flow is not declining because the services provided by an increasingly valuable location offset declining services from the structure. Location is correlated with age in our sample. Adding state, county and zip codes to the regression accelerated the depreciation rate by 0.2% per year. Older homes then are located in more valuable locations. Furthermore, BEA's chained price index for

[4] Limiting the model in column (1) of table 3 to houses age 15 or less, the depreciation rate jumps to 1.16%. For this set of houses, maintenance costs should be limited to relatively minor items and so prices should reflect the effect of age over a set of homes comparably maintained. Of course, the depreciation

new single family homes rose 5.2% annually between 1973 and 1999 while the PCE chained priced index rose 3.1% per year. If all homes follow patterns for new construction and the Baltimore-Washington area is similar to national patterns, the more rapid inflation for housing is indicative of rising service flows.

Ultimately, this is not sufficient to justify the observed near zero depreciation rate because homes would eventually decay completely. As part of the growth process for a city, alternative uses must arise. Homes are torn down both to allow for new homes and for conversion to commercial uses. In such examples, the opportunity cost of the land exceeds its value as a residence and the home owner sells at a price somewhere between its value as a house and its alternative use. If this is a common view for very old homes, then home buyers may view their home as an investment with service flows for many years followed by a lump-sum payment on retirement of the house. Depending on the lump-sum buyout price, the discount rate and the growth rate of real housing services, housing prices may rise or fall as houses age.

Section 4: Censored Prices

Hulten-Wykoff criticized the simple OLS estimates for being an unbalanced sample. For any cohort of assets only those still in service are observable. Hulten-Wykoff's solution was to implicitly include the unobservable portion of the cohort. For assets that have been retired, the current price of the retired asset is zero. Assuming that retired assets are otherwise identical to the observable assets, they then averaged the price

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rate may not be constant over the life of a house and so the faster depreciation rate may instead reflect rapid initial depreciation.

of surviving assets with comparable retired assets. The resultant price is the average price for the cohort.

The key to measuring depreciation in this manner, then, is to determine the discard pattern. One set can be obtained from modified S-3 Winfrey distributions of discards. These distributions are symmetric distributions centered about a mean discard age of one's choosing. Prior to BEA's switch to geometric depreciation, Winfrey distributions were used to allow investment of a given vintage to be discarded over a number of years. BLS also used this type of distribution for this purpose, but currently uses a truncated normal distribution.

Table 5 shows estimated age-price (land and structure) models using 80, 120 and 160 year mean lives.[5] For comparison, BEA now uses a geometric depreciation rate of 1.14% that corresponds to an average life of about 80 years.

The first point to be gleaned from Table 5 is that choosing a service life is tantamount to choosing a depreciation rate. Using an 80-year mean Winfrey discard distribution, the depreciation rate is 1.93%. Under 120-year mean discard distribution, the depreciation rate is 1.15% or about what BEA uses now. For 160-year mean discard distribution, the depreciation rate is only 0.47%.

Given almost no depreciation-in-place, we would have expected the mean discard age from the Winfrey distributions to dictate the corresponding estimated depreciation rate and the resulting estimated average age at discards. (In this section and the next, the weighted average of age at discard is the product of the estimated proportion of the cohort discarded (under the geometric or Weibull distribution) and the corresponding

[5] The modified S-3 distributions use lives slightly shorter than stated in the text. For example the 120-year mean life distribution is centered on 117 years and no depreciation takes place in the first 5 years.

age.) That is, for an 80-year Winfrey distribution, approximately an 80-year average age at discard and depreciation rate slightly in excess of 1% would be expected. Similarly, for 120-year Winfrey distribution, depreciation slower than 1% would seem logical. However, we find faster depreciation rates and younger lives for a given Winfrey distribution. Using an 80-year life and a 0.91 declining balance rate, BEA uses the corresponding 1.14% depreciation rate. Of course, a more flexible depreciation pattern may produce estimates closer to BEA's estimate.

Finally, our estimates do not imply a linear relationship between service lives and depreciation rates. Halving the mean life of the Winfrey distribution from 160 to 80 years produces a depreciation rate about 4 times as fast. Given the assumption of geometric decay, a linear relationship is not necessarily expected, but this is indicative of the sensitivity of the depreciation rate to the decay pattern and service life.

There are a number of reasons to find these results unsatisfying. First, information about the discard patterns for housing is not strong. Much of the original work on discards was based on 1942 Bulletin F tax lives and adjustments to the service lives have been made over time. Second, the discard pattern is symmetric and this may be a strong assumption. Third, the price of discarded assets in our work so far is assumed to be zero. This may be a reasonable assumption for equipment, but for residential structures this assumption can be problematic. The price of single family housing is a combination of land and structure. Essentially this is a bundled good and separate prices for each might be inferred or estimated but must be separated to measure the depreciation of the structure alone. Finally, while Hulten-Wykoff's method may produce a reasonable estimate of the average cohort price, the sample is still unrepresentative of the cohort as described earlier. That is, if only 1 in 10 houses built in 1920 is still standing, the average price is a tenth of the surviving house price, but this one observation now represents 10 houses and should be weighted accordingly in the estimation. Failure to weight observations appropriately should only lead to inefficient estimates and so it is a secondary concern. However, if depreciation rates vary by age, an unweighted sample will emphasize the depreciation rate for young homes and give too little weight to the pattern at the other end of the spectrum.

We can examine two of these issues using the Winfrey distributions. First, if the Winfrey distribution is a reasonable approximation of the discard pattern, the square root of the inverse of the surviving fraction of the cohort is the appropriate weight for observations of a given age. We re-estimate the model limiting the weight to 100 to avoid placing excess weight on very old houses.[6] Second, if a portion of the price includes land that does not depreciate, the model can be re-estimated using a Winfrey distribution modified to allow a remaining residual value. In this experiment we use a 30% residual value which is very close to the mean ratio of land to total tax assessed value in our sample.[7] Finally, the model is re-estimated combining the two effects.

The table below shows estimated depreciation rates and average discard age based on these various adjustments.[8] Given almost no depreciation for surviving houses, we should expect depreciation rates consistent with the mean life of the corresponding Winfrey distribution. First note that the standard Winfrey model reported above yields faster depreciation rates and shorter age at discard than the associated Winfrey distribution. Weighting the sample produces totally implausible results. One might have

[6] The results are very similar without this constraint.
[7] Alternatively, we could have used the sample with land removed and re-estimated.

expected allowing for a residual value would slow the depreciation rate a corresponding 30%. However, the depreciation rate slowed by at least 50% and average age at discard is much higher than the underlying Winfrey distribution. Weighting the sample and incorporating a 30% residual value does little to improve the estimates.

	80 Year Winfrey		120 Year Winfrey	
	Depreciation rate	Average Age at Discard	Depreciation rate	Average Age at Discard
Standard Winfrey	1.93%	52	1.15%	87
Weighted Winfrey	0.41%	189	2.68%	37
Winfrey with 30% residual value	0.86%	114	0.47%	178
Weighted Winfrey with 30% residual value	0.87%	113	0.63%	148

So what are we to make of these widely different estimates? First, our estimating form may be too restrictive. Hulten-Wykoff used a Box-Cox form that allows age-price profiles to fit a variety of patterns. They then determined which geometric pattern best approximated their estimated profile. We have imposed a geometric form. Even though they found that geometric was a good approximation, allowing a more flexible model initially may produce better and more consistent estimates. Second, there is little to guide us in the choice of an underlying Winfrey distribution. The 80, 120, and 160 year mean lives may not be the best choices. Third, the true discard distribution may not be symmetric. Imposing a particular shape may bias the estimates if the error in the adjusted prices is correlated with age. If the discard distribution is left-humped, then prices of newer houses adjusted by an incorrect symmetric distribution are too high and the depreciation rate will be too fast. Finally, if the simple OLS regressions on survivors are accurate, housing still in service barely depreciates. This is strictly consistent with nearly infinite-lived assets. While obviously impossible, such a price pattern might be observed

[8] The depreciation rate estimates without land are no more plausible than those found in the table.

if structures were fully insured against destruction and current buyers expected that houses would be discarded when an alternative higher-value use presented itself. Under such a scenario, it is unclear what a discard means let alone at what price to value a cohort.

Section 5: Estimated depreciation rates with an estimated Weibull discard distribution

If the Winfrey discard distributions produce largely implausible estimates of depreciation, we can see if estimating a discard function along with the age-price model improves the estimates. We choose the Weibull distribution to model discards because this distribution allows for a variety of shapes including skewed, symmetric, peaked and monotonically declining.

We estimate two price models. Equation (6) describes the first, which ignores residual values. In the second, assets are permitted to depreciate but asymptotically approach a residual value that represents some combination of land and structure. Equation (7) models this approach. These two forms are estimated for both the combined price of the land and structure (Table 6) and for the structure price only (Table 7).

The estimates in Tables 6 vary considerably across model, but less so than in the fixed Winfrey discard estimates. In column (1), the mean age at discard is 62.6 years. The coefficient on age 1.86% or about the same as the comparable rate using the 80 year Winfrey discard pattern. Chart 2 shows the resultant discard distribution. It clearly peaks to the left of the mean life and has a long tail to the right. Such a pattern would be consistent with a relatively small number of surviving older houses within a cohort, but enough of considerable age to yield a high average age.

This depreciation rate is for the land and structure combined, but the land would only depreciate under unusual circumstances. If land comprises 31% of the total value as our tax data suggest, sample mean depreciation rates can be calculated from a combination of the derivative of the model and the hazard function for discards.[9] The sample mean depreciation rate for the structure is 2.08%.

The direct way to glean the net depreciation rate is to estimate equation (7) and these results can be found in column (2). The coefficient on age is 1.20%, the sample mean depreciation rate is 1.25% and the mean age at discard is longer, 80.0 years. Chart 3 also shows a left humped discard function and a more reasonable mode age for discards. However, the residual value is extremely small, only 3.5% (.036/ (1 + .036)). Only if demolition and transaction costs were extremely high would this low estimate be plausible.

While we cannot know a priori the residual value, we can assume a more reasonable value and re-estimated. We set ρ in equation (7) at 0.4. This implies a 29% residual value or close to the 31% average value in our tax data. The resultant estimate (column 3) produces a much lower coefficient of 0.54%, a sample mean depreciation rate of 0.59% and a mean age at discard is even higher, 92.3 years. Chart 4 shows a much more symmetric discard pattern that is only slightly left humped. These results are similar to the 120 year fixed discard pattern that also allows for a 30% residual land value.

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[9] Depreciation rate is $\frac{\partial \ln \bar{q}}{\partial Age} = \frac{\partial \ln q^{v}}{\partial Age} \Big/ .(q_R / q_s) - h$, where q_R/q_0 is the ratio of residual (land) value to total value and h is the hazard rate for a Weibull distribution of discards, all evaluated at the sample mean.

Table 7 repeats this exercise for the estimate structure price only. Allowing no residual value (column 1) yields a coefficient on age of 1.18%, a sample mean depreciation rate of 1.24% and a very imprecisely measured average age at discard of 85.0. Finally, column (2) reports estimates allowing for a residual value for structures. The coefficient on age falls to 0.51% and the mean discard age is 100.3 years. The sample mean depreciation rate is 0.67%. Residual value is 21% (0.267/1.267). This value is much higher than the estimated residual value for the land and structure model. Charts 5 and 6 show the consistent pattern of left humped discards.

Based on these estimates, a few patterns emerge. While our expectations were that BEA's depreciation rate was too slow, our estimates, though not precise, suggest that the depreciation rate may be even slower than BEA's rate. The sample mean depreciation rates vary from 0.54% to 2.08 with most near or below BEA's rate of 1.14%. In fact, it is not so surprising (or it should not have been to us) that if the scrap value of land under a house is quite high that the depreciation rate would be correspondingly low.

Second, the discard patterns are consistently peak to the left of the mean discard age and some patterns are quite pronounced. This is inconsistent with the symmetric patterns of the modified S-3 Winfrey distributions. Furthermore, it has the effect of increasing the depreciation rate, ceterus paribus.

Third, our estimates of the depreciation rate vary considerably. So much so, that we do not yet have enough confidence in our estimates to suggest an alternative to BEA's rate of depreciation.

Fourth, the mean life of houses varies in a fairly wide range from 63 to 100 years. Even though all but one of the estimated mean discard ages are greater than the 80-year life currently used by BEA, the average of the 5 estimates is 84.1 years. Despite estimated discard age comparable to BEA' figure, the depreciation rate is low because the scrap value is estimated or assumed to be relatively large fraction of the original purchase price.

The estimates in sections 4 and 5 are weak and largely inconsistent. We have noted a number of potential causes and we will consider some improvements and refinements in our concluding section. However, the method of Hulten and Wykoff may have some limitations and we explore these in the next section. A Monte Carlo experiment also provides some insight into how to select a sample. After that, we consider a less restrictive model that may produce more plausible estimates.

Section 6: Monte Carlo Simulation of Housing Prices

Given the weak results of both sections 4 and 5, it may be useful to examine under what circumstances the Hulten-Wykoff method produces correct measures of depreciation. To preview our findings, the Hulten-Wykoff method will measure depreciation correctly when there is no depreciation-in-place. However, when depreciation occurs through both discards and depreciation-in-place, the Hulten-Wykoff method overstates depreciation rates because the average of the sample of survivors is younger than the cohort. The bias declines as the average life of the asset increases.

We start by creating a sample of 2200 "houses" age 0 to 10 according to the model:

Price = 10000 + 7500 * bedrooms + 3500 * bathrooms.

