An Optimizing Model of U.S. Wage and Price Dynamics

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First draft December 1999
Revised, March 2001

Abstract

The objective of this paper is to provide an optimizing model of wage and price setting consistent with U.S. data. The paper first investigates the predictions of an optimizing labor supply model for the aggregate nominal wage, taking as given the evolution of prices and quantities. In this part it seeks to determine whether a standard specification of consumption/leisure preferences is consistent with the data, and to what extent nominal or real rigidities in the wage setting process improve the fit with the data. Then it combines the evolution of wages predicted by this model with the evolution of prices predicted by staggered-price models to provide a model of the joint determination of prices and wages, given the evolution of real quantities. It thus supplies a "Phillips curve" specification that is consistent with intertemporal optimization and rational expectations.

JEL Classification: E31; E32.

Key words: Wage dynamics; Inflation; Phillips Curve.

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†I thank Michael Woodford for his invaluable advice, and seminar participants at the Board of Governors, New York University, Princeton University, and Rutgers University for their comments.

1. Introduction

This paper is an attempt to model the joint behavior of prices and wages in a way consistent with intertemporal optimization and rational expectations. Its ultimate goal is to construct a 'Phillips curve' specification that is consistent both with U.S. data and with optimizing behavior, to respond to the well known "Lucas critique".

The Phillips curve relationship has undergone a fruitful re-exploration in recent years. The effort has been devoted to explain the relation between nominal and real variables in rigorously specified general equilibrium, optimizing models[1]. For example, the so-called "New Keynesian" Phillips Curve (NKPC), which describes current inflation as a function of expected future inflation and a measure of output gap, is derived in the context of a general equilibrium, optimizing model, that allows some form of nominal rigidities, either by assuming staggered price-setting (for example, in the style of Calvo (1983) model), or by assuming staggered wage-setting, or both (for ex. Erceg et al. 1999)[2].

Models with nominal rigidities have been explored mostly in the context of monetary policy analysis. Providing a channel for real effects of monetary disturbances, staggered wage and price settings are in fact a suitable framework to investigate issues such as the optimality of alternative monetary policies.

However, the standard NKPC model predicts counterfactual comovement of output and inflation, unless there are large cyclical variations in potential output. For this reason, there have been some attempts to dismiss altogether the particular model of price-setting that lies at the heart of the model.

Some recent studies, in particular, have questioned the importance of the forward-looking component in pricing behavior, by focusing on the empirical failure of the inflation-output equation that it implies. For example, Fuhrer's (1997) empirical results point to a negligible role of future inflation in an estimated inflation-output relationship, specified in a way that is intended to nest the 'New Keynesian' Phillips Curve specification, the more complex variant proposed by Fuhrer and Moore (1995), and purely backward-looking Phillips Curve specifications. Roberts (1997, 1998) argues instead that the New Keynesian Phillips Curve fits reasonably well when survey measures are used to approximate inflation expectations, but that it does not fit well under the hypothesis of rational expectations. He thus proposes a model with an important backward-looking component in inflation expectations, which amounts to weakening the weight put on the forward-looking terms in his aggregate supply relation.

Some other recent work has shown, however, that, unlike tests of the standard NKPC model, tests of the pricing equation alone, derived from a staggering price model, seem to fit inflation data quite well, providing empirical support for the hypothesis of nominal price rigidity, and for the importance of forward-looking determinants of price-setting behavior[3].

[1] See the contributions in the special issue of the Journal of Monetary Economics (1999).

[2] An early estimation of such a curve is in Roberts (1995).

[3] See Sbordone (1998, 1999) and Gali and Gertler (1999). Both contributions use unit labor costs to proxy

In particular, Sbordone (1998) shows that, taking as given the evolution of unit labor costs, the dynamic of inflation predicted by sticky price models tracks actual data very closely, and imply a degree of price stickiness very much in line with that found through survey evidence.

But if one accepts the hypothesis that the evolution of inflation is well described by the evolution of future labor costs, then one should argue that the failure of NKPC models is not due to the theoretical link between inflation and real marginal costs, but may be due to the additional assumptions commonly made about the relationship between marginal costs, and therefore wages, and output.

In this paper I therefore seek to develop a more accurate optimizing model of wage dynamics: I first investigate, using a partial equilibrium approach, the predictions of an optimal labor supply model for the aggregate nominal wage, taking as given the evolution of prices and quantities. Together with the evolution of productivity, this model yields a quantitative model of the evolution of unit labor costs.

Then, combining the predictions of this model with the predictions of an optimizing price-setting model for the evolution of the aggregate price level, I provide a joint model of price and wage dynamics, taking as given the evolution of real quantities.

In developing the wage model, I start from the baseline optimizing model used in standard RBC models. In the wage-setting sector, a representative household chooses hours of work to maximize an expected lifetime utility function. The optimality condition for labor supply gives a desired real wage as a function of consumption and hours. Then I consider the hypothesis that the actual real wage adjusts only sluggishly to the desired wage, and compare the prediction of models with perfectly flexible wages to those of models with different kinds of wage rigidities[4].

The price-setting side of the model has one sector of production, monopolistic competition, and nominal price rigidity: these assumptions deliver the evolution of the price level as a function of expected future labor costs. The optimizing model of wage dynamics with wage rigidities, combined with the staggering price model, provides a complete optimizing model of wage-price dynamics.

The rest of the paper is organized as follows. In section 2 I discuss the inadequacy of the New Keynesian Phillips curve, and motivate the investigation of the behavior of labor costs. In section 3 I analyze the predictions of a baseline model of wage setting, and in Section 4 I study the implications of removing the flexible-wage assumption. I first introduce a model of nominal wage rigidity, then show how to write a model with real wage rigidity, and finally consider a general case of partially indexed nominal wages. Section 5 contains the central result of the paper: I discuss the joint modeling of wage and price dynamics, and present the fit of price and wage dynamics obtained with a set of calibrated parameters. Section 6

for variation in nominal marginal costs, but follow different estimation procedures.

[4] Although sticky wages are often postulated in theoretical models, the recent optimizing models with sticky wages have not yet been subject to much empirical testing. One recent piece of evidence for these models is Amato and Laubach (1999). Their empirical strategy is based on matching the impulse response functions to monetary shocks generated by the model with those estimated in the data.

concludes.

2. The Inadequacy of the New Keynesian Phillips Curve

An optimization based Phillips curve relationships results from the combination of the price setting behavior of the firms, which links the evolution of prices to the evolution of marginal costs, and the wage setting behavior of the households, which links the evolution of wages to the evolution of consumption and hours.

In the wage-setting sector, a representative household chooses hours of work to maximize an expected lifetime utility

$$E_0\{\Sigma_{t=0}^{\infty}\ \beta^t\ U(C_t, H_t; \xi_t)\}$$

subject to an intertemporal budget constraint

$$\sum_{t=0}^{\infty} E\{R_{0,t}C_t\} \leq \sum_{t=0}^{\infty} E_0\{R_{0,t}\omega_t H_t\} + a_0$$

where β is a subjective discount factor, ξ_t is a stochastic disturbance to household's preferences, ω_t is the real wage, a_0 is initial wealth, and $R_{t,T}$ is the product of stochastic discount factors. The first order condition for optimal labor supply gives a **desired** real wage, which I will denote throughout the paper by v_t

$$v_t \;= -\frac{U_H}{U_C}(C_t, H_t; \xi_t) \equiv w(C_t, H_t; \xi_t) \tag{2.1}$$

In the price-setting sector, a continuum of monopolistic firms, indexed by i, produce differentiated goods, also indexed by i, and face a demand curve for their product of the form:

$$Y_{it} = (P_{it}/P_t)^{-\theta}\, Y_t \tag{2.2}$$

where θ is the Dixit-Stiglitz elasticity of substitution among differentiated goods, and Y_t is the aggregator function defined as $Y_t = \left[\int_0^1 Y_{it}^{(\theta-1)/\theta} di\right]^{\theta/(\theta-1)}$. The production technology of each firm i is of the Cobb-Douglas form:

$$Y_{it} = (K_{it})^a(\Theta_t H_{it})^{1-a} \tag{2.3}$$

where Y_t is output, K_t and H_t are, respectively, capital and hours, and Θ_t is a stochastic labor augmenting technical progress.

To obtain a Phillips Curve in this optimization based model, NKPC models assume that not all firms adjust prices in full every period. This nominal price rigidity is typically introduced either by assuming that firms face some convex cost of adjusting prices (Rotemberg 1982) and therefore, although all firms are allowed to change prices at any time, it is not optimal to do so; or by assuming random intervals between price changes (Calvo 1983).

In a Calvo setting one assumes that, in every period, a fraction $(1-\alpha)$ of the firms can set a new price, independently of the past history of price changes, which will then be kept fixed until the next time the firm is drawn to change prices again. This set-up implies that the expected time between price changes is $\frac{1}{1-\alpha}$. By letting α vary between 0 and 1, the model nests a wide range of assumptions about the degree of price stickiness, from perfect flexibility ($\alpha = 0$) to complete price rigidity (the limit as $\alpha \to 1$).

The pricing problem of a firm that revises its price in period t is to choose its price, which I will indicate as X_{it}, to maximize its expected stream of profits

$$E_t\{\Sigma_{j=0}^{\infty} R_{t,t+j}\Pi_{it+j}\}$$

The solution to this problem leads to an optimal pricing condition of the form

$$\Sigma_{j=0}^{\infty}\ \alpha^j E_t \left\{ R_{t,t+j} Y_{t+j} \left(\frac{X_t}{P_{t+j}}\right)^{-\theta} \left[X_t - \frac{\theta}{\theta-1}\ S_{t+j,t}\right]\right\} = 0$$

where the subscript i on X_{it} is suppressed, since all the firms that change price solve the same problem, and $S_{t+j,t}$ denotes the marginal cost of producing , at date $t+j$, goods whose price was set at time t ($S_{t+j,t} \equiv \frac{1}{1-a}\frac{W_{t+j}H_{it+j}}{Y_{it+j}}$). Dividing this expression by P_t, and defining $x_t \equiv X_t/P_t$ and $s_{t+j,t} \equiv S_{t+j,t}/P_{t+j}$, one can rewrite it as

$$\Sigma_{j=0}^{\infty}\ \alpha^j E_t \left\{ R_{t,t+j} Y_{t+j} \left(\frac{X_t}{P_{t+j}}\right)^{-\theta} \left[x_t - \frac{\theta}{\theta-1}\ s_{t+j,t} \prod_{k=1}^{j} \pi_{t+k}\right]\right\} = 0 \tag{2.4}$$

Here $s_{t+j,t}$ is in general different from the average marginal cost s_{t+j} (which is equal to $\frac{1}{1-a}\frac{W_{t+j}H_{t+j}}{P_{t+j}Y_{t+j}}$) unless capital is instantaneously reallocated across firms, to equate the shadow price of capital services at all times.[5] Assuming instead that firms' relative capital stocks do not vary with their relative prices, or relative production levels, the real marginal cost at $t+j$ of firms that change price at t is related to the average marginal cost by

$$s_{t+j,t}\ \equiv \frac{1}{1-a}\frac{W_{t+j}H_{it+j}}{P_{t+j}Y_{it+j}} = s_{t+j}\ * \left[\left(\frac{X_t}{P_{t+j}}\right)^{-\theta}\right]^{\frac{a}{1-a}}$$

The optimal pricing condition (2.4), combined with the distribution of aggregate prices at any point in time

$$P_t = \left[(1-\alpha)X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{2.5}$$

allows one to describe the path of aggregate prices and inflation as a function of real marginal costs, shifted by expected inflation.

Specifically, combining the log-linear approximation of equations (2.4) and (2.5) around steady state values $x^*(\equiv 1)$, $s^*(\equiv \frac{\theta-1}{\theta})$, and $\pi^*(\equiv 1)$, with a log linear approximation of the

[5] This is the hypothesis made, for example, by Yun (1996) and Goodfriend and King (1997).

equation defining the marginal cost $s_{t+j,t}$ around the steady state values of $s_{t+j,t}$ and x_t, one obtains that the dynamics of inflation (deviation of inflation from long-run equilibrium) is described by an equation of the form[6]

$$\widehat{\pi}_t = \alpha_1 E_t \widehat{\pi}_{t+1} + \zeta \widehat{s}_t \tag{2.6}$$

where s_t indicates real marginal costs $(MC/P)_t$, the parameter ζ measures the degree of stickiness in the adjustment of prices[7], α_1 is a discount factor[8], and hat variables indicate deviations from steady state values.

This equation can be estimated using a proxy for real marginal costs: for example, one can approximate marginal costs by unit labor costs[9] and estimate the following Phillips curve relationship between inflation, expected inflation, and real unit labor costs u_t

$$\widehat{\pi}_t = \alpha_1 E_t \widehat{\pi}_{t+1} + \zeta \widehat{u}_t \tag{2.7}$$

The dynamic of inflation predicted by this model, taking as given the evolution of nominal unit labor costs, tracks very closely the actual dynamic of U.S. inflation, and the point estimate obtained for ζ implies a degree of nominal price inertia consistent with survey evidence (Sbordone 1998)[10].

To obtain the "New Keynesian Phillips Curve" in the familiar form of a relationship between inflation and output gap, where again expectations of future inflation are a shifting factor, one has to show that labor costs are proportional to output gap. Using the first order condition derived from the flexible wage model, the market clearing condition to substitute out C_t, and the production function to substitute out H_t, eq. (2.1) can be written as

$$v_t = \varpi(Y_t; \xi_t, \Theta_t, G_t) \tag{2.8}$$

Finally, using the definition of labor productivity $APL_t = \frac{Y_t}{H_t} = f(Y_t; \Theta_t)$, (2.8) implies that

$$\frac{v_t}{APL_t} \equiv \frac{ULC_t}{P_t} = \frac{\varpi(Y_t; \xi_t, \Theta_t, G_t)}{f(Y_t; \Theta_t)} \cong \eta(Y_t - Y_t^p) \tag{2.9}$$

[6] A complete derivation of this equation can be found in Sbordone ('98).

[7] In this staggered price setting a la Calvo (1983) the parameter ζ depends on the probability of changing prices (the fraction of firms that are allowed to change prices every period), on the capital share, and on the elasticity of substitution among differentiated goods. A similar equation can also be obtained under the assumption that the nominal rigidity stems from the existence of costs of adjusting prices a la Rotemberg (1982): in this case the parameter ζ is the inverse of the curvature of the adjustment cost function.

[8] $\alpha_1 = R\gamma_y^*$, where R is the steady state value of the discount factor, and γ_y^* is the steady state growth rate of output.

[9] This not only requires to assume a CRS technology, but also to assume away any other friction which may break the proportionality between average and marginal costs (for ex., the existence of costs of adjusting hours, that make the marginal cost of hours differ from the wage). See Sbordone ('99) for a discussion of the empirical implications of using alternative measures of marginal costs when estimating inflation dynamics.

[10] See also Gali-Gertler (1999), and Gali, Gertler, and Lopez-Salido (2000).

where Y_t^p is a function of the exogenous disturbances (ξ_t, Θ_t, G_t). Here Y_t^p indicates the level of output at each time for which real marginal cost would remain at a constant level. It can be thought of as a measure of potential output, so that $(Y_t - Y_t^p)$ is a measure of output gap. One then obtains the NKPC

$$\widehat{\pi}_t = \alpha_1 E_t \widehat{\pi}_{t+1} + \gamma(Y_t - Y_t^p) \tag{2.10}$$

where $\gamma = \eta\zeta$.

Note that potential output, in the above derivation, stands for the 'efficient' level of output, and therefore need not be a smooth trend, because it depends on a number of stochastic disturbances. However, empirical estimates of the NKPC curve usually approximate potential output Y_t^p by some deterministic function of time (for ex., Roberts '95 uses a quadratic trend), which is equivalent to arbitrarily assume that consumption and hours move in proportion of output.

To evaluate the empirical fit of the NKPC model I solve equation (2.10) forward to obtain an expression of inflation as a function of expected future output gaps. To follow the standard approach of NKPC I define the output gap as the deviation from a quadratic trend, and compute expected future output gaps by the forecast of this component derived from a multivariate VAR[11]. The result of this exercise is presented in figure 1. The solid line is inflation (in deviation from the mean), and the dotted line is the forecast of inflation obtained through eq. (2.10). The parameter γ is estimated to maximize the fit of the model (minimize the distance between actual inflation and inflation as predicted by the model). As the figure shows, the ability of this model to predict inflation is clearly poor; predicted and actual inflation are in fact negatively correlated.

Figure 2 shows further dimensions in which the model fails. The top panel compares the lead-lag correlations of inflation and output gap $corr(y_t^{gap}, \pi_{t+k})$: while in the data output gap leads inflation (the highest correlation occurs at $k = 4$), in the model output gap lags inflation (the highest correlation occurs at $k = -3$). Overall, the dynamic cross correlations predicted by the model lay outside the standard deviation bands, and can therefore judged to be significantly different from those computed in the data. The bottom panel, which compares the serial correlation of actual inflation with that of the predicted inflation, shows that the model clearly fails to reproduce the inflation persistence which is present in the data.

This result contrasts with the cited results of Sbordone (1998) and Gali-Gertler (1999). However, the reason why inflation dynamics is well explained when real marginal cost is approximated by unit labor costs, but is not well modeled when marginal cost is approximated by output gap, is easily seen by comparing these two measures. Figure 3 shows the behavior of output gap versus real unit labor costs. As the graph shows, U.S. data do not support the hypothesis that output gap should proxy the evolution of labor costs: the two series are

[11] The output measure I use is gdp for the private, non farm business sector. See later for the details of all the data used. The multivariate VAR includes detrended output, real unit labor cost, and the rate of growth of nominal unit labor cost.

in fact negatively correlated (-.34)[12]. As a result, if the model is true, the NKPC cannot fit the data well.

Taken together, these various pieces of evidence suggest that the empirical problems of NKPC models are not due to a misspecification of the price setting mechanism, but to the incorrect assumption of proportionality between marginal costs and output. Instead of using output gap (measured as deviation of output from trend) as forcing variable, one should use some measure closer to unit labor costs. But this requires to investigate the wage setting mechanism, in order to understand whether it is the case that the data in fact do not support the hypotheses that are needed to go from (2.6) to the conventionally used NKPC in terms of output gap. This is the task I am taking next.

3. A More General Flexible Wage Model

In the baseline optimizing model, as we saw before, the desired real wage is a function of the marginal rate of substitution between leisure and consumption.[13] For a given path of aggregate prices, and a given parametrization of the utility function this condition will determine a path of wages: an assessment of the model can then be given by comparing this path to the actual data.