There is a balanced sample of houses for each combination of bedrooms and bathrooms and for each cohort. The average new house cost $53,000.[10] Total observations are: 11 years (age 0-10) × 20 bed & bath combinations × 10 observations per combination = 2200.

Surviving assets also decline in value as they age. The price of used assets follows straight-line depreciation where $P_s = P_0(1-s/20)$. P_s is the price at age s and P_0 is the price model above. (Depreciation-in-place occurs at this artificially slow rate to distinguish between loss of value from discards.)

Retirements claim a tenth of the original cohort of houses every year, independent of wear and tear and independent of other traits. Of the 200 houses originally installed for each of the 10 cohorts, 20 are "gone" by the end of the first year although they had been otherwise identical to those that remain, 20 more are gone by the end of the second year, etc., until by the end of the ninth year only 20 remain, and none at all by the end of the tenth year. Moreover, the resale value of the vanished houses drops to zero. Hence, while the loss of *cohort* value is a sum—"depreciation-in-place" of houses that remain standing, plus the zeroing-out of houses that disappear—vanished houses cease to depreciate in place. Average depreciation per year of $5300 is sufficient to completely account for the decline in value over the 10 years.

We start with a sample typically available for measuring age-price profiles: survivors only. The results are:

...........*n* = 1100...........

Estimated Standard

[10] Adding an i.i.d. error term does not change the estimates in any significant way.

	Coefficient	Error
C	16455	205.7
Age	–2650	26.8
Bed	6375	30.6
Bath	2975	51.4

In this case the coefficient on age reflects just the loss in value of surviving assets, the problem Hulten-Wykoff sought to address. Coefficients on beds and baths are both 85 percent of their original values in the 1100-observation data: the percentage matches the ratio of the mean non-zero price of houses to the mean new price of houses.

Next we estimate the model on the data set that Hulten-Wykoff would ideally have used: the full cohort of 2200 houses. Data like these are usually unavailable because discards are not observable. The correct depreciation of $5300 is obtained.[11] The price of beds and baths again are the ratio of the mean price of used houses to the new house price.

	...........$n = 2200$...........	
	Estimated Coefficient	Standard Error
C	30750	1229.8
Age	–5300	118.2
Bed	3187.5	292.5
Bath	1487.5	174.4

Note that to obtain the correct depreciation, the sample must include houses of age 10. Limiting the sample to ages in which some survivors are present yields depreciation of $5565. This figure reflects average depreciation over houses age 0 to 9.[12]

[11] Depreciation-in-place is $1325 and depreciation from discards is $3975. Depreciation-in-place = 2650 per survivor * 0.5 (the surviving fraction). Depreciation from discards = $5300 * .75 (ratio of average used price to new price).

[12] Depreciation of 5565 for nine years results in cumulative depreciation of 50,085. The remaining depreciation of 2915 equals the average price of a 9 year old asset (survivors and discards).

Since depreciation is not constant by age because the value of discards per year declines with age, excluding the final year raises the depreciation rate.

The Hulten-Wykoff method limits the sample to survivors and adjusts the used asset price to include discards. The resulting depreciation rate is too high[13].

	...........$n = 1100$...........	
	Estimated Coefficient	Standard Error
C	24217	492.3
Age	–5989	64.1
Bed	4687.5	73.3
Bath	2187.5	122.9

The higher estimate is the result of two related factors. First, the sample can not include any assets of age 10 by construction. Second, the average age of the sample is much younger (3 years) than the average age of the cohort (5 years) because only survivors are included. The second effect can be eliminated by weighting observations by the square root of the inverse of their survival probability, but the first effect remains. The results of the weighted least squares are:

	...........$n = 1100$...........	
	Estimated Coefficient	Standard Error
C	29718	567.5
Age	–5565	60.2
Bed	3506.25	80.7
Bath	1636.25	135.3

Now the depreciation rate is \$5565, the same value as the full cohort of houses age 0 to 9. That is, the Hulten-Wykoff method, of course, can not weight houses age 10 because all are now discarded. Houses do not march lock step into discards as the

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[13] If surviving assets do not decline in price with age, the Hulten-Wykoff method produces the correct estimate of depreciation.

example we have set up implies and so the bias is exaggerated. For long-lived assets such as houses, the elimination of the final year is likely to have a trivial effect on the depreciation rate. We should expect that the Hulten-Wykoff method would produce almost the correct depreciation rate for houses.

However, the example does point out several considerations when choosing a sample. First, the average depreciation rate will depend on the average age of the sample. If the sample is relatively young, loss of value from discards is faster than for the entire cohort and depreciation is likely to be too rapid. Second, a few houses may live extremely long lives. Just as a sample of houses age 0-9 yields too fast a depreciation rate, a sample of houses age 0-11 produces too slow a depreciation rate. In the MRIS database, houses ranged in age from 0 to 300. If houses do not survive more than say 200 years with rare exception, including these very old houses may produce a depreciation rate that is too slow. This may be one (but not the only) reason that the weighted least squares of Section 4 were so implausible.

Section 7: Estimating depreciation with a distribution of lifetimes

Given that the Hulten-Wykoff method should produce almost correct estimates of depreciation, another cause of the weak estimation results may be an over-simplified model. In equations (6) and (7), the term δ*Age was intended to capture the cohort's depreciation, while $(Age/m)^n$ or $-\ln[(\rho + \exp(-Age/m)^n)/(1 + \rho)]$ was intended to adjust for discards (with or without residual values, respectively). But intending may not make them so: only their product (or in logs, their sum) is unambiguously the average age-price profile of assets still in play. As long as the two functions are separable, bias in one must transmit an equal and opposite bias to the other. Absent a survivorship schedule that is correct a priori, one way to make the functions less susceptible to misattribution bias is to make them nonseparable, so that the parameters of one show up in the other. To do this, interpret the cohort age-price function as an explicit average of the age-price functions of individual assets with different inherent discard ages. In this way the parameters of the survivor function appear also in the cohort age-price function, in the frequency weights that attend the member assets.

Plainly this is not the only source of retirements: unlucky accidents or sudden opportunities may also remove otherwise identical assets from play. Yet for many types of assets, and arguably houses, good or bad accidents cannot account for more than a small share of retirements. The assumption of no accidents is extreme, but less extreme than its opposite, and it permits the extraction of more information from observed transactions in used assets. In this section, then, we derive an estimating equation that allows a distribution of asset lives. So far, we keep the individual assets very simple--geometric--in order to develop a tractable form. We continue to work with this model and do not yet have estimates to report.

To begin, suppose the *purchase* price of an *s*-year-old asset declines geometrically:

$$q(s) = q_0 \, e^{-\delta s}, \tag{8}$$

where q_0, the new supply price, is assumed to be the same across all buyers (conditional on hedonic characteristics), and the minus sign in the exponent means the decay/depreciation rate δ must be a positive number. Note that δ differs randomly and permanently across assets, an unavoidable fact of nature. This alters our view of depreciation. There is no longer a single depreciation rate for which a few stubborn assets refuse to adhere to. Instead, assets have different expected lives when made and will reveal their lives over time.

The manufacturer and the initial purchaser are both "symmetrically misinformed": neither knows the δ_i inhering in a particular asset, but both know the distribution of δ. Each asset reveals its own character in use, in that *rental* prices start to differ from one another immediately according to their different δ_i's, although productivity shocks conceal the discrepancies for a time.

$$p(s) = (r + \delta_i)\, q_0\, e^{-\delta_i s}, \tag{9}$$

With Hulten-Wykoff, suppose that buyers and sellers in used-asset markets are symmetrically informed: by the time a particular asset is put on the block, both the seller and prospective buyers have formed tight, accurate estimates of δ_i.

Without the possibility of scrapage or maintenance costs, a geometrically decaying individual asset would remain in use forever, never quite wearing out entirely. However, a positive scrap price, q_R, induces retirements whenever $q(s)$ falls to q_R. Prices of assets that depreciate faster cross the q_R threshold sooner, so retirement ages are reciprocally related to depreciation rates. To see this explicitly, replace s in (8) by R, the retirement age, take logs and rearrange to find:

$$\delta = ln(q_0/q_R)/\mathrm{R}. \tag{10}$$

It follows that long-lived assets will be over-represented in typical resale data, particularly at great ages. Econometricians, who don't observe δ_i, should therefore treat any observed age-price pair (s, q) as a joint statistical outcome, and model depreciation to take advantage of the joint nature of the data. Standard age-price regressions don't do that, but rather begin (and end) with the logarithm of (1), plus an error term for haggling:

$$ln\, q(s) = ln\, q_0 - \delta s + \varepsilon. \tag{11}$$

—implicitly supposing δ is a constant uncorrelated with s. Instead, use (10) to replace δ by $ln(q_0/q_R)$/R, then model the joint distribution of ages and retirements. The *expectation* of $ln\, q(s)$ of surviving assets is:

$$E\, ln\, q(s) = ln\, q_0 + ln(q_0/q_R) \int_s^{\infty}(s/R) f(R \mid s)\, dR\,, \tag{12}$$

where $f(R|s)$ is the *conditional probability density function*: i.e., the probability that an s-year-old asset will be retired at age R. Suppose $f(R \mid s)$ is smooth and well behaved: $f(R \mid s)>0$ for $R>s$, $\partial[\int_s^t f(R \mid s)\, dR]/\partial t>0$ for age $t>s$, and $\int_s^{\infty} f(R \mid s)\, dR=1$. To derive $f(R \mid s)$, divide *unconditional* $f(R)$—the probability a *new* asset will be retired at age R—by 1−F(s), the fraction of assets still surviving at age s:

$$f(R \mid s) = \frac{f(R)}{1-F(s)} = f(R) \Big/ \int_s^{\infty} f(R)\, dR\,. \tag{13}$$

The conditional expectation of $ln\, q(s)$ becomes:

$E\, ln\, q(s)$

$$= ln\, q_0 + ln(q_R/q_0)\ s \int_s^{\infty} \frac{f(R)/R\, dR}{1-F(s)} = ln\, q_0 + ln(q_0/q_R)\ s \int_s^{\infty} f(R)/R\, dR \Big/ \int_s^{\infty} f(R)\, dR\,. \tag{14}$$

We show in the appendix that a suitable estimating form for (14) when individual assets depreciate geometrically is:

$$ln\, q(s) = ln\, q_0 - ln(q_0/q_R) \frac{n_1 ln(q_0/q_R)s + s^2}{d_0[ln(q_0/q_R)]^2 + d_1 ln(q_0/q_R)s + s^2} + \varepsilon \tag{15}$$

where q_0/q_R is the ratio of the new the scrap asset price and $d_1 \geq n_1 \geq 0$ and $d_1 n_1 \geq d_0 > 0$ are parameters of the model. The corresponding probability density function of the depreciation rates can be used to measure average depreciation rates once the parameters are estimated.

$$f(\delta) = \frac{1}{d_0^2 \delta^3} \cdot \left(1 + \frac{d_1 - n_1}{d_0} \cdot \frac{1}{\delta}\right)^{\frac{d_0}{(d_1-n_1)^2} - \frac{2d_1-n_1}{d_1-n_1}} \left\{(d_1 n_1 - d_0) + 2\frac{n_1}{\delta} + \left(\frac{1}{\delta}\right)^2\right\} \cdot e^{\frac{1}{(n_1-d_1)\delta}} . \qquad (16)$$

The cohort average depreciation rate is n_1/d_0.

Section 8: Conclusion, Summary and Much Needed Further Work

Using transaction price and characteristics for 160,000 houses in the Baltimore-Washington area, we find almost no depreciation (.09% per year). Location is an important element in the measurement of depreciation. Depreciation rates are estimated to be almost 0.2% per year faster when switching from state locations to 5-digit zip codes. Nevertheless, the incredibly small depreciation rate implies that survivors are either very long lived or purchasers have an expectation that houses will be converted to a more valuable alternative use because the land and location value rise over time as cities grow. Alternative interpretations are that age is a valued trait by some purchasers or that unmeasured valued traits are positively correlated with age.

We estimate the depreciation rate using the Hulten-Wykoff method with 80, 120 and 160-year Winfrey distributions. The choice of a distribution is tantamount to choosing a depreciation rate considering that surviving houses depreciate very little. Furthermore, our estimates using the Hulten-Wykoff method are not precise. Since the evidence on the appropriate discard distribution is scant, we devise a method to estimate the discard distribution along with the age-price model.

The resultant estimates of depreciation rates for single family housing are not sufficiently precise for us to recommend a rate different from BEA's current 1.14%. We have adapted the Hulten-Wykoff method of incorporating discard patterns into the estimation. Rather than adjust the price of survivors by modified S-3 Winfrey distributions, we estimate a Weibull discard pattern as part of the model. Since housing prices include the value of a depreciable structure and non-depreciable land, we further modify the Weibull distribution to allow a positive scrap value. We estimate depreciation rates from 0.54 to 2.08 percent although most estimates are near or below BEA's rate. Asset lives for single family houses range from 63 to 100 years with most near BEA's estimate of 80 years.

While the range of estimates is large, several other features are persistent. The distribution of discards peaks to the left of the mean with a substantial right tail. While our initial intuition was that the true depreciation rate was faster than BEA's current rate, BEA's figure is at the upper range of our estimates. The left humped distribution does help reconcile BEA's depreciation rate with our intuition that the proportion of older houses is not very large. Furthermore, BLS may be able to adopt BEA's depreciation rate provided BLS also employs a skewed discard pattern.

We next conducted a Monte Carlo simulation to investigate how well the Hulten-Wykoff method estimates a known depreciation rate. The Hulten-Wykoff method can be expected to misstate depreciation when used assets sell for less than new ones. The average age of the entire cohort differs from the average age of the sample unless survival is unrelated to age. In our experiment, younger assets depreciate faster, on average, than older assets and so the Hulten-Wykoff method overstated depreciation.

Weighting observations by the inverse of the survival probability reduces any bias, but does not eliminate it because the weighted average age of the sample is still younger than the cohort. For long-lived assets such as houses, the bias should be trivial.

The investigation also reveals that depreciation rates depend on the concept of depreciation being investigated. Chart 7 compares an age-price profile that declines slowly with one that declines more rapidly. For calculating the average depreciation over the lifetime of the cohort, only the asset life is relevant. Even though one asset appears to depreciate more rapidly, it merely depreciates more rapidly *early* in its life while the other asset depreciates more rapidly later in its life. Over the entire life, the average depreciation rate is the same. For depreciation of a stock of assets at a point in time, both depreciation-in-place and current period discards should be counted. In this case, the more rapidly declining profile depreciates, on average, faster. However, this is true only if surviving assets and current period discards are counted. If the entire cohort including all discards is counted, again, the lower curve depreciates more rapidly early in its life while the upper curve depreciates more rapidly later. Over the entire life, the depreciation rates are equal.

We suggest an alternative view to a distribution of discards around a single asset life. Specifically, we view asset as having a distribution of lives and derive an estimating equation using a quite general distribution of geometric depreciation rates and a conformable distribution of retirements.

Much more work is needed. We have specifically imposed a geometric depreciation rate to examine BEA's estimate. A better strategy would use a more flexible cohort age-price form in measuring depreciation. Neighborhood characteristics are currently summarized by zip codes, but a more detailed specification of schooling, crime and commuting distances may better define the location. Nor have we investigated models on the relatively small sub-sample that includes interior square footage. While the sample is small, it is still more than 10,000. Finally, a small sample of houses were sold more than once in the period of study. A fixed effects model may provide sharper estimates although houses resold in a short period of time may be lemons.

The MRIS database contains rental housing and commercial real estate. These data need to be investigated as well. It can be difficult to distinguish depreciation patterns from age-price profiles because most are concave even when the age-efficiency pattern is not (Sliker 2000). Even though rental housing is sometimes subject to price control, rental dwellings may provide more direct evidence on the age-efficiency pattern than age-price profiles. Since it is quite difficult to infer age-efficiency patterns from age-price patterns, rental housing may help determine the appropriate specification

In sum, we find no evidence that depreciation occurs at a rate faster than BEA's 1.14% rate for single family housing. While our estimates are imprecise, most of our estimates implied slower depreciation except when the residual value of land is not explicitly modeled. The average age at discard is also close to BEA's figure of 80 years. We do find that discards are not symmetric. Given the lack of precision in our estimates, much more work needs to be done.

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Appendix: Geometrically-Decaying Assets with General Statistical Retirement Patterns

This appendix derives a regression to fit the average age-price function of a cohort of assets that depreciate along separate paths drawn from an unknown but well-behaved distribution of Geometric depreciation rates. It is not necessary to assume in advance a particular distribution of rates—or, as it turns out, of the rates' reciprocal, retirement ages—to correct for censoring bias in age-price regressions: available resale data are sufficient to estimate the parameters of both depreciation and retirements consistently *in one step*. It would of course be still better to accommodate a more flexible depreciation pattern than Geometric while maintaining statistical generality of the lifespans, but we leave this for the future. The binding constraint here is to find estimating forms that do not need higher functions than those available in recent-vintage regression software. As software improves, the constraint will not bind for most practitioners.

First, a respectful nod toward "the engineers." Quality-control statisticians typically assume one-hoss shay efficiency—a device works until it breaks—and an Exponential, Gamma, or Weibull failure distribution; they also base their estimates on physical data obtained from well-designed experiments. Economists, by contrast, usually assume efficiency *declines* with age and seek to infer the shape of that decline indirectly, by comparing assets' resale *prices* to age; the comparison is hampered by the toll of retirements of assets that have already worn out. So far, corrections for retirements are not well-connected to efficiency declines; borrowing a sheet from the engineers' playbook might make the corrections more rigorous.

Many economists might think borrowing isn't necessary. We usually work with *cohorts* of assets, not individuals, and if the cohort Geometric model is not exactly right for summarizing the joint depreciation-in-place and retirements of many individual assets, it is often "about right," and it is easy to use. But black-box cohort models hide as much as they summarize; ambiguities in representing price changes in Geometric cohorts in terms of age *versus* time *versus* vintage are well known. Further, tests of cohort models in the literature have required *a priori* retirement distributions to correct for sample selection problems, but the distributions commonly used are now badly out of date. Revisiting the compound of individual age-price functions and retirement frequencies *within* the cohort would enable econometricians to use available data on used-asset transactions directly. The price of unpacking the data, however, is greater mathematical complication: it will turn out to be difficult to model depreciation and the distribution of retirements jointly using the engineers' favored failure distributions, but efforts to do so will point up tractable estimating forms from which the distribution of retirements can be inferred *ex post*.

To begin, suppose the purchase price of an *s*-year-old individual asset declines Geometrically:

$$q(s) = q_0\, e^{-\delta s}, \tag{A1}$$

where q_0, the new supply price, is assumed to be the same across all buyers (conditional on hedonic characteristics), and the minus sign in the exponent means the decay/depreciation rate δ must be a positive number. Note that δ differs randomly and permanently across assets, an unavoidable fact of nature. The manufacturer and initial buyer are both "symmetrically misinformed": neither knows the δ_i inhering in a particular asset, but both know the distribution of δ. Each asset reveals its own character in use, in that *rental* prices:

$$p(s) = (r + \delta_i)\, q_0\, e^{-\delta_i s}, \tag{A2}$$

start to differ from one another immediately according to their different δ_i's, although productivity shocks conceal the discrepancies for a time. With Hulten-Wykoff, suppose buyers and sellers in used-asset markets are symmetrically informed: by the time a particular asset is put on the block, both the seller and prospective buyers have formed tight, accurate estimates of δ_i.

Without the possibility of scrappage, a Geometric asset would remain in use forever, never quite wearing out entirely. However, a positive scrap price, q_R, induces retirements whenever $q(s)$ falls to q_R. Prices of assets that depreciate faster cross the q_R threshold sooner, so retirement ages are reciprocally related to depreciation rates. To see this explicitly, replace s in (A1) by R, the retirement age, take logs and rearrange to find:

$$\delta = ln(q_0/q_R)/\mathrm{R}. \tag{A3}$$

It follows that that long-lived assets will be over-represented in typical resale data, particularly at great ages. Econometricians, who don't observe δ_i, should therefore treat any observed age-price pair (s, q) as a joint statistical outcome, and model depreciation to take advantage of the joint nature of the data. Standard age-price regressions don't do that, but rather begin (and end) with the logarithm of (1), plus an error term for haggling:

$$ln\, q(s) = ln\, q_0 - \delta s + \varepsilon. \tag{A4}$$

—implicitly supposing δ is a constant uncorrelated with s. Instead, use (A3) to replace δ by $ln(q_0/q_R)$/R, then model the joint distribution of ages and retirements. The *expectation* of $ln\ q(s)$ of surviving assets is:

$$E\ ln\ q(s) = ln\ q_0 - ln(q_0/q_R) \int_s^{\infty}(s/R) f(R \mid s)\, dR\,, \qquad \text{(A5)}$$

where $f(R|s)$ is the *conditional probability density function*: i.e., the probability that an s-year-old asset will be retired at age R. Suppose $f(R \mid s)$ is smooth and well behaved: $f(R \mid s)>0$ for $R>s$, $\partial[\int_s^t f(R \mid s)\, dR]/\partial t > 0$ for age $t>s$, and $\int_s^{\infty} f(R \mid s)\, dR = 1$. To derive $f(R \mid s)$, divide *unconditional* $f(R)$—the probability a *new* asset will be retired at age R—by 1−F(s), the fraction of assets still surviving at age s:

$$f(R \mid s) = \frac{f(R)}{1-F(s)} = f(R) \Big/ \int_s^{\infty} f(R)\, dR\,. \qquad \text{(A6)}$$

The conditional expectation of $ln\ q(s)$ becomes:

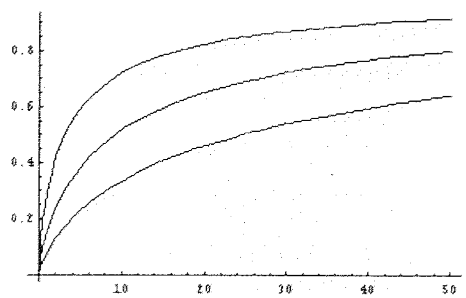
$$E\ ln\ q(s) = ln\ q_0 - ln(q_0/q_R)\ s \int_s^{\infty} \frac{f(R)/R\ dR}{1-F(s)} = ln\ q_0 - ln(q_0/q_R)\ s \int_s^{\infty} f(R)/R\ dR \Big/ \int_s^{\infty} f(R)\, dR\,. \qquad \text{(A7)}$$

Since $ln\ q_0$ and $ln(q_0/q_R)$ are constants or at least hedonic functions, the only problem now is to express the "moving part"—i.e., $s \int_s^{\infty} \frac{f(R)/R\ dR}{1-F(s)}$. The table on the next page examines three attempts, using retirement distributions taken from engineering: the Exponential, with a modal retirement age of zero, and the Gamma and Weibull, which extend the Exponential by adding a "shape parameter" $n \geq 0$ to the "scale parameter" $m \geq 0$. Similarities among the three distributions are instructive. In particular, the nine curves

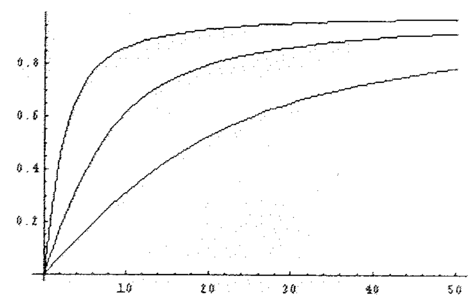
FEATURES OF THREE COMMON RETIREMENT DISTRIBUTIONS

	Exponential Distribution	Gamma Distribution	Weibull Distribution
…Probability Density Functions for Retirement Ages…	$e^{-R/m}/m$	$e^{-R/m}(R/m)^{n-1}/(m\,\Gamma(n))$	$(n/m)(R/m)^{n-1}e^{-(R/m)^n}$
…Cumulative Distribution Functions for Retirement Ages…	$1-e^{-R/m}$	$1-\Gamma\left(n,\frac{R}{m}\right)/\Gamma(n)$	$1-e^{-(R/m)^n}$
…Means and Variances…	$\{m, m^2\}$	$\{mn, m^2 n\}$	$\{m\,\Gamma(\frac{n+1}{n}),\ m^2[\,\Gamma(\frac{n+2}{n})-\Gamma^2(\frac{n+1}{n})\,]\}$
…"Moving Parts": $s\int_s^\infty \frac{f(R)/R\,dR}{1-F(s)}$ …	$(s/m)\,e^{s/m}\,\Gamma(0, s/m)$	$(s/m)\Gamma(n-1,\frac{s}{m})/\Gamma(n,\frac{s}{m})$	$(s/m)e^{(s/m)^n}\Gamma\left(1-\frac{1}{n},(\frac{s}{m})^n\right)$

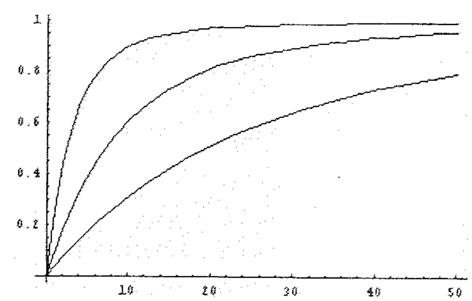
…Example Plots of "Moving Parts"…



Top curve: mean=standard deviation=5
Middle curve: mean=standard deviation=15
Bottom curve: mean=standard deviation=40



Top curve: mean=5, standard deviation=2.5
Middle curve: mean=15, standard deviation=7.5
Bottom curve: mean=40, standard deviation=20



Top curve: mean=5, standard deviation=2.5
Middle curve: mean=15, standard deviation=7.5
Bottom curve: mean=40, standard deviation=20

shown all start at zero, increase—at decreasing rates—and tend toward one.[14] Further, the math behind each curve involves the Incomplete Gamma function:

$$\Gamma(a, x) = \int_x^{\infty} e^{-z} z^{a-1} dz ,$$

which is neither available in most econometric packages nor easily approximated. We found one simple representation only, but it will suggest a way to proceed: for the Gamma distribution with integer $n \geq 2$, the "moving part" reduces to a rational function in s:

$$\frac{s}{m}\frac{\Gamma(n-1,\frac{s}{m})}{\Gamma(n,\frac{s}{m})} = \frac{s}{m(n-1)} \sum_{j=0}^{n-2} (s/m)^j / j! \Big/ \sum_{j=0}^{n-1} (s/m)^j / j! .$$

For example:

$n=2$: $$\frac{s}{m}\frac{\Gamma(1,s/m)}{\Gamma(2,s/m)} = \frac{s}{m+s}$$

$n=3$: $$\frac{s}{m}\frac{\Gamma(2,s/m)}{\Gamma(3,s/m)} = \frac{ms+s^2}{2m^2+2ms+s^2}$$

$n=4$: $$\frac{s}{m}\frac{\Gamma(3,s/m)}{\Gamma(4,s/m)} = \frac{2m^2s+2ms^2+s^3}{6m^3+6m^2s+3ms^2+s^3}$$

$n=5$: $$\frac{s}{m}\frac{\Gamma(4,s/m)}{\Gamma(5,s/m)} = \frac{6m^3s+6m^2s^2+3ms^3+s^4}{24m^4+24m^3s+12m^2s^2+4ms^3+s^4} .$$

Note each right-side ratio has only positive coefficients. The numerator lacks a constant, giving a zero at $s=0$. Terms of highest degree are equal (and normalized to 1), forcing the ratio to 1 as $s \to \infty$. Coefficients on middling denominator terms dominate their counterparts in the numerator, keeping the ratio securely less than 1 for "medium" s. More subtly, the second derivative with respect to s is never positive for $s \geq 0$.