To examine the prediction of this model, I directly examine the joint behavior of real wages, consumption, and hours. I start by making the hypothesis that the shock ξ_t is a random walk, and derive a log-linear approximation to eq. (2.1)

$$\widehat{v}_t = \lambda \widehat{c}_t + \nu \widehat{h}_t$$

I then denote the empirical counterpart of this equation as

$$v_t^{cyc} = \lambda c_t^{cyc} + \nu h_t^{cyc} \tag{3.1}$$

where the superscript '*cyc*' indicates that I proxy $\widehat{v}_t$, $\widehat{c}_t$, and $\widehat{h}_t$ with the cyclical components, respectively, of real wage, consumption, and hours, which are in turn defined as the log deviation from their trend (as explained below, real wages and consumption share a stochastic trend, while hours are trend stationary). The parameters λ and ν , respectively the elasticity of the marginal rate of substitution with respect to consumption and hours, are preference

[12] A qualitatively similar, although less dramatic result, obtains if one alternatively measures output gap as deviation from a stochastic trend (as discussed below, this specification would seem more appropriate with the data used here). This measure of output gap has a smaller negative correlation with unit labor cost, -.08, but still misses the lead-lag correlation with inflation.

[13] A class of preferences for consumption and leisure common in standard RBC models, because consistent with balanced growth (see King-Plosser-Rebelo 1988), has the form:

$$u(c, L) = \frac{1}{1-\sigma}\{[Cv(L)]^{1-\sigma} - 1\}$$

parameters to be estimated. $1/\lambda$ represents the elasticity of consumption to the real wage (holding hours constant), and $1/\nu$ represents the elasticity of hours to the real wage (holding consumption constant).

The estimation of this equation consists of two steps. I first construct the cyclical components of real wages, consumption and hour; then, denoting by $\underline{\phi}$ the vector of parameters to be estimated, $\underline{\phi} = [\lambda \ \ \nu]'$, I define the distance between the model and the data as

$$\varepsilon_t^v = v_t^{\mathsf{mod}}(\underline{\phi}) - v_t^{data}$$

and compute the value of the parameters λ and ν that minimize (a square measure of) this distance

$$\widehat{\underline{\phi}} = \arg\min var(\varepsilon_t^v) \tag{3.2}$$

3.1. Data description

In all estimates, I use U.S. data for the non-farm private business sector (NFB). Nominal wage is hourly compensation, real wage is nominal compensation divided by the implicit deflator of GDP[14], output is value added, and hours is total hours of work. The price index is the implicit deflator of real output. All data are from BLS. Consumption is the NIPA aggregate for nondurables and services[15].

The solid lines in the graphs of figure 4 (a-d) are the (logs of) the main series.

3.2. Constructing the cyclical components

A first problem to address is the presence of stochastic trends. I tested for the presence of unit roots in all the variables of interest[16], and found that the presence of unit roots is rejected only for hours, and for the labor share. Consumption/output ratio also has no stochastic trend, but is stationary around a small, although significant, negative deterministic trend. To estimate the permanent component of the nonstationary series I use the Beveridge-Nelson definition of the stochastic trend as the forecasting profile of a random variable. Applying this definition I obtain a trend-cycle decomposition of each series of interest. The forecasting profile is constructed using a multivariate forecasting model which includes productivity, hours (output/productivity ratio), the consumption/output ratio, the labor share and inflation. Specifically, the forecasting model is

$$A(L)\ X_t = u_t$$

[14] Note that this is different from what is reported in the statistics as 'real compensation' in the same sector, which is instead obtained by deflating the nominal compensation by a consumer price index.

[15] These data are retrieved from the FRED database at the St. Louis Fed.

[16] I conducted univariate unit root tests on these variables, allowing for the presence of a deterministic trend. Specifically, I test the joint hypothesis of a zero coefficient on the deterministic trend, and a unit coefficient on the first lag, in a regression of the level of the variable on its lagged level and two lags of its first difference.

where the vector X_t is

$$X_t = [\Delta q_t \quad y_t - q_t \quad c_t - y_t \quad sh_t \quad \Delta p_t]'$$

Lowercase letters denote the natural log of the corresponding upper case variables; Q_t is labor productivity, Y_t is real output, C_t is real consumption expenditures on non durables and services, SH_t is the labor share, defined as the ratio of total compensation to nominal output ($SH_t = \frac{W_t H_t}{P_t Y_t}$),[17] and Δ is the first difference operator. Since hours are stationary around a deterministic trend, output and productivity must share the same stochastic trend, and for this reason I include the ratio Y/Q in the VAR, as well as the consumption/ output ratio C/Y (both ratios are in deviation from a small, but significant deterministic trend).

The VAR matrix polynomial is $A(L) = I - A_1 L - A_2 L^2$, and u_t are i.i.d. innovations.

The estimated forecast profile, which defines the Beveridge and Nelson stochastic trend, is plotted as a dotted line in figure 4, panels a-c, for productivity, real wage and consumption. The deterministic trend of hours is plotted in panel d. Figure 5 plots the cyclical components of the same series. The real wage is in the upper right corner of the figure (panel b): this cyclical component is the one that the model of real wage below will try to approximate.

3.3. Parameter estimates

The criterion (3.2) leads to the estimates reported in the first row of table 1. The table also reports the correlation between the estimated cyclical component of the real wage, and the cyclical component of the real wage predicted by the model. Finally, column five reports the variance of the distance between these two series - this variance is minimized by the estimated parameters, and will serve as a benchmark for evaluating, in the next section, whether introducing wage rigidities will improve the fit with the data.

The fitted value of the cyclical wage, constructed using these parameter values, is plotted against the cyclical component of the actual real wage in the top panel of figure 6. The model fits the data significantly well, and, as the bottom panel of the figure shows, the generated real wage and the actual real wage share very similar serial correlation properties.[18] The implied real wage growth, however, is too volatile, as panel a. of figure 7 shows (the standard deviation of the theoretical series is .0062 vs. .0055 of the data). Moreover, the correlation of the actual growth rate and the one projected is only 33%.

3.3.1. Implied nominal wage

The implied growth rate of nominal wages is plotted in figure 7b: the graph shows that the nominal wage, as well as the real wage, is more volatile than in the data (the standard deviation of the actual nominal wage growth is .0069, vs. .008 of the growth of the predicted

[17] In logs, $sh_t = \omega_t - q_t$, where I use the symbol ω to denote the log of the real wage.

[18] The graph shows the serial correlation of the actual data and that of the series predicted by the model, with two standard errors bands around the empirical correlation.

nominal wage). However, the correlation between the nominal wage series is quite high, at .64.

3.4. A Stronger Hypothesis on Preference Shock

The above analysis is conditional upon the assumption that the preference shock follows a random walk. However, since the VAR model contains a single real unit root, we may wish to interpret this single source of non stationarity as a technology shock, that should not affect preferences. Here therefore I make the alternative assumption that ξ_t is a deterministic trend: in this case the model implies

$$v^{trend} = \lambda c^{trend} + \nu h^{trend} + trend$$

As a consequence, $(v_t - \lambda c_t)$ should be a trend stationary series, and the parameter λ can be determined from the cointegrating vector, without reference to cyclical components of the series. Since the estimated VAR model implies that the variable $(v_t - c_t)$ is trend stationary, this hypothesis about the preference shock requires that $\lambda = 1$.

I estimated the model parameters in this restricted case. The estimate still gives a negative value for the elasticity ν, although lower in absolute value: $\nu = -.465$ ($s.e.$ 0.05); the restriction on λ is however strongly rejected, and the resulting estimated cyclical component of the real wage is approximated to a much lower degree, as fig. 8a shows. In terms of the criterion function used for estimation, the variance of the distance between predicted and actual cyclical wage ($var(\varepsilon_t^v)$), going from the restricted to the unrestricted estimate by removing the constraint on λ, this variance is reduced by 66%.

Before turning to the interpretation of the estimated parameter values, I examine another possible restriction, a non-negativity constraint on the elasticity ν. Not surprisingly, the optimal value of ν under such a restriction is zero, and the estimated value for λ is reduced to 1.29 ($s.e.$0.05). Under this restriction as well, the distance between the cyclical component of the real wage predicted by the model and the one estimated in the data increases (see fig. 8b). Relaxing this constraint helps to reduce it by about 76%. The statistics for the two restricted models are reported in the second and third row of table 1.

3.5. Evaluating the Baseline Wage-Setting Model

3.5.1. Interpretation of the estimated parameter values

The estimates obtained, both in the unconstrained and the constrained case, imply that the elasticity of hours to wages, keeping consumption constant, is negative, and the elasticity of consumption to wages, keeping hours constant, is less than 1. One way to understand which kind of preferences are consistent with such values is to use the correspondence between the parameters of this 'cyclical wage' model and the more familiar Frisch elasticities.

Appropriately transforming the economy into a stationary one, one can solve the first order conditions of the consumer maximization problem of the transformed economy to obtain the Frisch demand for consumption

$$\widetilde{C}_t = C(\widetilde{v}_t, \widetilde{\mu}_t)$$

and the Frisch supply of hours

$$H_t = H(\widetilde{v}_t, \widetilde{\mu}_t)$$

Here I denote stationary variables with a tilde, and denote by $\widetilde{\mu}_t$ the (transformed) marginal utility of income. Denoting by (v^*, μ^*, c^*, h^*) the steady state value of $(\widetilde{v}_t, \widetilde{\mu}_t, \widetilde{c}_t, \widetilde{h}_t)$, a log-linearization of the Frisch demands around the steady state values gives

$$\widehat{c}_t = \epsilon_{cw}\widehat{v}_t + \epsilon_{c\mu}\widehat{\mu}_t \tag{3.3a}$$

and

$$\widehat{h}_t = \epsilon_{Hw}\widehat{v}_t + \epsilon_{H\mu}\widehat{\mu}_t \tag{3.4}$$

where ϵ_{ij} denote Frisch elasticities. Solving for $\widehat{\mu}_t$ in eq.(3.3a), and substituting it in (3.4), gives an expression for the desired real wage as a function of consumption and hours

$$\widehat{v}_t = \frac{\epsilon_{H\mu}}{\epsilon_{H\mu}\epsilon_{cw} - \epsilon_{c\mu}\epsilon_{Hw}}\widehat{c}_t - \frac{\epsilon_{c\mu}}{\epsilon_{H\mu}\epsilon_{cw} - \epsilon_{c\mu}\epsilon_{Hw}}\widehat{h}_t \tag{3.5}$$

The parameters λ and ν are therefore the following transformations of the Frisch elasticities

$$\begin{aligned} \lambda &= \tfrac{\epsilon_{H\mu}}{\epsilon_{H\mu}\epsilon_{cw} - \epsilon_{c\mu}\epsilon_{Hw}} \\ \nu &= -\tfrac{\epsilon_{c\mu}}{\epsilon_{H\mu}\epsilon_{cw} - \epsilon_{c\mu}\epsilon_{Hw}} \end{aligned}$$

The concavity of the utility function requires that $\frac{\partial H}{\partial w} > 0$, which means that $\epsilon_{Hw} > 0$ as well. The assumption that consumption is a normal good requires that $\frac{\partial C}{\partial \mu} < 0$, which implies that $\epsilon_{c\mu} < 0$ as well.

In order for λ and ν to have opposite signs (as it results in the estimation), since they share the same denominator, it must be the case that the denominator is negative, $(\epsilon_{H\mu}\epsilon_{cw} - \epsilon_{c\mu}\epsilon_{Hw}) < 0$, and $\epsilon_{H\mu} < 0$.

What do these theoretical restrictions imply on the form of the preferences?

First, and most obvious, preferences should be non-separable in consumption and leisure: were the utility function separable, $\epsilon_{cw} = 0$, and one could not obtain opposite signs for the two parameters. So work must increase the marginal utility of consumption.

Moreover, from the above derivation, it results that leisure should be an inferior good[19].

These two results necessarily follow in the representative household context assumed here, so a first conclusion is that these empirical results are not consistent with the theoretical framework of a representative household for which both consumption and leisure are normal goods.

[19] Alternatively, a negative ν and a positive λ could be obtained by assuming that leisure is a normal good and consumption is the inferior good: in this case in fact $\epsilon_{H\mu} > 0$ and $\epsilon_{c\mu} > 0$ (in that case the denominator of the two parameters needs to be positive, and it is required that $(\epsilon_{H\mu}\epsilon_{cw} - \epsilon_{c\mu}\epsilon_{Hw}) > 0$).

3.5.2. Alternative interpretations

There are a number of ways, however, to rationalize these results. One alternative, more simplistic interpretation, is that part of consumers are 'rule of thumb' consumers. These consumers will tend to increase consumption when income increases; as a result, keeping consumption constant, all increases in hours must be accompanied by a decline in wages.

Alternatively, one can assume that the economy is populated by a number of heterogeneous households, with different preferences for consumption and leisure, but for whom both consumption and leisure are normal goods. One can then show that, at least in some particular cases, the aggregation of consumption and labor supply behavior of these heterogeneous agents may as well deliver the estimated signs of the parameters (an example is provided in appendix A).

Another alternative is to maintain the representative household framework, but specify its preferences as in the "high substitution economy" of King and Rebelo (2000), a generalized indivisible-labor model. In this economy, there is a stand-in representative agent whose preferences are

$$u(c,N) = \frac{1}{1-\sigma}\{c^{1-\sigma}v^*(1-N)^{1-\sigma}-1\}$$

where

$$v^*(1-N) = \left[\frac{N}{H}v_1^{\left(\frac{1-\sigma}{\sigma}\right)} + \left(1-\frac{N}{H}\right)v_2^{\left(\frac{1-\sigma}{\sigma}\right)}\right]^{\frac{\sigma}{\sigma-1}}$$

where H is the shift length of those who work, N indicates the average hours of work in the economy, and $v_1 = v(1-H)$ and $v_2 = v(1)$ are respectively the utility of leisure of those who work and those who do not work. A log-linear approximation to the first order conditions of the consumer[20] can be written as

$$\begin{aligned} -\sigma\widehat{c}_t - (1-\sigma)\eta\widehat{N}_t &= \widehat{\mu}_t && (3.6)\\ (1-\sigma)\,\widehat{c}_t + \frac{(1-\sigma)^2}{\sigma}\eta\widehat{N}_t &= \widehat{\mu}_t + \widehat{w}_t && (3.7)\end{aligned}$$

where $\widehat{\mu}_t$ is the marginal utility of consumption, $\widehat{w}_t$ is the real wage, and $\eta = \frac{v^{*\prime}(1-N)}{v^*(1-N)}N^*$. Substituting (3.6) into (3.7) one gets

$$\widehat{w}_t = \lambda\widehat{c}_t + \nu\widehat{N}_t$$

It is clear then that $\nu = \frac{1-\sigma}{\sigma}\eta$ has a negative value for any $\sigma > 1$. This model rationalizes the empirical result that non separable preferences are a necessary condition to obtain a negative value for the parameter ν, but also implies that, contrary to the empirical result obtained here, λ should be equal to 1.

[20] These are eqs. (6.8) and (6.9) in King and Rebelo, rewritten as function of hours, as opposed to leisure.

3.5.3. Concluding comments

To summarize, the empirical estimates suggest two major departures from the standard parametrization of preferences.

$\lambda \neq 1$ implies that preferences are not consistent with balanced growth, unless they have a secular drift in them - which I have assumed with both my alternative hypotheses about the preference shock ξ, ruling out any predictable component in the shock.

$\nu < 0$ implies that preferences are non separable in consumption and leisure, and hours increase the marginal utility of consumption. Rationalizing these preferences requires either relaxing the representative agent framework, or generalizing further the indivisible labor model.

4. Introducing sluggish wage adjustment

Although the baseline model of the real wage discussed so far fits the cyclical component of real wage quite well, it implies too much volatility in the rate of growth of both real and nominal wage. This result suggests that it is worth attempting to incorporate some degree of inertia in the adjustment either of the real or the nominal wage, and examine whether allowing for such inertia improves the fit with the data. I therefore consider the possibility that the actual wage departs in some way from the 'desired' wage that would hold under perfectly flexible wages.

4.1. Nominal wage stickiness

I assume a wage setting structure of the kind described by Erceg et al. (2000), which is the analogue to the structure developed by Calvo to model price stickiness. The model has monopolistic competition among households with respect to the supply of labor, by assuming that households offer differentiated types of labor services to firms; I further assume that households stipulate wage contracts in nominal terms, and that at the stipulated wage they supply as many hours as are demanded. Total labor employed by any firm j is an aggregation of individual differentiated hours

$$H_t^j = \left[\int_0^1 h_{it}^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} \tag{4.1}$$

where $\theta > 1$ is the Dixit-Stiglitz elasticity of substitution among differentiated labor services. The wage index is defined as

$$W_t = \left[\int_0^1 W_{it}^{1-\theta} di \right]^{1/(1-\theta)}$$

Household i faces the following demand function for her labor services from each firm j[21]

$$h_{it}^{j} = \left(W_{it}/W_t\right)^{-\theta} H_t^j \tag{4.2}$$

which, aggregated across firms, gives the total demand of labor hours h_{it} equal to

$$h_{it} = \left(W_{it}/W_t\right)^{-\theta} H_t \tag{4.3}$$

where $H_t = \left[\int_0^1 H_t^j dj\right]$.[22]

To introduce staggered wage changes, I assume that at each point in time only a fraction $(1-\psi)$ of the households can set a new wage, which I denote by X_{it}, independently of the past history of wage changes, and this wage will then remain fixed until the next time the household is drawn to change wages again. This assumption means that the expected time between wage changes is $\frac{1}{1-\psi}$; letting ψ vary between 0 and 1, the model nests a wide range of assumptions about the degree of wage inertia, from perfect wage flexibility ($\psi = 0$) to complete nominal wage rigidity ($\psi \longrightarrow 1$).

The wage setting problem is defined as the optimal choice of the wage X_{it}, namely the choice of X_{it} that maximizes the expected stream of discounted utility from the new wage, where the latter is defined as the difference between the gain (measured in terms of the marginal utility of consumption) derived from the hours worked at the new wage and the disutility of working the number of hours associated with the new wage

$$E_t \left\{ \Sigma_{j=0}^{\infty} \left(\beta\psi\right)^j \left[\frac{U_C(C_{it+j}, h_{t+j,t})}{P_{t+j}} \left(X_t h_{t+j,t} - P_{t+j} C_{it+j}\right) + U(C_{it+j}, h_{t+j,t}) \right] \right\} \tag{4.4}$$

where $h_{t+j,t}$ denotes the hours worked at $t+j$ at the wage set at time t (I eliminate the index i on X_t since all the households that change wage at t solve the same problem).

The first order condition for this problem can be written as

$$E_t \left\{ \Sigma_{j=0}^{\infty} \ \left(\beta\psi\right)^j \left(x_t \Pi_{k=1}^{j} \left(\pi_{t+k}^{w}\right)^{-1} \right)^{-\theta} H_{t+j} \left[x_t \omega_{t+j} \Pi_{k=1}^{j} \left(\pi_{t+k}^{w}\right)^{-1} - \frac{\theta}{\theta - 1} mrs_{t+j,t} \right] \right\} = 0 \tag{4.5}$$

where $x_t \equiv X_t/W_t$, π_t^w is the wage inflation $\pi_t^w \equiv W_t/W_{t-1}$, and $mrs_{t+j,t} \equiv MRS_{t+j,t}/W_{t+j}$, with MRS indicating the marginal rate of substitution between consumption and leisure[23].

[21] This demand is obtained by solving firm $j's$ problem of allocating a given wage payment among the differentiated labor services, i.e. the problem of maximizing (4.1) for a given level of total wages to be paid.