[14] We would go a step further and surmise that for *any* reasonable continuous distribution describing nonnegative R, the following hold:

$$\lim_{s\to 0}\left(s\frac{\int_s^{\infty} f(R)/R\,dR}{\int_s^{\infty} f(R)\,dR}\right) = 0,\ \lim_{s\to \infty}\left(s\frac{\int_s^{\infty} f(R)/R\,dR}{\int_s^{\infty} f(R)\,dR}\right) = 1,\ \frac{\partial}{\partial s}\left(s\frac{\int_s^{\infty} f(R)/R\,dR}{\int_s^{\infty} f(R)\,dR}\right) \geq 0,\text{ and } \frac{\partial^2}{\partial^2 s}\left(s\frac{\int_s^{\infty} f(R)/R\,dR}{\int_s^{\infty} f(R)\,dR}\right) \leq 0 .$$

Unfortunately rational forms do not hold for non-integer *n*, and in any case it is inconvenient to fit a model by iterating over the number of terms in addition to the parameter values. An alternative that does not lock in the assumption of a Gamma distribution would *start from* a rational function with a fixed number of terms, allow the terms' values to vary vis-à-vis each other subject to some reasonable constraints, then infer the retirement distribution *ex post*. Suppose, then, a "Gamma-*inspired*" estimating form:

$$ln\ q(s) = ln\ q_0 - ln(q_0/q_R) \cdot \left(\sum_{i=1}^{T-1} n_i \left(\frac{s}{ln(q_0/q_R)} \right)^i + \left(\frac{s}{ln(q_0/q_R)} \right)^T \right) \Bigg/ \left(\sum_{j=0}^{T-1} d_j \left(\frac{s}{ln(q_0/q_R)} \right)^j + \left(\frac{s}{ln(q_0/q_R)} \right)^T \right) + \varepsilon \quad \text{(A8)}$$

where *T*, the number of numerator terms, is set by the researcher, ε is the zero-mean difference between $ln\ q(s)$ and its expectation, and

$$\left(\sum_{i=1}^{T-1} n_i \left(\frac{s}{ln(q_0/q_R)} \right)^i + \left(\frac{s}{ln(q_0/q_R)} \right)^T \right) \Bigg/ \left(\sum_{j=0}^{T-1} d_j \left(\frac{s}{ln(q_0/q_R)} \right)^j + \left(\frac{s}{ln(q_0/q_R)} \right)^T \right) \text{ equals } s \int_s^{\infty} \frac{f(R)/R\ dR}{1-F(s)},$$

although just what $f(R)$ *is*, precisely, is left unstated for the moment. For "good behavior"—i.e., to make the moving part "look like" the nine previous plots—restrict all the n_i's and d_j's to be positive, and impose $n_i \leq d_j$ when $i=j$.[15] Fit (A8) fit by nonlinear least squares. To interpret the fitted coefficients, start with the relation between the statistical moving part and the rational function:

[15] We have neglected second-order conditions, which increase and become more complicated with T: e.g., $d_1 n_1 \geq d_0$ for $T=2$, but $d_1 n_1 \geq d_0 n_2$, $d_2 n_1 \geq d_0$, and $3n_1 - d_1 \geq d_2 n_2$ for $T=3$. Moreover, the connection between the moving part's good behavior and the shape of the implicit retirement distribution is not yet clear.

$$s\frac{\int_s^\infty \frac{f(R)}{R}dR}{\int_s^\infty f(R)dR} = \frac{\sum_{i=1}^{T-1} n_i\left(s/ln(q_0/q_R)\right)^i + \left(s/ln(q_0/q_R)\right)^T}{\sum_{j=0}^{T-1} d_j\left(s/ln(q_0/q_R)\right)^j + \left(s/ln(q_0/q_R)\right)^T}. \tag{A9}$$

Rearrange, canceling an s:

$$\left(\int_s^\infty \frac{f(R)}{R}dR\right)\left(\sum_{j=0}^{T-1} d_j ln(q_0/q_R)^{T-j}s^j + s^T\right) = \left(\int_s^\infty f(R)dR\right)\left(\sum_{i=1}^{T-1} n_i ln(q_0/q_R)^{T-i}s^{i-1} + s^{T-1}\right).$$

Use Leibniz' Rule to differentiate both sides with respect to s:

$$-\frac{f(s)}{s}\left(\sum_{j=0}^{T-1} d_j ln(q_0/q_R)^{T-j}s^j + s^T\right) + \left(\int_s^\infty \frac{f(R)}{R}dR\right)\left(\sum_{j=1}^{T-1} jd_j ln(q_0/q_R)^{T-j}s^{j-1} + Ts^{T-1}\right)$$
$$= -f(s)\left(\sum_{i=1}^{T-1} n_i ln(q_0/q_R)^{T-i}s^{i-1} + s^{T-1}\right) + \left(\int_s^\infty f(R)dR\right)\left(\sum_{i=2}^{T-1} (i-1)n_i ln(q_0/q_R)^{T-i}s^{i-2} + (T-1)s^{T-2}\right),$$

then substitute for $\int_s^\infty \frac{f(R)}{R}dR$ from (A9):

$$-\frac{f(s)}{s}\left(\sum_{j=0}^{T-1} d_j ln(q_0/q_R)^{T-j}s^j + s^T\right) + \int_s^\infty f(R)dR\,\frac{\sum_{i=1}^{T-1} n_i ln(q_0/q_R)^{T-i}s^{i-1} + s^{T-1}}{\sum_{j=0}^{T-1} d_j ln(q_0/q_R)^{T-j}s^j + s^T}\left(\sum_{j=1}^{T-1} jd_j ln(q_0/q_R)^{T-j}s^{j-1} + Ts^{T-1}\right).$$
$$= -f(s)\left(\sum_{i=1}^{T-1} n_i ln(q_0/q_R)^{T-i}s^{i-1} + s^{T-1}\right) + \left(\int_s^\infty f(R)dR\right)\left(\sum_{i=2}^{T-1} (i-1)n_i ln(q_0/q_R)^{T-i}s^{i-2} + (T-1)s^{T-2}\right),$$

and rearrange again to express $-f(s)\Big/\int_s^\infty f(R)dR$ as a rational function in s:

$$\frac{\left(\sum_{i=2}^{T-1}(i-1)n_i\left(\frac{s}{ln(q_0/q_R)}\right)^i + (T-1)\left(\frac{s}{ln(q_0/q_R)}\right)^T\right)\left(\sum_{j=0}^{T-1} d_j\left(\frac{s}{ln(q_0/q_R)}\right)^j + \left(\frac{s}{ln(q_0/q_R)}\right)^T\right) - \left(\sum_{i=1}^{T-1} n_i\left(\frac{s}{ln(q_0/q_R)}\right)^i + \left(\frac{s}{ln(q_0/q_R)}\right)^T\right)\left(\sum_{j=1}^{T-1} jd_j\left(\frac{s}{ln(q_0/q_R)}\right)^j + T\left(\frac{s}{ln(q_0/q_R)}\right)^T\right)}{s\left(\sum_{j=0}^{T-1} d_j\left(\frac{s}{ln(q_0/q_R)}\right)^j + \left(\frac{s}{ln(q_0/q_R)}\right)^T\right)\left(d_0 + \sum_{i=0}^{T-1}(d_i - n_i)\left(\frac{s}{ln(q_0/q_R)}\right)^i\right)} \tag{A10}$$

Since $\frac{\partial}{\partial s}\int_s^\infty f(R)dR = -f(s)$, think of $\int_s^\infty f(R)dR$ as "H(s)" and $-f(s)$ as dH/ds, so the left side of (A10) is $\frac{dH}{ds}\Big/H$. Integrate: the left with respect to H, the right with respect to s. The left-side integral is simply $ln(H) = ln\left(\int_s^\infty f(R)dR\right) = ln(1-F(s))$. The right-side integral looks daunting, but a rational function integral can *always* be solved in terms of rational functions, natural logarithms and arctangents. Given the results of integration:

$$ln(1-F(s)) = [\text{right-side integral } (s)] + C,$$

the retirement distribution follows almost immediately. Set s=0 to solve the constant of integration:

$$C = -[\text{right-side integral } (0)],$$

then exponentiate both sides, replace s by R, and solve for the cumulative distribution function:

$$F(R) = 1 - e^{[\text{right-side integral } (R)]-[\text{right-side integral } (0)]}.$$

Take the derivative for the probability density function:

$$f(R) = -\partial[\text{right-side integral } (R)]/\partial R \cdot e^{[\text{right-side integral } (R)]-[\text{right-side integral } (0)]}. \qquad \text{(A11)}$$

The probability density function of δ is a direct transformation[16] of the probability density function of R: substitute $ln(q_0/q_R)/\delta$ into (A11) at each occurrence of R, then multiply the whole thing by $ln(q_0/q_R)/\delta^2$.

At this point some examples will both make the method outlined here more familiar and build up a small kit of functional forms for later use "off the shelf." For the simplest case, T=1, the regression equation is:

$$ln\ q(s) = ln\ q_0 - ln(q_0/q_R)\frac{s}{d_0 ln(q_0/q_R)+s} + \varepsilon. \qquad \text{(A12)}$$

[16] By Theorem V.11 of Mood, Graybill, and Boes, *Introduction to the Theory of Statistics*, 3rd Edition (McGraw-Hill, 1974), p. 200: if X is a continuous random variable with a positive probability density function $f(x)$ over some region, and if y=$g(x)$ is a one-to-one transformation of X onto Y with $d\,g^{-1}(y)/d\,y$ continuous and nonzero wherever Y is defined, then the probability density function for Y is:

$$f(y) = \left|d\,g^{-1}(y)/d\,y\right| f(g^{-1}(y))$$

wherever Y is defined. In the present application, R is X, δ is Y, and $g^{-1}(\delta)$ is $ln(q_0/q_R)/\delta$.

Rearrange the moving part:

$$(d_0 ln(q_0/q_R)+s)\int_s^{\infty}\frac{f(R)}{R}dR=\int_s^{\infty}f(R)dR\,,$$

differentiate by s:

$$-\frac{f(s)}{s}(d_0 ln(q_0/q_R)+s)+\left(\int_s^{\infty}\frac{f(R)}{R}dR\right)=-f(s)\,,$$

substitute for $\int_s^{\infty}\frac{f(R)}{R}dR$:

$$-\frac{f(s)}{s}(d_0 ln(q_0/q_R)+s)+\left(\int_s^{\infty}f(R)dR\right)\Big/(d_0 ln(q_0/q_R)+s)=-f(s)\,,$$

rearrange again:

$$\frac{-f(s)}{\int_s^{\infty}f(R)dR}=-\frac{s/(d_0 ln(q_0/q_R))}{d_0 ln(q_0/q_R)+s}=\frac{1}{d_0 ln(q_0/q_R)+s}-\frac{1}{d_0 ln(q_0/q_R)}\,,$$

and integrate to find:

$$ln(1-F(s)) = ln[d_0\, ln(q_0/q_R)+ s] - s/[d_0\, ln(q_0/q_R)] + C.$$

Set $C = -ln[d_0\, ln(q_0/q_R)]$ so that $F(0) = 0$, then exponentiate, replace s by R, and rearrange for the cumulative distribution function:

$$F(R) = 1-\left(1+\frac{R}{d_0\, ln(q_0/q_R)}\right)e^{-\frac{R}{d_0\, ln(q_0/q_R)}}\,. \qquad \text{(A13)}$$

The probability density function of retirements follows as:

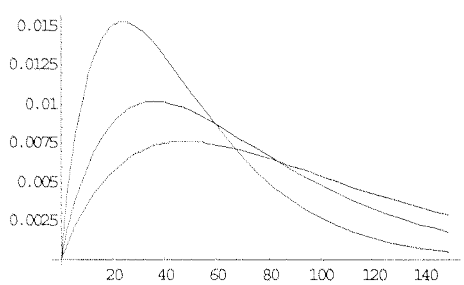
$$f(R) = \frac{R}{d_0^{\,2}\,[ln(q_0/q_R)]^2}\,e^{-\frac{R}{d_0\, ln(q_0/q_R)}}\,, \qquad \text{(A14)}$$

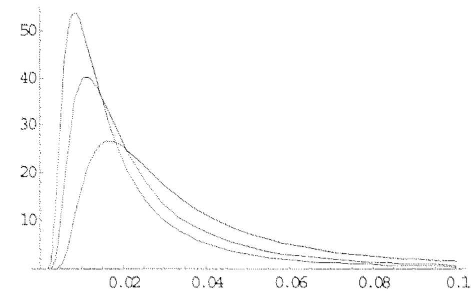
while the probability density function of depreciation rates is:

$$f(\delta) = 1\Big/\left(d_0^{\,2}\,\delta^3\,e^{\frac{1}{d_0\delta}}\right). \qquad \text{(A15)}$$