[22] Erceg et al. ('99) posit a representative labor aggregator, which they call 'employment agency' that combines hours in the same proportions as firms would choose to do, so that the aggregator demand is equal to the sum of all firms' demands.

[23] See section 7.2.1 of the appendix for a complete derivation of this expression.

A log-linear approximation of (4.5) around $x^*(\equiv 1)$, $mrs^*(\equiv \frac{\theta}{\theta-1})$, and $\pi^{w*}(\equiv 1)$, combined with a similar approximation to the distribution of aggregate real wages, allows to obtain the following equation for the Calvo model of adjustment of nominal wage contracts

$$\Delta w_t = \gamma((v_t + p_t) - w_t) + \beta E_t \Delta w_{t+1} \tag{4.6a}$$

where v_t is the desired real wage, defined as before as the real wage at which the marginal benefit of an increase in real wage is zero, and whose cyclical component is determined according the model above in (2.1). The parameter $\gamma \equiv \frac{(1-\psi)(1-\beta\psi)}{\psi(1+\chi\theta)}$, which I will refer to as the inertia parameter, is a measure of the degree of stickiness in the nominal wage.[24]

The solution of this model can be written as

$$w_t = \lambda_1 w_{t-1} + (1-\lambda_1)(1-\lambda_2^{-1}) \sum_{j=0}^{\infty} \lambda_2^{-j} E_t(v_{t+j} + p_{t+j}) \tag{4.7}$$

where $|\lambda_1| < 1$ and $|\lambda_2| > 1$ denote now the roots of the polynomial associated with the difference equation (4.6a), satisfying $\lambda_1 + \lambda_2 = \frac{1+\gamma+\beta}{\beta}$ and $\lambda_1\lambda_2 = \frac{1}{\beta}$.

The approach I use for estimation is to take as given the evolution of the real variables that determine the evolution of the desired real wage v_t, and the evolution of prices, and construct the path of expected future desired nominal wage. The structural parameters λ and ν, and the roots λ_1 and λ_2, are then estimated by minimizing the distance between the model and the data. From these estimates, fixing the subjective discount factor at $\beta = .99$, one can then retrieve an estimate for the inertia parameter γ. Since the desired real wage is modeled, as before, as a function of consumption and hours, its expected future value is constructed using forecasts of hours and consumption according to the VAR model discussed above.

The estimated parameters are reported in row b. of table 1. They show a statistically significant degree of nominal wage inertia while the elasticities λ and ν are not statistically different from those estimated for the flexible wage model. The last row of the table indicates the gain, in terms of goodness of fit, of removing the assumption of perfect wage flexibility: the model improves significantly over the flexible wage model, by reducing the discrepancy between actual and estimated cyclical wage by slightly more than 30%. The contemporaneous correlation between the two series is also slightly higher than in the flexible wage case (.96 vs. .93). The implied growth of nominal wage has virtually the same volatility of the actual nominal wage growth, and the two series have a correlation of .78. Assuming nominal wage rigidity also smooths real wage growth (the volatility of the projected series is about 85% of that of the actual series).

4.2. Real wage stickiness

One can alternatively assume that households stipulate their wage contracts in real terms (i.e. they are able to fully index their nominal contracts), so that the appropriate model to

[24] For the derivation, see again section 7.2.1 of the appendix.

consider is one with staggered real wages. In this case a similar version of the Calvo model delivers the following model for the dynamic adjustment of real wages

$$\Delta w_t - \Delta p_t = \gamma(v_t - (w_t - p_t)) + \beta E_t(\Delta w_{t+1} - \Delta p_{t+1}) \tag{4.8}$$

where γ is now a measure of the degree of stickiness in the real wage.

Denoting the (log of) real wage by ω_t ($\omega_t \equiv w_t - p_t$) the solution of this model can be written as

$$\omega_t = \lambda_1 \omega_{t-1} + (1-\lambda_1)(1-\lambda_2^{-1}) \sum_{j=0}^{\infty} \lambda_2^{-j} E_t v_{t+j} \tag{4.9}$$

where $|\lambda_1| < 1$ and $|\lambda_2| > 1$ are the roots of the polynomial associated with the difference equation (4.8), satisfying $\lambda_1 + \lambda_2 = \frac{1+\gamma+\beta}{\beta}$ and $\lambda_1 \lambda_2 = \frac{1}{\beta}$.[25]

With similar approach to estimation, I obtain the parameter estimates reported in row c. of table 1. The estimated consumption and hour elasticities are very similar to those estimated in the case of sticky nominal wages, and there is a statistically significant degree of real wage inertia.

This model has a slightly worse fit than the model with nominal wage rigidity, in that it improves upon the fit of the flexible wage model by 22%. It succeed as well in increasing the correlation of the actual and predicted cyclical components of the real wage, and in smoothing the growth rate of the predicted real wage. Real wage growth is now actually too smooth (its volatility is about 60% of the volatility of the actual series), but it has a much stronger correlation (.64) with the actual growth rate than the real wage growth predicted by the flexible wage model.

Taking as given the evolution of prices, the predicted path for the real wages implies a predicted path for the nominal wage as well. Real wage inertia translates into a similar smoothing of the nominal wage: the volatility of the predicted and actual series is virtually the same (.0067 in the model and .0069 in the data), and the two nominal wage series have a quite strong correlation (.80).

4.3. Partially Indexed Nominal Wages

While the two cases considered above illustrate extreme cases in which wage stipulations occur either in real or nominal terms, a more reasonable assumption is to allow an endogenous determination of the degree to which households are able to index their nominal contracts to the price level. A more general specification of the wage setting structure in this sense delivers the following equation for the evolution of wages

$$\Delta w_t - \vartheta \Delta p_t = \gamma(v_t - (w_t - p_t)) + \beta E_t(\Delta w_{t+1} - \vartheta \Delta p_{t+1})$$

where the parameter $\vartheta \in [0,1]$ represents the degree of indexation. Such a formulation nests what I called the 'real' wage stickiness case ($\vartheta = 1$) and the 'nominal' wage stickiness

[25] This solution is derived in section 7.2.2 of the appendix.

case ($\vartheta = 0$). With the same methodology used above, I obtained the parameter estimates reported in the last row of table 1.

Allowing partial indexation reduces the distance between model and data, compared to the flexible wage model, by more than is obtained in either the 'real' wage stickiness case (22%) or in the 'nominal' stickiness case (31%). This model marginally outperforms the other two models in all the other dimensions considered. Predicted and actual cyclical components have a correlation of .96, while predicted and actual real wage growths have a correlation of .64. The predicted real wage is still smoother than the actual, but to a lesser degree than in the real sticky wage model. The volatility of theoretical and actual nominal wage is the same in this model (Std (Δw^{data}) = Std ($\Delta w^{\text{mod}\,el}$) = .0069), and the correlation of actual and predicted nominal wage growth is .81. Figures 9 and 10 show the extent to which the partially indexed wage model approximate actual data.

4.4. Interpretation of wage stickiness

From the expression for γ

$$\gamma = \frac{(1-\psi)(1-\beta\psi)}{\psi(1+\chi\theta)}$$

it is apparent that the 'inertia' parameter is a combination of various structural parameters: the parameter that drives the frequency of wage changes, ψ , the elasticity of substitution among differentiated labor services, θ , and a parameter, χ, which depends upon the elasticity of the marginal rate of substitution between leisure and consumption

$$\chi = \frac{-U_{cH}H}{U_{cc}C}\eta_{mrs,c} + \eta_{mrs,h} = \frac{-U_{cH}H}{U_{cc}C}\lambda + \nu \tag{4.10}$$

Given χ, β and θ, a higher γ implies a lower degree of stickiness. To interpret χ, note that, everything else equal, this parameter increases with the degree of non-separability between consumption and hours. It can also be noted that the estimates of λ and ν are very similar in all the sticky wage models, so that the value of the parameter χ is also approximately the same.

To assess how much wage rigidity is implied by the estimate of γ, I need a way to parametrize χ. First, I consider a slight transformation of expression (4.10)

$$\chi = \frac{-U_{cH}U_c}{U_{cc}U_H}\left(\frac{U_H H}{U_c C}\right)\lambda + \nu \tag{4.11}$$

Then, I write the expression for λ as

$$\lambda = -\frac{U_{cc}C}{U_c} + \frac{U_{Hc}C}{U_H} = \sigma + \frac{U_{Hc}C}{U_H} = \sigma + \frac{U_{Hc}}{U_{cc}}\left(\frac{U_{cc}C}{U_c}\right)\frac{U_c}{U_H} = \sigma\left(1 - \frac{U_{Hc}}{U_{cc}}\frac{U_c}{U_H}\right) \tag{4.12}$$

where, with conventional notation, I indicate with σ the inverse of the intertemporal elasticity of substituition in consumption. Expression (4.12) implies that

$$\frac{U_{Hc}}{U_{cc}}\frac{U_c}{U_H} = \frac{\sigma - \lambda}{\sigma}$$

Substituting this result in (4.11), I obtain

$$\chi = \left(\frac{\sigma - \lambda}{\sigma} * \tau\right)\lambda + \nu$$

Therefore, the value of χ can be determined by assigning a value to σ, and to the ratio wH/C, which I denote by τ.

Table 2 reports various estimates of inertia for the partial indexation model (the one that best fits the data), based on three different assumptions about the value of the intertemporal elasticity of substituition in consumption ($\sigma = 4$, 5, or 10), and three possible values of wage mark-up (10%, 30% and 50%, respectively). Note that every value of σ implies in turn a different degree of non-separability in preferences.

The table shows that the implied wage inertia ranges from 3.4 to 5.6 months. In particular, the estimates show that, for any given degree of wage mark up, a higher χ (i.e. a higher degree of non separability in preferences) is consistent with a lower degree of wage inertia.

Finally, it is interesting to underline that the introduction of wage rigidities, although relevant, doesn't affect the implications of the empirical results for the specification of preferences, that we discussed above. The parameters λ and ν remain of very similar size, and again the hypotheses that $\lambda = 1$, and $\nu \geq 0$ are strongly rejected.

5. A complete wage-price system with sticky prices and wages

While in the analysis of the wage process conducted so far I took as given the path of prices, here I try to analyze the simultaneous determination of prices and wages, allowing for possible inertia in both processes. In this exercise only the paths of quantities (c_t, q_t, y_t) are taken as given .

A complete wage - price model can be obtained by combining the wage equation discussed in the previous sections

$$\Delta w_t - \vartheta \Delta p_t = \gamma(v_t - (w_t - p_t)) + \beta E_t(\Delta w_{t+1} - \vartheta \Delta p_{t+1}) \tag{5.1}$$

where $\vartheta \; \epsilon \; [0, 1]$ indicates the degree of indexation of wages, with the price equation (2.7), which can be more conveniently rewritten as

$$\Delta p_t = \zeta((w_t - p_t) - q_t) + \alpha E_t \Delta p_{t+1} \tag{5.2}$$

I denote by q_t the average labor productivity. The desired real wage v_t is in turn the sum of a stochastic trend and a cyclical component which is, according to eq. (3.1), a function of the cyclical components of consumption and hours

$$v_t = v_t^{tr} + (\lambda c_t^{cyc} + \nu h_t^{cyc}) \tag{5.3}$$

Instead of specifying all the remaining equations of a fully general model, I take as given the evolution of a number of real variables: in particular I take as given the evolution of productivity and the desired real wage, which according to (5.3) is a function of hours and consumption. I therefore assume that the evolution of these real variables is well described by the stochastic process

$$Z_t = \Gamma Z_{t-1} + \varepsilon_{zt} \tag{5.4}$$

where, as before,

$$Z_t = [X_t \quad X_{t-1}]'$$

and

$$X_t = [\Delta q_t \quad (y_t - q_t) \quad (c_t - y_t) \quad sh_t \quad \Delta p_t]'$$

Eqs. (5.1), (5.2), and (5.4) form a system that can be solved for equilibrium processes $\{w_t, p_t\}$, given stochastic processes for $\{v_t, q_t\}$, and initial conditions $\{w_{-1}, p_{-1}\}$.

This system can be written in the form of a first order expectational difference equation system. First, using the identities

$$\begin{aligned} q_t &= q_{t-1} + \Delta q_t && (5.5) \\ w_t - p_t &= w_{t-1} - p_{t-1} + \Delta w_t - \Delta p_t && (5.6) \end{aligned}$$

the wage equation and the price equation can be written respectively as

$$E_t \Delta w_{t+1} - \vartheta E_t \Delta p_{t+1} = \frac{1+\gamma}{\beta} \Delta w_t - \frac{\gamma + \vartheta}{\beta} \Delta p_t + \frac{\gamma}{\beta}(w_{t-1} - p_{t-1}) - \frac{\gamma}{\beta} v_t \tag{5.7}$$

and

$$E_t \Delta p_{t+1} = \frac{1+\zeta}{\alpha} \Delta p_t - \frac{\zeta}{\alpha} \Delta w_t - \frac{\zeta}{\alpha}(w_{t-1} - p_{t-1}) + \frac{\zeta}{\alpha} q_{t-1} + \frac{\zeta}{\alpha} e_1' Z_t \tag{5.8}$$

Then, from (5.3), using the definition of stochastic trend, and the model for the cyclical components of hours and consumption, v_t can be written as a function of the variables in Z_t

$$v_t = q_t + \Xi Z_t \tag{5.9}$$

where $\Xi = ((1-\lambda) e_1' (I - \Gamma)^{-1} \Gamma + (\lambda + \nu) e_2' + \lambda e_3')$, and e_i' denotes a 10-dimensional row vector which has a 1 in the i-th position, and zeros elsewhere. Finally, substituting (5.8) and (5.9) into (5.7), the nominal wage becomes the following function of observables

$$\begin{aligned} E_t \Delta w_{t+1} = {} & \left(\frac{\alpha (1+\gamma) - \beta \vartheta \zeta}{\alpha \beta} \right) \Delta w_t + \left(\frac{\beta \vartheta (1+\zeta) - \alpha (\gamma + \vartheta)}{\alpha \beta} \right) \Delta p_t \\ & + \left(\frac{\alpha \gamma - \beta \vartheta \zeta}{\alpha \beta} \right) (w_{t-1} - p_{t-1}) + \left(\frac{\beta \zeta \vartheta - \alpha \gamma}{\alpha \beta} \right) q_{t-1} + \Psi Z_t \qquad (5.10) \end{aligned}$$

where $\Psi = \left(\frac{\beta\zeta\vartheta-\alpha\gamma}{\alpha\beta}\right) e_1' - \frac{\gamma}{\beta}\Xi$.

Defining

$$Y_t = [\Delta w_t \ \Delta p_t \ (w_{t-1} - p_{t-1}) \ q_{t-1} \ Z_t]',$$

the system of equations (5.10), (5.8), (5.4), and identities (5.5) and (5.6) can be written as

$$E_t Y_{t+1} = M \ Y_t \tag{5.11}$$

where the matrix M is

$$M = \begin{bmatrix} \frac{\alpha(1+\gamma)-\beta\vartheta\zeta}{\alpha\beta} & \frac{\beta\vartheta(1+\zeta)-\alpha(\gamma+\vartheta)}{\alpha\beta} & \frac{\alpha\gamma-\beta\vartheta\zeta}{\alpha\beta} & -\frac{\alpha\gamma-\beta\vartheta\zeta}{\alpha\beta} & \Psi \\ -\frac{\zeta}{\alpha} & \frac{(1+\zeta)}{\alpha} & -\frac{\zeta}{\alpha} & \frac{\zeta}{\alpha} & \frac{\zeta}{\alpha}e_1' \\ 1 & -1 & 1 & 0 & \underline{0} \\ 0 & 0 & 0 & 1 & e_1' \\ 0 & 0 & 0 & 0 & \Gamma \end{bmatrix}$$

This system has a unique solution since the matrix M has exactly two unstable eigenvalues[26]. Letting μ_1 and μ_2 denote these two eigenvalues, and x_1 and x_2 denote respectively the eigenvectors associated with them, the solution is given by the two equations

$$x_1'[\Delta w_t \ \Delta p_t \ (w_{t-1} - p_{t-1}) \ \ q_{t-1} \ Z_t] = 0 \tag{5.12}$$

and

$$x_2'[\Delta w_t \ \Delta p_t \ (w_{t-1} - p_{t-1}) \ \ q_{t-1} \ Z_t] = 0 \tag{5.13}$$

5.1. A calibration exercise

As a first experiment with the complete model, I simulate the series of wages and prices by assigning to the parameters of interest the values estimated with the single equation procedures used above. Specifically, I calibrated the parameter as in table below

α	=	.99
β	=	.99
γ	=	.76
λ	=	2.36
ν	=	−.996
ζ	=	.046
ϑ	=	.5

and compute the series of wages and prices according to the solution (5.12)-(5.13). Figure 11 graphs the U.S. series of inflation (panel a) and nominal wage growth (panel b) against the corresponding series that are predicted by the model, usingthe calibrated parameters.

The fit of the inflation process appears quite good: actual and predicted inflation have a correlation of .85. The model seems to be able to reproduce also quite closely the major

[26] The conditions for uniqueness are verified in the Appendix, sect. 7.3.

features of the wage process, although slightly overpredicting wage growth in ‘74-‘75 and before the ‘82 recession. The correlation between actual and predicted nominal wage growth is .78.

At this point it is possible to evaluate whether the modeling of the wage behavior allows to overcome the shortcomings of the standard NKPC discussed in section 2 above. Comparing the fit of inflation dynamics of figure 11a with that of figure 1, it is easy to see the significant improvement obtained by modeling the evolution of labor costs, instead of assuming their proportionality to a measure of output gap. Unlike the standard NKPC model, the model presented here accounts for most of the major swings in the inflation process. Moreover, it is also able to reproduce both the inflation persistence shown by the data, and the dynamic correlations of inflation and output gap. The top panel of figure 12 plots the lead-lag correlations with the output gap[27]: estimated and actual dynamic correlations peak at the same lag, and are overall statistically close. A comparison of this figure with figure 2, discussed previously, shows that this model clearly accounts for both the inflation persistence and the lead of output over inflation observed in the data. The bottom panel shows the serial correlation of the estimated inflation vs. that of actual inflation: the estimated serial correlation is well within the confidence band.

6. Conclusion

This paper provides some evidence that, taking as given the evolution of real variables, a fully microfounded model with staggered prices and wages is able to reproduce quite closely the features of U.S. data on the evolution of prices and wages.

I view the contribution of the paper as twofold. First, the paper shows that it is possible to fit to U.S. data a Phillips curve specification consistent with rational expectations and optimizing behavior.

Secondly, the empirical investigation of the wage setting mechanism carries some clear implications about household preferences, which are at odd with the standard form of preferences used in business cycle literature. In this respect, this contribution adds to a line of papers which have recently investigated the theoretical consequences of alternative forms of preferences (for example, Baxter and Jermann (‘99), King and Rebelo (‘99)), and can be related to the empirical results obtained in early estimates of intertemporal substitution models (for ex. Mankiw, Rotemberg, and Summers (1985), Eichenbaum, Hansen, and Singleton (1988)).

[27] Consistent with the assumption, made in estimating the VAR, that output contains a unit root, the appropriate measure of output gap used here is the deviation from a stochastic trend, and it is therefore indicated as y^{cyc}.