Positive d_0 guarantees good behavior. The r^{th} uncentered moment of the retirement distribution is $[d_0\, ln(q_0/q_R)]^r[(1+r)!]$, so the cohort-mean retirement age is $2d_0\, ln(q_0/q_R)$ and the coefficient of variation is $1/\sqrt{2}$. The modal retirement age is $d_0\, ln(q_0/q_R)$, so retirements are right-skewed. The cohort-mean depreciation rate is $1/d_0$; there are no higher moments. The modal depreciation rate is $1/(3d_0)$. Sample plots retirement-age and depreciation-rate densities follow for $d_0 = \{20, 30, 40\}$ and $q_0/q_R = 10/3$:

`retirement ages:` $f(R)$ □ `depreciation rates:` $f(\delta)$





To treat the spread and skew of the distributions more flexibly, turn to the T=2 case. The regression equation is:

$$ln\ q(s) = ln\ q_0 - ln(q_0/q_R)\frac{n_1 ln(q_0/q_R)s + s^2}{d_0[ln(q_0/q_R)]^2 + d_1 ln(q_0/q_R)s + s^2} + \varepsilon, \tag{A16}$$

which is expression (15) in the main body of the text. Rearrange the moving part and differentiate by s:

$$-\frac{f(s)}{s}\left(d_0 ln(q_0/q_R)^2 + d_1 ln(q_0/q_R)s + s^2\right) + \left(\int_s^\infty \frac{f(R)}{R}dR\right)\left(d_1 ln(q_0/q_R) + 2s\right) = -f(s)\left(n_1 ln(q_0/q_R) + s\right) + \int_s^\infty f(R)dR \cdot$$

Replace $\int_s^\infty \frac{f(R)}{R}dR$ and rearrange again:

$$\frac{-f(s)}{\int_s^\infty f(R)dR} = \frac{d_0 - d_1(d_1 - n_1)}{(d_1 - n_1)[d_0 ln(\frac{q_0}{q_R}) + (d_1 - n_1)s]} + \frac{d_1 ln(\frac{q_0}{q_R}) + 2s}{d_0\left(ln(\frac{q_0}{q_R})\right)^2 + d_1 ln(\frac{q_0}{q_R})s + s^2} - \frac{1}{(d_1 - n_1)ln(\frac{q_0}{q_R})},$$

integrate:

$$ln(1-F(s)) = \frac{d_0 - d_1(d_1 - n_1)}{(d_1 - n_1)^2} ln\left[ln(\tfrac{q_0}{q_R})\left(d_0 ln(\tfrac{q_0}{q_R}) + (d_1 - n_1)s\right)\right] + ln[d_0\left(ln(\tfrac{q_0}{q_R})\right)^2 + d_1 ln(\tfrac{q_0}{q_R})s + s^2] - \frac{s}{(d_1 - n_1)ln(\frac{q_0}{q_R})} + C,$$

set s=0 to find C, and finally solve for the cumulative distribution function of retirement ages:

$$F(R) = 1 - \left(1 + \frac{d_1 - n_1}{d_0 ln(q_0 / q_R)} R\right)^{\frac{d_0}{(d_1-n_1)^2} - \frac{d_1}{d_1-n_1}} \left(1 + \frac{d_1 ln(q_0 / q_R) R + R^2}{d_0 [ln(q_0 / q_R)]^2}\right) e^{\frac{R/ln(q_0/q_R)}{(n_1-d_1)}} . \quad \text{(A17)}$$

Differentiate for the probability density function:

(A18)

$$f(R) = \frac{R}{d_0{}^2 [ln(q_0 / q_R)]^2} \cdot \left(1 + \frac{d_1 - n_1}{d_0 ln(q_0 / q_R)} R\right)^{\frac{d_0}{(d_1-n_1)^2} - \frac{2d_1-n_1}{d_1-n_1}} \left\{(d_1 n_1 - d_0) + \frac{2n_1 R}{ln(q_0 / q_R)} + \left(\frac{R}{ln(q_0 / q_R)}\right)^2\right\} \cdot e^{\frac{R/ln(q_0/q_R)}{(n_1-d_1)}} .$$

The probability density function of the depreciation rates—expression (16) in the text—follows as:

$$f(\delta) = \frac{1}{d_0^2 \delta^3} \cdot \left(1 + \frac{d_1 - n_1}{d_0} \cdot \frac{1}{\delta}\right)^{\frac{d_0}{(d_1-n_1)^2} - \frac{2d_1-n_1}{d_1-n_1}} \left\{(d_1 n_1 - d_0) + 2\frac{n_1}{\delta} + \left(\frac{1}{\delta}\right)^2\right\} \cdot e^{\frac{1}{(n_1-d_1)\delta}} . \quad \text{(A19)}$$

The shapes of $f(R)$ and $f(\delta)$ are quite flexible for various values of d_0, d_1, and n_1 and domains of R and δ. Inspection of $f(R)$ reveals three critical pieces for establishing *global* coefficient restrictions: for R non-negative, $f(R)$ is *real* if $\left(1 + \frac{d_1 - n_1}{d_0 ln(q_0 / q_R)} R\right)$ is positive[17], *positive* if $\left\{(d_1 n_1 - d_0) + \frac{2n_1 R}{ln(q_0 / q_R)} + \left(\frac{R}{ln(q_0 / q_R)}\right)^2\right\}$ is the same, and *bounded* if $\lim_{R \to 0} Exp\left[\frac{R / ln(q_0 / q_R)}{(n_1 - d_1)}\right] = 0$. To meet the first condition, make $d_1 - n_1$ and d_0 have the same sign. To meet the second condition, impose either $n_1 \geq 0$ and $d_0 \leq d_1 n_1$ (to keep the parabola's roots, if real, sufficiently negative) or $n_1 < 0$ and $d_0 \leq d_1 n_1 - n_1{}^2$ (to prevent repeated real roots in the first place). To meet the third condition, constrain $n_1 < d_1$. To meet all three, impose $0 \leq n_1 < d_1$ and $0 \leq d_0 \leq d_1 n_1$. The same joint restrictions force the first derivative of $\frac{n_1 ln(q_0/q_R)s+s^2}{d_0[ln(q_0/q_R)]^2+d_1 ln(q_0/q_R)s+s^2}$ to be always positive and the second derivative to be always nonpositive. The resulting $f(R)$ curves all start at zero, increase to one or two modes, peter out for very long lifespans, and integrate properly to 1. The curves may be more or less skewed than in the T=1 case (to which the T=2 case reduces when d_0=n_1=0), although the right tail never looks quite as small as the left. Starting from the "zero moment" of 1 and the mean of

$2(d_1-n_1)\,ln(\frac{q_0}{q_R})\,e^{\frac{d_0}{(d_1-n_1)^2}}\left(\frac{d_0}{(d_1-n_1)^2}\right)^{-\frac{d_0-n_1(d_1-n_1)}{(d_1-n_1)^2}}\Gamma\left[1+\frac{d_0-n_1(d_1-n_1)}{(d_1-n_1)^2},\frac{d_0}{(d_1-n_1)^2}\right]$, higher-order uncentered moments follow recursively:

$$\mu_r = ln\left(\frac{q_0}{q_R}\right)\frac{r+1}{r-1}\left\{ln\left(\frac{q_0}{q_R}\right)rd_0\mu_{r-2}+[(r-1)d_1-rn_1]\mu_{r-1}\right\}.$$

The cohort-mean depreciation rate is n_1/d_0, again without higher moments.

Two special restrictions deserve comment. First, if d_0=$d_1 n_1$ *and* $n_1 = ½\, d_1$, then the moving part:

$$\frac{½\, d_1 ln(q_0/q_R)s+s^2}{½\, d_1^2[ln(q_0/q_R)]^2+d_1 ln(q_0/q_R)s+s^2},$$

is consistent with one of the integer-n Gamma distributions that inspired the rational estimating forms:

$$F(R)=1-\frac{1}{2}\Gamma\left(3,\frac{2R}{d_1\, ln(q_0/q_R)}\right) \qquad f(R)=4\frac{e^{\frac{-2R}{d_1 ln(q_0/q_R)}}R^2}{d_1^3[ln(q_0/q_R)]^3} \qquad f(\delta)=\frac{4}{d_1^3\,\delta^4\,e^{2/d_1\delta}},$$

with cohort-mean depreciation rate $1/d_1$, "scale parameter" $½\, d_1\, ln(q_0/q_R)$,and "shape parameter" n=3. Second, in the unconstrained form, if d_0 is large and imprecisely

[17] If $\frac{d_0}{(d_1-n_1)^2}-\frac{2d_1-n_1}{d_1-n_1}$ is an integer, then $\left(1+\frac{d_1-n_1}{d_0 ln(q_0/q_R)}R\right)$ may be negative, but this is very unlikely.

estimated and d_1 is near zero (sothis probably violates $n_1 \leq d_1$ and possibly $d_0 \leq d_1 n_1$), normalize the moving part by $d_0[ln(q_0/q_R)]^2$:

$$\frac{\frac{n_1}{d_0 ln(q_0/q_R)} s + \frac{s^2}{d_0[ln(q_0/q_R)]^2}}{1 + \frac{d_1}{d_0 ln(q_0/q_R)} s + \frac{s^2}{d_0[ln(q_0/q_R)]^2}},$$

If the composite parameters d_1/d_0 and $1/d_0$ are not statistically different from zero, then the hypothesis of a single Geometric rate across all members ($\delta = n_1/d_0$) cannot be rejected. Variations in retirement ages would depend solely on variations in q_R.

For T=3, the regression equation is:

$$ln\ q(s) = ln\ q_0 - ln(q_0/q_R) \frac{n_1[ln(q_0/q_R)]^2 s + n_2 ln(q_0/q_R) s^2 + s^3}{d_0[ln(q_0/q_R)]^3 + d_1[ln(q_0/q_R)]^2 s + d_2 ln(q_0/q_R) s^2 + s^3} + \varepsilon.$$

The same steps as in the T=1 and T=2 cases give the cumulative distribution function of retirement ages:

$$F(R) =$$

$$1 - e^{\frac{R/ln(q_0/q_R)}{(n_2-d_2)}} \left(1 + \frac{d_1[ln(\frac{q_0}{q_R})]^2 R + d_2 ln(\frac{q_0}{q_R}) R^2 + R^3}{d_0[ln(\frac{q_0}{q_R})]^3}\right) \cdot \left(1 + \frac{(d_1 - n_1) ln(\frac{q_0}{q_R}) R + (d_2 - n_2) R^2}{d_0[ln(\frac{q_0}{q_R})]^2}\right)^{\frac{d_1 - n_1 - (d_2 - n_2)(2d_2 - n_2)}{2(d_2 - n_2)^2}}$$

$$\cdot exp\left\{-\frac{(d_1 - n_1)^2 + (d_2 - n_2)\{2[n_1(d_2 - n_2) - d_0] - n_2(d_1 - n_1)\}}{(d_2 - n_2)^2 \sqrt{4d_0(d_2 - n_2) - (d_1 - n_1)^2}} arctan\left[\frac{R}{ln(\frac{q_0}{q_R})} \frac{\sqrt{4d_0(d_2 - n_2) - (d_1 - n_1)^2}}{2d_0 + (d_1 - n_1)R/ln(q_0/q_R)}\right]\right\},$$

the probability density function of retirement ages:

$$f(R) = \frac{R}{d_0^2[ln(\frac{q_0}{q_R})]^2} \cdot \left\{1 + \frac{d_1 - n_1}{d_0} \frac{R}{ln(\frac{q_0}{q_R})} + \frac{d_2 - n_2}{d_0}\left(\frac{R}{ln(\frac{q_0}{q_R})}\right)^2\right\}^{\frac{d_1 - n_1 - (d_2 - n_2)(4d_2 - 3n_2)}{2(d_2 - n_2)^2}}$$

$$\cdot \left[(d_1 n_1 - d_0 n_2) + 2(d_2 n_1 - d_0)\frac{R}{ln(\frac{q_0}{q_R})} + (3n_1 - d_1 + d_2 n_2)\left(\frac{R}{ln(\frac{q_0}{q_R})}\right)^2 + 2n_2\left(\frac{R}{ln(\frac{q_0}{q_R})}\right)^3 + \left(\frac{R}{ln(\frac{q_0}{q_R})}\right)^4\right]$$

$$\cdot Exp\left\{\frac{R/ln(q_0/q_R)}{(n_2-d_2)}-\frac{(d_1-n_1)^2+(d_2-n_2)\{2[n_1(d_2-n_2)-d_0]-n_2(d_1-n_1)\}}{(d_2-n_2)^2\sqrt{4d_0(d_2-n_2)-(d_1-n_1)^2}}Arctan\left[\frac{R}{ln(\frac{q_0}{q_R})}\frac{\sqrt{4d_0(d_2-n_2)-(d_1-n_1)^2}}{2d_0+(d_1-n_1)R/ln(q_0/q_R)}\right]\right\}.$$

and the probability density function of depreciation rates:

$$f(\delta)=$$

$$\frac{1}{d_0^2\delta^3}\cdot\left\{1+\frac{d_1-n_1}{d_0\delta}+\frac{d_2-n_2}{d_0\delta^2}\right\}^{\frac{d_1-n_1-(d_2-n_2)(4d_2-3n_2)}{2(d_2-n_2)^2}}\left\{(d_1n_1-d_0n_2)+2\frac{d_2n_1-d_0}{\delta}+\frac{3n_1-d_1+d_2n_2}{\delta^2}+2\frac{n_2}{\delta^3}+\frac{1}{\delta^4}\right\}$$

$$\cdot Exp\left\{\frac{1}{(n_2-d_2)\delta}-\frac{(d_1-n_1)^2+(d_2-n_2)\{2[n_1(d_2-n_2)-d_0]-n_2(d_1-n_1)\}}{(d_2-n_2)^2\sqrt{4d_0(d_2-n_2)-(d_1-n_1)^2}}Arctan\left[\frac{\sqrt{4d_0(d_2-n_2)-(d_1-n_1)^2}}{2d_0\delta+(d_1-n_1)}\right]\right\}.$$

While it is possible to construct distribution functions for $T>3$, in practice the algebraic forms the functions take—i.e., when natural logs appear *versus* arctangents—will depend on the fitted values of the d_i's and n_j's. Moreover, $T=2$ is probably flexible enough, as it even allows double-humped retirement distributions.