7. Appendix

7.1. Aggregation of heterogeneous consumers[28]

Consider the simple example of two heterogeneous consumers, for whom risk-sharing in financial markets implies that the marginal utility of wealth (μ) is the same (up to a constant). Using the notation in the text, their first order conditions for optimal consumption and hours supply can be written (suppressing the subscript t) as

$$\begin{aligned} u_c^i(c_i, h_i) &= \mu & (7.1) \\ -u_h^i(c_i, h_i) &= \mu w_i & (7.2) \end{aligned}$$

with $i = 1, 2$. With usual notation, log-linearizing equations (7.1) around steady state values (c_i^*, h_i^*), and defining the elasticity of the marginal utilities as $\eta_{xy}^i \equiv \frac{u_{xy}^i \, y_i^*}{u_x^i}$, for $x, y = c, h$, we get

$$\eta_{cc}^1 \widehat{c}_1 + \eta_{ch}^1 \widehat{h}_1 = \widehat{\mu} = \eta_{cc}^2 \widehat{c}_2 + \eta_{ch}^2 \widehat{h}_2 \tag{7.3}$$

Assume that labor demands for the two type of labor are complements:

$$\widehat{h} = \widehat{h}_1 = \widehat{h}_2$$

and let s_1^c and s_2^c denote respectively the shares of each consumer in total consumption

$$\widehat{c} = s_1^c \widehat{c}_1 + s_2^c \widehat{c}_2$$

Then

$$\eta_{cc}^1 \widehat{c}_1 = \eta_{cc}^2 \widehat{c}_2 + \left(\eta_{ch}^2 - \eta_{ch}^1\right) \widehat{h} = \eta_{cc}^2 \left(\frac{\widehat{c} - s_1^c \widehat{c}_1}{s_2^c}\right) + \left(\eta_{ch}^2 - \eta_{ch}^1\right) \widehat{h}$$

so that $\widehat{c}_1$ can be expressed as a function of $(\widehat{c}, \widehat{h})$

$$\begin{aligned} \left(\frac{s_2^c \eta_{cc}^1 + s_1^c \eta_{cc}^2}{s_2^c}\right) \widehat{c}_1 &= \frac{\eta_{cc}^2}{s_2^c} \widehat{c} + \left(\eta_{ch}^2 - \eta_{ch}^1\right) \widehat{h} \\ \widehat{c}_1 &= \frac{\eta_{cc}^2}{s_2^c \eta_{cc}^1 + s_1^c \eta_{cc}^2} \widehat{c} + \frac{s_2^c \left(\eta_{ch}^2 - \eta_{ch}^1\right)}{s_2^c \eta_{cc}^1 + s_1^c \eta_{cc}^2} \widehat{h} & (7.4) \end{aligned}$$

A similar expression can be derived for $\widehat{c}_2$

$$\widehat{c}_2 = \frac{\eta_{cc}^1}{s_2^c \eta_{cc}^1 + s_1^c \eta_{cc}^2} \widehat{c} + \frac{s_1^c \left(\eta_{ch}^1 - \eta_{ch}^2\right)}{s_2^c \eta_{cc}^1 + s_1^c \eta_{cc}^2} \widehat{h} \tag{7.5}$$

From the first equality in (7.3), substituting in (7.4) we then get

$$\begin{aligned} \widehat{\mu} &= \eta_{cc}^1 \left(\frac{\eta_{cc}^2}{s_2^c \eta_{cc}^1 + s_1^c \eta_{cc}^2} \widehat{c} + \frac{s_2^c \left(\eta_{ch}^2 - \eta_{ch}^1\right)}{s_2^c \eta_{cc}^1 + s_1^c \eta_{cc}^2} \widehat{h}\right) + \eta_{ch}^1 \widehat{h} \\ &= \left(\frac{\eta_{cc}^1 \eta_{cc}^2}{s_2^c \eta_{cc}^1 + s_1^c \eta_{cc}^2}\right) \widehat{c} + \left(\frac{s_1^c \eta_{ch}^1 \eta_{cc}^2 + s_2^c \eta_{cc}^1 \eta_{ch}^2}{s_2^c \eta_{cc}^1 + s_1^c \eta_{cc}^2}\right) \widehat{h} \end{aligned}$$

[28] This example was suggested to me by Mike Woodford.

which can be written as

$$\widehat{\mu} = \gamma_1 \widehat{c} + \gamma_2 \widehat{h} \tag{7.6}$$

where the coefficients γ_i are [29]

$$\begin{aligned}
\gamma_1 &= \frac{\eta_{cc}^1 \eta_{cc}^2}{s_2^c \eta_{cc}^1 + s_1^c \eta_{cc}^2} \\
\gamma_2 &= \frac{s_1^c \eta_{cc}^2}{s_2^c \eta_{cc}^1 + s_1^c \eta_{cc}^2} \eta_{ch}^1 + \frac{s_2^c \eta_{cc}^1}{s_2^c \eta_{cc}^1 + s_1^c \eta_{cc}^2} \eta_{ch}^2
\end{aligned}$$

A log-linearization of the labor supply conditions (7.2) gives

$$\widehat{\mu} + \widehat{w}_i = \eta_{hc}^i \widehat{c}_i + \eta_{hh}^i \widehat{h}_i \tag{7.7}$$

so that

$$\begin{aligned}
\widehat{w}_1 &= \eta_{hc}^1 \widehat{c}_1 + \eta_{hh}^1 \widehat{h}_1 - \widehat{\mu} \\
\widehat{w}_2 &= \eta_{hc}^2 \widehat{c}_2 + \eta_{hh}^2 \widehat{h}_2 - \widehat{\mu}
\end{aligned}$$

Defining the aggregate wage as the average of the wages of the two households, with respective weights s_1^w and s_2^w obtain

$$\widehat{w} = s_1^w \widehat{w}_1 + s_2^w \widehat{w}_2 = s_1^w (\eta_{hc}^1 \widehat{c}_1 + \eta_{hh}^1 \widehat{h}_1) + s_2^w (\eta_{hc}^2 \widehat{c}_2 + \eta_{hh}^2 \widehat{h}_2) - \widehat{\mu}$$

Substituting in this expression the results in (7.4), (7.5), and (7.6), we derive $\widehat{w}$ as a function of $\widehat{c}$ and $\widehat{h}$

$$\begin{aligned}
\widehat{w} &= \left(s_1^w \eta_{hc}^1 \frac{\eta_{cc}^2}{s_2^c \eta_{cc}^1 + s_1^c \eta_{cc}^2} + s_2^w \eta_{hc}^2 \frac{\eta_{cc}^1}{s_2^c \eta_{cc}^1 + s_1^c \eta_{cc}^2} \right) \widehat{c} \\
&\quad + \left(s_1^w \eta_{hc}^1 \frac{s_2^c \left(\eta_{ch}^2 - \eta_{ch}^1\right)}{s_2^c \eta_{cc}^1 + s_1^c \eta_{cc}^2} + s_2^w \eta_{hc}^2 \frac{s_1^c \left(\eta_{ch}^1 - \eta_{ch}^2\right)}{s_2^c \eta_{cc}^1 + s_1^c \eta_{cc}^2} \right) \widehat{h} \\
&\quad + \left(s_1^w \eta_{hh}^1 + s_2^w \eta_{hh}^2 \right) \widehat{h} - (\gamma_1 \widehat{c} + \gamma_2 \widehat{h}) \\
&= \lambda \widehat{c} + \nu \widehat{h}
\end{aligned} \tag{7.8}$$

The question is whether the coefficient ν can be negative even though both consumption and leisure are normal goods for both households.

From (7.3) and (7.7)

$$\begin{aligned}
\widehat{\mu} &= \eta_{cc}^i \widehat{c}_i + \eta_{ch}^i \widehat{h}_i \\
\widehat{\mu} + \widehat{w}_i &= \eta_{hc}^i \widehat{c}_i + \eta_{hh}^i \widehat{h}_i
\end{aligned}$$

[29] These coefficients can also be conveniently written as weighted averages, respectively, of the elasticity of the marginal utility of consumption to consumption of the two consumer, and the elasticity of the marginal utility of consumption to hours of the two consumers: $\gamma_1 = \alpha_1 \eta_{cc}^1 + \alpha_2 \eta_{cc}^2$ and $\gamma_2 = \alpha_1 \eta_{ch}^1 + \alpha_2 \eta_{ch}^2$, where $\alpha_1 = \frac{s_1^c \eta_{cc}^2}{s_2^c \eta_{cc}^1 + s_1^c \eta_{cc}^2} = 1 - \alpha_2$.

we derive

$$\widehat{c}_i = \frac{\eta^i_{hh} - \eta^i_{ch}}{\eta^i_{cc}\eta^i_{hh} - \eta^i_{ch}\eta^i_{hc}}\widehat{\mu} - \frac{\eta^i_{ch}}{\eta^i_{cc}\eta^i_{hh} - \eta^i_{ch}\eta^i_{hc}}\widehat{w}$$

$$\widehat{h}_i = \frac{\eta^i_{cc} - \eta^i_{hc}}{\eta^i_{cc}\eta^i_{hh} - \eta^i_{ch}\eta^i_{hc}}\widehat{\mu} + \frac{\eta^i_{cc}}{\eta^i_{cc}\eta^i_{hh} - \eta^i_{ch}\eta^i_{hc}}\widehat{w}$$

Normality requires $\partial\widehat{c}_i/\partial\widehat{\mu} < 0$, and $\partial\widehat{h}_i/\partial\widehat{\mu} > 0$. The denominator of these expressions is necessarily negative because of the concavity of the utility function:

$$\eta^i_{cc}\eta^i_{hh} - \eta^i_{ch}\eta^i_{hc} = \frac{ch}{u_c u_h}(u_{cc}u_{hh} - u_{ch}u_{hc}) < 0$$

so

$$\partial\widehat{c}_i/\partial\widehat{\mu} < 0 \qquad if \qquad \eta^i_{hh} > \eta^i_{ch}$$

and

$$\partial\widehat{h}_i/\partial\widehat{\mu} > 0 \qquad if \qquad \eta^i_{cc} < \eta^i_{hc}$$

for $i = 1, 2$.

Consider, for simplicity, the extreme case where all consumption goes to household 2, and all wages go to household 1: $s^w_1 = 1$, $s^c_2 = 1$.

Then (7.8) simplifies as

$$\widehat{w} = \left(\eta^1_{hc}\frac{\eta^2_{cc}}{\eta^1_{cc}} - \eta^2_{cc}\right)\widehat{c} + \left(\eta^1_{hc}\frac{\eta^2_{ch} - \eta^1_{ch}}{\eta^1_{cc}} + \eta^1_{hh} - \eta^2_{ch}\right)\widehat{h}$$

The coefficient of $\widehat{h}$, ν, simplifies to

$$\nu = \frac{\eta^1_{cc}\eta^1_{hh} - \eta^1_{hc}\eta^1_{ch}}{\eta^1_{cc}} + \frac{\eta^1_{hc} - \eta^1_{cc}}{\eta^1_{cc}}\eta^2_{ch}$$

The first term is positive because, by the concavity of the utility function, both numerator and denominator are negative; the second term is negative by the assumption that both leisure and consumption are normal goods.

Therefore $\nu < 0$ can be obtained for a large enough, positive elasticity of the marginal utility of consumption with respect to hours for consumer 2.

7.2. Derivation of the sticky wage equations

7.2.1. Sticky nominal wage

In this section I first derive the first order condition (4.5) and then obtain the log-linearizations that lead to eq. (4.6a). To be consistent with the empirical results of the flexible wage model, which implies that preferences should be non-separable in consumption and leisure, I allow the marginal utility of consumption to vary with hours of work.

Derivation of eq. (4.5) To derive the first order condition for optimal wage, observe that, by (4.3), $h_{t+j,t} = \left(\frac{X_t}{W_{t+j}}\right)^{-\theta} H_{t+j}$, and therefore

$$\frac{\partial h_{t+j,t}}{\partial X_t} = -\frac{\theta}{X_t}\left(\frac{X_t}{W_{t+j}}\right)^{-\theta} H_{t+j} = -\frac{\theta}{X_t} h_{t+j,t}$$

Using $U_{A,t+j}$ as short notation for $U_A(C_{it+j}, h_{t+j,t})$, $A = C, H$, the derivative of the terms in square brackets of the objective function (4.4) with respect to X_t is

$$\frac{\partial[.]}{\partial X_t} = h_{t+j,t}\frac{U_{C,t+j}}{P_{t+j}} + h_{t+j,t}\frac{X_t}{P_{t+j}}\frac{\partial U_{C,t+j}}{\partial X_t} + X_t\frac{U_{C,t+j}}{P_{t+j}}\frac{\partial h_{t+j,t}}{\partial X_t} - C_{it+j}\frac{\partial U_{C,t+j}}{\partial X_t} + U_{H,t+j}\frac{\partial h_{t+j,t}}{\partial X_t}$$

Efficient risk sharing implies that the marginal utility of consumption is the same across households, and therefore $\frac{\partial U_{C,t+j}}{\partial X_t} = 0$, so

$$\begin{aligned}
\frac{\partial[.]}{\partial X_t} &= h_{t+j,t}\frac{U_{C,t+j}}{P_{t+j}} + X_t\frac{U_{C,t+j}}{P_{t+j}}\frac{\partial h_{t+j,t}}{\partial X_t} + U_{H,t+j}\frac{\partial h_{t+j,t}}{\partial X_t} \\
&= h_{t+j,t}\frac{U_{C,t+j}}{P_{t+j}} + \frac{\partial h_{t+j,t}}{\partial X_t}(X_t\frac{U_{C,t+j}}{P_{t+j}} + U_{H,t+j}) \\
&= h_{t+j,t}\frac{U_{C,t+j}}{P_{t+j}} - \frac{\theta}{X_t}h_{t+j,t}(X_t\frac{U_{C,t+j}}{P_{t+j}} + U_{H,t+j}) = h_{t+j,t}\left[(1-\theta)\frac{U_{C,t+j}}{P_{t+j}} - \theta\frac{U_{H,t+j}}{X_t}\right] \\
&= h_{t+j,t}\left[1 - \frac{\theta}{1-\theta}\frac{U_{H,t+j}/X_t}{U_{C,t+j}/P_{t+j}}\right] = h_{t+j,t}\left[\frac{X_t}{P_{t+j}} - \frac{\theta}{\theta-1}\left(\frac{-U_{H,t+j}}{U_{C,t+j}}\right)\right]
\end{aligned}$$

where $\frac{\theta}{\theta-1}$ denotes a wage mark up.

The first order condition can therefore be written as

$$E_t\left\{\Sigma_{j=0}^{\infty}\ (\beta\psi)^j\left(\frac{X_t}{W_{t+j}}\right)^{-\theta} H_{t+j}\left[\frac{X_t}{P_{t+j}} - \frac{\theta}{\theta-1}\left(\frac{-U_{H,t+j}}{U_{C,t+j}}\right)\right]\right\} = 0 \tag{7.9}$$

which has the usual interpretation that the optimal wage sets the discounted sum of labor income equal to the discounted expected sum of future marginal rates of substitution between consumption and leisure.

This optimal wage condition, combined with the distribution of aggregate wages at any point in time, allows one to describe the path of aggregate wages and wage inflation in this model.

First, consider that the distribution of nominal wages at time t is a mixture of the distribution of wages of the previous period (since all previous wages have the same probability of being changed), with weight ψ, and the new wage X_t, with weight $(1-\psi)$

$$W_t = \left[(1-\psi)X_t^{1-\theta} + \psi W_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{7.10}$$

Dividing both sides by W_t, and defining the real wage inflation as $\pi_t^w \equiv W_t/W_{t-1}$, a log linear approximation of this expression is:

$$0 = (1-\psi)\widehat{x}_t - \psi\widehat{\pi}_t^w$$

or

$$\widehat{x}_t = \frac{\psi}{1-\psi}\ \widehat{\pi}_t^w \tag{7.11}$$

Next, denote by $MRS_{t+j,t}$ the marginal rate of substitution, at date $t+j$, between consumption and hours, when the marginal utilities are evaluated at the level of hours $h_{t+j,t}$ (those offered at the new wage X, i.e. $MRS_{t+j,t} = \frac{-U_{H,t+j}}{U_{C,t+j}}(\widetilde{c}_{t+j}, h_{t+j,t})$, divide expression (7.9) by W_t, and define $x_t \equiv X_t/W_t$, $\pi_t^w \equiv W_t/W_{t-1}$, $mrs_{t+j,t} \equiv MRS_{t+j,t}/W_{t+j}$, and $\omega_t = W_t/P_t$. Observing that $\frac{X_t}{P_{t+j}} = \frac{X_t}{W_t}\frac{W_t}{W_{t+j}}\frac{W_{t+j}}{P_{t+j}}$ and $\frac{X_t}{W_{t+j}} = \frac{X_t}{W_t}\frac{W_t}{W_{t+j}} = x_t\Pi_{k=1}^{j}\left(\pi_{t+k}^w\right)^{-1}$ one obtains eq. (4.5) in the text.

Derivation of eq. (4.6a) Taking a log-linear approximation of (4.5) around $x^*(\equiv 1)$, $mrs^*(\equiv \frac{\theta}{\theta-1})$, and $\pi^{w*}(\equiv 1)$, one obtains

$$\Sigma_{j=0}^{\infty}\ (\beta\psi)^j \left(\widehat{x}_t - \Sigma_{k=1}^{j}\widehat{\pi^w}_{t+k} + \widehat{\omega}_{t+j}\right) = \Sigma_{j=0}^{\infty}(\beta\psi)^j\ E_t\left(\widehat{mrs}_{t+j,t}\ + \widehat{\omega}_{t+j}\right)$$

This gives

$$\frac{1}{1-\beta\psi}\widehat{x}_t = \Sigma_{j=0}^{\infty}(\beta\psi)^j\ E_t\left(\widehat{mrs}_{t+j,t}\ + \Sigma_{k=1}^{j}\widehat{\pi^w}_{t+k}\right)$$

or

$$\widehat{x}_t = (1-\beta\psi)\ \Sigma_{j=0}^{\infty}(\beta\psi)^j\ E_t\left(\widehat{mrs}_{t+j,t}\ + \Sigma_{k=1}^{j}\widehat{\pi^w}_{t+k}\right) \tag{7.12}$$

To obtain a relationship between wage inflation and the deviation between desired and actual aggregate wage, I solve for $mrs_{t+j,t}$ in terms of the marginal rate of substitution evaluated at average aggregate consumption and hours, which I will call mrs_{t+j}; this in turn, according to (2.1), is equal to the desired real wage in the baseline model. To define such "average marginal rate of substitution", I rewrite $mrs_{t+j,t}$ as

$$mrs_{t+j,t} \equiv \frac{MRS_{t+j,t}}{W_{t+j}} = \frac{\widetilde{MRS}_{t+j,t}}{\widetilde{MRS}_{t+j}}\frac{\widetilde{MRS}_{t+j}}{\widetilde{W}_{t+j}} = \frac{\widetilde{MRS}_{t+j,t}}{\widetilde{MRS}_{t+j}} mrs_{t+j} \tag{7.13}$$

where a tilde on the variables indicates that they are appropriately transformed to be stationary:

$$\widetilde{MRS}_{t+j,t} = \frac{-U_H}{U_C}(\widetilde{c}_{t+j,t}, h_{t+j,t}),\ \text{while}\ \widetilde{MRS}_{t+j} = \frac{-U_H}{U_C}(\widetilde{c}_{t+j}, h_{t+j})$$

A log-linearization of (7.13) gives therefore

$$\widehat{mrs}_{t+j,t}\ = \eta_{mrs,c}(\widehat{c}_{t+j,t} - \widehat{c}_{t+j}) + \eta_{mrs,h}\left(\widehat{h}_{t+j,t} - \widehat{H}_{t+j}\right) + \widehat{mrs}_{t+j} \tag{7.14}$$

where $\eta_{mrs,x}$ $(x = c, h)$ indicates the elasticity of the marginal rate of substitution between consumption and leisure with respect to x, evaluated at the steady state.