To sum up, this appendix offers a way to estimate the whole distribution of Geometric depreciation rates inherent in a cohort of assets from available resale price data, without prior knowledge of the pattern of retirements. The next steps are to generalize the depreciation function without sacrificing the generality of the implied retirement function and to test the stability of the estimated parameters.

Table 1. Selected Summary Statistics from the MRIS Database

Variable	Mean	Standard Error	Minimum	Maximum
Net Price	222650.7	169423.9	10000	14361193
Gross Price	225170.3	169429.1	10000	14370018
Seller Subidy	2519.6	3398.8	0	200000
Age	22.908	23.574	0	300
1 Bedroom Exactly	0.001	0.027	0	1
2 Bedroom Exactly	0.042	0.201	0	1
3 Bedrooms Exactly	0.453	0.498	0	1
4 Bedrooms Exactly	0.392	0.488	0	1
5+ Bedrooms	0.112	0.315	0	1
1 Full Bath Exactly	0.157	0.364	0	1
2 Full Baths Exactly	0.574	0.495	0	1
3 Full Baths Exactly	0.228	0.419	0	1
4 Full Baths Exactly	0.033	0.178	0	1
5+ Full Baths	0.009	0.093	0	1
1 Half Bath Exactly	0.772	0.419	0	1
2 Half Baths Exactly	0.117	0.321	0	1
3 Half Baths Exactly	0.002	0.045	0	1
4+ Half Baths	0.000	0.016	0	1
New house Dummy	0.077	0.267	0	1
Acreage	0.424	0.902	0.032002	10
Ln($ value of Improvements)	3.769	10.119	-9.21034	19.10473
1996 Year Dummy	0.119	0.324	0	1
1997 Year Dummy	0.196	0.397	0	1
1998 Year Dummy	0.254	0.435	0	1
1999 Year Dummy	0.301	0.459	0	1
2000 Year Dummy	0.053	0.224	0	1
2 Master BR	0.020	0.139	0	1
2nd Stry Fam Ovrlk	0.007	0.086	0	1
2nd Stry Fam Rm	0.013	0.111	0	1
Attic-Finished	0.016	0.125	0	1
Attic-Unfinished	0.110	0.313	0	1
Den/Stdy/Lib	0.230	0.421	0	1
Encl Glass Prch	0.023	0.150	0	1
Family Room	0.475	0.499	0	1
Florida/Sun Rm	0.044	0.206	0	1
Game/Exer Rm	0.043	0.202	0	1
Great Room	0.020	0.140	0	1
In-Law/auPair/Ste	0.047	0.212	0	1
Laundry-BR Lvl	0.068	0.251	0	1
Laundry-Kit Lvl	0.202	0.402	0	1
Loft	0.023	0.149	0	1
Maids Rm/Quart	0.013	0.112	0	1
Main Lvl BR	0.086	0.281	0	1
MBR w/Sit Rm	0.074	0.261	0	1
Mud Room	0.047	0.212	0	1
Other	0.011	0.105	0	1
Photo Lab/Darkroom	0.002	0.045	0	1
Professional Off	0.013	0.113	0	1

Recreation Room	0.341	0.474	0	1
Solarium	0.006	0.074	0	1
Storage Room	0.202	0.401	0	1
Utility Room	0.320	0.467	0	1
Workshop	0.091	0.288	0	1
Attach/Row House	0.048	0.214	0	1
Back-to-Back	0.000	0.014	0	1
Detached House	0.664	0.472	0	1
Double Wide	0.000	0.013	0	1
Duplex	0.004	0.066	0	1
Dwelling w/Rental	0.000	0.012	0	1
Mobile	0.000	0.014	0	1
Barn/Stable	0.005	0.071	0	1
Cabana/Pool Hse	0.002	0.045	0	1
Carriage House	0.001	0.031	0	1
Gazebo	0.006	0.076	0	1
Greenhouse	0.002	0.042	0	1
Guest House	0.001	0.039	0	1
Office/Studio	0.004	0.060	0	1
Shed	0.165	0.371	0	1
Tenant House	0.000	0.020	0	1
Balcony	0.024	0.154	0	1
Deck	0.484	0.500	0	1
Fenced - Fully	0.095	0.294	0	1
Fenced - Partially	0.115	0.319	0	1
Fenced - Rear	0.323	0.468	0	1
Hot Tub	0.022	0.148	0	1
Other	0.045	0.208	0	1
Patio	0.288	0.453	0	1
Pool (Above Ground)	0.007	0.084	0	1
Pool (In-Ground)	0.028	0.164	0	1
Porch-front	0.156	0.363	0	1
Porch-rear	0.017	0.129	0	1
Porch-screened	0.062	0.241	0	1
Porch-wraparound	0.010	0.099	0	1
Private Beach	0.001	0.032	0	1
Private Pier	0.003	0.056	0	1
Private Road	0.007	0.085	0	1
Roof Deck	0.004	0.065	0	1
Secure Storage	0.001	0.028	0	1
Side Porch	0.001	0.024	0	1
Sidewalks	0.268	0.443	0	1
Sport Court	0.000	0.010	0	1
Number of fire places	0.955	0.802	0	7
Number of garage spaces	0.949	0.977	0	8
Number of floors	2.745	1.268	0	6
A-Frame	0.001	0.026	0	1
Art Deco	0.000	0.009	0	1
Beaux Arts	0.000	0.015	0	1
Bilevel	0.004	0.061	0	1
Bungalow	0.006	0.075	0	1
Cape Cod	0.036	0.185	0	1

Chalet	0.001	0.028	0	1
Colonial	0.629	0.483	0	1
Contemporary	0.057	0.232	0	1
Cottage	0.002	0.045	0	1
Farm House	0.004	0.060	0	1
Federal	0.008	0.089	0	1
International	0.000	0.018	0	1
Log Home	0.001	0.024	0	1
Other	0.048	0.214	0	1
Provincial	0.001	0.026	0	1
Raised Rambler	0.002	0.049	0	1
Raised Rancher	0.001	0.035	0	1
Rambler	0.072	0.259	0	1
Rancher	0.020	0.139	0	1
Spanish	0.000	0.018	0	1
Split Foyer	0.034	0.182	0	1
Split Level	0.052	0.221	0	1
Tudor	0.004	0.065	0	1
Victorian	0.015	0.123	0	1
Center Hall	0.100	0.300	0	1
Foyer	0.646	0.478	0	1
Hall	0.050	0.218	0	1
Living Room	0.143	0.350	0	1
Lower Level	0.014	0.118	0	1
Other	0.005	0.073	0	1
Side	0.006	0.078	0	1
Split Foyer	0.012	0.107	0	1
Two Story Foyer	0.051	0.221	0	1
Alum/Steel Siding	0.195	0.396	0	1
Brick	0.268	0.443	0	1
Brick and Siding	0.342	0.474	0	1
Brick Front	0.076	0.265	0	1
Cedar Siding	0.020	0.139	0	1
Composition	0.008	0.086	0	1
Concrete/Block	0.007	0.082	0	1
Frame	0.019	0.138	0	1
Metal	0.000	0.019	0	1
Mod/Manuf	0.000	0.016	0	1
Other	0.012	0.110	0	1
Shake	0.006	0.079	0	1
Shingle	0.022	0.146	0	1
Stone	0.028	0.165	0	1
Stucco	0.020	0.139	0	1
Vinyl Siding	0.175	0.380	0	1
Wood	0.063	0.243	0	1
District of Columbia	0.038	0.192	0	1
Maryland	0.361	0.480	0	1
West Virginia	0.012	0.108	0	1
Pennsylvania	0.007	0.084	0	1
Virginia	0.581	0.493	0	1
Anne Arundel County	0.031	0.173	0	1
Arlington County	0.027	0.162	0	1

Baltimore City	0.077	0.266	0	1
Baltimore County	0.024	0.153	0	1
Fairfax County	0.306	0.461	0	1
Howard County	0.046	0.210	0	1
Montgomery County	0.144	0.351	0	1
Prince Georges County	0.039	0.194	0	1
Prince William County	0.080	0.272	0	1

Table 2. Selected Summary Statistics from the MRIS Database

Variable	Mean	Standard Error	Minimum	Maximum
Net Price excluding land	149559.1	108523.2	5571.43	3165616
Land value	72209.4	56888.4	2833.33	2214306
Gross Price	224476.5	156739.3	13000	3750000
Sell Subsidy	2708.0	3257.3	0	200000
Age	24.887	23.119	0	300
Land assessment in $	64320.0	45292.5	2000	1221000
Total assessment in $	198532.9	127309.1	7000	3370520
1 Bedroom Exactly	0.000	0.021	0	1
2 Bedroom Exactly	0.039	0.193	0	1
3 Bedrooms Exactly	0.469	0.499	0	1
4 Bedrooms Exactly	0.374	0.484	0	1
5+ Bedrooms	0.118	0.323	0	1
1 Full Bath Exactly	0.165	0.372	0	1
2 Full Baths Exactly	0.568	0.495	0	1
3 Full Baths Exactly	0.226	0.418	0	1
4 Full Baths Exactly	0.033	0.178	0	1
5+ Full Baths	0.009	0.092	0	1
1 Half Bath Exactly	0.806	0.395	0	1
2 Half Baths Exactly	0.134	0.341	0	1
3 Half Baths Exactly	0.002	0.047	0	1
4+ Half Baths	0.000	0.016	0	1
New Home Dummy	0.007	0.084	0	1
Acreage	0.355	0.766	0.032002	10
Ln($ value of Improvements)	11.668	0.508	-9.21034	19.10473
District of Columbia	0.046	0.210	0	1
Maryland	0.468	0.499	0	1
West Virginia	0.004	0.065	0	1
Pennsylvania	0.000	0.005	0	1
Virginia	0.482	0.500	0	1

Table 3. Estimated Parameters of the Single Family House Price Model with Different Location Dummy Variables, Land & Structure Value

Variable	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
Depreciation rate * 100%	-0.09	0.003	-0.01	0.004	0.10	0.004
Ln($value of improvments)	-0.00054	0.00006	-0.00038	0.000075	0.0007	0.0009
2 Bedroom Exactly	0.085	0.017	0.062	0.020	0.093	0.024
3 Bedrooms Exactly	0.145	0.017	0.100	0.020	0.107	0.023
4 Bedrooms Exactly	0.197	0.017	0.148	0.020	0.160	0.024
5+ Bedrooms	0.226	0.017	0.185	0.020	0.190	0.024
2 Full Baths Exactly	0.153	0.002	0.188	0.002	0.208	0.002
3 Full Baths Exactly	0.257	0.002	0.328	0.002	0.365	0.003
4 Full Baths Exactly	0.381	0.004	0.503	0.004	0.539	0.005
5+ Full Baths	0.528	0.006	0.668	0.007	0.671	0.008
1 Half Bath Exactly	0.086	0.002	0.096	0.002	0.094	0.003
2 Half Baths Exactly	0.143	0.003	0.172	0.002	0.183	0.003
3 Half Baths Exactly	0.178	0.010	0.198	0.012	0.176	0.014
4+ Half Baths	0.105	0.028	0.085	0.034	0.092	0.039
acreage	0.038	0.001	0.031	0.001	-.004	0.001
New construction dummy	0.159	0.003	0.180	0.003	0.193	0.003
Contract Year Dummies						
1996	-0.007	0.002	-0.011	0.002	-0.016	0.003
1997	-0.013	0.003	-0.017	0.003	-0.046	0.004
1998	0.019	0.003	0.014	0.003	-0.021	0.004
1999	0.075	0.003	0.067	0.003	0.022	0.004
2000	0.133	0.004	0.117	0.004	0.065	0.005
State Dummies	NO		NO		**YES**	
County Dummies	NO		**YES**		NO	
Zip code dummies	**YES**		NO		NO	
Contract Month Dummies	YES		YES		YES	
Other buildings dummies	YES		YES		YES	
Other rooms by type dummies	YES		YES		YES	
Heating & cooling dummies	YES		YES		YES	
lot description dummies	YES		YES		YES	
acreage * county dummies	YES		YES		YES	
view dummies	YES		YES		YES	
water proximity dummies	YES		YES		YES	
house style dummies	YES		YES		YES	
construction materials dummies	YES		YES		YES	
N	163310		163310		163310	
DF	162341		162846		162900	
R-squared	0.8905		0.8392		0.7830	

Table 4 Estimated Parameters of the Single Family House Price Model with Different Location Dummy Variables, Structure Value Only