By the assumption that changes in consumption occur in a way to maintain the marginal utility of consumption equal across households, $\widehat{c}_{t+j,t}$ and $\widehat{c}_{t+j}$ are respectively function of $\widehat{h}_{t+j,t}$ and $\widehat{H}_{t+j}$. Moreover, since $h_{t+j,t} = \left(\frac{X_t}{W_{t+j}}\right)^{-\theta} H_{t+j}$,

$$\widehat{h}_{t+j,t} = -\theta\left(\widehat{x}_t - \Sigma_{k=1}^{j}\widehat{\pi^w}_{t+k}\right) + \widehat{H}_{t+j}$$

so that (7.14) becomes

$$\widehat{mrs}_{t+j,t} = -\chi\ \theta\left(\widehat{x}_t - \Sigma_{k=1}^{j}\widehat{\pi^w}_{t+k}\right) + \widehat{mrs}_{t+j} \tag{7.15}$$

where $\chi = \frac{-U_{cH}H}{U_{cc}C}\eta_{mrs,c} + \eta_{mrs,h}$ and [30] S

$$\widehat{mrs}_{t+j} = \eta_{mrs,c}\widehat{c}_{t+j} + \eta_{mrs,h}\widehat{H}_{t+j} - \widehat{w}_{t+j} = \widehat{v}_{t+j} - \widehat{w}_{t+j} \tag{7.16}$$

Finally, substituting (7.11), (7.15) and (7.16) into (7.12), one gets

$$\frac{\psi(1+\chi\ \theta)}{1-\psi}\ \widehat{\pi^w}_t = (1-\beta\psi)\ \Sigma_{j=0}^{\infty}(\beta\psi)^j E_t\left(\widehat{mrs}_{t+j}\ + (1+\chi\ \theta)\Sigma_{k=1}^{j}\widehat{\pi^w}_{t+k}\right)$$

so that

$$\widehat{\pi^w}_t = \gamma\ \Sigma_{j=0}^{\infty}(\beta\psi)^j\ E_t\left(\widehat{mrs}_{t+j}\ + (1+\chi\ \theta)\Sigma_{k=1}^{j}\widehat{\pi^w}_{t+k}\right) \tag{7.17}$$

where $\gamma \equiv \frac{(1-\psi)(1-\beta\psi)}{\psi(1+\chi\theta)}$.[31]

I compute now $(\beta\psi)E_t\widehat{\pi}^w_{t+1}$ (by evaluating expression (7.17) at $t+1$, and pre-multiplying it by $(\beta\psi)$) and subtract the resulting expression from (7.17), to obtain

$$\widehat{\pi}^w_t - (\beta\psi)E_t\widehat{\pi}^w_{t+1} = \frac{(1-\psi)(1-\beta\psi)}{\psi(1+\chi\theta)}J_t \tag{7.18}$$

---

[30] I use here the fact that

$$\widehat{mrs}_{t+j} = \ln\left(\frac{\widetilde{MRS}}{\widetilde{W}}\right)_{t+j} - \ln(mrs)^*$$

and

$$\frac{\widetilde{MRS(c,H)}}{\widetilde{W}} = \frac{MRS(\widetilde{c},H)}{\widetilde{W}}$$

[31] The last equality follows from the fact that the coefficient on current wage inflation simplifies to

$$1 + \frac{(1-\psi)(1-\beta\psi)}{\psi}\Sigma_{j=0}^{\infty}(\beta\psi)^j\left(\eta_{mrs,h}\theta\frac{\psi}{1-\psi}\right) = 1 + \eta_{mrs,h}\theta$$

where

$$
\begin{aligned}
J_t &= \Sigma_{j=0}^{\infty}(\beta\psi)^j\ E_t\left(\widehat{mrs}_{t+j}\ + (1+\chi\theta)\Sigma_{k=1}^{j}\widehat{\pi^w}_{t+k}\right) - \\
&\quad \Sigma_{j=0}^{\infty}(\beta\psi)^{j+1}\ E_t\left(\widehat{mrs}_{t+1+j}\ + (1+\chi\theta)\Sigma_{k=1}^{j}\widehat{\pi^w}_{t+1+k}\right) \\
&= \Sigma_{j=0}^{\infty}(\beta\psi)^j E_t\left[(\widehat{mrs}_{t+j}\ - (\beta\psi)\widehat{mrs}_{t+1+j}) + (1+\chi\theta)\left(\Sigma_{k=1}^{j}\widehat{\pi^w}_{t+k} - (\beta\psi)\Sigma_{k=1}^{j}\widehat{\pi^w}_{t+1+k}\right)\right] \\
&= \widehat{mrs}_t + \frac{\beta\psi(1+\chi\theta)}{1-\beta\psi}E_t\widehat{\pi^w}_{t+1}
\end{aligned}
$$

Expression (7.18) becomes then

$$
\widehat{\pi}_t^w - (\beta\psi)E_t\widehat{\pi}_{t+1}^w = \frac{(1-\psi)(1-\beta\psi)}{\psi(1+\chi\theta)}(\widehat{mrs}_t + \frac{\beta\psi(1+\chi\theta)}{1-\beta\psi}E_t\widehat{\pi^w}_{t+1})
$$

so that wage inflation is

$$
\widehat{\pi}_t^w = \beta E_t\widehat{\pi}_{t+1}^w + \frac{(1-\psi)(1-\beta\psi)}{\psi(1+\chi\theta)}\widehat{mrs}_t \tag{7.19}
$$

Finally, using the fact that $\widehat{mrs}_t = \widehat{v}_t - \widehat{\omega}_t$, one obtains the wage equation (4.6a) of the text.

Solving for the optimal path of nominal wage I first write explicitly the wage inflation equation as

$$
w_t - w_{t-1} = \gamma(v_t + p_t) - \gamma w_t + \beta E_t w_{t+1} - \beta w_t
$$

so that

$$
\begin{aligned}
v_t + p_t &= \frac{1+\gamma+\beta}{\gamma}w_t - \frac{1}{\gamma}w_{t-1} - \frac{\beta}{\gamma}E_t w_{t+1} = -\frac{\beta}{\gamma}E_t\left[1 - \frac{1+\gamma+\beta}{\beta}L + \frac{1}{\beta}L^2\right]w_{t+1} \\
&= -\frac{\beta}{\gamma}E_t\left[L^2 P(L^{-1})\right]w_{t+1} = -\frac{\beta}{\gamma}E_t\left[(1-\lambda_1 L)(1-\lambda_2 L)\right]\ w_{t+1}
\end{aligned}
$$

where $P(L^{-1}) = L^{-2} - \frac{1+\gamma+\beta}{\beta}L^{-1} + \frac{1}{\beta}$ has real roots λ_1, λ_2 satisfying $0 < \lambda_1 < 1$, and $\lambda_2 > \beta^{-1} \geq 1$.

Then, defining $x_{t+1} = (1-\lambda_1 L)\ w_{t+1}$, I rewrite $v_t + p_t$ as

$$
v_t + p_t = -\frac{\beta}{\gamma}E_t(1-\lambda_2 L)\ x_{t+1} = -\frac{\beta}{\gamma}E_t\ x_{t+1} + \frac{\beta\lambda_2}{\gamma}x_t
$$

from which

$$
x_t = \frac{\gamma}{\beta\lambda_2}(v_t + p_t) + \lambda_2^{-1}E_t x_{t+1}
$$

Solving forward

$$
x_t = \frac{\gamma}{\beta\lambda_2}\sum_{j=0}^{\infty}\lambda_2^{-j}E_t\left(v_{t+j} + p_{t+j}\right) = (1-\lambda_1)(1-\lambda_2^{-1})\sum_{j=0}^{\infty}\lambda_2^{-j}E_t\left(v_{t+j} + p_{t+j}\right)
$$

where the equality $\frac{\gamma}{\beta\lambda_2} = (1-\lambda_1)(1-\lambda_2^{-1})$ follows from the fact that $\lambda_1 + \lambda_2 = \frac{1+\gamma+\beta}{\beta}$ and $\lambda_1\lambda_2 = 1/\beta$. Finally, from the definition of x_t, I obtain

$$w_t = \lambda_1 w_{t-1} + (1-\lambda_1)(1-\lambda_2^{-1}) \sum_{j=0}^{\infty} \lambda_2^{-j} E_t \left(v_{t+j} + p_{t+j} \right)$$

which is expression (4.7) in the text.

7.2.2. Sticky real wage

The wage setting structure is analogous to the one developed before, except that the wage contracts are set in real terms. The objective function is modified to be

$$E_t \left\{ \Sigma_{j=0}^{\infty} \left(\beta\psi\right)^j \left[U_C(C_{it+j}, h_{t+j,t}) \left(Y_t h_{t+j,t} - C_{it+j} \right) + U(C_{it+j}, h_{t+j,t}) \right] \right\} \tag{7.20}$$

where I have indicated by Y_t the real wage set at time t by those workers who are allowed to set a new real wage (the average real wage is instead indicated by W_t^r). The first order condition of this problem can be written as

$$E_t \left\{ \Sigma_{j=0}^{\infty} \ \left(\beta\psi\right)^j \left(\frac{Y_t}{W_{t+j}^r} \right)^{-\theta} H_{t+j} \left[y_t - \frac{\theta}{\theta-1} mrs_{t+j,t} \prod_{k=1}^{j} \pi_{t+k}^{rw} \right] \right\} = 0$$

where $y_t \equiv Y_t/W_t^r$, and π_t^{rw} indicates real wage inflation. A log-linear approximation of this condition around $y^*(\equiv 1)$, $mrs^*(\equiv \frac{\theta}{\theta-1})$, and $\pi^{rw*}(\equiv 1)$, gives

$$\widehat{y}_t = (1-\beta\psi) \ \Sigma_{j=0}^{\infty} (\beta\psi)^j \ E_t \left(\widehat{mrs}_{t+j,t} \ + \Sigma_{k=1}^{j} \widehat{\pi^{rw}}_{t+k} \right) \tag{7.21}$$

Combining this expression with a log-linear approximation of the distribution of real wages, gives

$$\widehat{\pi}_t^{rw} = \gamma \Sigma_{j=0}^{\infty} (\beta\psi)^j \ E_t \left(\widehat{mrs}_{t+j} + (1+\chi\theta) \Sigma_{k=1}^{j} \widehat{\pi^{rw}}_{t+k} \right)$$

where $\gamma = \frac{(1-\psi)(1-\beta\psi)}{\psi(1+\chi\theta)}$ as before.

With the same procedure used before, one then obtains

$$\widehat{\pi}_t^{rw} = \beta E_t \widehat{\pi}_{t+1}^{rw} + \gamma \ \widehat{mrs}_t \tag{7.22}$$

which, by the fact that $\widehat{mrs}_t = \widehat{v}_t - \widehat{\omega}_t$, is equation (4.8) in the text.

7.3. Solution of the system (5.11)

For the system (5.11) to have a unique solution, the matrix M must have two unstable eigenvalues. From inspection of the matrix M it is easy to see that it is enough to check that the upper left 3x3 matrix, call it $\widetilde{M}$, has two eigenvalues with modulus strictly bigger than one. The eigenvalues of $\widetilde{M}$ solve $P(\mu) = \mid \widetilde{M} - \mu I \mid = 0$, which gives the following polynomial equation

$$P(\mu) = \mu^3 + \mu^2 M_2 + \mu M_1 + M_0 = 0$$

where

$$\begin{aligned} M_2 &= -(1 + \tfrac{1+\gamma}{\beta} + \tfrac{1}{\alpha}(1 + \zeta(1-\vartheta))) \\ M_1 &= \tfrac{1}{\alpha} + \tfrac{1}{\beta} + \tfrac{1}{\alpha\beta}(1 + \gamma + \zeta(1-\vartheta)) \\ M_0 &= -\tfrac{1}{\alpha\beta} \end{aligned}$$

The coefficients (M_0, M_1, M_2) satisfy the following necessary and sufficient conditions for determinacy[32]

$$\begin{array}{ll} i. & 1 + M_2 + M_1 + M_0 > 0 \\ ii. & -1 + M_2 - M_1 + M_0 < 0 \end{array}$$

and either

$$iii. \quad M_0^2 - M_0 M_2 + M_1 - 1 > 0$$

or

$$\begin{array}{ll} iv. & M_0^2 - M_0 M_2 + M_1 - 1 < 0 \\ v. & |M_2| > 3 \end{array}$$

[32] These conditions are stated in Woodford (2000).

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Table 1
Estimates of wage models†

	λ	ν	γ	ϑ	$corr(w^a, w^p)$	$var(\varepsilon_t^w)$	% var red
a. Flexible wage model	2.15	-.84			.93	$8.5{*}e^{-5}$	
	(.066)	(.037)					
$\lambda-$ restricted	1	-4.65			.92	$2.5{*}e^{-4}$	-195
		(.030)					
$\nu-$ restricted	1.29	0			.68	$3.6{*}e^{-4}$	-328
	(.054)						
b. Sticky nominal wage	2.32	-.987	1.699		.96	$5.8{*}e^{-5}$	31.6
	(.077)	(.045)	(.26)				
c. Sticky real wage	2.40	-.988	.266		.95	$6.6{*}e^{-5}$	22.3
	(.058)	(.058)	(.079)				
d. Partially indexed wage	2.32	-.996	.76	.5	.96	$5.5{*}e^{-5}$	34.8
	(.077)	(.047)	(.14)	(.012)			

† Standard errors are in parenthesis. $corr(w^a, w^p)$ indicates the correlation between the cyclical component of the real wage estimated from the data (w^a), and the one predicted by each model (w^p).

Table 2
Estimated average time between wage changes (months)

Partially indexed wages

	low wage mark-up ($\mu^{w*} = 1.1$)	mid wage mark-up ($\mu^{w*} = 1.3$)	high wage mark-up ($\mu^{w*} = 1.5$)
$\tau = 1$; $\sigma = 4$ (low non-sep.)	5.6	5.4	5.3
$\tau = 1$; $\sigma = 5$ (mid non-sep.)	3.8	4.3	4.5
$\tau = 1$; $\sigma = 10$ (high non-sep.)	3.4	3.7	3.9

Fig. 1 — Sticky price model with output gap
Inflation

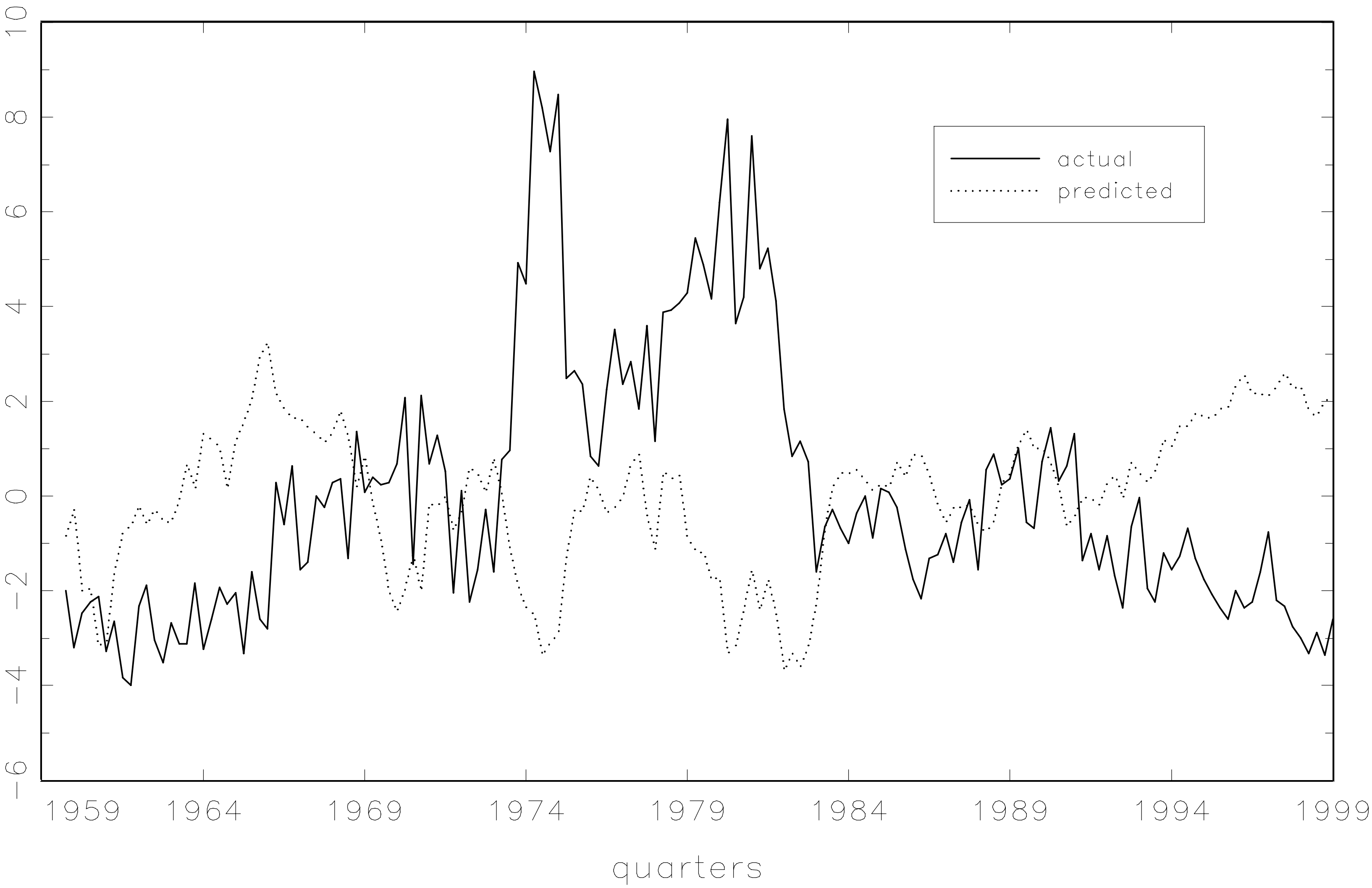
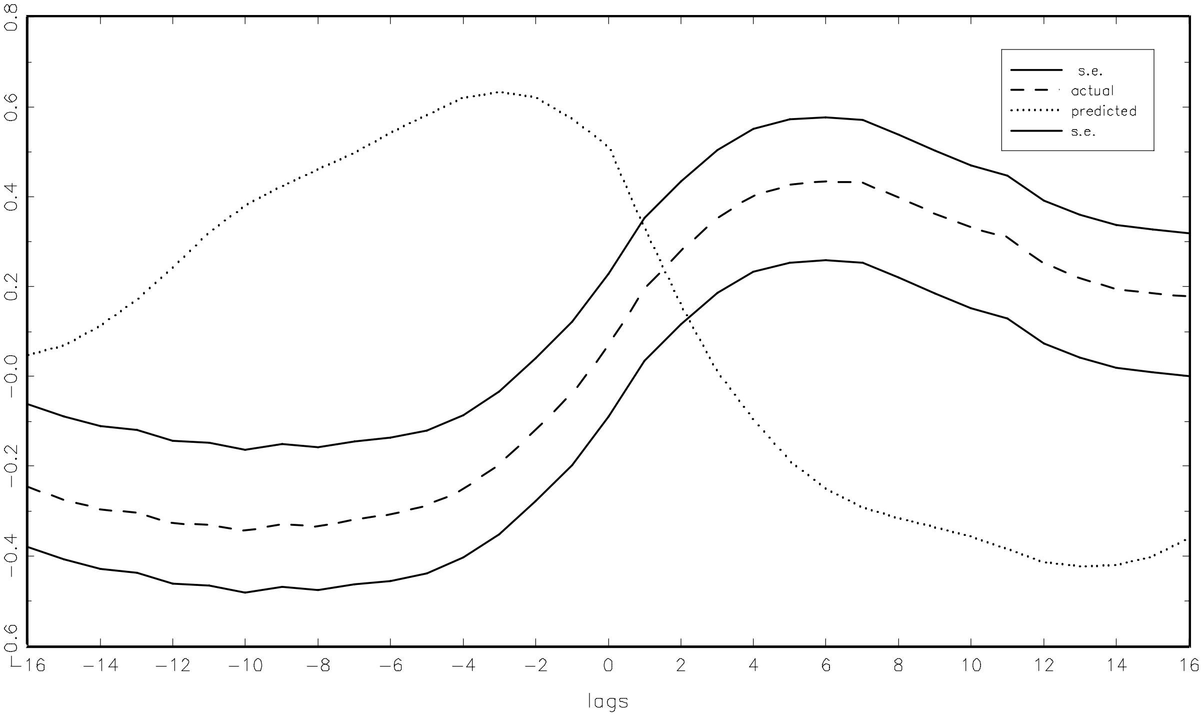