Variable	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
Depreciation rate * 100%	0.01	0.003	0.05	0.003	0.10	0.003
Ln($value of improvments)	0.655	0.005	0.731	0.002	0.778	0.002
2 Bedroom Exactly	0.006	0.019	-0.020	0.020	-0.034	0.022
3 Bedrooms Exactly	0.048	0.019	0.012	0.020	-0.013	0.022
4 Bedrooms Exactly	0.076	0.019	0.033	0.020	0.003	0.022
5+ Bedrooms	0.095	0.019	0.049	0.020	0.015	0.022
2 Full Baths Exactly	0.067	0.002	0.063	0.002	0.057	0.002
3 Full Baths Exactly	0.109	0.002	0.107	0.002	0.100	0.002
4 Full Baths Exactly	0.159	0.003	0.153	0.003	0.142	0.004
5+ Full Baths	0.227	0.005	0.215	0.006	0.195	0.006
1 Half Bath Exactly	0.034	0.002	0.030	0.002	0.014	0.002
2 Half Baths Exactly	0.056	0.002	0.052	0.002	0.035	0.003
3 Half Baths Exactly	0.082	0.009	0.070	0.009	0.041	0.010
4+ Half Baths	0.041	0.025	0.015	0.026	-0.012	0.028
acreage						
New construction dummy	0.078	0.004	0.077	0.005	0.072	0.005
Contract Year Dummies						
1996	0.019	0.019	0.025	0.020	0.016	0.021
1997	0.027	0.019	0.037	0.019	0.034	0.021
1998	0.054	0.019	0.064	0.019	0.057	0.021
1999	0.110	0.019	0.119	0.019	0.103	0.021
2000	0.166	0.019	0.172	0.020	0.151	0.021
State Dummies	NO		NO		**YES**	
County Dummies	NO		**YES**		NO	
Zip code dummies	**YES**		NO		NO	
Contract Month Dummies	YES		YES		YES	
Other buildings dummies	YES		YES		YES	
Other rooms by type dummies	YES		YES		YES	
Heating & cooling dummies	YES		YES		YES	
lot description dummies	YES		YES		YES	
acreage * county dummies	NO		NO		NO	
view dummies	YES		YES		YES	
water proximity dummies	YES		YES		YES	
house style dummies	YES		YES		YES	
construction materials dummies	YES		YES		YES	
N	96300		96300		96300	
DF	95525		95986		95934	
R-squared	0.9461		0.9397		0.9318	

Table 5. Estimated Parameters for Age-Price Model using Hulten-Wykoff Adjustment, Land & Structure Value

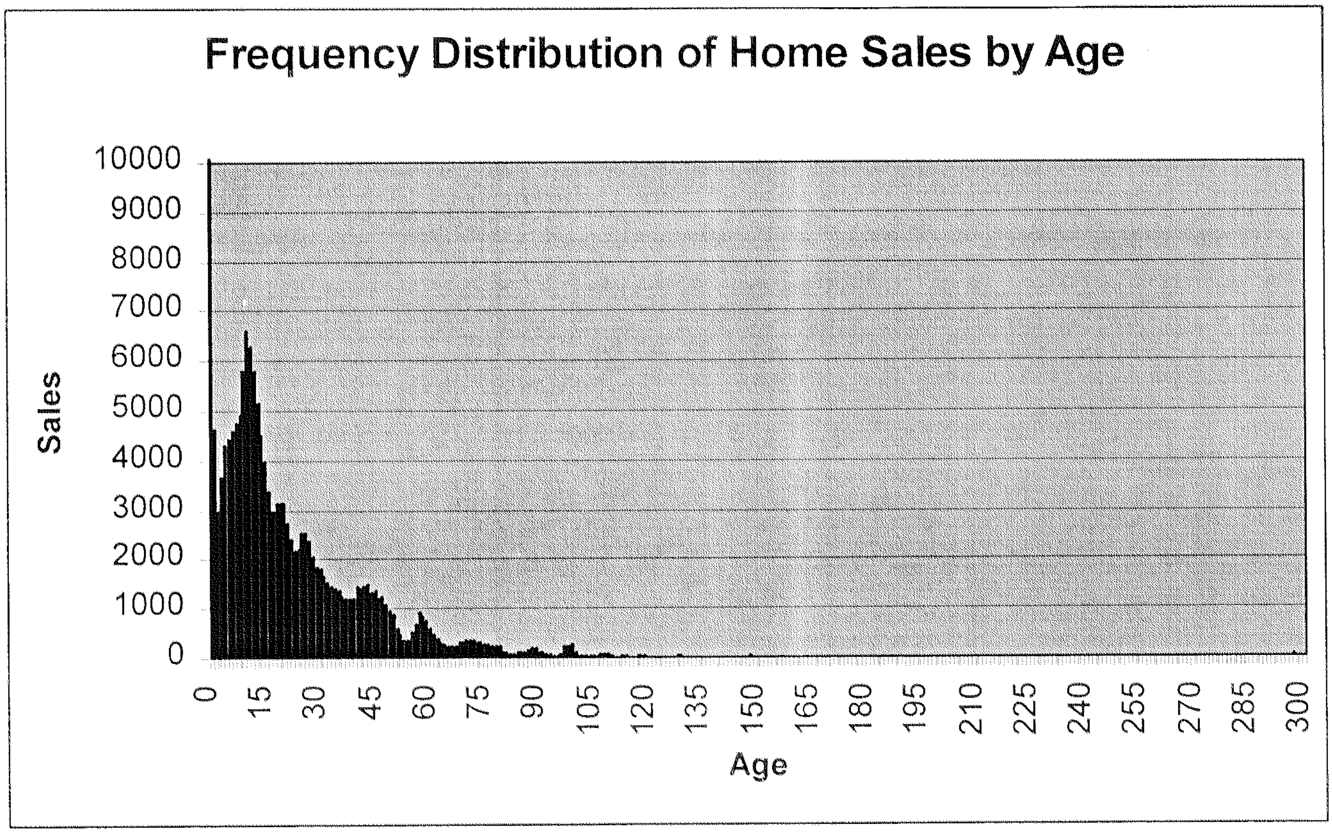
Variable	80 year mean discard Estimate (1)	Std. Error	120 year mean discard Estimate (2)	Std. Error	160 year mean discard Estimate (3)	Std. Error
Depreciation rate * 100%	-1.93	0.006	-1.15	0.005	-0.47	0.004
Ln($value of improvments)	0.000083	0.00008	-0.000014	0.00008	-0.000366	0.00007
2 Bedroom Exactly	0.135	0.023	0.117	0.02	0.115	0.018
3 Bedrooms Exactly	0.209	0.022	0.183	0.02	0.174	0.018
4 Bedrooms Exactly	0.268	0.023	0.24	0.02	0.228	0.018
5+ Bedrooms	0.304	0.022	0.28	0.02	0.261	0.018
2 Full Baths Exactly	0.069	0.002	0.096	0.002	0.133	0.002
3 Full Baths Exactly	0.128	0.003	0.174	0.003	0.228	0.003
4 Full Baths Exactly	0.219	0.005	0.277	0.004	0.344	0.004
5+ Full Baths	0.353	0.008	0.426	0.007	0.498	0.006
1 Half Bath Exactly	0.032	0.002	0.051	0.002	0.075	0.002
2 Half Baths Exactly	0.075	0.003	0.101	0.003	0.13	0.003
3 Half Baths Exactly	0.122	0.013	0.158	0.012	0.174	0.011
4+ Half Baths	0	0.037	0.091	0.034	0.126	0.029
acreage	0.036	0.001	0.038	0.001	0.037	0.001
New construction dummy	0.003	0.003	0.063	0.003	0.125	0.003
Contract Year Dummies						
1996	0.009	0.002	0.001	0.003	-0.004	0.002
1997	0.015	0.003	0.004	0.003	-0.007	0.003
1998	0.059	0.003	0.046	0.003	0.027	0.003
1999	0.127	0.004	0.111	0.003	0.085	0.003
2000	0.195	0.004	0.178	0.004	0.146	0.004
State Dummies	NO		NO		NO	
County Dummies	NO		NO		NO	
Zip code dummies	YES		YES		YES	
Contract Month Dummies	YES		YES		YES	
Other buildings dummies	YES		YES		YES	
Other rooms by type dummies	YES		YES		YES	
Heating & cooling dummies	YES		YES		YES	
lot description dummies	YES		YES		YES	
acreage * county dummies	YES		YES		YES	
view dummies	YES		YES		YES	
water proximity dummies	YES		YES		YES	
house style dummies	YES		YES		YES	
construction materials dummies	YES		YES		YES	
N	162829		163308		163308	
DF	161862		162240		162340	
R-squared	0.8635		0.8584		0.8811	
Average Discard Age	51.81		86.69		177.8	

Table 6. Single Family House and Land Pricing Model With Weibull Distribution of Discards

Variable	No scrap value Estimate (1)	Std. Error	Scrap Value Estimate (2)	Std. Error	Scrap Value Estimate (3)	Std. Error
Depreciation rate * 100%	-1.86	0.089	-1.20	0.073	-0.54	0.095
M	67.79	2.97	88.26	3.21	102.98	0.52
n	1.354	0.017	1.542	0.045	2.952	0.039
Scrap value			0.036	0.008	0.4	fixed constant
Ln($value of improvements)	-0.000194	0.00006	-0.000285	0.000062	-0.0004	0.00061
2 Bedroom Exactly	0.092	0.017	0.091	0.017	0.089	0.017
3 Bedrooms Exactly	0.156	0.017	0.154	0.017	0.152	0.016
4 Bedrooms Exactly	0.215	0.017	0.212	0.017	0.207	0.016
5+ Bedrooms	0.249	0.017	0.244	0.017	0.238	0.017
2 Full Baths Exactly	0.132	0.002	0.134	0.002	0.139	0.002
3 Full Baths Exactly	0.221	0.002	0.226	0.002	0.234	0.002
4 Full Baths Exactly	0.333	0.004	0.34	0.004	0.351	0.004
5+ Full Baths	0.476	0.006	0.484	0.006	0.496	0.006
1 Half Bath Exactly	0.075	0.002	0.076	0.002	0.078	0.002
2 Half Baths Exactly	0.128	0.003	0.13	0.002	0.132	0.002
3 Half Baths Exactly	0.168	0.010	0.168	0.01	0.169	0.01
4+ Half Baths	0.085	0.029	0.092	0.028	0.095	0.027
acreage	0.038	0.001	0.038	0.00096	0.038	0.00096
New construction dummy	0.077	0.003	0.098	0.003	0.125	0.002
Contract Year Dummies						
1996	-0.001	0.002	-0.003	0.002	-0.004	0.002
1997	-0.003	0.003	-0.004	0.003	-0.007	0.002
1998	0.033	0.003	0.031	0.003	0.027	0.003
1999	0.093	0.003	0.09	0.003	0.085	0.003
2000	0.157	0.004	0.153	0.003	0.146	0.003
State Dummies	NO		NO		NO	
County Dummies	NO		NO		NO	
Zip code dummies	YES		YES		YES	
Contract Month Dummies	YES		YES		YES	
Other buildings dummies	YES		YES		YES	
Other rooms by type dummies	YES		YES		YES	
Heating & cooling dummies	YES		YES		YES	
lot description dummies	YES		YES		YES	
acreage * county dummies	YES		YES		YES	
view dummies	YES		YES		YES	
water proximity dummies	YES		YES		YES	
house style dummies	YES		YES		YES	
construction materials dummies	YES		YES		YES	
N	163310		163310		163310	
DF	162339		162338		162339	
R-squared	0.9998		0.9998		0.9998	
Average Discard Age	62.63		80.02		92.39	

Table 7. Single Family House (Structure Only) Pricing Model With Weibull Distribution of Discards

Variable	No scrap value Estimate (1)	Std. Error	Scrap Value Estimate (2)	Std. Error
Depreciation rate * 100%	-1.18	0.005	-0.51	0.055
m	90.63	38.75	111.93	1.34
n	1.13	0.062	1.71	0.09
Scrap value			0.267	0.049
Ln($value of improvments)	0.639	0.002	0.642	0.002
2 Bedroom Exactly	0.006	0.02	0.003	0.019
3 Bedrooms Exactly	0.052	0.02	0.049	0.019
4 Bedrooms Exactly	0.084	0.02	0.08	0.019
5+ Bedrooms	0.106	0.02	0.101	0.019
2 Full Baths Exactly	0.062	0.002	0.062	0.002
3 Full Baths Exactly	0.1	0.002	0.101	0.002
4 Full Baths Exactly	0.147	0.003	0.15	0.003
5+ Full Baths	0.216	0.006	0.218	0.005
1 Half Bath Exactly	0.033	0.002	0.033	0.002
2 Half Baths Exactly	0.054	0.002	0.054	0.002
3 Half Baths Exactly	0.083	0.009	0.083	0.009
4+ Half Baths	0.036	0.025	0.04	0.025
acreage				
New construction dummy	0.044	0.005	0.056	0.005
Contract Year Dummies				
1996	0.021	0.02	0.02	0.019
1997	0.033	0.019	0.031	0.019
1998	0.061	0.02	0.059	0.019
1999	0.119	0.02	0.116	0.019
2000	0.178	0.02	0.175	0.019
State Dummies	NO		NO	
County Dummies	NO		NO	
Zip code dummies	YES		YES	
Contract Month Dummies	YES		YES	
Other buildings dummies	YES		YES	
Other rooms by type dummies	YES		YES	
Heating & cooling dummies	YES		YES	
lot description dummies	YES		YES	
acreage * county dummies	NO		NO	
view dummies	YES		YES	
water proximity dummies	YES		YES	
house style dummies	YES		YES	
construction materials dummies	YES		YES	
N	96301		96301	
DF	95523		95522	
R-squared	0.9999		0.9999	
Average Discard Age	84.99		100.26	


Frequency Distribution of Home Sales by Age
Sales
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
0
15
30
45
60
75
90
105
120
135
150
165
180
195
210
225
240
255
270
285
300
Age