Fig. 2 – Sticky price model with output gap

a. Dynamic cross–correlations $\rho(y_t^{gap}, \pi_{t+k})$



b. Inflation autocorrelations

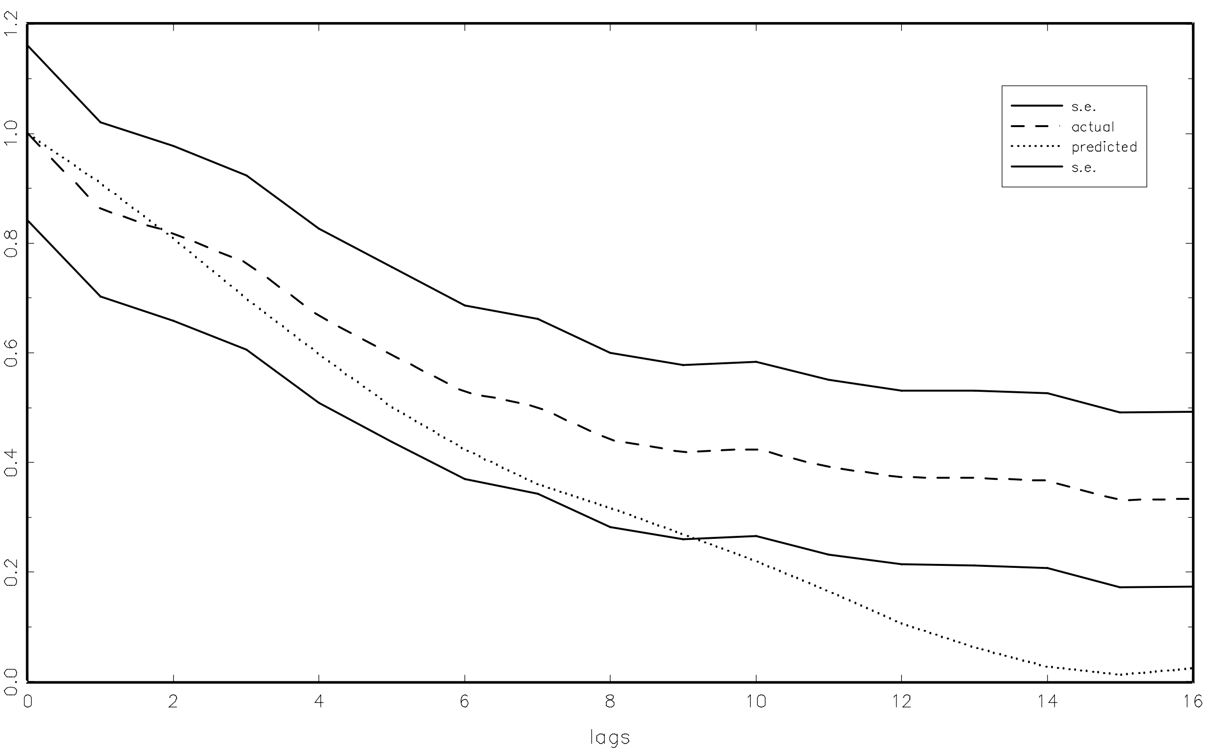


Fig. 3
Output Gap and Real Unit labor Cost

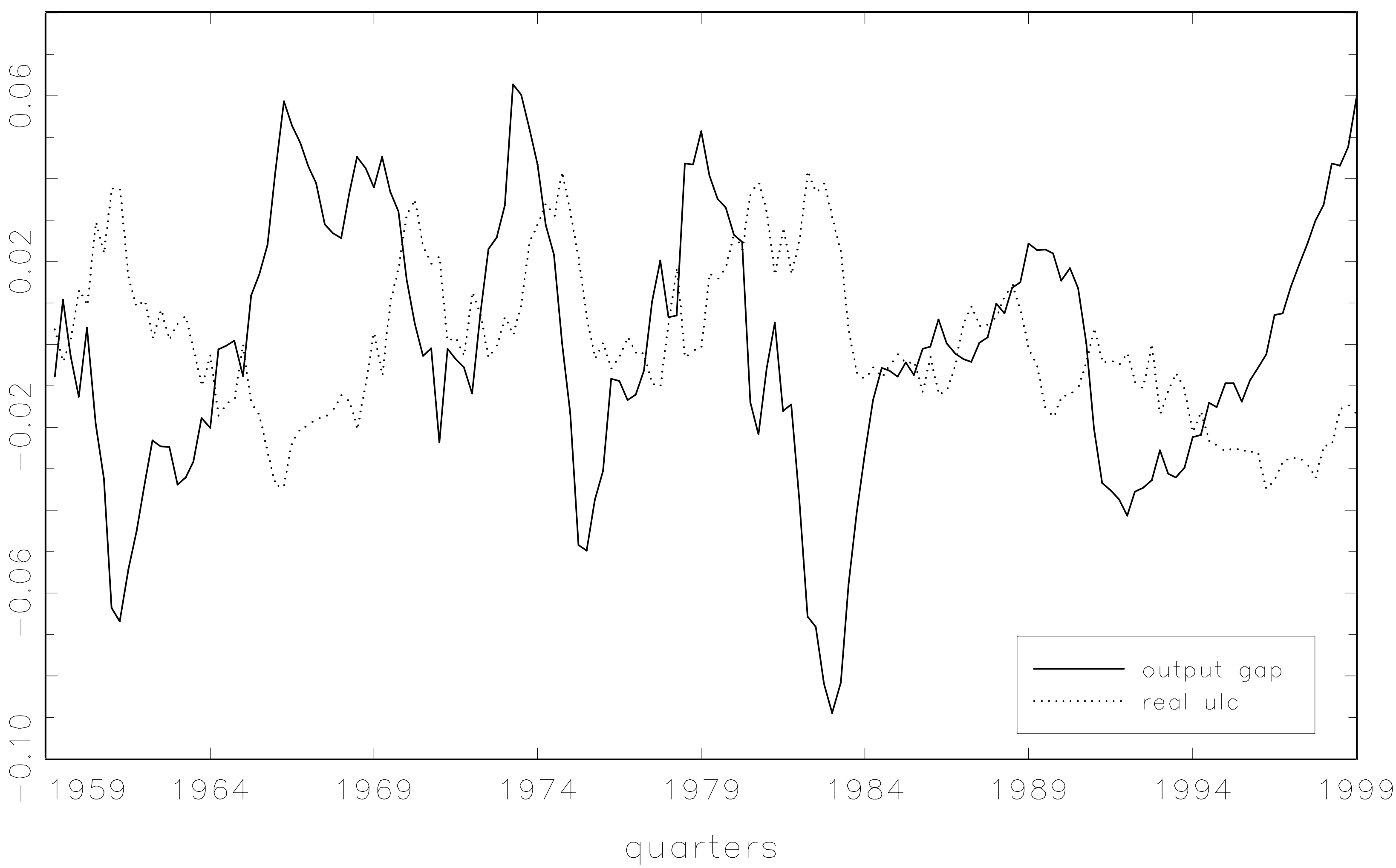
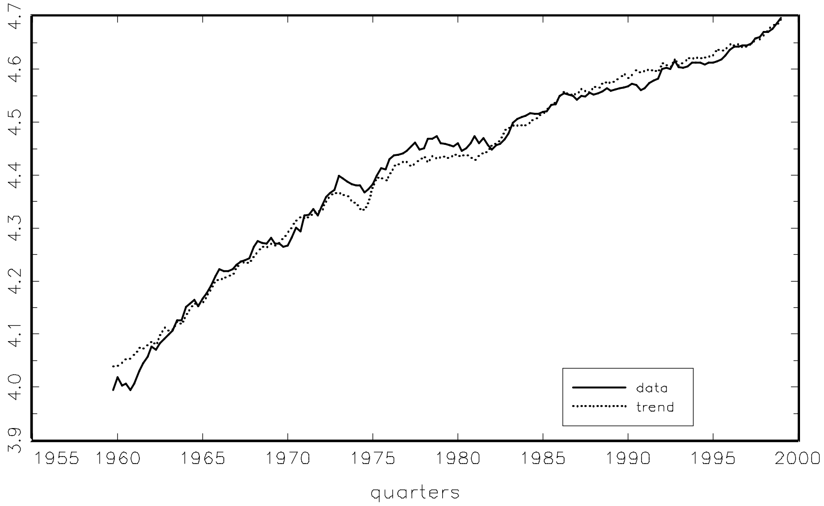
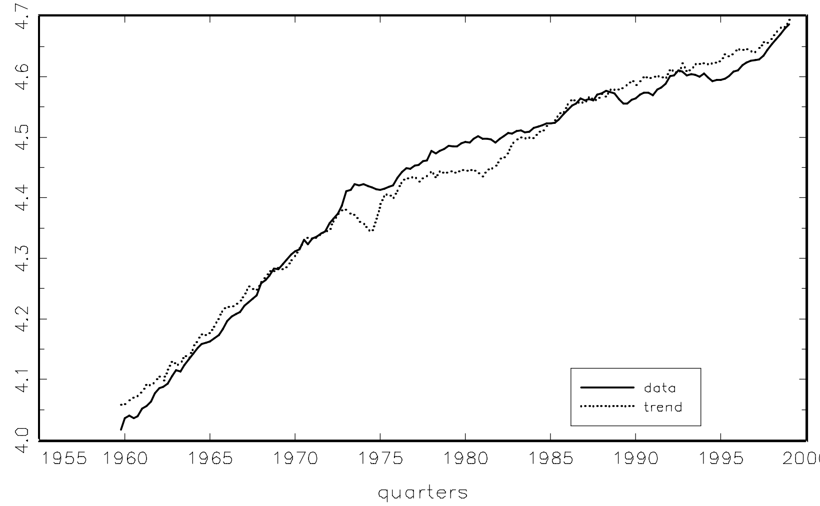


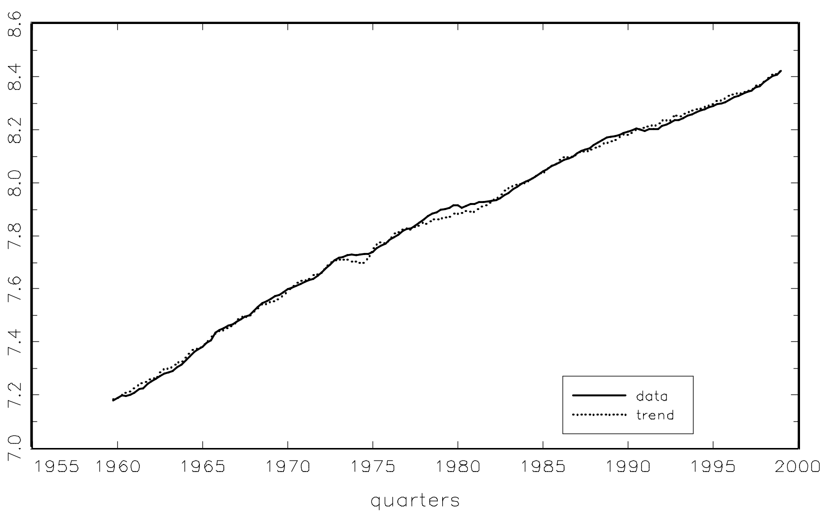
Fig. 4 — a. Productivity and its trend
VAR(2): (dq, y−q, c−y, sh, dp)', 59:4−99:1



b. Real wage and its trend
VAR(2): (dq, y−q, c−y, sh, dp)', 59:4−99:1



c. Consumption and its trend
VAR(2): (dq, y−q, c−y, sh, dp)', 59:4−99:1



d. Hours and its trend
VAR(2): (dq, y−q, c−y, sh, dp)', 59:4−99:1

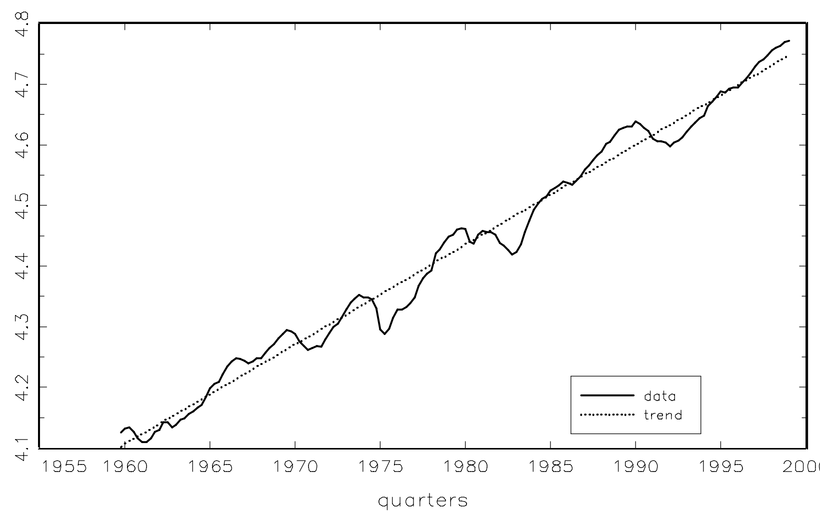
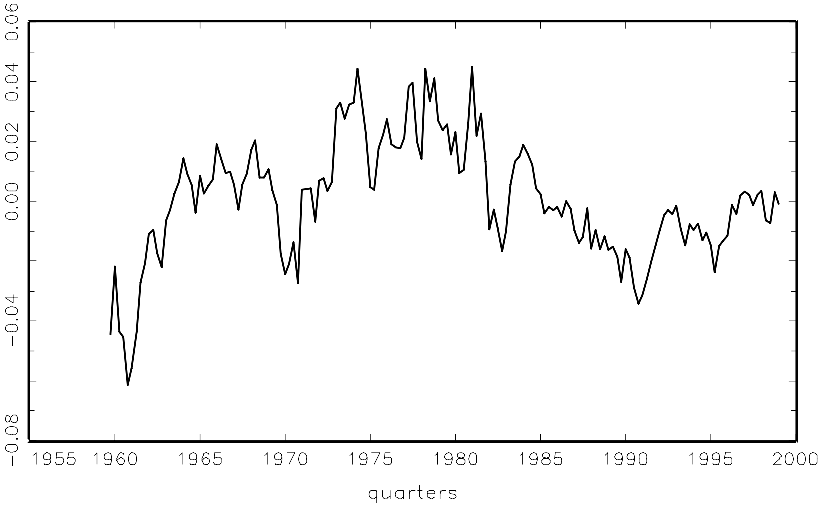
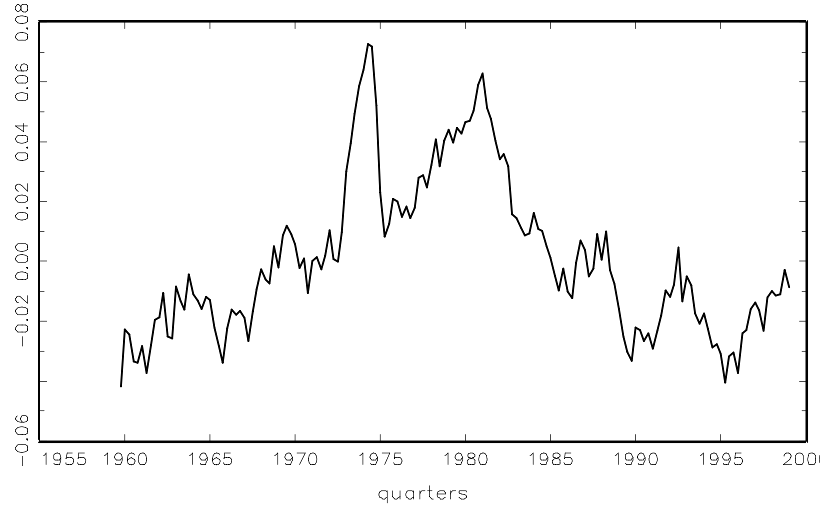


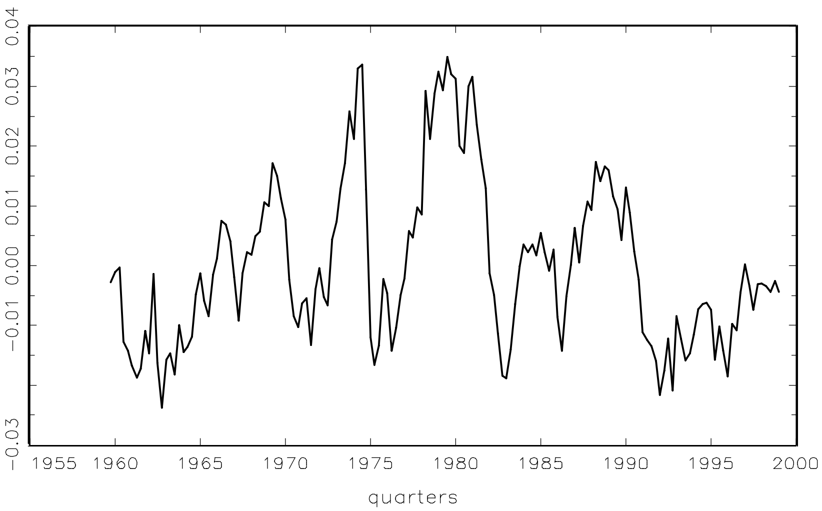
Fig. 5 – a. Cyclical Component of Productivity
VAR(2): (dq, y–q, c–y, sh, dp)', 59:4–99:1



b. Cyclical Component of Real Wage
VAR(2): (dq, y–q, c–y, sh, dp)', 59:4–99:1



c. Cyclical Component of Consumption
VAR(2): (dq, y–q, c–y, sh, dp)', 59:4–99:1



d. Cyclical Component of Hours
VAR(2): (dq, y–q, c–y, sh, dp)', 59:4–99:1

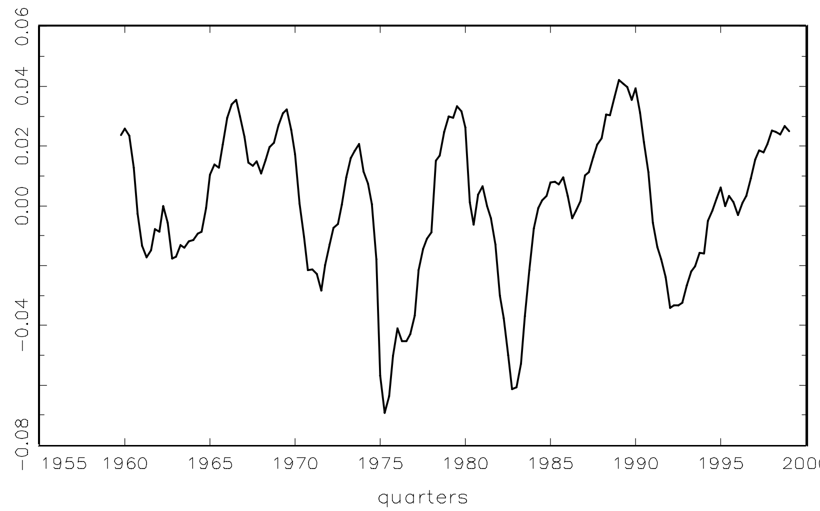
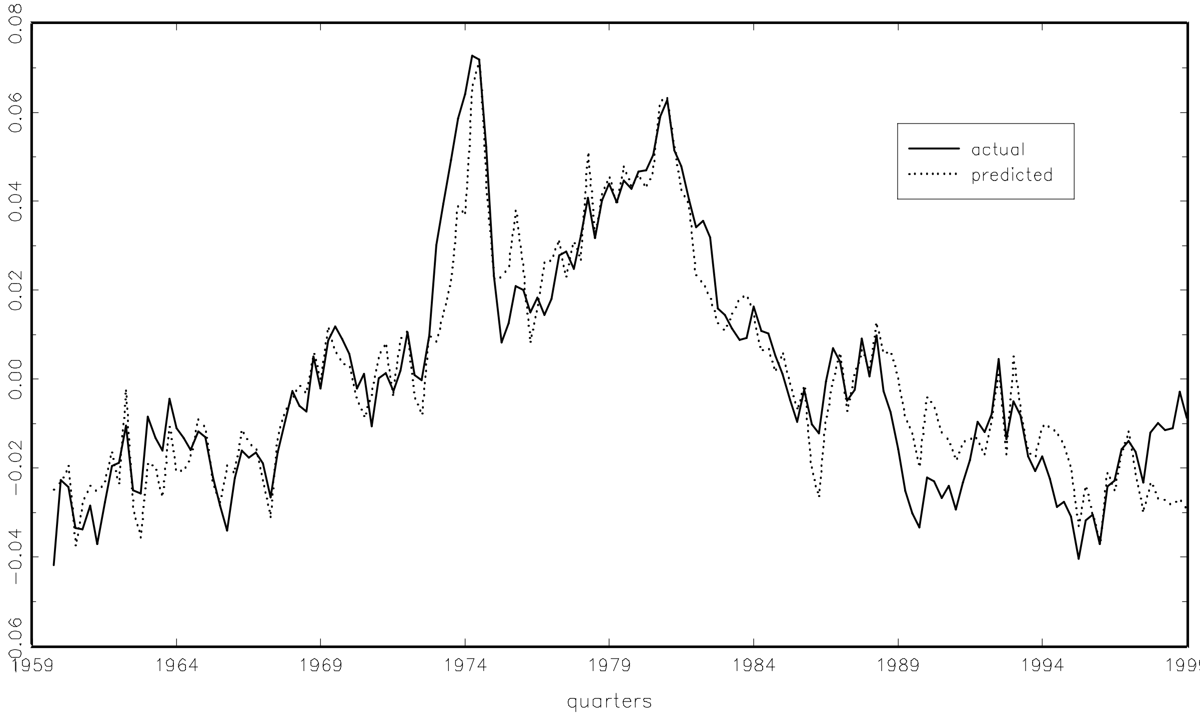


Fig. 6 – Flexible real wage model

a. Cyclical real wage



b. Cyclical real wage autocorrelations

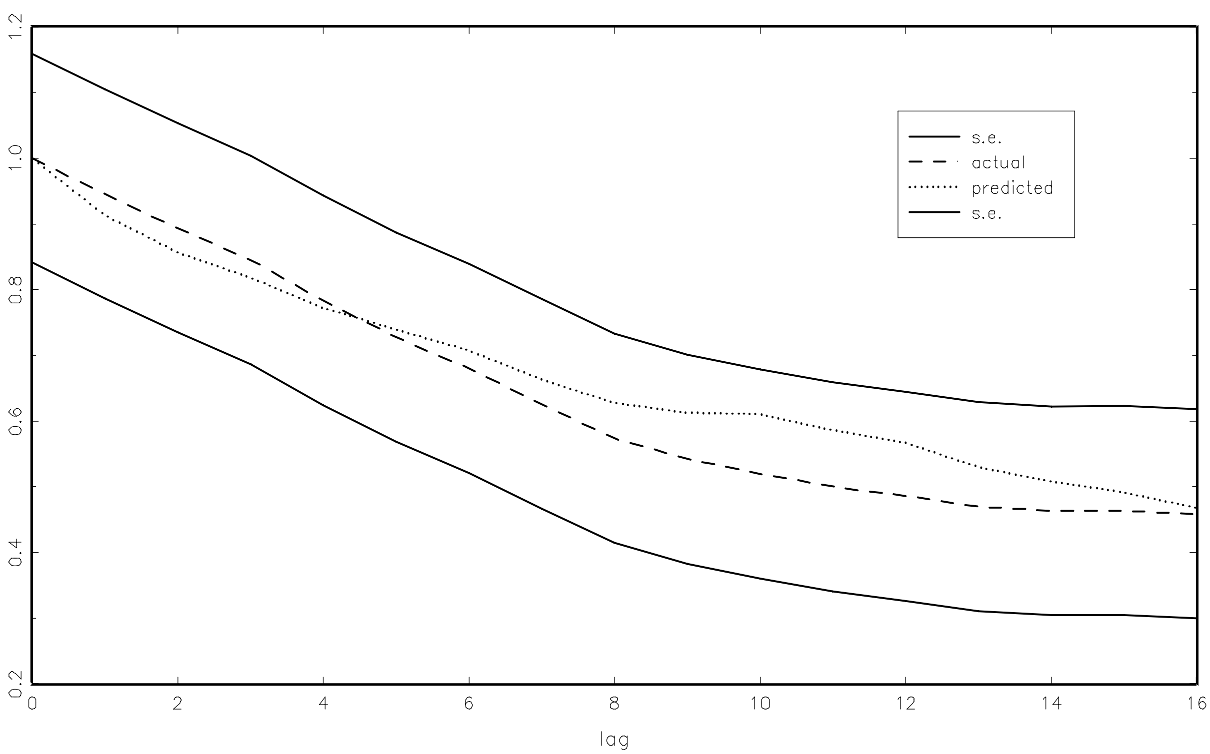
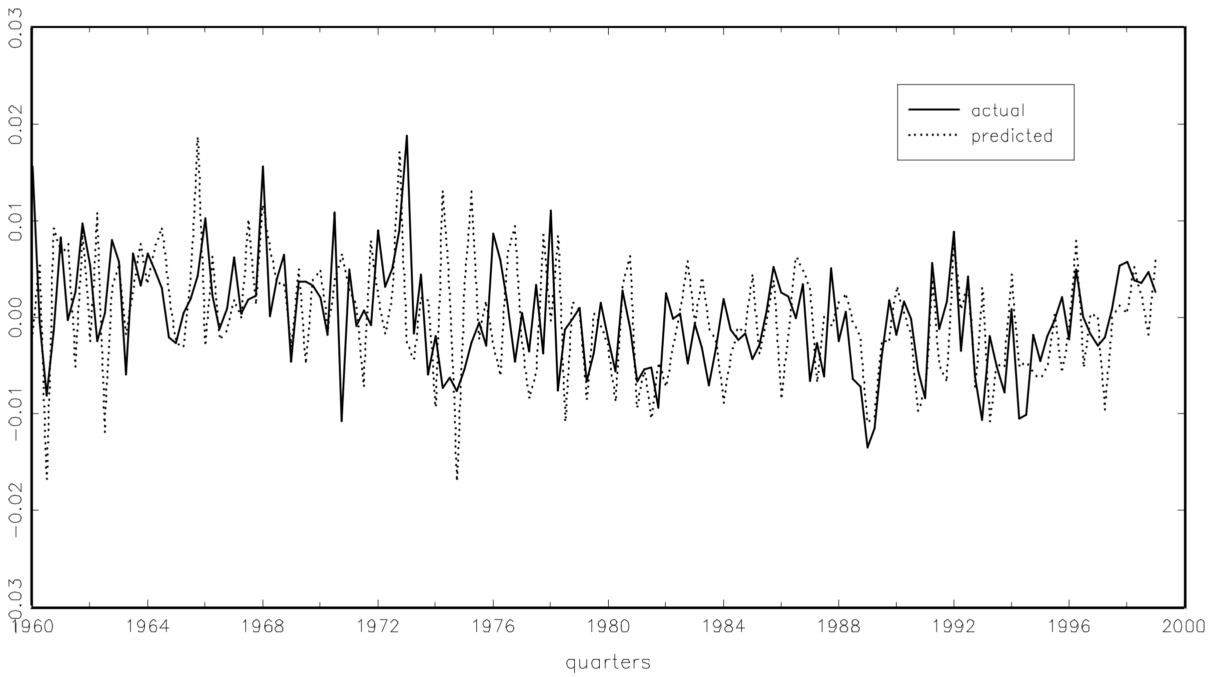


Fig. 7 – Flexible real wage model

a. Real wage growth



b. Nominal wage growth

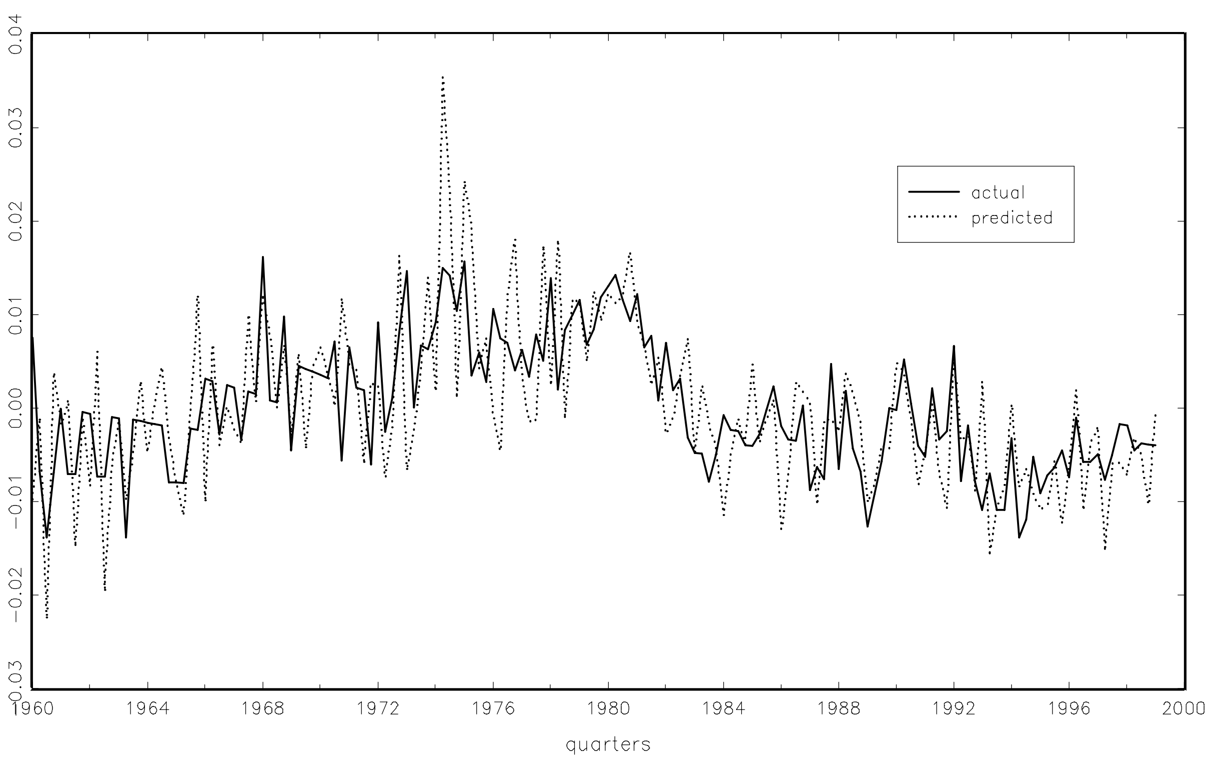
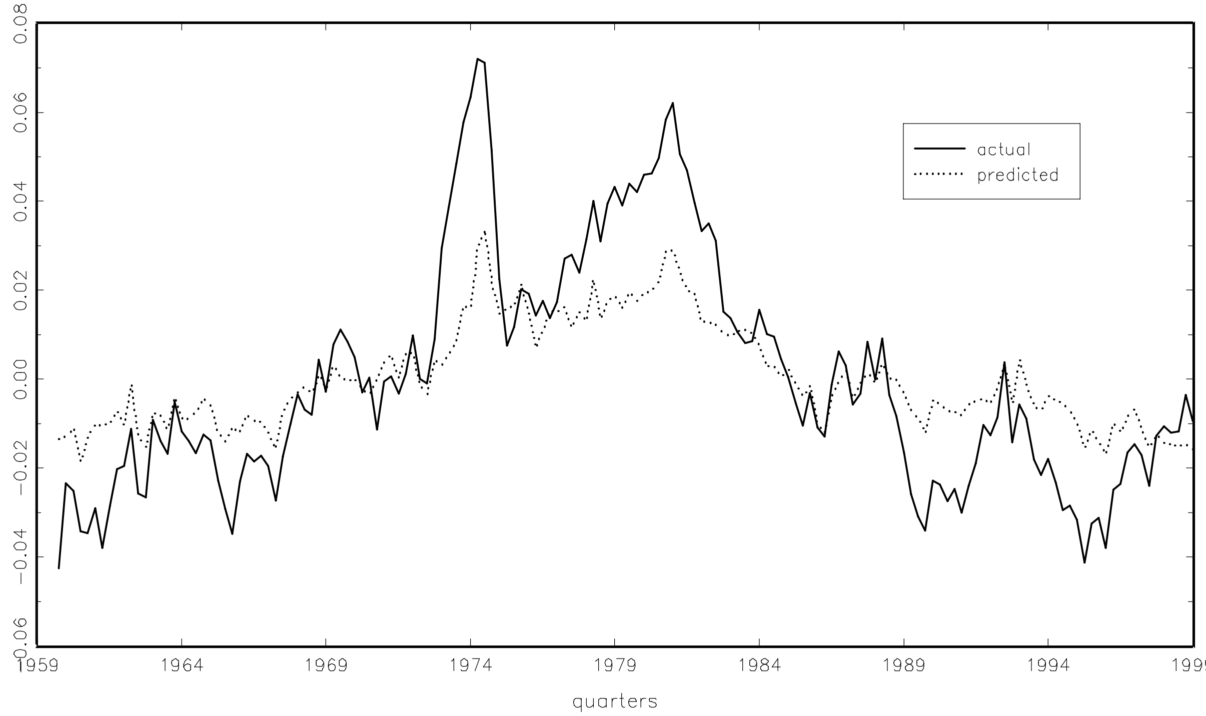


Fig. 8 – Restricted flexible wage model

a. Cyclical real wage, λ–restricted (λ^r=1, ν^r=–.465)



b. Cyclical real wage, ν–restricted (λ^r=1.29, ν^r=0)