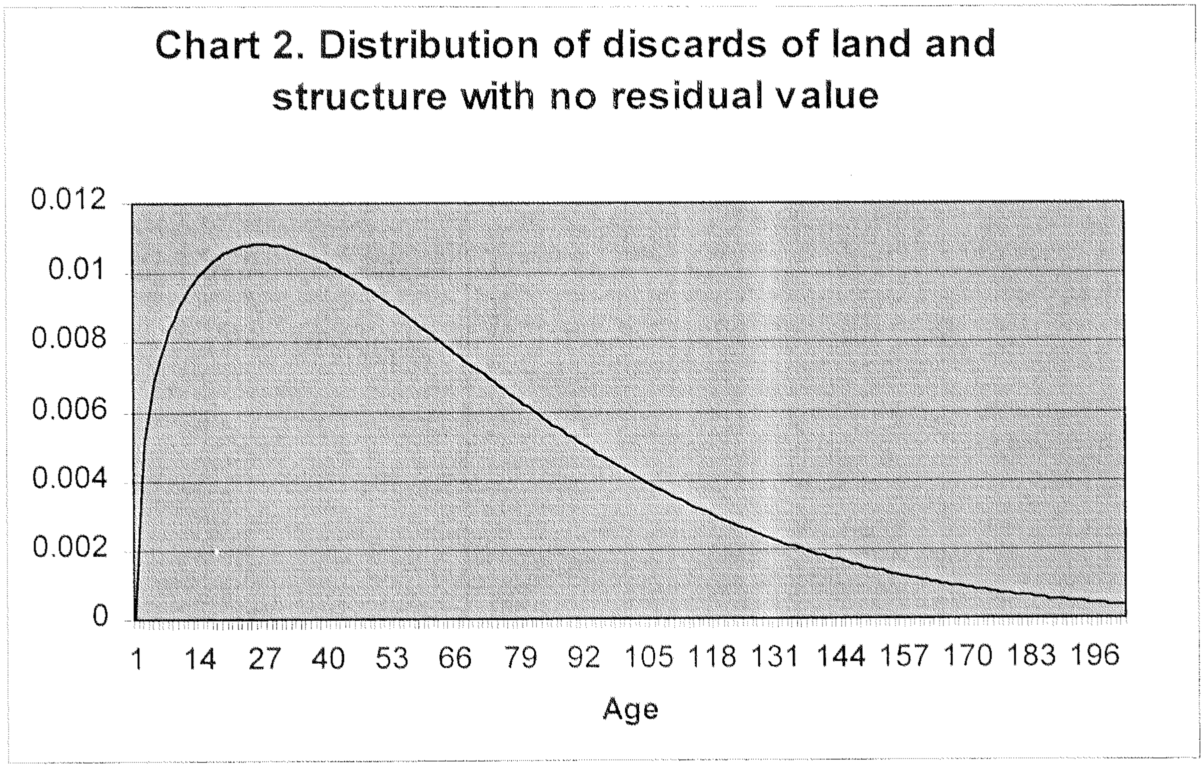


Chart 2. Distribution of discards of land and structure with no residual value
0.012
0.01
0.008
0.006
0.004
0.002
0
1 14 27 40 53 66 79 92 105 118 131 144 157 170 183 196
Age

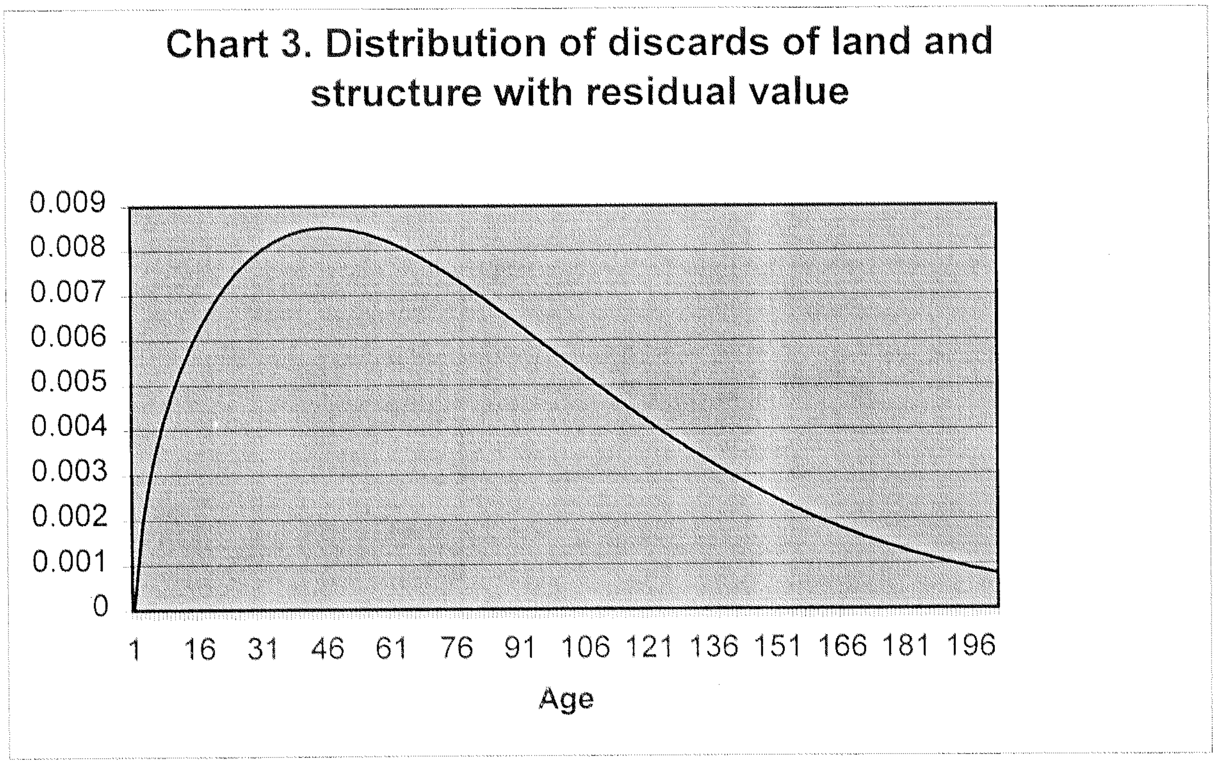


Chart 3. Distribution of discards of land and structure with residual value
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
1 16 31 46 61 76 91 106 121 136 151 166 181 196
Age

Chart 4. Distribution of discards of land and structure with residual value set at 29%

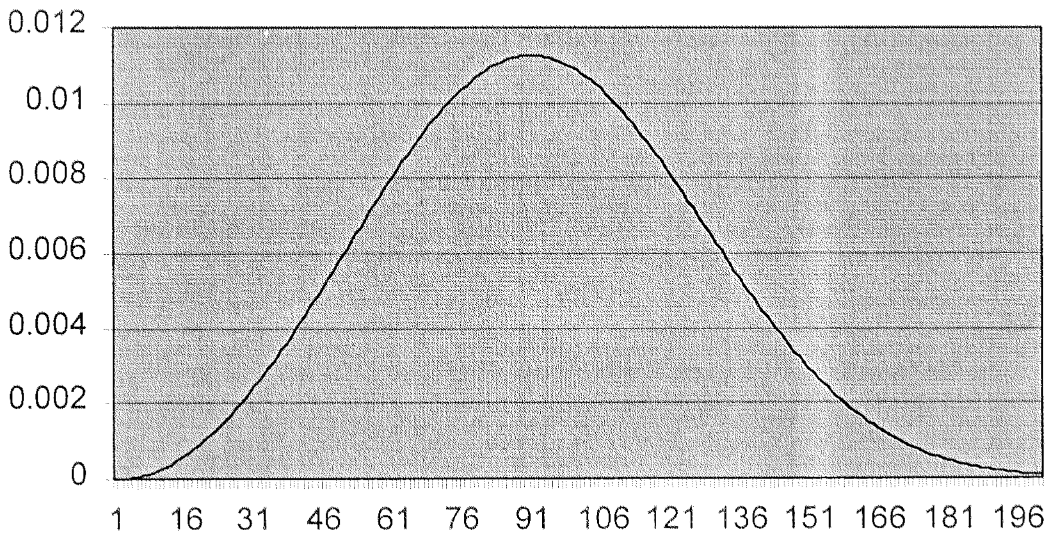
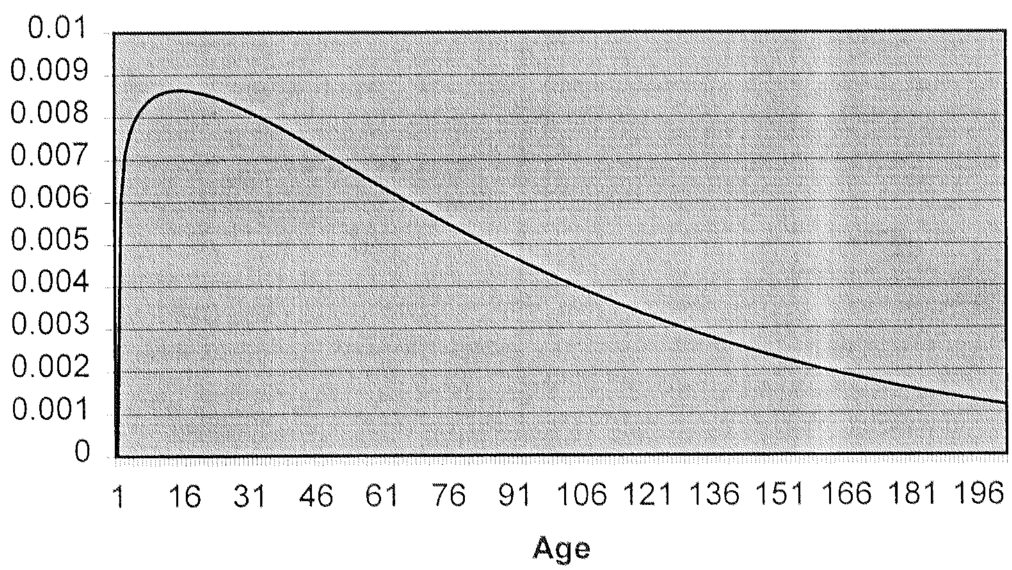


Chart 5. Distribution of discards of structures only with no residual value



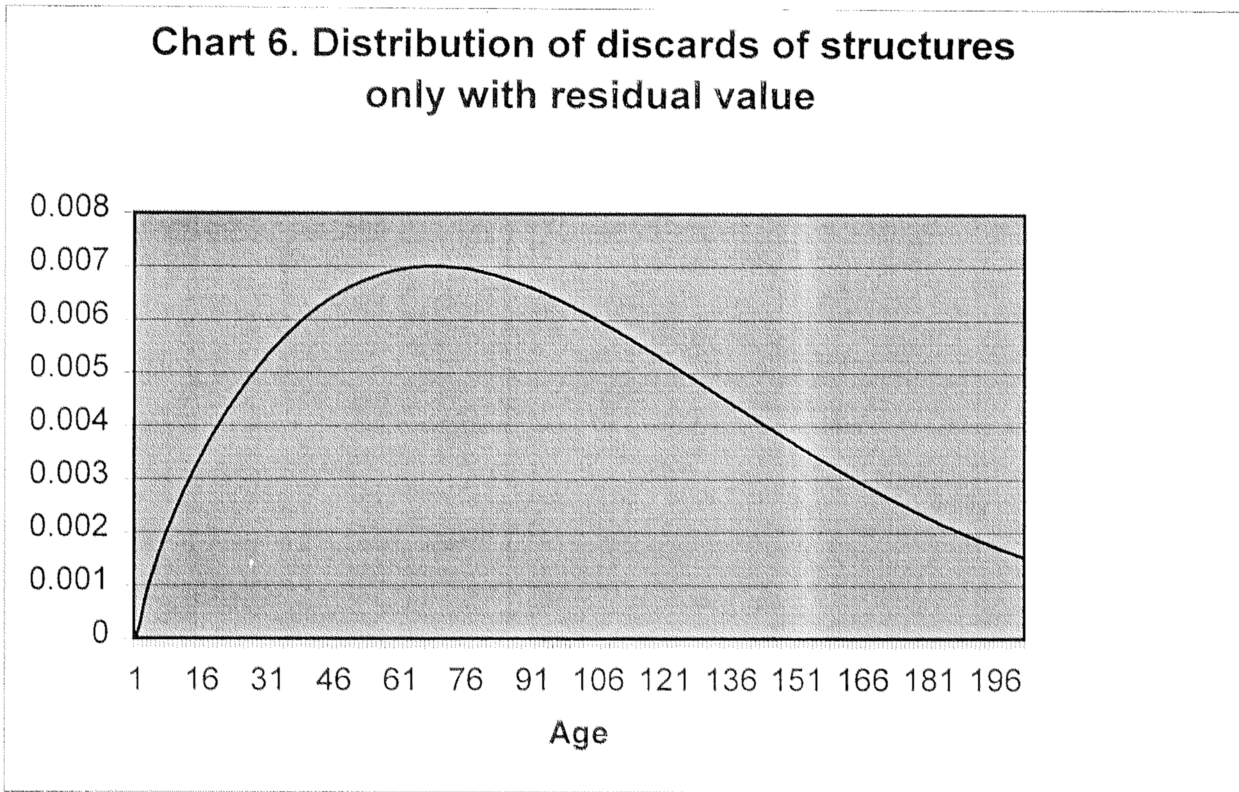


Chart 6. Distribution of discards of structures only with residual value
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
1 16 31 46 61 76 91 106 121 136 151 166 181 196
Age

Chart 7: Hypothetical age-price patterns