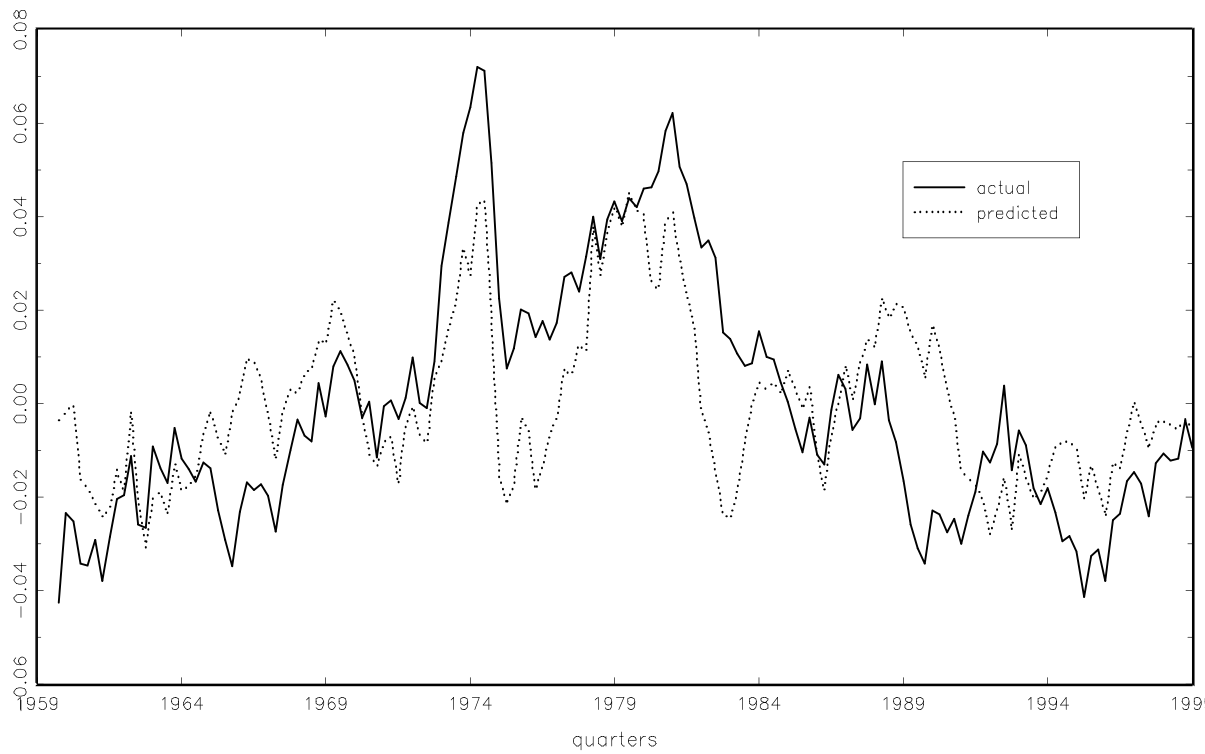


Fig. 9 – Partially indexed model

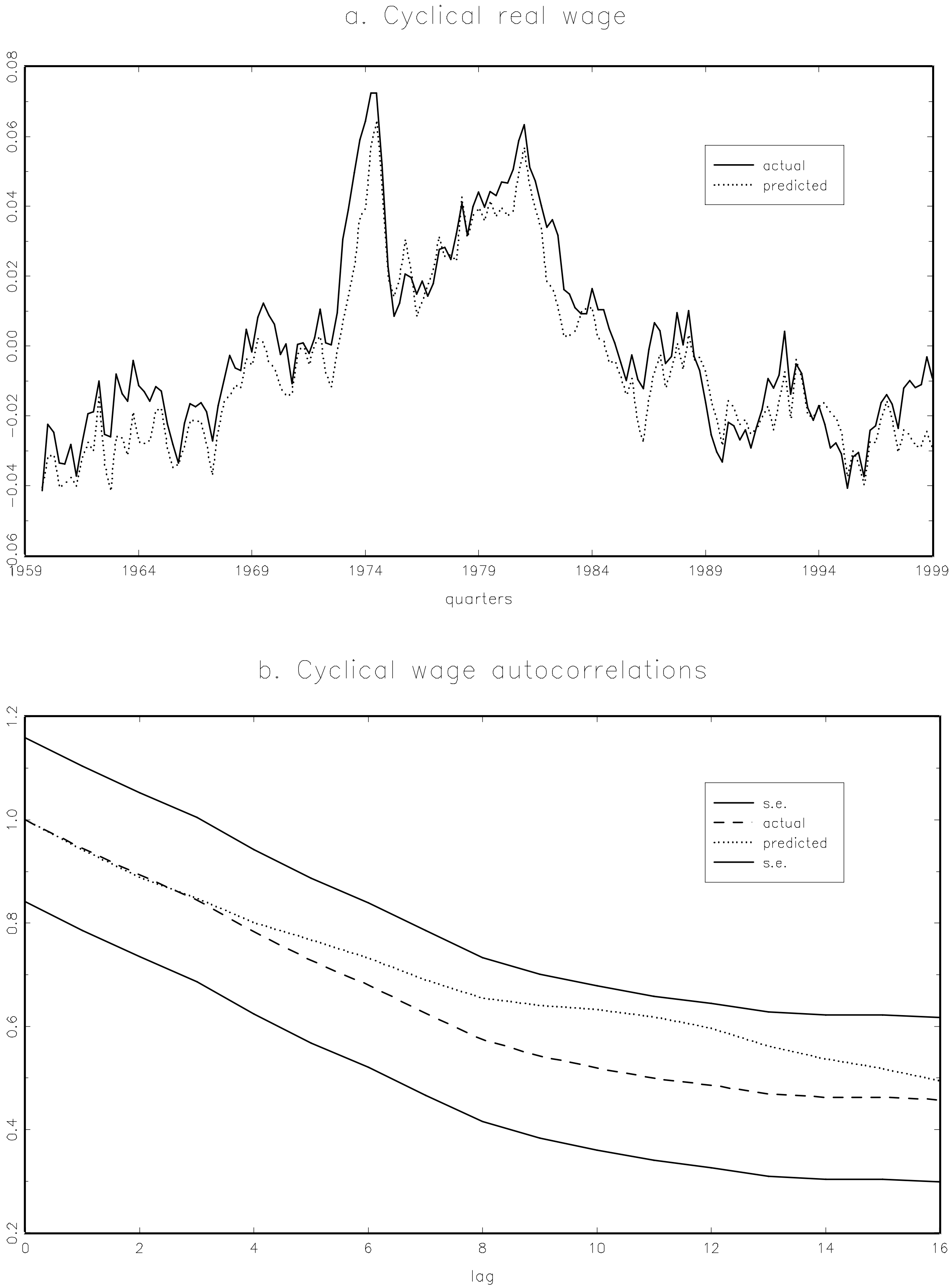
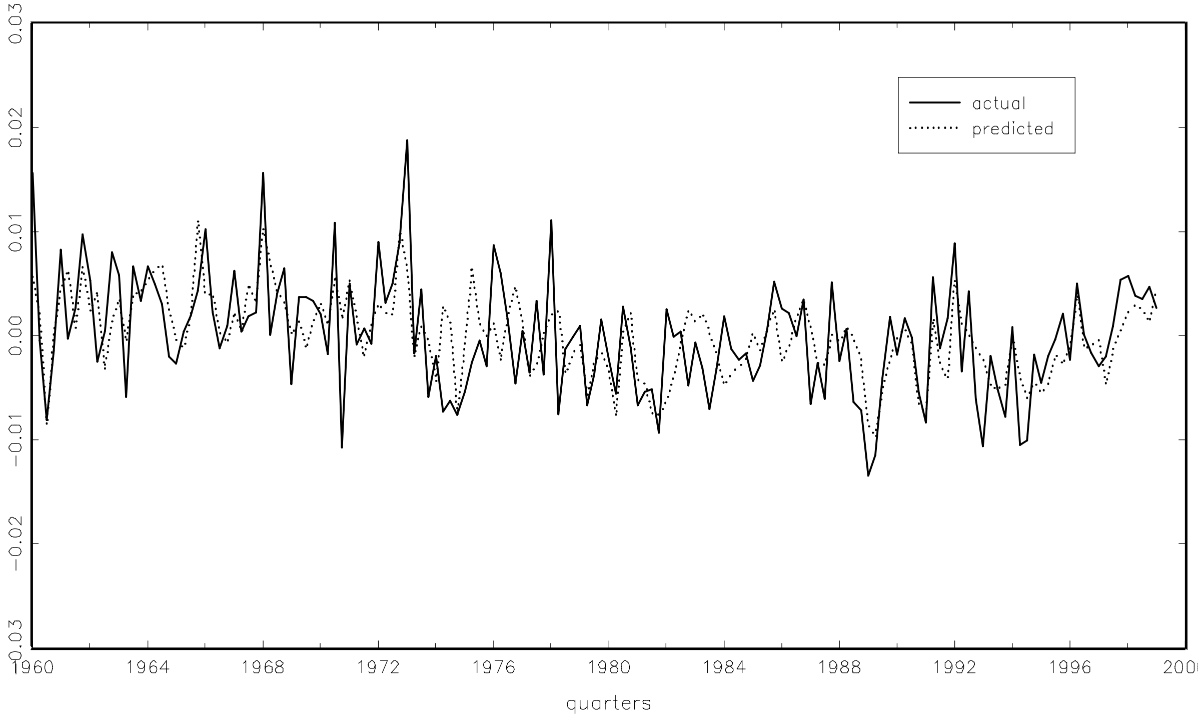


Fig. 10 – Partially Indexed wage model

a. Real wage growth



b. Nominal wage growth

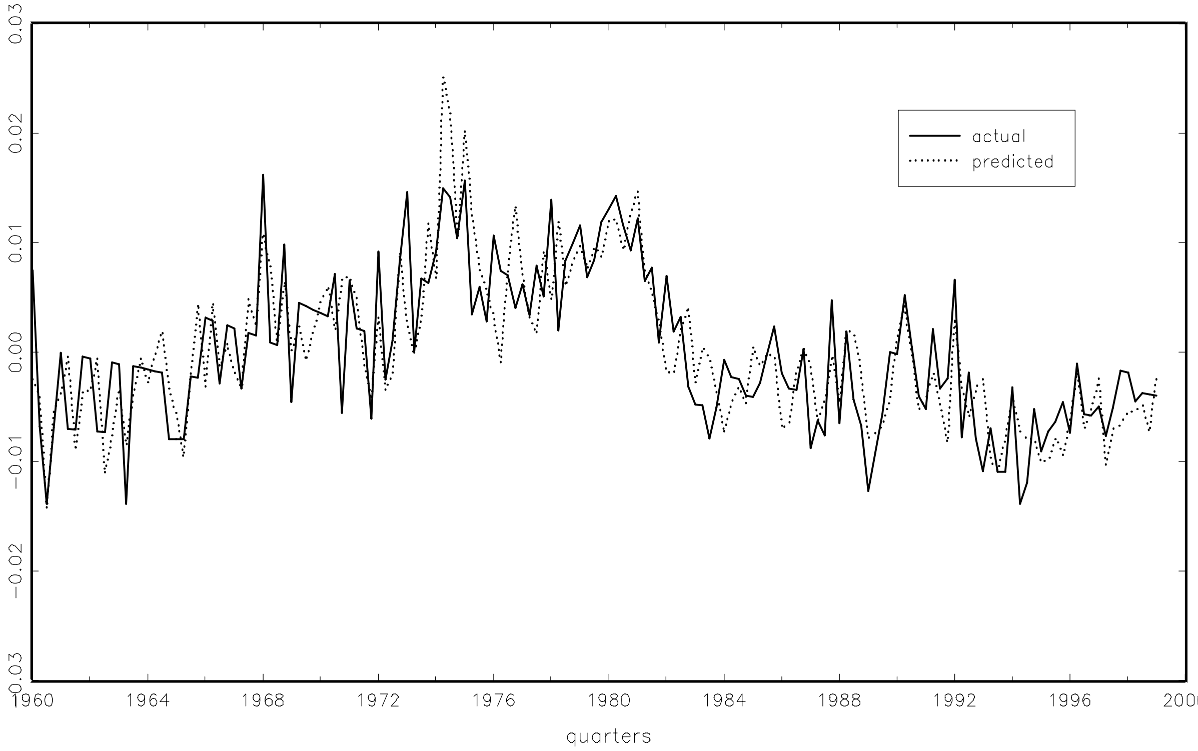
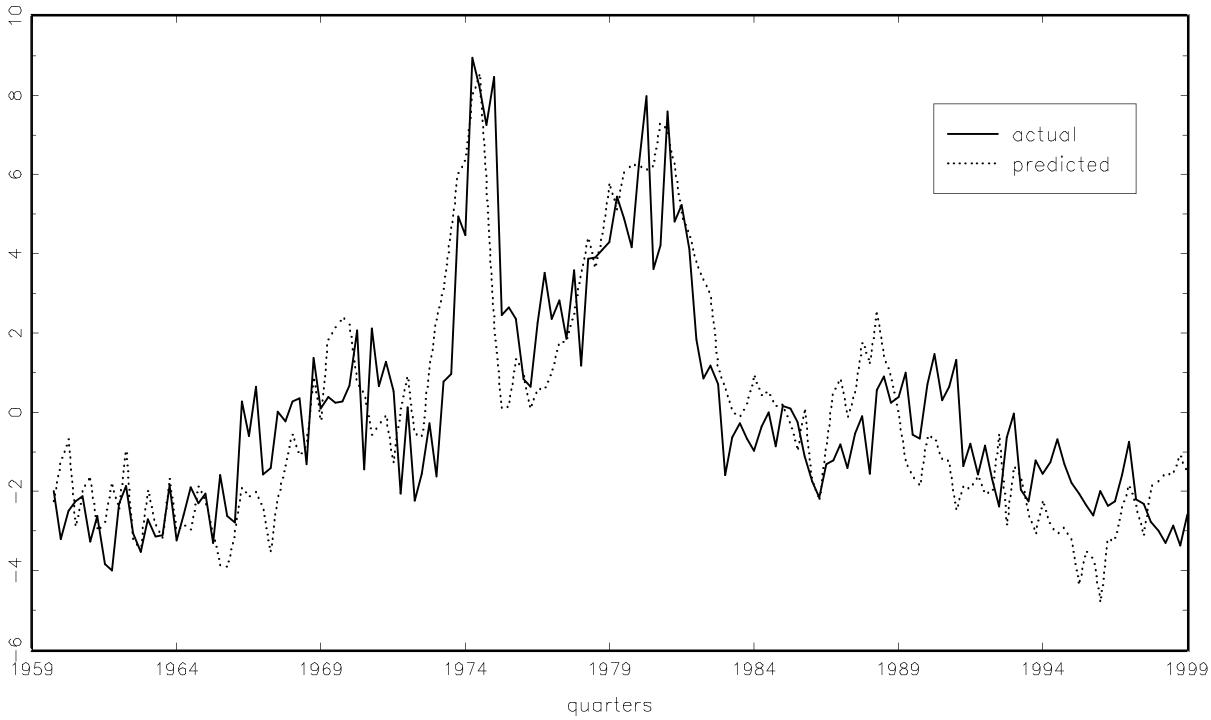


Fig. 11 – Simultaneous Wage and Price Dynamics

a. Inflation



b. Nominal Wage Growth

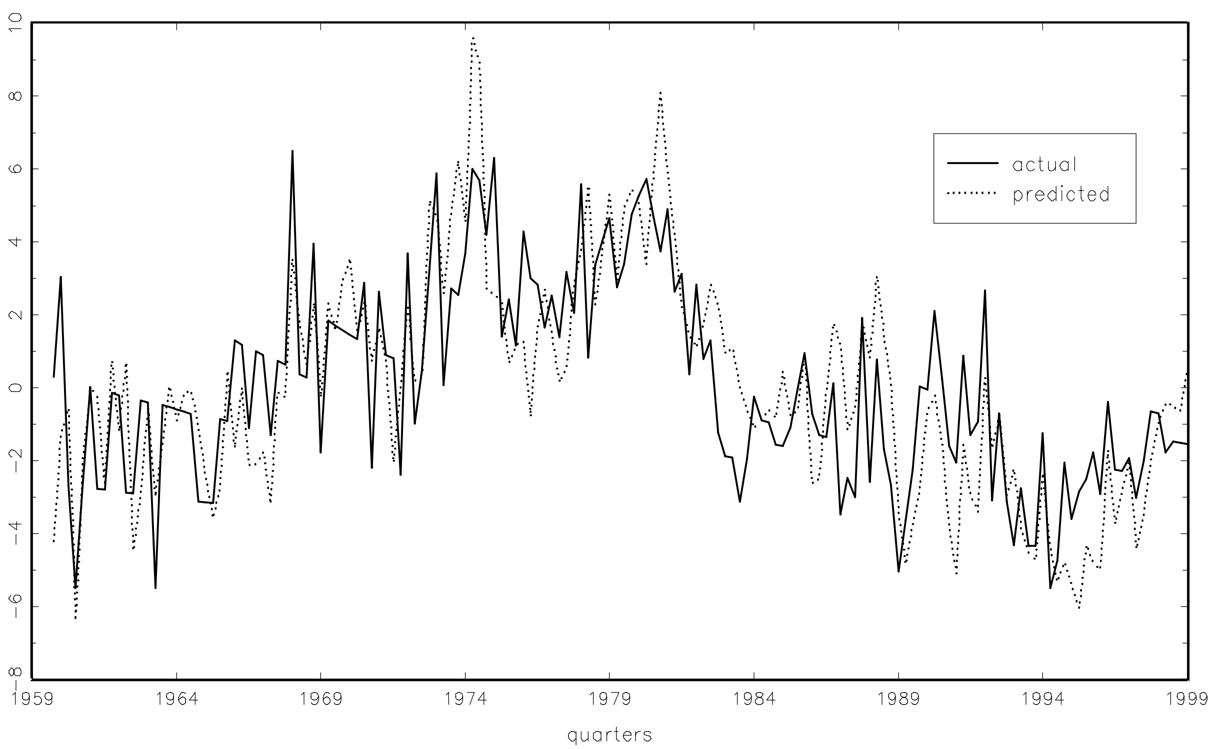
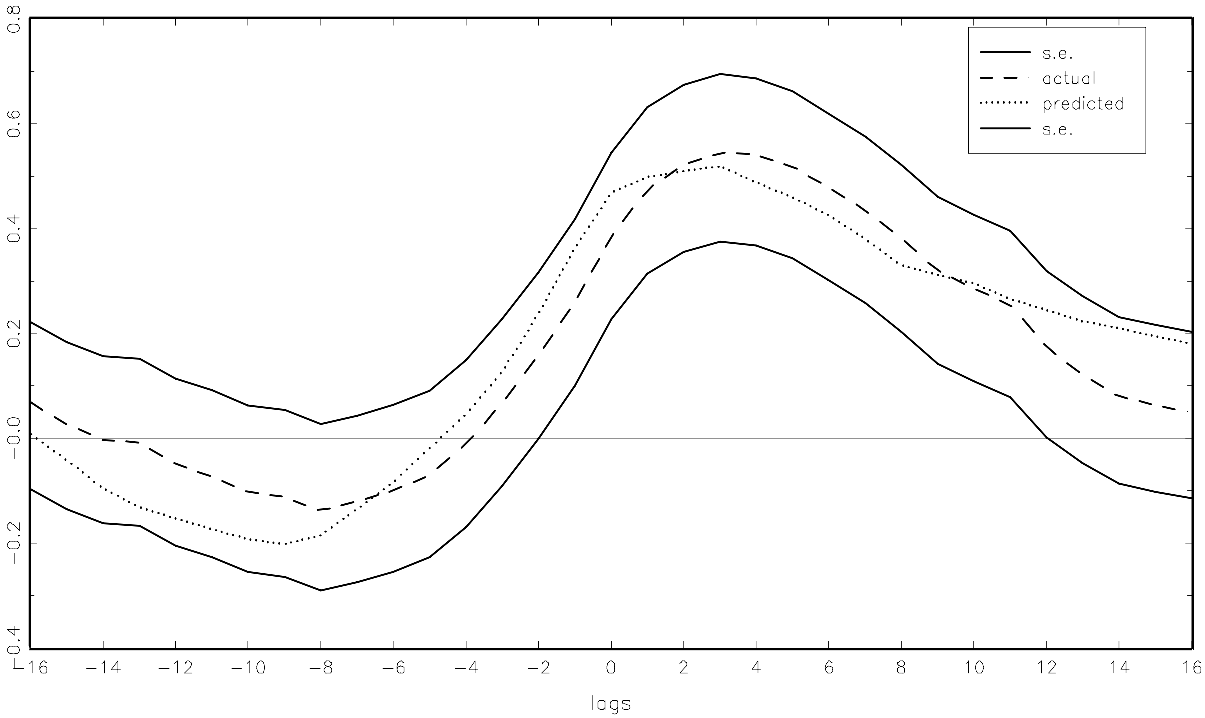


Fig. 12 – Simultaneous Wage and Price Dynamics

a. Dynamic cross-correlations $\rho(y_t^{cyc}, \pi_{t+k})$



b. Inflation autocorrelations